

EC 313 Spring 2000 Problem Set 2

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Due Thursday 2 March 2000 at classtime

Produce a Mathematica notebook containing the answers to these questions, label it with your name, and email it (as an email attachment) to `baum@bc.edu` to submit your problem set. If you have any questions, please send me email.

1. Consider a profit-maximizing firm that produces units of good x with the technology $f(x) = \sqrt{x}$ and faces costs that are proportional to x . Define the profit function for the firm as a function of x , p (the price of output) and w (the cost per unit of output). For $p=w=1$, plot profits over the unit interval.
2. Analytically solve for the profit-maximizing level of output, as a function of arbitrary levels of p and w , by differentiating the profit function with respect to x (hint: see *D[]*) and solving the resulting first order condition for x (hint: *Solve[]*). The result should be a rule.
3. Substitute the profit-maximizing level of output (call it $xhat$) into the profit function to express profit as a function of p and w . Hint: use the `'/.'` postfix operator.
4. Modify the profit function to contain a fixed cost term, k . Demonstrate that the profit maximizing level of output (per #2) is invariant to the level of k .
5. Now consider a Cobb-Douglas production function where output is related to two factors, x_1 and x_2 , via $f(x) = x_1^{1/4} x_2^{1/4}$. Define the profit function for this firm, where output sells for p and costs are linear in the two factors with factor prices w_1 and w_2 , respectively. Use *ContourPlot* to generate isoprofit contours for $p=20$, $w_1=2$, $w_2=1$ over the range $\{x_1, 0.01, 80\}$, $\{x_2, 0.01, 80\}$.
6. Analytically solve for the profit-maximizing level of output, as a function of arbitrary levels of p , w_1 , and w_2 , by differentiating the profit function with respect to each factor input. The result will be two equations in two unknowns (x_1, x_2), which should then be Solved.
7. Evaluate this solution to calculate the profit-maximizing level of output for $p=20$, $w_1=2$, $w_2=1$, and evaluate the level of profit that will be generated at that output level.