Complexity, Contract and the Employment Relationship

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Abstract

This paper introduces a model of contract incompleteness and bounded rationality based on the multi-tasking model of Holmström and Milgrom (1991). It is shown that the trade-off between the use of an employment relationship versus an explicit state contingent contract depends on number of tasks or complexity of the services provided by the individual.

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1 Introduction

This paper introduces a model of contract incompleteness based on the underlying characteristics of the relationship. Using the multi-tasking model of Holmström and Milgrom (1991), I show that the trade-off between the use of an employment relationship versus an explicit state contingent contract depends on number of tasks or complexity of the services provided by the individual. The model also provides a foundation for the common assumption that *ex ante* agents cannot contract upon all states of nature, even though *ex post* this information becomes common knowledge and can be used for further renegotiation (See Hart (1995)).

Macauley (1963) points out that an important ingredient to any contract is the "rational planing of the transaction with careful provision for as many future contingencies as can be foreseen"[1]. Herbert Simon has long argued that such planning is limited due to the bounded computational resources of the mind[2]. In recent years there has been tremendous advances in our understanding of human decision making, with the current the state of art characterizing thought as a complex, hybrid process involving a variety of techniques, including search, learned reflex responses and thinking by analogy (See for example Newall and Simon (1972) and Newell (1990)). In this paper I follow Allan Newell and take the process of search as a first order approximation for the thinking and planning process[3]. He states:

> "Search is fundamental for intelligent behavior. It is not just another method or cognitive mechanism, but a fundamental process."[4]

I argue that the problem of writing a contract involves search over a very large space, so large that in practice contracts are necessarily incomplete. This is because the construction of a complete contract requires the use of an algorithm of exponential computational complexity, which is intractable by the Cook-Karp thesis[5].

Modelling contract choice as a search process generates a natural distinction between the *ex ante* and the *ex post* states of the world that is central to the recent literature on incomplete contracts[6]. This literature

[1] Page 56.

[2] See Simon (1957) and the collection of readings in Simon (1982).

[3] See MacLeod (1995) for a more detailed model of search and its relationship to decision making.

[4] Newell (1990), page 96.

[5] See Davis, Sigal, and Weyuker (1994) and Garey and Johnson (1979).

[6] See Grout (1984), Grossman and Hart (1986) and the recent review by Hart (1995).

typically assumes that *ex ante* parties to an agreement cannot write a contract contingent upon mutually relevant states of the world, even though *ex post* agents are assumed to have symmetric information. Maskin and Tirole (1995) point out that the rationality assumption implicit in these models is inconsistent with the assumed impossibility agreeing upon and implementing a state contingent contracts does not restrict the set of possible outcomes. The current paper resolves this problem by focusing on the impossibility of forming beliefs or judgements over all possible future contingencies. Thus contract incompleteness does not arise due to a lack of enforceability, but due to bounded rationality that makes it impossible to anticipate or even think about all possible future events.

To see how bounded rationality creates a natural division between the *ex ante* and *ex post* states of the world consider the proverbial needle in a haystack. Before we have found the needle, one has a long and costly search process. However once the location of the needle is known, then it can be recovered speedily. Other, more natural, examples might include finding the proof to a theorem, or finding a new way to produce a product. Many years may past before a solution is found, but upon discovery one often asks why it was not thought of before! The cost of implementing a known solution is dramatically less than trying to discover it.

In the context of contracts, if one knew before hand the future state of the world, then the contract could simply specify the desired terms and conditions. It was Simon (1951)'s insight that a distinctive feature of the employment relationship, in contrast to a sales contract, is the potential to delay decision making until after the state of the world has been revealed. In the context of the current model, I show that such a delay is computationally much less complex than the construction of an explicit contract *ex ante.*

Consider an example that is familiar to most readers, the academic tenure decision. One would not doubt that the university employs an assistant professor, as opposed to buying professorial services in a spot market. What is crucial is that the employer has the right to terminate employment based on a judgement of performance that occurs after the fact. The decision to tenure a professor is made at the end of the period, and is based on past performance. I argue that this occurs for the simple reason that it is impossible to write even a small subset of the sufficient criteria that would eventually result in promotion. Thus the employer uses a contract in which a high level of performance is expected *ex ante*, but the evaluation of this performance is delayed until after the employee has chosen her effort.

The basic model follows Dye (1985)'s idea that transactions costs arise from adding contract contingencies. However, in his model it is the cost of writing additional clauses, while here the costs arising from simply thinking about or considering different possible contingencies. In section 2 I begin with the multi-tasking model of Holmström and Milgrom (1991), and show that even with infinitesimally small costs per contract contingency, contracts are likely to be incomplete because their complexity is an exponential function of the number of tasks. Therefore contract incompleteness arises from bounded rationality and the costs of carrying out the careful planning that Macauley (1963) cites as being key to a good contract[7].

In section 3 the characteristics of an optimal incomplete contract are discussed. When the number of different tasks (or equivalently measures of output) is very small then we should expect to see contingent contracts. Examples of this include piece rates or commissions. The models also predicts that as the value of the relationship increases, then the number of contingent clauses should increase, a result that is consistent with Macauley's observations of the complexity of written contracts.

Section 5 discusses how the employment relationship can provide an efficient solution to the problem of contract complexity[8]. Using the employment model of MacLeod and Malcomson (1989) I show that if there are sufficient rents in a relationship, then an agreement that does not explicitly specify expectations *ex ante,* but judges the worker's performance *ex post,* may be an efficient alternative to an explicit contingent contract. This result also illustrates how the current model may provide a foundation for the incomplete contracts model introduced by Grout (1984) and Grossman and Hart (1986). This relationship is discussed in the conclusion.

2 A Multi-Tasking Model of Exchange

Consider the following multi-period version of the Holmström and Milgrom (1991) multi-tasking model. Each period a principal hires an agent to produce a complex good described by a vector of input tasks $\mathbf{y} \in \Re^k$. The agent has a total time (or effort) endowment Y that must be divided among the k tasks. The benefit from allocating y_i units to task i is αy_i, where α is a random variable representing the marginal benefit of task i with values taken from $\{a^1, ..., a^n\}$. It is assumed that $a^1 = 0$, and $a^{i+1} > a^i$, for $i = 1, ..., n-1$. The personal costs of allocating $y_i > 0$ units of effort to

[7]Note that this model is different from a standard agency model where contract incompleteness followes frm asymmetric information (See Hart and Holmström (1987)).

