

PROBLEM SET 2: SOLUTIONS

Problem 1 (20 points) We can write the system as $Z = XA$, where

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -3 \\ 0 & 4 & 5 \end{pmatrix}$$

You can solve for its inverse as

$$A^{-1} = \begin{pmatrix} 17 & 8 & -2 \\ 10 & 5 & -1 \\ -8 & -4 & 1 \end{pmatrix}$$

To show that the fitted values and the residuals are the same in the two regressions it is sufficient to show that the projection matrices P_X and P_{XA} are the same:

$$P_{XA} = (XA)[A'X'XA]^{-1}(XA)' = XA(A^{-1}(X'X)^{-1}A'^{-1}A'X') = X(X'X)^{-1}X' = P_X$$

The fitted values of the two regressions are $Xb = P_X y = P_{XA} y = XAa$, which implies that $a = A^{-1}(X'X)^{-1}X'y$ and $b = (X'X)^{-1}X'y$. Therefore,

$$a = A^{-1}b \leftrightarrow b = Aa$$

This shows that the estimate $b_1 = a_1 + 2a_3$.

Problem 2 (10 points)

use <http://fmwww.bc.edu/ec-p/data/greene2008/tbrate>

```
. regress D.r L.pi LD.y LD.r L2D.r
```

Source	SS	df	MS	Number of obs =	185
Model	22.1971507	4	5.54928768	F(4, 180) =	6.99
Residual	142.934504	180	.794080577	Prob > F =	0.0000
				R-squared =	0.1344
				Adj R-squared =	0.1152

Total | 165.131655 184 .897454645 Root MSE = .89111

D.r	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
pi						
L1.	.0160647	.0200335	0.80	0.424	-.023466	.0555955
y						
LD.	18.38055	5.758924	3.19	0.002	7.016859	29.74423
r						
LD.	.2374557	.0740703	3.21	0.002	.0912979	.3836135
L2D.	-.1540175	.0725383	-2.12	0.035	-.2971523	-.0108828
_cons	-.2319403	.1256143	-1.85	0.066	-.4798063	.0159256

. predict rhat
(option xb assumed; fitted values)
(3 missing values generated)

. predict uhat, residuals
(3 missing values generated)

. twoway (connected rhat yq, msize(vsmall)) (line uhat yq)

. reg uhat rhat

Source	SS	df	MS	Number of obs = 185		
Model	5.6843e-14	1	5.6843e-14	F(1, 183)	=	0.00
Residual	142.934505	183	.781062871	Prob > F	=	1.0000
Total	142.934505	184	.776817964	R-squared	=	0.0000
				Adj R-squared	=	-0.0055
				Root MSE	=	.88378

uhat	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
rhat	7.53e-09	.1875834	0.00	1.000	-.3701043	.3701043
_cons	1.93e-10	.0650252	0.00	1.000	-.1282955	.1282955

Since the residuals are by construction orthogonal to the fitted values, we verify via the above OLS regression that the mean of the residuals is zero and that the fitted values are uncorrelated with the residuals.

```
reg rhat uhat
```

Source	SS	df	MS			
Model	0	1	0	Number of obs =	185	
Residual	22.1971507	183	.121295905	F(1, 183) =	0.00	
Total	22.1971507	184	.120636688	Prob > F =	1.0000	
				R-squared =	0.0000	
				Adj R-squared =	-0.0055	
				Root MSE =	.34828	

rhat	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
uhat	1.17e-09	.0291309	0.00	1.000	-.0574757	.0574757
_cons	.0133946	.0256057	0.52	0.602	-.0371258	.063915

As explained earlier, the residuals and the fitted values are uncorrelated, which is borne out by the above OLS regression results. Additionally, we obtain a non-zero constant that is the mean of the fitted values, which is also the mean of the dependent variable in the original regression, since the residuals are constructed to be mean-zero.

Problem 3 (10 points)

```
regress D.r LD.y LD.r L2D.r
```

Source	SS	df	MS			
Model	21.6865324	3	7.22884414	Number of obs =	185	
Residual	143.445122	181	.792514488	F(3, 181) =	9.12	
Total	165.131655	184	.897454645	Prob > F =	0.0000	
				R-squared =	0.1313	
				Adj R-squared =	0.1169	
				Root MSE =	.89023	

D.r	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
LD.	17.47536	5.641639	3.10	0.002	6.343518	28.6072
r						
LD.	.2437294	.0735833	3.31	0.001	.0985381	.3889208
L2D.	-.1471644	.071962	-2.05	0.042	-.2891568	-.0051721
_cons	-.158009	.0852318	-1.85	0.065	-.3261847	.0101668

