# A Comparison of an Oligopoly Game and the N-person Iterated Prisoner's Dilemma

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## Abstract

Oligopolies are difficult to be modelled, unlike the extremes of monopolies or perfect competition. In this experiment, a similar NIPD model and GA based evolution procedures is investigated to simulate the Oligopoly situation [4]. This oligopoly game model should get more trends to cooperate than the NIPD and would not be the NIPD in any case. We use the useful and efficient calculation [5] to check the tendency of results to cooperation or to defection in these games instead of checking the total payoff scheme. Checking the real data, we can prove some thinking from the common sense about this economic field and also find the critical factors affecting the price competition. The profit margin parameters that is a choice variable for the company are the most important factors to force the results tending to cooperation or defection. The objective factors, i.e., the environment, that the players involving in this game can not change also affect the results. The more companies involving in the market will make the cooperation situation more difficult to be reached, i.e., the difference about the price competition between the prefect competition and oligopoly could be proved through this programme.

KEYWWORD: **Oligopoly**, **Price competition**, and **N-person Iterated Prisoner's Dilemma**.

### 1. INTRODUCTION

Oligopolies are notoriously unpredictable. By contrast with other forms of market structure such as perfect competition and monopoly, there is no unified framework for the analysis of competition among the few. Possible outcomes of oligopolistic competition include implicit collusion through tacit recognition of standard industry practices, explicit collusion through the formation of a cartel, and cyclical periods of intense price competition followed by price stability. One way of modelling competition among the few is in terms of the Genetic Algorithm based Iterated Prisoner's Dilemma. In this approach, the behaviour of the firms that participate in the oligopoly is determined by the evolutionarily fittest strategy – i.e. the strategy which is best adapted to the competitive environment. We take as our basis for this modelling exercise, the framework outlined in [4]
.

The main objectives of our investigation is to analyse the importance for competitive cooperation environmentally 'given' factors (such as number of competitors or market growth), by contrast with those factors which are under the firm's control (i.e. price, cost and profit margin). In addition, a notable result of our analysis is the finding that [4]'s model diverges slightly from the classic IPD.

### 2. N-ITERATED PRISONER'S DILEMMA

The 2-player Iterated Prisoner's Dilemma (2IPD) [1][2] is a non-zero sum game in which each of the two players has the choice of defecting or cooperating on each round of the game. Given that each participant cooperates, it is in each participant's best interests to defect – this maximises his/her gain – but if both participants cooperate, their joint gain exceeds the sum of any possible individual gains should both defect.

[3] investigated an extension of that game in which there are N players in the game, with N>2. The game is qualitatively different from the 2-IPD in that the best strategies for the 2-IPD do not

necessarily scale to the N-IPD. However the payoff matrices must satisfy the same principles as the 2-IPD:

1. Each player must decide whether to Cooperate (C ) or Defect (D) at the same time as every other player.
2. Deciding to Defect (D) is always optimal for each individual given extensive cooperation on the part of others.
3. The payoff to the whole group is maximised when all players cooperate.

Thus the payoff matrix can be represented as in Table 1 which shows the gain for a single prisoner in a population of N-players. It is important to note that the return is dependent on the actions of the other N-1 players in the population. The term $C_i$ ($D_i$) refers to the payoff to the current strategy if it cooperates (defects) when there are i *other* cooperators in the population.

| Number of Cooperators | 0 | 1 | 2 | … | N-1 |
|---|---|---|---|---|---|
| Cooperate | $C_0$ | $C_1$ | $C_2$ | … | $C_{N-1}$ |
| Defect | $D_0$ | $D_1$ | $D_2$ | … | $D_{N-1}$ |

Table 1: The payoff matrix for a single prisoner in a population of N players. There may be 0,1,…,N-1 cooperators in the remainder of the population.

The payoff matrix must satisfy

1. It pays to defect: $D_i > C_i$ for all i in 0,…,N-1.
2. Payoffs increase when the number of cooperators in the population increases: $D_{i+1} > D_i$ and $C_{i+1} > C_i$ for all i in 0,…,N-1
3. The population as a whole gains more, the more cooperators there are in the population: $C_{i+1} > (C_i + D_{i+1})/2$ for all i in 0,…,N-2. Notice that this last gives a transitive relationship so that the global maximum is a population of cooperators.

## 3. THE OLIGOPOLY GAME MODEL

[4] provides the basis for our NIPD. It should be noted, that although, as we will demonstrate, its payoffs do not conform to the standard NIPD, it does allow more realistic modelling of price competition. We will initially consider a situation where there are three competing companies. Each company has the choice of setting a high price (implicitly cooperating with the other two companies) or a low price (defecting) If it defects, it will win market share from the other (high price setting) companies though with a reduced profit margin. A genetic algorithm is used to investigate whether cooperation can evolve.

The profit of one company in this game can be simplified as,

$$\text{Profit} = (\text{Price} - \text{Cost}) * \text{Number\_of\_customers}.$$

These two factors which affect the profit that company would get in this game, price and customer number, are not mutually dependent. We can think of the profit margin *(price-cost)* as a choice variable for the company with the number of customers as an environmentally determined parameter.

[4] models the oligopoly game using the following key parameters;

1. High Price ($P_h$): The price when the company cooperates with others.
2. Low Price ($P_l$): The price when the company defects against others.
3. Cost (C): The cost of one goods or service.
4. Change in market share ($\alpha$): The percentage of customers who are lost to rivals when a company charges a high price when its competitors are charging a low price.
5. Overall market growth or decline ($\beta$): A function to modify the customers attracted by this market or leaving this market by the prices' change of these companies.

Therefore, the profit of one unit sale for one cooperating company with one customer is ($P_h$-C) and if for defecting one is ($P_l$-C). That means the choice variables are the prices that the company can choose and the per unit profit. However, the change in market share ($\alpha$) and market growth ($\beta$) are environmentally determined. When the environment changes, the company adapts its strategy in such a way as to maintain maximum profit.

For n companies, the overall change in the customer base may be expressed in the following matrix form.

$$M_t = \begin{bmatrix} m^t{}_{11} & m^t{}_{12} & m^t{}_{13} & \ldots\ldots & m^t{}_{1n} \\ m^t{}_{21} & m^t{}_{22} & \ldots\ldots & \ldots\ldots & m^t{}_{2n} \\ m^t{}_{31} & m^t{}_{32} & \ldots\ldots & \ldots\ldots & m^t{}_{3n} \\ \vdots & & & \ldots\ldots & \vdots \\ m^t{}_{n1} & & & \ldots\ldots & m^t{}_{nn} \end{bmatrix}$$

Where $m^t_{ij}$ denotes the proportion of the customers of firm *i* move to firm *j* at the period *t*. Let $n^t_i$ be the number of customers of firm *i* at period *t*, and $N_t$ as the vector $[n^t_1, n^t_2, n^t_3]$. The number of customers at period t+1 can be expressed as,

$$N_{t+1} = M_t N_t$$

Let vector $(a_1, a_2, a_3, \ldots, a_n)$ represent the choice of three firms, with $a_i = 1$ for cooperation and $a_i = 0$ for the defection, $i$=1, 2,….,n. The $M_i$ $(a_1, a_2, a_3, \ldots, a_n)$ denotes the probability state of customer change at period *t* when these three company choose the operation $a_i$. The market share-change matrices $M_t$ when there are all cooperators or defectors can be written as,

$$M_t(1 \quad 1 \quad \ldots\ldots \quad 1) = M_t(0 \quad 0 \quad \ldots\ldots \quad 0) = \begin{bmatrix} 1 & 0 & 0 & \ldots. & 0 \\ 0 & 1 & 0 & \ldots. & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \ldots. & 1 \end{bmatrix}$$

When there are some defectors among the cooperators, the representation of changes in the customer base becomes more complicated. There are *m* companies defecting in this *n*-companies' market, i.e., there are *m* cooperators and *(n-m)* defectors, *n*>*m*>0. From the definition of Market Share Change, when a company chooses cooperation, it will lose α percentage of customers to defectors, regardless of the number of other firms which are defecting. The defectors will share the α companies between them. The cooperator will maintain (1-α) of customers in the next round and the defector will gain $m/(n-m)*\alpha$ in the next round.