[8]See Williamson (1975) and Williamson, Wachter, and Harris (1975) for insightful dicussions of the employment relationship, the form of which is motivated in part by the problem of contractual incompleteness.

task i is $\beta_i y_i^2 + f$, where $\beta \in \{b^1, ..., b^m\}$,represents a random variable with $b^{i+1} > b^i > 0$ for $i = 1, .., m-1$, and $f \geq 0$ is a fixed cost. If no effort is allocated to a task then the fixed cost is zero. Let $C(\mathbf{y}) = \left[y_1^2, \ldots, y_k^2\right]^T$, $\boldsymbol{\alpha} = [\alpha_1, ..., \alpha_k]$ and $\boldsymbol{\beta} = [\beta_1, ..., \beta_k]$ represent the vectors of cost functions, productivities and cost parameters respectively. Let $F(\mathbf{y})$ be the total fixed costs associated with $\mathbf{y}$. Then the total cost of effort to the worker is given by $\boldsymbol{\beta} C(\mathbf{y}) + F(\mathbf{y})$, and the value of the product is given by $\boldsymbol{\alpha}\mathbf{y}$. For simplicity suppose that the underlying parameters are independently and identically distributed each period.

The relevant state space in period t is $\Omega = \left\{\{a^1, ..., a^n\} \times \{b^1, ..., b^m\}\right\}^k$, with the probability of each state given by $\mu : \Omega \to \Re_+$. If $\omega \in \Omega$, then ω_i denotes the state $(\alpha_i, \beta_i) \in \{a^1, ..., a^n\} \times \{b^1, ..., b^m\}$ that is relevant to task i. Furthermore we suppose that the set of possible states relevant for task i is can indexed by the set $\{\omega^{i\,1}, \omega^{i\,2}, ..., \omega^{i\,nm-1}, \omega^{i\,nm}\}$.

The vector $\mathbf{y}$ represents the set of services provided by the agent. Labor services are differentiated from a physical commodity in that the principle receives the benefits as they are produced. If the agent were producing a physical commodity, then even in the absence of a contract, the agent could withhold the goods until a satisfactory price is agreed upon. With service goods trade occurs as it is produced, as for example in the cases of typing services, teaching services, or medical services. This implies that the good cannot be produced before negotiation over contract terms and conditions, and hence a contract or agreement must be reached before production begins. This creates a motive for contract formation that does not depend on a trade-off between risk and incentive.

The focus of the Holmström and Milgrom (1991) model is on the trade-off between the provision of insurance and incentives in a multi-task environment. In their model once the contract is fixed, the allocation of effort to tasks is deterministic. This model integrates the multi-task setup with the Simon (1951) model where at the time of contracting it is not known how the worker should allocate her time. To highlight the role of contract complexity, as opposed to problem of measurement precision, I make the extreme assumption that a worker's allocation of effort to tasks is perfectly observed *ex post,* an assumption that is reasonable for a number of situations.

For example consider the job of a surgeon. An operation is a situation where unexpected events often occur during an operation that require immediate decision making and reallocation of effort, all of which is observed by the other individuals in the operating theatre. Another common example are athletes on a competitive team, who must constantly decide how to respond to the actions of their opponents. A fireman does not know in advance when he or she will called upon to fight a fire, nor it is known in advance how difficult it will be to fight the fire. Despite this, the services of the fireman are contracted in advance before any fires occur. In these examples one cannot predict before hand exactly what the employee will be called upon to do, however *ex post* a great deal of information concerning the actions taken is available.

In summary, I assume that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are observed, as is the choice $\mathbf{y}$ of the agent. This extreme assumption serves to highlight the source of contract complexity. The timing of the actions in period t proceeds as follows:

1. The principal and agent enter into an agreement on compensation and expectations for performance.

2. The state of the world $\omega_t \in \Omega$ is revealed.

3. The agent decides on a vector of tasks $\mathbf{y}_t$.

4. The principal pays the agent W_t.

5. Both principal and agent decide whether to continue the relationship or not.

The payoff to the agent after the state of nature has been revealed is given by:

$$U_t = W_t - \boldsymbol{\beta} C\left(\mathbf{y}_t\right) - F\left(\mathbf{y}_t\right) + \delta E\left\{U_{t+1}\right\},$$

where $\delta \in (0,1)$ is the discount factor and $E\left\{U_{t+1}\right\}$ the expected future utility. The present discounted value of a principal's future profits from employing the agent is:

$$\Pi_t = \boldsymbol{\alpha}\mathbf{y}_t - W_t + \delta E\left\{\Pi_{t+1}\right\}.$$

It is assumed that the actions and states in one period do not affect the actions and states in the next period. Each period the agent and principal have available a market return of $\bar{u}_t$ and $\bar{\pi}_t$ respectively. They are also assumed to be infinitely lived.

3 Spot Market Contracting and Complexity

Given that production in each period is independent of production in other periods and there are no motives for income smoothing, then the relationship can in principle be governed by a sequence of spot contracts. Such a contract would specify what the agent is expected to do in every state that occurs with positive probability. In exchange for signing this contract the agent is paid a sum that ensures her utility from this contract is better that then her next best alternative. A complete contract is a function $\delta : \Omega \rightarrow X = \Re \times \Re^k$, where for each state $\omega \in \Omega$, the function $\delta(\omega) = (w(\omega), y(\omega)) \in X$ defines the wage payment and the output expected from the agent. An efficient complete contract, $\delta^*(\omega) = (w(\omega), \mathbf{y}(\omega))$, is the solution to the following program:

$$\mathbf{y}(\omega) \in \arg\max_{\mathbf{y}'} \boldsymbol{\alpha}\mathbf{y}' - \boldsymbol{\beta}C(\mathbf{y}') - F(\mathbf{y}'), \text{ subject to:} \tag{1}$$

$$|\mathbf{y}| \equiv \sum_{i=1}^{k} y_i' = Y, \text{ and} \tag{2}$$

$$w(\omega) = \bar{u} + F(\mathbf{y}(\omega)) + \boldsymbol{\beta}C(\mathbf{y}(\omega)). \tag{3}$$

Observe that due to the assumption of risk neutrality, the optimal solution does not depend on the probability distribution of states. Without loss of generality it is assumed that the agent allocates all effort from the time allocation Y. Any leisure time can be defined as allocating effort to an unproductive task.

The solution to this program begins with an index set $K(\omega) \subset \{1, .., k\}$ denoting the tasks that have positive effort levels. The effort assigned to task $i \in K(\omega)$ in state $\omega \in \Omega$ given by

$$y_i(\omega) = (\alpha_i - \lambda(\omega))/2\beta_i, \tag{4}$$

where $\lambda(\omega)$ is the multiplier associated with the time constraints, and is given by:

$$\lambda(\omega) = \sum_{i \in K(\omega)} \alpha_i/2\beta_i - Y \Bigg/ \sum_{i \in K(\omega)} 1/2\beta_i.$$

I do not wish to discuss the complexity of this subproblem except to observe that finding its solution is not without cost. In particular even with the available data, the existence of a fixed cost implies that there is a combinatorial problem required to find the mix of tasks that should be given a positive weight. Let $\gamma > 0$ denote the processing cost of finding an optimal task allocation for each state. For simplicity we suppose that writing it down as part of a contract has no cost, and hence all costs in this model are due to thinking about or considering the different possible events relevant to the relationship.

