Market Design for Distributional Objectives in (Re)assignment: An Application to Improve the Distribution of Teachers in Schools*

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Abstract

Centralized (re)assignment of workers to jobs is increasingly common in public and private sectors. These markets often suffer from distributional problems. To alleviate these, we propose two new strategy-proof (re)assignment mechanisms. While they both improve individual and distributional welfare over the status quo, one achieves two-sided efficiency and the other achieves a novel fairness property. We quantify the performance of these mechanisms in teacher (re)assignment where unequal distribution of experienced teachers in schools is a widespread concern. Using French data, we show that our efficient mechanism reduces the teacher experience gap across regions more effectively than benchmarks, including the current mechanism, while also effectively increasing teacher welfare. As an interesting finding, while our fairness-based mechanism is very effective in reducing teacher experience gap, it prevents the mobility of tenured teachers, which is a detrimental teacher welfare indicator.

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1 Introduction

The centralized (re)assignment of workers, which involves first time assignment of new employees and reassignment of existing ones, to jobs, tasks, or managers is increasingly common in both public and private sectors. In many countries, doctors are centrally

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(re)assigned to hospitals, police officers to precincts (Sidibe et al., 2021), teachers to public schools (Combe et al., 2020, Bobba et al., 2021, and Bates et al., 2021), and civil servants to regional jobs (Thakur, 2020). Centralized (re)assignment also exists in the private sector. Within large corporations, rotation procedures are commonly used to (re)assign workers to jobs (Cheraskin and Campion, 1996).[1]

From a market design perspective, these labor markets are different from other two-sided matching markets studied in the literature in various aspects. First, they are characterized by the presence of both *new workers* who need to be assigned their first job and *existing workers* who already hold a job and might therefore free up their initial position in the (re)assignment process. As a result, there are already occupied positions as well as vacant ones. Second, existing workers are usually allowed to keep their job if they do not obtain a better one, which might be at odds with employer preferences over workers. Mechanism designers need to carefully consider these novel aspects.

They are also interesting from a policy and design perspective because many of them suffer from distributional problems. Rural hospitals have difficulties recruiting doctors (Kamada and Kojima, 2015). Police officers tend to shy away from urban city centers that are prone to violence (Sidibe et al., 2021). Public administrators in India prefer and are assigned jobs close to their home states, which is seen as an obstacle to national integration objectives (Thakur, 2020). Moreover, good teachers are almost never equally distributed among schools (Hanushek et al., 2004 and Jackson, 2009). Some countries and cities try to solve this unequal distribution problem by making disadvantaged jobs or locations more attractive through higher salaries or better working conditions (Bobba et al., 2021, Biasi, 2021, and Falch, 2010).

Addressing the unequal distribution of workers through higher salaries is often limited by two fundamental constraints. In the public sector, teachers, doctors, police officers, etc., are civil servants whose salaries are regulated by a rigid pay schedule (often based on experience) that prevents policy makers from using this incentive tool as a compensating factor. Wage elasticities are also often low in professions that attract workers based on intrinsic motivation, which makes wage policies ineffective or very costly (Bobba et al., 2021 and Bates et al., 2021). In such contexts, countries that use centralized (re)assignment mechanisms can benefit from an additional tool to mitigate the unequal distribution: the mechanism itself.

This motivates the research question of this paper: In the rich two-sided environments we mentioned above, how can we design (re)assignment mechanisms that fulfill certain distributional objectives? This paper introduces a model and two mechanisms,

[1] There are other examples of (re)assignment, such as the reallocation of students to schools in inter-district school choice in the US (Hafalir et al., 2019) as well as in intra-district school choice as a new application, see Appendix C.

and then empirically assesses how much the new mechanisms can improve the distribution of workers.

As our empirical application, we use the (re)assignment of public school teachers to regions of France; a market that suffers from large and persistent distributional problems. Good teachers, worldwide, tend to work in schools that serve more affluent students and schools with a higher share of native and high-achieving students (Bobba et al., 2021, Bates et al., 2021, Biasi, 2021, Hanushek et al., 2004, Jackson, 2009, Bonesrønning et al., 2005, and Allen et al., 2018). Despite a rich emerging literature that investigates how effective wage policies can fare in attracting good teachers to disadvantaged schools in countries with decentralized hiring practices, our understanding of the role played by mechanisms is still limited for centralized (re)assignment.[2] Our paper sheds light on this question. Although we present the theory with a focus on teachers, all theoretical results apply more broadly to the applications mentioned earlier.[3]

Fundamental design desiderata. We start by introducing a two-sided matching framework in which we explicitly model each school's preference over the distribution of types of teachers who are matched with it. Each teacher's type captures her observable characteristics such as her experience, education, past performance, etc. In addition, teachers can either be tenured or new. Tenured teachers are initially assigned to positions, which is captured by a *status-quo matching*. New teachers do not have initial assignments. As in the standard matching settings, teachers have preferences over schools.[4] For schools, we assume that a school is better off if the type distribution of teachers who are assigned to this school *Lorenz dominates* its status-quo teacher distribution. In other words, a school is better off if, for each teacher type, the number of matched teachers with a weakly more-preferred type weakly increases after the match. We impose that our mechanisms improve upon the status quo not only for teachers, but also for schools, that is, to be *status-quo improving*. This is an important criterion that ensures an increased welfare of teachers is not achieved at the cost of a poorer distribution of teachers, and vice versa.

There are two separate and equally valid interpretations of school preferences. While

[2]Countries that use a centralized process to assign teachers to schools include Germany, the Czech Republic (Cechlárová et al., 2015), Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru (Bobba et al., 2021), Uruguay (Vegas et al., 2006), and Portugal.

[3]We discuss some of these other applications in more detail in Appendix C.

[4]This assumption is valid in settings like ours, where individuals' preferences do not depend endogenously on their (re)assigned peers but mostly on other exogenous factors such as where the school is located (see Section 5.4). In some other applications, preferences over (re)assigned peers may be as or, even more, important, such as in police officer assignment (Sidibe et al., 2021). In such applications, if the number of individuals seeking (re)assignment is relatively small, then our setup can still be used. For example, in France there were more than 700,000 teachers in primary and secondary education while 25,100 teachers were saught (re)assignment in 2013 (slightly more than 3.5% of the whole body). Thus, it is safe to assume that preferences of teachers are not endogenously affected by the (re)assigned peers. See Pycia (2012) for a theoretical treatment and Cox et al. (2021) for an empirical and large market treatment of such peer effects in other markets where they can be more relevant.

they can be intrinsic to the school reflecting real preferences, they can also be induced by a centralized authority to fulfill a distributional objective especially when schools are part of a resource pool that is collectively managed by this authority. In such cases, they correspond to designed *priorities* reflecting the centralized authority's distributional objectives. This dual interpretation in our theoretical framework allows us to be fully agnostic while referring to school preferences: they may truly be intrinsic preferences of schools or a policy tool by the central authority to fulfill its distributional objectives.[5,6]

In addition to status-quo improvement, we require *strategy-proofness for teachers* as teacher preferences are private information. We do not consider preference revelation for schools as their preferences are either based on a known metric about teacher types or customized as priorities by the central authority, and hence, are known.

We also consider two additional criteria: *two-sided efficiency*—where welfare entities are both teachers and schools—and *fairness*. As school preferences can be intrinsic or customized by the designer as priorities, the motivation behind two-sided efficiency is straightforward. In the first case, two-sided efficiency literally pays attention to each school's welfare. In the second case, if one could Pareto-improve the assignment for schools, the distribution of teachers across schools could be further improved. The motivation for a fairness-based axiom comes from the current teacher assignment procedures, for example, in France. However, the standard fairness notion based on the elimination of justified envy (and on stability) (see Balinski and Sönmez, 1999, Abdulkadiroğlu and Sönmez, 2003, and Gale and Shapley, 1962) is not suitable for our desiderata as there may not be a status-quo improving matching that eliminates justified envy. We, therefore, introduce a novel fairness notion that is neither weaker nor stronger than the standard elimination of justified envy.

Two-sided efficiency and our fairness criteria are in conflict.[7] We introduce two mechanisms that are status-quo improving and strategy-proof for teachers such that one is also two-sided efficient while the other satisfies our novel fairness concept.

The status-quo improving efficient mechanism. We focus on a concept that is stronger than the combination of status-quo improvement and two-sided efficiency which we refer to as *status-quo improving teacher optimality* (or SI teacher optimality). An SI teacher optimal matching is status-quo improving and Pareto undominated for teachers among all status-

[5] We also weaken status-quo improvement through *improvement over a target lower bound distribution* rather than strict status quo (see Appendix A).

[6] In this latter interpretation which we adopt in our empirical work, starting from an unequal distribution of teachers, we believe that a good design of schools' priorities to reflect central authority's objectives together with weaker or stricter versions of status-quo improvement allows to achieve a more even distribution. One of the main goals of our empirical analysis, as we will discuss below, is to provide support for this claim.

[7] See Example A.6 in Appendix D. This is different from the one-sided efficiency and (standard) fairness conflict (Roth, 1982 and Balinski and Sönmez, 1999), which also exists in our setup.

quo improving matchings.[8]

The SI teacher optimal mechanism we introduce, the *status-quo improving cycles and chains* (hereafter, SI-CC), is related to top-trading cycles (TTC) mechanisms (especially Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, and Roth et al., 2004). However, it has one key difference: while TTC-variant mechanisms ensure that only teachers necessarily are better off as we execute exchanges, under SI-CC both teachers and schools become better off with respect to the status quo.

The SI-CC outcome is determined through an algorithm which runs iteratively on directed graphs and assigns a group of teachers new positions in each round. When a cycle or an appropriately defined chain of the directed graph is encountered, we assign each teacher in it to the school to which she is pointing.

Compared to TTC algorithms, we introduce two main innovations in the SI-CC algorithm by endogenously defining pointing rules to form the directed graphs. First, we define the *school pointing rule* that determines the order in which a school would like to send out its status-quo employees. By pointing, a school effectively gives permission to one of its status-quo employees to be assigned to a different school. We define the pointing order such that a school points to one of its least-preferred-type employees first (therefore, she leaves first) and more-preferred-type employees later in the directed graph.[9] The second innovation pertains to the *teacher pointing rule* that determines to which schools a teacher can point and, therefore, be assigned. We allow a teacher to point to a school if either one of the two following conditions are satisfied: replacing with her the teacher pointed to by that school does not make the school worse off with respect to the status quo or if there is a vacant position at the school that can be filled. In the directed graph, a teacher points to the best school to which she is allowed to point.

We carefully tailor the pointing rules and the order in which cycles and chains are cleared. If we allowed substantial changes to these orderings, the mechanism might not be two-sided efficient, strategy-proof or status-quo improving. Our precise formulation

[8] Proposition 1 shows that any SI teacher optimal matching is also two-sided efficient. There are two rationales for focusing on a notion seemingly favoring the teacher side in two-sided efficiency. First, we show that no status-quo improving mechanism is strategy-proof if it chooses a different two-sided efficient and status-quo improving matching from the SI teacher optimal one whenever it can (see Proposition A.1 in Appendix B). Moreover, empirically this notion helps a large number of tenured teachers move; an important pre-requisite for any mechanism that could replace the current mechanism in France, which leads to large movement but does not have good distributional properties.

[9] This is practically in the reverse order of how a traditional priority order would be embedded under TTC.

makes SI-CC satisfy these three properties (Theorem 1).[10,11,12]

The status-quo improving fair mechanism. Next, we turn our attention to fairness, noting that the standard fairness notion—elimination of justified envy (Balinski and Sönmez, 1999)—and status-quo improvement may be in conflict (for example, see Compte and Jehiel, 2008 and Pereyra, 2013). Indeed, status-quo improvement for teachers implies that a tenured teacher stays at her status-quo assignment if she cannot obtain any better school. For example, if she is one of the less preferred teachers of her status-quo school, justified envy may form. To overcome the conflict between fairness and status-quo improvement for teachers, the standard approach consists of weakening elimination of justified envy by ignoring the envy caused by assigning a tenured teacher to her status-quo school. For instance, this weakened fairness notion is currently satisfied by the French teacher assignment mechanism.

However, we show that to impose status-quo improvement, in contexts where schools initially have vacant positions, this weakening alone does not resolve the conflict. We introduce a novel notion, *status-quo improving fairness* (or SI fairness), which implicitly gives new teachers rights over the vacant positions of a school. Under a mild overdemand assumption involving new teachers and schools with vacant positions, which is empirically satisfied in many applications (including ours as discussed below), we prove that an SI fair matching exists. We introduce a strategy-proof, non-wasteful, and SI fair mechanism, the *status-quo improving deferred acceptance mechanism* (hereafter, SI-DA).[13]

To find the outcome of the SI-DA mechanism, we first design novel auxiliary choice rules for schools defined through slot-specific priorities and a linear processing order of these slots (see Kominers and Sönmez, 2016). Then we employ a variant of the teacher-proposing deferred acceptance algorithm of Gale and Shapley (1962) that is tailored to be

[10] We earlier mentioned that for strategy-proofness and status-quo improvement, we need to focus on SI teacher optimal mechanisms among two-sided Pareto efficient ones (see Footnote 7). The key observation for the SI teacher optimality of the mechanism is that a school does not need to improve from one round to the next to improve upon status quo, for example, its welfare can sometimes decrease across rounds; yet, by using a buffer function, we make sure it is always weakly better off with respect to the status quo. Therefore, teachers can move more freely with less constraints, which in turn increases their welfare and achieves SI teacher optimality.

[11] The key observation for strategy-proofness that is different from TTC is the following. The set of schools to which a teacher can point weakly shrinks through rounds as she may no longer be allowed to point to some schools even if these schools still have positions to fill. However, we show that no future assignment opportunity can be changed by this teacher through a preference manipulation even if she points early to a school that will leave this set later.

[12] Status-quo improvement is ensured for schools through the coupling of their pointing rule and teacher pointing rule that uses the aforementioned buffer function in Footnote 10. This feature does not exist in TTC-variant mechanisms. Status-quo improvement for teachers is ensured as they are guaranteed to be at least matched with their status-quo assignment.

[13] The overdemand assumption combined with giving higher priority to new teachers for vacant positions restrict tenured teachers from fleeing away from their status-quo assignments without being replaced. In the absence of such a restriction some schools might be worse off compared to their status-quo assignment.

used with complex choice rules (see Kelso and Crawford, 1982 and Roth and Sotomayor, 1990).

One of our contributions is the design of appropriate auxiliary choice rules for schools with intricate individual slot rankings and the processing order of slots, which have both fairness and welfare consequences. In the previously studied domains, all slot rankings consistent with school preferences and slots' processing orders would lead to fair outcomes, unlike ours. Moreover, the effects of the slot processing order on the welfare of the applicants were ambiguous unless special conditions were also satisfied (see Dur et al., 2018).

We prove that SI-DA is SI fair when our overdemand assumption is satisfied, which requires an independent proof without invoking previously known results, and is also strategy-proof (Theorem 2).[14]

Empirical application: Improving unequal teacher distribution in France. In the second part of the paper, we quantify the gains that our mechanisms can bring by using data on the annual centralized (re)assignment of teachers to the regions of France.[15] This labor market is particularly appropriate to study our question because it suffers from severe imbalance in the distribution of teachers (see map in Figure 1).

About 50% of the tenured teachers who ask to change region come from two regions (out of 25) in the suburbs of Paris—called Créteil and Versailles—that are particularly disadvantaged and unattractive for teachers. To compensate for the large exit flows, a majority of the new teachers are assigned to one of these two regions. This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France and it is seen as one of the structural determinants of the large achievement inequalities that France suffers from.[16] Reducing the unequal distribution of teachers across regions has become one of the priorities of French policy makers, as illustrated by this quote from the Ministry that evaluated the teacher transfer process in 2015 (IGAEN, 2015):

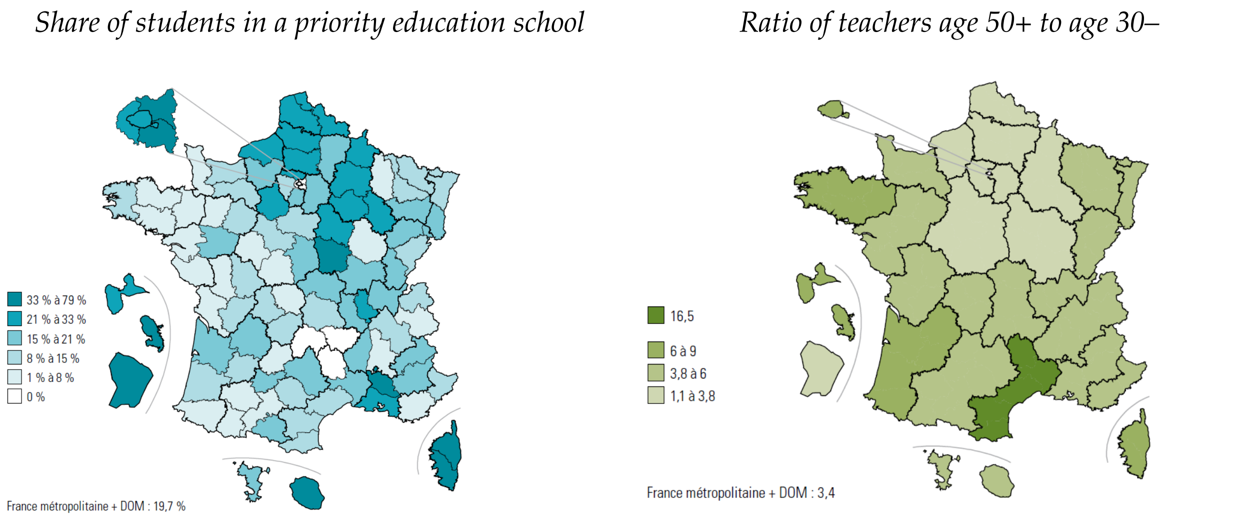
> "Inequalities among regions is not considered [by the current transfer process]. When data on mobility is provided, the statistics mostly focus on whether teachers' transfer requests are satisfied, with particular attention being paid to spousal reunion and

[14] We show that the auxiliary choice rules possess substitutes and law of aggregate demand properties (Proposition 4). These are sufficient properties for the deferred acceptance algorithm to be strategy-proof (see Hatfield and Milgrom, 2005).

[15] We consider the (re)assignment of teachers to regions instead of individual schools for reasons that will be clear in the empirical section.

[16] The PISA results show that, in OECD countries, a more socio-economically advantaged student scores 39 points higher in Math than a less-advantaged student, which is equivalent to one year of schooling. There is a large variation among countries in how much a student's social background predicts her school achievement, and France is one of the worst countries on this inequality indicator, ranking fourth position from the bottom.

Figure 1: Share of Disadvantaged Students and Experienced to Inexperienced Teacher Ratio in France



Notes: The left map plots the share of students enrolled in a "priority education" school in each region, a label given to the most disadvantaged schools in France. The right map plots the ratio of the number of teachers older than 50 to the number of teachers younger than 30. Column (3) of Table A.2 provides the underlying statistics. The ratio is equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 times more teachers older than 50 than teachers younger than 30.

> teachers with a disability. These statistics are necessary, but they do not allow to account for inequalities among regions."

This observation lead them to the following recommendation:

> "Focus the discussion on mobility around a reduction of inequalities among regions."

Counterfactuals. We first empirically estimate preferences separately for tenured and new teachers under a *stability assumption* used by Fack et al. (2019). We then use the estimated teacher preferences, along with data on region priorities and vacant positions, to evaluate the performance of the two mechanisms we propose, SI-CC and SI-DA. We define a teacher type as her experience and assume that regions staffed with relatively younger teachers—referred to as *younger regions*—have a ranking over types that ranks teachers by decreasing levels of experience, that is, the more experienced teachers are ranked higher than the less experienced teachers. On the other hand, the regions staffed with relatively older teachers—referred to as *older regions*—rank teachers by increasing levels of experience.

Our motivation for designing type rankings as above is to ensure that the distribution of teachers becomes more even under the status-quo improvement property after the match, as SI-CC and SI-DA are both status-quo improving. The change in teacher distribution is the first empirical outcome we consider. On the other hand, imposing status-quo improvement may have a cost in terms of teacher welfare for some teachers, which motivates looking at the mobility of tenured teachers and the ranks of the regions teachers obtain in their preferences as additional empirical outcomes. To measure the effect of imposing status-quo improvement on both the distribution of teacher experience as well as on teacher welfare, we consider two alternative mechanisms, TTC* and DA*, which benchmark SI-CC and SI-DA, respectively, when we only impose status-quo improvement for teachers but not for regions. Our results vastly differ for SI-CC and SI-DA in comparison to their benchmarks.

Empirical performance of SI-CC versus its benchmark. When focusing on SI-CC, we observe that imposing status-quo improvement makes the distribution of teacher types more equitable with respect to status quo and also reduces the experience gap between younger and older regions with respect to its benchmark. For instance, SI-CC only assigns 1,344 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them.

We then investigate whether a better distribution is achieved at the cost of lower welfare for teachers, as measured by their mobility and the rank of the region they obtain in their estimated preferences. In line with the existence of a distribution–efficiency trade-off, fewer teachers manage to obtain an assignment other than the status quo under SI-CC than under the benchmark TTC* (5,356 versus 6,382 teachers). The distribution of ranks that tenured teachers obtain under the benchmark also Lorenz dominates that under SI-CC.

To conclude, our results show that SI-CC effectively improves teacher distribution. We empirically quantify the expected trade-off that exists between the distribution of teacher experience and teacher mobility. This brings supporting empirical evidence for our new approach where school preferences can be used as instruments to reduce the inequalities in teacher distribution in a context where SI-CC is the mechanism in use.

Empirical performance of SI-DA versus its benchmark. The picture is radically different for SI-DA. One of the most interesting insights of our empirical exercise is that imposing status-quo improvement to mechanisms designed to sustain fairness can backfire. We first show that imposing status-quo improvement has a tremendous mobility cost for SI-DA: no tenured teachers move from their initial position under SI-DA, compared to 894 under its benchmark DA*. In addition, in the three youngest regions of France, SI-DA produces a distribution of teacher experience which does not Lorenz dominate the distribution under DA*. In the three oldest regions, our results are even more striking: DA* produces a distribution of teacher experience which Lorenz dominates the distribution under SI-DA. Put differently, under SI-DA, status-quo improvement fails to achieve its goal: the distribution of teachers does not improve and the experience gap between younger and older regions does not decreases as intended. This is because, under status-quo improvement, many teachers cannot leave the disadvantaged regions. Now, because these teachers do not move, other tenured teachers are not able to move either since, otherwise, it would violate our fairness requirement from these teachers who had to stay in the disadvantaged regions.

Relaxing status-quo improvement and comparisons to the current mechanism in France. We finally compare our mechanisms' performance to that of the current French mechanism, which is similar to DA* with the exception that the preferences of regions are constructed using the Ministry's priority points (primarily increasing based on teacher experience level) rather than the teacher type rankings we introduced which can be either decreasing or increasing based on experience level.

Our empirical analysis so far identifies an important trade-off between tenured teacher mobility from disadvantaged regions and status-quo improvement. If, for a policy maker, the trade-off goes in favor of teacher mobility, that is, one is willing to allow tenured teachers to leave disadvantaged regions at the expense of status-quo improvement for regions, our mechanisms can easily be modified to accommodate a more permissive requirement. In this weakened version of status-quo improvement, experienced teachers can leave the three youngest regions as long as they are replaced by teachers with any experience level. Apart from this modification, status-quo improvement property is still satisfied for all other regions.[17]

Our results show that the modified SI-CC mechanism that we call *weak* SI-CC (wSI-CC for short) leads to 5,554 teachers being assigned to a region other than their status-quo assignment versus 5,864 with the current mechanism in France. While this is a small difference in favor of the French mechanism, the average rank of the region that teachers are assigned in their preferences is smaller (7.3) under wSI-CC than under the French mechanism (8.3), such that on average, there is a slight teacher welfare gain under wSI-CC on this dimension. Therefore, taking different measures into account, the two mechanisms are comparable in terms of teacher welfare. On the other hand, wSI-CC yields a far more desirable distribution of teacher experience across regions than the current French mechanism. Younger regions get more experienced teachers and older regions get more inexperienced teachers. This distribution is very close to that of SI-CC, which is only slightly better for the younger regions. As a result, wSI-CC provides a convincing alternative to the current mechanism in France that would improve the teacher experience distribution without hurting teacher mobility.

[17] Appendix A gives a formal treatment of this property.

2 Model

Let T be a finite set of **teachers**. Each teacher t has a **type** which captures her observable characteristics that matter for the schools, such as experience, education, past performance, etc., or only a subset of these.[18] Let Θ be the finite type space and $\tau : T \to \Theta$ be the **type function** such that $\tau(t)$ is the type of teacher t. For any $\hat{T} \subseteq T$, we denote type θ teachers in $\hat{T}$ with $\hat{T}^\theta$, i.e.,

$$\hat{T}^\theta = \{t \in \hat{T} \,:\, \tau(t) = \theta\}.$$

Let S be a finite set of **schools**. Each school s has a **capacity** of q_s. Let $q = (q_s)_{s\in S}$. Each teacher t has a **strict preference relation**, which is a linear order and denoted by P_t, over the schools and **outside option** denoted by $\emptyset$. Let $P = (P_t)_{t\in T}$. We denote the at least as good as relation related with P_t by R_t for all $t \in T$: $s \; R_t \; s'$ if and only if $s = s'$ or $s \; P_t \; s'$.

