Real Implications of the Zero Bound on Nominal Interest Rates

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Abstract

When price-setting is staggered and firms choose their prices optimally, low inflation regimes (where the nominal interest rate is occasionally zero) do not entail significant distortions to the real economy. By targeting the price level, the monetary authority can generate temporary expected inflation when nominal rates are zero, pushing real rates down. In contrast, when firms choose their prices according to the Fuhrer-Moore [1995] rule, the zero bound causes real distortions. By targeting the price level instead of inflation, however, the monetary authority can lessen those distortions.

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1 Introduction

The nominal interest rate cannot be less than zero: no one would choose to hold assets bearing a certain negative nominal return when they could instead hold money, which bears a certain zero nominal return. This fact has implications for the behavior of nominal interest rates when average inflation is very low, or negative. For example, consider a world of moderate average inflation where the nominal interest varies stochastically, always well above zero. Below some level of average inflation, the zero bound makes it impossible for the de-meaned behavior of the nominal rate to be the same as at the moderate inflation rate. This paper addresses the question of whether the zero bound also has implications for real variables, in particular the real interest rate. If the zero bound were to make real interest rates behave differently at very low average inflation rates, then the zero bound would be a *real* distortion, effectively constituting an argument against very low inflation.

Previous work has argued that the zero bound *is* a real distortion when price-setting is staggered and prices are set in the manner of Fuhrer and Moore [1995].[1] There are two reasons to pursue the issue further. First, the pricing policy assumed in that work is suboptimal, and second, the models do not contain money. The pricing policy is important because it affects the extent to which there can be temporary, sharp movements in expected inflation; the behavior of expected inflation in turn determines whether the real interest rate is affected by the zero bound on nominal rates. Money is important – in principle – because its existence is the source of the zero bound. The staggered price-setting model in this paper includes money, and is analyzed with both optimal and Fuhrer-Moore pricing policies. In addition, this paper departs from existing work by solving the entire model nonlinearly, including imposing the zero bound on nominal interest rates. Previous work has either worked with models that are mostly linear, or linearized the models and analyzed the zero bound indirectly.

Our principal finding is that with prices chosen optimally, the zero bound does not have to represent a real distortion. A policy rule that targets the price level generates temporarily high expected inflation in states where the

[1] The articles by Fuhrer and Madigan [1997] and Orphanides and Wieland [1998] will be discussed below. Earlier arguments not based one explicit models are due to Vickrey [1954], Okun [1981] and Summers [1991].

nominal rate is zero and the real rate needs to fall.[2] If the policy rule instead targets inflation, it does not generate enough temporary expected inflation in these states to prevent the real rate from being constrained. Fuhrer-Moore pricing introduces persistence in the inflation rate that is absent when prices are set optimally. This persistence plays a role similar to inflation targeting; we confirm that within a class of simple policy rules, the real rate is distorted under Fuhrer-Moore pricing. However, the standard inflation targeting rules exacerbate the real rate distortion in comparison to a price-level targeting rule. As for money, it is the main factor behind our results on welfare. However, the result that the zero bound does not have to entail a real distortion with optimal pricing holds with or without money in the model.

The paper proceeds as follows. Section 2 puts the analysis in context by discussing other recent work on the topic. Section 3 describes the basic modelling framework. Section 4 contains the principal results, which are for the case where prices are set optimally and the monetary authority targets the price level. For that case we provide a detailed comparison of low and high inflation regimes where the zero bound is, and is not "active." Section 5 briefly describes extensions involving Fuhrer-Moore pricing and inflation targeting (this section is work in progress). Section 6 concludes.

2 Related Literature

Declining inflation in many industrialized countries and near-zero nominal interest rates in Japan, have given rise to a large amount of recent research on the zero bound. This paper is most closely related to work by Fuhrer and Madigan [1997], Rotemberg and Woodford [1997], and Orphanides and Wieland [1998].

Because Fuhrer and Madigan [1997] and Orphanides and Wieland [1998] use similar models, we will consider them together. In addition – and like the analysis below – they assess the zero bound's importance by comparing their models' behavior at a moderate inflation target and at an inflation target low enough to make the nominal interest rate occasionally zero. Fuhrer and Madigan use a small model that contains: (i) a backward looking IS curve, (ii) a Fuhrer-Moore pricing specification, and (iii) a monetary policy reaction function. Orphanides and Wieland's model shares the same pricing

[2] Mishkin [1996] stresses this possibility.

specification, but disaggregates the IS curve into separate spending equations for consumption, fixed investment, inventory investment, net exports and government spending. Neither model includes money. Monetary policy operates by changing the short-term nominal interest rate. Long-term real interest rates enter the spending equations, but because the pricing specification makes inflation sticky, persistent changes in the short-term nominal rate generate changes in the long-term real rate. Thus, monetary policy can affect real spending, and hence output.

Fuhrer and Madigan evaluate the zero bound's importance by comparing their model's responses to IS curve shocks at inflation targets of zero and 4%. In contrast, Orphanides and Wieland simulate their model using estimated shock processes, and compare the variance of output at different inflation targets. These papers conclude that at a zero inflation target, monetary policy is significantly constrained by the zero bound: the zero bound is encountered regularly, and because of the zero bound output is more variable than at a moderate inflation target. The easiest explanation for this result comes from Fuhrer and Madigan's first example, which involves a permanent shock to the IS curve. The monetary authority responds to this shock by lowering short-term nominal interest rates. When the inflation target is zero, the monetary authority cannot lower the nominal rate by as much as it would choose if the inflation target were 4%. With sticky inflation, this translates into a smaller decline in the real interest rate, and hence – via the interest rate effect on spending – a larger fall in output at the zero inflation target.

The principal virtue of the analysis performed by Fuhrer and Madigan [1997] and Orphanides and Wieland [1998] is that it is performed using models which fit a particular sample of data quite well. However, because the low inflation experiments they perform involve an economic environment quite different from the data sample, the fact that the models' equations are not derived from explicit objective functions makes it doubtful that those equations would be stable in the face of the contemplated policy experiments. Although the model we will use in section 4 has not been shown to fit recent data well, it has the virtue of being set up in terms of explicit objective functions for individuals and firms. This means that the model can legitimately be used for policy and welfare analysis.

Rotemberg and Woodford [1997] reach a more qualified conclusion about the importance of the zero bound as a constraint on monetary policy, using a different model and approach from those of Fuhrer and Madigan and Orphanides and Wieland. As we do, Rotemberg and Woodford use a sticky-price model whose equations are derived from explicit optimization problems, and they use the utility function of agents to measure the welfare associated with different monetary policy rules. However, Rotemberg and Woodford linearize their model to simplify the analysis, and this precludes them from directly imposing the zero bound. They account for the zero bound indirectly, by assuming that the variability of the monetary authority's interest rate instrument is constrained by the average level of interest rates, that is by the inflation target. Specifically, they assume that the ratio of the standard deviation of the nominal interest rate to its average level can be no greater than the ratio that describes their 1980-1995 U.S. sample. Thus, policy rules that generate high variability of nominal rates are incompatible with low inflation targets. Since a generic implication of models such as theirs is that stable inflation requires volatile nominal interest rates, their assumption implies a sharp trade-off between the level of inflation and its variability. While this assumption has the effect of increasing the optimal inflation target from zero, they find that the optimum does not move far from zero.

Money is absent from all the models discussed above. Rotemberg and Woodford correctly state that the behavior of their model would be unchanged if they used a money-in-the-utility function specification where money was additively separable in the period utility function. However, ignoring money demand also means ignoring the welfare costs of positive nominal interest rates. That is, while the behavior of real and nominal variables may be invariant to adding money in a separable way, the welfare implications of different monetary policies are not invariant to this modification. Since concern about the zero bound on nominal interest rates boils down to concern about the welfare level associated with very low inflation targets, leaving money out of the model may be an important omission.

In another recent paper related to the zero bound, Krugman [1998] analyzes the zero bound's role in Japan's current economic situation. Krugman argues that in Japan, the zero bound is a real distortion; private agents do not believe that the monetary authority will generate inflation, so expected inflation does not rise, and the real interest rate cannot fall. This scenario involves a credibility problem that is absent from this paper and the three discussed above. As we see below, however, just as the credibility problem prevents expected inflation from moving as needed, so can a poorly chosen policy rule, or a particular pricing environment.

3 The General Model

Our analysis of the zero bound's importance for monetary policy is based on an explicit optimizing sticky price model, similar to, but simpler than that in Rotemberg and Woodford [1997]. As in Fuhrer and Madigan [1997] and Orphanides and Wieland [1998], we impose the zero bound directly, rather than measuring its importance indirectly.[3] Two features of the analysis are novel in comparison to each of the papers discussed above. First, we explicitly model money demand (using a shopping time technology), so there is a force working in *favor* of zero nominal interest rates. Second, no linear approximations are employed in solving the model, which is fundamentally nonlinear.