If there are three players in the oligopoly market, the market share-change matrices are:

$$M_t(1 \quad 0 \quad 0) = \begin{bmatrix} 1-\alpha & \frac{\alpha}{2} & \frac{\alpha}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_t(0 \quad 1 \quad 0) = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha}{2} & 1-\alpha & \frac{\alpha}{2} \\ 0 & 0 & 1 \end{bmatrix} \quad M_t(0 \quad 0 \quad 1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\alpha}{2} & \frac{\alpha}{2} & 1-\alpha \end{bmatrix}$$

$$M_t(1 \quad 1 \quad 0) = \begin{bmatrix} 1-\alpha & 0 & \alpha \\ 0 & 1-\alpha & \alpha \\ 0 & 0 & 1 \end{bmatrix} \quad M_t(1 \quad 0 \quad 1) = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 1 & 0 \\ 0 & \alpha & 1-\alpha \end{bmatrix} \quad M_t(0 \quad 1 \quad 1) = \begin{bmatrix} 1-\alpha & 0 & 0 \\ \alpha & 1-\alpha & 0 \\ \alpha & 0 & 1-\alpha \end{bmatrix}$$

Overall market growth or decline is determined by the behaviour of prices. Relatively low prices attract new customers, while relatively high prices cause existing customers to exit the market. Thus, for example, in the three firms case, following [4], the total number of customers will increase if there are three (D) or two (D) defectors charging low prices, and will decline if there are two or three cooperators charging high prices.

$$\beta=\begin{cases} \delta_W & if \quad \sum_{i=1}^{3} a_i = 0 \\ \delta_w & if \quad \sum_{i=1}^{3} a_i = 1 \\ \delta_c & if \quad \sum_{i=1}^{3} a_i = 2 \\ \delta_C & if \quad \sum_{i=1}^{3} a_i = 3 \end{cases}$$

Where $\delta_W \geq \delta_w \geq 0 \geq \delta_c \geq \delta_C$ , δ is a probability function that describes what percentage of customer will move at this special situation. Therefore, the customer number vector at time $t$ in this market, $N_t$ can be described as,

$$N_{t+1} = N_{t+1} M_t (1+\beta_t)$$

And the profits that each company in this game can get can be shown as,

$$p^t_i = (P^t_i - C) n^t_i$$

Where $i$ = 1, 2, 3, i.e., three players, and t is the period among these rounds.

## 4. IMPLEMENTATION OF OLIGOPOLY GAME

Genetic algorithms use processes which are analogous to biological evolution to find 'fit' solutions. This is done by creating chromosomes which represent potential solutions to the problem to be addressed. In our case, the chromosomes represent strategies which will be played by participants in oligopolistic price competition. The fitness of each strategy is defined in terms of the profits that it would produce in competition with two other strategies.

The strategies produced by the GA process can be thought of as a firm's strategic response to the price competitive behaviour of rivals in an oligopoly game. The plays of the oligopoly gam are alternated with the GA in order to evolve the strategy which is best adapted to the conditions of the competitive environment.

For the oligopoly game part, the parameters we set are,

- High Price ($P_h$ >1) for cooperation and Low Price ($P_l$ >1) for the defection;
- Cost of each goods or service(C=1);
- Internal Market Share Change (α), $0 \leq \alpha \leq 1$;
- Total customer number at the first round, N=1000;
- The number of rounds in every game, r;
- And External Customer influence (β). If there are three competitors, the overall market growth or decline should be, $1 > \delta_W \geq \delta_w \geq 0 \geq \delta_c \geq \delta_C > -1$.

Each bit in the chromosome determines whether the company chose high price (1) or low price (0) given the current situation. The length of chromosome depends on prices of how many previous rounds keep in the memory of the company. If there are $n$ companies in this oligopoly market, each company should meet 2$n$ competition situations in one round, i.e., $C_{n-1}$, $C_{n-2}$, …..,$C_1$, $C_0$, $D_0$, $D_1$, ….., $D_{n-2}$, $D_{n-1}$. If the company should keep results of $p$ previous rounds in memory to decide the strategy, the length of chromosome should be spent $(2n)^p$ bits to describe it. Including the acts of company in the initial rounds when it has not got the memory enough. The total length of the chromosome when the player keeps $p$ histories of previous rounds should be $1+2n+(2n)^2+\ldots+(2n)^P$.

We evolve this GA-based game using a starting population of 1000 customers and a memory length of 1 in most cases then try expanding the memory length to 2. Because the optimum strategy should be found quickly, we set the population size to 30, i.e., each company has 30 strategies in this game.

The GA-based oligopoly game should be run by the following steps:

Step 1 Initialize the 30 chromosomes of each company randomly for one population. Therefore, there are three populations in this game for three different companies.

Step 2 Match 30 pairs of the chromosomes from three populations.

Step 3 Let these 30 pairs of chromosomes to run r rounds of this game. Calculate the profits of each chromosome get after r rounds of the game. Set fitness function to maximise profits.

Step 4. The evaluation technique used is 'fitness is evaluation'. That is fitness is equal to the profit generated by the strategy encoded in the chromosome. There is an independent population of chromosomes for each firm.

Step 5 The simulations are run for 1000 generations.

In order to determine the effect of the $\alpha$ and $\beta$ parameters on the extent to which cooperation is achieved, three experiments are performed. In the first experiment (*absolute loyalty without external effects*) customers are absolutely loyal to one particular firm ($\alpha$=0), while there is no growth or decline in the number of customers ($\beta$=0). In the second experiment (*the captive market*) customers choose between rival firms partially on the basis of relative prices ($\alpha$>0), but there is no overall growth or decline in the market ($\beta$=0). In the third experiment (*price competition*), customers transfer their loyalties between markets ($\alpha$>0), and also exit and enter the market ($\beta \neq 0$).

## 5. THE OLIGOPOLY GAME AS AN N-PERSONS IPD

A basis for the comparison between the payoff scheme in the 3-person oligopoly game and the 3-person IPD is to be found in [4]. Nevertheless, Its method is an approximation and it is possible to improve the precision.

Firstly [4] calculated the long term payoff of $[D_2 \quad C_1 \quad C_1]$ as

$\sum_{t=1}^{r}[P_l - C \quad P_h - C \quad P_h - C][M(0 \quad 1 \quad 1)]^t$ and similarly with the other matrices. However in making the approximation in calculating the values of the payoffs, we remove the slight differences between these payoffs.

For example, using $D_2 = (P_l - C)[r + 2\sum_{t=1}^{r}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s] \approx (P_l - C)[3r - 2\frac{1-\alpha}{\alpha}]$ instead of the true value $D_2 = (P_l - C)[r + 2\sum_{t=1}^{r}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s] = (P_l - C)[3r - 2\frac{(1-\alpha)[1-(1-\alpha)^r]}{\alpha}]$ disguises the similarities between different parameter settings and suggests differences between the Oligopoly Game and the NIPD. Another way of modifying the previous analysis of the Oligopoly Game is as follows. [5]

There are r rounds of game before running GA, the market share lost should be based on (r-1) trials. At the first round, the number of customers of the three companies should be the same and customer will only subsequently choose to go to the defecting (low price) companies. Therefore, there are (r-1) customer change rounds. An example payoff is calculated as:

$$[D_2 \quad C_1 \quad C_1] = [P_l - C \quad P_h - C \quad P_h - C] + \sum_{t=1}^{r-1}[P_l - C \quad P_h - C \quad P_h - C][M(0 \quad 1 \quad 1)]^t$$

which gives us payoffs:

$$D_2 = (P_l - C) + (P_l - C)[r + 2\sum_{t=1}^{r-1}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s]$$

and similarly with the other payoffs.

