

sts15	Tests for stationarity of a time series
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**Abstract:** Implements the Elliott–Rothenberg–Stock (1996) DF-GLS test and the Kwiatkowski–Phillips–Schmidt–Shin (1992) KPSS tests for stationarity of a time series. The DF-GLS test is an improved version of the augmented Dickey–Fuller test. The KPSS test has a null hypothesis of stationarity and may be employed in conjunction with the DF-GLS test to detect long memory (fractional integration).

**Keywords:** stationarity, unit root, time series.

### Syntax

```
dfgls varname [if exp] [in range] [, maxlag(#) notrend ers ]
```

```
kpsss varname [if exp] [in range] [, maxlag(#) notrend ]
```

Both tests are for use with time series data; you must `tsset` your data before using these tests; see [R] `tsset`. `varname` may contain time series operators; see [U] **14.4.3 Time series varlists**.

### Options

`maxlag(#)` specifies the maximum lag order to be considered. The test statistics will be calculated for each lag up to the maximum lag order (which may be zero). If not specified, the maximum lag order for the test is by default calculated from the sample size using a rule provided by Schwert (1989) using  $c = 12$  and  $d = 4$  in his terminology. Whether the maximum lag is explicitly specified or computed by default, the sample size is held constant over lags at the maximum available sample.

`notrend` specifies that no trend term should be included in the model. The critical values reported differ in the absence of a trend term.

`ERS` (`dfgls` only) specifies that the ERS (and Dickey–Fuller) values are to be used for all levels of significance (eschewing the response surface estimates).

### Description

`dfgls` performs the Elliott–Rothenberg–Stock (ERS, 1996) efficient test for an autoregressive unit root. This test is similar to an (augmented) Dickey–Fuller  $t$  test, as performed by `dfuller`, but has the best overall performance in terms of small sample size and power, dominating the ordinary Dickey–Fuller test. The `dfgls` test “has substantially improved power when an unknown mean or trend is present” (ERS, 813).

`dfgls` applies a generalized least squares (GLS) detrending (demeaning) step to the `varname`

$$y_t^d = y_t - \hat{\beta}' z_t$$

For detrending,  $z_t = (1, t)'$  and  $\hat{\beta}_0, \hat{\beta}_1$  are calculated by regressing

$$[y_1, (1 - \bar{\alpha}L)y_2, \dots, (1 - \bar{\alpha}L)y_T]$$

onto

$$[z_1, (1 - \bar{\alpha}L)z_2, \dots, (1 - \bar{\alpha}L)z_T]$$

where  $\bar{\alpha} = 1 + \bar{c}/T$  with  $\bar{c} = -13.5$ , and  $L$  is the lag operator. For demeaning,  $z_t = (1)'$  and the same regression is run with  $\bar{c} = -7.0$ . The values of  $\bar{c}$  are chosen so that “the test achieves the power envelope against stationary alternatives (is asymptotically MPI (most powerful invariant)) at 50 percent power” (Stock 1994, 2769; emphasis added). The augmented Dickey–Fuller regression is then computed using the  $y_t^d$  series

$$\Delta y_t^d = \alpha + \gamma t + \rho y_{t-1}^d + \sum_{i=1}^m \delta_i \Delta y_{t-i}^d + \epsilon_t$$

where  $m = \text{maxlag}$ . The `notrend` option suppresses the time trend in this regression.

Approximate 5% and 10% critical values, by default, are calculated from the response surface estimates of Table 1, Cheung and Lai (1995, 413), which take both the sample size and the lag specification into account. Approximate 1% critical values for

the GLS detrended test are interpolated from Table 1 of ERS (page 825). Approximate 1% critical values for the GLS demeaned test are identical to those applicable to the no-constant, no-trend Dickey–Fuller test and are computed using the `dfuller` code. The ERS option specifies that the ERS (and Dickey–Fuller) values are to be used for all levels of significance (eschewing the response surface estimates).

If the maximum lag order exceeds one, the optimal lag order is calculated by the Ng and Perron (1995) sequential  $t$  test on the highest order lag coefficient, stopping when that coefficient's  $p$ -value is less than 0.10. The lag minimizing the Schwarz criterion (SC, or BIC) is printed with its minimized value.