The model is in the tradition of Taylor [1980], in that price-setting is staggered: each firm sets its price for 2 periods, with 1/2 of the firms adjusting each period.[4] There are a continuum of firms, and they produce differentiated consumption goods using, as the sole input, labor provided by consumers at a competitive wage. Consumers are infinitely lived, and purchase consumption goods using income from their labor, which is supplied elastically. Consumers hold money in order to economize on transactions time, as in McCallum and Goodfriend [1987].

3.1 Consumers

Consumers have preferences over a consumption aggregate (c_t) and leisure (l_t) given by

$$E_t \sum_{t=0}^{\infty} \beta^t \cdot (\ln(c_t) + \chi_t \cdot l_t). \tag{1}$$

[3]Fuhrer and Madigan use three different approaches, one of which involves directly imposing the zero bound.

[4]The remainder of this section is loosely based on section 2 in King and Wolman [1998]. The model analyzed here differs in that it explicitly motivates money demand with a shopping-time technology.

The discount factor β is set to 0.985, and the variable χ_t is a random preference shock.[5] The consumer's budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{B_t/P_t}{1+R_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + w_t n_t + d_t + \frac{S_t}{P_t},$$

and the time constraint is

$$n_t + l_t + h(M_t/(P_t c_t)) = E, \tag{2}$$

where P_t is the price level, M_t is nominal money balances chosen in period t, to carry over to $t+1$, B_t is holdings of one-period nominal zero-coupon bonds maturing at $t+1$, R_t is the interest rate on nominal bonds, w_t is the real wage, n_t is time spent working, d_t is real dividend payments from firms, S_t is a lump sum transfer of money from the monetary authority, $h(M_t/(P_t c_t))$ is time spent transacting, and E is the time endowment. Defining real balances to be $m_t \equiv M_t/P_t$, the function $h(\cdot)$ is parameterized as in Wolman [1997]:

$$h(m_t/c_t) = \phi \cdot (m_t/c_t) - \tfrac{\nu}{1+\nu} A^{-1/\nu} (m_t/c_t)^{\frac{1+\nu}{\nu}} + \Omega, \text{ for } m_t/c_t < A \cdot \phi^{\nu},$$

$$h(m_t/c_t) = \Omega, \text{ for } m_t/c_t \geq A \cdot \phi^{\nu}, \tag{3}$$

with $\phi = 1.4 \times 10^{-3}$, $A = 1.7 \times 10^{-2}$, and $\nu = -0.7695$. Transactions time is thus decreasing in real balances and increasing in consumption, up to a satiation level of the ratio of real balances to consumption.[6]

Goods market structure. As in Blanchard and Kiyotaki [1987], we assume that every producer faces a downward sloping demand curve, with constant elasticity ε.[7] With a continuum of firms, the consumption aggregate is an integral of differentiated products $c_t = [\int c(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega]^{\frac{\varepsilon}{\varepsilon-1}}$, as in Dixit and Stiglitz [1977].

Since all producers that adjust their prices in a given period choose the same price, it is easier to write the consumption aggregate as

$$c_t = c(c_{0,t}, c_{1t}) = (\frac{1}{2} \cdot c_{0,t}^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} \cdot c_{1,t}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

[5] This value of β implies a steady state real interest rate of 6.5% per annum, and hence a steady state nominal interest rate of about 11.5% when there is 5% annual inflation. While the number assigned to β has quantitative implications for the results reported below, it does not have qualitative implications.

[6] Because the parameters of the transactions time technology were estimated with annual data, they may imply too high an interest elasticity of money demand.

[7] We assume $\varepsilon = 10$.

where $c_{j,t}$ is the quantity consumed in period t of a good whose price was set in period $t-j$. The constant elasticity demands for each of the goods take the form:

$$c_{j,t} = \left(\frac{P^*_{t-j}}{P_t}\right)^{-\varepsilon} \cdot c_t, \tag{5}$$

where P^*_{t-j} is the nominal price at time t of any good whose price was set j periods ago, and P_t is the price index at time t, given by

$$P_t = \left[\frac{1}{2} \cdot (P^*_t)^{1-\varepsilon} + \frac{1}{2} \cdot \left(P^*_{t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{6}$$

Optimization. If we attach Lagrange multipliers λ_t and μ_t to the budget and time constraints, respectively, so that λ_t is the marginal value of real wealth and μ_t is the marginal value of time, the first order conditions for the individual's maximization problem, with respect to c_t, l_t, n_t, B_t and M_t, are

$$\frac{1}{c_t} = \lambda_t - \mu_t \cdot h'(\cdot)(\frac{m_t}{c_t^2}) , \tag{7}$$

$$\chi_t = \mu_t, \tag{8}$$

$$\mu_t = w_t \cdot \lambda_t \tag{9}$$

$$\frac{\lambda_t}{P_t} = \beta \cdot (1 + R_t) \cdot E_t \frac{\lambda_{t+1}}{P_{t+1}}, \tag{10}$$

and

$$\frac{\lambda_t}{P_t} + \frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t}) = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}. \tag{11}$$

In choosing consumption optimally (7), the individual weighs the benefit of consuming a marginal unit, which is the left hand side of (7), against the cost, which consists of both forfeited real wealth (the first term on the right hand side) and time spent transacting (the second term on the right hand side). In choosing leisure and labor supply optimally (8 and 9), the individual balances the marginal value of time against both the marginal utility of leisure and the wage earnings that the time would yield. The choice of bond holdings (10) balances the marginal value of nominal wealth today against $(1 + R_t)$ times the marginal value of nominal wealth tomorrow. And finally, optimal money holdings (11) imply that the individual balances the transactions-facilitating benefit against the foregone interest cost of holding money.[8]

3.2 Firms

Each firm produces with an identical technology:

$$c_{j,t} = n_{j,t}, \quad j = 0, 1, \tag{12}$$

where $n_{j,t}$ is the labor input employed in period t by a firm whose price was set in period $t - j$. Given the price that a firm is charging, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their price in a given period can thus be thought of as passive. Given that it has set a relative price $\frac{P^*_{t-j}}{P_t}$, real profits for a firm of type j are

$$\frac{P^*_{t-j}}{P_t} \cdot c_{jt} - w_t \cdot n_{jt}, \tag{13}$$

that is, revenue minus cost.

Price setting: We will analyze two specifications for the determination of P^*_t. In the first, firms choose P^*_t optimally, to maximize the present discounted value of profits over the two periods that the price will be charged. In the second, P^*_t is determined by the "sticky inflation" specification of Fuhrer and Moore.

- *Optimal*: Maximization of present value implies that a firm chooses its current relative price taking into account the effect on current and expected future profits. Substituting into (13) the demand curve (5) and the technology (12), the present discounted value of expected profits is

[8] The transactions facilitating benefit is given by $\frac{\mu_t}{P_t} \cdot h'(\cdot)(\frac{1}{c_t})$, and the foregone interest cost is $\frac{\lambda_t}{P_t} - \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}$ (see (10)). A conventional money demand equation can be derived by combining (9)-(11): $m_t/c_t = A \cdot ((R_t/(1 + R_t)) \cdot (c_t/w_t) + \phi)^{\nu}$.

given by

$$c_t \cdot \left[\left(\tfrac{P_t^*}{P_t} \right)^{1-\varepsilon} - w_t \cdot \left(\tfrac{P_t^*}{P_t} \right)^{-\varepsilon} \right] + \\ \beta E_t \tfrac{\lambda_{t+1}}{\lambda_t} \cdot c_{t+1} \cdot \left[\left(\tfrac{P_t^*}{P_{t+1}} \right)^{1-\varepsilon} - w_{t+1} \cdot \left(\tfrac{P_t^*}{P_{t+1}} \right)^{-\varepsilon} \right], \tag{14}$$

for the two periods over which a price will be in effect. Differentiating (14) with respect to P_t^* and setting the resulting expression equal to zero, one sees that the optimal relative price satisfies

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\sum_{j=0}^{1} \beta^j E_t \{ \lambda_{t+j} \cdot w_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon} \cdot c_{t+j} \}}{\sum_{j=0}^{J-1} \beta^j E_t \{ \lambda_{t+j} \cdot (P_{t+j}/P_t)^{\varepsilon - 1} \cdot c_{t+j} \}}. \tag{15}$$

Essentially, the optimal relative price equates discounted marginal revenue with discounted marginal cost; the numerator of (15) represents marginal revenue and the denominator marginal cost.[9] In a non-inflationary steady state, the firm would choose a markup over marginal cost of $\frac{\varepsilon}{\varepsilon-1}$. In an inflationary or deflationary steady state, the markup would differ from $\frac{\varepsilon}{\varepsilon-1}$, as adjusting firms would take into account the future erosion (or inflation) of their relative price (see King and Wolman [1998] for details). With uncertainty, the markup becomes time varying: it depends on the current and expected future marginal utility of wealth, price level, aggregate demand and real wage.

