EC 327 Financial Econometrics
Problem Set 3: Solutions

Prof. Baum, Mr. Yurtseven

April 3, 2008

Exercise 11.2

(i) $E(x_t) = E(e_t)(1/2)E(e_{t-1}) + (1/2)E(e_{t-2}) = 0$ for t = 1,2,...Also, because the e_t are independent, they are uncorrelated and so $Var(x_t) = Var(e_t) + (1/4)Var(e_{t-1}) + (1/4)Var(e_{t-2}) = 1 + (1/4) + (1/4) = 3/2$ because $Var(e_t) = 1$ for all t.
(ii) Because x_t has zero mean:

$$Cov(x_t, x_{t+1}) = E(x_t x_{t+1}) = E[(e_t - (1/2)e_{t-1} + (1/2)e_{t-2})(e_{t+1} - (1/2)e_t + (1/2)e_{t-1})] =$$

$$= E(e_t e_{t+1}) - (1/2)E(e_t^2) + (1/2)E(e_t e_{t-1}) - (1/2)E(e_{t-1}e_{t+1}) + (1/4)E(e_{t-1}e_t) - (1/4)E(e_{t-1}^2) +$$

$$+ (1/2)E(e_{t-2}e_{t+1}) - (1/4)E(e_{t-2}e_t) + (1/4)E(e_{t-2}e_{t-1}) = -(1/2)E(e_t^2) - (1/4)E(e_{t-1}^2) = -3/4$$

The third to last equality follows because the e_t are pairwise uncorrelated and $E(e_t^2)$ =1 for all t. Thus:

$$Corr(x_t, x_{t+1}) = -(3/4)/(3/2) = 1/2.$$

Computing $Cov(x_t, x_{t+2})$ is even easier because only one of the nine terms has expectation different from zero: $(1/2)E(e_t^2) = \frac{1}{2}$. Therefore, $Corr(x_t, x_{t+2}) = (1/2)/(3/2) = 1/3$.
(iii) $Corr(x_t, x_{t+h}) = 0$ for $h > 2$ because, for $h > 2$, x_{t+h} depends at most on e_{t+j} for $j > 0$, while x_t depends on e_{t+j}, $j \leqslant 0$.
(iv) Yes, because terms more than two periods apart are actually uncorrelated, and so it is obvious that $Corr(x_t, x_{t+h}) = 0$ as h tends to ∞.

Exercise 11.6

The t statistic for $H_0 : \beta_1 = 1$ is $t = (1.1041)/.039 \approx 2.67$. Although we must rely on asymptotic results, we might as well use $df = 120$. So the 1 per cent critical value against a two-sided alternative is about 2.62, and so we reject $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 \neq 1$ at the 1 per cent level. It is hard to know whether the estimate is practically different from one

without comparing investment strategies based on the theory ($\beta_1 = 1$) and the estimate ($\beta_1 = 1.104$). But the estimate is 10 per cent higher than the theoretical value.
(ii) The t statistic for the null in part (i) is now $(1.0531)/.039 \approx 1.36$, so $H_0 : \beta_1 = 1$ is no longer rejected against a two-sided alternative unless we are using more than a 10 per cent significance level. But the lagged spread is very significant (contrary to what the expectations hypothesis predicts): $t = .480/.109 \approx 4.40$. Based on the estimated equation, when the lagged spread is positive, the predicted holding yield on six-month T-bills is above the yield on three-month Tbills (even if we impose $\beta_1 = 1$), and so we should invest in six-month T-bills.
(iii) This suggests unit root behavior for $hy3_t$, which generally invalidates the usual ttesting procedure.
(iv) With dummy variables for seasons

Exercise C11.2

The estimated equation is

$$\hat{ghrwage_t} = -0.010 + .728 goutphr_t + .458 goutphr_{t-1}$$

$$n = 39, R^2 = .493.$$

The t statistic on the lag is about 2.76, so the lag is very significant.
(ii) We follow the hint and write the LRP as $\theta = \beta_1 + \beta_2$, and then plug $\beta_1 = \theta - \beta_2$ into the original model

$$ghrwage_t = \beta_0 + \theta goutphr_t + \beta_2(goutphr_{t-1} - goutphrt_t) + u_t.$$