3.1 Complexity

A contract is a function from the states of the world to the set of possible outcomes. As such it is an algorithm that takes as an input the specification of the problem, the number of tasks k, and the number of cost and productivity levels, m and n. The *complexity* of the contract is a measure of the cost of designing, writing and implementing the contract as a function of the data describing the relationship. The contract is *complete* if it describes the actions that are to be taken for each state of nature. In most employment relationships there do not exist numerical measures of the costs and benefits for each task. For example a document needs to be typed up immediately by a typist with a severe cold. How does one measure need or cost in this case? Thus it not possible to write a contract of the form: "Given the observed state, the agent is required to compute and apply the optimal mix of effort". Rather, I assume that for each carefully specified state it is possible to describe the appropriate set of actions. In the case of the typist, the contract would explicitly describe the scenario and possibly state that the document be typed up, followed by the typist going home for the rest of the day. The cost of formulating this contract is the number of states times the cost of finding the optimal effort allocation for each state. In this spirit this cost of complete contract is summarized in the following proposition.

Proposition 1 *The cost of implementing the complete contract procedure when all states occur with positive probability is* $n^k m^k \gamma$.

What is immediately clear is that in practice one rarely implements such a contract. Suppose that the cost of writing a contract clause for each state is 1 cent regardless of the number of tasks. Further suppose that the number of cost and performance levels are the same ($n = m$). The following table presents the cost of a complete contract as a function of the number of tasks and effort levels.

As one can see from table 1, when there are a reasonable number of tasks, even with a small number of performance levels, the cost of a complete state contingent contract is astronomical. Observe that contract costs are an exponential function of the number of tasks. Since the important work of Edmonds (1965), Cook (1971), and Karp (1971) it is widely recognized that problems whose complexity is exponential in problem size are effectively intractable. This table provides a vivid illustration of this point. As in Dye (1985), I suppose that is the cost of considering each contingency that explains why contracts are incomplete. However in Dye's model the space of

Number of Cost and Performance Levels	Number of Tasks			
	2	5	10	15
2	$0.16	$10	$10,000	$10 million
3	$0.81	$600	$35 million	$2 trillion
4	$2.56	$10,000	$11 billion	$11,000 trillion
5	$6.25	$100,000	$1000 billion	$10 million trillion
Cost of considering a contingency:	1 cent			

Table 1: Cost of a Complete State Contingent Contract

contingencies is infinite, and therefore contracts are necessarily incomplete in all cases. In this model I am highlighting the role of multi-tasking as the source of contracting costs. In table (2) the same data is presented in terms of time costs, with the same evident implication.

Number of Cost and Performance Levels	Number of Tasks		
	2	5	10
2	0.0 16 sec.	1 sec.	17 min.
3	0.0 81 sec	1 min.	40 days
4	0. 256 sec.	17 min.	36 years
5	0. 625 sec	2. 7 hr	3135 years
Time to consider a single state:	1 millisecond		

Table 2: Time Required to Think About All Possible States

4 The Formation of Incomplete Contracts

Contracts for services that require an *ex post* allocation of effort among several tasks are necessarily incomplete. In the this section I explore one way to model the process of contract formation under bounded rationality and characterize some of the properties of the resulting incomplete contract. It is unlikely that one can capture the richness of the contracting procedure in any formal model. Rather my aim is to illustrate a way of modelling boundedly rational choice that has implications that are not inconsistent with some of the characteristics of contracts we observe in practice. The field of cognitive science has highlighted the fact that humans use a variety of techniques to think about and structure the world around them. Here I consider the implication of modelling thought as a search process, the one technique, as pointed out by Newell (1990), that is considered central to any model of decision making.

Thought must provide a way to organize reflection over a small subset of possible states, a process that can modelled using search on a graph[9]. A paradigm example of this thought process is the game of chess. If one were to analyze the game of chess from a Bayesian perspective, then there must exist a probability measure over all possible positions. This is clearly impossible. When making a move, a player uses the rules of chess to construct a very small subset of possible scenarios, and then chooses a move consistent with one of these scenarios.

Bounded rationality is modelled by supposing that agents are unable to consider *ex ante* how to allocate effort for all relevant events. This is modelled by supposing that the principal and agent begin to think about the possible events that might occur for each task in turn. I shall not model how tasks are ordered, it might be random or experience might suggest that some tasks are more important and hence should be considered first. Simply let the tasks be labelled so that they correspond to the order of examination, that is the agents consider first task 1, then task 2 and so on.

As an example consider the case with two productivity levels $(n = 2)$ and two cost levels $(m = 2)$. Beginning at the root of the search tree with the sure event Ω, the agent considers the different possible states that are relevant to this task $1 : \{\omega^{11}, \omega^{12}, \omega^{13}, \omega^{14}\}$ (see the search graph illustrated in figure (1). If search were to stop at this point, then the contract would specify that all effort be allocated to task one, with compensation depending on the cost of effort $(\beta_1 Y^2 + f)$. Let this allocation be denoted $x^{1j} \in X$. Limited search implies that the contract would not depend on the productivity and cost realizations for the other tasks. Therefore $\delta(\omega) = x^{1j}$ for all $\omega \in A_j$, where $A_j = \{\omega \in \Omega | \omega_1 = \omega^{1j}\}$, and hence the contract is measurable with respect to the partition $\{A_1, A_2, A_3, A_4\}$.

This limited search process has a number of interpretations. For example, on a construction site, the employment contract may require workers to take certain precautions. These precautions may not cover all possible precautions that one may take, and hence the level of safety is likely to be incomplete. In practice what we observe is that precautions, such as wearing

[9] See Pearl (1984) for an excellent introduction to algorithms that use search in decision making. See MacLeod (1995) for a more detailed application of these ideas to economic decision making.

a safety hat, are carried out as one gathers experience with different kinds of accidents.

This approach is consistent with the Savage (1972) view that individuals think about the world in terms of a limited number of events relative to the complete state space. I assume that agents can assign probabilities to events that have been examined, but that these probabilities are explicitly not conditional probabilities in the sense that they are derived from the probability distribution μ.