- *Fuhrer-Moore*: The second specification we will consider has firms choosing their relative price as a function of the relative price chosen by firms in the previous period, and that expected to be chosen in the next period, as well as "output gaps" in the current and immediate future periods:

$$\ln \left(\frac{P_t^*}{P_t} \right) = \frac{1}{2} \cdot \left(q_t + \gamma \ln \left(\frac{c_t}{c} \right) \right) + \frac{1}{2} \cdot E_t \left(q_{t+1} + \gamma \ln \left(\frac{c_{t+1}}{c} \right) \right) + z_t, \tag{16}$$

[9] Note that in this sentence, marginal revenue and cost are with respect to price, not quantity.

where

$$q_t = \frac{1}{2} \cdot \ln\left(\frac{P_t^*}{P_t}\right) + \frac{1}{2} \cdot \ln\left(\frac{P_{t-1}^*}{P_{t-1}}\right). \tag{17}$$

In (16), the term z_t is an increasing function of the real wage, and is absent from other papers using this general specification. This term is necessary in the context of the optimizing model used here, as without it the model would not be completely specified.[10]

3.3 Monetary Policy

We assume that policy is given by a feedback rule for the nominal interest rate. One component of the feedback rule is a "target" inflation rate, that is an inflation rate that the rule would deliver in the absence of shocks. In general, the feedback rule makes the nominal rate a differentiable function of observable variables. In certain states of the world, however, that differentiable function would make the nominal rate negative. In those states of the world we assume that the policy rule sets $R_t = 0$. Given the nominal interest rate implied by the policy rule, the monetary transfer (S_t) is determined by money demand. Note that money demand is an integral part of the model. It is sometimes asserted that when the monetary authority follows an interest rate rule, money demand can be left out of the model, as it only serves to determine the value of the money supply. Here that is not the case, because the quantity of money enters other equations of the model in addition to the money demand equation (specifically, (7) and (2)).

The nominal interest rate is the rate on one-period bonds, which are assumed to be in zero net supply. This is somewhat problematic from the standpoint of justifying the zero bound. That is, the zero bound is a necessary characteristic of nominal bonds which are willingly held, but nominal bonds are not actually held in the model (they are priced). This inconsistency can be rectified by assuming that there is a fixed real quantity of outstanding government bonds, and the government pays the interest on those bonds by levying lump sum taxes as necessary.

[10] To see this, note that in steady state, (16) collapses to $0 = 0$ without z_t. We specify z_t so that in steady state (16) is equivalent to (15).

3.4 Driving Process

The only exogenous variable in the model is the preference shock χ_t, and it is assumed to follow a two-state Markov process:

$$\begin{aligned} \Pr\left(\chi_t = \overline{\chi} \mid \chi_{t-1} = \overline{\chi}\right) &= 0.8 \\ \Pr\left(\chi_t = \underline{\chi} \mid \chi_{t-1} = \underline{\chi}\right) &= 0.8, \quad \underline{\chi} \leq \overline{\chi}. \end{aligned} \tag{18}$$

Thus, χ_t varies between high and low values, and on average each value persists for 5 periods before switching.[11] This process is not meant to replicate actual features of the U.S. economy. Rather, it is chosen to make the economy alternate between periods of high and low output in a way that makes the real interest rate vary over time. It is by no means the only process that would yield such behavior, and the equilibrium behavior of the real interest rate will also be affected by monetary policy.

4 The Zero Bound with Optimal Staggered Price-Setting and a Price Level Target

Using the model described above, one can determine whether the zero bound means that a very low inflation target significantly modifies economic performance relative to a moderate inflation target. For a policy rule that involves targeting the price level, we will simulate the model with optimal price-setting at moderate inflation and then at moderate deflation, and compare the results along three dimensions. The first involves simulating the model for 30 periods with the same shocks at high and low inflation, and informally comparing the results. The second involves the variances of inflation and output, which has been the conventional metric in the literature on monetary policy rules (see the papers in Taylor [forthcoming]). Given that the model yields an obvious choice for a welfare function (the representative agent's expected utility), we also compare the two regimes in terms of welfare.

[11] The inequality in (18) is weak because in section 5 we focus on an environment where χ_t is constant.

4.1 Policy Rule

Recent research on monetary policy has emphasized "Taylor rules," that is specifications of policy where the monetary authority sets a short term interest rate as a linear combination of deviations of inflation from a target and deviations of output from some trend or potential level. These rules, popularized by John Taylor [1993], have been shown to be parsimonious approximations to the behavior of some central banks. The rule considered here is similar to a Taylor rule, except that instead of inflation on the right hand side it uses the price level. Concretely,[12]

$$R_t = \max \left\{ \begin{array}{c} R^* + 1.5 \cdot \left(\ln\left(P_t\right) - \ln\left(\bar{P}_t\right)\right) + 1.0 \cdot \left(\ln\left(c_t\right) - \ln\left(\bar{c}\right)\right), \\ 0 \end{array} \right\}, \quad (19)$$

where R^* is the steady state nominal interest rate consistent with the chosen inflation target, $\bar{P}_t$ is a target price level path that grows at the targeted inflation rate, and $\bar{c}$ is the steady state level of consumption associated with the inflation target.[13] This rule has the implication that the price level will always be expected to return to the same trend path. In contrast, the standard Taylor rules imply that inflation will always be expected to eventually return to target, but the price level will be expected to drift away from any previous trend path. This distinction turns out to have important implications.

4.2 Functions Describing General Equilibrium.

As background to analyzing simulation results, Figure 1 displays the relationships between key endogenous variables and the state variable, which is the price set last period by adjusting firms.[14] Figure 1 is generated with an inflation target of 5%.[15] The solid lines show the relationship between P^*_{t-1} (detrended by the targeted inflation rate) and each endogenous variable when the preference shock takes on a high value, and the dashed lines shows the

[12] The interest rate in (19) is a quarterly interest rate, whereas the rates plotted in figures 1-4 are annual rates.

[13] The inflation target affects steady state consumption for two reasons. First, the markup chosen by adjusting firms varies with the inflation target in a way that does not exactly offset the inflation erosion of non-adjusting firms' markups. Second, by lowering real balances, higher inflation effectively makes consumption more expensive.

[14] An appendix describes the model solution procedure.

[15] Even though the policy rule targets the price level, there is still an inflation target, defined as the growth rate of the price level target.

relationships when the preference shock takes on a low value. Using panel B, and with knowledge of P_0^*, one can trace out a path for P_t^* by drawing values of χ_t from the stochastic process governing it. Then, with the path for P_t^* in hand, the relationships in panels A, C and D can be used to generate paths for the other variables for the given sequence of χ_t. What follows is a discussion of the model's principal mechanisms in light of the relationships shown in Figure 1.

There are essentially two determinants of current period variables in the model. One is the value of the stochastic preference parameter (χ_t), and the other is the value of the price that adjusting firms set last period. When χ_t takes on a high value, the marginal utility of leisure is high. Agents react by supplying less labor to the market, and this necessarily decreases consumption. Thus, in panel A, the level of consumption is low when $\chi_t = \bar{\chi}$. For low values of P_{t-1}^*, the lower level of consumption causes the monetary authority to set a lower value for the nominal interest rate, as in the left-hand part of panel C, and the lower nominal interest rate in turn drives up money demand (panel D). However, when P_{t-1}^* is especially high, the nominal rate is lower in the $\underline{\chi}$ (high consumption) state. Why is this? The feedback rule for monetary policy sets the nominal rate as an increasing function of both consumption and the price level, so it must be that in the high-P_{t-1}^* region the price level effect dominates in the feedback rule. The policy functions for the price level (not shown) indeed reflect this fact. The price level is higher in the $\bar{\chi}$ state than in the $\underline{\chi}$ state, and the gap between the price levels in the two states is increasing in P_{t-1}^*.

Another perspective on the nominal interest rate functions in panel C comes from thinking about two relationships emphasized by Irving Fisher. "Fisher Equation: I" states that the nominal interest rate is approximately equal to the sum of the real interest rate and expected inflation. When utility is logarithmic in consumption, as it is here, "Fisher Equation: II" implies that the real interest rate is approximately equal to expected consumption growth.[16] From panel A, we know that consumption and the preference parameter move in opposite directions. Further, the stochastic process for the preference parameter is mean reverting, so that when it is low it is expected to increase, and therefore consumption is expected to fall. From Fisher's second

[16]The relationship is only approximate here because the shopping time requirement means that the marginal utility of consumption is greater than the marginal value of a unit of real wealth. To derive this approximate relationship, combine (7) and (10) above.

equation, this means a low real interest rates when the preference parameter is low. Note, however, that the policy rule typically makes nominal rates high in those cases when we have just argued that real rates are low. Referring to Fisher's first equation, it must then be that high nominal rates correspond to high enough expected inflation to counteract the low real rates. From panel D we can see that monetary policy does in fact deliver high expected inflation when the preference parameter is low. The money supply is low when the preference parameter is low, and mean reversion implies that the money supply is expected to increase in those periods, generating high expected inflation.

Note that the behavior of real interest rates conflicts with the behavior displayed in the other articles discussed above. There the monetary authority lowers nominal interest rates when output is low, and real rates fall as well. Here, real interest rates are to a great extent determined by the shock process in conjunction with Fisher's second equation. For a large class of such processes, including the one used here, real interest rates are low when output is *high*. More generally, without resorting to reduced forms, it has proven difficult to produce models where the cyclical behavior of real rates matches the data.

