EC821: Time Series Econometrics, Spring 2003

Notes Section 9

Panel Unit Root Tests

A variety of procedures for the analysis of unit roots in a panel context have been developed. The emphasis in this development is the attempt to combine information from the time–series dimension with that obtained from the cross–sectional dimension, in the hope that inference about the existence of unit roots and cointegration can be made more straightforward and precise by taking account of the latter. Given that many interesting relations involve relatively short time–series dimensions, and the well–known low power of conventional unit root tests when applied to a single time series, there may be considerable potential for tests that can be employed in an environment where the time series may be of limited length, but very similar data may be available across a cross–section of countries, regions, firms, or industries. With the increasing availability of quite rich panel data sets in a number of contexts, tests that can be applied to these data would seem very attractive.

However, a variety of issues arise when panel data are employed in testing for unit roots. Some of the tests proposed require a balanced panel (no missing data for any i nor t) whereas others allow for an unbalanced panel setting. In a panel context—that is, with a set of time series—one may form the null hypothesis as a generalization of the standard Dickey–Fuller test, in that all series in the panel are assumed to exhibit nonstationary behavior. This null might be rejected if a fraction of the series in the panel appear to be stationary. Conversely, one could form the null hypothesis of the sort employed in the KPSS test or the Leybourne–McCabe test, in which one presumes that all of the series in the panel are $I(0)$ processes, rejecting when there is sufficient evidence of nonstationarity. In any case, the consideration of a set of timeseries lead to a “box score” concept, wherein one makes an inference on the set of series depending on the predominance of the evidence.

An important theoretical consideration in the development of this literature is the issue of the asymptotic behavior of the panel’s two dimensions, N and T. Various assumptions may be made about the rates at which those parameters tend to infinity. For instance, one may fix N and let T tend to infinity, and subsequently let N tend to infinity. Alternatively, one may allow the two indices to pass to infinity at a controlled rate, such as $T = T(N)$. A third possibility, as expressed in Phillips and Moon’s work, is to allow both indices to tend to infinity simultaneously.

The MADF Test

Sarno and Taylor developed a multivariate analogue to the ADF test (1998, 1998) as an extension of a test developed by Abuaf and Jorion several years earlier. In this test, a single autoregressive parameter is estimated over a panel, by applying Zellner’s SUR estimator to N equations, corresponding to the N units of the panel. Since SUR can only be employed where $T >> N$, the test may only be used where this condition is satisfied. Thus, it is not a suitable test for small-T, large-N

panels, such as those often employed in a cross–country context. Each equation is specified as a $k^{th}-$order autoregression, and the test involves testing the hypothesis that the sum of the coefficients on the autoregressive polynomial is unity. The null hypothesis states that this condition is satisfied over the N equations. Thus, this null will be violated if even one of the series in the panel is stationary. A rejection should thus not be taken to indicate that each of the series in the panel is stationary, but rather an indication that the condition that all series are $I(1)$ does not receive empirical support. Critical values are nonstandard, and have been generated by simulation of a response surface. The Sarno and Taylor paper also present Johansen's likelihood ratio test, which has the null that at least one of the series in the panel is a nonstationary process. The MADF test (Baum, 2001) is available in Stata (version 7 or later) as routine `madfuller` (`findit madfuller`).

The Levin–Lin–Chu Test

One of the first unit root tests to be developed for panel data is that of Levin and Lin, as originally circulated in working paper form in 1992 and 1993. Their work was finally published, with Chu as a coauthor, in 2002. Their test is based on analysis of the equation:

$$\begin{aligned}\Delta y_{i,t} &= \alpha_i + \delta_i t + \theta_t + \rho_i y_{i,t-1} + \varsigma_{i,t}, \\ i &= 1, 2, ..N, t = 1, 2, ...T.\end{aligned}$$

This model allows for two–way fixed effects (α and θ) and unit–specific time trends. The unit–specific fixed effects are an important source of heterogeneity, since the coefficient of the lagged dependent variable is restricted to be homogeneous across all units of the panel. The test involves the null hypothesis $H_0 : \rho_i = 0$ for all i against the alternative $H_A : \rho_i = \rho < 0$ for all i, with auxiliary assumptions under the null also being required about the coefficients relating to the deterministic components. Like most of the unit root tests in the literature, LLC assume that the individual processes are cross–sectionally independent. Given this assumption, they derive conditions (and correction factors) under which the pooled OLS estimate of ρ will have a standard normal distribution under the null hypothesis. Their work focuses on the asymptotic distributions of this pooled panel estimate of ρ under different assumptions on the existence of fixed effects and homogeneous time trends.

The LLC test may be viewed as a pooled Dickey–Fuller (or ADF) test, potentially with differing lag lengths across the units of the panel. Unlike the MADF test, it is applicable to small–T, large–N panels. The LLC test (Bornhorst and Baum, 2001) is available in Stata (version 7 or later) as routine `levinlin` (`findit levinlin`).