However the payoff numbers for all three companies for the first round should be identical and so

$[D_2 \quad C_1 \quad C_1]$ should be equal to $[P_l - C \quad P_h - C \quad P_h - C] + \sum_{t=1}^{r-1}[P_l - C \quad P_h - C \quad P_h - C][M(0 \quad 1 \quad 1)]^t(1+\delta_c)^t$

which will give us payoff, $D_2 = (P_l - C) + (P_l - C)[r + 2\sum_{t=1}^{r-1}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s(1+\delta_c)^t]$

Using these values we may still show that while $C_1 \neq C_0$, the prerequisite of the NIPD game that $C_1 > C_0$ is not satisfied in all cases. Therefore, we can say this Oligopoly Game is not a strict NIPD.

A final suggested modification is as follows. In the above, we have used the calculation method suggested by [4] to check the payoffs in order to compare this model with NIPD. However, we can divide this GA-base oligopoly game as two parts, game and genetic algorithm.

For the true IPD, each strategy will play a game with another strategy for several rounds, then play game with another. Each strategy plays games with every other strategy once, where the other strategies are selected in a random order. However, the Oligopoly Game lets the strategies of three companies combine in sets of three, so the strategies will not play game randomly. This aspect of the oligopoly game decreases the exploitation of environmental fitness through the game and makes premature convergence to local fitness more likely.

In addition, the payoff that each strategy can get in IPD games at every round is independent which means that the score that the strategy gets in this round is independent of its behaviour at previous rounds and will not affect the score that it can get during subsequent rounds. However, in the oligopoly game the customers' behaviour is affected by the price that the companies adopt at the previous round. Therefore, the payoff structure of each round is not independent of previous payoffs. It makes the calculation of payoff structure somewhat more difficult. For the IPD game, all possible payoffs can be calculated with respect to only one round which means that checking all the payoffs to define the evolution of behaviors in the game is possible. However, in the Oligopoly Game, it is impossible to check the payoff structure from the results of one round. In order to illustrate how the interdependence of payoffs considerably complicates their calculation, consider the following. If we run 8 rounds in one game for three oligopolists, there are 6 outcomes, i.e., $C_2$, $C_1$, $C_0$, $D_0$, $D_1$, and $D_2$, for one round, so we should get the total $6^8$ payoffs at the end of all these 8 rounds instead of 6 outcomes to check the payoff scheme which is not enough to describe the entire structure of the game.

However, in theory, The method in [4] can be used to approximate the payoffs as follows;

$$\text{All } D_2 = (P_l - C) + (P_l - C)[r + 2\sum_{t=1}^{r-1}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s(1+\delta_c)^t]$$

$$\text{All } D_1 = (P_l - C) + (P_l - C)[r + \frac{1}{2}\sum_{t=1}^{r-1}\sum_{s=0}^{t-1}\alpha(1-\alpha)^s(1+\delta_w)^t]$$

$$\text{All } D_0 = (P_l - C) + (P_l - C)\sum_{t=1}^{r-1}(1+\delta_W)^t$$

$$\text{All } C_2 = (P_h - C) + (P_h - C)\sum_{t=1}^{r-1}(1+\delta_C)^t$$

$$\text{All } C_1 = (P_h - C) + (P_h - C)[\sum_{t=1}^{r-1}(1-\alpha)^t(1+\delta_c)^t]$$

And all $C_0 = (P_h - C) + (P_h - C)[\sum_{t=1}^{r-1}(1-\alpha)^t (1+\delta_w)^t]$

## 6. PRE-TEST

Let $[a^t_1, a^t_2, a^t_3, \ldots, a^t_n]$ denote the operation of these *n* companies at t time, and $a^t_i = 1$ with cooperation, $a^t_i = 0$ with defection. Therefore, when there are three competitors, there are 8 possible outcomes, i,e,. [1,1,1], [1,0,1], [1,1,0], [0,1,1], [1,0,0], [0,1,0], [0,0,1], and [0,0,0]. We can classify these 8 outcomes into 4 groups, i.e., total cooperation (**C**) [1,1,1], one defection (**c**) [1,0,1] [0,1,1] [0,0,1], one cooperation (**w**) [1,0,0] [0,1,0] [0,0,1], and total defection (**W**) [0,0,0]. We may then calculate the percentage of each group at the each stage of each game.

One of the problems which was encountered in the simulations was the fact that convergence often took place prematurely, with the GA achieving local rather than global fitness. This is a particular difficulty given the path dependency of the payoffs. i.e. Given the initial randomisation of the strategies, some will simulations will achieve global fitness, while others will 'fall into' a sink of local fitness, even with the same parameter values. In order to cope with this, simulations are run with the same parameter many times. The simulation is then analysed in terms of the distributions of generation numbers in which total cooperation was achieved.

The results of some trials are shown in Figure 1 and Figure 2.

Figure 1 PH=1.4 PL=1.2 a=0.2 Tiral 1 Muation Rate=0.001

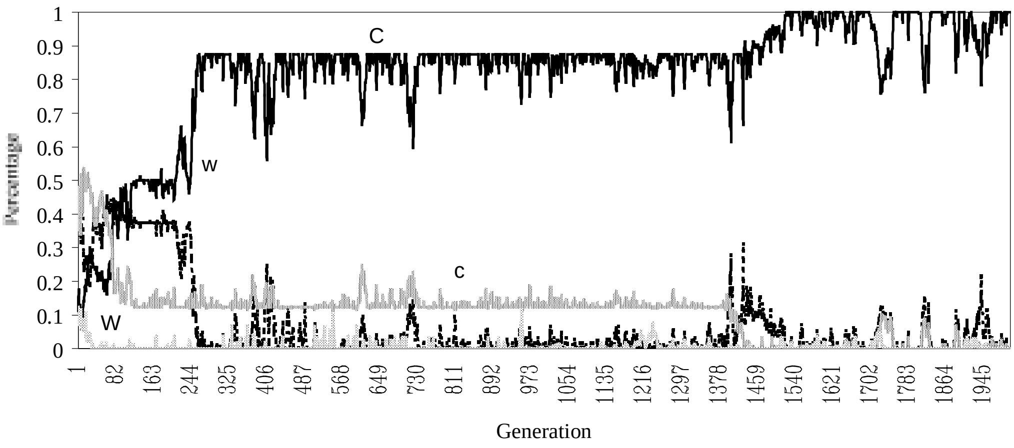
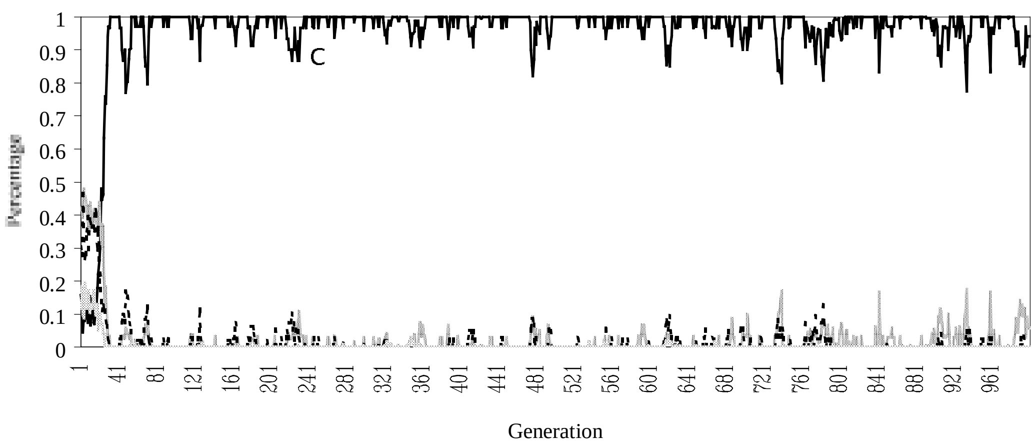



Figure 2 PH=1.4 PL=1.2 a=0.2 trail 2 Mutation Rate =0.001



## 7 THE COMPARATIVE EXPERIMENTS

The parameters for the GA are shown below

| | |
|---|---|
| Number of oligopolists | 2-10 |
| Memory (Histories kept) | 1(2) |
| Population size | 30 |
| Number of round in a game (r) | 8(16) |
| Selecting scheme | Roulette-wheel |
| Fitness function | Profits |
| Number of generations evolved | 1000 |
| Crossover Style | One-point |
| Crossover rate | 0.8 |
| Mutation rate | 0.001 |

Table 2 The GA parameters

In most cases, the memory length is set to one and the 8-round games are used.