4.3 Simulated Time Paths

Figure 2 displays the time paths of the variables from Figure 1 other than P_t^*, as well as the price level, the real interest rate, and expected inflation, for a sequence of 30 χ_t drawn from the stochastic process described above. This sequence will be a benchmark for comparison with the low inflation target case below. Focusing first on consumption (panel A), note that there are essentially three regions: low, high and intermediate. The high consumption region is attained with any sequence of at least two consecutive low values for χ_t (the realizations of χ_t are plotted in panel B). Likewise, the low consumption region is attained with any sequence of at least two consecutive high values for χ_t. These regions correspond to the points marked with **x**'s in figures 1.A and B. The intermediate consumption region corresponds to the transition from one value of the preference shock to the other; these are the points marked **y** in figure 1.A and B. The fact that it takes two periods to transit between the high and low consumption regions is an implication of two-period price stickiness. To see this, suppose the economy had been in the low preference parameter/high consumption state for several periods. If

χ_t then took on a high value, in the initial period the state variable (P^*_{t-1}) would be at the level associated with $\underline{\chi}$, so that the economy could not immediately transit to low consumption. If χ_t remained high in the next period, consumption would settle at a lower level, because the state variable had changed; by the period after the shift in χ_t, all firms would have had a chance to adjust their price. If prices were flexible, the transition would be immediate, whereas with prices set for more than two periods the transition would be correspondingly longer.

Note that in some of the periods when consumption takes on an intermediate value, the real rate is negative (Figure 2.F). Specifically, this occurs in periods when $\chi_t = \bar{\chi}$ and $\chi_{t-1} = \underline{\chi}$ (periods 12,17,20). Referring back to Figure 1, one can see that in this situation consumption is expected to fall towards the low level associated with $\bar{\chi}$. With consumption expected to fall significantly, the real rate must be negative. Because the inflation target is 5%, the zero bound does not inhibit the real rate from going negative. However, one might expect that with a very low inflation target, the real rate would be inhibited from going negative, and thus the zero bound would interfere with the economy's "natural" behavior.

Figures 3 and 4 correspond to Figures 1 and 2, with an inflation target of minus five percent. From Figure 3.A-C, we see that for a wide range of values of the state variable, including the region corresponding to high consumption, the nominal rate is zero. However, this drastically different behavior of the nominal rate does not correspond to significantly different functions for consumption (Figure 3.A). The simulation in Figure 4 confirms these results. Whereas we surmised that the nominal rate might hit the zero bound when $\chi_t = \bar{\chi}$, $\chi_{t-1} = \underline{\chi}$, in fact it hits the bound *whenever* $\chi_t = \bar{\chi}$. However, the behavior of consumption is almost indistinguishable from Figure 2, the 5% inflation target. From Fisher's second equation, we know that similar consumption behavior must correspond to similar real rate behavior, and this is confirmed in Figure 4.F. How is a zero nominal rate consistent with a negative real rate in periods 12, 17 and 20? From Fisher's first equation, the real rate is the difference between the nominal rate and expected inflation, so in those periods the monetary authority is making expected inflation positive (Panel C). The targeted rate of deflation is consistent with periods of high expected inflation, because the policy rule unambiguously makes the expected inflation temporary, and there is no uncertainty about whether the monetary authority will adhere to the policy rule.

Simulations such as those in Figures 2 and 4 are an informal basis for evaluating whether the zero bound is important. However, those simulations provide clear evidence – in the model used here – that monetary policy can offset the zero bound by generating temporary expected inflation. With real rates thus unconstrained, the existence of the zero bound does not appear to constitute an argument against a low inflation target. Figure 4 illustrates an additional feature of the model that *favors* a very low inflation target. In Panels A and C, the series for consumption and real rates from Figure 2, corresponding to a 5% inflation target, are reproduced along with the new series corresponding to 5% deflation. In panel A, we see that consumption is actually higher in every period with the 5% deflation target than it is with the 5% inflation target. The lower inflation target corresponds to lower nominal interest rates on average, as is clear from Panel D of figures 2 and 4. Lower nominal interest rates in turn correspond to a smaller money demand distortion, as in Bailey [1956] and Friedman [1969]. Individuals hold higher real balances because the opportunity cost of real balances has fallen, and higher real balances effectively make consumption cheaper, because they decrease the time that an individual must spend transacting.

4.4 Variances

The simulations in Figures 2 and 4 provide strong evidence on the importance of the zero bound, and the welfare results below give the bottom line. To enhance comparability with the articles by Rotemberg and Woodford [1997] and Orphanides and Wieland [1998], we also provide information on variability at high and low inflation targets. Table 1 shows the standard deviations of some of the main variables in the model for both regimes, based on simulations of 5000 periods. As suggested by Figures 1-4, the variability of consumption is barely affected by the inflation target. On the other hand, the nominal interest rate is much less variable when the inflation target makes zero occasionally binding. There is a trade-off in the model between the average level of inflation and the minimum feasible variability of inflation, just as described in Rotemberg and Woodford [1998]. Also as in that paper, the large difference in nominal interest rate variability in the two regimes translates into only a small difference in inflation variability. A striking feature of Table 1 is the tremendous increase in money supply variability in the deflation regime. This can be traced to the fact that the money demand function exhibits increasing sensitivity to nominal interest rates as the nominal interest rate falls.

Table 1: standard deviations in the two policy regimes

	consumption	inflation	nominal rates	money
5% inflation	.0427	.0706	.0145	.0910
5% deflation	.0435	.0786	.0093	.7562

4.5 Welfare

It is clear from the simulations we have already looked at that the real (as opposed to nominal) distortions associated with the zero bound are small. Nevertheless, it is interesting to know whether the inflation or deflation regime is preferred on welfare grounds. With the zero bound not a factor, a welfare comparison will hinge on the other distortions present in the model. Those other distortions involve the inflation tax and the interaction between sticky prices and monopolistic competition. The inflation tax distortion makes deflation preferable to inflation. Sticky prices and monopolistic competition make the optimal inflation target near zero, so neither 5% inflation nor deflation targets would obviously be preferred to the other on that basis. It thus seems likely that the unambiguous effect of the inflation tax will dictate that the lower inflation regime is preferred. However, to definitively resolve the issue we must compare the representative individual's expected utility in the inflation and deflation regimes.

We calculate expected utility by performing 1000 simulations of 1000 periods each, with each simulation beginning from a random value for the state variable. The initial condition is chosen by simulating the model for 50 periods, starting from the steady state, and then setting $P_0 = P_{50}$. Each simulation ($k = 1$ to 1000) yields a value for $U_k \equiv \sum_{t=0}^{1000} \beta^t \cdot (\ln (c_t) + \chi_t \cdot l_t)$, and then expected utility is given by $E(U) = 1000^{-1} \cdot \sum_{k=1}^{1000} U_k$. With values for expected utility in both regimes, we compare them by pretending that they were generated in a steady state. This involves calculating first the average per period utility in the two regimes, and then the percentage increase in consumption that would make an agent living in the lower utility regime just as well-off as an agent in the higher utility regime. The results of this exercise are that an agent living in the inflationary regime would be indifferent between receiving a 2.6% increase in per period consumption and switching to the deflationary regime.

To illustrate how important the inflation tax is in these results, we can repeat the comparison of the two inflation regimes with a slight modification. That modification is to eliminate the money demand distortion; we modify (7) to $\lambda_t = 1/c_t$, and replace (11) with $M_t = P_t \cdot c_t$. With the inflation tax eliminated, the 5% inflation target regime is marginally preferred to the 5% deflation target regime, although the difference in welfare is minuscule compared to the difference found (with opposite sign) when the inflation tax played a role. The results from eliminating the money demand distortion mean that money demand is crucial in making the deflationary regime welfare-superior to the inflationary regime. However, even without the money demand distortion, the fact that the nominal interest rate is occasionally zero in the deflationary regime does not significantly affect the behavior of real variables. In particular, the policy rule is still able to generate temporarily high expected inflation when real rates need to be negative.

5 Inflation Targeting and Inflation Persistence

The preceding section shows that if prices are staggered, but chosen optimally, a policy rule that targets the price level prevents the zero bound from "contaminating" the real economy. It may be, however, that this result is not robust to policies that target inflation. This section thus considers inflation targeting. We also look at the Fuhrer-Moore pricing specification, to determine whether including money in the model affects the results of Fuhrer and Madigan [1997] and Orphanides and Wieland [1998], and again to learn about the role of the policy rule. For each of these modifications, we will simplify the framework by looking at the models' pure transitional dynamics. That is, we eliminate variation in the preference parameter, and focus on the resulting equilibrium functions (hereafter to be referred to as policy functions) corresponding to Figures 1 and 3. The reason for doing this is that a simple comparison of the real rate policy functions in high inflation and low inflation regimes reveals whether the zero bound is introducing a real distortion. As a prelude, figure 5 plots the policy functions for nominal rates, real rates, and expected inflation that correspond to the specification in figures 1-4, except that there is no variation in the preference parameter. Note that while the nominal rate policy function has a kink and a horizontal portion reflecting the zero bound, the real rate appears to be smooth. The steep function for expected inflation reconciles these facts.