. predict ehat, residuals
(3 missing values generated)

. regress L.pi LD.y LD.r L2D.r

Source	SS	df	MS	Number of obs = 185	
Model	127.410931	3	42.4703102	F(3, 181) =	3.89
Residual	1978.56248	181	10.9312844	Prob > F =	0.0101
Total	2105.97341	184	11.4455077	R-squared =	0.0605
				Adj R-squared =	0.0449
				Root MSE =	3.3062

L.pi	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
LD.	-56.34641	20.95257	-2.69	0.008	-97.68912	-15.00371
r						
LD.	.3905278	.2732821	1.43	0.155	-.1487006	.9297562
L2D.	.4265952	.2672608	1.60	0.112	-.1007524	.9539427
_cons	4.602094	.3165437	14.54	0.000	3.977503	5.226684

. predict vhat, residuals
(3 missing values generated)

. regress ehat vhat

Source	SS	df	MS			
Model	.510618184	1	.510618184	Number of obs =	185	
Residual	142.934503	183	.781062856	F(1, 183) =	0.65	
Total	143.445121	184	.779593048	Prob > F =	0.4198	
				R-squared =	0.0036	
				Adj R-squared =	-0.0019	
				Root MSE =	.88378	

ehat	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
vhat	.0160647	.0198686	0.81	0.420	-.0231363	.0552658
_cons	4.07e-11	.0649766	0.00	1.000	-.1281996	.1281996

The value of the coefficient on vhat is the same as that of π_{t-1} in the original regression in the previous problem, a consequence of the Frisch-Waugh Theorem (also known as the Frisch-Waugh-Lovell Theorem). See Greene 3.3 for details.

Problem 4 (10 points)

The covariance matrix is positive semidefinite, hence its determinant is nonnegative:

$$\begin{aligned} \text{Var}(b_1)\text{Var}(b_2) - \text{Cov}(b_1, b_2)^2 &\geq 0 \\ \Rightarrow \rho_{b_1, b_2}^2 &= \frac{\text{Cov}(b_1, b_2)^2}{\text{Var}(b_1)\text{Var}(b_2)} \leq 1 \\ \Rightarrow -1 &\leq \rho_{b_1, b_2} \leq 1 \end{aligned}$$

Problem 5 (10 points)

We can estimate the model by enforcing the constraint $\beta_3 = 1 - \beta_2$. Thus, the restricted model can be estimated as

$$(y - x_{t3}) = b_1 + b_2(x_{tx} - x_{t3}) + u_t$$

We obtain the estimates for β_1 and β_2 directly and then use the constraint to find β_3 . For the second part we can estimate the model instead by running:

$$(y - x_{t3}) = b_2(x_{tx} - x_{t3}) + (b_3 + b_2 - 1)x_{t3} + u_t$$

where the coefficient on x_{t3} will be 0 if the restriction held exactly in the data.

Problem 6 (10 points)

(a) Let A be the original T by m matrix of quarterly observations. Form the "averaging" matrix

$$B = I \otimes \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right)$$

where I is the mxm identity matrix. Then the desired matrix of annual averages will be A*B.

(b)

```
bigmat = runiform(24,6)
: eye = I(rows(bigmat) / 4)
: bee = J(1, 4, 1/4)
: transmat = eye # bee
: reduced = transmat * bigmat
```

reduced	1	2	3	4	5
1	.3611000053	.6375933131	.4779701745	.6089173277	.3895759421
2	.6638042688	.3569173898	.4979834579	.3683052972	.5017640498
3	.449947984	.5332581371	.2735498206	.6168799133	.3844898993
4	.3874929606	.4261613118	.5332233117	.5727609355	.4038432156
5	.4412368214	.4155914657	.793594785	.5651997102	.4112042366
6	.6238966074	.6294545996	.2663315696	.4389555661	.475714832
	6				
1	.3598862696				
2	.5533621304				
3	.5813695368				
4	.2729719419				
5	.559078523				
6	.6554834227				