A **matching** $\mu : T \to S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$. With a slight abuse of notation, we use μ_t and μ_s instead of $\mu(t)$ and $\mu^{-1}(s)$, respectively.[19] We refer to μ_t as the **assignment** of teacher t and μ_s as the **assignment** of school s in matching μ. Also for a subset of teachers $\hat{T}$, we denote the set of their matches in μ, $\mu(\hat{T})$ as $\mu_{\hat{T}}$.

Initially some teachers are already employed by some schools. This is captured by a **status-quo matching** ω. If $\omega_t = s$, then teacher t is a **status-quo employee** of school s. If $\omega_t = \emptyset$, then teacher t is called a **new teacher**. She is seeking employment for the first time and she is unemployed at the status quo. By definition, $|\omega_s| \leq q_s$ for each school s. We denote the set of new teachers with N, i.e.,

$$N = \{t \in T \,:\, \omega_t = \emptyset\}.$$

The rest of the teachers, that is, those who are status-quo employees of schools, are referred to as **tenured teachers**.

We make one assumption on teacher preferences: We assume that each tenured teacher finds her status-quo assignment acceptable, i.e. $\omega_t \; P_t \; \emptyset$ for each $t \in T \setminus N$.

Finally, we define school preferences over subsets of teachers. Unlike teacher preferences, these preferences are typically weak and allow indifferences. Typically $\succcurlyeq_s$ denotes the preferences of a school s over subsets of teachers. Let $\sim_s$ and $\succ_s$ be the associated indifference and strict preference relation with $\succcurlyeq_s$, respectively, denoting symmetric and asymmetric portions of the preferences.

In reality, are schools agents who have preferences over subsets of teachers or are they government-mandated entities or resources that have mandated priority orders over teachers, who are public servants, as in the French application? In our model, we are ag-

[18] For example, in the French application, the experience level of a teacher can be considered as the type of a teacher.

[19] Thus, μ_s^θ is the set of teachers of type θ that are assigned school s.

nostic about this as either interpretation works. A school can be either seen as an agent that cares about its teacher quality or it can be seen as part of a resource pool that is collectively managed by a central authority, which aims at improving the quality of employed teachers in every school as much as possible with an eye on not making any school worse than its status quo. It is therefore only a semantic exercise to call them as agent or resource and these binary relations as preferences or priority orders of the school dictated by a centralized authority. In the latter case, we propose these binary relations as a new priority design that can be used by a centralized authority to replace an existing one that solely relies on some other criteria, as in the French application.

To this end, each school s has a **type ranking**, which is a linear order and denoted by $\triangleright$, over the types of teachers and an **individual rationality threshold** type denoted by $\theta_\emptyset$: $\theta \triangleright_s \theta' \triangleright_s \theta_\emptyset \triangleright_s \theta''$ means school s ranks type θ teachers over type θ' teachers and finds both types of teachers **acceptable** to hire but it considers type θ'' teachers **unacceptable** to hire. Let $\theta \trianglerighteq_s \theta'$ if either $\theta \triangleright_s \theta'$ or $\theta = \theta'$. We assume that each school finds the types of its current teachers acceptable, i.e. if $\omega_s^\theta \neq \emptyset$, then $\theta \triangleright_s \theta_\emptyset$.

We make two assumptions on the preferences of schools.

1. We assume that when a school compares two subsets of teachers it uses **Lorenz domination** relation based on first-order stochastic domination of teacher distributions using its type ranking. In particular, school s weakly prefers subset of teachers $\bar{T}$ to $\hat{T}$, i.e., $\bar{T} \succcurlyeq_s \hat{T}$, if
   (i) there does not exist an unacceptable teacher in $\bar{T}$, i.e., $\bar{T}^\theta = \emptyset$ for any $\theta \triangleleft_s \theta_\emptyset$, and
   (ii) for any $\theta \triangleright_s \theta_\emptyset$ we have
   $$\sum_{\theta' \trianglerighteq_s \theta} |\bar{T}^{\theta'}| \geq \sum_{\theta' \trianglerighteq_s \theta} |\hat{T}^{\theta'}|.$$
   Moreover, the preference is strict if at least one of the inequalities is strict.
   If Lorenz comparison does not hold between two sets in either direction, then the school preferences do not compare them. Therefore, school preferences are incomplete. Consequently, a school only unambiguously prefers groups whenever two groups of teachers can be ranked based on this Lorenz comparison.
   In the rest of our analysis, we compare different matchings with the status-quo matching. Lorenz domination relation will be sufficient and we will achieve unambiguous comparisons for our purposes.[20]
2. We assume that for any subset of teachers T', if $\theta_\emptyset \triangleright_s \tau(t)$ for some $t \in T'$, then $\omega_s \succ_s T'$. That is, each school prefers its status-quo assignment to any teacher set with an unacceptable teacher in it.

We refer to the list $\langle T, S, q, \omega, P, \succcurlyeq \rangle$ as a **teacher (re)assignment market**. Typically,

[20]We refer to any preference relation that satisfies this assumption as **Lorenz preferences**.

$T, S, q, \omega, \succcurlyeq$ are commonly known in our applications. Only teacher preferences are private information. For the rest of our analysis, we fix $T, S, q, \omega, \succcurlyeq$ and denote a market with teacher preferences P.

We are seeking a matching given a market P.

The most basic property of matchings we consider is *status-quo improvement*. Perhaps surprisingly, this is sometimes in conflict with many other standard desiderata used in the literature for matching market design. A matching μ is **status-quo improving** if $\mu_t \; R_t \; \omega_t$ for all $t \in T$ and $\mu_s \succcurlyeq_s \omega_s$ for all $s \in S$. That is, each agent should be weakly better off in a status-quo improving matching with respect to the status-quo matching.

We inspect rules that select a matching for each market. Formally, a (**direct**) **mechanism** φ is a function that chooses a matching for any market P. Let $\varphi(P)$, $\varphi_t(P)$, and $\varphi_s(P)$ denote the matching selected by mechanism φ in market P, the assignment of teacher t, and the assignment of school s in that matching, respectively.

A mechanism φ is **strategy-proof** if truth telling is a weakly dominant strategy for all teachers, that is, for all markets P, for all teachers t, for all possible alternative preference reports P'_t,

$$\varphi_t(P_t, P_{-t}) \; R_t \; \varphi_t(P'_t, P_{-t}),$$

where $P_{-t} = (P_{t'})_{t' \in T \setminus \{t\}}$.

As we assume that schools' rankings over the types of teachers, and therefore, their preferences over the teachers are commonly known, schools do not need to report them.

In the next two sections, we provide two different mechanisms to achieve two different desiderata: a refinement of two-sided Pareto efficiency or an appropriate fairness concept for our applications together with status-quo improvement, respectively.

3 Efficiency and Status-quo Improving Cycles and Chains

3.1 Status-quo Improving Teacher Optimality

We start by defining our efficiency notion. Fix a market P. A matching μ **Pareto dominates** a matching ν **for teachers** if

$$\mu_t \; R_t \; \nu_t \;\text{ for all } t \in T, \text{ and} \tag{1}$$

$$\mu_{t'} \; P_{t'} \; \nu_{t'} \;\text{ for some } t' \in T. \tag{2}$$

Matching μ **Pareto dominates** a matching ν **for schools** if

$$\mu_s \succcurlyeq_s \nu_s \text{ for all } s \in S, \text{ and} \tag{3}$$

$$\mu_{s'} \succ_{s'} \nu_{s'} \text{ for some } s' \in S. \tag{4}$$

Finally, matching μ **Pareto dominates** a matching ν if (i) Equations 1 and 3 hold, and (ii) Equation 2 or Equation 4 holds. A matching is **two-sided Pareto efficient** if it is not Pareto dominated by any other matching.

A matching μ is **status-quo improving teacher optimal** (**SI teacher optimal** for short) if it is status-quo improving and not Pareto dominated for teachers by any other status-quo improving matching. While SI teacher optimality seems to care mostly about the welfare of teachers, any SI teacher optimal matching is also two-sided Pareto efficient, since teacher preferences are strict.

Proposition 1. *Any SI teacher optimal matching is two-sided Pareto efficient.*

All proofs are provided in Appendix B.

We will introduce a strategy-proof and SI teacher optimal mechanism. Why do we focus on SI teacher optimality rather than a dual concept such as SI school optimality or different two-sided Pareto efficient matchings?[21] There are two reasons: First, in most of the applications we have in mind, the side we characterize as schools are quasi-agents rather than full agents unlike the side we characterize as teachers. Their welfare has been the main efficiency measure both in practice and in literature (for example, see Abdulkadiroğlu and Sönmez, 2003 and Combe et al., 2020).

Second, if we wanted to implement other status-quo improving two-sided Pareto efficient matchings that are not SI teacher optimal whenever possible, we would not be able to find a strategy-proof mechanism (Proposition A.1 in Appendix B).

3.2 Pointing Rule Design and Status-quo Improving Cycles and Chains

Next, we will introduce a strategy-proof and SI teacher optimal mechanism. To achieve this goal, we introduce additional tools.

Our mechanism will iteratively construct a sequence of directed graphs in which teachers, schools, and outside option are the nodes. Teachers can only point to schools or the outside option and schools can only point to teachers in their status-quo assignment in each of these graphs. When node x points to node y, then a directed arc from x to y is activated.

Our mechanism relies on *executing* two types of multi-lateral exchanges based on the constructed directed graphs.

A **cycle** is a directed path of distinct teachers $t_1, \ldots, t_k$ and distinct options $x_1, \ldots, x_k \in S \cup \{\emptyset\}$, i.e., schools or the outside option,

$$(x_1, t_1, x_2, t_2, \ldots, x_k, t_k)$$

[21] Because of indifference classes in schools' Lorenz preferences regarding same-type teachers, this concept defined as a simple dual notion of SI teacher optimality may not be two-sided Pareto efficient and has to be refined even further.

such that whenever x_m is a school, $\omega_{t_m} = x_m$ for all m, each node points to the next node in the path, and t_k points back to x_1.

A **chain** is a directed path of distinct teachers $t_0, \ldots, t_{k-1}$ and (not necessarily distinct) schools $s_1, \ldots, s_k$

$$(t_0, s_1, t_1, \ldots, s_{k-1}, t_{k-1}, s_k)$$

such that $\omega_{t_m} = s_m$ for all $m = 1, \ldots k-1$, each node points to the next node in the path. Here, we say the chain starts with t_0 and ends with s_k. In other words, s_k is the head of the chain and t_0 is the tail of the chain.

As certain cycles and chains are encountered in the constructed graph, we will **execute** the exchanges in them by assigning each teacher to the school or the outside option to which she is pointing and remove her.

Our main theoretical innovation relies on designing *pointing rules* that designate which possible directed arcs in a graph will be endogenously activated through the algorithm.

The pointing rule of teachers will be introduced within the definition of the mechanism below as it uses the endogenous working of the mechanism's algorithm. On the other hand, the pointing rule of schools relies on their type rankings and an exogenously given tie breaker.

Formally, a **tie breaker** is a linear order $\vdash$ over teachers.[22] It can be randomly determined or can be the mandated priority orders for a particular application, such as in the French case, or can be exogenously fixed in some other manner.

The tie breaker for the new teachers and tie breaker regarding teachers employed at the status quo are utilized differently in the algorithm. For each school s, using tie breaker $\vdash$ and its type ranking $\rhd_s$, we first construct a **pointing order** $\gtrdot_s$ over teachers in ω_s, which is a linear order as well: For any two distinct teachers $t, t' \in \omega_s$,

$$t \gtrdot_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } \left[\tau(t) = \tau(t') \text{ and } t \vdash t'\right].$$

Note that a worse-type teacher is prioritized over a better-type teacher under $\gtrdot_s$, and only when two same-type teachers are compared, we use the tie breaker to prioritize one over the other.

As the mechanism will iteratively assign and remove teachers, the **pointing rule of schools** is "point to the highest remaining priority teacher in its pointing order."

Now, we are ready to define our mechanism through an iterative algorithm:

Definition 1. ***Status-quo Improving Cycles and Chains (SI-CC) Mechanism***

We will construct a matching μ dynamically through the following algorithm. Initially, μ is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in μ, they will be removed from the algorithm; similarly schools whose all positions are

[22] Technically, each tie breaker induces a new mechanism in our class.

filled in μ and also some other schools chosen by the algorithm will be removed.

For each school s and type θ, let b_s^θ track the ***current balance*** *of type θ teachers at school s in* ***current matching*** *μ, which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type θ teachers assigned to s in μ and the number of type θ teachers in its status-quo assignment assigned to any school in μ: $b_s^\theta = |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset\}^\theta|$. Thus, we initialize $b_s^\theta = 0$.*

A general step k is defined as follows:

Step k:

- *Each remaining school s points to the highest priority remaining teacher in ω_s under $\succ_s$, if not all teachers in ω_s are already assigned in μ; let t_s^k be the teacher pointed to by school s in step k. Otherwise, school s does not point to any teacher.*
- *We define the* ***pointing rule of teachers*** *as follows: Any remaining teacher t is allowed to point to a remaining school s if at least one of the following two school* ***improvement conditions*** *holds for school s via teacher t:*
  1. *(Improvement for s by teacher trades) if the school points to a teacher t_s^k and*

  $$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t),$$

  or[23]

  2. *(Improvement for s by only incoming teachers) $\tau(t) \triangleright_s \theta_\emptyset$, school s currently has a vacant position, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.*

  Let A_t^k be the opportunity set for a remaining teacher t, that is, the set of schools to which t can point in this step together with the outside option $\emptyset$.[24]

  Each remaining teacher t points to her most preferred option in A_t^k.
- *Outside option $\emptyset$ points to all teachers who are pointing to it.*

By definition, there always exists a chain. There are two possible cases:

Case (i): *there is a cycle in which all schools in the cycle satisfy Improvement Condition 1 or a cycle between a single teacher and the outside option $\emptyset$. Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered by assigning the teachers in that cycle to the school or the outside option to which she is pointing, update current matching μ and current balances accordingly, remove assigned teachers and filled schools in μ, and go to step $k+1$.*

Case (ii): *there exists no such cycle. In that case, there must be a chain and at least one new teacher left.*[25] *We select the chain to be executed as follows:*

[23] Condition 1 is trivially satisfied if no such θ exists, i.e., if $\tau(t) \trianglerighteq_s \tau(t_s^k)$.

[24] Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo since their type must be weakly higher than $\tau(t_s^k)$ under $\trianglerighteq_s$ by construction. Hence, Condition 1 is satisfied.

[25] Since there is no cycle, then no teacher points to the outside option. If all teachers point due to Im-

- *Select as the tail of the chain the new teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school to which she is pointing.*[26] *If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:*
- *Include to the chain the teacher who is pointed to by the last included school.*[27] *If we include a teacher, we also include next in the chain the school to which she is pointing. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.*[28]

The last school included is the head of the selected chain.

We execute the exchanges in the selected chain by assigning each teacher in the chain to the school to which she is pointing, update current matching μ and current balances accordingly, remove assigned teachers and filled schools, and go to step $k+1$.

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ.

The name of the mechanism suggests that both teachers and schools become better off through the mechanism with respect to the status quo. Indeed, this is the case. We introduced several innovations in the mechanism that exploit different Pareto improvement possibilities for teachers and schools over the status-quo matching.

Pareto improvement of teachers is straightforward in the algorithm. Teachers who are employed at the status quo are eventually assigned to a school at least as good as their status-quo assignment. Moreover, all teachers are assigned the best option to which they can point in the step they are assigned.

What is more delicate is the Pareto improvement of schools, that is, how we make sure that they always weakly improve with respect to their status-quo assignment in every step. This is ensured through the introduction of both teacher and school pointing rules.

A school's pointing order designates in which order the school would like to *send out* its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned possibly to a different school. Thus, we make sure that this priority order is in the reverse order of its preferences: Less preferred-type employees are pointed to first and more preferred-type employees are pointed to later. This is the first innovation.

On the other hand, the teacher pointing rule designates which teachers can be assigned

provement Condition 1, there should be a cycle, a contradiction. Therefore, at least one teacher points due to Improvement Condition 2 and, by definition of this condition, there must be a new teacher left.

[26] Such a school exists, because if she does not point to a school, then she pointed to the outside option $\emptyset$ and was removed previously.

[27] Such a teacher exists by Improvement Condition 1.

[28] This iterative procedure is guaranteed to terminate. Otherwise, we would have a cycle in which all schools involved in the cycle satisfy Improvement Condition 1, a contradiction with the assumption that Case (i) does not hold.

to a school. Therefore, we only allow teachers who can improve the school's welfare with respect to its status-quo assignment after the currently pointed employee of the school is sent out.

The two school improvement conditions make sure of this.

Condition 1 has two cases: If the type of the possibly incoming teacher is at least as good as the type of the possibly outgoing teacher, the school has no danger of becoming worse off in this trade. The second case, on the other hand, is more delicate: As trades that strictly improve a school's welfare occur over steps, schools acquire new teachers who are actually of better types than the types of outgoing status-quo employees. Therefore, they may build up a *buffer*. If such a buffer exists, a worse-type teacher than its currently outgoing employee can still be assigned to the school, although this trade makes the school worse off with respect to the previous step. However, the school is still weakly better off with respect to the status quo thanks to the buffer. Only the buffer gets thinner. The existence of the buffer is tracked by checking whether the sums of the relevant type balances are positive through Condition 1. The use of this buffer ensures teacher optimality.

While the first condition is about a trade the school will make by exchanging an outgoing teacher with an incoming teacher, Condition 2 is only relevant as long as new teachers remain in the algorithm. When Condition 2 holds for a school via some teacher, but not Condition 1, the school will not send out an employee as it has extra capacity: it will only hire one additional acceptable teacher.

We illustrate how the algorithm of the SI-CC mechanism works in Example A.1 in Appendix D. We are ready to state our main result in this section.

Theorem 1. *The SI-CC mechanism is SI teacher optimal and strategy-proof.*

SI teacher optimality of the mechanism is delicate to show. Note that SI teacher optimality implies that the mechanism outcome is Pareto undominated for teachers among all status-quo improving matchings. However, the pointing rule of teachers has restrictions imposed by the school improvement conditions. That is, a teacher cannot arbitrarily point to the best school she likes. We show that the restrictions imposed by these conditions are the necessary and sufficient conditions for keeping status-quo improvement for schools without affecting the outcome being Pareto undominated for teachers. Therefore, implementing any further Pareto improvement for teachers would make the schools worse off with respect to the status quo. Moreover, imposing further restrictions for teacher pointing would prevent SI teacher optimality.

Strategy-proofness of the mechanism relies on several observations: First, once a teacher is pointed to by a school, she will continue to be pointed to until she is assigned. We show that the opportunity set for each teacher t, A_k^t, weakly shrinks across steps k. Although Improvement Conditions 1 or 2 may stop holding for a school via a teacher t across steps, we show that teacher t cannot affect which schools leave and stay in A^t_k before she is assigned by submitting different preferences.

An immediate corollary to the theorem is that the SI-CC mechanism is also two-sided Pareto efficient by Proposition 1.

Under the pointing rule of schools, in each step of SI-CC, each school points to one of its employees who has the lowest ranked type. One can wonder whether Theorem 1 holds when we consider alternative school pointing rules under SI-CC mechanism. Example A.2 in Appendix D shows that SI-CC can be manipulated by a teacher and it is not SI teacher optimal under an alternative pointing rule.

4 Status-quo Improving Fairness and Deferred Acceptance

4.1 Status-quo Improving Fairness

Although two-sided Pareto efficiency is a very appealing property of matchings, some real-life mechanisms are adopted to have a fairness notion, which conflicts with SI teacher optimality and in general with two-sided Pareto efficiency. For example, in the French teacher (re)assignment application, the mechanism currently used is not two-sided Pareto efficient, while it satisfies a particular fairness condition that is not necessarily status-quo improving.[29]

To this end, we also introduce a novel fairness concept that is consistent with status-quo improvement under a mild assumption about the number of new teachers in a market. Our notion has different requirements than standard elimination of justified envy (Balinski and Sönmez, 1999) (and the mathematically related property of stability due to Gale and Shapley, 1962), which is extensively used in the literature, because in our setting we have a status-quo matching while most of the literature focuses on not having a status quo or equivalently an empty matching as the status quo.

Consider a market P. We interpret the preferences induced by type rankings of the schools over individual teachers as *priorities* in the notions we introduce below.

First, we formally define acceptability for schools and teachers. Given a school s, a matching μ is **acceptable for** s if there is no $t \in \mu_s$ such that $\theta_\emptyset \rhd_s \tau(t)$. Similarly, given a teacher t, a matching μ is **acceptable for** t if $\mu_t \; P_t \; \emptyset$.

Next, we introduce a standard weak efficiency property. A matching μ is **non-wasteful** if, for each teacher t and each school s, $\tau(t) \rhd_s \theta_\emptyset$, and $s \; P_t \; \mu_t$ imply $|\mu_s| = q_s$.

Our fairness notion relies on respect of priorities: Given two teachers t, t' and a school

[29]The current mechanism in France is the teacher-proposing deferred acceptance algorithm (see Definition 2 below) using the strict priorities mandated by the French Ministry of Education as preferences of schools with the following modification: the status-quo employees of each school are ranked at the top of the priority rankings of the school before the other teachers, each group is ranked according to their true priorities (see Appendix E for the definition of the current mechanism in France).

s, a matching μ **respects the priority of t over t' at s** if, $s = \mu_t$ and $s \; P_{t'} \; \mu_{t'}$ together imply $\tau(t) \trianglerighteq_s \tau(t')$. We call a matching μ **Gale-Shapley stable** (Gale and Shapley, 1962) if it is acceptable for all schools and teachers, non-wasteful, and respectful of priorities of all teachers (see Balinski and Sönmez, 1999). This classical concept potentially conflicts with our most basic property, status-quo improvement:

Proposition 2. *A Gale-Shapley stable matching always exists. However, a matching that is respectful of priorities and status-quo improving may not exist.*

One may think that the cause of incompatibility of status-quo improvement with respect to priorities is not giving employment rights to teachers at their status-quo schools. Indeed the current system in France uses a strategy-proof mechanism that satisfies the following fairness concept implicitly.[30]

A matching μ is **teacher–status-quo improving (teacher-SI) fair** if it is non-wasteful, acceptable for schools, status-quo improving for teachers, and respectful of priorities of teachers at each school over teachers who are not status-quo employees of the school, i.e., $s = \mu_t$, $s \neq \omega_t$, and $s \; P_{t'} \; \mu_{t'}$ together imply $\tau(t) \trianglerighteq_s \tau(t')$.

This concept ignores respect for priorities of teachers over status-quo employees at a school so that it is compatible with status-quo improvement for teachers. However, this concept still does not resolve the main problem regarding status-quo improvement for schools.

Proposition 3. *If there are vacant positions at some schools at status quo, then a teacher-SI fair and status-quo improving matching may not exist.*[31]

We introduce our main fairness concept, which satisfies status-quo improvement for both sides of the market. Given a school s, recall that $T \setminus \omega_s$ is the set of teachers who are not status-quo employees of this school and $T \setminus (N \cup \omega_s)$ is the set of *tenured* teachers who are not status-quo employees of the school. A matching μ is **status-quo improving fair** (**SI fair**) if the following conditions hold:

- it is non-wasteful,
- it is status-quo improving,
- at each school s, it respects priorities of teachers over those who are not status-quo employees of s (i.e., those in $T \setminus \omega_s$) with the following possible exception: if it does not respect the priority of a teacher $t \in T \setminus (N \cup \omega_s)$ over a new teacher (i.e., one in N) then
$$|\{t' \in \mu_s \cap N : \tau(t) \triangleright_s \tau(t')\}| \leq q_s - |\omega_s|.$$

[30]The current French setting, which is a standard school choice problem (Abdulkadiroğlu and Sönmez, 2003), can be easily embedded into our framework: each teacher has a different type and the type ranking of each school is given by the strict priorities of the schools.

[31]Even when there are no vacant positions at schools at status quo and there are no new teachers, it is teacher-SI fair but not status-quo improving for schools.

While all other conditions are self explanatory the latter point of the last condition requires some intuition. This condition gives implicit rights of initially vacant seats to new teachers: If at some school, the priority of a tenured teacher who is not a status-quo employee over a new teacher is not respected, then there are less of such new teachers than the number of empty seats. Examples A.7 and A.8 in Appendix D show how matchings satisfying more stringent versions of SI fairness are not guaranteed to exist.[32]

4.2 Auxiliary Choice Rule Design and Status-quo Improving Deferred Acceptance

One natural candidate for an SI fair mechanism is a version of the teacher-proposing deferred acceptance (DA) algorithm (Gale and Shapley, 1962). However, the notion of status-quo improvement is difficult to fulfill by such a mechanism. Intuitively, status-quo improvement would require that whenever a teacher applies to a school, she should be accepted by that school *only if* it is guaranteed that her departure will not eventually make her initial school worse-off compared to the status-quo. This acceptance decision depends on later applications received by the initial school of that teacher and therefore. Hence, such a variant mechanism needs to be forward-looking while the design of iterative steps of DA relies on myopic decisions. In this section, we explain how a mild overdemand assumption—satisfied in many empirical applications including ours—allows us to go around this issue. Our main contribution here is to introduce auxiliary choice rules for schools that—under our overdemand assumption—will achieve SI fairness and strategy-proofness when they are used in conjunction with the teacher-proposing DA algorithm adopted for complex matching terms by Kelso and Crawford (1982) and refined by Roth and Sotomayor (1990).