5.1 Inflation Targeting with Optimal Pricing

Modifying (19) to

$$R_t = \max \left\{ \begin{array}{c} R^* + 1.5 \cdot (\ln(P_t/P_{t-1}) - \ln(1+\pi)) + 0.5 \cdot (\ln(c_t) - \ln(\bar{c})) \,, \\ 0 \end{array} \right\} \tag{20}$$

yields a rule that targets inflation instead of the price level, where π is the target rate of inflation. Figure 6 is the analogue of figure 5, with policy given by (20).[17] In stark contrast to price-level targeting, with inflation targeting the horizontal portion of the nominal rate function generates a horizontal portion of the real rate function. Necessarily, there is a corresponding horizontal portion of the expected inflation function. Figures 5 and 6 reveal that whether the zero bound is a real distortion hinges in large part on what policy rule the monetary authority follows when rates are not at the bound. With a price level target, low values of the price level generate expectations of a higher price level in the future, i.e. expected inflation. Because states in which the nominal rate is zero are states in which the price level is low, the rule's implications for future price movements allow decreases in the real rate when the nominal rate is zero. In contrast, with an inflation target, low values of inflation carry with them only an expected *increase* in inflation, as opposed to a high *level* of future inflation. This induced persistence in inflation makes the real rate downwardly rigid when the nominal rate is at zero.

5.2 Price-Level Targeting with Fuhrer-Moore Pricing

Next, we modify the pricing specification to Fuhrer-Moore, and return to a policy rule of price-level targeting. Price level targeting made the zero bound innocuous with optimal price-setting, but figure 7 shows that this does not carry over to Fuhrer-Moore.[18] As is well known, this pricing specification

[17] In the case of inflation targeting, the price level is nonstationary. This means that to solve the model we detrend by dividing nominal variables by P_{t-1}, whereas with the price level target we divided by $(1+\pi)^t$. The state variable is thus P^*_{t-1}/P_{t-1} instead of $P^*_{t-1}/(1+\pi)^t$.

[18] When the policy rule targets inflation and price-setting is Fuhrer-Moore, there are two state variables in the model (P_{t-1} and P^*_{t-1}). Figure 7 is drawn for one value of P_{t-1}. The complete policy function is three-dimensional.

induces persistence in the inflation rate. The simple price-level targeting rule is unable to overcome this pricing behavior, and the real rate is constrained when the nominal rate is zero.

5.3 Inflation Targeting with Fuhrer-Moore Pricing

For completeness, Figure 8 displays the combination of an inflation targeting rule with Fuhrer-Moore pricing. The Figure confirms the work of Fuhrer and Madigan [1997] and Orphanides and Wieland [1998] in showing a significant real distortion associated with the zero bound. The real rate policy function actually gets “bent” in the region where the nominal rate is zero. In this case, the inflation persistence introduced by the policy rule compounds that already present because of the pricing specification, and little expected inflation is forthcoming in those states where the nominal rate is zero and the real rate needs to be low. The fact that our model contains money and generates similar results to FM and OW confirms the finding of section 3: including money in the model, while important for welfare calculations, does not seem to alter conclusions regarding the effect of the zero bound on real variables.

6 Conclusions

Staggered price-setting is currently one of the most popular frameworks for studying monetary policy. Yet, there is no consensus about the zero bound’s implications even within the staggered price-setting framework. This paper, along with those repeatedly referenced above, will hopefully bring us closer to such a consensus. Toward that consensus, we make the following contributions:

- When prices are set optimally, the simple policy rule of targeting the price level with an interest rate instrument succeeds in keeping the zero bound a nominal phenomenon.

- When prices are set as in Fuhrer and Moore, the same simple rule fails at keeping the zero bound a nominal phenomenon. However, that rule is more successful than its commonly analyzed inflation targeting cousin. Furthermore, there may be some rule that does “nominalize” the zero bound with Fuhrer-Moore pricing.

- Since the ultimate purpose of studying the zero bound is to better understand the welfare consequences of different policies, money demand is an important element in the analysis. However, if one is not interested in welfare analysis or in the behavior of money per se, then it is reasonable to omit money from the model. As a caveat, it would be useful to explore whether different ways of incorporating money have more of an impact than what is done here.

We conclude with some suggestions for future research. For the first point above, the assumption that agents have perfect information about the policy rule is crucial. Zero nominal interest rates need not prevent the real rate from falling, because the monetary authority can generate expected inflation. These occasional periods of high expected inflation do not trigger inflation scares, because agents know that any inflation that ensues will be temporary, and that the monetary authority remains committed to its stated inflation target. In practice, it may be difficult for central banks to generate occasional episodes of high expected inflation without endangering the credibility of their low inflation target. In principle, it would be possible to analyze this issue in an extension of the current framework.

Regarding the second point, for both pricing specifications it would be useful to know what the optimal policy rule is. While King and Wolman [1998] and Rotemberg and Woodford [1998] have studied optimal policy in similar models, neither have directly imposed the zero bound or considered money demand distortions. Related to optimal policy is the question of whether there is any role for focusing on distortions associated with the zero bound, as opposed to simply looking at the overall welfare implications of different policy rules. At this point, with much confusion about the "pure" implications of the zero bound, some focus on the zero bound seems warranted. Ultimately, however, the zero bound is just one feature of an economic environment. Statements about what is good policy must take into account many features, among them the inflation tax.

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Solving the Model

We solve the model using the finite element method (see McGrattan [1996]). This involves picking a grid of points for the model's state variables, and then finding values of the "control" variables numerically for each grid point and each value of the preference shock such that the model's equations are satisfied. The solution consists of mappings from the state variable to each of the other variables. Those mappings can be used in conjunction with the stochastic process for the preference shock to simulate the model. Because this solution method involves a finite number of grid points, it necessarily yields only an approximate solution. However, to the extent that the true mappings from the state variables to the other variables are smooth functions, the grid method can yield an extremely accurate solution. Furthermore, to the extent that the mappings appear nonlinear, we have an indication of the error that would be associated with linearization methods.

Figure 1.
Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5% Inflation Target

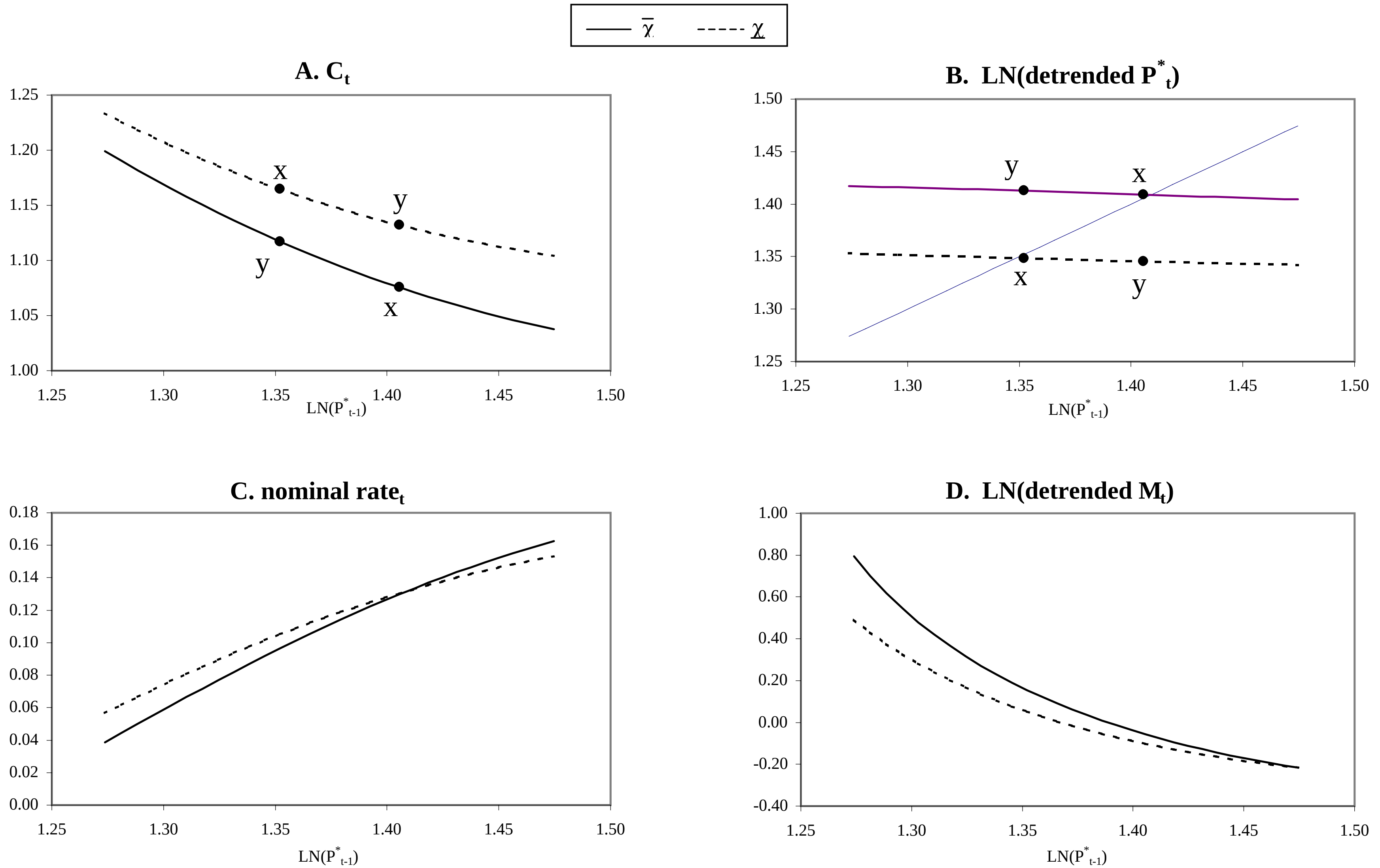


Figure 2.
Time Paths From 30 Period Simulation (5% Inflation Target)