-----+

Problem 7

(a) 5 points

```
xi i.year
i.year _Iyear_66-73 (naturally coded; _Iyear_66 omitted)
. ivreg2 lw expr s (iq = age kww med)
IV (2SLS) estimation
-----
Number of obs = 758
F( 3, 754) = 105.26
Prob > F = 0.0000
Total (centered) SS = 139.2861498 Centered R2 = 0.2886
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9960
Residual SS = 99.0915462 Root MSE = .3616
-----
lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+-----
iq | -.0012932 .0047482 -0.27 0.785 -.0105995 .0080132
expr | .0442341 .0065777 6.72 0.000 .0313421 .057126
s | .1107632 .0157675 7.02 0.000 .0798595 .1416668
_cons | 4.259495 .3124346 13.63 0.000 3.647134 4.871855
-----
Anderson canon. corr. LR statistic (underidentification test): 43.846
Chi-sq(3) P-val = 0.0000
-----
Cragg-Donald F statistic (weak identification test): 14.927
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 13.91
10% maximal IV relative bias 9.08
20% maximal IV relative bias 6.46
30% maximal IV relative bias 5.39
10% maximal IV size 22.30
15% maximal IV size 12.83
20% maximal IV size 9.54
25% maximal IV size 7.80
```

Source: Stock-Yogo (2005). Reproduced by permission.

Sargan statistic (overidentification test of all instruments): 84.806
Chi-sq(2) P-val = 0.0000

Instrumented: iq
Included instruments: expr s
Excluded instruments: age kww med

The Anderson canonical correlation test rejects at the 5 percent level the null hypothesis of underidentification. However, the rejection of the null of the Sargan test suggests that one or more of the instruments is not uncorrelated with the disturbance process.

b) (5 points)

. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med)
IV (2SLS) estimation

Number of obs = 758
F(9, 748) = 47.13
Prob > F = 0.0000
Total (centered) SS = 139.2861498 Centered R2 = 0.3621
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9964
Residual SS = 88.85241753 Root MSE = .3424

lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+-----
iq | .007033 .0040735 1.73 0.084 -.0009509 .0150169
expr | .0398175 .0067903 5.86 0.000 .0265086 .0531263
s | .0565379 .0139059 4.07 0.000 .0292829 .0837929
_Iyear_67 | -.0725177 .0497367 -1.46 0.145 -.1699999 .0249644
_Iyear_68 | .0504323 .0465702 1.08 0.279 -.0408436 .1417082
_Iyear_69 | .1605229 .045594 3.52 0.000 .0711604 .2498854
_Iyear_70 | .2097466 .053631 3.91 0.000 .1046318 .3148614
_Iyear_71 | .183241 .0456348 4.02 0.000 .0937985 .2726836
_Iyear_73 | .2792134 .0420477 6.64 0.000 .1968014 .3616254
_cons | 4.013944 .2761018 14.54 0.000 3.472795 4.555094

Anderson canon. corr. LR statistic (underidentification test): 54.386
Chi-sq(3) P-val = 0.0000

Cragg-Donald F statistic (weak identification test): 18.497
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 13.91
10% maximal IV relative bias 9.08
20% maximal IV relative bias 6.46
30% maximal IV relative bias 5.39
10% maximal IV size 22.30
15% maximal IV size 12.83
20% maximal IV size 9.54
25% maximal IV size 7.80
Source: Stock-Yogo (2005). Reproduced by permission.

Sargan statistic (overidentification test of all instruments): 91.950
Chi-sq(2) P-val = 0.0000

Instrumented: iq
Included instruments: expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
_Iyear_73
Excluded instruments: age kww med

The year dummies for years after 1968 are all significant and positive, suggesting some unmodeled change in the underlying process determining the wage that isn't captured by the included characteristics of workers. IQ now has a positive coefficient, but one that is still not statistically significantly different from 0 at the 5 percent level. The Anderson test and the Sargan test produce similar results as in part (a).

c) (5 points)

```
. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med),  
robust  
IV (2SLS) estimation  
-----  
Statistics robust to heteroskedasticity  
Number of obs = 758  
F( 9, 748) = 42.35  
Prob > F = 0.0000  
Total (centered) SS = 139.2861498 Centered R2 = 0.3621  
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9964  
Residual SS = 88.85241753 Root MSE = .3424
```

```

| Robust
lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+-----
iq | .007033 .004181 1.68 0.093 -.0011616 .0152276
expr | .0398175 .0068121 5.85 0.000 .0264659 .053169
s | .0565379 .0141939 3.98 0.000 .0287185 .0843574
_1year_67 | -.0725177 .0474303 -1.53 0.126 -.1654794 .0204439
_1year_68 | .0504323 .046312 1.09 0.276 -.0403376 .1412021
_1year_69 | .1605229 .0426472 3.76 0.000 .0769361 .2441098
_1year_70 | .2097466 .0563248 3.72 0.000 .099352 .3201412
_1year_71 | .183241 .0433592 4.23 0.000 .0982585 .2682235
_1year_73 | .2792134 .0420768 6.64 0.000 .1967443 .3616824
_cons | 4.013944 .285412 14.06 0.000 3.454547 4.573341
-----+-----
Anderson canon. corr. LR statistic (underidentification test): 54.386
Chi-sq(3) P-val = 0.0000
Test statistic(s) not robust
-----+-----
Cragg-Donald F statistic (weak identification test): 18.497
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 13.91
10% maximal IV relative bias 9.08
20% maximal IV relative bias 6.46
30% maximal IV relative bias 5.39
10% maximal IV size 22.30
15% maximal IV size 12.83
20% maximal IV size 9.54
25% maximal IV size 7.80
Test statistic(s) not robust
Source: Stock-Yogo (2005). Reproduced by permission.
-----+-----
Hansen J statistic (overidentification test of all instruments): 72.328
Chi-sq(2) P-val = 0.0000
-----+-----
Instrumented: iq
Included instruments: expr s _1year_67 _1year_68 _1year_69 _1year_70 _1year_71
_1year_73
Excluded instruments: age kww med
-----+-----