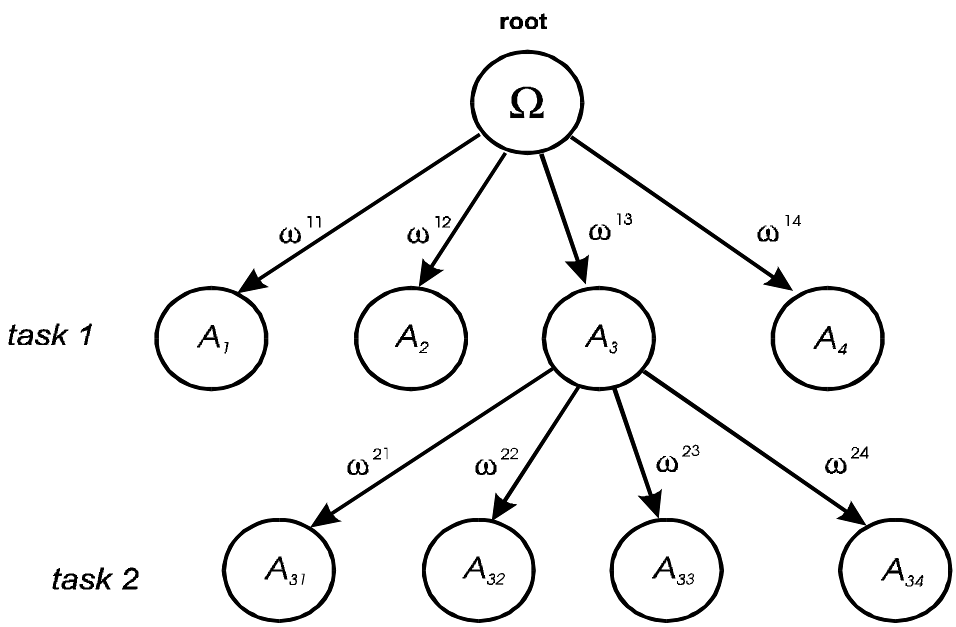

Figure 1: Search Graph

This contract is incomplete in the sense that further reflection may result in more detail. For example the agents may feel that event A_3 is highly likely to occur, thus they consider how effort might be allocated to a second task given this event. As illustrated in figure (1), event A_3 can be further divided into events of the form A_{3i}, corresponding to the state ω^{13} for task 1 and ω^{2i} for task 2. In this case a clause for event A_3 is replaced by four clauses for events A_{31} to A_{34}. As illustrated in figure (1) the total number of clauses in the contract is then 7. For events in $\{A_1, A_2, A_4\}$ all effort is allocated to task 1, while for events in $\{A_{31}, A_{32}, A_{33}, A_{34}\}$ the contract allocates effort optimally between tasks 1 and 2. This the search process in this case results in a contract that is measurable with respect to the partition $\{A_1, A_2, A_4, A_{31}, A_{32}, A_{33}, A_{34}\}$. The number sets in this partition is used to measure the complexity of the contract.

To summarize contract formation is modelled with the following two step procedure:

1. Beginning with the root Ω, a limited search over the set of possible events occurs resulting in a partition P of Ω. Let $|P|$ be the number of events in P. This defines a measure of the *complexity* of the contract.

2. A contract is signed that is measurable with respect to P. That is each event in P corresponds to a well defined allocation of effort.

Notice that this definition of complexity is different from Dye (1985), where complexity is equal to the number of written contingencies. A contract may, after a great deal of consideration, specify the same allocation for each event. In that case the contract itself would have a small number of clauses, however it may have taken a great deal of time to reach an agreement due to the many cases that were considered. .

The definition is also different from the notion of complexity implicit in Segal (1995). He shows that when the number of states in a particular model of exchange is very large the optimal allocation approaches the outcome given by a simple fixed price contract. The proof of this result is ingenious and difficult. From the point of view of the current model I would argue that though the optimal contract has a simple form, it is very complex. At every point before the limit is achieved, the optimal contract would in principle depend on all states of nature. Thus the underlying reasoning used to determine the contract is difficult and complex.

It is very unlikely that in practice agents in the world modelled by Segal would use a simple contact because they understood it was close to optimal. In order that his model explain the use of simple incomplete contracts one would need to understand to what extent his model is a generic representation of complex exchange. If it were generic, then his result suggests that in a competitive world agents using simple fixed price rules would survive, even if they did not understand the reasons for using these rules.

In contrast, the role of the current model to understand the limits imposed by a conscious process of contract choice. In this regard, a contact is incomplete if $|P| < |\Omega|$. From the tables in the previous section it is clear that contracts are likely to be very incomplete when there is a significant element of multi-tasking. Two issues can be addressed within the context of this model. First what is the trade-off between the cost of adding more contract terms and the benefit in terms of an increased payoff? Secondly, how can an employment relationship mitigate some of the costs associated with incomplete contracts?

Consider the following brute force approach to contract formation. Search occurs as illustrated in figure (1), with P_l denoting the partition of the state space after the events characterizing the productivity and cost levels for the first l tasks have been examined. More formally:

$$P_l = \left\{ A \in 2^{\Omega} | \text{ for all } \{\alpha, \beta\}, \{\alpha', \beta'\} \in A, \ \alpha_i = \alpha_i', \beta_i = \beta_i', \ i \in \{1, .., l\} \right\}.$$

In this case the complexity as a function of l is $|P_l| = (nm)^l$. In constructing the partition in this way, one is focusing on the effects of the first l tasks. Therefore the explicit contract based on P_l ignores the impact of effort assigned to tasks $l+1$ and above. This ensures that the computation of rewards is itself feasible because we do not require expectations to be taken with respect to the full state space. The optimal contract with respect to events $A \in P_l$ is denoted $\delta_l^* (A) = (w(A), \mathbf{y}_l^* (A))$, and is defined as the solution to:

$$\begin{aligned} \mathbf{y}^{l*} (A) &= \arg \max_{|\mathbf{y}|=Y} \left\{ \sum_{i=1}^{l} \left\{ \alpha_i y_i - \beta_i y_i^2 \right\} - F(\mathbf{y}) \right\}, \\ & \{(\alpha_1, \beta_1), ..., (\alpha_l, \beta_l), \omega_{l+1}, ..., \omega_{n^k m^k}\} \in A, \\ & w(\omega) = \bar{u} + t(\mathbf{y}(\omega)) f + \boldsymbol{\beta} C(\mathbf{y}(\omega)). \end{aligned}$$

Notice that no information concerning task l to k is used to determine the contract, nor do agents need beliefs over the probability of different events.

Beliefs are important for the determination of how much search is optimal. In a manner that is consistent with the search process, suppose that agents construct probability beliefs as events are explored. Let ν_l denote these beliefs over the partition P_l. Given this measure then the *ex ante* return to the contract δ_l^* gross of contract costs is defined by:

$$V_l^* = \sum_{A \in P_l} \left\{ \sum_{i=1}^{l} \left\{ \alpha_i y_i^{l*} (A) - \beta_i y_i^{l*} (A)^2 \right\} - F\left(\mathbf{y}^{l*} (A)\right) \right\} \nu_l (A).$$

I do not propose a model of belief formation here, but I do need some structure on beliefs if the payoffs with different levels of search are to be comparable. Consistent with Savage (1972)'s small world model of the large world, I suppose that refinements of the information sets do not change an individuals belief concerning an already explored event[10]:

Definition 2 *The beliefs ν_l are* coherent *for all $l \geq 1$ if for all $k < l$,* $\nu_k (A) = \nu_l (A) \ \forall A \in P_k$.

[10] See section 5.5 of Savage (1972).