Given a school s, an **auxiliary choice rule** is a function $C_s : 2^T \to 2^T$ such that for any $\hat{T} \subseteq T$, (i) $C_s(\hat{T}) \subseteq \hat{T}$ and $|C_s(\hat{T})| \leq q_s$.

Using the auxiliary choice rules that we will design below, we will employ the well-known teacher-proposing DA algorithm. We consider the sequential version of this algorithm also known as the cumulative offer process (Hatfield and Milgrom, 2005) for more complex contractual matching terms:

Definition 2. ***Teacher-Proposing Deferred Acceptance Algorithm (DA):***

Step 1*: Some teacher t' proposes to her favorite acceptable school, denoted by s', if such a school exists. In this case, define $B^2_{s'} = \{t'\}$ and $B^2_s = \emptyset$ for each school $s \neq s'$. Otherwise, define $B^2_s = \emptyset$ for each school s.*

Each school s holds teachers in $C_s(B^2_s)$ and rejects all other teachers in B^2_s.

In general,

[32] Also it is useful to note that SI fairness also potentially conflicts with two-sided Pareto efficiency (see Example A.6 in Appendix D).

***Step** $k > 1$: Some teacher t'' who is not currently held by any school proposes to her most favorite acceptable school that has not rejected her yet, denoted by s'', if such a school exists. In this case, define $B_{s''}^{k+1} = B_{s''}^{k} \cup \{t''\}$ and $B_s^{k+1} = B_s^k$ for each $s \neq s''$. Otherwise, define $B_s^{k+1} = B_s^k$ for each school s.*

Each school s holds $C_s(B_s^{k+1})$ and rejects all teachers in $B_s^{k+1} \setminus C_s(B_s^{k+1})$.

The algorithm terminates when each teacher is either rejected by all of her acceptable schools or currently held by some school. We assign each school the students it is holding.

Our main contribution in this subsection is the construction of an auxiliary choice rule for each school. Fix a school s. First, we need some additional concepts.

A **tie breaker** is a linear order over teachers $\vdash$ as before. Given this tie breaker, we construct a new linear order over the teachers in ω_s denoted by $\blacktriangleright_s$:
For any $t, t' \in \omega_s$,

$$t \blacktriangleright_s t' \iff \tau(t) \rhd_s \tau(t') \text{ or } \left[\tau(t) = \tau(t') \text{ and } t \vdash t'\right].$$

Observe that a better-type teacher is prioritized over a worse-type teacher, and when two same-type teachers are compared, we use the tie breaker to prioritize one over the other.[33]

The auxiliary choice rule will use a lexicographic decision structure within a school by dividing the school into independent slots where each **slot** eventually represents a position at the school.[34]

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \ldots, s^{q_s}\}$ be the set of slots at school s. We define a ranking for each slot over $T \cup \{\emptyset\}$ where $\emptyset$ denotes keeping the slot unfilled. The **ranking of slot** s^k, $\succ_s^k$, is defined separately for the slots representing the filled positions at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the vacant positions at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:

- For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$, we define $\succ_s^k$ as follows:
  - the teacher $t \in \omega_s$ who is ranked k^{th} under $\blacktriangleright_s$ has the highest ranking,
  - any teacher t' with $\tau(t) \rhd_s \tau(t')$ is ranked below $\emptyset$, and
  - the rest of the ranking is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.
- For vacant slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$, we define $\succ_s^k$ as follows:
  - a teacher t is ranked above $\emptyset$ if and only if she is acceptable, i.e., $\tau(t) \rhd_s \theta_\emptyset$,
  - any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \rhd_s \theta_\emptyset$) is ranked above any

[33]Thus, linear order $\blacktriangleright_s$ effectively reverses the ordering of status-quo employees of different types in the pointing order $\gg_s$ used in the SI-CC mechanism, while it respects the school's type ranking.

[34]Such a model was previously introduced by Kominers and Sönmez (2016) in one-sided priority-based matching context for more complex contractual matching terms. In their approach, the choice rule of a school was a primitive of the model, unlike in our setup.

teacher t' employed at status quo by some school (i.e., $t' \notin N$), and
  - the rest of the ranking is determined according to $\triangleright_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.

Thus, the set of acceptable teachers for slot s^k is a superset of the set of acceptable teachers for slot s^{k-1}.

We will make the following mild overdemand assumption in the rest of this section involving new teachers and schools with excess status-quo capacity:

Assumption 1. *There exists a subset of new teachers $N' \subseteq N$ such that (i) there are at least as many new teachers in N' as vacant positions at status quo, i.e., $|N'| \geq \sum_{s \in S} \left(q_s - |\omega_s|\right)$, and (ii) each teacher $t \in N'$*

- *considers all schools with excess capacity acceptable, i.e., if $q_s > |\omega_s|$, then $s \; P_i \; \emptyset$, and*
- *is acceptable for all schools with excess capacity, i.e., if $q_s > |\omega_s|$, then $\tau(t) \triangleright_s \theta_\emptyset$.*

In the absence of either part of Assumption 1, we can come up with examples such that some schools end up with fewer teachers than what they have under the status-quo matching and status-quo improvement is violated for schools (see Examples A.11 -A.13 in Appendix D).

Since the auxiliary choice rule is defined through filling one slot at a time, we need to determine in which **processing order** the slots are filled. We process the slots in the following natural order[35]

$$s^1, s^2, \ldots, s^{q_s}.$$

Therefore, the above natural order first fills the occupied slots following the order of their status-quo teachers using $\blacktriangleright_s$ (from the most preferred teacher types to the least preferred ones) and second, fills the vacant slots.

Definition 3. *The **auxiliary choice rule** C_s of school s is defined through an iterative procedure. The chosen set from the set of teachers $\hat{T}$ by school s, denoted by $C_s(\hat{T})$, is determined as follows:*

- ***Step** 1: The most preferred acceptable teacher under $\succ_s^1$ in $\hat{T}_1 = \hat{T}$ is assigned to slot s^1 and she is removed. If there is no such teacher, then s^1 remains vacant. Denote the remaining teachers with $\hat{T}_2$.*

  In general,
- ***Step** $k \geq 2$: The most preferred acceptable teacher under $\succ_s^k$ in $\hat{T}_k$ is assigned to slot s^k and she is removed. If there is no such teacher, then s^k remains vacant. Denote the remaining teachers with $\hat{T}_{k+1}$.*

The procedure terminates when all slots are processed; thus, step q_s is the last step. Chosen set $C_s(\hat{T})$ is the set of teachers assigned to the slots of school s.

[35]Notice that, one can define the auxiliary choice rule with an alternative processing order of the slots. Later we will explain why this order is chosen.

We illustrate how a chosen set is found in the Example A.3 in Appendix D.

We define the following notions for the auxiliary choice rules that will be crucial for our mechanism to be both strategy-proof and SI fair.

The auxiliary choice rule C_s satisfies **substitutes** (Kelso and Crawford, 1982) if for all $\bar{T} \subseteq T$ and distinct teachers $t, t' \in \bar{T}$,

$$t \in C_s(\bar{T}) \implies t \in C_s\big(\bar{T} \setminus \{t'\}\big).$$

The auxiliary choice rule C_s satisfies the **law of aggregate demand** (Alkan and Gale, 2003 and Hatfield and Milgrom, 2005) if for all $\bar{T}, \hat{T} \subseteq T$,

$$\bar{T} \subseteq \hat{T} \implies \big|C_s(\bar{T})\big| \leq \big|C_s(\hat{T})\big|.$$

Next, we show that C_s satisfies these two properties.

Proposition 4. *The auxiliary choice rule C_s satisfies the substitutes and law of aggregate demand conditions.*

We refer to the mechanism that selects the outcome of the DA algorithm using the auxiliary choice rules $(C_s)_{s \in S}$ that we designed as the **status-quo improving deferred acceptance** (**SI-DA** for short) mechanism. The logic behind naming will be clear with the following result:

Theorem 2. *The SI-DA mechanism is strategy-proof, and under Assumption 1, it is also SI fair.*

Notice that, when there is no new teacher and no empty seats, Theorem 2 holds without Assumption 1 since this assumption trivially holds. We conclude this section discussing the role of the processing order we used to fill the slots in our construction of the auxiliary choice rules.

Processing order and tie-breaking. A **processing order** is a linear order of the slots of a school in which one can process the slots in the choice procedure in Definition 3 as an alternative to the natural order we defined. In the proof of Proposition 4 the processing order of the slots does not play any role. Hence, Proposition 4 holds for any order we use in the procedure to find the chosen teachers. As a result, the SI-DA mechanism continues to be strategy-proof independently of the processing order of the slots. Moreover, the proof of SI fairness of SI-DA does not rely on the processing order. Hence, the SI-DA mechanism continues to be SI fair independently of the processing order of the slots. However, the processing order has an impact on the mobility and the welfare of the teachers as illustrated by the following proposition.

Proposition 5. *Consider a school s. Let p_s and $\hat{p}_s$ be two processing orders of slots at school s such that $\hat{p}_s$ is obtained from p_s by swapping two adjacent slots s^k and s^ℓ where $k < \ell \leq |\omega_s|$ and $p_{\bar{s}} = \hat{p}_{\bar{s}}$ for every school $\bar{s} \neq s$. Let $D_{s'}$ and $\hat{D}_{s'}$ be the auxiliary choice rules induced by $p_{s'}$ and $\hat{p}_{s'}$ for every school s' through the procedure in Definition 3, and μ and $\hat{\mu}$ be the outcomes of the DA algorithm by using $(D_{s'})_{s' \in S}$ and $(\hat{D}_{s'})_{s' \in S}$, respectively. Then each teacher t (weakly) prefers μ_t to $\hat{\mu}_t$.*

Proposition 5 implies that the natural processing order that we originally used in Definition 3 for the first $|\omega_s|$ slots at any school s increases the welfare of teachers compared to any alternative processing order of those slots. However, Proposition 5 does not say anything about the impact of the relative order of vacant positions on teacher welfare. Indeed Example A.9 in Appendix D shows that we cannot find an optimal processing order when vacant slots are also considered. [36]

5 Empirical Analysis: The Case of France

5.1 Institutional Background

Teacher recruitment is highly centralized in France. Once teachers get tenure after one year probation, teachers in public schools become civil servants. The government manages both the first assignment of newly tenured teachers to a school and the transfer process of tenured teachers who previously received an assignment but wish to move.[37] In the rest of the empirical analysis, as we did in the theory section, we refer to newly tenured teachers who ask for their first assignment as *new teachers* and tenured teachers who have a status-quo assignment and would like to move to a new school as *tenured teachers*, with a slight abuse of terminology.

The assignment procedure takes place in two successive steps. In the first step, teachers are assigned to one of the 31 French regions using a mechanism. Tenured teachers who wish to change regions and new teachers submit a preference list over regions. In the second step, teachers who are newly assigned to a region and tenured teachers who wish to change schools within their region submit a preference list over schools and the same first step mechanism is run for each region with the new inputs. Our empirical analysis focuses on the first regional assignment because of potential strategic reports of preferences during the second phase.[38] From now on we treat each region like a large single school. Participation in the assignment mechanism is compulsory for all new teachers (as

[36] We also used a tie-breaker $\vdash$ in the construction of the linear order $\blacktriangleright_s$ which is used to define the slot priorities. The inclusion of this exogenous tool causes both two-sided Pareto efficiency and SI teacher optimality loss as illustrated in Example A.10 in Appendix D.

[37] This centralized assignment process is used for public school teachers only. Private schools employ 16% of teachers in France.

[38] Preferences reported during the second phase of the assignment are more difficult to interpret because

they do not have a status-quo position) but optional for tenured teachers who cannot be forced to change their school.

We use data from 2013 on the (re)assignment of teachers to one of the 31 French regions. There were 700,000 secondary public school teachers in France that year. When organizing the annual (re)assignment process, the central administration has to take into account not only a large pool of tenured teachers who wish to change positions, but also some vacant positions that need to be filled—9,793 public secondary school teachers retired in 2013—and new teachers who have passed the recruitment exam, validated their probation year, and need to be assigned to their first jobs. In 2013, about 25,100 teachers took part in the centralized regional (re)assignment process. Among them 17,200 were tenured teachers and 7,900 were new teachers.

5.2 Data and Descriptive Statistics

In our analysis, we use data on the (re)assignment of teachers to one of the 25 out of (a total of) 31 French regions.[39] We have information on reported preferences and status-quo assignments (if any) of teachers, Ministry-mandated priorities of regions,[40] and the number of vacant positions in each region. We use *single* teachers from the 8 largest subjects, such as French, Math, English, Sports, etc., and discard *couples* because they benefit from special treatment in the assignment process.[41] Finally, in order to keep the market structure balanced, we drop one seat for each teacher we omit.[42] Our final sample contains 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers. Table A.1[43] shows the decomposition for each subject.

A central motivation of our analysis is to rebalance the unequal distribution of teachers across regions. Part of this large imbalance stems from differences in attractiveness of regions. Table 1 reports descriptive statistics of teachers (Panel A), their status-quo assignment (Panel B), and the region they rank first (Panel C). Two regions surrounding Paris,

teachers can only rank up to 20 or 30 schools (depending on the region) and because, in addition to ranking schools, teachers can also rank larger geographic areas, such as cities for instance.

[39] We discard the 6 overseas regions because of their specificities in terms of (i) teacher preferences—in contrast to what we find in our estimates, distance from the current location often becomes an attractive feature—and (ii) region Ministry-mandated priorities—some of these regions, like Mayotte, give teachers who grew up in these regions bonus points when they rank it first.

[40] The Ministry-mandated priorities are determined by a point system which is mainly based on teachers' experience, whether they ask for a spousal reunification in a region, and whether their status-quo school is disadvantaged. We refer the reader to Combe et al. (2020) for more details on the point system as this system is not key for understanding our results.

[41] Spouses in two different subjects can submit a joint transfer application (by submitting two identical lists). This creates dependencies across markets for different fields.

[42] For each tenured teacher we discard, we drop her corresponding position. For new teachers, we find the share discarded among new teachers (denoted as K%) and we delete K% of vacant positions in each region.

[43] Tables and figures with prefix "A" are in Appendix F.

Table 1: Descriptive Statistics on Teachers and Regions

	Tenured Teachers			New Teachers		
	French (1)	Math (2)	English (3)	French (4)	Math (5)	English (6)
Panel A. Characteristics of teachers						
Female (%)	76.1	47.0	85.4	80.3	41.7	80.4
Married (%)	48.5	45.0	46.8	41.1	39.4	40.9
Is in priority education school (%)	10.4	13.2	4.4	-	-	-
Experience (yrs)	7.48	7.23	7.18	2.76	2.24	2.30
Has advanced teaching qualif. (%)	7.9	29.1	8.8	16.8	31.7	15.2
Panel B. Characteristics of the regions to which teachers are assigned at status-quo						
Is the teacher's birth region (%)	8.7	8.6	9.3	-	-	-
Is Créteil or Versailles (%)	37.7	52.3	35.6	-	-	-
Is in South of France (%)	5.6	9.3	12.7	-	-	-
Students in urban areas (%)	61.7	67.4	64.0	-	-	-
Disadvantaged students (%)	52.5	54.0	53.5	-	-	-
Students in priority education (%)	26.0	24.5	22.7	-	-	-
Students in private school (%)	15.2	16.3	17.4	-	-	-
Teachers younger than 30 yrs (%)	11.9	13.3	11.3	-	-	-
Panel C. Characteristics of the regions teachers rank first						
Distance to status-quo region (km)	2,148.7	1,316.9	1,521.9	-	-	-
Is the teacher's birth region (%)	36.5	35.8	40.0	35.8	38.4	39.3
Is in South of France (%)	25.2	25.2	25.4	20.1	18.6	20.0
Is Créteil or Versailles (%)	2.8	3.3	3.4	14.3	11.9	12.5
Students in urban area (%)	60.2	56.2	51.7	61.9	58.6	59.2
Disadvantaged students (%)	52.8	53.1	53.5	53.3	53.1	53.0
Students in priority education (%)	20.3	19.9	17.9	21.8	20.6	21.0
Students in private school (%)	23.7	22.9	25.8	22.2	21.8	21.7
Teachers younger than 30 yrs (%)	6.5	6.4	6.0	8.2	8.1	8.2
Observations (#)	859	605	628	786	958	746

Notes: This table reports descriptive statistics for teachers and regions in three subjects: French, Math, and English. Statistics are reported for the sample of teachers we use for the demand estimations. Columns (1) to (3) report statistics for tenured teachers. Columns (4) to (6) report statistics for new teachers. New teachers have missing values for statistics related to the region of status-quo assignment. We discard teachers who submit a joint list with their partner, teachers who are from one of the six regions that are overseas, and teachers for whom one of the individual characteristics is missing. The last row reports the number of teachers in each subject. Panels A, B, and C, respectively, present descriptive statistics of teachers, of the region to which they are assigned at status quo, and of the region they rank first. Appendix G provides a detailed description of each variable.

called Créteil and Versailles, are particularly unattractive. The imbalance is blatant when we compare the number of teachers asking to leave the region and the number asking to enter. For instance, in Math, 52.3% of the tenured teachers who ask to change region come from Créteil or Versailles, but only 3.3% rank one of these two regions as their first choice. Table A.2 provides additional evidence on attractiveness differences and their determinants for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Créteil, Versailles, and Amiens), and three intermediate regions (Paris, Aix-Marseille, and Grenoble).

The large share of transfer requests that originate from unattractive regions have a direct consequence on annual mobility flows as they result in a large number of teachers exiting these regions and each vacating a position that eventually needs to be filled. About 50% of the new teachers get their first assignment in one of the three least attractive regions (Créteil, Versailles, or Amiens). This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France.[44] In addition, it creates large differences in the age profile of teachers across regions. As reported in column (3) of Table A.2 of Appendix F, the ratio of the number of teachers older than 50 to the number of teachers younger than 30 was equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 times more teachers older than 50 than teachers younger than 30. In Bordeaux and Toulouse, this ratio was 6.5 and 5.3, respectively.

Several papers have found that teachers contribute less to the educational development of their students during the initial years of their career than when they have more experience (Bates et al., 2021, Chetty et al., 2014, and Rockoff, 2004). Reducing the unequal distribution of teachers based on experience levels across regions and reducing the assignment chance of new teachers to a disadvantaged region initially became important objectives of the French policy makers. They consider these as ways to reduce the achievement gap among students and improve the attractiveness of the teaching profession in the longer run.

5.3 Specifications of the Empirical Analysis

Mechanisms. Our counterfactual analysis aims to both formally define possible inputs and quantify the performance of SI-CC and SI-DA; the mechanisms we introduced in Sections 3 and 4, respectively. We also benchmark them with variants of two widely studied mechanisms:

- **Benchmark for SI-CC:** A variant of SI-CC, which we refer to as **TTC***, that relaxes the mechanism features that ensure status-quo improvement while still accounting

[44] In 2014, 24% of the positions for the most common recruitment exam (CAPES) in France remained vacant because of both a shortage of applicants and the poor quality of those applying. The shortage situation has not improved since.

for teacher types (as defined in the theoretical analysis). More precisely, this mechanism differs from SI-CC in two aspects: (1) we lift the restrictions on the set of regions to which a teacher can point, and (2) tenured teachers can now start a chain and potentially leave their position without being replaced (see Appendix E for a formal definition). This benchmark is close to the well-known TTC-variant mechanism "You request my house – I get your turn" (YRMH-IGYT) introduced by Abdulkadiroğlu and Sönmez (1999). This mechanism is strategy-proof, Pareto efficient, and status-quo improving for teachers, but not status-quo improving for regions. Intuitively, TTC* is expected to generate higher teacher welfare and more mobility than our mechanisms at the cost of a potentially more unequal teacher distribution.

- **Benchmark for SI-DA:** A mechanism that uses Gale and Shapley (1962)'s original version of the teacher-proposing DA algorithm with the modification to type rankings of regions that each tenured teacher is moved to the top of her status-quo region's ranking (but otherwise type rankings are respected among status-quo teachers and among non-status-quo teachers, respectively). Thus, it does not employ complex auxiliary choice rules as our SI-DA does and relaxes the features of SI-DA that ensure status-quo improvement. We refer to this benchmark as **DA***. More precisely, DA* differs from SI-DA in two main aspects: (1) we lift the restrictions on region priority rankings over teacher types (i.e, an applicant teacher with a less-preferred type than a status-quo teacher will no longer be considered as unacceptable by a region), and (2) vacant positions in a region no longer prioritize new teachers over tenured teachers (see Appendix E for a formal definition). If the Ministry-mandated priorities are used for regions, this mechanism becomes equivalent to that used in each step of the current French assignment process. However, we use our regional teacher type rankings to account for priorities, making DA* different from the current French mechanism. Incorporating teacher types as school preference provides an interesting benchmark that satisfies teacher-SI fairness, potentially at the cost of efficiency and distributional objectives.

We run our different mechanisms using teacher preferences, teacher types, and region type rankings as inputs.

Teacher type. A teacher type corresponds to the teacher's experience level. We classify teachers into 12 experience bins, where the first bin corresponds to teachers with one or two years of experience, the second bin to teachers with three or four years of experience, and so on. Each bin represents a *teacher experience type*. When it is relevant for our discussion in some graphs, we further refine the experience type for the least experienced teachers and assume that new teachers have a lower experience type than any tenured teacher, resulting in 13 effective experience types. Figure A.4 shows a histogram of teacher experience types.

Region type ranking. We design type rankings of regions to reflect the central authority's distributional objectives rather than treating them as intrinsic preferences of regions.[45] We start by identifying which regions would benefit most from receiving more experienced teachers. We compute average teacher type in each region at status quo (see Figure A.5) and classify regions into two groups:

- The first group contains all regions whose average types are strictly below the median, that is, *younger regions* that could benefit from receiving more experienced teachers. We design the type rankings of these regions to rank types by *decreasing* level of experience, that is, higher experience types are preferred to lower experience types.
- The second group contains all regions whose average types are above the median, that is, *older regions*. We design the type rankings of these regions to rank types by *increasing* level of experience, i.e., lower experience types are preferred to higher experience types.

Our ultimate goal in designing type rankings in this manner is to ensure that the outcomes of our mechanisms eventually yield more equal distributions of teachers across regions. Both SI-DA and SI-CC, which are both status-quo improving, would achieve this goal through these type rankings. Note that the way we designed type rankings is tailored to the objectives of French policy makers (as discussed in Section 5.2). Evidence from the US also supports our assumption that younger regions value teacher experience more than older regions.[46] To further support our choice of type rankings, we also conduct robustness tests and show in Appendix H results in which all regions rank experienced teachers over inexperienced teachers.[47]

We further assume that all regions find all types acceptable, that is, that any teacher is always preferred to leaving a position vacant. Indeed, in the French system, by law, all teachers are acceptable for all regions.[48] Finally, running SI-CC (and its variants) requires an additional tie-breaker over teachers to determine which chain will be selected.[49] We

[45]Recall that in the theoretical analysis we were agnostic about how school type rankings were formed. Here, we use them as policy variables designed by the central authority.

[46]By estimating principal preferences over teachers, Bates et al. (2021) find that, when hiring teachers, Title I principals have a stronger preference for high-value-added teachers than principals in non-Title I schools. Combined with the very large increase in teacher value-added they observe during teachers first year of experience (by 0.09 student standard deviations), this suggests a larger preference of disadvantaged schools for experienced teachers.

[47]We obtain similar improvements in terms of teacher experience in the three youngest regions. However, unsurprisingly, allowing older regions to prefer experienced teachers over inexperienced teachers leads to fewer inexperienced teachers being assigned to these regions.

[48]For tie-breaking, we need an additional ordering over teachers for the SI-CC, SI-DA, and two benchmark mechanisms. We use the tie-breaking rule used by the French Ministry which uses the date of birth of teachers and some extra conditions for the rare cases with the same date of birth.