**Experiment 1** *absolute loyalty without external effects*

In this case, the environment is static. As the number of customers does not change, there is no path dependency. This makes the payoffs of this particular oligopoly game comparable with the NIPD The payoff scheme of oligopoly game is, $D_n = D_{n-1} \ldots = D_1 = D_0 = (P_l - C)$, and $C_n = C_{n-1} = \ldots C_1 = C_0 = (P_h - C)$. It is evident that the cooperation payoff is greater than the cooperate payoff, and that it is almost inevitable that cooperation should ensue.

We ran the 3-players' game with the price settings shown in Table 3. The results are shown in Figure 3.

| | High Price (C) | Low Price (D) | Cost | Market share lost (α) | Rounds (r) |
|---|---|---|---|---|---|
| Set 1 | 1.4 | 1.2 | 1 | 0 | 8 |
| Set 2 | 2.0 | 1.2 | 1 | 0 | 8 |

Table 3 specific parameter regimes for absolute loyalty of 3 players

Although cooperation breaks out here, it is not really cooperation in the true sense. It is simply firms taking advantage of their local monopoly to charge as high a price as possible. There would be no point in charging a low price.

Figure 3 The loyalty of customers, That is no market share lost, a=0, r=8

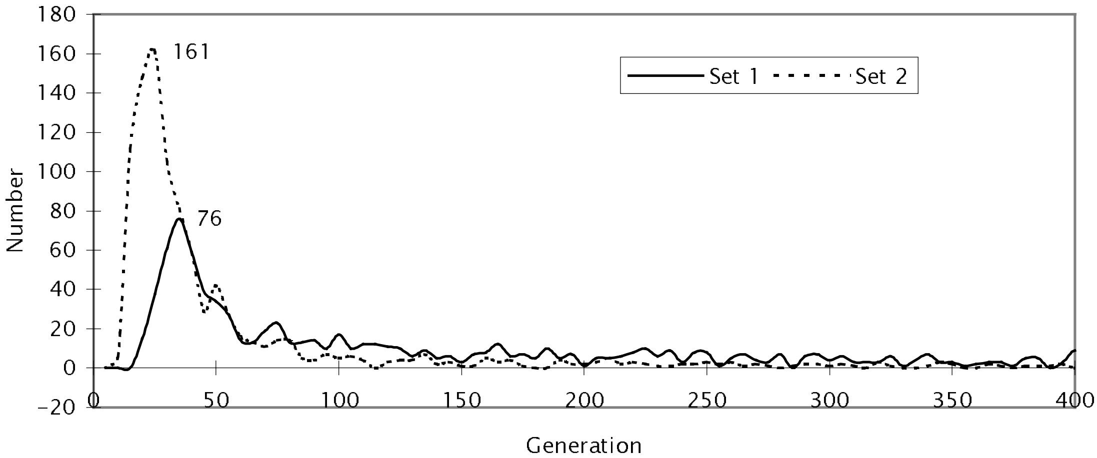


Figure 3 shows that 'cooperation' emerges more slowly when the difference between high and low prices is smaller. Thus, the 'choice variable is the most important determinant of oligopolistic behaviour. When there is absolute loyalty, there is a direct relationship between price and profit. Figures 4 and 5 show what happens when more players are added to the game.

Figure 4 Customer Loyalty, (PH-C)/(PL-C)=5 a=0 r=8

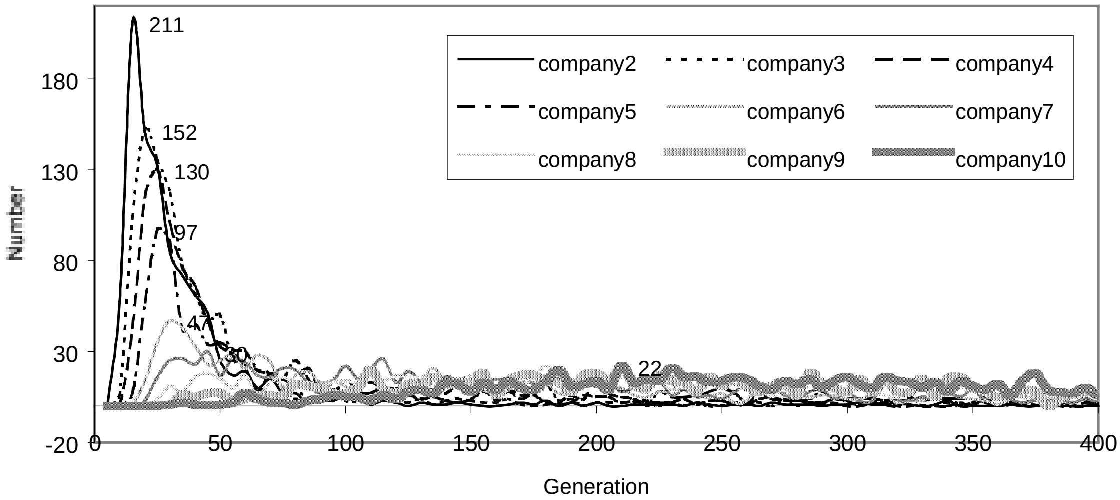
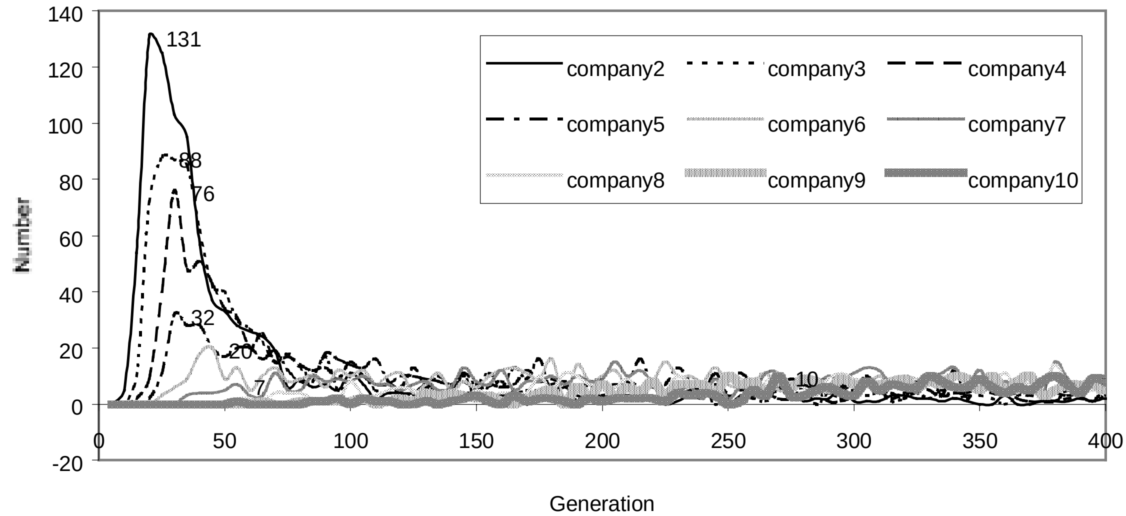



Figure 5 Customer Loyalty, (PH-C)/(PL-C)=2 a=0 r=8




It is evident that the more players involved in the game, the more difficult it is for total cooperation to emerge. When the number of players exceeds eight, cooperation does not emerge at all. This is a result which is of some note, as the defect payoff is always inferior to the cooperation payoff, and both are independent of the number of cooperators.