The Im–Pesaran–Shin Test

The Im–Pesaran–Shin (IPS, 1997) test extends the LLC framework to allow for heterogeneity in the value of ρ_i under the alternative hypothesis. Given the same equation:

$$\Delta y_{i,t} = \alpha_i + \delta_i t + \theta_t + \rho_i y_{i,t-1} + \varsigma_{i,t},$$
$$i = 1, 2, ..N, t = 1, 2, ...T.$$

The null and alternative hypotheses are defined as:

$H_0 : \rho_i = 0 \, \forall i$

$$H_A\ :\ \rho_i < 0, i = 1, 2, ..., N_1; \rho_i = 0, i = N_1 + 1, N_1 + 2, ...N$$

Thus under the null hypothesis, all series in the panel are nonstationary processes; under the alternative, a fraction of the series in the panel are assumed to be stationary. This is in contrast to the LLC test, which presumes that all series are stationary under the altenative hypothesis. The errors $\varsigma_{i,t}$ are assumed to be serially autocorrelated, with different serial correlation properties and differing variances across units. IPS propose the use of a group–mean Lagrange multiplier statistic to test the null hypothesis. The ADF regressions (perhaps of differing lag lengths) are computed for each unit, and a standardized statistic computed as the average of the LM tests for each equation. Adjustment factors (available in their paper) are used to derive a test statistic that is distributed standard Normal under the null hypothesis. IPS also propose the use of a group–mean $t-$bar statistic, where the t statistics from each ADF test are averaged across the panel; again, adjustment factors are needed to translate the distribution of $t-$bar into a standard Normal variate under the null hypothesis. IPS demonstrate that their test has better finite sample performance than that of LLC.

The IPS test (Bornhorst and Baum, 2001) is available in Stata (version 7 or later) as routine `ipshin (findit ipshin)`.

The Hadri LM Test

The Lagrange multiplier test of Hadri (2000) differs from the other tests in that its null hypothesis is that all series in the panel are stationary. Just as the null of the KPSS test differs from that of Dickey–Fuller style tests in assuming stationarity rather than nonstationarity, Hadri's test generalizes this notion to the panel context. The test statistic is distributed as standard Normal under the null hypothesis. As in the univariate KPSS test, the series may be stationary around a deterministic level (specific to the unit–i.e. a fixed effect) or around a unit–specific deterministic trend. The error process may be assumed to be homoskedastic across the panel, or heteroskedastic across units. Serial dependence in the disturbances may also be taken into account using a Newey–West estimator of the long–run variance. The residual–based test is based on the squared partial sum process of residuals from a demeaning (detrending) model of level (trend) stationarity.

The Hadri LM test (Baum, 2001) is available in Stata (version 7 or later) as routine `hadrilm (findit hadrilm)`.

The Nyblom–Harvey Test of Common Stochastic Trends

Nyblom and Harvey (2000) have developed a number of tests for common stochastic trends. They test the validity of a specific value of the rank of the covariance matrix of the disturbances driving the multivariate random walk, which is equal to the number of common trends in the set of series. As they show, this test is very simple, since it does not require the specification of a model (in contrast to, say, the Johansen approach).

The test may be considered as a generalization of the Nyblom and Makelainen (1983) and KPSS univariate tests for stationarity of a series. Those tests consider the null hypothesis that the series is stationary, or stationary around a deterministic trend, against the alternative that a random walk component is present. The Nyblom–Harvey test considers the same structure in the context of multiple time series. The “random walk with noise” model, for a univariate time series, can be written as:

$$\begin{aligned} y_t &= \mu_t + \epsilon_t \\ \mu_t &= \mu_{t-1} + \eta_t \end{aligned}$$

where each error process is $i.i.d.$ normal with variances σ^2_ϵ and σ^2_η, respectively. If $\sigma^2_\eta = 0$, the random walk becomes a constant level, and y is a (level) stationary series.

The multivariate analog, for a vector of time series processes, may be written in the same notation; now the vector ϵ_t has covariance matrix Σ_ϵ, while the vector η_t has covarance matrix Σ_η. We assume that Σ_ϵ is a positive definite $n \times n$ matrix. The null hypothesis is then $H_0 : \Sigma_\eta = 0$, which a test for whether there is any nonstationarity in the system. Under the condition that there are N independent random walks in the set of timeseries, Σ_η is of full rank (N). The test involves roots of the matrix equation:

$$|\Sigma_\eta - q_i \Sigma_\epsilon| = 0, i = 1, N$$

where $Q = diag(q_1 ... q_{N),}$ $P\Sigma_\eta P' = Q$, and P is defined from the factorization $P\Sigma_\epsilon P' - I$. The null hypothesis involves the test that all of these roots are zero; without any beliefs about relative magnitudes under the alternative, assume $q_1 = q_2 = ... = q_N = q > 0$, corresponding to the “homogeneous” model $\Sigma_\eta = q\Sigma_\epsilon$. The test has critical values, for the constant and constant–trend case, tabulated by the authors, which depend only on the number of series and the hypothesized number of common trends.

One form of the Nyblom–Harvey test (Baum and Bornhorst, 2001) is available in Stata (version 7 or later) as routine `nharvey` (`findit nharvey`). This program considers the special case that the rank of the covariance matrix, k, equals zero: that is, there are no common trends among the variables. In that context, it may be considered a test for cointegration, since in the authors' words, "common trends imply cointegration, and vice versa." Thus, a failure to reject the null hypothesis of zero common trends is also an indication that the variables do not form a cointegrated combination. The test could also be extended to consider the null hypothesis that k takes on a certain value, less than N (the number of series), versus the alternative that it takes on a greater value. Another useful reference in this field is Nyblom and Harvey (2001).

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