[49]This ordering applies only to new teachers under SI-CC but to all teachers under TTC* as chains can

use the true point system of the French Ministry and sort teachers by decreasing level of the maximum points they obtain over all regions.[50]

5.4 Estimation of Teacher Preferences

Teachers can rank all regions when they submit their preference list in the first step of the assignment process and the Ministry uses a modified version of the DA mechanism to assign teachers to regions, as mentioned earlier. If the whole assignment were done in a single step, this mechanism would induce being truthful as a dominant strategy for teachers. In our setting, if teachers have lexicographic preferences first over regions and then over schools or, more generally, if preferences over regions are well-defined, the mechanism assigning teachers to regions in the first step is strategy-proof. Combe et al. (2020) find evidence of such lexicographic teacher preferences. Yet, even under strategy-proof mechanisms, a number of experimental and empirical papers show that truthfulness is a strong assumption (Chen and Sönmez, 2006, Pais and Pinter, 2008, Rees-Jones, 2018, Chen and Pereyra, 2019, and Hassidim et al., 2017). In our context, French teachers have reasonably accurate information on their admission probabilities to each region, which might encourage some teachers to discard from their preference list the regions where their chances of being accepted are too low.[51] These omissions could introduce a bias in any counterfactual analysis done using reported preferences of teachers. Combe et al. (2020) previously rejected truth telling among French teachers. To avoid this potential bias, instead of using the reported preferences, we estimate preferences of teachers using an identifying assumption (presented below) that does not require teachers to be fully truthful.

Model. We estimate preferences of teachers over regions using the following utility function:

$$u_{t,r} = \delta_r + Z'_{t,r}\beta + \varepsilon_{t,r} \tag{5}$$

Teacher t's utility for region r is a function of region fixed effect δ_r, teacher-region-specific observables $Z_{t,r}$ (with coefficients β) and a random shock $\varepsilon_{t,r}$ which is i.i.d. over t and r and follows a type-I extreme value distribution, Gumbel$(0,1)$. The region fixed effect captures region characteristics such as average socio-economic and academic level of students in the region, cultural activities, housing prices, facilities, etc. We estimate preferences separately for tenured teachers and new teachers. This allows us to include a richer set of

start from any teacher.

[50]We show in Appendix F that the performance of the SI-CC mechanism is sometimes sensitive to the ordering chosen. We report results in which we flip the ordering to rank teachers by increasing level of their maximum priority points.

[51]Cutoffs for entry in each region are published every year. Combe et al. (2020) show that these cutoffs are relatively persistent over time, so they provide reasonably accurate information to teachers on their chances of being admitted to each region.

variables for the former group. The vector $Z_{t,r}$ includes a dummy specifying whether region r is the birth region of teacher t. If teacher t is tenured, it also includes a dummy showing whether r is the status-quo region of teacher t, as well as the distance between region r and the status-quo region of teacher t. Vector $Z_{t,r}$ additionally includes interaction terms between teacher t's and region r school characteristics (that are presented in Panels A and B of Table 1). We apply standard scale and position normalization, setting the scale parameter of the Gumbel distribution to 1 and the fixed effect of the Paris region to 0.

Identifying assumptions. To avoid the potential bias generated by teachers omitting regions they consider as infeasible, we estimate preferences of teachers under a weaker *stability assumption* developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2020).[52]

We start by defining the *feasible set* of each teacher as the set of regions that has a cutoff—that is, the lowest priority of the teacher assigned to a region—smaller than her own priority. These are the regions a teacher could be assigned to if she ranked the region first in her preference list. The key identifying assumption is that, for each teacher, the region obtained is her most preferred region among all regions that are in her feasible set.[53] Hence, we estimate a discrete choice model with personalized choice sets. Choice probabilities have closed form solutions and we estimate parameters using maximum likelihood.

Estimation results. Table 2 reports preference estimates for tenured teachers and new teachers for a selected group of coefficients. We run the estimations in each of the 8 subjects separately and report results for Math and French teachers in the table. The first 9 rows report coefficients for a selected set of region fixed effects. They reveal an interesting difference between the preferences of tenured and new teachers. While the Créteil and Versailles regions are very unattractive for tenured teachers (as indicated by the negative coefficient of their fixed effect relative to the Paris region), these regions are less unattractive for new teachers, who often see a first position in a disadvantaged school as a stepping stone for better positions in the future.[54] The fact that Créteil and Versailles

[52] Combe et al. (2020) provide an in-depth discussion of the two alternative identifying assumptions (truthfulness versus stability), as well as statistical tests in favor of the latter. For more references on estimations that do not require truth telling see Akyol and Krishna (2017), Artemov et al. (2019), Agarwal and Somaini (2018), and Calsamiglia et al. (2020). They mainly focus on the estimation of the preferences of tenured teachers and we use the same estimation in our analysis. Here, we provide an additional detailed discussion on the estimation of the preferences of new teachers.

[53] This assumption is theoretically founded: Artemov et al. (2019) show that, in a large market environment, any (regular) equilibrium outcome of DA* must have this property. Since a variant of DA* with the Ministry-mandated priorities is the current mechanism for the regional assignment step, this result also applies to our setup.

[54] Teachers who stay in a disadvantaged school for at least five years benefit from additional priority when

Table 2: Teacher Preference Estimates

	Tenured Teachers				New Teachers			
	French		Math		French		Math	
	coef. (1)	s.e. (2)	coef. (3)	s.e. (4)	coef. (5)	s.e. (6)	coef. (7)	s.e. (8)
Region BESANCON	-3.88***	(0.99)	0.37	(0.80)	-1.52*	(0.65)	-0.32	(0.78)
Region BORDEAUX	-1.36	(0.95)	1.12	(0.66)	1.14	(0.62)	-0.40	(0.64)
Region DIJON	-5.08***	(0.97)	-2.88***	(0.73)	-1.39*	(0.59)	-1.88**	(0.61)
Region LILLE	-5.09***	(0.95)	-1.55	(0.81)	-1.7**	(0.62)	-1.26	(0.67)
Region REIMS	-6.22***	(1.00)	-3.60***	(0.74)	-2.39***	(0.61)	-3.04***	(0.65)
Region AMIENS	-6.44***	(1.06)	-3.31***	(0.75)	-1.88**	(0.59)	-2.35***	(0.62)
Region ROUEN	-5.96***	(0.97)	-2.17**	(0.69)	-1.64**	(0.59)	-1.53*	(0.62)
Region CRÉTEIL	-6.66***	(1.00)	-3.65***	(0.71)	-0.34	(0.60)	-1.33*	(0.58)
Region VERSAILLES	-5.12***	(0.89)	-2.13***	(0.60)	-0.50	(0.55)	-1.14*	(0.56)
Status-quo region	4.97	(6.72)	-15.72	(8.27)				
Birth region	10.21**	(3.41)	14.89***	(3.53)	10.46***	(2.48)	8.31**	(2.69)
Dist. to status-quo region	-23.33***	(4.61)	-23.52***	(5.47)				
% stud. urban x Status-quo region	-5.82***	(0.85)	-5.40***	(1.15)				
% stud. urban x Teach. from CV	2.81***	(0.71)	0.12	(0.68)				
% stud. in priority ed. x Married	-7.61***	(1.60)	-3.89*	(1.65)	-7.13***	(1.58)	-4.93***	(1.31)
% stud. in priority ed. x Status-quo region	11.26***	(2.99)	0.67	(3.80)				
% stud. in private sch. x Teach. in disadv. sch.	5.48**	(2.01)	6.58***	(1.83)				
% teach. younger than 30 x Advanced qualif.	10.59**	(3.65)	0.24	(3.06)	10.80**	(2.77)	5.58**	(1.99)
% teach. younger than 30 x Status-quo region	52.42***	(5.19)	54.15***	(6.47)				
% teach. younger than 30 x Birth region	-22.08***	(3.73)	-19.10***	(4.79)	-8.42**	(2.79)	-9.26***	(2.61)
Region in South of France x Teach. from CV	-1.27***	(0.37)	0.35	(0.36)				
Number of teachers	859		605		786		958	
Fit measure	0.669		0.674		0.632		0.624	

Notes: This table reports selected coefficients from estimations of teacher preference for region characteristics based on Equation 5. We set the fixed effect of the Paris region to 0. The last row reports our goodness of fit measure that we compute by looking at the top two regions that a teacher has included in her submitted preference list. We measure, for each teacher, the probability of observing this particular preference ordering in the preference list predicted with our estimations. We then average these probabilities across teachers. Stars correspond to the following p-values: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$. Variable "Teach. from CV" refers to whether the status-quo region of the teacher is Créteil or Versailles.

are more attractive for new teachers than for tenured teachers surely contributes to the unequal distribution of teachers denounced by policy makers. Yet, this is not the only ex-

they ask to change region or school.

planation for teacher unequal distribution. The counterfactual analysis we present in the next section shows that the assignment mechanism also shapes the distribution of teachers in important ways. The fact that preferences alone do not drive the unequal distribution is fundamental for our ability to improve both teacher distribution and teacher welfare. Appendix G reports goodness of fit measures for preference estimation.

Simulations. We use our estimates of utility coefficients to draw preferences of teachers 1,000 times using Equation 5. After having drawn them, we keep the entire set of regions without imposing any truncation for their simulated preference lists so that teachers find all regions acceptable, while for the tenured teachers status-quo improvement implies being assigned to a region no worse than her status-quo region.[55] In each of the 8 subjects and for each draw, we use these simulated preferences to run the mechanisms. The results reported in the next section correspond to averages over these 1,000 draws, aggregated over 8 subjects.

5.5 Relative Performance of the SI-CC and SI-DA Mechanisms

We start by discussing the relative performance of SI-CC and SI-DA. Recall that while SI-CC is SI teacher optimal (i.e., Pareto undominated for teachers among all status-quo improving matchings) and two-sided Pareto efficient, SI-DA is SI fair[56] and not two-sided Pareto efficient. Comparing the performance of these two mechanisms that respectively target teacher welfare (and two-sided Pareto efficiency) and fairness is important for various reasons. First, neither mechanism satisfies any obvious optimality property for the distribution of teacher experience, which is the measure of regional welfare.[57] Therefore, it is important to understand which one leads to a more desirable teacher experience distribution. Second, the mechanism that is currently used by the French Ministry of Education is teacher-SI fair (with respect to their Ministry-mandated priorities). This suggests that policy makers consider fairness as an important feature of the assignment process. As there is no mechanism that is both SI teacher optimal and SI fair, and there is even no Pareto comparison of teacher welfare between these two mechanisms, understanding the tradeoff between these properties is important.

Distribution of teacher experience. We start by comparing, for different regions, the cumulative distribution of teacher experience under SI-CC and SI-DA. We classify the

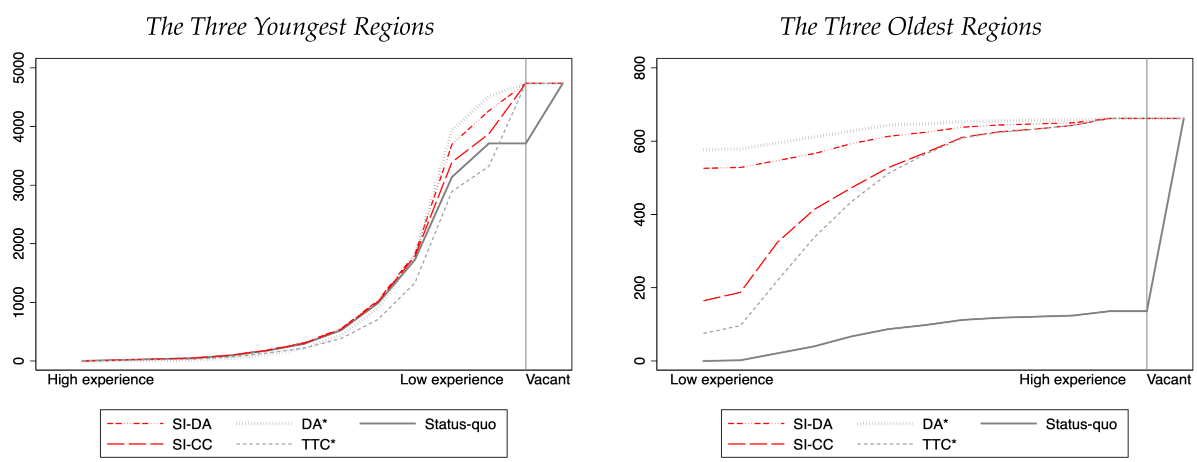
[55]This implicit assumption about new teachers is in line with the policy of the Ministry. Teachers are indeed not required to rank all regions when they submit their lists, but the Ministry fills the incomplete lists of new teachers to make sure that all of them get an assignment; even those who ranked few regions.

[56]SI-DA is SI fair under Assumption 1 (see Section 4), which is satisfied when all regions are acceptable by new teachers, all new teachers are acceptable by regions, and there are at least as many new teachers as empty seats. Table A.1 shows that these conditions are met in each of the 8 subjects we consider. The matching obtained under SI-DA is, therefore, SI fair.

[57]In this section, with a slight abuse of terminology, our discussion of regional welfare under different matchings refers to comparisons of teacher experience type distributions (at these matchings).

assigned teachers to regions into 13 types based on experience as explained before. The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France (Créteil, Versailles, and Amiens). Teachers in these regions represent 78% of teachers in disadvantaged regions.[58] SI-DA slightly outperforms SI-CC in these regions, that is, it assigns fewer inexperienced teachers: 1,041 teachers with up to two years of experience are assigned to one of the three regions under SI-DA versus 1,344 under SI-CC. SI-DA also improves the experience distribution in the three oldest regions by assigning them a larger number of inexperienced teachers (see the right panel of Figure 1).

Figure 1: Cumulative Distribution of Teacher Experience Types



Notes: This figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14th type corresponds to vacant positions. The mechanisms that satisfy status-quo improvement are plotted in red. Those that do not are in gray. The thick dark gray line (marked as "Status-quo" in the legend) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Fairness – teacher welfare trade-off. However, SI-DA's better distributional performance comes at a large cost in terms of teacher mobility. Panel A of Table 3 shows that only 3,912 teachers obtain a new assignment under SI-DA, compared to 5,356 under SI-CC. The lack of mobility under SI-DA is particularly striking for tenured teachers: none of them move from their status-quo region, compared to 1,444 under SI-CC. The very low level of mobility under SI-DA is due to the strength of the requirements imposing status-quo improvement and SI fairness. Indeed, status-quo improvement together with low demand from tenured teachers for unattractive regions imply that many tenured teachers

[58]See Figure A.5 for the type distribution in regions.

from these regions will be unable to move. Given this, any teacher entering a more attractive region almost automatically induces a priority violation for a tenured teacher stuck in an unattractive region.[59] This holds because the least attractive regions are also the youngest and the relatively low experience of the tenured teachers stuck in these regions are preferred by the older regions like Bordeaux. The SI fairness requirement prevents such assignments from happening and, thus, blocks the mobility of tenured teachers.

This intuition and the results from our counterfactual analysis show that under fairness-based mechanisms satisfying different notions of fairness, imposing status-quo improvement can have the unintended consequence of dramatically reducing mobility. Said differently, prioritizing SI fairness (under SI-DA) over SI teacher optimality (under SI-CC) entails a very large efficiency cost for teachers in our context.

Concerning fairness measures, SI-DA is, by construction, SI fair contrary to SI-CC. As reported in Panel E of Table 3, the latter leads to 8,122 teachers whose priorities are not respected via violations that are authorized in the definition of SI fairness. Note that neither SI-DA nor SI-CC satisfy Gale-Shapley stability, which requires full respect of priorities: SI-CC leads to 8,206 teachers whose priorities are not respected compared to 8,438 teachers under SI-DA. This reduction can be explained by the important teacher welfare gains that SI-CC has achieved, compared to SI-DA. Since many more tenured teachers move under SI-CC, the number of priority violations against tenured teachers staying at their status-quo assignment decreases. The small differences in the numbers of priority violations among our three fairness notions under SI-CC mean that the vast majority of such violations are caused by less preferred teachers being assigned to a new region despite more preferred teachers requesting that region (for SI-DA, only 2,171 teachers' priorities are not respected due to this last reason while the remaining 6,267 teachers' priorities are not respected because of a tenured teacher staying at her status-quo region). We conclude this subsection with the following summary of our main findings so far:

Fact 1. *Despite a slightly better distributional performance and obtaining an SI fair matching, SI-DA has a tremendous mobility cost compared to SI-CC. No tenured teacher moves from her status-quo region under SI-DA, compared to 1,444 under SI-CC. Imposing status-quo improvement together with (constrained) fairness come at a large teacher welfare cost.*

5.6 Benefits and Costs of Status-quo Improvement Requirement

We now inspect the benefits and costs of requiring status-quo improvement in designing assignment mechanisms. To do so, we compare SI-CC to TTC*; the benchmark mechanism which is not status-quo improving. We also compare SI-DA to DA*, although

[59] Using the terminology of Section 4, at a given matching, teacher t induces priority violation for teacher t' if the matching does not respect priority of teacher t over t' at teacher t's assigned region/school.

Table 3: Teacher Welfare and Violations of Priorities

	Suggested mech.		Benchmark mech.		Other mech.	
	SI-CC	SI-DA	TTC*	DA*	wSI-CC	Current French
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Teacher mobility						
Tenured teachers moved and new teachers assigned	5,356	3,912	6,382	4,806	5,554	5,864
Tenured teachers moved - from the 3 oldest regions	239	0	1,018	316	464	980
Tenured teachers moved - from the 3 youngest regions	116	0	117	117	116	72
Tenured teachers moved - from all regions	1,444	0	2,470	894	1,642	1,952
New teachers assigned	3,912	3,912	3,912	3,912	3,912	3,912
New teachers unassigned	715	715	715	715	715	715
Panel B. Cumulative distribution of ranks of regions that tenured teachers are assigned						
Rank = 1	749	250	1,112	338	761	833
Rank $\leq$ 2	1,604	919	2,203	1,050	1,632	1,775
Rank $\leq$ 3	2,080	1,352	2,758	1,531	2,113	2,300
Rank $\leq$ 4	2,485	1,745	3,168	1,967	2,522	2,737
Rank any	5,833	5,833	5,833	5,833	5,833	5,833
Panel C. Cumulative distribution of ranks of regions that new teachers are assigned						
Rank = 1	1,243	967	645	803	1,207	621
Rank $\leq$ 2	1,780	1,557	1,010	878	1,723	990
Rank $\leq$ 3	2,128	1,942	1,289	1,397	2,059	1,263
Rank $\leq$ 4	2,404	2,242	1,535	1,735	2,327	1,502
Rank any	4,627	4,627	4,627	4,627	4,627	4,627
Panel D. Average rank of region assigned						
All teachers	7.3	8.5	7.5	8.4	7.3	8.3
Teachers from the 3 youngest regions	7.2	7.5	5.8	7.1	6.9	5.8
Teachers from the 3 oldest regions	2.2	9.2	2.0	6.6	2.2	5.0
Tenured teachers	6.6	8.5	5.3	7.5	6.5	6.0
New teachers	8.2	8.6	10.2	9.5	8.4	11.1
Panel E. Number of teachers whose priorities are not respected						
for Gale-Shapley stability	8,206	8,438	8,474	8,126	8,233	8,697
for Teacher-SI fairness	8,205	2,171	8,473	0	8,232	8,665
for SI fairness	8,122	0	8,408	0	8,146	8,627

Notes: Panel A of this table reports statistics on teacher mobility: numbers of tenured teachers who moved to a new region and assigned and unassigned new teachers. Panels B and C present the cumulative distribution of the ranks of the regions teachers are assigned to in their preferences. Panel D reports statistics on the average rank of the region teachers obtain. Panel E reports the numbers of teachers whose priorities are not respected under different fairness notions.

we devote less time to this comparison due to the relatively poor performance of SI-DA identified in the previous section in mobility.

Better distribution of teacher experience. The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France.[60] Every year a very large number of teachers with a few years of experience leave Créteil, Versailles, and Amiens. They are mostly replaced by inexperienced teachers. This structural imbalance means that the outcomes under DA* and TTC* are unlikely to improve these regions upon the status-quo. Our counterfactual analysis confirms this (see the left panel of Figure 1).[61] Indeed, the distributions of teacher experience under DA* and TTC* do not Lorenz dominate the status quo.[62,63]

On the other hand, when compared to their status-quo improving counterparts, the picture is more complicated for these mechanisms.

Fact 2. *In the three youngest regions of France, the distribution of teacher experience under SI-CC Lorenz dominates that under TTC*. SI-CC assigns only 1,344 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. On the contrary, the distribution of teacher experience under SI-DA does not Lorenz dominate that under DA* (or vice versa).*

Interestingly, the distributional benefits of status-quo improvement we find for SI-CC do not hold for SI-DA compared to their respective benchmarks. In the three youngest regions of France, SI-DA produces a distribution of teacher experience which does not Lorenz dominate the distribution under DA*. SI-DA also assigns more teachers with one or two years of experience (1,041) to the three youngest regions than DA* (801). This finding confirms that imposing status-quo improvement to (constrained) fair mechanisms can backfire. In general, when two-sided Pareto efficiency is satisfied, we expect that an increase in mobility upon a status-quo improving matching can only be done at the expense of the distribution of teacher experience. This is indeed what we observe while comparing SI-CC to TTC*. However, when two-sided Pareto efficiency is not satisfied and mobility is extremely low, as under SI-DA, this trade-off disappears. In essence, SI-DA just as-

[60]See Figure A.5 for the type distribution in all regions at status-quo.

[61]Although we defined status-quo improvement through the Lorenz domination relation for groups of teachers regarding preferences of regions, it is trivial to extend Lorenz domination to any arbitrary cumulative distribution obtained from histograms of statistics. This is what we use here.

[62]Figures A.6 and A.7 show the Lorenz domination of TTC* over the status quo when we consider only the younger regions.

[63] Figure A.8 shows that the distributional performance of TTC* depends on the tie breaker. For instance, when the teachers starting a chain are selected by increasing order of their maximum Ministry-mandated priority points, the resulting distribution Lorenz dominates those when they are ordered by decreasing order or randomly. However, none of the distributions of teacher experience under TTC* Lorenz dominates the status quo.

signs new teachers to vacant positions and leaves all tenured teachers at their status-quo regions. The improvement upon the status-quo distribution in terms of teacher types is, thus, minimal among tenured teachers under SI-DA. In principle, further movement may help improve these distributions in many regions. Indeed, even though DA* does not impose status-quo improvement, the higher mobility it creates improves the distribution of teacher experience for medium to lower experience types in these three youngest regions in France. However, this improvement comes at the expense of losing higher experience-type teachers with respect to SI-DA and the status-quo. Finally, note that the two fairness-based mechanisms assign fewer inexperienced teachers to Créteil, Versailles, and Amiens than the two efficiency-based mechanisms (1,041 for SI-DA and 801 for DA*, while 1,344 for SI-CC and 1,844 for TTC*). Again, this is due to the severe lack of mobility from these regions.

To complement the results for the three youngest regions of France, we also report the results for the three oldest regions of France. The objective is now to assign younger teachers.

Fact 3. *In the three oldest regions of France, the teacher distribution under SI-CC Lorenz dominates that under TTC* (see the right panel of Figure 1). SI-CC assigns 187 teachers with one or two years of experience to these regions, while TTC* only assigns 96 of them. On the contrary, the distribution of teacher experience under DA* Lorenz dominates that under SI-DA.*

Two channels exist to explain SI-CC's performance in the oldest and attractive regions. For example, take one of the youngest and least attractive regions; Créteil. SI-CC prevents relatively inexperienced, yet, tenured teachers leaving Créteil as more experienced teachers do not want to come to the region. This limits the possibility of assigning these younger teachers to the attractive regions. Moreover, SI-CC prevents new teachers replacing tenured teachers in Créteil due to new teachers' even lower experience (even if they rank Créteil higher than the attractive regions). Thus, these new teachers are directed to the attractive regions. Indeed, by preventing relatively inexperienced but tenured teachers from leaving Créteil for attractive regions, SI-CC lowers competition for vacant seats in the attractive regions. New teachers, who prefer these regions to Créteil, are able to get assigned to these vacant seats. (As we discuss below, new teachers are on average and in distribution assigned to more preferred regions under SI-CC than under TTC*.)