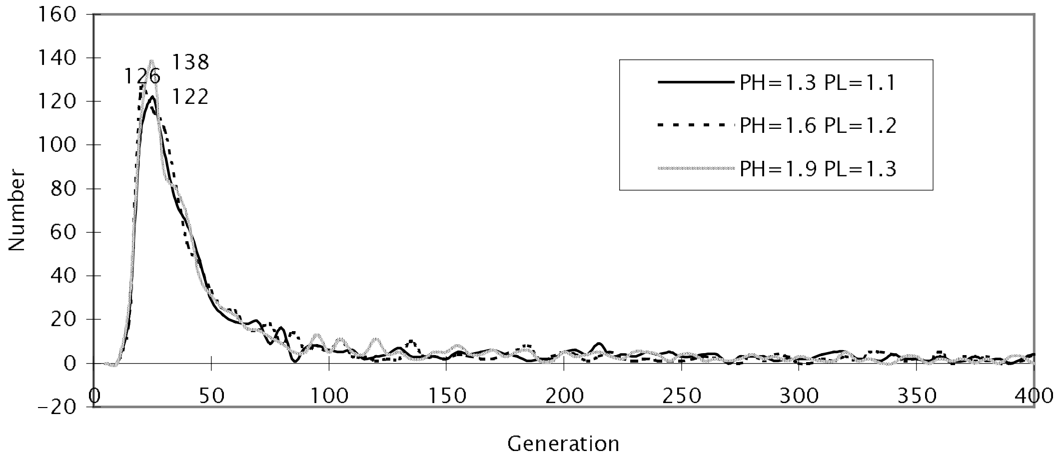
**Experiment 2** *The captive market*

In this simulation, there is no growth or decline in the overall market, although customers may change suppliers depending on relative prices. We firstly check the effect of the choice variables, price and cost. The impact on the simulation of the variation in price is set out in Table 4 and Figure 6.

| Company(n)=3<br>Market share lost α=0.2<br>Cost (C)=1<br>Round (r)=8 | The number of trials that can not reach total cooperation before 400 generation | The number of trials that can not reach total cooperation before 1000 generation |
|---|---|---|
| $P_h$=1.3$P_l$=1.1 | 123 | 34 |
| $P_h$=1.6$P_l$=1.2 | 119 | 31 |
| $P_h$=1.9$P_l$=1.3 | 107 | 26 |

Table 4 $(P_h\text{-}C)/(P_l\text{-}C)=3$

Figure 6 3 companies, (PH-C)/(PL-C)=3, the graphs are similiar




This experiment clearly demonstrates very similar outcomes. This is because the critical factor here is not really prices and costs as such, but the ratio of profitability with a high price to profitability with a low price, $(P_h - C)/(P_l - C)$. Because roulette wheel selection is used, this ratio is directly equal to the probability of a chromosome being selected for the next round. Therefore, we replace the choice variables, prices and costs, with a single variable, profitability ratio. The results are given in Table 5 and Figure 7. It is evident that the higher the profit ratio, the more quickly cooperation emerges. Ceteris paribus, a higher profitability ratio leads to more rapid cooperation. Nevertheless, a higher profitability ratio for a particular firm, in the absence of cooperation implies a greater loss of custom to rivals. Hence it is necessary also to examine the impact of a key environmental factor, α, the extent to which customers are prepared to switch firms as a result of price differentials. The result of variation in α, the parameter controlling the loss of market share to other firms as a result of charging high price, is shown in Table 6 and Figure 8.

| Company(n)=3<br>Market share lost α=0.2<br>Round (r)=8 | The number of trials that can not reach total cooperation before 400 generation | The number of trials that can not reach total cooperation before 1000 generation |
|---|---|---|
| Profit Ratio=1.5 | 716 | 559 |
| Profit Ratio=2 | 310 | 146 |
| Profit Ratio=3 | 119 | 31 |
| Profit Ratio=4 | 70 | 17 |
| Profit Ratio=5 | 48 | 11 |

Table 5 $(P_h\text{-}C)/(P_l\text{-}C)=3$

| Company(n)=3<br>Profit Ratio=2<br>Round (r)=8 | The number of trials that can not reach total cooperation before 400 generation | The number of trials that can not reach total cooperation before 1000 generation |
|---|---|---|
| Market share lost=0.1 | 198 | 43 |
| Market share lost=0.2 | 310 | 146 |
| Market share lost=0.3 | 462 | 287 |
| Market share lost=0.4 | 679 | 566 |
| Market share lost=0.5 | 803 | 730 |

Table 6 The affect of market share on cooperation

Figure 7 The difference of Profit Ratio would affect the cooperation

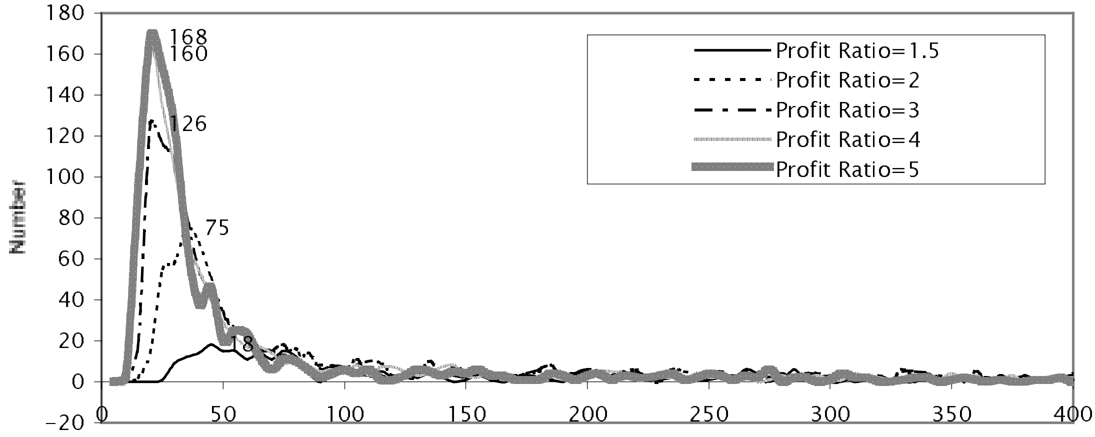
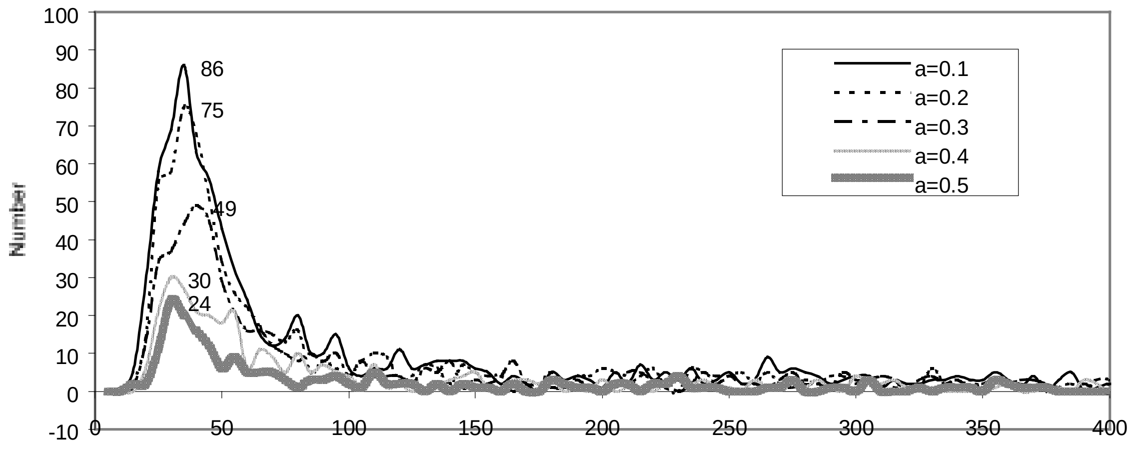