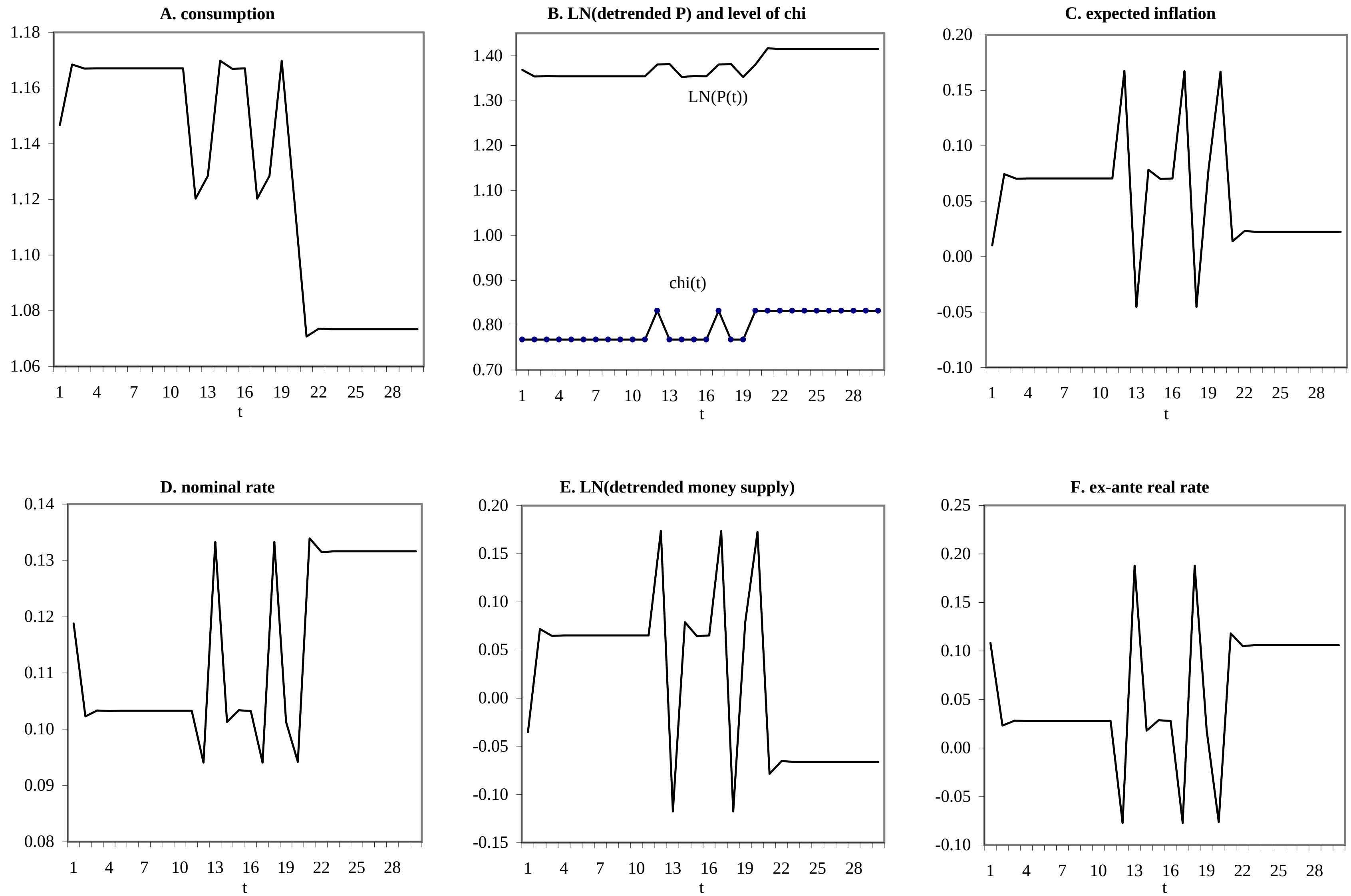


Figure 3.

Functions Mapping State Variable (P^*_{t-1}) to Other Variables at 5% Deflation Target

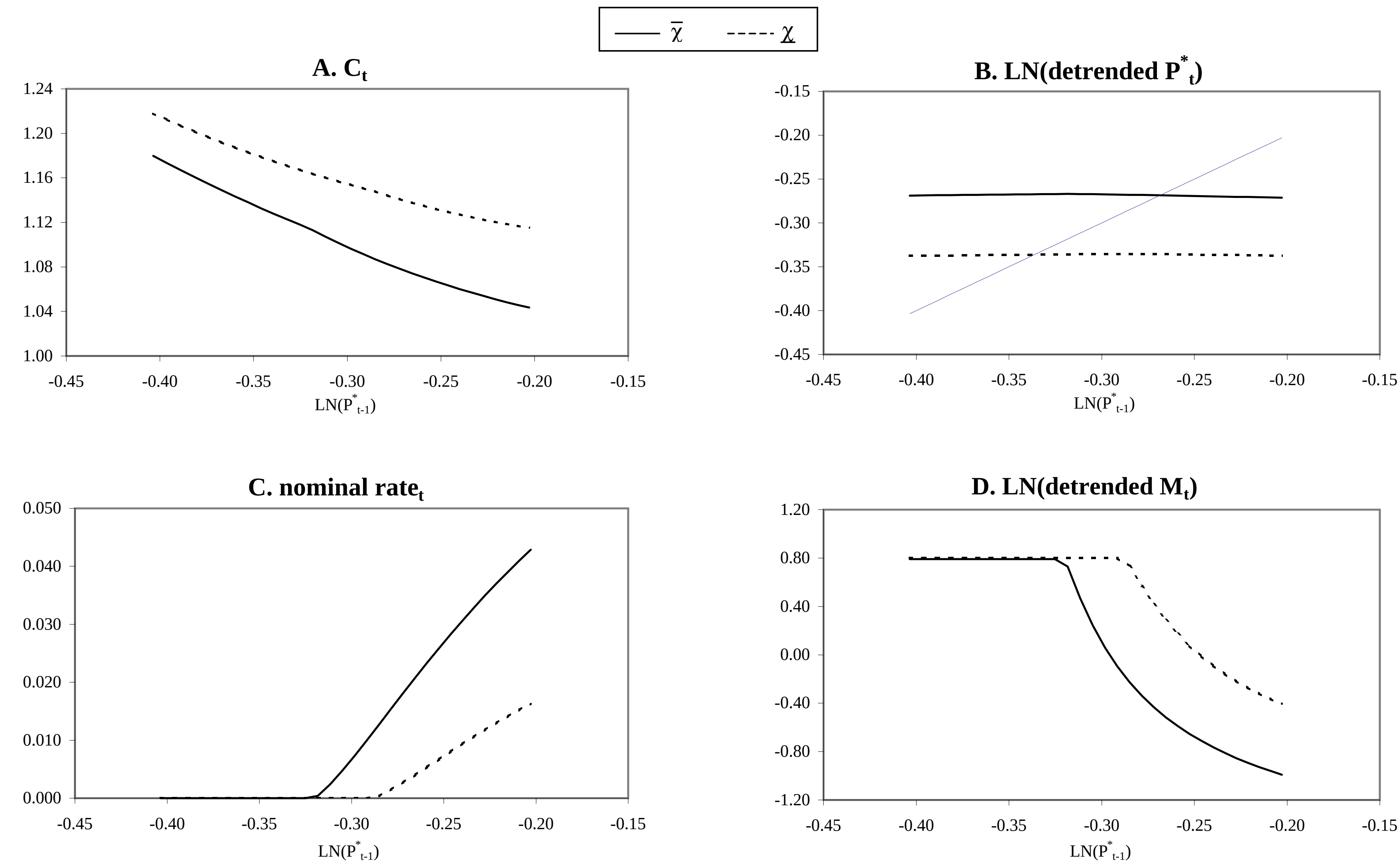


Figure 4.
Time Paths From 30 Period Simulation (5% Deflation Target)