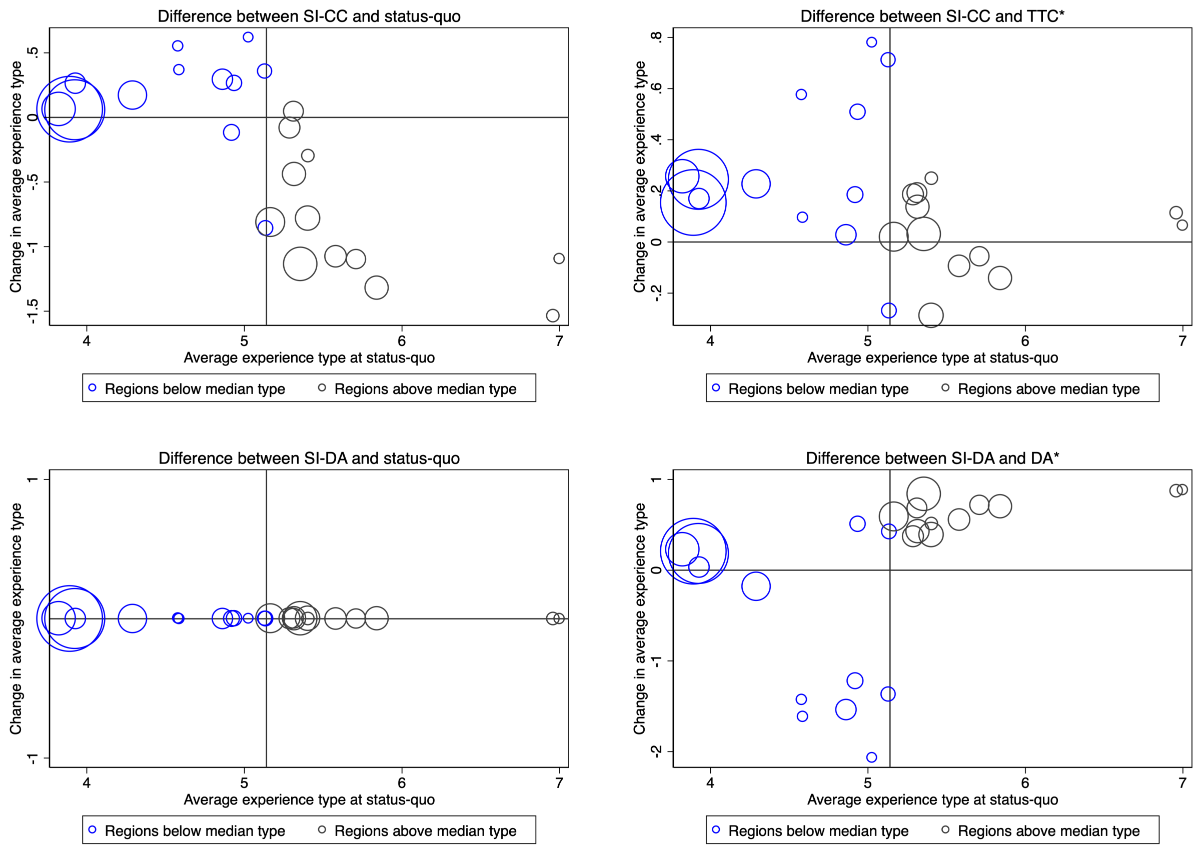
A salient fact emerges when comparing distributional performances in the youngest and oldest regions. In the oldest regions, all mechanisms easily produce a distribution of teacher experience that Lorenz dominates the status-quo, while in the youngest regions only mechanisms that respect status-quo improvement do. This finding reflects the very different levels of attractiveness of these regions. As the oldest regions are highly demanded by teachers, it is easier to improve their teacher experience distribution. This is much more difficult in the youngest regions as they are not as highly demanded, especially by experienced teachers. The ratio of entry to exit requests is equal to 15.5 in Rennes but 0.03 in Créteil (see Table A.2). This large difference in demand also explains why the distributions of teacher experience are very compressed in the youngest regions, but not in the oldest ones (see Figure 1). Due to the limited room for improvement in disadvantaged regions, most mechanisms have a similar *capped* performance. Last, the performance of DA* is very good for the oldest regions since it produces a distribution of teacher experience which Lorenz dominates those of all other mechanisms. For these regions, DA* gives priority to the youngest teachers among those applying. Since these regions are overdemanded, the regions accept the youngest teachers, and, hence, status-quo improvement for these regions is fulfilled as younger teachers are preferred to the older ones in the type rankings of these regions. Note that, under SI-DA, many young (tenured) teachers are not accepted by these regions even though they apply and get stuck in the youngest regions such as Créteil. Thus, the effective demand for the oldest regions under SI-DA is lower. The better performance of DA* in the oldest regions is achieved by accepting young tenured teachers from other regions at the expense of the youngest regions. Thus, as we mentioned before and seen in Figure 1, DA* violates status-quo improvement in the three youngest regions.

Lower inequality among regions. The status-quo improvement requirement makes sure that regions are not harmed by the reassignment of teachers. Older regions become relatively younger, and younger regions become relatively older, reducing the initial difference in teacher experience among regions. While the previous paragraph discussed the distributional performance of the mechanisms for the three youngest and oldest regions, we now consider their performance across all regions. Figure 2 plots, for each region, the change in tenured teacher experience (proxied by the average type) between SI-CC and the status-quo matching (top left figure) and between SI-DA and the status-quo matching (bottom left figure).[64] Regions are ordered, along the horizontal axis, by the average experience of their teachers at the status quo, so that all regions on the left are the younger regions that need more experienced teachers.

The top left panel of Figure 2 shows that, compared to the status-quo matching, SI-CC increases the average experience of tenured teachers in the younger regions (by 0.07 experience types on average, i.e., about 0.14 years) and reduces the average experience of tenured teachers in the older regions (by 0.13 experience types on average, i.e., 0.26 years).

[64]Since after the assignment mechanisms are run new teachers are assigned for the first time, the average experience falls in all regions with respect to status quo. Therefore, first we only inspect the distribution of tenured teachers after reassignment across regions under different mechanisms and status quo.

Figure 2: Change in Average Tenured Teacher Experience Types Across Regions: SI-CC and SI-DA



Notes: This figure shows the difference in the average tenured teacher experience types between the matching obtained by SI-CC and the status-quo matching (top left figure) and between SI-CC and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained by SI-DA and the status-quo matching (bottom left figure) and between SI-DA and its benchmark DA* (bottom right figure). Each circle represents a French region. Circle size reflects region's size. Regions are ordered (on the horizontal axis) by the average experience type of their teachers at status quo. The vertical line represents the median type. All regions on the left of the vertical line have an average type that is strictly below the median. This is the group of regions we identified as *younger regions*. All regions on right of the vertical line are regions whose average type is above the median, i.e., *older regions*. In regions above the horizontal line, the average experience of tenured teachers after reassignment is larger than that under the benchmark matching to which it is compared.

Therefore, SI-CC effectively lowers existing inequalities between younger and older regions by about 0.4 years of experience. The top right panel of Figure 2 shows that SI-CC reduces inequality compared to not only the status-quo matching, but also TTC*.[65] As discussed when comparing SI-DA and its benchmark DA* for the oldest and youngest regions, the same conclusion may not hold for those mechanisms. Indeed, relaxing the status-quo improvement requirement creates a more equal distribution of teacher expe-

[65]We reach similar (if not better) conclusions regarding SI-CC's better performance than its benchmark TTC* when considering both new and tenured teachers together (see Figure A.9).

rience. In the bottom right panel of Figure 2, we observe that SI-DA does not reduce the average experience gap between the younger and older regions compared to DA* for tenured teachers. We summarize these findings as follows:

Fact 4. *SI-CC reduces the large gap in average tenured teacher experience that exists at the status-quo matching between the younger regions and the older regions. This gap goes down by 0.4 years of experience. SI-CC also reduces the gap by 0.3 years compared to TTC*. In contrast, SI-DA is less effective than DA* at reducing the gap in average tenured teacher experience between younger and older regions.*

Limited trade-off between teacher distribution and teacher welfare. Next, we investigate whether the better distributional performance of SI-CC comes at the cost of poorer welfare for teachers, as measured by the number of teachers who obtain a new region and the rank of assigned region in the teacher preference list. Table 3 shows that the number of tenured teachers who move is larger under the benchmark mechanisms. Under SI-CC 1,444 tenured teachers move and under TTC* 2,470 move; a difference of 1,026 teachers. For fairness-based mechanisms, the difference is lower: 894 additional tenured teachers move under DA* compared to SI-DA.

The cost in terms of teacher mobility is larger in the three youngest regions (Créteil, Versailles, and Amiens) than in the three oldest regions (Rennes, Bordeaux, and Lyon). Moreover, similar numbers, 116 vs 117, tenured teachers leave Rennes, Bordeaux, and Lyon under SI-CC and TTC*, respectively, while more tenured teachers leave Créteil, Versailles, and Amiens under TTC* (1,018) than under SI-CC (239). The lower demand for these three youngest and least attractive regions from tenured teachers compared to the oldest regions explains the large difference in outflow under TTC*. The status-quo improvement requirement considerably bounds the outflow from the youngest regions under SI-CC; a potential concern we investigate in Section 5.7.

Welfare differences between tenured and new teachers. The differences we observe between the youngest and oldest regions might explain an interesting finding: despite a larger movement under TTC*, the rank distribution of the region that new teachers are assigned is Lorenz dominated by the distribution under SI-CC (see Panel C of Table 3). This confirms our prior explanations when comparing the mechanisms with their respective benchmarks.

Fact 5. *The distribution of the ranks of the regions that tenured teachers are assigned under TTC* Lorenz dominates that under SI-CC. The opposite Lorenz comparison holds for new teachers.*

On average, new teachers are assigned their 8.2^{th} ranked region under SI-CC and their 10.2^{th} ranked region under TTC* (see Panel D of Table 3). This is because a much larger number of tenured teachers leave the youngest regions of Créteil, Versailles, and Amiens under TTC* (1,018) than under SI-CC (239). These exiting teachers have to be replaced, and new teachers are the most likely substitutes due to lower demand from other tenured teachers. This is because very few tenured teachers ask to enter younger regions and, under TTC*, new teachers can replace tenured teachers in younger regions, even if they have less experience. In practice, we see that 1,415 new teachers are assigned to Amiens, Créteil, or Versailles under TTC* versus 862 under SI-CC. The large share of new teachers being assigned to unattractive regions under TTC* explains why these teachers are assigned to lower ranked regions than under SI-CC.[66]

Effects on fairness measures. Last, we investigate whether imposing the status-quo improvement requirement has an impact on the number of priority violations under different fairness notions. For fairness-based mechanisms, imposing the status-quo improvement requirement can only create priority violations among teachers since it forbids certain teachers to move from their initial position or gives top priority to new teachers over empty slots. This is indeed what we observe since SI-DA has 385 more teachers involved in priority violations compared to its benchmark. The latter being teacher-SI fair, the only priority violations are caused by teachers staying at their initial position. In addition, SI-DA has 2,171 teachers whose priorities are not respected because of new teachers and vacant positions.

For SI-CC, the opposite happens. Indeed, since the latter and its benchmark do not impose any respect of priorities requirement, the additional mobility created by relaxing the status-quo improvement requirement is done at the further expense of fairness under TTC*, which has 268 additional teachers whose priorities are not respected compared to SI-CC. The smaller differences among the three fairness notions show that priority violations among teachers under the TTC* and SI-CC matchings are mainly driven by their high mobility rates implying that less preferred teachers are assigned to a new region at the expense of more preferred ones who also requested that region.

5.7 Relaxing Distributional Objectives

This last empirical subsection investigates by how much we can increase mobility if we relax the status-quo improvement requirement, which has a large role in sustaining distributional objectives as we have seen so far. The results we have presented so far show that SI-CC's better distributional performance (compared to its benchmark mechanism TTC*) comes at the cost of lower mobility, especially for teachers in the three youngest

[66] Our preference estimates reveal that new teachers dislike unattractive regions less than tenured teachers. Yet, only 11.9% of Math teachers rank Créteil or Versailles as their first choice and 14.3% of French teachers. That mild preference for unattractive regions is not large enough to justify that assigning a large share of the new teachers to these two regions will improve the ranking of the region they obtain.

regions. We show in this section that a weakened status-quo improving version of our efficient mechanism, **weak SI-CC** (**wSI-CC**), which is formally defined in Appendix A, can increase mobility, while preserving good distributional results.

Currently tenured teachers accumulate priority points during their tenure at a region, and these priority points are utilized as higher priorities for all regions in the current mechanism. Therefore, from a policy and market design perspective, a design intervention should be sensitive to these accumulated points, as the abrupt change of the mechanism from one that does not require status-quo improvement to one that does may have the undesirable consequences of retroactively eliminating earned rights. In that regard, adopting a milder distributional objective, at least initially, by partially relaxing the status-quo improvement requirement for these regions meets this requirement. Such a policy can be adopted temporarily in a transition period from the old mechanism to the new one until these priority points are fully used up. Or it can be adopted permanently, as we show below that the weaker requirement still has very good distributional outcomes.

The wSI-CC mechanism relaxes the distributional constraint on status-quo teachers by allowing some of them to leave their school while being replaced by a teacher with a lower experience type. The number of teachers replaced by a less experienced teacher is finely controlled at the school level by the threshold acceptability distribution concept presented in Appendix A. In this section, we explore what teacher allocations would look like if schools in the three youngest regions (Créteil, Versailles, and Amiens) were setting the vector of thresholds $d_s = (d_s^{\theta_1}, d_s^{\theta_2}, \ldots, d_s^{|\Theta|})$ to $(0, \ldots, 0, |\omega_s|)$. In other words, in three regions (out of 25) we allow all status-quo teachers to be replaced by any teacher, irrespective of her experience type.[67]

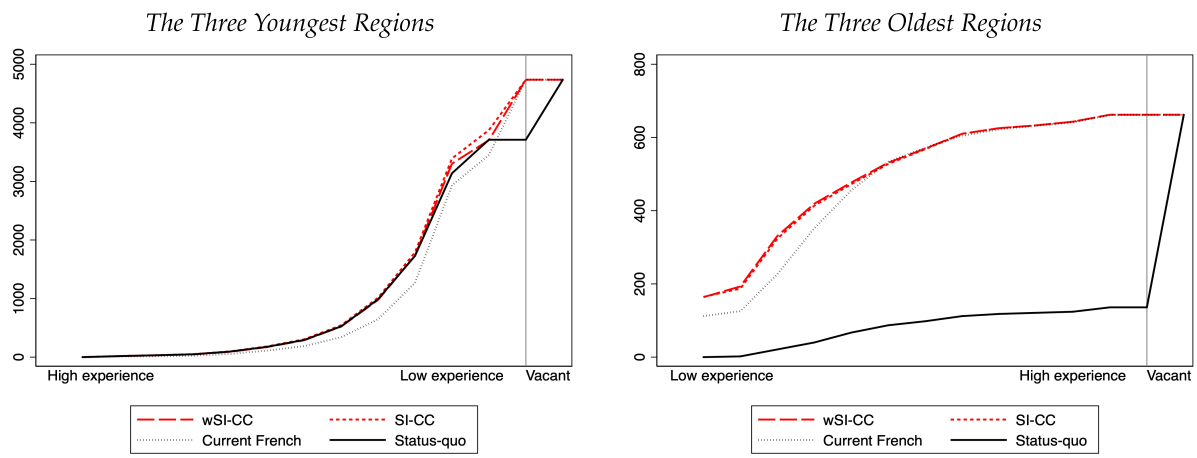
There are two natural benchmarks to which we compare the wSI-CC teacher assignment. First, we compare the matchings under wSI-CC and SI-CC to quantify the mobility benefits and the potential distributional costs of relaxing the distributional objectives. Another interesting benchmark is the matching obtained with the DA* mechanism and Ministry-mandated priorities, that is, priorities that use the Ministry's formula in determining priorities (see Footnote 40) and ignore school preferences based on teacher type rankings.[68] We refer to this mechanism as **Current French** (see also Appendix E). This second benchmark allows us to check whether partially relaxing SI-CC's distributional constraints boosts mobility while maintaining a better distribution of teachers than that

[67]Note, however, that in contrast to TTC*, a teacher cannot leave her position without being replaced. The relaxation only implies that the incoming teacher can have a less preferred experience type. This is an important difference which explains the mobility gap that we observe for the three youngest regions between wSI-CC and TTC*.

[68]This latter approach, used so far, only uses the Ministry priorities as a tie-breaker for the same type teachers.

which prevails under the current French mechanism.

Figure 3: Cumulative Distribution of Teacher Experience Types — Weaker Distributional Objectives

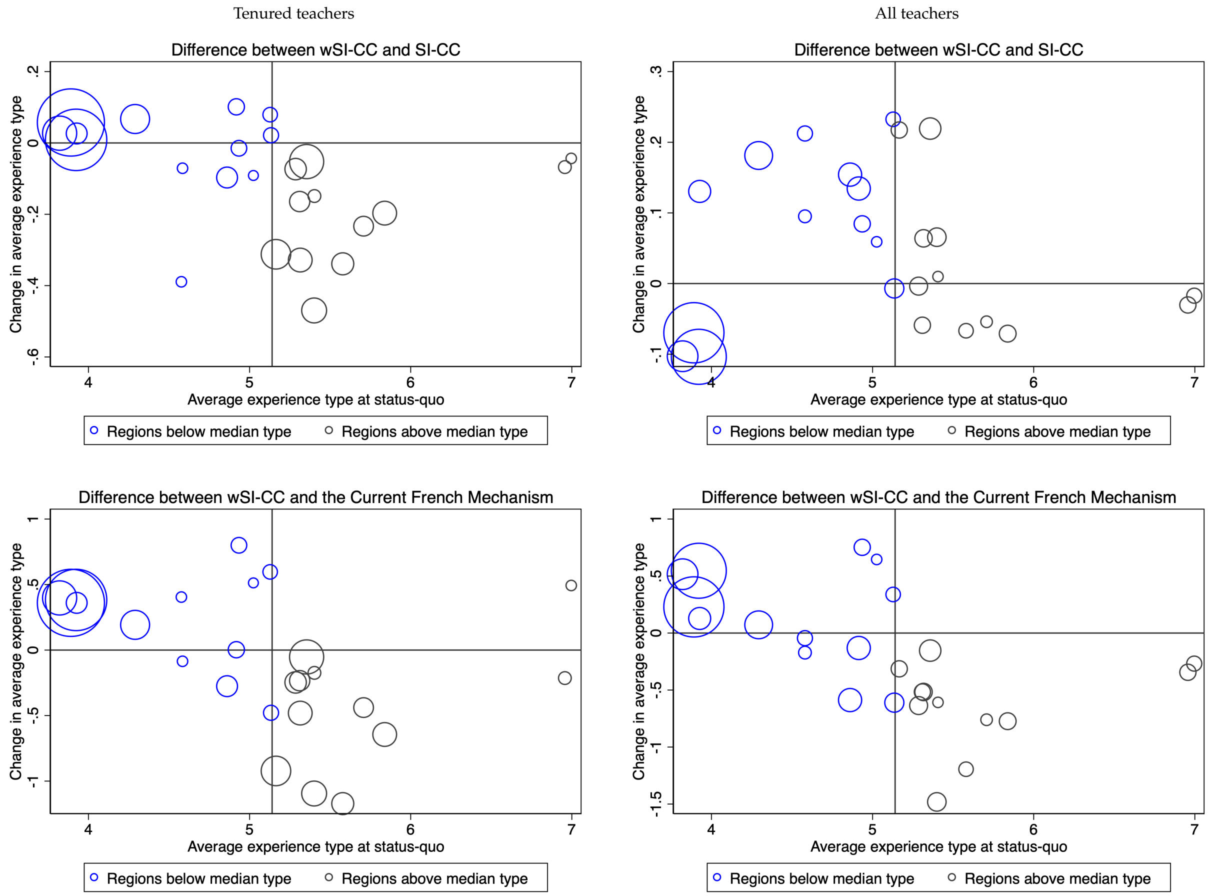


Notes: This figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14$^{\text{th}}$ type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in gray. The thick dark gray line (with legend Status-quo) corresponds to the cumulative distribution of teacher types at the status-quo matching.

The results are reported in the last two columns of Table 3. Several findings stand out. First, partially relaxing the distributional objectives allows significantly more teachers to move away from the three youngest regions compared to the original SI-CC studied in the previous sections. Mobility from these regions goes up from 239 under SI-CC to 464 under wSI-CC. Second, as expected, partially relaxing distributional objectives leads to a smaller improvement in the distribution of teachers. While SI-CC and wSI-CC produce almost the same distributions in the three oldest regions—in which distributional objectives have not changed— a small difference emerges in the three youngest regions (see Figure 3). wSI-CC assigns 96 more teachers with one or two years of experience to the three youngest regions than SI-CC. As a result, wSI-CC leads to a reduction in the average teacher experience in the three youngest regions (see top right results of Figure 4 in which the three youngest regions are represented by the three large blue circles on the left). The mobility gain we observe under wSI-CC in these three regions is mainly driven by new teachers who replace tenured teachers. They would have been prevented from doing so under SI-CC due to their lower experience type.

Finally, note that, for all distributional metrics considered, wSI-CC has a much better performance than the current French mechanism. In both younger and older regions,

Figure 4: Change in Average Tenured Teacher Experience Type Across Regions — Weaker Distributional Objectives



Notes: This figure shows, for each region, the difference in the average tenured experience of teachers under wSI-CC and under SI-CC. The left column reports the experience of tenured teachers, while the right column reports the experience of all teachers. Each circle represents a French region. Circle size reflects a region's size. Regions are ordered (on the horizontal axis) by the average experience of their teachers at status-quo. The vertical line represents the median type. All regions on the left of the vertical line have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on the right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teacher average experience post reassignment is larger than that of the benchmark matching to which it is compared.

the cumulative distribution of teacher exprience types clearly dominates the distribution under the current French mechanism (see Figure 3). As a result, wSI-CC better fulfills the twofold objective of (i) making younger regions older and (ii) making older regions younger (bottom panel of Figure 4). This is true when considering tenured teachers only but also when considering all teachers, that is, tenured and new teachers. These last results are important as they show that there is significant room to improve upon the distribution that the Ministry of Education reaches every year after the annual assignment process. This improvement comes at a small cost in terms of overall teacher mobility, 5,864 teachers move under the current French mechanism versus 5,554 under wSI-CC. For tenured teachers in the three youngest regions, 980 teachers move under the current French mechanism versus 464 under wSI-CC. However, the average rank of the region teachers are assigned in their preferences is 7.3 under wSI-CC and is better than the current French mechanism, which is 8.3.

The role of tie breaking. The tie breaker rule under SI-CC and wSI-CC that determines the order in which the new teacher chains are implemented influences the final allocations (as mentioned in Footnote 63).[69] All the results presented so far order teachers by decreasing order of their maximum Ministry-mandated priority points. This choice is conservative. It leads to lower mobility under both SI-CC and wSI-CC than the mobility we obtain under alternative tie breaker rules. For instance, reversing the order and ranking teachers by increasing order of their maximum Ministry-mandated priority points increases mobility under wSI-CC from 5,554 to 5,687 (and from 464 to 597 in the three youngest regions), which mitigates the mobility cost of wSI-CC compared to the current French mechanism while preserving the good distributional results. This highlights that, in practice, policy makers have multiple tools that can be used to tailor the mechanism to their objectives. Investigating precisely how the different tools can impact the outcomes would be of practical interest for future research.

6 Related Literature

The SI-CC mechanism has its roots in the top-trading cycles algorithms (see Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Pápai, 2000, Roth et al., 2004, and Dur and Ünver, 2019) and the SI-DA mechanism has its roots in the teacher-proposing deferred acceptance algorithm (see Gale and Shapley, 1962, Kelso and Crawford, 1982, Roth and Sotomayor, 1990, Abdulkadiroğlu and Sönmez, 2003, and Hatfield and Milgrom, 2005).

The design of efficient mechanisms in two-sided matching markets with a balanced exchange constraint was previously studied by Dur and Ünver (2019) in the context of student and worker exchange programs. The main difference from the current model is that status-quo improvement was not a requirement in the previous paper. Status-quo improvement substantially changes modeling choices and mechanism design. For example, we have school preferences based on the Lorenz dominance relation over distributions of teacher types leading to a new class of pointing rules in SI-CC that is substantially novel with respect to the variants of TTC. As a result, our design of efficient mechanisms in this domain gives, in general, higher welfare for the schools. We additionally focus on a fairness-based approach with SI-DA and conduct a thorough empirical analysis, which

[69] In contrast, the tie breaker used for tenured teachers does not lead to significant differences in the outcomes.

has no antecedent in this previous paper.

Our paper is also related to Combe et al. (2020) who study teacher reassignment, introduce a class of mechanisms that is two-sided Pareto efficient, and show that a unique selection in this class is teacher optimal. Although both papers consider teacher reassignment problems, our paper differs in three important respects. First, the mechanisms introduced in this previous study do not embed distributional objectives. Their paper focuses on efficiency-based design and efficiency gains, which stands in stark contrast with the objective of our paper, namely to design mechanisms that lead to a better distribution of workers while satisfying incentive properties. Second, the theoretical and empirical results in this previous study focus on teachers with a status-quo assignment and, therefore, largely ignore new teachers. Accounting for the entire market, as we do in this paper, is important because new teachers are a key driver of the unequal distribution of teachers in schools. Additionally, many of the desirable mechanism properties, such as fairness notions and status-quo improvement, do not easily translate to markets with vacancies and new teachers, and as a result theoretical and conceptual treatment in our paper is substantially more complex along this dimension. In that respect, compared to this previous study, (i) we consider a realistic and highly policy-relevant *full* market, (ii) we carefully adapt properties to this market, and (iii) we design mechanisms that fulfill these properties.[70,71,72] Finally, our paper introduces both a novel fairness property that satisfies status-quo improvement and a new mechanism (SI-DA) that fulfills this property. This focus on fairness is absent from Combe et al. (2020), yet SI-DA is an important mechanism. It is a key benchmark for SI-CC in our paper and a relevant fairness-focused mechanism in other applications.

Despite these two previous studies and the current paper, the study of efficient mechanisms under distributional constraints is still rare. Suzuki et al. (2018) and its generalization by Hafalir et al. (2019) provide sufficient conditions on policy goals to get a version of TTC that takes constraints into account and satisfies desirable properties. In particu-

[70] Although Combe et al. (2020) propose a generalization of their mechanisms to account for new teachers, the two step procedure they suggest is not two-sided Pareto efficient and is only strategy-proof when new teachers rank all the schools. Their generalization significantly differs from our SI-CC mechanism. In a first step, new teachers can only point to an employed teacher, excluding vacant positions. In a second step, the unassigned new teachers from the first step are assigned to the remaining vacant positions using the DA mechanism. In contrast, our one-step mechanism simultaneously deals with the first-time assignment of new teachers and the reassignment of tenured teachers.