`kpss` performs the Kwiatkowski–Phillips–Schmidt–Shin test introduced in Kwiatkowski et al. (1992) for stationarity of a time series. This test differs from those in common use (such as `dfuller` and `pperron`) by having a null hypothesis of stationarity. The test may be conducted under the null hypothesis of either trend stationarity (the default) or level stationarity. Inference from this test is complementary to that derived from those based on the Dickey–Fuller distribution (such as `dfgls`, `dfuller` and `pperron`). The KPSS test is often used in conjunction with those tests to investigate the possibility that a series is fractionally integrated; that is, neither  $I(1)$  nor  $I(0)$ ; see Lee and Schmidt (1996).

The series is detrended (demeaned) by regressing  $y$  on  $z_t = (1, t)'$  ( $z_t = (1)'$ ), yielding residuals  $e_t$ . Let the partial sum series of  $e_t$  be  $s_t$ . Then the zero-order KPSS statistic  $k_0 = T^{-2} \sum_{t=1}^T s_t^2 / T^{-1} \sum_{t=1}^T e_t^2$ . For `maxlag` > 0, the denominator is computed as the Newey–West estimate of the long run variance of the series; see [R] `newey`.

Approximate critical values for the KPSS test are taken from Kwiatkowski et al. (1992).

## Examples

Data from Terence Mills' *Econometric Analysis of Financial Time Series* on the UK FTA All Share Index of stock prices (`ftap`) and stock returns (`ftaret`) are analyzed.

```
. use http://fmwww.bc.edu/ec-p/data/Mills2d/fta.dta
. tsset
      time variable: month, 1965m1 to 1995m12
. dfgls ftap
Number of obs =   355
Maxlag = 16 chosen by Schwert criterion
```

	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
DF-GLS (tau) [16]	-0.068	-3.480	-2.818	-2.536
DF-GLS (tau) [15]	-0.155	-3.480	-2.824	-2.542
DF-GLS (tau) [14]	-0.046	-3.480	-2.829	-2.547
DF-GLS (tau) [13]	-0.234	-3.480	-2.835	-2.552
DF-GLS (tau) [12]	-0.131	-3.480	-2.840	-2.557
DF-GLS (tau) [11]	-0.196	-3.480	-2.846	-2.562
DF-GLS (tau) [10]	-0.251	-3.480	-2.851	-2.566
DF-GLS (tau) [9]	-0.173	-3.480	-2.856	-2.571
DF-GLS (tau) [8]	-0.107	-3.480	-2.861	-2.575
DF-GLS (tau) [7]	-0.361	-3.480	-2.865	-2.580
DF-GLS (tau) [6]	-0.391	-3.480	-2.870	-2.584
DF-GLS (tau) [5]	-0.476	-3.480	-2.874	-2.588
DF-GLS (tau) [4]	-0.524	-3.480	-2.879	-2.592
DF-GLS (tau) [3]	-0.484	-3.480	-2.883	-2.595
DF-GLS (tau) [2]	-0.507	-3.480	-2.887	-2.599
DF-GLS (tau) [1]	-0.789	-3.480	-2.891	-2.602

```
Opt Lag (Ng-Perron sequential t) = 15 with RMSE  35.59803
Min SC =  7.275482 at lag  2 with RMSE   37.0745
. kpss ftap
KPSS test for ftap
Maxlag = 16 chosen by Schwert criterion
Critical values for H0: ftap is trend stationary
10%: 0.119  5% : 0.146  2.5%: 0.176  1% : 0.216
Lag order  Test statistic
  0         7.90141
  1         4.18402
  2         2.86036
  3         2.18027
  4         1.76579
```

```

5      1.48676
6      1.2861
7      1.13475
8      1.01642
9      .921225
10     .84288
11     .777242
12     .721428
13     .673349
14     .631492
15     .594708
16     .562121

. dfgls ftaret
Number of obs = 355
Maxlag = 16 chosen by Schwert criterion

      Test          1% Critical    5% Critical    10% Critical
      Statistic      Value          Value          Value
-----
DF-GLS(tau) [16]   -4.161         -3.480         -2.818         -2.536
DF-GLS(tau) [15]   -4.119         -3.480         -2.824         -2.542
DF-GLS(tau) [14]   -4.413         -3.480         -2.829         -2.547
DF-GLS(tau) [13]   -4.733         -3.480         -2.835         -2.552
DF-GLS(tau) [12]   -4.663         -3.480         -2.840         -2.557
DF-GLS(tau) [11]   -4.392         -3.480         -2.846         -2.562
DF-GLS(tau) [10]   -4.653         -3.480         -2.851         -2.566
DF-GLS(tau) [9]    -4.795         -3.480         -2.856         -2.571
DF-GLS(tau) [8]    -4.931         -3.480         -2.861         -2.575
DF-GLS(tau) [7]    -6.006         -3.480         -2.865         -2.580
DF-GLS(tau) [6]    -6.203         -3.480         -2.870         -2.584
DF-GLS(tau) [5]    -6.911         -3.480         -2.874         -2.588
DF-GLS(tau) [4]    -7.614         -3.480         -2.879         -2.592
DF-GLS(tau) [3]    -7.769         -3.480         -2.883         -2.595
DF-GLS(tau) [2]    -9.176         -3.480         -2.887         -2.599
DF-GLS(tau) [1]   -13.075        -3.480         -2.891         -2.602

Opt Lag (Ng-Perron sequential t) = 8 with RMSE .0593867
Min SC = -5.566828 at lag 2 with RMSE .0603119

. dfgls ftaret,notrend
Number of obs = 355
Maxlag = 16 chosen by Schwert criterion

      Test          1% Critical    5% Critical    10% Critical
      Statistic      Value          Value          Value
-----
DF-GLS(mu) [16]    -3.165         -2.580         -1.952         -1.637
DF-GLS(mu) [15]    -3.161         -2.580         -1.955         -1.640
DF-GLS(mu) [14]    -3.430         -2.580         -1.958         -1.643
DF-GLS(mu) [13]    -3.725         -2.580         -1.962         -1.646
DF-GLS(mu) [12]    -3.711         -2.580         -1.965         -1.649
DF-GLS(mu) [11]    -3.528         -2.580         -1.968         -1.652
DF-GLS(mu) [10]    -3.776         -2.580         -1.971         -1.655
DF-GLS(mu) [9]     -3.933         -2.580         -1.974         -1.658
DF-GLS(mu) [8]     -4.087         -2.580         -1.977         -1.660
DF-GLS(mu) [7]     -5.039         -2.580         -1.980         -1.663
DF-GLS(mu) [6]     -5.278         -2.580         -1.982         -1.665
DF-GLS(mu) [5]     -5.966         -2.580         -1.985         -1.668
DF-GLS(mu) [4]     -6.679         -2.580         -1.988         -1.670
DF-GLS(mu) [3]     -6.928         -2.580         -1.990         -1.672
DF-GLS(mu) [2]     -8.312         -2.580         -1.993         -1.675
DF-GLS(mu) [1]    -12.060         -2.580         -1.995         -1.677

Opt Lag (Ng-Perron sequential t) = 8 with RMSE .0600067
Min SC = -5.53158 at lag 2 with RMSE .0613843

