BOSTON COLLEGE
Department of Economics

EC 771
Econometrics
Spring 2004
Prof. Baum

Midterm Exam
9 March 2004

Answer all questions. Total of 160 points. Partial credit given for partial answers.

1. (35 pts) Briefly explain each term. Use examples to illustrate your explanation.
a. constant elasticity model
b. bootstrap standard error
c. outer product of gradient estimator
d. likelihood ratio test
e. strict exogeneity
f. adjusted R-squared
g. spherical disturbances

2. (30 pts) Suppose our model of y is $y = \mu + v$, and we have a sample of size N.
a. Write out the least squares (OLS) criterion for the estimation of this model, and derive the least squares estimator of $\hat{\mu}$ as that value minimizing the criterion.
b. Demonstrate that your estimator satisfies the Gauss–Markov theorem.
c. An alternative estimator for $\hat{\mu}$ is defined as $m = 0.5(y_1 + y_N)$. Is this estimator unbiased? consistent? efficient?

3. (20 pts) Consider the OLS regression of y on k regressors contained in X (which includes a column ι). Consider an alternative set of regressors $Z = XP$ where P is a nonsingular matrix. Prove that the residual vectors in the regressions of y on X and y on Z are identical. Discuss the implications of your findings for the assertion that one cannot affect the fit of a regression by changing the units of measurement of the regressors.

4. (20 pts) Suppose that the regression model is $y = \mu + \epsilon$, where $E[\epsilon_i | x_i] = 0$, $Cov[\epsilon_i, \epsilon_j | x_i, x_j] = 0 \; \forall i \neq j$, but $Var[\epsilon_i | x_i] = \sigma^2 x_i^2, x_i > 0$.

Given a sample of observations $\{y_i, x_i\}$, what is the most efficient estimator of μ? What is the OLS estimator of μ? Do they coincide?

5. (25 pts) Suppose that we want to estimate the k-variable linear regression model $y = Xb + u$ subject to a set of linear restrictions which may be expressed as $Rb = q$.

a. Matrix R has j rows. What restrictions must be placed on j? Upon the elements of R? Why?

b. The problem may be expressed as the Lagrangean

$$L = (y - Xb)'(y - Xb) - \lambda(Rb - q)$$

Derive the vector λ.

c. Derive the estimator of b in terms of the unrestricted vector b_{OLS}.

d. What is the intuition for the relation between b and b_{OLS} in your solution?

6. (15 pts) Write a short essay discussing the advantages and disadvantages of using maximum likelihood estimation.

7. (15 pts) Write a short essay on the issue of near–perfect collinearity in multiple regression. Discuss the nature of the problem, its consequences, and how it might be detected.