[71] The estimation of teacher preferences is also primarily carried out on tenured teachers. The discussion of teacher preferences ignores the difference between the preferences of tenured and new teachers, which is central in this paper.

[72] Note that, even in the pure reassignment market, which is a special case of our full market, we show that our SI-CC mechanism does not belong to the class of TO-BE mechanisms defined in Combe et al. (2020), and vice-versa. Our SI-CC mechanism is therefore another strategy-proof and teacher optimal mechanism (see Example A.4 in Appendix D).

lar, these sufficient conditions involve a notion of discrete convexity on the policy goals, namely, M-convexity. In our context with new teachers and vacant positions at schools, we show that the M-convexity of the policy goals is no longer sufficient to ensure a well-behaved version of TTC (see Example A.5 in Appendix D).

Our methodology for constructing the SI-DA mechanism and the auxiliary choice rule is inspired by the choice rule constructions in the slot-specific priorities model of Kominers and Sönmez (2016), which are also used in Dur et al. (2018), Sönmez and Yenmez (2019), and Dur et al. (2020) to address fairness within distributional constraints. On the other hand, Hafalir et al. (2013), Kamada and Kojima (2016), and Kojima et al. (2018) use different setups to address distributional concerns within the fair assignment framework. As we consider status-quo improvement that is substantially different from the standard distributional constraints, our fairness concept, auxiliary choice-rule construction, and results on fairness do not have any correspondence in this previous literature.

On the empirical side, our paper also complements a fast-growing literature that explores wage-based solutions to the unequal distribution of quality teachers in schools. Several recent papers have developed equilibrium models of the labor market for teachers, and used these models to inspect the effect of compensation policies on the distribution of teacher quality (Biasi et al., 2021, Bobba et al., 2021, Bates et al., 2021, and Tincani, 2021). Despite the tremendous progress made by these papers to shed light on price-based solutions to distributional concerns, much less is known on solutions for labor markets that do not rely on prices, or that do so imperfectly. Yet, several countries use a centralized process to assign teachers to schools, like Germany, Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru (Bobba et al., 2021), Uruguay (Vegas et al., 2006), Portugal, and the Czech Republic (Cechlárová et al., 2015).[73] Understanding how to address distributional concerns in these regulated markets is important. The evidence also points to a large cost of wage-based policies to attract good teachers in disadvantaged schools, which might encourage countries to rely on more centralized solutions (Bobba et al., 2021).[74]

Finally, our paper builds on a recent literature developing demand estimation methods in school choice environments (Abdulkadiroğlu et al., 2017, Agarwal and Somaini, 2018,

[73]Beside these fully centralized markets, in most teacher labor markets (like in the US), wage variations are strongly limited by rigid pay scales that determine teacher salary as a function of experience. Biasi et al. (2021) provides insightful discussions on non-flexible wage policies in the US: "Most US public school districts pay teachers according to *steps-and-lanes* schedules, which express a teacher's salary as a function of their experience and education."

[74]Bobba et al. (2021) finds that "it would take six times the current budget to equalize access to teacher quality across Peru". Thakur (2020) also investigates the distributional consequences of centralized assignment for Indian Administrative Service jobs, the top-tier government jobs located across the country before and after a mechanism change.

and Calsamiglia et al., 2020). In particular, we build on techniques based on discrete choice models with personalized choice sets which are relevant for preference estimation when reported preferences might fail to be truthful even under strategy-proof mechanisms (Fack et al., 2019, Akyol and Krishna, 2017, and Artemov et al., 2019).

7 Conclusions

Beyond our novel design axioms such as SI teacher optimality, SI fairness, status-quo improvement and its relaxation, we introduced two novel mechanisms that satisfy the two possibly conflicting properties, SI teacher optimality and SI fairness, respectively, in addition to strategy-proofness and status-quo improvement. The first mechanism, SI-CC, aims at reaching an efficient assignment. As far as we know, it is the first time that an efficient mechanism takes the preferences of both sides of the market into account in its design, in addition to status-quo improvement. This approach relies on a novel *pointing rule* design for both schools and teachers in its algorithm.

The second mechanism, motivated by the current French teacher (re)assignment market, uses a fairness-based solution. Our novel concept, SI fairness, overcomes the non-existence of matchings that respect all priorities and at the same time satisfy status-quo improvement under a mild overdemand assumption that holds in our application. This concept is neither weaker nor stronger than Gale-Shapley stability. The SI-DA mechanism utilizes novel auxiliary choice rules for schools that are not fundamental to our problem domain unlike in other fairness-based mechanisms that utilize the DA algorithm.

We also test these solutions against various benchmarks using field data from France. An important finding of our empirical analysis for the French teacher (re)assignment market is that SI-DA, that satisfies SI fairness, decreases the movement of tenured teachers considerably in spite of decreasing the experience gap. Consequently, fairness-based approaches, like SI-DA or that employed by the French Ministry of Education, may be less suitable for decreasing the teacher experience gap among regions. On the other hand, SI-CC, which satisfies our two-sided Pareto efficiency refinement called SI teacher optimality, performs much better than the current French mechanism and other benchmarks in decreasing the experience gap of teachers while facilitating mobility. The weaker version of SI-CC leads to even more movement of tenured teachers, helping them to exercise their pre-existing accumulated priority points better. Therefore, to decrease the experience gap, SI-CC or its wSI-CC variant are ideal mechanisms.

The applicability of our theoretical and empirical framework is not limited to centralized teacher (re)assignment. It can be applied to any centralized two-sided matching market that aims to improve upon a status-quo matching for both sides of the market. Examples of such markets include, but are not limited to, student exchange programs among colleges, public school districts targeting racial balance among schools, corporate job rotations, and other civil service sectors. We provide more concrete details for three applications in Appendix C. Moreover, our model does not restrict the way schools value teacher experience. Hence, our results hold as long as each school has rankings based on a coarse metric of teacher characteristics in which different schools use possibly different metrics. It remains as a future policy and research question to explore whether SI-DA and SI fairness are more suitable for these other applications.

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Appendices

A Weakening Status-quo Improvement for Schools

In this appendix, we introduce a relaxation of the status-quo improvement requirement for schools since there is an inherent tension between the mobility of teachers and status-quo improvement for schools. We also modify our mechanisms to respect this new requirement.

In the case in which the central authority would like to favor teacher mobility more, instead of status-quo we consider improvements with respect to a *hypothetical teacher distribution*. Consider a school $s \in S$. We relabel types as $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ for school s such that its ranking over teacher types is given as

$$\theta_1 \rhd_s \theta_2 \rhd_s \ldots \rhd_s \theta_m \rhd_s \theta_\emptyset$$

for some $m \leq n$. Then any type $\theta_{\hat{m}}$ with $n \geq \hat{m} > m$ is unacceptable for school s. We refer to a vector $d_s = (d_s^{\theta_1}, d_s^{\theta_2}, \ldots, d_s^{\theta_n}) \in \mathbb{Z}_+^n$ as a **threshold acceptability distribution for school s** if for each $\hat{m} \in \{1, 2, \ldots, m\}$,

$$\sum_{\ell=1}^{\hat{m}} d_s^{\theta_\ell} \leq \sum_{\ell=1}^{\hat{m}} |\omega_s^{\theta_\ell}|.$$

A matching μ is **d_s-improving for school s** if types of teachers in μ_s are acceptable for school s and for each $\hat{m} \in \{1, 2, \ldots, m\}$,

$$\sum_{\ell=1}^{\hat{m}} d_s^{\theta_\ell} \leq \sum_{\ell=1}^{\hat{m}} |\mu_s^{\theta_\ell}|.$$

As the threshold values are weakly smaller than the number of status-quo teachers from the best type to the worst in a cumulative sense, this concept is a weakening of status-quo improvement for school s. Let a profile $d_S = (d_s)_{s \in S}$ be such that for each school s, d_s is a threshold acceptability distribution. A matching μ is **(d_S, ω_T)-improving** if it is d_s-improving for each school $s \in S$ and for each teacher $t \in T$, $\mu_t \; R_t \; \omega_t$. Observe that, if matching μ is status-quo improving, then μ is (d_S, ω_T)-improving.

Under this weakening, we can use the SI-CC and SI-DA mechanisms after we relabel the types of teachers for each school. Given a school s, we need a linear order over its status-quo teachers that rank them according to their types under the school's type ranking $\rhd_s$ and then for the teachers of the same type according to a tie-breaker. Without loss of generality, we can use the linear order $\blacktriangleright_s$ (constructed in Section 4 for SI-DA using a tie breaker $\vdash$) such that for any two status-quo teachers $t, t' \in \omega_s$,

$$t \blacktriangleright_s t' \iff \tau(t) \rhd_s \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

To determine the pointing rule of school s and the pointing rule of teachers under SI-CC and priority rankings of seats under SI-DA, we introduce a school-specific type relabeling function over teachers: Let $\hat{\tau}_s : T \to \Theta$ be the **pseudo type function for school s** which is defined as follows:

- For any status-quo teacher $t \in \omega_s$,

$$\hat{\tau}_s(t) = \begin{cases} \theta_1 & \text{if } \; d_s^{\theta_1} > \left|\{t' \in \omega_s \, : \, t' \blacktriangleright_s t\}\right| \\ \theta_{\hat{m}} & \text{if } \; d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}}} > \left|\{t' \in \omega_s \, : \, t' \blacktriangleright_s t\}\right| \geq d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}-1}} \end{cases},$$

  i.e., the first $d_s^{\theta_1}$ status-quo teachers under $\blacktriangleright_s$ are relabeled with pseudo type θ_1 and status-quo teachers ordered between $d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}-1}} + 1$ and $d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}}}$ under $\blacktriangleright_s$ are relabeled with pseudo type $\theta_{\hat{m}}$ where $m \geq \hat{m} > 1$.
- For any non-status-quo teacher $t \in T \setminus \omega_s$,

$$\hat{\tau}_s(t) = \tau(t),$$

  i.e., each non-status-quo teacher for school s is relabeled with her real type.

Observe that the real type of each status-quo teacher is ranked at least as high as her pseudo type in school's type ranking $\rhd_s$: For any $t \in \omega_s$, we have $\tau(t) \trianglerighteq_s \hat{\tau}_s(t) \rhd_s \theta_\emptyset$.

The **weak status-quo improving cycles and chains** mechanism (or **wSI-CC**) is defined using the school-specific pseudo type functions $(\hat{\tau}_s)_{s\in S}$ in the definition of school pointing orders $(\gtrdot_s)_{s\in S}$ and in the SI-CC mechanism definition, which is Definition 1 in Section 3, with the exception that when a tenured teacher is assigned to her own status-quo school in the algorithm her type used in the balance calculation is not her pseudo type but her real type.[75]

The **weak status-quo improving deferred acceptance** mechanism (or **wSI-DA**) uses the school-specific pseudo type functions $(\hat{\tau}_s)_{s\in S}$ instead of the real type functions τ in the construction of the slot-specific ranking definition in the auxiliary choice rule construction in Definition 3 in Section 4 with the exception that when a tenured teacher is ranked in the ranking of one of her own status-quo school's seats—whenever she is not ranked as the top teacher for this seat—her real type is used but not her pseudo type.[76] Then the DA

[75]This exception ensures that she is treated like non-status-quo teachers when she is assigned to her own school, i.e., using her real type, for efficiency reasons.

[76]This exception similarly ensures that she is treated like non-status-quo teachers in case she is considered for this seat, i.e., using her real type, for efficiency reasons.

algorithm is executed using these choice rules.

The new mechanisms inherit the desired properties of their respective base mechanisms under the modification that each "status-quo improvement regarding schools" should be replaced with "d_S-improvement" in the property definitions (see Section 3 for SI-CC and see Section 4 for SI-DA). We use wSI-CC in our empirical simulations besides the original SI-CC in Section 5.7.

B Omitted Result and Proofs

Proposition A.1. *Let ψ be a status-quo improving and two-sided Pareto efficient mechanism which selects a matching that is not SI teacher optimal whenever such a matching exists. Then, ψ is not strategy-proof.*

Proof. On the contrary suppose ψ is strategy-proof. Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}, \omega_{s_2} = \{t_2\}, \omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_{\emptyset}$; $\tau(t_1) \rhd_{s_2} \tau(t_3) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_{\emptyset}$; $\tau(t_2) \rhd_{s_3} \tau(t_1) \rhd_{s_3} \tau(t_3) \rhd_{s_3} \theta_{\emptyset}$; $s_3\, P_{t_1}\, s_2\, P_{t_1}\, s_1\, P_{t_1}\, \emptyset$; $s_1\, P_{t_2}\, s_3\, P_{t_2}\, s_2\, P_{t_2}\, \emptyset$; and $s_2\, P_{t_3}\, s_1\, P_{t_3}\, s_3\, P_{t_3}\, \emptyset$.

There exists a unique SI teacher optimal matching, denoted by ν, in which each teacher is assigned to her top choice. In any other status-quo improving two-sided Pareto efficient matching at most one teacher is assigned to her top choice and at least one teacher is assigned to her second choice. Suppose ψ selects two-sided Pareto efficient and status-quo improving matching μ in which t_1 is assigned to her second choice s_2. If t_1 reports only s_3 preferred to s_1 such that s_3 and s_1 are the only acceptable schools, then ν is the unique SI teacher optimal matching and, given the properties of mechanism ψ, it must assign t_1 to s_1, t_2 to s_3 and t_3 to s_2. In this updated market, if t_2 reports only s_1 and s_2 acceptable, then ν is the unique SI teacher optimal matching and, given the properties of mechanism ψ, it must assign t_1 to s_3, t_2 to s_2 and t_3 to s_1. Finally, in this updated market, if t_3 reports only s_2 and s_3 acceptable, then ν is the unique two-sided Pareto efficient and status-quo improving matching and ψ needs to select it. However, this contradicts with the strategy-proofness of ψ, i.e., it is manipulated by t_3.

By symmetry, we can show the same result for any two-sided Pareto efficient and status-quo improving matching in which at least one teacher is assigned to her second choice under the original market. ■

Proof of Proposition 1. On the contrary, suppose μ is SI teacher optimal and it is Pareto dominated by ν. Since μ status-quo improves ω so does ν. Hence, ν is status-quo improving. In addition, since ν Pareto dominates μ, all teachers weakly prefer ν to μ. Because, $\nu \neq \mu$ and teachers' preferences are strict, some teachers strictly prefer ν to μ. However, this violates SI teacher optimality of μ. This is a contradiction. ∎

Proof of Theorem 1. **SI teacher optimality:** Recall that the requirement of status-quo improvement is embedded in the definition of SI teacher optimality. Consider an arbitrary market P. Let $\hat{\mu}$ be the outcome of SI-CC under this market. We proceed in two parts.

1. We first show that $\hat{\mu}$ is status-quo improving.
   First, consider teachers. Under SI-CC, each school s points to all teachers in ω_s one by one. When a teacher $t \in \omega_s$ is pointed to by s in some Step k, then $s \in A_t^k$ and she can always form a one-school cycle (s,t) whenever she points to s. Similarly, any new teacher $t \in N$ can form a cycle with $\emptyset$ in any step of SI-CC. Hence, $\hat{\mu}_t \; R_t \; \omega_t$ for all $t \in T$.
   Next, consider schools. The tail of any executed chain is a new teacher. Hence, if in some step of SI-CC, a school s is sending out a teacher then it is simultaneously acquiring another teacher; as a result $|\hat{\mu}_s| \geq |\omega_s|$. In any step k of SI-CC, when we consider the set of remaining status-quo employees and teachers assigned in the first $k-1$ steps, because of the positive balance requirement in Improvement Condition 1, the previous observation and the fact that a teacher cannot be assigned a school if she is unacceptable, each school s is weakly better off compared to ω_s. Hence, $\hat{\mu}_s \; \succsim_s \; \omega_s$ for all $s \in S$.
   We showed that $\hat{\mu}$ is status-quo improving.
2. Before proving $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers, we first state a claim that will be used in the proof.

   Claim 1. For a school s, suppose step K is a step in which school s is assigned through a chain with s as a head. Let the set of remaining status-quo employees of s at the end of step K be denoted as ω_s^K and all assigned teachers to s from the beginning of step 1 until the end of step K as μ_s^K. Let ν be a matching such that all teachers assigned in the first K steps of SI-CC are matched with their assignment under SI-CC and $|\nu_s| < |\omega_s^K \cup \mu_s^K|$. Then, ν is not a status-quo improving matching.

   Proof. Suppose teacher t is assigned to s in this chain in step K. By the construction of SI-CC, it must be that t points to s under Condition 2. First observe that, by construction, $\mu_s^K \subseteq \nu_s$. Also notice that, if $\omega_s^K = \emptyset$, then $|\nu_s| \geq |\omega_s^K \cup \mu_s^K|$, a contradiction with the assumption of the claim. Hence, $\omega_s^K \neq \emptyset$. Since, by the definition of SI-CC, Condition 1 does not hold for school s via teacher t, there exists some type θ such that $\tau(t_s^K) \; \trianglerighteq_s \; \theta \vartriangleright_s \; \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step K of SI-CC.[77] That is, the number of teachers with a weakly better type than θ in $\mu_s^K \cup \omega_s^K$ cannot be more than what it is in ω_s. Moreover, by the definition of SI-CC, all teachers in ω_s^K have a weakly better type than θ. Hence, $|\nu_s| < |\omega_s^K \cup \mu_s^K|$ and $\mu_s^K \subseteq \nu_s$ imply that the number of teachers with a weakly better type than θ in ν_s is strictly less than this number in ω_s. Therefore, ν_s is not preferred to ω_s by school s. $\diamond$

Next, we show that $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers.

On the contrary, suppose there exists a status-quo improving matching ν that Pareto dominates $\hat{\mu}$ for teachers. By considering the teachers assigned in each step of SI-CC inductively, we show that such a matching cannot exist, in particular we should have $\nu = \hat{\mu}$.

We denote the set of teachers assigned in step k of SI-CC in market P with T_k and the union of these sets up to step k as $\bar{T}_k = \cup_{k'=1}^{k} T_{k'}$.

Step 1: Each teacher $t \in T_1$ is assigned in $\hat{\mu}_t$ to the best school in A_t^1. If $\nu_t \; P_t \; \hat{\mu}_t$ for some $t \in T_1$, then $\nu_t \notin A_t^1$. Thus, for school $s = \nu_t$ either improvement condition does not hold for teacher t. Since this is Step 1, the current matching satisfies $\mu = \emptyset$, and hence, the current balances $b_s^\theta = 0$ for all schools s and types θ. If Condition 1 does not hold, then two cases are possible:

- There exists a teacher $t_s^1 \in \omega_s$ to whom s is pointing. Then, it has type $\tau(t_s^1) \rhd_s \tau(t)$: thus, t has a worse type than the worst type status-quo employees of this school; and
- There does not exist a teacher to whom s is pointing. Then, $\omega_s = \emptyset$.

Thus, in either case, $\nu_s \setminus \{t\} \;\not\succcurlyeq_s\; \omega_s$ and $|\nu_s| > |\omega_s|$ as otherwise ν is not status-quo improving for s by Lorenz preferences. The violation of Condition 2 for s via t, on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no new teachers: in this case, as we showed $|\nu_s| > |\omega_s|$ implies that there exists some school s' such that $|\omega_{s'}| > |\nu_{s'}|$; as a result in this case status-quo improvement for s' under ν would be violated by Lorenz preferences; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

[77] Actually, the sum of balances $\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'}$ never becomes negative in the mechanism for any type θ, as the sum starts at zero at the beginning of Step 1, and whenever it is zero, we do not admit a teacher with a type worse than θ by sending out a teacher with a type better than θ by Improvement Condition 1.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher t cannot exist with $\nu_t P_t \hat{\mu}_t$. Since $\nu_t \; R_t \; \hat{\mu}_t$ for all t then for all $t \in T_1$, $\nu_t = \hat{\mu}_t$.

Inductive assumption: For any $k > 1$, assume that for all $k' < k$ and $t \in T_{k'}$, $\nu_t = \hat{\mu}_t$. We show that the same holds for teachers in T_k:

Step k: Each teacher $t \in T_k$ is assigned in $\hat{\mu}_t$ to the best school in A_t^k. If $\nu_t \; P_t \; \hat{\mu}_t$ for some $t \in T_k$, then $\nu_t \notin A_t^k$. Thus, for school $s = \nu_t$ both improvement conditions are violated via teacher t. Noting μ is the current matching determined until the end of step $k-1$, the violation of Condition 1 implies that

- if there exists a teacher $t_s^k \in \omega_s$ to whom s is pointing, then it has type $\tau(t_s^k) \rhd_s \tau(t)$ and there exists an intermediate type θ such that $\tau(t_s^k) \; \trianglerighteq_s \; \theta \rhd_s \tau(t)$ with

  $$\sum_{\theta' \trianglerighteq_s \theta} |\mu_s^{\theta'}| - |\{t' \in \omega_s : \mu_{t'} \neq \emptyset\}^{\theta'}| \leq 0.$$

  By the inductive assumption for the current matching $\mu_{t'} = \nu_{t'}$ for all t' assigned until this step (i.e., those in $\bar{T}_{k-1}$), and hence we also have

  $$\sum_{\theta' \trianglerighteq_s \theta} \left| \left(\nu_s \cap \bar{T}_{k-1}\right)^{\theta'} \right| - \left| \left(\omega_s \cap \bar{T}_{k-1}\right)^{\theta'} \right| \leq 0.$$

  By the definition of SI-CC, teacher t has a worse type than the remaining worst-type status-quo employee of this school, i.e., those in $\omega_s \setminus \bar{T}_{k-1}$. Thus, in ν replacing any of these employees with t would violate status-quo improvement for s in ν, as this would have led to a Lorenz violation for type θ:

  $$\sum_{\theta' \trianglerighteq_s \theta} \left|\nu_s^{\theta'}\right| - \left|\omega_s^{\theta'}\right| < 0.$$

  Then t does not replace any of the remaining status-quo employees, but she is an additional teacher acquired: $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.
- if such a teacher does not exist, then $\omega_s \setminus \bar{T}_{k-1} = \emptyset$, and hence, as $t \in \nu_s \setminus \bar{T}_{k-1}$ we have $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

Observe that in the algorithm at each step we make sure that each school acquires at least as many teachers as it sends out and hence, for the current matching $|\mu_s| \geq |\omega_s \cap \bar{T}_{k-1}|$. Since $\mu_s = \nu_s \cap \bar{T}_{k-1}$ by the inductive assumption, we have $|\nu_s \cap \bar{T}_{k-1}| \geq |\omega_s \cap \bar{T}_{k-1}|$. Therefore, as we also showed that $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$ above, we obtain $|\nu_s| > |\omega_s|$.

The violation of Condition 2 for s via t, on the other hand, implies that one of the following conditions holds:

- either t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no remaining new teachers: we know that

$$|\nu_s \backslash \bar{T}_{k-1}| > |\omega_s \backslash \bar{T}_{k-1}| \,.$$

If there are no remaining new teachers at step k, this implies that for some school s'

$$|\nu_{s'} \backslash \bar{T}_{k-1}| < |\omega_{s'} \backslash \bar{T}_{k-1}| \,.$$

Hence, for school s', the set of tenured teachers in $T \backslash \bar{T}_{k-1}$ it gets is smaller than the set of tenured teachers that it has in $\omega_{s'} \backslash \bar{T}_{k-1}$. If status-quo improvement is satisfied for school s', we must have

$$|\nu_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| > |\omega_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| \,.$$

By the induction hypothesis,

$$|\nu_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| = |\mu_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| \,.$$

Hence, we must have

$$|\mu_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| > |\omega_{s'} \cap \bar{T}_{k-1} \cap (T \backslash N)| \,.$$

Thus, a chain was implemented in a previous step and s' was the head of this chain. Let $\ell \leq k-1$ be the last step at which such a chain was implemented. Now, recall that

$$|\nu_{s'} \backslash \bar{T}_{k-1}| < |\omega_{s'} \backslash \bar{T}_{k-1}|$$

and the induction hypothesis yield

$$|\nu_{s'}| < \left|(\omega_{s'} \backslash \bar{T}_{k-1}) \cup \mu_{s'}^{k-1}\right|$$

where $\mu_{s'}^{k-1}$ stands for all assigned teachers to s' from the beginning of step 1 until the end of step $k-1$. Note that the previous inequality is true for all steps k' with $\ell \leq k' \leq k-1$. Indeed, by the definition of ℓ, either a cycle or a chain in which s' was not the head was implemented at step k'. In both cases, if s' was part of this cycle or chain, the number of status-quo teachers leaving equals the number of teachers entering so that the right hand side of the previous inequality remains the same as at step k. So at the end of step ℓ: a chain was just implemented where s' was the head of the chain and $|\nu_{s'}| < \left|(\omega_{s'} \backslash \bar{T}_{\ell-1}) \cup \mu_{s'}^{\ell-1}\right|$. We can now apply

Claim 1 to step ℓ and school s' and conclude that ν is not SI-improving, or,

- all vacant seats of school s have been assigned to teachers in $\bar{T}_{k-1}$. By the induction hypothesis, this implies that no vacant seats of school s are assigned to teachers in $T \setminus \bar{T}_{k-1}$ under ν_s. However, we showed that $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$ which contradicts the feasibility of ν.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher $t \in T_k$ with $\nu_t \; P_t \; \hat{\mu}_t$ cannot exist.