Figure 8 Profit Ratio=2, the variable results of different market share lost




It is evident that the greater the sensitivity of market share to relative price, as reflected in the $\alpha$ parameter, the more difficult it is to get cooperation. However the impact of the $\alpha$ parameter on cooperation is mitigated by the effect of the profit ratio, as can be seen in Figures 9 and 10 which show that the declining distributions of cooperates is much less marked in the case where the profit ratio is 3 and 5. This result is particularly notable in the case where the profit ratio is equal to 5. Even when the parameter is increased to 0.5, there is negligible impact on cooperation. Using the calculation method described [5], it is possible to analyse the payoffs in this case.

| Profit Ratio=3 | All $D_2$ | All $D_1$ | All $D_0$ | All $C_0$ | All $C_1$ | All $C_2$ |
|---|---|---|---|---|---|---|
| $\alpha$=0.1 | 1.576 | 1.144 | 1 | 2.136 | 2.136 | 3 |
| $\alpha$=0.2 | 1.960 | 1.240 | 1 | 1.550 | 1.560 | 3 |
| $\alpha$=0.3 | 2.215 | 1.304 | 1 | 1.178 | 1.178 | 3 |
| $\alpha$=0.4 | 2.385 | 1.347 | 1 | 0.922 | 0.922 | 3 |
| $\alpha$=0.5 | 2.502 | 1.375 | 1 | 0.747 | 0.747 | 3 |

Table 7 The payoffs of variable market share lost when Profit Ratio=3

Figure 9 Profit Ratio=3, the variable results of different internal market share change

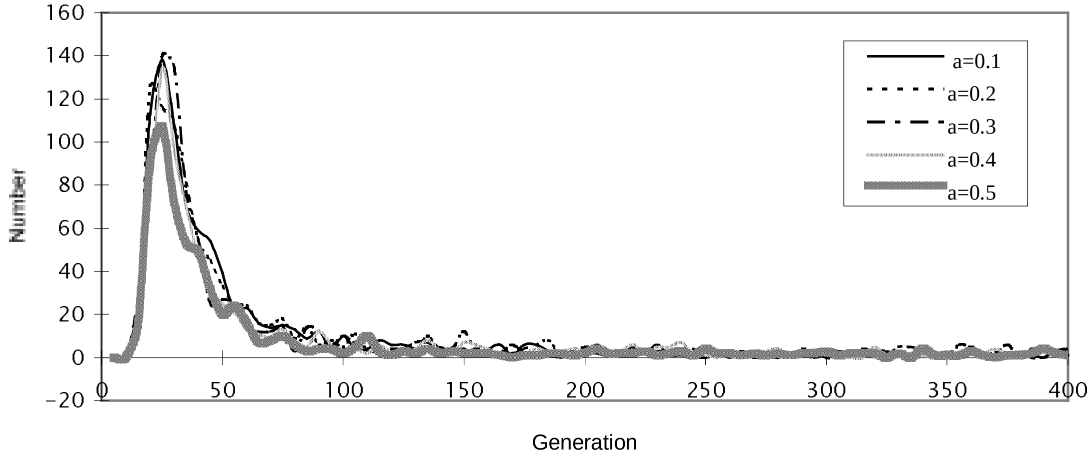
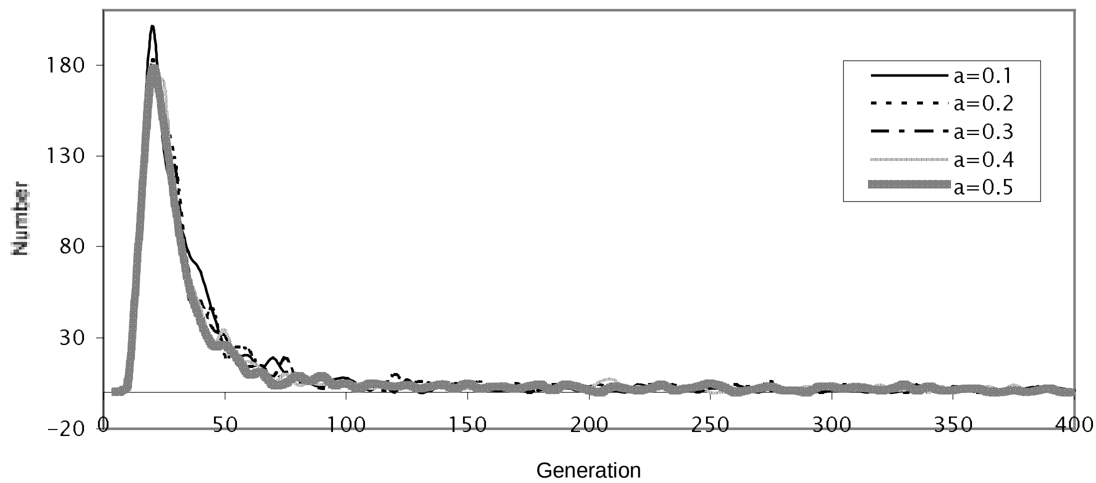



Figure 10 Profit Ratio=5, the variable results of different internal market share chagne

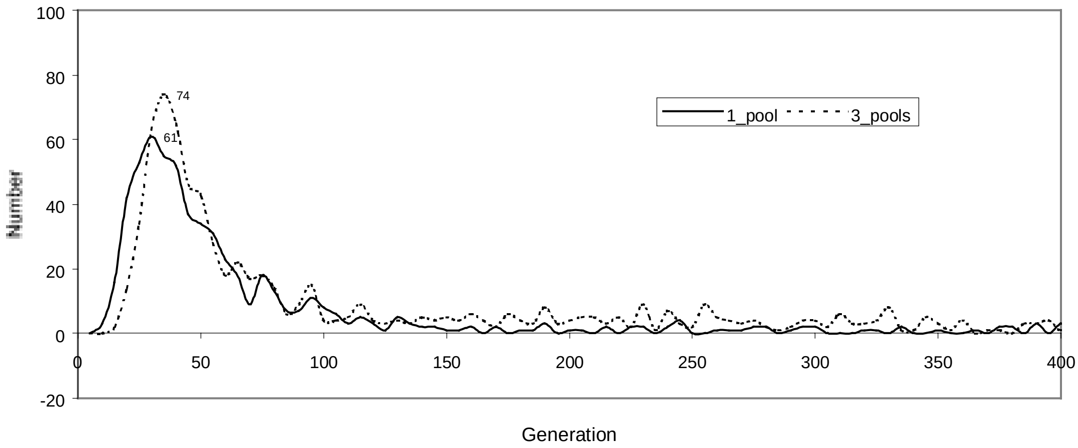



The results of the calculation are given in Table 7 which shows how the payoff changes when the value of the $\alpha$ parameter increases at profit ratio equal to 3. It is can be seen that at low values for $\alpha$, defection is highly unlikely, as it is not justified by the payoffs available. It is only as the value of $\alpha$ approaches 0.5 that defection may become justified. In the case where the profit ratio is 5, the incentive for defection does not really begin to appear until the parameter $\alpha$ is equal to 0.9.

Thus far, the strategies pursued by the three participants to the game have been evolved in three separate pools. In a further experiment, chromosomes were grouped in one pool, in order to assess whether this had any discernible effect.