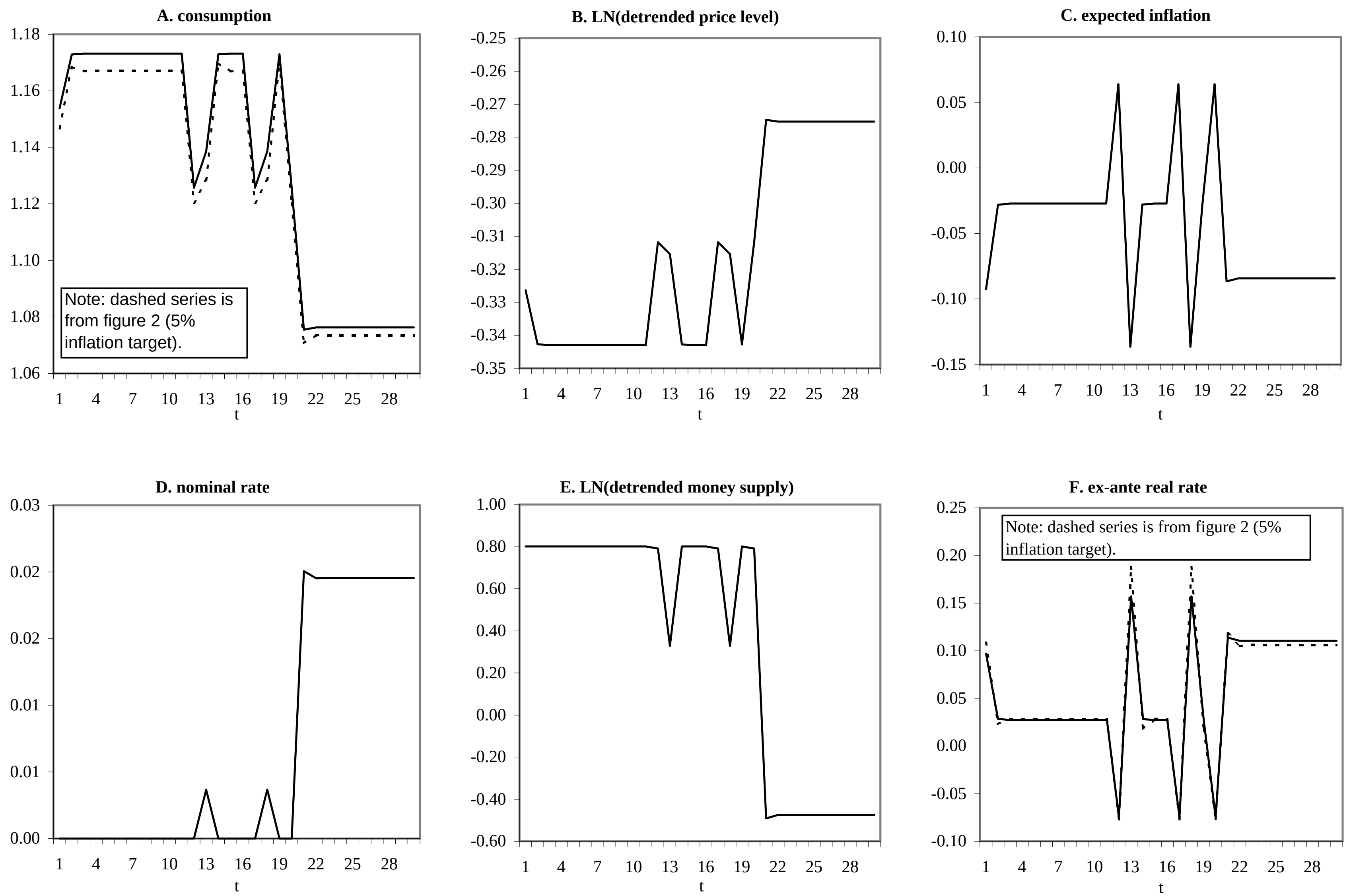
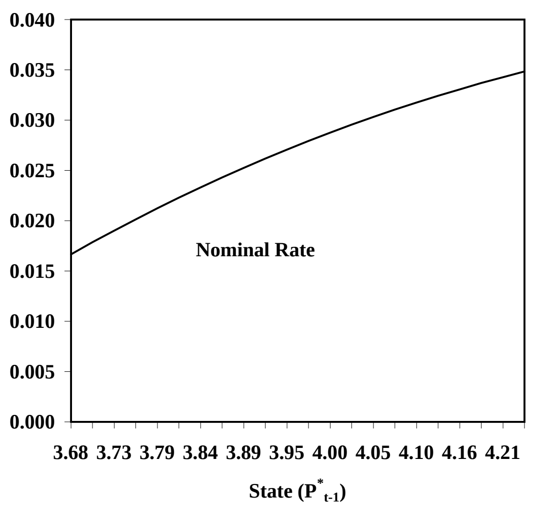
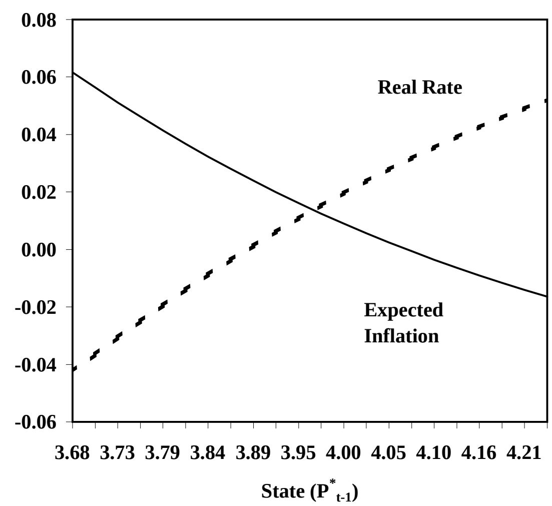


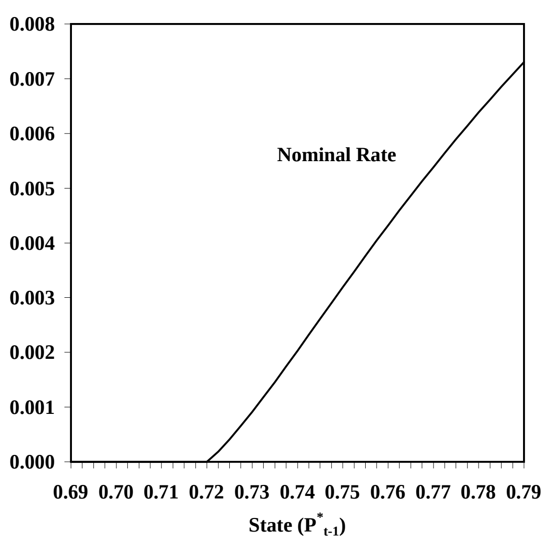
Figure 5.
Optimal Pricing; Price- Level Target

A. High Inflation





B. Low Inflation



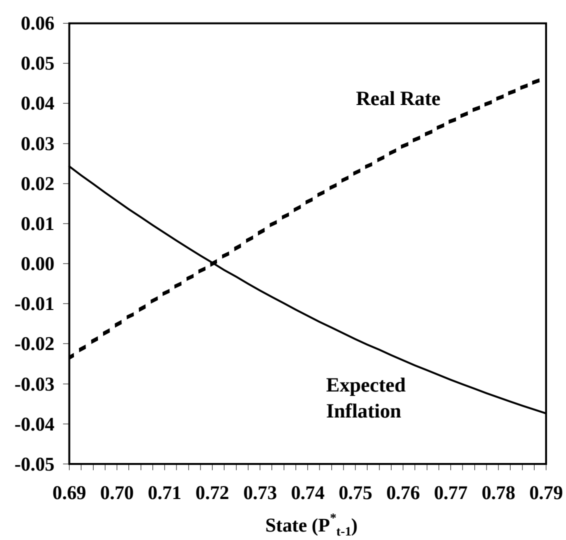
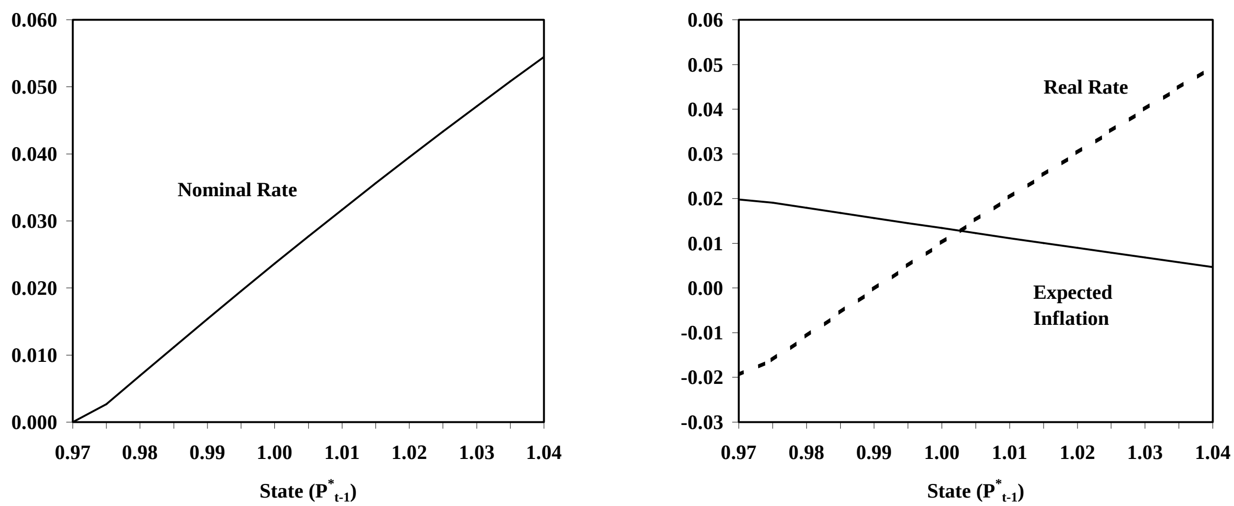