Since $\nu_t \; R_t \; \hat{\mu}_t$ for all t then for all $t \in T_k$, $\nu_t = \hat{\mu}_t$, completing the induction and showing that $\nu = \hat{\mu}$.

Strategy-proofness: We state two claims that we will use in the proof.

Claim 2. Suppose teacher t is assigned in step K of SI-CC. For any $k < K$, $A_t^{k+1} \subseteq A_t^k$.

Proof. Let $s \notin A_t^k$. We will show that $s \notin A_t^{k+1}$. We consider two possible cases.

Case 1. s does not have a vacant position at step k: First notice that, if there is no remaining status-quo employee of s in step k, then s should have been removed in an earlier step of SI-CC. Then, there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If school s is part of the executed cycle or chain in step k, then the teacher assigned to s has a type weakly better than type θ under $\triangleright_s$ and similarly, the teacher leaving school s, namely, t_s^k also has a type weakly better than type θ. Hence, after executing the cycle in step k the relation above still holds. Moreover, s cannot send out a status-quo employee without getting a new one by the definition of SI-CC. Similarly, s cannot get a teacher without sending a status-quo employee. If school s is not part of the executed cycle or chain in step k, the equation above still holds. In either case, $s \notin A_t^{k+1}$.

Case 2. s has a vacant position at step k: Either t is unacceptable for s or t is acceptable for s but there does not exist a remaining new teacher in step k. If the former case holds, then $s \notin A_t^{k+1}$ by definition.

If the latter case holds, then either s does not have a remaining status-quo employee or there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If the former subcase holds, then neither Condition 1 nor Condition 2 holds for t in step $k+1$. For the latter condition, we refer to Case 1 above. Hence, $s \notin A_t^{k+1}$. ◇

Claim 3. Consider a step k of SI-CC mechanism such that there exists a path of schools and teachers $(s_1, t_1, s_2, t_2, \ldots, s_\ell, t_\ell)$ in which for each $\ell' \leq \ell$, school $s_{\ell'}$ points to teacher $t_{\ell'}$ and teacher $t_{\ell'-1}$ points, according to the improvement condition 1, to school $s_{\ell'}$ for each $\ell' < \ell$ and $s_1 \in A_{t_\ell}^k$. If none of the schools in this path are assigned a teacher in this step, the same path forms in step $k+1$ and $s_1 \in A_{t_\ell}^{k+1}$.

Proof. As no teacher is assigned to the schools of the path in step k, the teachers in the path remain in the step $k+1$. Since $t_{\ell'} = t_{s_{\ell'}}^k$ is the highest priority remaining status-quo employee under the pointing order in step k of school $s_{\ell'}$, she continues to be so in step $k+1$, thus, school $s_{\ell'}$ points to $t_{\ell'}$ in step $k+1$. Moreover, no other status quo employee of these schools is assigned to any other school in step k, either, because the assignment of status-quo employees requires the school pointing to them and each school points to at most one teacher in this step. Thus, as Condition 1 or Condition 2 holds for each school $s_{\ell'}$ via teacher $t_{\ell'-1}$ (in modulo ℓ, thus $t_0 = t_\ell$) in step k, the same condition continues to hold in step $k+1$ via the same teacher. Hence, $s_{\ell'} \in A_{t_{\ell'-1}}^{k+1}$ for each ℓ'. Since $A_{t_{\ell'-1}}^{k+1} \subseteq A_{t_{\ell'-1}}^k$ by Claim 2, and $s_{\ell'}$ is the favorite school of teacher $t_{\ell'-1}$ in the opportunity set in step k, we still have $s_{\ell'}$ as the favorite school of teacher $t_{\ell'-1}$ in step $k+1$ and she continues to point to $s_{\ell'}$ in Step $k+1$. ◇

We are ready to finish the proof for the strategy-proofness of SI-CC. Recall that we denote the set of teachers assigned in step k of SI-CC with T_k. First, notice that a teacher t' cannot change the schools in $A_{t'}^1$ by misreporting her preferences since $A_{t'}^1$ does not depend on the submitted preferences. Moreover, by Claim 2, A_t^k, the opportunity sets for teacher t, weakly shrink throughout SI-CC. Hence, a teacher t cannot be assigned to a school $s \notin A_t^1$ under SI-CC. We first consider the teachers in T_1. Each $t \in T_1$ is assigned to her best choice in A_t^1. Hence, any teacher $t \in T_1$ cannot benefit from misreporting her preferences.

Next, we consider a teacher $t \in T_2$. As explained above, teacher t cannot be assigned to school $s \notin A_t^1$ under SI-CC. Teacher $t \in T_2$ is assigned to best school in A_t^2 when she submits her true preferences. We denote the best school in A_t^2 according to P_t with s'. By Claim 2, $A_t^2 \subseteq A_t^1$. Hence, if $t \in T_2$ can benefit from misreporting her preferences, then she is assigned to some school $s \in A_t^1$. If $A_t^1 = A_t^2$, then t cannot benefit from misreporting her preferences. Suppose $A_t^1 \setminus A_t^2 \neq \emptyset$. We will show that t cannot be assigned to a school $s \in A_t^1 \setminus A_t^2$ such that $s \, P_t \, s'$ by misreporting. Particularly, we show t cannot prevent the cycle or chain executed in step 1 without hurting herself.

First notice that, if t forms a cycle in step 1 by misreporting and pointing to some school $s'' \in A_t^1$, then by Claim 3, $s'' \in A_t^2$ and the path leading to t in this cycle starting with school s'' forms again when she submits P_t, which does not match her in step 1. Hence, any such school s'' cannot be preferred to s', i.e., t's assignment under truthtelling.

If a chain is executed in step 1 when teacher t is truthful, teacher t cannot be a part of a new executed chain by misreporting and pointing to some other school in A_t^1. This follows from the fact that the executed chain starts with a specific new teacher and a teacher $\bar{t}$, who is pointed to by her status-quo school $\bar{s}$, can only be added to the executed chain if a previously included teacher points to $\bar{s}$, independent of $\bar{t}$'s preferences. Teacher t can prevent the executed chain by only forming a cycle by misreporting. However, as explained above, under such a cycle t will be assigned to a school weakly worse than s'.

Moreover, with a similar reasoning to a chain, teacher t cannot affect the executed cycles in step 1 by submitting a different preference list without being matched in step 1 in a new cycle (and therefore, making her weakly worse off as we showed above).

By using similar arguments, we can show that any teacher in T_k where $k > 2$ cannot benefit from misreporting her preferences. ■

Proof of Proposition 2. Recall that Gale-Shapley stability is equivalent to acceptability, non-wastefulness, and respect for priorities altogether. Thus, we first show the existence of a Gale-Shapley stable matching. Consider a market P. We construct a strict rank order list, $\hat{\succ}_s$, for each school s over the teachers as follows: for any $t, t' \in T$

- if $\tau(t) \rhd_s \tau(t')$ then $t \hat{\succ}_s t'$;
- if $\tau(t) = \tau(t')$, then the relative order between t and t' is determined arbitrarily;
- $\tau(t) \rhd_s \theta_\emptyset$ if and only if $t \hat{\succ}_s \emptyset$.

It is easy to verify that the outcome of teacher-proposing DA algorithm (Abdulkadiroğlu and Sönmez, 2003) under $(P, \hat{\succ})$ is Gale-Shapley stable.

Next, we show that for some market there does not exist a matching that is both respectful of priorities and status-quo improving. Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},$$

with quotas $q_s = q_{s'} = 1$, type rankings $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$, and $\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s'\ P_{t_1}\ s\ P_{t_1}\ \emptyset,$$

$$s'\ P_{t_2}\ s\ P_{t_2}\ \emptyset,$$

Under this market, unique status-quo improving matching is ω. However, ω does not respect the priority of t_1 at s'. ■

Proof of Proposition 3. We show via an example that, when there are vacant positions at some school, then there exists no teacher-SI fair and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1\}$, $\omega_s = \{t_1\}$, $\omega_{s'} = \emptyset$. Each school has one available position and t_1 prefers s' to s. The unique status-quo improving matching is ω but it is wasteful. Hence, it is not teacher-SI fair. ■

Proof of Proposition 4. **Substitutes:** On the contrary, we suppose there exist $\bar{T} \subseteq T$ and distinct $t, t' \in \bar{T}$ such that $t \in C_s(\bar{T})$ and $t \notin C_s(\bar{T} \setminus \{t'\})$. There exists some other teacher t'' who was assigned to s^k under $C_s(\bar{T} \setminus \{t'\})$, where s^k is t's slot under $C_s(\bar{T})$. Consider the execution of the algorithm to determine $C_s(\bar{T} \setminus \{t'\})$ in step k when t'' is assigned to s^k: as t is not assigned in $C_s(\bar{T} \setminus \{t'\})$, she is still available and is not picked by slot s^k; thus, $t'' \succ_s^k t$. As a consequence, when the algorithm was executed to determine $C_s(\bar{T})$, teacher t'' was already assigned to a slot $s^{k''}$ such that $k'' < k$ so that she was not available when t was assigned s^k.

We will show that such a teacher t'' cannot exist, leading to a contradiction and completing the proof for the substitutes condition.

Claim. There is no teacher $\bar{t}$ such that she is assigned to a slot $s^{\hat{k}}$ in $C_s(\bar{T} \setminus \{t'\})$ and to a slot $s^{\bar{k}}$ in $C_s(\bar{T})$ such that $\bar{k} < \hat{k}$.

Proof. Suppose on the contrary such a teacher $\bar{t}$ exists. Let $\bar{t}$ be chosen such that $\bar{k}$ is the smallest such index among the index of slots filled by such teachers in $C_s(\bar{T})$.

If slot $s^{\bar{k}}$ is unfilled in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T} \setminus \{t'\})$ by the supposition, we should have $\emptyset \succ_s^{\bar{k}} \bar{t}$. But then teacher $\bar{t}$ cannot be assigned to $s^{\bar{k}}$ in $C_s(\bar{T})$.

If a teacher $\hat{t}$ is assigned to $s^{\bar{k}}$ in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T} \setminus \{t'\})$ by the supposition, we should have $\hat{t} \succ_s^{\bar{k}} \bar{t} \succ_s^{\bar{k}} \emptyset$. Because $\bar{k}$ is the smallest index k of the slots filled by teachers in $C_s(\bar{T} \setminus \{t'\})$ who get a slot $s^{\bar{k}}$ in $C_s(\bar{T})$ such that $\bar{k} < k$, teacher $\hat{t}$ is not assigned a slot preceding $s^{\bar{k}}$ in $C_s(\bar{T})$. Therefore, she is available when $s^{\bar{k}}$ is filled in $C_s(\bar{T})$. Yet she is not picked even though $\hat{t} \succ_s^{\bar{k}} \bar{t} \succ_s^{\bar{k}} \emptyset$, a contradiction. Thus, such a teacher $\bar{t}$ cannot exist. ◇

Law of Aggregate Demand: On the contrary, we suppose there exists $\bar{T} \subseteq T$, $t \notin \bar{T}$ and $|C_s(\bar{T})| > |C_s(\bar{T} \cup \{t\})|$. Then, there exists a slot s^k which is filled under $C_s(\bar{T})$ but not under $C_s(\bar{T} \cup \{t\})$. However, due to the above Claim in the proof for the substitutes condition, the teacher who was assigned s^k in $C_s(\bar{T})$ is available when s^k is being filled in

$C_s(\bar{T} \cup \{t\})$. Then this slot cannot be vacant in $C_s(\bar{T} \cup \{t\})$ as this teacher is acceptable for the slot, which is a contradiction. We showed that $|C_s(\bar{T})| \leq |C_s(\bar{T} \cup \{t\})|$.

By the repeated application of this argument, we conclude that whenever $\bar{T} \subseteq \hat{T}$, $|C_s(\bar{T})| \leq |C_s(\hat{T})|$. ■

Proof of Theorem 2. Strategy-proofness: It was shown by Hatfield and Milgrom (2005) that whenever the choice rules of schools satisfy the substitutes and law of aggregate demand conditions, the resulting mechanism through DA is strategy-proof for teachers. Since for each school s, auxiliary choice rule C_s satisfies these conditions, SI-DA is strategy-proof.

SI Fairness: Suppose Assumption 1 holds. Let SI-DA outcome be μ. We will first show that it is status-quo improving. By our construction of the slot priorities, a teacher t will be accepted by ω_t whenever she applies and she will never be rejected in the further steps. Hence, it is status-quo improving for teachers. Consider the schools. First, we prove the following claim.

Claim. Each school fills all its positions in μ.

Proof. To see this, notice that no teacher t is assigned to a school s that is less preferred to ω_t in μ. Therefore, all teachers who were employed at the status quo are assigned to some school in μ. Moreover, we claim that exactly $\sum_{s \in S}(q_s - |\omega_s|)$ new teachers are assigned in μ. On the contrary, suppose this claim does not hold. Then, at least one position of a school s is unfilled in μ and this matching leaves at least one new teacher $t \in N$ unmatched who, by Assumption 1, considers all schools with a vacant position acceptable and is acceptable at all schools with vacant positions at status quo. In determining $C_s(B_s^{K+1})$, where K is the final step of the DA algorithm, if the slot corresponding to this vacant position was unfilled at the status quo, then an unassigned new teacher would have applied to that school and have been assigned to that slot by Assumption 1. Thus, this slot is filled at the status quo.

Then as all employed teachers at status quo are assigned to some school in μ, there exists a teacher $\hat{t} \notin N$ assigned in μ to a slot $\hat{s}^{\hat{k}}$ that was unfilled at the status quo at some school $\hat{s}$.

Since new teacher t is unassigned in μ, again by Assumption 1, she should have applied to all schools with vacant positions at status quo including $\hat{s}$. Since $\hat{s}$ has a vacant position at status quo, it considers t acceptable. Moreover, at the slots that are unfilled at the status quo, acceptable new teachers have a higher ranking than employed teachers at the status quo by the construction of the slot rankings: $t \succ_{\hat{s}}^{\hat{k}} \hat{t}$. Thus, slot $\hat{s}^{\hat{k}}$ should have held t instead of $\hat{t}$, a contradiction.

Hence, all positions are filled in μ. $\diamond$

Since all positions are filled in μ, by our construction of the rankings of the slots, the matching Lorenz dominates the status-quo matching ω. Hence, μ is also status-quo improving for schools.

Given all positions are filled in μ, it is non-wasteful.

Next, we will show that the third bullet in the definition of SI fairness is satisfied, i.e. there is no violation of priorities from non status-quo employees except for new teachers via the empty seats. By construction of the slot rankings for a school s, for teachers neither in ω_s nor in N, the type ranking of the school is respected.

Suppose there exists a teacher t whose priority is not respected at some school s under μ, i.e., t prefers s to μ_t and there exists a teacher $\hat{t} \in \mu(s)$ such that $\tau(t) \rhd_s \tau(\hat{t}) \rhd_s \theta_\emptyset$.

Then t should have made an offer to s that was rejected in the DA algorithm. All teachers assigned to all slots of s would have a higher ranking in that slot than t. If $\hat{t}$ was assigned in μ to a filled slot at status quo then, by the construction of the slot priorities, $\hat{t} \in \omega_s$ and $t \notin \omega_s$ as $\tau(t) \rhd_s \tau(\hat{t})$. If $\hat{t}$ was assigned in μ to a vacant position at status quo so that $\hat{t} \in N$ and $t \notin N$ as $\tau(t) \rhd_s \tau(\hat{t})$. Moreover, the number of such $\hat{t}$ teachers cannot exceed the number of vacant positions at the status quo. Hence, there are at least $q_s - |\omega_s|$ teachers in μ_s who have a type no worse than t's type according to the type ranking of school s. As a result, the third bullet in the definition of SI fairness is satisfied. ■

Proof of Proposition 5. Remember that D_s is the choice rule before swapping the two positions s^k and s^ℓ with $k < \ell$ in the processing order p_s and $\hat{D}_s$ is the one after the swap in the processing order $\hat{p}_s$. We use the following claim in our proof.

Claim. For any $\bar{T} \subseteq T$, $\hat{D}_s(\bar{T}) \subseteq D_s(\bar{T})$.

Proof. Let s^k and s^ℓ (s^ℓ and s^k) be m^{th} and $(m+1)^{\text{th}}$ positions under p_s ($\hat{p}_s$), respectively. Since the relative positions of the first $(m-1)$ positions are the same under p_s and $\hat{p}_s$, the same teachers are assigned to the first $(m-1)$ positions by D_s and $\hat{D}_s$. Therefore, we consider the same set of teachers for the m^{th} position under both p_s and $\hat{p}_s$. Let $\tilde{T}$ be the set of teachers considered for the m^{th} position under both p_s and $\hat{p}_s$.

Recall that, by our construction, the set of teachers acceptable for position s^ℓ is a (weak) superset of the teachers acceptable for position s^k. Hence, if there does not exist an acceptable teacher in $\tilde{T}$ for position s^ℓ, then there does not exist an acceptable teacher in $\tilde{T}$ for position s^k. If there is no acceptable teacher in $\tilde{T}$ for position s^k but there is some acceptable teacher for position s^ℓ who is assigned to it under $D_s(\bar{T})$ that teacher is assigned to s^ℓ under both auxiliary choice rules D_s and $\hat{D}_s$. Since the relative positions of the remaining positions are the same under p_s and $\hat{p}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$ whenever the set of acceptable teachers in $\tilde{T}$ for either s^k or s^ℓ is vacant.

Now suppose there exist acceptable teachers in $\tilde{T}$ for positions s^k and s^ℓ. Let t^k and t^ℓ be the highest ranked teachers for positions s^k and s^ℓ among those in $\tilde{T}$, respectively.

If $t^k \neq t^\ell$, then under both auxiliary choice rules D_s and $\hat{D}_s$ t^k and t^ℓ are assigned to positions s^k and s^ℓ, respectively. Since the relative positions of the remaining positions are the same under p_s and $\hat{p}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$.

If $t^k = t^\ell = t'$, then t' is assigned to positions s^k and s^ℓ under auxiliary choice rules D_s and $\hat{D}_s$, respectively. Next, we consider the teachers in $\tilde{T} \setminus \{t'\}$. First notice that, the status-quo employees who have the highest priority among all teachers in T for s^k and s^ℓ cannot be in $\tilde{T} \setminus t'$. This would conflict with the fact that t' has the highest priority for both positions among the teachers in $\tilde{T}$. Then, there is one teacher in $\tilde{T} \setminus t'$ who has highest priority for both s^k and s^ℓ. We denote such a teacher with t''. If t'' is acceptable for both s^k and s^ℓ, then t'' is assigned to s^ℓ and s^k under both auxiliary choice rules D_s and $\hat{D}_s$, respectively. If t'' is unacceptable for both s^k and s^ℓ, then no teacher is assigned to s^ℓ and s^k under both auxiliary choice rules D_s and $\hat{D}_s$, respectively. Under both cases, since the relative positions of the remaining positions are the same under p_s and $\hat{p}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$. We are left with one remaining case: t'' is acceptable for s^ℓ but not for s^k. Then, t'' is assigned to s^ℓ under D_s but s^k is not filled under $\hat{D}_s$. Then, when we consider the remaining positions under both auxiliary choice rules and the remaining teachers, we can treat the assignment of these positions as being done via the standard DA mechanism where each teacher ranks these positions in the same way as the processing order p_s (and $\hat{p}_s$ since the two have the same ordering after slot s^ℓ). Since the DA mechanism is population monotonic[78] and individually rational, any teacher assigned under $\hat{D}_s$ is assigned under D_s. But the other way is not always true. $\diamond$

Now, consider a sequential application of the DA algorithm in which we allow each teacher to apply one by one as long as they do not apply to school s. If they do need to apply to s, we leave the teacher unmatched and move to the next teacher (see for instance Dur et al., 2018 who also use this approach). Then, eventually, we will have a set of teachers $\bar{T}$ who have been rejected from all of their choices that they prefer to school s. Once we let all these teachers apply to s, the Claim implies that the rejected teachers under D is a subset of the rejected teachers under $\hat{D}$. Then, we repeat the procedure of letting each teacher apply one by one but only those teachers who have been rejected under both auxiliary choice rules from s and those who have not applied to s yet. Following

[78] A mechanism is population monotonic if everybody is weakly better-off when some agents are removed from the problem.

this procedure will give us a matching μ that is defined for all schools except s under both auxiliary choice rules. Moreover, because the rejected teachers under D is a subset of the rejected teachers under $\hat{D}$, then DA algorithm so-defined that way terminates under the auxiliary choice rule D. In other words, μ is the final outcome under the rule D. However, there might be teachers rejected from s and who have not applied to their next best choice under $\hat{D}$. That is, we may observe that some teachers are rejected from their assignment under μ. Hence, no teacher t prefers $\hat{\mu}_t$ to μ_t. ■

C Other Applications

In this appendix, we explain other applications of our mechanisms in more detail. We give three concrete applications.

Intra-district school choice after a status-quo assignment. In the US, Austin Independent School District (AISD) of Texas assigns students to schools through an address based matching procedure.[79,80] Unfortunately, address based assignment ends up with segregated schools.[81] In order to eliminate the segregation and fill the empty seats at the under-demanded schools, AISD runs a transfer procedure in which a student who is in relative demographic majority in her assigned school can apply to the schools in which she belongs to the minority demographic group. Moreover, new students who arrive at the district after the matching procedure is run can also participate in this transfer procedure.[82]

In addition to the transfer programs to achieve racial diversity at schools, many school districts, including Davenport, IA (DCS, 2019), and Seminole, FL (SCPS, 2021), run transfer programs to achieve diversity in terms of the socioeconomic status (SES) of the students. Diversity transfer programs based on SES are also suggested to the school district by The United States Department of Education and the United States Department of Justice (ED, 2011).

Job rotation. Job rotation is defined as the horizontal movement of employees among different positions in a company. It is a well-established and commonly practiced human resource management program in many settings. It benefits companies through employee

[79] Although we mentioned earlier possible application of our approach for inter-district-school choice (see Hafalir et al., 2019), a direct application of our methodology exists in intra-district school choice, which we discuss here.

[80] There are 128 school programs in AISD. In the 2020-2021 school year, the total enrolment in AISD is more than 75,000 (AISD, 2021).

[81] In 2019, the student body at 15% and 63% of the elementary schools were composed of more than 60% white and hispanic-black students, respectively.

[82] The majority minority transfer program is used in many school districts in the US including Huntsville, AL (HCS, 2020), Suffolk, VA (SPSK12, 2021), and Florence, SC (F1S, 2019).

enrichment and success of developing future managers as well as decreased worker turnover due to increased job satisfaction of the participants (Cheraskin and Campion, 1996).

Job rotation programs can also be used as a means to obtain certain distributional goals of a company such as achieving gender balance across different departments of the company and retention of female employees and increasing the development of more female leaders through rotation programs.[83]

Other civil services. There are other centralized matching procedures for civil servants from different professions. For example, police officers are assigned to neighborhoods by centralized procedures in several US cities such as Chicago (Sidibe et al., 2021); doctors are assigned to government hospitals in some countries such as Turkey and for their first residency jobs in many countries including Canada, the U.K., and the U.S. (Roth, 1984); civil administrators are assigned centrally for example in India (Thakur, 2020).

In police officer (re)assignment, senior officers are known to shy away from urban centers, while the police departments would like more of officers in urban centers due to disproportionate crime rates, similar to our distributional problems in teacher assignment.

Another concrete example in this domain is the Indian Administrative Services, the top-tier government jobs in India. This selective service conducts first time assignment of officials to regional government jobs in states of India every year, while reassignment is conducted separately. The state has distributional objectives based on spread of talent across states, constitutional affirmative action, and respect of preferences for home states. Our procedures can be used in these domains as well to achieve different distributional objectives.