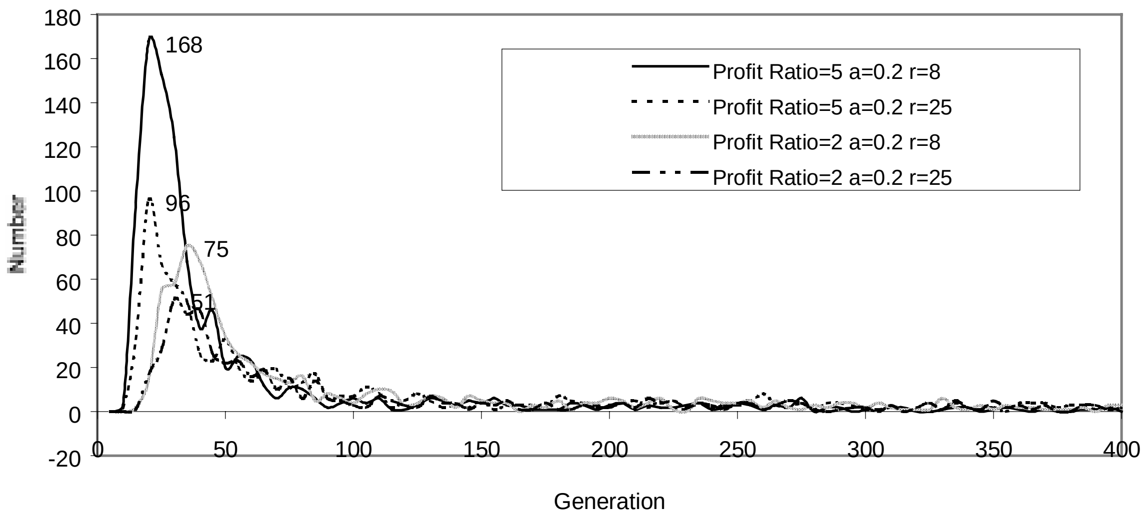
One of the reasons for performing this experiment was that it was thought that it might improve the performance of the GA by decreasing the attraction of local fitness and increasing the attraction of global fitness. As can be ascertained from Figure 11, this actually does little to improve the probability of the emergence of global fitness. In the three-pool case, cooperation is achieved in 122/1000 cases by generation 400 and 312/1000 by generation 1000. In the three-pool case, global fitness is achieved in 389/1000 cases by generation 400, with not much improvement thereafter.

Figure 11 Compare the results of evolution in 1_pool and 3_pools



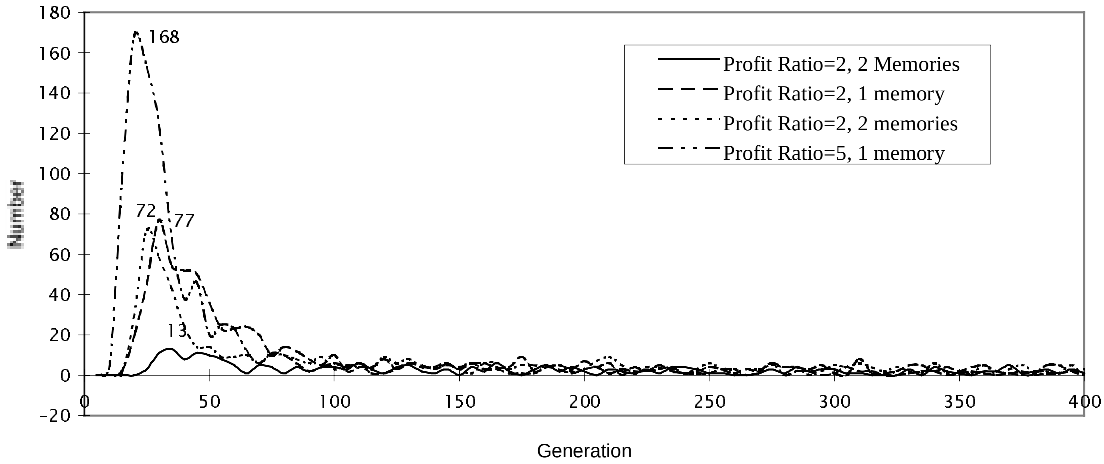

Another experiment that was performed was to increase the number of rounds from 8 to 25. An increase in the number of rounds might be thought of as analogous to an increase in the time between strategy reviews by firms. The greater the number of rounds, the lower the frequency of review and the slower the reaction to competitors. Hence one would expect cooperation to emerge more slowly when the number of rounds are increased. This is borne out by the results illustrated in Figure 12.

Figure 12 The more rounds run before the strategy review, the more difficult is it to get cooperation (3 companies)



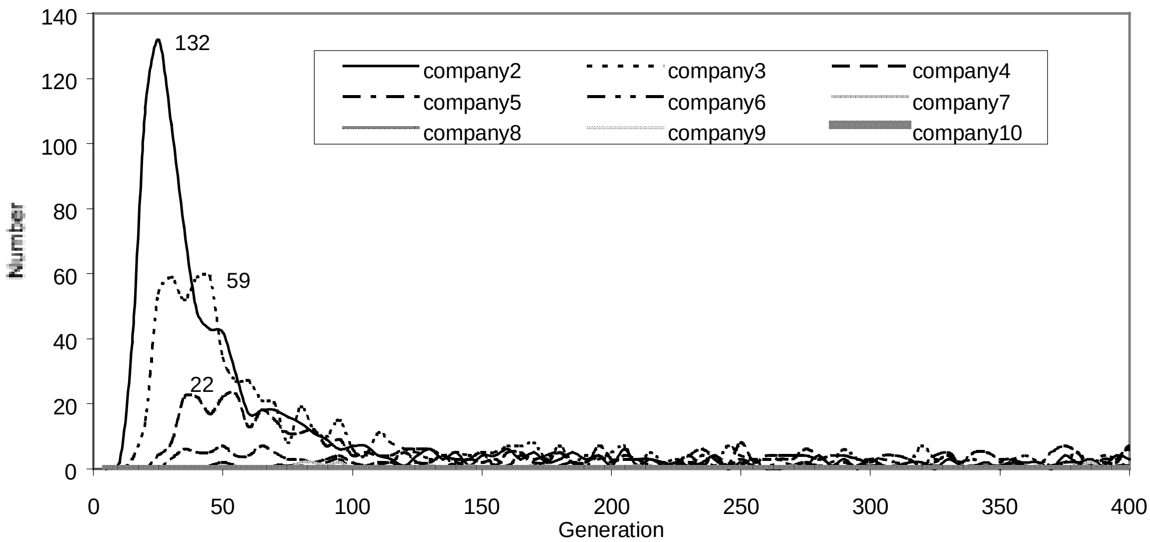

An additional experiment which was undertaken was to check the effect of the memory length. In all of the previously reported cases, the memory length is 1. An increase in the memory length should make the simulated agents more intelligent. However, the increase in memory length also increases the problem of complexity and path dependency which was discussed earlier. It is perhaps for the latter reason that cooperation takes longer to emerge in the memory length 2 case, as can be seen in Figure 13.

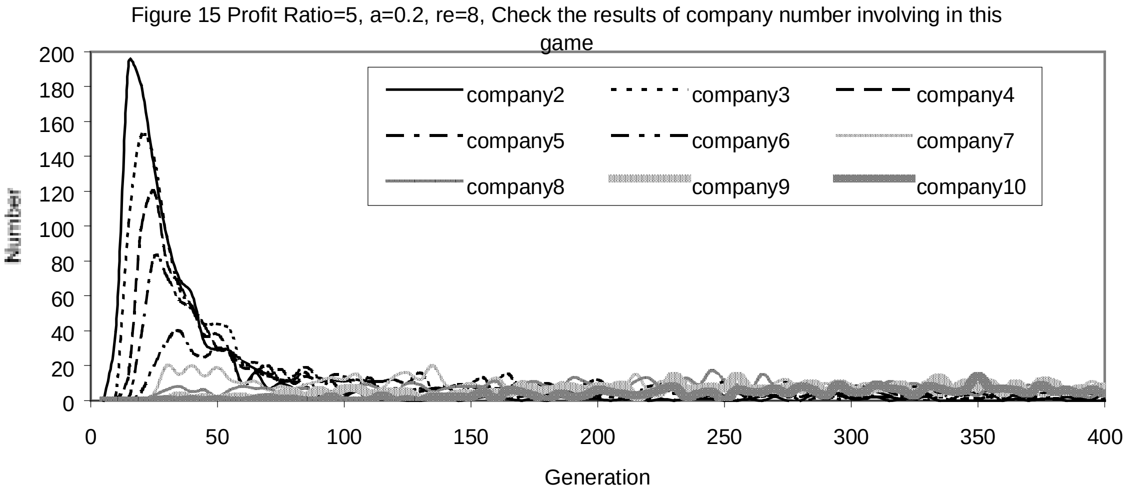
Figure 13 Comparison of the affects of different menories on the cooperation (3 companies, a=0.2)