Therefore, we regress $ghrwage_t$ onto $goutphr_t$, and $(goutphr_{t-1} - goutphr_t)$ and obtain the standard error for θ. Doing this regression gives 1.186 (as we can compute directly from part (i)) and $se(\theta) = .203$. The t statistic for testing $H_0 : \theta = 1$ is $(1.1861)/.203 \approx .916$, which is not significant at the usual significance levels (not even 20 per cent against a two-sided alternative).
(iii) When $goutphr_{t-2}$ is added to the regression from part (i), and we use the 38 observations now available for the regression, $\hat{\beta}_3 \approx .065$ with a t statistic of about .41. Therefore, $goutphr_{t-2}$ need not be in the model.

Exercise C11.6

The estimated accelerator model is

$$hatDeltainven_t = 2.59 + .152 \Delta GDP_t$$

$$n = 36, R^2 = .554.$$

$\hat{\beta}_1$ is very statistically significant, with $t =\approx 6.61$.
(ii) When we add $r3_t$, we obtain

$$\Delta inven_t = 3.00 + .159\Delta GDP_t - .895r3_t$$

$$n = 36, R^2 = .562.$$

The sign of $\hat{\beta}_2$ is negative, as predicted by economic theory, and it seems practically large. However, $\hat{\beta}_2$ is not statistically different from zero. (Its t statistic is less than one in absolute value.)
If $\Delta r3_t$ is used instead, the coefficient becomes about -.470, $se = 1.540$. So this is even less significant than when $r3_t$ is in the equation. But, without more data, we cannot conclude that interest rates have a ceteris paribus effect on inventory investment.

Exercise C11.7

(i) If $E(gc_t|I_{t-1}) = E(gc_t)$ - that is, $E(gc_t|I_{t-1})$ - does not depend on gc_{t-1}, then $\beta_1 = 0$ in $gc_t = \beta_0 + \beta_1 gc_{t-1} + u_t$. So the null hypothesis is $H_0 : \beta_1 = 0$ and the alternative is $H_1 : \beta_1 \neq 0$. Estimating the simple regression gives

$$gc_t = .011 + .446gc_{t-1}$$

$$n = 35, R^2 = .199.$$

The t statistic for β_1 is about 2.86, and so we strongly reject the PIH. The coefficient on gc_{t-1} is also practically large, showing significant autocorrelation in consumption growth.

(ii) When gy_{t-1} and $i3_{t-1}$ are added to the regression, the R-squared becomes about .288.
The F statistic for joint significance of gy_{t-1} and $i3_{t-1}$, obtained using the Stata test command, is 1.95, with p-value .16. Therefore, gy_{t-1} and $i3_{t-1}$ are not jointly significant at even the 15% level.

Exercise C11.8

(i) The estimated AR(1) model is:

$$unem_t = 1.57 + .732unem_{t-1}$$

In 2003 the unemployment rate was 5.9, so the predicted unemployment rate is ≈ 5.89. From the 2004 Economic Report of the President, the U.S. civilian unemployment rate was

5.4. Therefore, the equation overpredicts the 2004 unemployment rate.
(ii) When we add inf_{t-1} to the equation we get:

$$unem_t = 1.30 + .647unem_{t-1} + .183inf_{t-1}$$

Lagged inflation is very statistically significant, with a t statistic of about 4.7.
(iii) To use the equation from part (ii) to predict unemployment in 2004, we also need the inflation rate for 2003. Therefore, the prediction of unemployment in 2004 is 1.30 + .647(5.9) + .184(2)$\approx$ 5.48. While still large, it is pretty close to the actual rate of 5.4 percent, and it is certainly better than the predication from part (i).
(iv) The confidence interval is (5.263, 5.819).