```

Both tests indicate that `ftap` appears to be nonstationary. `ftaret` appears to be both trend and level stationary.

## Saved Results

`dfgls` saves the following scalars in `r()`:

<code>r(N)</code>	number of observations
<code>r(optlag)</code>	optimal lag order
<code>r(sc<math>n</math>)</code>	Schwarz criterion at lag $n$
<code>r(rmse<math>n</math>)</code>	root mean square error at lag $n$
<code>r(dft<math>n</math>)</code>	DF-GLS statistic at lag $n$

`kpss` saves the following scalars in `r()`:

<code>r(N)</code>	number of observations
<code>r(dft<math>n</math>)</code>	KPSS statistic at lag $n$

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## References

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sts16	Tests for long memory in a time series
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**Abstract:** Implements the Geweke/Porter-Hudak log periodogram estimator (1983), the Phillips modified log periodogram estimator (1999b) and the Robinson log periodogram estimator (1995) for the diagnosis of long memory, or fractional integration, in a time series. The Robinson estimator may be applied to a set of time series.

**Keywords:** fractional integration, long memory, stationarity, time series.

## Syntax

```
gphudak varname [if exp] [in range] [, powers(numlist) ]
```

```
modlpr varname [if exp] [in range] [, powers(numlist) notrend ]
```

```
roblpr varlist [if exp] [in range] [, powers(numlist) l(#) j(#) constraints(numlist) ]
```

These tests are for use with time series data; you must `tsset` your data before using these tests; see [R] `tsset`. `varname` or `varlist` may contain time series operators; see [U] **14.4.3 Time-series varlists**.

## Options

`powers(numlist)` indirectly specifies the number of ordinates to be included in the regression. A number of ordinates equal to the integer part of  $T$  raised to the `powers(numlist)` will be used. Powers ranging from 0.50 to 0.75 are commonly employed for `gphudak` and `modlpr`. These routines use the default power of 0.5. `roblpr` uses the default power of 0.9. For `roblpr`, multiple powers may only be specified if a single variable appears in `varlist`.

`notrend` specifies that detrending is not to be applied by `modlpr`. By default, a linear trend will be removed from the series.