A final experiment which was carried out was to investigate the impact of increasing the number of competitors. It can be seen from Figures 14 and 15 that the outcome is very similar to the result which was found in the first experiment. Even with as few as 5 competitors, only 10% of games get cooperation before 1000 generations ($\alpha$=0.2, profit ratio=2, Figure 14). As can be ascertained from figure 15, the difficulty of getting cooperation is somewhat abated when the profit ratio is increased to 5. In the latter case, even when there are 10 companies, 90% of games will cooperate by generation 1000 and 50% before generation 400. Hence cooperation still breaks out when there are many competitors, but it does take much longer to emerge.

Figure 14 Profit Ratio=2, a=0.2, r=8, Check the results of company number involving in this game





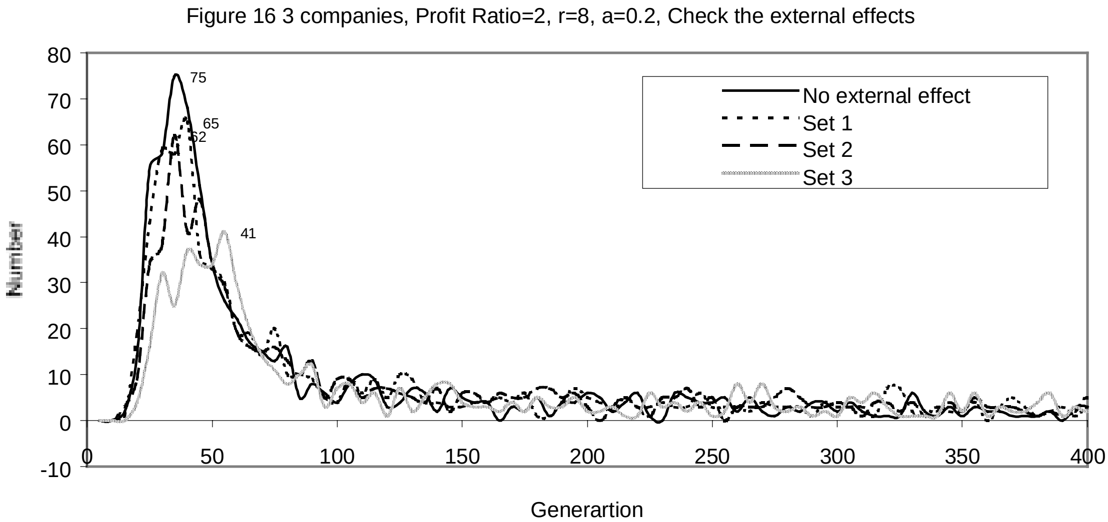

**Experiment 3** *the price competition*

In these experiments, we let the β function work to check if the barriers of the market will affect the cooperation. We set Additional Customer Rate (b) to 0, to see what the relationship is between the external effect and cooperation. The parameter settings for this experiment are shown in Table 6.

| | No Effect | Set 1 | Set 2 | Set 3 |
|---|---|---|---|---|
| Rounds | 8 | 8 | 8 | 8 |
| $\alpha$ | 0.2 | 0.2 | 0.2 | 0.2 |
| $\delta_C$ | 0 | 0 | -0.015 | -0.03 |
| $\delta_c$ | 0 | 0 | 0 | -0.015 |
| $\delta_w$ | 0 | 0 | 0 | 0.015 |
| $\delta_W$ | 0 | 0.015 | 0.015 | 0.03 |

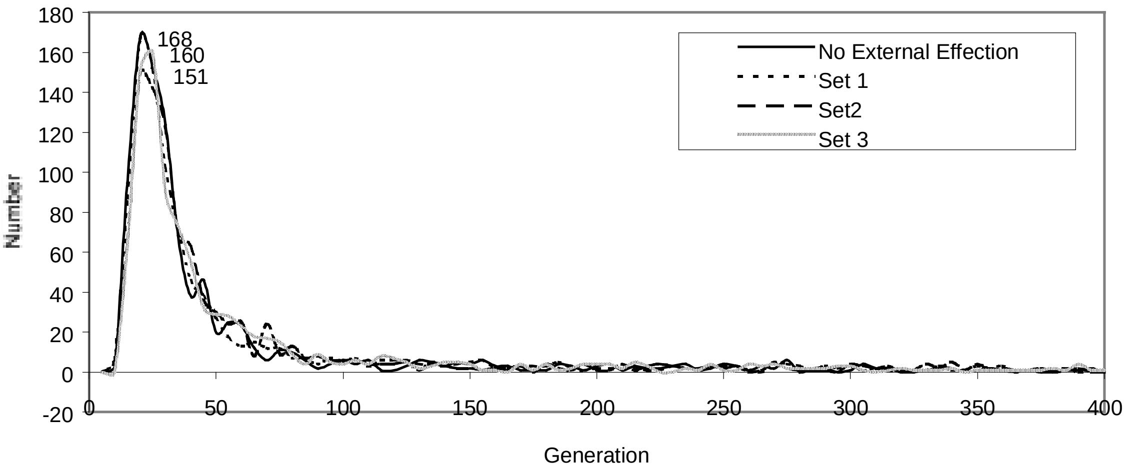
Table 6: *Parameters used for investigating the Additional Customer Rate ($\beta$)*

When the ratio is set to 2, this model is very sensitive to small fluctuations, so we can find the cooperation will be more difficult when external affection come. But we can show that the graph of set 1 and set 2 are similar in Figure 16. We can not say that means $\delta_C$ has no any effect on the cooperation situation but we can say the $\delta_W$ is more useful to break the cooperation situation than $\delta_C$.



We also wish to investigate if there are parameter values that are robust against the effect of Additional Customer Rate (β). We use Profit Ratio 5 to check the ease of cooperation in this model and find that, when the ratio is high enough, the cooperation situation is robust to defection: the graphs of all 4 sets are very similar. This means that even though the customer will move out in greater numbers when three oligopolists choose cooperation always, the companies still stay in cooperation situation because they can get more profit from that joint strategy.

Figure 17 3 companies, Profit Rtio=5, r=8, Check the external effects




## 8. CONCLUSION

1. The oligopoly game is similar to the NIPD but not the same as NIPD in any case.
2. The more companies involve in the oligopoly market, the more defection strategies will be chosen by the players even though the customers are absolutely loyal.
3. Higher difference between the profits of the cooperation and of the defection makes the cooperation situation much easier to be reached through the evolution procedure.
4. More customers moving from the companies with high price to those with the low price will make the cooperation much more difficult to be reached.
5. The frequency that the companies review their price strategies really affects the competition situation. The more frequent review makes the competitors choose the cooperation more quickly.
6. When the environment does not change, more difference between the profits of the cooperation and those of the defection will make the cooperation situation easier to be reached. In that case, even though the market share will lose much, the companies still insist to cooperate.
7. More histories kept by the price strategies make the strategies more complicated and the cooperation situation more difficult to be reached.
8. The price strategies developed by the companies themselves would not be disadvantage to them that can be exchanged by the companies. Even though companies will negotiate the strategies with one another, they still can not get the global fitness more quickly.
9. The number of customers moving into the market is much more important than that of customers moving away from this market to affect the competition situation.

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