```

Using robust standard errors does not seem to affect the standard errors very much, suggesting that heteroskedas-

ticity is not an issue.

d)(10 points)

```
. ivreg2 lw expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (iq = age kww med),  
gmm
```

```
2-Step GMM estimation
```

```
-----  
Statistics robust to heteroskedasticity
```

```
Number of obs = 758
```

```
F( 9, 748) = 41.49
```

```
Prob > F = 0.0000
```

```
Total (centered) SS = 139.2861498 Centered R2 = 0.3562
```

```
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9964
```

```
Residual SS = 89.67457928 Root MSE = .344
```

```
-----  
| Robust
```

```
lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
```

```
-----+-----  
iq | .0077785 .004178 1.86 0.063 -.0004103 .0159673  
expr | .0457042 .0067758 6.75 0.000 .032424 .0589845  
s | .0550396 .0141922 3.88 0.000 .0272233 .0828558  
_Iyear_67 | -.0602284 .0474059 -1.27 0.204 -.1531422 .0326854  
_Iyear_68 | .0569231 .0463035 1.23 0.219 -.0338301 .1476763  
_Iyear_69 | .1601399 .0426317 3.76 0.000 .0765833 .2436965  
_Iyear_70 | .1794522 .0561917 3.19 0.001 .0693184 .289586  
_Iyear_71 | .1548847 .04323 3.58 0.000 .0701555 .2396139  
_Iyear_73 | .2763517 .0420029 6.58 0.000 .1940274 .358676  
_cons | 3.940908 .285036 13.83 0.000 3.382248 4.499568
```

```
-----  
Anderson canon. corr. LR statistic (underidentification test): 54.386
```

```
Chi-sq(3) P-val = 0.0000
```

```
Test statistic(s) not robust
```

```
-----  
Cragg-Donald F statistic (weak identification test): 18.497
```

```
Stock-Yogo weak ID test critical values: 5% maximal IV relative bias 13.91
```

```
10% maximal IV relative bias 9.08
```

```
20% maximal IV relative bias 6.46
```

```
30% maximal IV relative bias 5.39
```

```
10% maximal IV size 22.30
```

```
15% maximal IV size 12.83
```

20% maximal IV size 9.54
 25% maximal IV size 7.80
 Test statistic(s) not robust
 Source: Stock-Yogo (2005). Reproduced by permission.

 Hansen J statistic (overidentification test of all instruments): 72.328
 Chi-sq(2) P-val = 0.0000

Instrumented: iq
 Included instruments: expr s _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
 _Iyear_73
 Excluded instruments: age kww med

None of the results are markedly different from that obtained in part b). In the GMM model we estimate here, we do not maintain the assumption of conditional homoskedasticity, but rather allow arbitrary heteroskedasticity. The GMM model also delivers efficient estimates. The Hansen J statistic allows a test of overidentification similar to that provided by the Sargan statistic in the 2SLS model; the Hansen J is consistent in the presence of heteroskedasticity. The rejection of the null in this test suggests that one or more of the instruments is not uncorrelated with the disturbance process. The Anderson test as before indicates that the model is not underidentified.