This assumption ensures that the investigation of more tasks does not affect the beliefs concerning the probability of previously examined events. In the case that productivity shocks are independent events, then the examination of additional tasks would not lead to a revision of earlier beliefs. In this case the assumption does not implicitly impose additional rationality. Of course this assumption is always satisfied in the case that agent's beliefs are characterized by a probability measure over the full state space. However this later assumption is much stronger than we need, and inconsistent with bounded rationality. An implication of this minimal level of consistency on beliefs is that increasing the complexity of a contract unambiguously increases the expected gains.

Proposition 3 *Suppose that beliefs are coherent then the total gain, V_l^*, is increasing in l. If all states occur with positive probability, the realizations for each task, ω_k are independent and identically distributed, and $f = 0$ then V_l^* is strictly increasing in l.*

Proof. The assumption of coherence ensures that increasing the number of tasks studied does not result in a revision of previous beliefs, and hence $E\{V_k^* | l > k \text{ tasks have been examined}\} = V_k^*$. The first part follows immediately from the fact that increasing l, increases the choice set, without imposing additional costs. When fixed costs f are zero, then from the first order conditions 4 it is efficient to spread effort over all equally productive tasks. The i.i.d. assumption ensures that additional tasks are expected to be as productive as the earlier tasks, thus increasing the number of tasks available strictly increases the expected returns. ∎

Thus agents have an incentive to increase the complexity of the contract until the benefits outweigh the costs. As discussed section (3), I suppose that the only contracting cost is the one associated with determining the optimal action given an event A. This cost is likely to increase as the number of tasks to be considered increases. However, the essential point can be illustrated under the hypothesis that the cost of a contract is a linear function of the time required to consider each event, or $\gamma |P_l|$. The *best* incomplete contract an agent is defined as follows:

Definition 4 *The* best *contract is given by $\delta_{l^*}^*$, where l^* satisfies:*

$$\begin{aligned} V_l^* - \gamma_l |P_l| &< V_{l+1}^* - \gamma_{l+1} |P_{l+1}|, \forall l < l^*, \\ V_{l^*}^* - \gamma_{l^*} |P_{l^*}| &< V_{l^*+1}^* - \gamma_{l^*+1} |P_{l^*+1}|. \end{aligned}$$

Given that agents do not know all the possible states, then they cannot globally optimize as would a Bayesian decision maker. The best contract is found by the agents continuing to consider additional tasks and events until the cost of further search outweighs the benefit. The benefit and cost as a function of the number of tasks is illustrated in figure (2) for the case of two performance and cost levels when γ is $\$10^{-8}$. The bars on the benefit line indicate one standard deviation (it is assumed that each state occurs with equal probability).

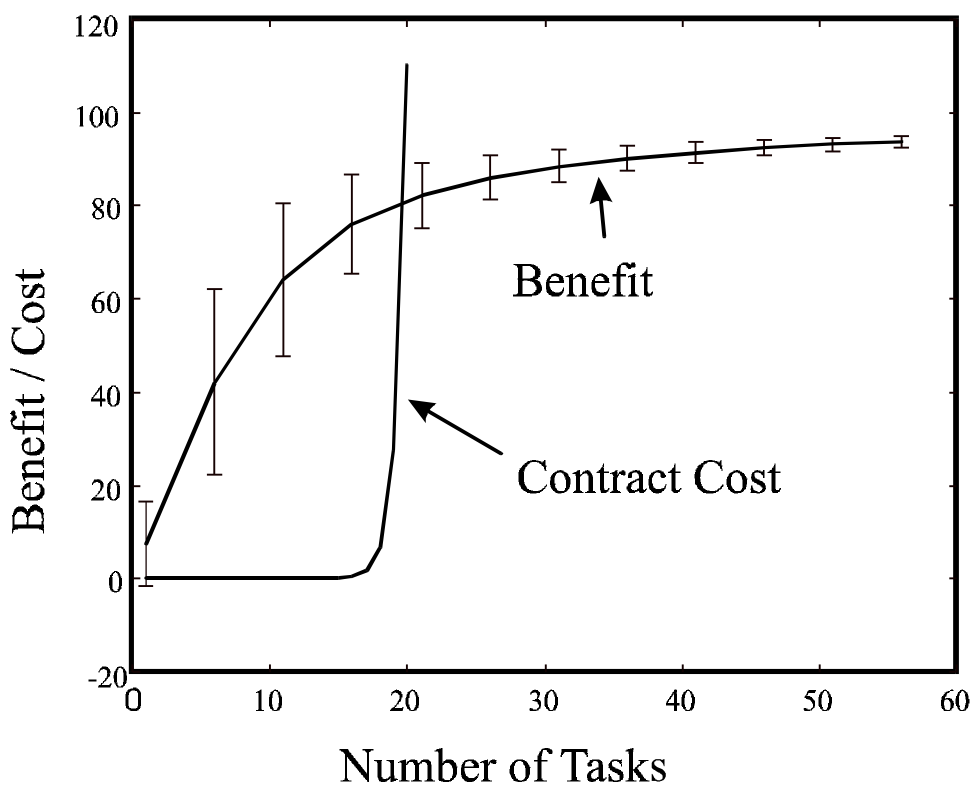

Figure 2: Benefit and Cost of Contract Complexity

Individuals have a limited time resource to allocate to different tasks, and therefore the benefits from increasing contract complexity are bounded. Given that contracting costs rise without limit as the number of tasks increase one can immediately deduce:

Proposition 5 *Suppose that for each task there are at lease two relevant* *states ($nm \geq 2$), then for a sufficiently large number of tasks the best contract is incomplete.*

Proof. Given that task productivities and costs have a bounded support, and the amount of effort available is constrained by Y this implies that $\lim_{l\to\infty} V_l = V^{\max} \leq \alpha^n Y$. This combined with the monotonic increase in contract costs as function of l implies that the best contract in incomplete. ∎

In the work of Dye (1985) it is not clear why transactions costs should significantly limit the completeness of contracts. This model highlights that fact that even with negligibly small transactions costs, we can expect contracts to be very incomplete in the presence of multi-tasking. However, Macauley (1963) also observes that relationships that involve the exchange of more valuable commodities tend to have more complex contracts (he cites the example of the 400 page sales contract for the Empire State Building). This property is consistent with this model.

Proposition 6 *Suppose that the productivity levels are parameterized by $p\alpha^i$. Further suppose that the productivity realizations for each task are independent, there is full support over the set of productivities, and the number of productivity levels satisfies $n \geq 2$. If the best contract given p is incomplete, then for a sufficiently large p', the best contract is more complex.*

Proof. Under the full support assumption there is a strictly positive probability, ρ, that the lowest productivity level is realized for tasks 1 to $l^*(p)$. Let π be the probability of obtaining the highest productivity level, then the benefit from adding a new task is at least $\rho\pi\left(p'\alpha^n Y - \beta^m Y^2 - f - p'\alpha^1 Y\right) - \gamma\left((nm)^{l^*(p)+1} - (nm)^{l^*(p)}\right)$. This is monotonically increasing without limit in p', and therefore for a sufficiently large p' it is best to consider an additional task. ∎

One aspect that makes the task allocation complex is the need to reallocate effort among a variety of tasks as a function of the state. When the fixed costs of assigning effort to a task is low, then the optimal solution requires the distribution of varying quantities of effort among many different tasks, a situation that is typical for many jobs. Conversely, if fixed costs are high then efficiency may be achieved with a simple contract.