Figure 6.
Optimal Pricing; Inflation Target

A. High Inflation



B. Low Inflation

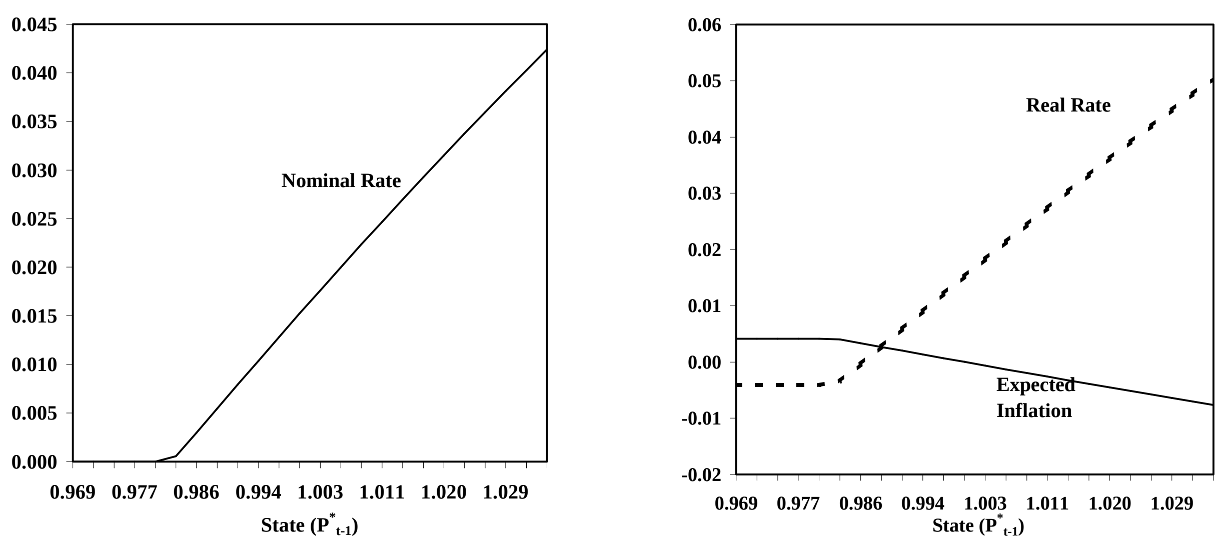
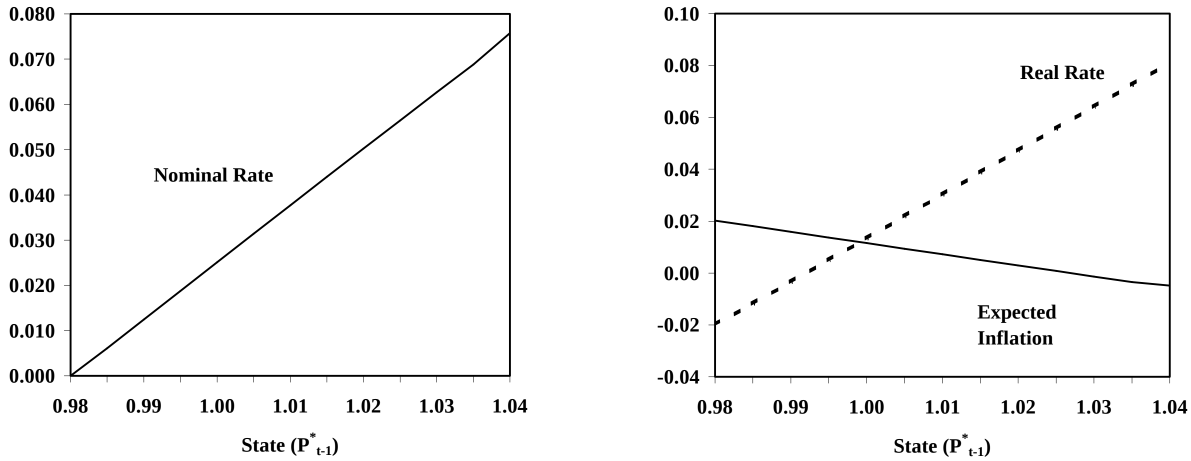


Figure 7.
Fuhrer-Moore Pricing; Price-Level Targeting

A. High Inflation



B. Low Inflation

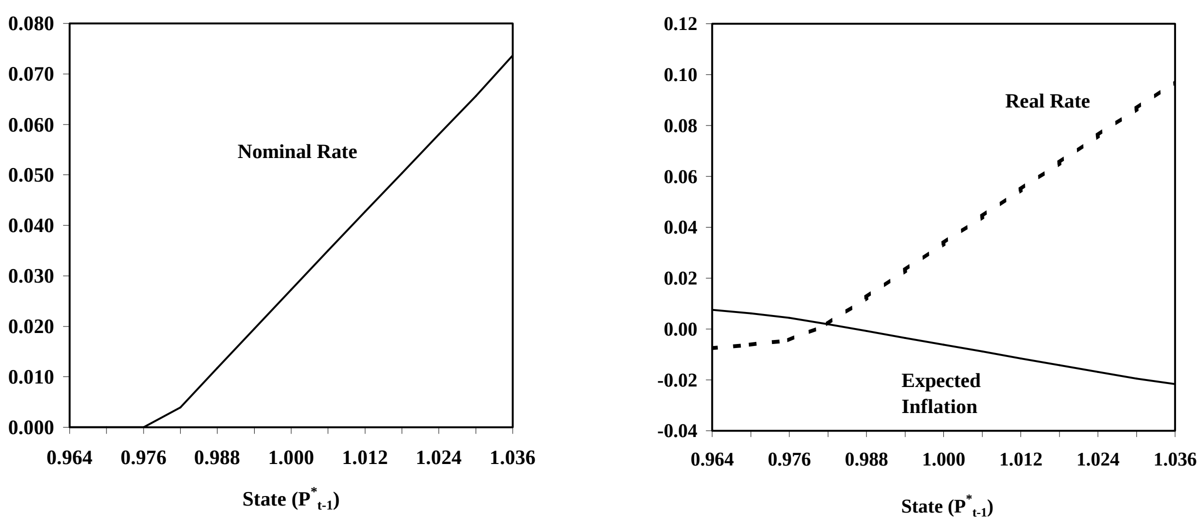
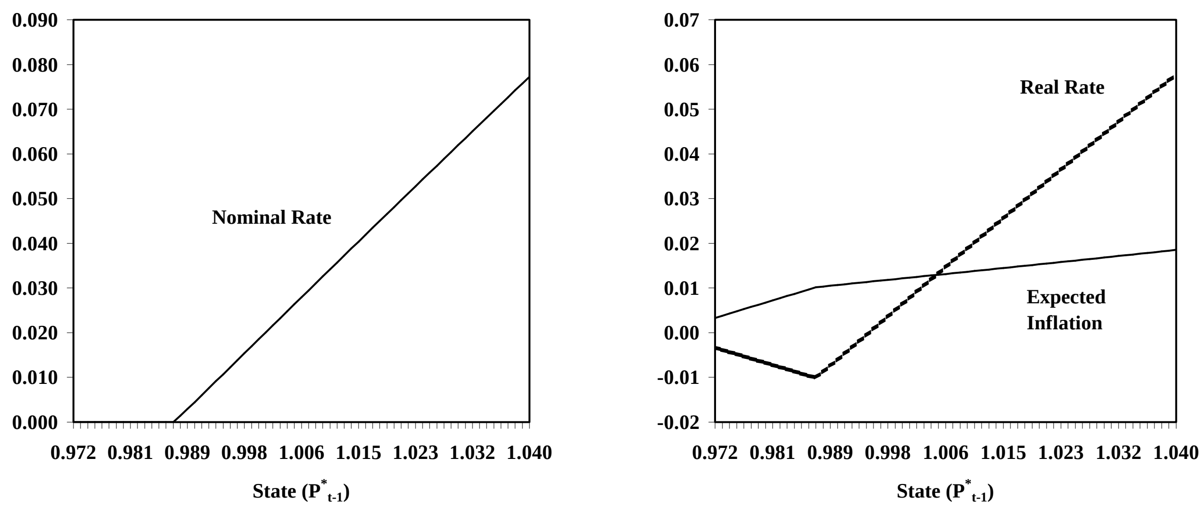


Figure 8.
Fuhrer-Moore Pricing; Inflation Target

A. High Inflation



B. Low Inflation