e) (5 points)

```
. ivreg2 lw expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71 _Iyear_73 (s iq = age kww med),
gmm endog(s)
2-Step GMM estimation
```

```
-----
Statistics robust to heteroskedasticity
Number of obs = 758
F( 9, 748) = 37.83
Prob > F = 0.0000
Total (centered) SS = 139.2861498 Centered R2 = 0.0906
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9949
Residual SS = 126.6665339 Root MSE = .4088
```

```
-----
| Robust
lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+-----
s | .1993476 .0254187 7.84 0.000 .1495279 .2491674
```

```

iq | -.0089693 .0054021 -1.66 0.097 -.0195573 .0016187
expr | .0630694 .0081395 7.75 0.000 .0471162 .0790225
_1year_67 | -.0753593 .0560256 -1.35 0.179 -.1851675 .0344488
_1year_68 | .012483 .0531677 0.23 0.814 -.0917237 .1166897
_1year_69 | .0967016 .050023 1.93 0.053 -.0013417 .1947449
_1year_70 | .1450002 .0670161 2.16 0.030 .013651 .2763494
_1year_71 | .0198738 .0584071 0.34 0.734 -.094602 .1343495
_1year_73 | -.0100273 .0670913 -0.15 0.881 -.1415238 .1214693
_cons | 3.81719 .3332255 11.46 0.000 3.16408 4.4703

```

```

-----
Anderson canon. corr. LR statistic (underidentification test): 45.115
Chi-sq(2) P-val = 0.0000
Test statistic(s) not robust

```

```

-----
Cragg-Donald F statistic (weak identification test): 15.270
Stock-Yogo weak ID test critical values: 10% maximal IV size 13.43
15% maximal IV size 8.18
20% maximal IV size 6.40
25% maximal IV size 5.45
Test statistic(s) not robust
Source: Stock-Yogo (2005). Reproduced by permission.

```

```

-----
Hansen J statistic (overidentification test of all instruments): 0.482
Chi-sq(1) P-val = 0.4873
-endog- option:
Endogeneity test of endogenous regressors: 71.528
Chi-sq(1) P-val = 0.0000
Regressors tested: s

```

```

-----
Instrumented: s iq
Included instruments: expr _1year_67 _1year_68 _1year_69 _1year_70 _1year_71
_1year_73
Excluded instruments: age kww med
-----

```

The endogeneity test rejects the null hypothesis of exogeneity of the variable s, years of schooling.

f)(10 points)

```
. ivreg2 lw expr _1year_67 _1year_68 _1year_69 _1year_70 _1year_71 _1year_73 (s iq = age kww med),
```

```

gmm
2-Step GMM estimation
-----
Statistics robust to heteroskedasticity
Number of obs = 758
F( 9, 748) = 37.83
Prob > F = 0.0000
Total (centered) SS = 139.2861498 Centered R2 = 0.0906
Total (uncentered) SS = 24652.24662 Uncentered R2 = 0.9949
Residual SS = 126.6665339 Root MSE = .4088
-----
| Robust
lw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
-----+-----
s | .1993476 .0254187 7.84 0.000 .1495279 .2491674
iq | -.0089693 .0054021 -1.66 0.097 -.0195573 .0016187
expr | .0630694 .0081395 7.75 0.000 .0471162 .0790225
_1year_67 | -.0753593 .0560256 -1.35 0.179 -.1851675 .0344488
_1year_68 | .012483 .0531677 0.23 0.814 -.0917237 .1166897
_1year_69 | .0967016 .050023 1.93 0.053 -.0013417 .1947449
_1year_70 | .1450002 .0670161 2.16 0.030 .013651 .2763494
_1year_71 | .0198738 .0584071 0.34 0.734 -.094602 .1343495
_1year_73 | -.0100273 .0670913 -0.15 0.881 -.1415238 .1214693
_cons | 3.81719 .3332255 11.46 0.000 3.16408 4.4703
-----
Anderson canon. corr. LR statistic (underidentification test): 45.115
Chi-sq(2) P-val = 0.0000
Test statistic(s) not robust
-----
Cragg-Donald F statistic (weak identification test): 15.270
Stock-Yogo weak ID test critical values: 10% maximal IV size 13.43
15% maximal IV size 8.18
20% maximal IV size 6.40
25% maximal IV size 5.45
Test statistic(s) not robust
Source: Stock-Yogo (2005). Reproduced by permission.
-----
Hansen J statistic (overidentification test of all instruments): 0.482
Chi-sq(1) P-val = 0.4873
-----
Instrumented: s iq

```

Included instruments: expr _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70 _Iyear_71
_Iyear_73
Excluded instruments: age kww med

Unlike for the previous regression models, the Hansen J test fails to reject the null hypothesis of instruments uncorrelated with the disturbance process, suggesting, together with the successful rejection of underidentification via the Anderson test, that the instrument set and endogenous variables set used are valid.