Proposition 7 *Suppose that all tasks have the realization $(\bar{\alpha}, \bar{\beta})$ or $\left(0, \underline{\beta}\right)$, and $\bar{\alpha}Y - \bar{\beta}Y^2 > f > \bar{\beta}Y^2/2$, then there is a first best contract with complexity $k+1$.*

Proof. Under these assumptions should some task be in state $(\bar{\alpha}, \bar{\beta})$ then efficiency implies that all effort should be assigned to this task. Should no task be in this state then effort should be allocated equally among all tasks. The complexity of the contract is determined by the complexity of the search protocol. In this case a very simple protocol that carries out a sequential search of tasks results in a simple contract. Define the events:

$$\begin{aligned} B_i &= \left\{\omega \in \Omega | \left(\alpha_j, \beta_j\right) = \left(0, \underline{\beta}\right), j = 1, ..i\right\} \\ A_i &= \left\{\omega \in \Omega | \left(\alpha_j, \beta_j\right) = \left(0, \underline{\beta}\right), j = 1, ..i-1 \text{ and } (\alpha_i, \beta_i) = (\bar{\alpha}, \bar{\beta})\right\}. \end{aligned}$$

Event B_i corresponds to having low productivity for tasks 1 to i, while A_i corresponds to having low productivity for the first $i-1$ tasks, and high productivity for task i. Let A_0 denote the event that all tasks have zero productivity. The search process for this case is illustrated in figure (3), resulting in the partition $P = \{A_0, A_1, ..., A_k\}$. The first best contract with complexity $k+1$ is achieved by allocating all effort to task i if event A_i occurs, $i \neq 0$, and to allocate effort equally over $\bar{n}$tasks should event A_0 occur, where $\bar{n}$ is determined by the level of fixed costs (that is $\bar{n} \approx \sqrt[3]{2\underline{\beta}Y^2/f}$). ∎

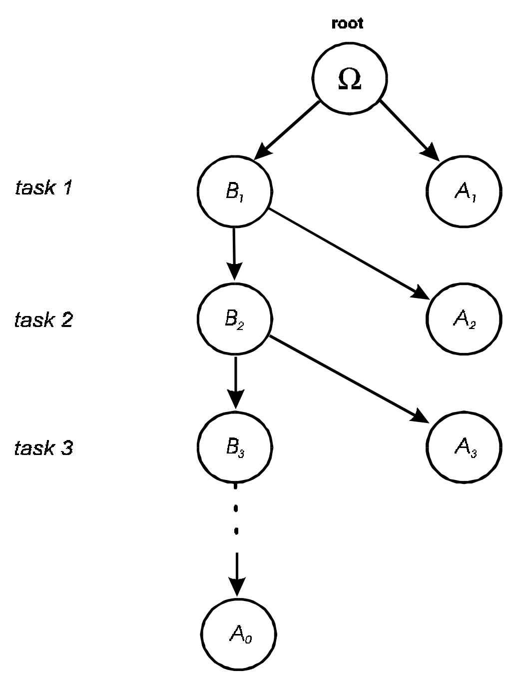

Figure 3: Search Graph with Simple Task Assignment

In summary, explicit contracts for services that require effort to be allocated to a number of different tasks are likely to be very incomplete due to the complexity of considering all contingencies. In these cases, increasing the productivity of output, keeping all other parameters fixed, increases the number of contingencies used by the "best" contract. When the fixed cost of allocating effort to a task is sufficiently high, the optimal contract involves all effort to be allocated to a single task. In this case contract costs are linear in the number of tasks, and hence a efficient complete state contingent contract is feasible.

5 The Employment Relationship

This section illustrates how the employment relationship provides a natural solution to the problem of contract incompleteness. I begin with Simon (1951)'s idea that employment is characterized by a delay in decision making until the state of the world has been revealed[11]. Rather than specifying what the employee should do in every state of the world, she is simply directed to do a good job. Then given the state of nature and the actions chosen by the employee, the employer decides upon whether to keep the employee, or provide additional compensation. Observe that in the current setup such a process is computationally much less complex that a state contingent contract written *ex ante* because the employee and employer need to decide upon an appropriate course of action for a single state (at a cost γ), rather than for all potential states (with costs as illustrated in tables (1) and (2))[12].

[11] Simon's view of authority is somewhat different from the concept in this paper. Here I follow Grossman and Hart (1986) and view authority as a residual claim that gives the employer the right to hire, fire and provide additional compensation to the worker. It is these instruments that provide the employee with an incentive to carry out the wishes of the employer.

[12] Another way to think about the computational simplicity of the employment relationship is terms of the distinction between P and NP problems as developed by Cook (1971), and Karp (1971).

Problems of class P can be solved in polynomial time, and are considered tractable. In the case that $f = 0$, the problem of task assignment is a convex optimization problem and hence of class P. Problems of class NP can be solved by a *non-deterministic* polynomial time algorithm. In this class of problems the algorithm first "guesses" a possible answer, and then uses a polynomial time algorithm to check if a solution is found. For example consider the problem of coloring a map so that no two adjacent regions have the same color. It is very easy to guess a pattern of colors and then check if a solution has been found. The problem is that the number of possible colorings is an exponential function of the number of regions, so that for large maps this algorithm is not tractable. It is still not

However, in the absence of an explicit contract the employment relationship must be structured so that the agreement is self-enforcing[13]. I adapt the model of MacLeod and Malcomson (1989) to show that when the value of the relationship is sufficiently large relative to market alternatives, then an efficient self-enforcing contract exists implementing the first best. In this case an employment relationship strictly dominates the use of an explicit contract. To illustrate these points consider a simple repeated employment relationship, with stages as illustrated in figure (3). The job of the employee, given the realization ω_t, is to decide how to allocate effort $\mathbf{y}_t$. The employer observes both the state and the choice of the employee, and then decides on compensation, part of which might be contracted upon.

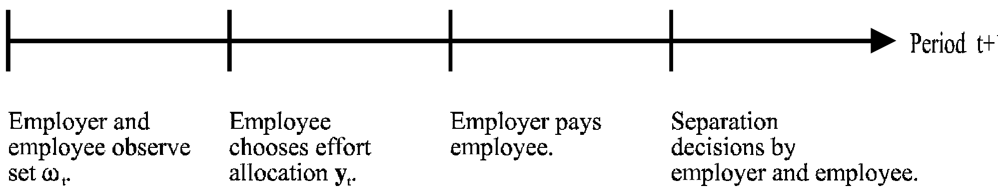

Figure 4: Sequence of Decisions for Employment Relationship in Period t

Following MacLeod and Malcomson (1989), suppose that the employment contract consists of a base wage w_t, that the firm pays whatever the worker's performance, combined with a bonus $b(\mathbf{y}_t,\omega_t)$ that the firm pays at the end of the period as a function of the effort vector, $\mathbf{y}_t$, and the state of the world, ω_t. Thus the end of period wage is $W_t = w_t + b(\mathbf{y}_t,\omega_t)$. Without loss of generality suppose that the bonus payment is bounded below by zero. One can achieve a negative bonus payment by setting $w_t < 0$.

The firm evaluates the worker at the end of the period, and then decides upon the bonus pay. The worker and firm then simultaneously decide whether or not to continue the relationship[14]. The firm is required to pay

know whether or not the hardest problems in the class NP, called $NP-hard$ problems, are of exponential complexity.

In the context of contracting, agents may decide to contract for a small set of contingencies in the hope that the realized state corresponds to one of the anticipated states. In this case complexity is reduced by trying to "guess" what will be the future outcome.

[13]In the legal literature it well understood that contracts are not only incomplete, but that reputation effects may provide a solution. See Posner (1986), page 79. The analysis here highlights the role of multi-tasking for the generation of contractual incompleteness.

[14]MacLeod and Malcomson (1989) demonstrate that a simultaneous move is not stricty necessary, however it greatly simplifies the characterization of the set of equilibrium allocations.

the wage w_t, however given that there is no explicit contract in place the worker is free to choose effort as she wishes. Incentives to perform are provided by the firm's bonus pay and the threat of termination. Should the employee decide to shirk then the maximum penalty would be to pay no bonus and fire the worker. Given this threat, a shirking employee would allocate effort among the least costly activities to solve:.

$$V(\boldsymbol{\beta}_t) = \min_{|\mathbf{y}|=Y} \boldsymbol{\beta}_t C(\mathbf{y}) + F(\mathbf{y}) \tag{5}$$

Let $\bar{U}_t$ denote the utility for the worker in the next best alternative, then under the maximal punishment for shirking the worker's incentive constraint is:

$$U_t = W_t - \boldsymbol{\beta}_t^T C(\mathbf{y}_t) - F(\mathbf{y}_t) + \delta E\{U_{t+1}\} \geq w_t - V(\boldsymbol{\beta}_t) + \delta E\{\bar{U}_{t+1}\}.$$

This can be rewritten:

$$E\{U_{t+1} - \bar{U}_{t+1}\} \geq \frac{\left\{V(\boldsymbol{\beta}_t) - \boldsymbol{\beta}_t^T C(\mathbf{y}_t) - F(\mathbf{y}_t)\right\} - b(\mathbf{y}_t,\omega_t)}{\delta}. \tag{6}$$

Notice that if no bonus payment is made then the worker's future utility from employment must be strictly greater than the market alternative, in which case the firm would have to pay an efficiency wage to provide incentives. This can be avoided with the use of a bonus payment.

However, when the employer is expected to pay a bonus (approximately 1/3 of workers receive some form of bonus pay in the U.S.), the lack of an explicit contract creates an incentive to renege because only the wage payment w_t is enforceable. The employee provides the employer with an incentive to pay a bonus with the threat to leave or shirk should it not be paid. Such a threat is a practical possibility. Stewart (1993) describes a group of securities traders who left First Boston Bank because they felt that their admittedly high bonus were not large enough.

Let $\bar{\Pi}_t$ be the firm's market opportunity should the worker leave, then with incorporating the employee's threat, the firm's incentive constraint is given by:

$$E\{\Pi_{t+1} - \bar{\Pi}_{t+1}\} \geq b(\mathbf{y}_t,\omega_t)/\delta. \tag{7}$$

Notice that increasing the bonus relaxes the worker's incentive constraint while making the firm's constraint more binding. These effects are exactly offsetting, and hence the existence of an efficient self-enforcing agreement depends only on the existence of sufficient future gains from trade.

Proposition 8 *An effort level $\mathbf{y}_t$ can be implemented in an employment relationship without state contingent contracts if and only if*

$$S_{t+1} \equiv E\left\{U_{t+1} + \Pi_{t+1} - \bar{U}_{t+1} - \bar{\Pi}_{t+1}\right\} \geq \frac{V(\boldsymbol{\beta}_t) - \boldsymbol{\beta}_t C(\mathbf{y}_t) - F(\mathbf{y}_t)}{\delta} \quad (8)$$

MacLeod and Malcomson (1989) prove this result using a version of the perfect equilibrium concept[15]. This result demonstrates that when it is not possible to contract explicitly for effort, the provision of incentives in period t require the return from continuing the relationship, S_{t+1}, be at least as large as net gains from shirking, $B_t \equiv V(\boldsymbol{\beta}_t) - \boldsymbol{\beta}_t C(\mathbf{y}_t) - F(\mathbf{y}_t)$. The use of bonus payments affects who should be receiving the rent, but not the level of the over all rent needed to enforce an implicit contract.

At this point one may wonder if the amount of rationality implicit in these equations is inconsistent with the assumed constraints on rationality assumed thus far. In terms of the bonus payments what is crucial is not that the agent knows the likely payments *ex ante*, but rather she believes that the firm will provide the appropriate reward for performance. These equations can be viewed as providing limits on the size of the payments that would be appropriate. If the worker discovers an unusually productive way of carrying out a task, or has performed well in the face of adverse conditions, then these events should affect the bonus payments.

For example the firm may offer a wage such that $w_t \approx \bar{u}_t + V(\boldsymbol{\beta}_t)$, the alternative utility in period t, plus an amount approximately equal to the dis-utility of effort when shirking. It is not important that there is strict equality, only that there are sufficient rents and that both the worker and firm can agree on a wage that they feel is appropriate. In this case the bonus payment can be defined to ensure the worker has no incentive to shirk:

$$b(\mathbf{y}_t, \omega_t) = \{V(\boldsymbol{\beta}_t) - \boldsymbol{\beta}_t C(\mathbf{y}_t) - F(\mathbf{y}_t)\}.$$

Observe that neither the firm nor the work need to know the state of nature when implicitly agreeing upon this bonus payment. All this says is should the worker be required to put forward effort that is more costly than $V(\boldsymbol{\beta}_t)$, then she should be compensated. If an appropriate bonus is not forthcoming, then, like the traders at First Boston Bank described by Stewart (1993), the employee can leave and impose retraining costs upon the firm. Given that all evaluation takes place *ex post*, neither the worker nor the firm are required

[15] They use a version of perfection that is stronger than subgame perfect. Essentially their result demonstrates that the timing of the separation decision, that is who gets to quit first, does not affect the set of equilibrium payoffs.

to anticipate or think about all possible contingencies at the beginning of the relationship.

A second issue concerns the existence of a rent S_t need to enforce the contract. Even if workers are paid competitive wages, Klein, Crawford, and Alchian (1978) observe that search costs, retraining costs and moving costs all imply that there exist specialized quasi-rents ensuring that $S_t > 0$. Other ways that such a rent may be created include including unemployment (Shapiro and Stiglitz (1984)), reputation (MacLeod and Malcomson (1988)) and the existence of overlapping generations of workers (Crémer (1986)). Suppose then that the surplus is strictly positive, then the issue is the conditions under which self-enforcing contracts regulated by an employment relationship are more efficient than an incomplete contingent contract. To highlight the role of multi-tasking I make the following assumption concerning the payoff structure.

Assumption 1 *The productivity and cost levels for the k tasks are independently and identically distributed with supports $\alpha_i \in \{0, \bar{\alpha}\}$, $\beta_i \in \{b^1, b^2\}$, where $\bar{\alpha} > 0$, $b^2 > b^1 > 0$.*

The distortions in the model arise from the worker's desire to allocate effort to the easier tasks. These distortions are smaller in environments where all tasks are of equal difficulty or equivalently, the worker enjoys doing them all. This is a situation for which an employment relationship is likely to be more efficient.

Proposition 9 *Given assumption 1, suppose the rent to the relationship, S_t, is strictly positive then for b^2 sufficiently small an employment relationship can achieve the first best.*

Proof. If no task has positive productivity the it is efficient for the worker to minimize effort, in which no surplus is needed to implement the contract. If at least one task has positive effort then for b^2 sufficiently small, allocating all effort to that task is approximately efficient. As b^2 is made smaller, then

$$\lim_{b^2 \to \mathbf{0}} V(\boldsymbol{\beta}) - b^2 \mathbf{Y}^2 - f = 0,$$

from which it follows the incentive constraint 8 is satisfied with a strictly positive S_t, and an efficient task assignment can be implemented with a self enforcing contract. ∎

This result has the intuitive interpretation that in situations with low agency costs then an employment relationship is more efficient that an explicit contract. For example jobs that involve mainly decision making, such as managerial work, the relative cost of different types of decisions may be very similar, thus it is the judgement of where to allocate time that is most important. In this case it may more efficient for the manager to be judged on the outcomes of these decisions, rather than trying to construct a contract that delineates how these decisions are to be carried out. Moreover, observe that for strictly positive b^2 the cost of the first best explicit contract does not fall because the number of contingencies to be considered does not change.

Consider now the case for which the number of tasks are increased, while the number of cost and benefit levels is fixed. Under assumption 1 and the assumption of quadratic cost of effort employed prefer spreading effort over more tasks. In this case increasing the number of tasks ensures that an employment relationship is strictly more efficient that an explicit contract.

Proposition 10 *Given assumption 1, $f = 0$ and $S_t > 0$. Further suppose that there is a strictly positive probability that each task has a positive productivity. Then for a sufficiently large number of tasks the expected value of trade with an employment relationship is greater than with an state contingent contract.*

Proof. This results follows from the quadratic nature of the effort costs. Let

$$p(\epsilon, k) = prob\left\{\max_{1\leq i\leq k} y_i^*(\omega) \leq \epsilon\right\},$$

the probability that the effort assigned to task i at an optimal effort choice is greater than ϵ when there are k tasks. Due to the time constraint, the independence assumption and the quadratic effort costs one has $\lim_{k\to\infty} p(\epsilon, k) = 1$. The concavity of costs implies

$$prob\left\{\boldsymbol{\beta} C(\mathbf{y}^*(\omega)) \leq \left(\frac{Y}{\epsilon}\right)\beta^2\epsilon^2 = Y\beta^2\epsilon\right\} \geq p(\epsilon, k).$$

This implies that for large k, one can make the effort costs at the optimal allocation arbitrarily close to zero, and hence given that the rent is strictly positive one may use an employment relationship to implement the optimal contract at least $p\left(S_t/Y\beta^2, k\right)$ fraction of the time. With strictly positive costs of using an explicit contract then in expected value a pure employment relationship is strictly more efficient. ∎

Therefore in jobs where there are a large variety of tasks they are most efficiently regulated with self-enforcing contracts rather than with an explicit contingent contract. Of course the current setup does not preclude the use of a mixture of *ex ante* and *ex post* criteria in the employment relationship. Baker, Gibbons, and Murphy (1994) have an example of a simple repeated relationship that uses both implicit and explicit contracts. In their model a contingent contract in some cases increases the size of the surplus S_t, and hence lowers the cost of using a self-enforcing contract for other aspects of performance. The critique of Maskin and Tirole (1995) also applies to their model, therefore an interesting question for future research is to understand how agents in practice might mix contingent contracts with implicit contracts when bounded rationality constrains the set of feasible contracts.

6 Discussion

In this essay I have illustrated how multi-tasking can leading naturally to contracts that are incomplete because of bounded rationality. Employment relationships can dramatically reduce the complexity of governance by delaying judgement of an employee's performance until after the state of the world is revealed. Institutions such as tenure, promotion and bonus pay are all examples of employment institutions that base rewards on an *ex post* evaluation of employee performance.

Thus complexity considerations naturally generate a distinction between the *ex ante* and *ex post* states of the world that underlie the recent work on incomplete contracts. In this paper I have illustrated this in the context of the employment model of MacLeod and Malcomson (1989) which highlights the need for a match specific rent to ensure the existence of an efficient incentive contract. This creates a trade off between the use of an explicit *ex ante* contract and an implicit contract governed by *ex post* rewards. If the number of dimensions of effort or output are small, then *ex ante* contracting is likely to be more efficient. It is in these situations that we are likely to observe the use of agency contracts that depend on explicit state contingent contracts (for example Holmstrom (1979) and Holmström and Milgrom (1987)).

When there is significant multi-tasking then the number of contingencies is astronomical, making it impossible for individuals to consider or even think about all possibilities. This observation, combined with the results of Maskin and Tirole (1995), highlight the role of bounded rationality as a foundation for incomplete contracts[16]. If agents were able to consider all possible events then, as Maskin and Tirole (1995) show, that the fact that

[16] See also the discussion in Tirole (1994).

contingent contract cannot be written does not affect the set of possible allocations.

This suggests that it is the lack of foresight that underlies the incomplete contract models used for the theory of the firm developed in Hart (1995). An interesting implication is that bounded rationality may provide a way to integrate the incomplete contracts approach with the Knightian theory of the firm based on the distinction between risk and uncertainty (Knight (1921)). The theory of bounded rationality used in the paper is explicitly based on the idea that one cannot anticipate possible future events, and hence having probability beliefs over these events is not possible. Therefore the unfolding of these events can be formally identified with Knightian uncertainty, whereas anticipated events would be assigned subjective probabilities, treated as risk and explicitly contracted upon as in the Arrow-Debreu model. The incomplete contract model proposed here suggests that interpreting residual rights and ownership as a reallocation of "uncertainty" may provide a way to formally differentiate the theory of the firm from the agency problem. These are issues that await future research.

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