D Examples

We illustrate how SI-CC works with the following example:

Example A.1. *Let* $S = \{s_1, s_2, s_3, s_4\}$, $T = \{h_1, \ell_1, m_2, h_3, m_2, h_3, m_3, h_N\}$. *The status-quo matching* ω *is given as*

$$\omega_{s_1} = \{h_1, \ell_1\}, \;\; \omega_{s_2} = \{m_2\}, \;\; \omega_{s_3} = \{h_3, m_3\}, \;\; \omega_{s_4} = \emptyset,$$

[83] Observe that companies use professionally designed centralized matching software for job rotations, for example see `https://www.tws-partners.com/corporate-functions/hr/`.

and h_N is a new teacher. Let $q_{s_1} = q_{s_2} = 2$ and $q_{s_3} = q_{s_4} = 1$. The preferences of teachers are:

$$
\begin{gathered}
s_4 \, P_{h_1} \, s_2 \, P_{h_1} \, s_3 \, P_{h_1} \, s_1 \, P_{h_1} \, \emptyset \\
s_2 \, P_{\ell_1} \, s_3 \, P_{\ell_1} \, s_1 \, P_{\ell_1} \, s_4 \, P_{\ell_1} \, \emptyset \\
s_4 \, P_{m_2} \, s_3 \, P_{m_2} \, s_2 \, P_{m_2} \, s_1 \, P_{m_2} \, \emptyset \\
s_1 \, P_{h_3} \, s_3 \, P_{h_3} \, s_2 \, P_{h_3} \, s_4 \, P_{h_3} \, \emptyset \\
s_2 \, P_{m_3} \, s_1 \, P_{m_3} \, s_3 \, P_{m_3} \, s_4 \, P_{m_3} \, \emptyset \\
s_2 \, P_{h_N} \, s_1 \, P_{h_N} \, s_3 \, P_{h_N} \, s_4 \, P_{h_N} \, \emptyset
\end{gathered}
$$

Let $\Theta = \{h, m, \ell\}$, $\tau(h_1) = \tau(h_3) = \tau(h_N) = h$, $\tau(m_2) = \tau(m_3) = m$ and $\tau(\ell_1) = \ell$. The type rankings of schools are:

$$
\begin{gathered}
h \rhd_{s_1} m \rhd_{s_1} \ell \rhd_{s_1} \theta_\emptyset \\
h \rhd_{s_2} m \rhd_{s_2} \ell \rhd_{s_2} \theta_\emptyset \\
\ell \rhd_{s_3} m \rhd_{s_3} h \rhd_{s_3} \theta_\emptyset \\
\ell \rhd_{s_4} m \rhd_{s_4} h \rhd_{s_4} \theta_\emptyset
\end{gathered}
$$

In Step 1 of SI-CC, we obtain the graph in Figure A.1 (such that the status-quo employees of each school are placed in a dashed bubble around the school). Notice that, ℓ_1 does not point to her top choice, s_2, since neither improvement condition holds for s_2 via her. There exists a cycle, (h_3, s_1, ℓ_1, s_3), in which every school satisfies Improvement Condition 1. We execute that cycle by assigning h_3 and ℓ_1 to s_1 and s_3, respectively.

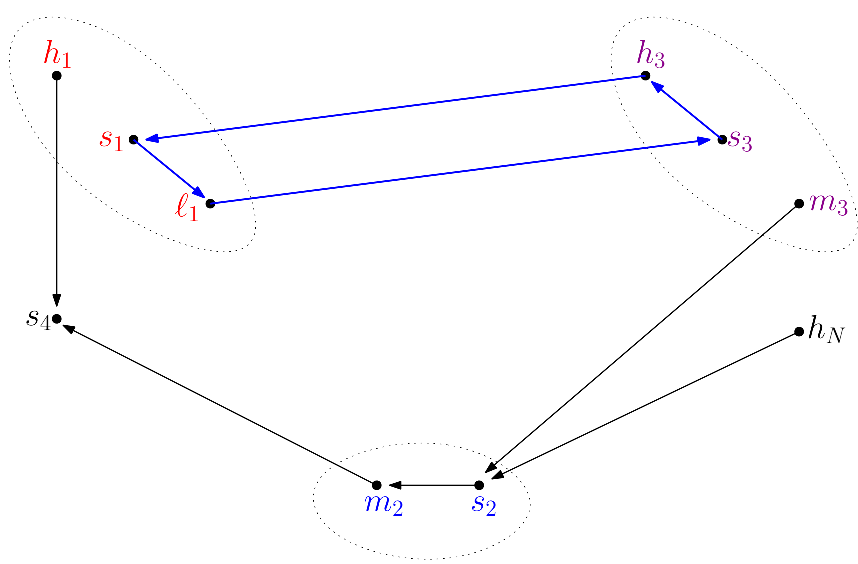


Figure A.1: Graph of Step 1 of SI-CC

In Step 2 of SI-CC, we obtain the graph in Figure A.2. There exists no cycle. We execute *chain (h_N, s_2, m_2, s_4), whose tail is the only new teacher h_N, by assigning h_N and m_2 to s_2 and s_4, respectively.*

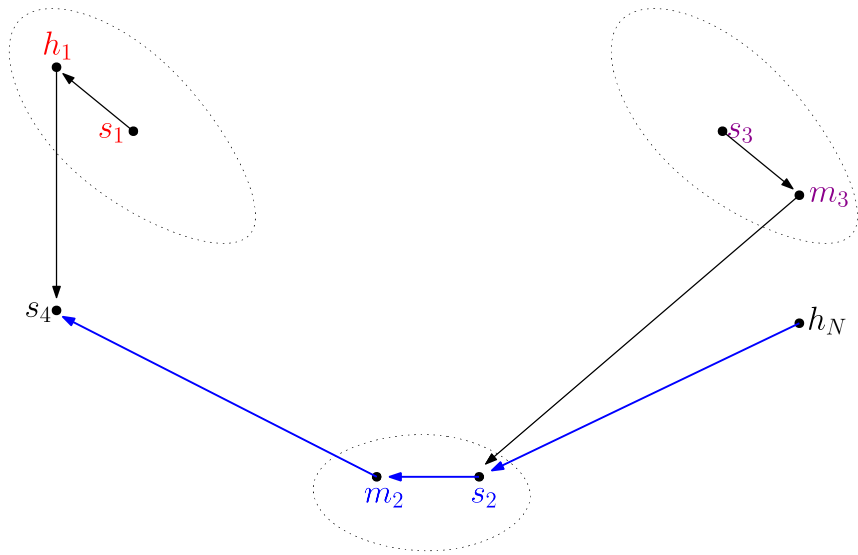


Figure A.2: Graph of Step 2 of SI-CC

In Step 3 of SI-CC, we obtain the graph in Figure A.3. Notice that, h_1 (m_3) points to s_3 (s_1) even though she has a worse type than the teacher pointed by s_3 (s_1). Such a situation is possible due to the positive balance buffer achieved as a result of the exchanges executed in the earlier steps. There exists a cycle, (h_1, s_3, m_3, s_1), in which every school satisfies Improvement Condition 1. We execute that cycle by assigning h_1 and m_3 to s_3 and s_1, respectively.

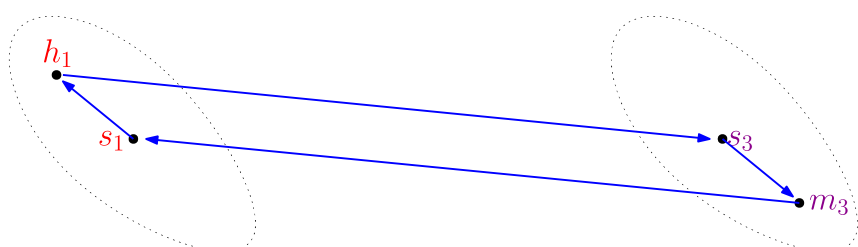


Figure A.3: Graph of Step 3 of SI-CC

The outcome matching of SI-CC is μ such that

$$\mu_{s_1} = \{h_3, m_3\}, \quad \mu_{s_2} = \{h_N\}, \quad \mu_{s_3} = \{\ell_1, h_1\}, \quad \mu_{s_4} = \{m_2\}.$$

Example A.2 below shows that SI-CC can be manipulated by a teacher and it is not SI teacher optimal under an alternative pointing rule.

Example A.2. *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_1, t_2\}, \omega_{s'} = \{t_3\}, \omega_{s''} = \{t_4\},$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$ and $\tau(t_1) \rhd_s \tau(t_2) = \tau(t_3) = \tau(t_4)$. The preferences of the teachers are

$$sP_{t_1}s'P_{t_1}s''P_{t_1}\emptyset, \qquad s'P_{t_2}sP_{t_2}s''P_{t_2}\emptyset,$$

$$sP_{t_3}s'P_{t_3}s''P_{t_3}\emptyset, \qquad s''P_{t_4}sP_{t_4}s'P_{t_4}\emptyset.$$

If in the first step of SI-CC school s points to t_1, the best school t_3 can point is s'. Therefore, she will be assigned to s'. In particular, under true preferences SI-CC assigns all employees to their status-quo schools. This outcome is not SI teacher optimal because it is Pareto dominated by another status-quo improving matching ν for teachers where $\nu_{t_1} = \nu_{t_3} = s$, $\nu_{t_2} = s'$ and $\nu_{t_4} = s''$. Moreover, if t_3 swaps the rankings of s' and s'', then SI-CC selects ν, i.e., t_3 manipulates SI-CC when s points t_1 in the first step.

In the next example, we illustrate how the auxiliary choice rule works.

Example A.3. *Suppose there are five teachers one of whom is new: $T = \{t_1, t_2, t_3, t_4, t_5\}$ and $t_5 \in N$. The status-quo assignment of school s, which has capacity $q_s = 3$ is $\omega_s = \{t_1, t_2\}$. The type ranking of school s is*

$$\tau(t_1) = \tau(t_3) \rhd_s \tau(t_2) \rhd_s \tau(t_4) \rhd_s \tau(t_5) \rhd_s \theta_\emptyset.$$

The slot set of s is $\mathcal{S}_s = \{s^1, s^2, s^3\}$ such that s^1 and s^2 correspond to filled positions at status quo and s^3 corresponds to the vacant position.

Let the tie breaker $\vdash$ be such that $t_1 \vdash t_3$.

We construct the rankings for each slot as follows:

$$\begin{aligned}
&t_1 \succ_s^1 t_3 \succ_s^1 \emptyset \succ_s^1 t \quad \text{for any } t \notin \{t_1, t_3\},\\
&t_2 \succ_s^2 t_1 \succ_s^2 t_3 \succ_s^2 \emptyset \succ_s^2 t \quad \text{for any } t \notin \{t_1, t_2, t_3\},\\
&t_5 \succ_s^3 t_1 \succ_s^3 t_3 \succ_s^3 t_2 \succ_s^3 t_4 \succ_s^3 \emptyset.
\end{aligned}$$

Suppose $\hat{T} = \{t_2, t_3, t_4, t_5\}$. Then, the set of chosen teachers $C_s(\hat{T})$ is found as follows:

- ***Step** 1: Teacher t_3 is the highest ranked teacher for slot s^1 among the teachers in $\hat{T}_1 = \hat{T}$. Hence, t_3 is assigned to slot s^1 and she is removed. We set $\hat{T}_2 = \hat{T}_1 \setminus \{t_3\}$.*
- ***Step** 2: Teacher t_2 is the highest ranked teacher for slot s^2 among the teachers in $\hat{T}_2$. Hence, t_2 is assigned to slot s^2 and she is removed. We set $\hat{T}_3 = \hat{T}_2 \setminus \{t_2\}$.*
- ***Step** 3: Teacher t_5 is the highest ranked teacher for slot s^3 among the teachers in $\hat{T}_3$. Hence, t_5 is assigned to slot s^3 and she is removed. We set $\hat{T}_4 = \hat{T}_3 \setminus \{t_5\}$.*

Hence, $C_s(\hat{T}) = \{t_3, t_2, t_5\}$.

In Example A.4, we show that, in the same setting as Combe et al. (2020), SI-CC is not equivalent to the teacher optimal selection of TO-BE they propose.[84]

Example A.4. *Let* $S = \{s_1, s_2\}$, $T = \{t_1, t_2, t'_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $q_{s_1} = 1$ *and* $q_{s_2} = 2$. *Let* $\tau(t_2) \rhd_{s_1} \tau(t'_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_{\emptyset}$ *and* $\tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \tau(t'_2) \rhd_{s_2} \theta_{\emptyset}$.

One can check that the matching selected by the teacher optimal selection of TO-BE matches[85] t_1 *to* s_1 *and* t_2 *to* s_1 *while SI-CC matches* t_1 *to* s_1 *but* t'_2 *to* s_1.

In Example A.5, we show that the M-convexity of the policy goals is no longer sufficient to ensure the existence of an SI teacher optimal and strategy-proof mechanism.

Example A.5. *Let* $S = \{s_1, s_2\}$, $T = N = \{t_1, t_2\}$, $\omega_{s_1} = \omega_{s_2} = \emptyset$, $q_{s_1} = q_{s_2} = 1$ *and* $\tau(s_1) = \tau(s_2) = \theta$. *Suppose the constraint over the distribution of teachers requires that a teacher of type* θ *is assigned to* s_1. *This is a constraint fixing a floor which is known to be M-convex. Suppose both teachers rank* s_2 *ahead of* s_1 *and* s_1 *ahead of* $\emptyset$. *If teachers report their true preferences, then there will be a teacher assigned to* s_1 *under any SI teacher optimal (or two-sided Pareto efficient) matching. Then, the teacher assigned to* s_1, *say* t_1, *has an incentive to claim that* s_1 *is unacceptable to her. Indeed, any SI teacher optimal mechanism must then assign* t_2 *to* s_1 *and* t_1 *to* s_2.

In Example A.6, we show that in some markets there is no two-sided Pareto efficient and SI fair matching; hence, a SI teacher optimal and SI fair matching does not exist, either, even when there is no new teacher and all teachers have different types.

Example A.6. *Let* $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ *and* $q_{s_1} = q_{s_2} = q_{s_3} = 1$. *Let all types be acceptable for schools and all schools be acceptable for teachers and*

$$\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1), \qquad \tau(t_3) \rhd_{s_2} \tau(t_1) \rhd_{s_2} \tau(t_2), \qquad \tau(t_1) \rhd_{s_3} \tau(t_2) \rhd_{s_2} \tau(t_3),$$

$$s_2 \, P_{t_1} \, s_1 \, P_{t_1} \, s_3 \qquad s_1 \, P_{t_2} \, s_2 \, P_{t_2} \, s_3 \qquad s_1 \, P_{t_3} \, s_2 \, P_{t_3} \, s_3.$$

Let μ *be a matching such that* $\mu_{s_1} = t_2$, $\mu_{s_2} = t_1$ *and* $\mu_{s_3} = t_3$. *Under this market,* ω *is the unique SI fair matching and* μ *Pareto dominates* ω. *Here,* μ *is SI teacher optimal (and hence, two-sided Pareto efficient by Proposition 1).*

[84] Combe et al. (2020) already noted that their class of TO-BE mechanisms did not entirely define the class of status-quo improving, strategy-proof and two-sided Pareto efficient mechanisms. However, they did not investigate it further. Our example suggests that other non-trivial mechanisms, such as SI-CC, exist outside their class.

[85] One can easily check that this example is well defined in their setting. Just set the preferences of the schools over the teachers being equivalent to the schools' ranking over their corresponding types.

In Examples A.7 - A.8, we inspect possible relaxations in the definition of SI fairness. First, we show that if we require the priority of any teacher over another teacher in ω_s at s to be respected, then for some market there is no SI fair matching.

Example A.7. *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \rhd_s \tau(t_1) \rhd_s \theta_\emptyset$ for both $s \in S$ and $s_1 \, P_t \, s_2 \, P_t \, \emptyset$ for all $t \in T$.*

In this market, the unique status-quo improving matching is ω. However, it fails to respect the priority of t_2 over t_1 at s_1. Hence, a SI fair matching does not exist in this market when we require the priority of any teacher over a status-quo employee to be respected.

Next, via example we show that if we require the priority of any teacher to be respected over more than $q_s - |\omega_s|$ new teachers at any school s,then for some market there is no SI fair matching.

Example A.8. *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ for both $s \in S$, $s_2 \, P_{t_1} \, s_1 \, P_{t_1} \, \emptyset$ and $s_2 \, P_{t_2} \, \emptyset \, P_{t_2} \, s_1$.*

In this market, in any status-quo improving matching t_1 is assigned to s_1. However, any such matching dos not respect the priority of t_1 at s_2 over t_2. Hence, any SI fair matching does not exist in this market when the second condition is excluded.

Example A.9 shows that we cannot find an optimal processing order when vacant slots are also considered in SI-DA.

Example A.9. *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$ with status-quo matching*

$$\omega_s = \{t_4\}, \, \omega_{s'} = \{t_2\}, \, \omega_{s''} = \emptyset,$$

capacities $q_s = 2$ and $q_{s'} = q_{s''} = 1$, and type rankings and preferences

$$\tau(t_1) \rhd_s \tau(t_2) \rhd_s \tau(t_3) \rhd_s \tau(t_4),$$
$$\tau(t_1) \rhd_{s'} \tau(t_3) \rhd_{s'} \tau(t_4) \rhd_{s'} \tau(t_2),$$
$$\tau(t_1) \rhd_{s''} \tau(t_2) \rhd_{s''} \tau(t_3) \rhd_{s''} \tau(t_4),$$
$$s \, P_{t_1} \, s' \, P_{t_1} \, s'' \, P_{t_1} \, \emptyset, \qquad s \, P_{t_2} \, s'' \, P_{t_2} \, s' \, P_{t_2} \, \emptyset, \qquad s \, P_{t_3} \, s'' \, P_{t_3} \, s' \, P_{t_3} \, \emptyset, \qquad s' \, P_{t_4} \, s'' \, P_{t_4} \, s \, P_{t_4} \, \emptyset.$$

If s^1 is filled before s^2, then under DA t_1 and t_3 are assigned to s, t_2 is assigned to s'' and t_4 is assigned to s'. If s^2 is filled before s^1, then under DA t_1 and t_2 are assigned to s, t_3 is assigned to s'' and t_4 is assigned to s'. Hence, we cannot have the same conclusion as in Proposition 5.

The next example shows that SI-DA does not select a Pareto-undominated SI fair matching.

Example A.10. *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be*

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \{t_3\},$$

$q_s = q_{s'} = q_{s''} = 1$, $\tau(t_1) = \tau(t_2) = \tau(t_3)$, and all teachers are acceptable for all schools. The preferences of the teachers are

$$s' \; P_{t_1} \; s \; P_{t_1} \; s'' \; P_{t_1} \; \emptyset, \qquad s \; P_{t_2} \; s' \; P_{t_2} \; s'' \; P_{t_2} \; \emptyset, \qquad s' \; P_{t_3} \; s'' \; P_{t_3} \; s \; P_{t_3} \; \emptyset.$$

Let $t_2 \vdash t_3 \vdash t_1$ be the tie breaker. Then the rankings of the slots are given as:

$$t_1 \succ_s^1 t_2 \succ_s^1 t_3, \qquad t_2 \succ_{s'}^1 t_3 \succ_{s'}^1 t_1, \qquad t_3 \succ_{s''}^1 t_2 \succ_{s''}^1 t_1.$$

SI-DA assigns t_1 to s, t_2 to s' and t_3 to s''. However, this outcome is Pareto dominated by another SI fair matching in which t_1, t_2 and t_3 are assigned to s', s, and s'', respectively.

In Examples A.11 -A.13, we relax the conditions of Assumption 1 one by one and show that the existence of an SI fair outcome may not be guaranteed.

Example A.11. *We consider a market in which there does not exist $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$.*

Let $S = \{s, s'\}$, $T = \{t_1\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, and teacher t_1 is acceptable for both schools. The preferences of teacher t_1 is

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is wasteful: s' has a vacant seat that t_1 wants.

Example A.12. *We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$ and each teacher in N' is acceptable for all schools with excess capacity but not all teachers in N' consider all schools with excess capacity acceptable.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ and $\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset, \qquad s \; P_{t_2} \; \emptyset \; P_{t_2} \; s'.$$

In this market, ω is the unique status-quo improving matching but it is wasteful as s' has a vacant seat that t_1 wants.

Example A.13. *We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$ and all teachers in N' consider all schools with excess capacity acceptable but some teacher in N' is not acceptable for some school with excess capacity.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ and $\tau(t_1) \rhd_{s'} \theta_\emptyset \rhd_{s'} \tau(t_2)$. The preferences of the teachers are

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset,$$
$$s \; P_{t_2} \; s' \; P_{t_2} \; \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is wasteful as s' has a vacant seat that t_1 wants.

E Descriptions of Benchmark Mechanisms

E.1 Description of TTC*

Definition A.1. ***TTC* Mechanism***

*Let $\vdash$ be a tie breaker over tenured teachers and $\vdash^*$ be a tie breaker over all teachers. For each school s, we construct a **pointing order** $\gtrdot_s$ over teachers in ω_s using its type rankings $\rhd_s$ and $\vdash$: For any two distinct teachers $t, t' \in \omega_s$*

$$t \gtrdot_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

The second tie breaker $\vdash^*$ will be used for chain selection below. It is a separate tie-breaker as in TTC, chains can be started by any teacher, i.e., not only by new teachers, and thus, choice of it presents another policy tool.*

A general step k is defined as follows:

Step k:

- *Each remaining school s points to the highest priority remaining teacher in ω_s under $\gtrdot_s$, if not all students in ω_s are already removed. Otherwise, school s does not point to any teacher.*
- *Each remaining teacher t points to her most preferred remaining option.*
- *Outside option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle, or

(ii) a chain.

Then:

- ***If Case (i) holds:*** *Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle by assigning the teachers in that cycle to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.*
- ***If Case (i) does not hold:*** *Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. Then we select the chain such that the tail of the chain is the highest priority teacher under* ***chain tie breaker*** *$\vdash^*$ and the head of the chain is a school which does not point to a teacher.*

  We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.

The mechanism terminates when all teachers are removed.

E.2 Descriptions of DA* and the Current French Mechanism

Mechanism **DA*** uses a version of the DA algorithm that is modified to ensure status-quo improvement for teachers. School preferences are modified such that each teacher t, with a status-quo assignment s, is ranked in the (modified) ranking of her status-quo school $s := \omega_t$, above any teacher $t' \notin \omega_s$. Other than this modification, the preference relations of schools remain unchanged among the status-quo teachers and among the the non-status-quo teachers. That is, the school has Lorenz preferences over teacher experience types introduced in the empirical section. Then it runs the DA algorithm using these modified school preferences and the submitted teacher preferences. Also see Guillen and Kesten (2012), Pereyra (2013), and Compte and Jehiel (2008) regarding the use of this algorithm in another context and teacher assignment context.

The French Ministry of Education's current mechanism, which we refer to as **Current French** in the empirical analysis, uses the same algorithm as DA*. However, instead of regional type rankings to rank status-quo teachers among themselves and non-status-quo teachers among themselves, it uses the Ministry-mandated priorities of regions over teachers. See Combe et al. (2020) for a more detailed presentation of this mechanism and its properties.

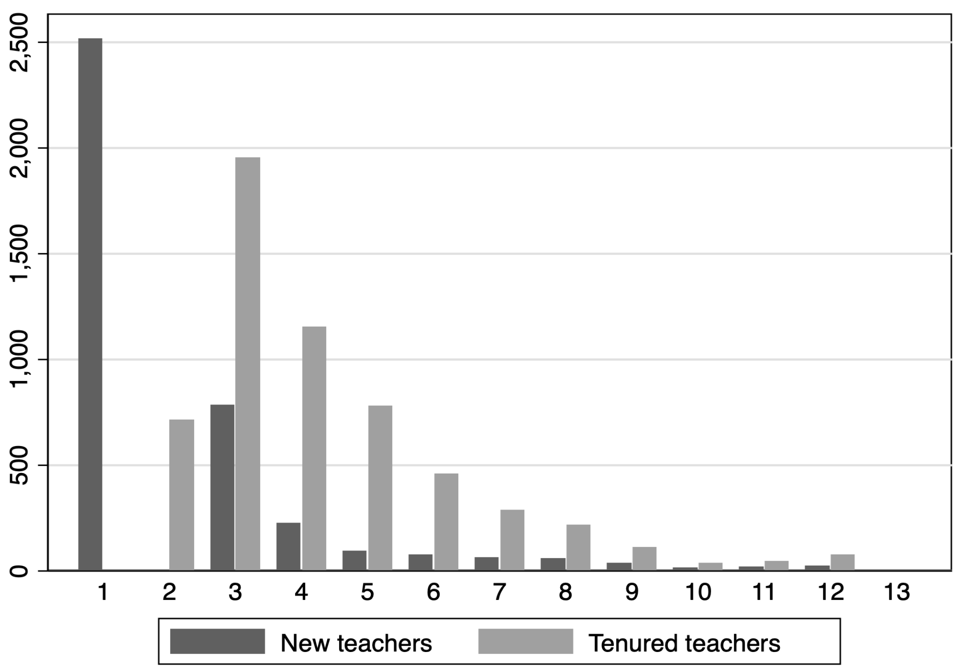
By construction, DA* and Current French are both status-quo improving for teachers. They do not satisfy status-quo improvement for schools and are not Gale-Shapley stable. However, DA* and Current French are teacher-SI fair using regional teacher type rankings as priorities and Ministry-mandated regional priorities, respectively.

Under the current French mechanism, when there are no vacant positions, each school fills its capacity and only the status-quo employees are assigned to the schools. This follows from the fact that teachers in ω_s have the q_s highest priority at school s and they consider their status-quo school acceptable. Hence, by the definition of the mechanism, if there is a priority violation of a teacher t at some school s, i.e., $\tau(t) \vartriangleright_s \tau(t')$ and t' is assigned to s, then $t' \in \omega_s$. Hence, the Current French Mechanism is teacher SI fair.

Next, by slightly modifying the example in the proof of Proposition 2, we show that when there are no vacant positions at schools and there are no new teachers the current French mechanism is not status-quo improving. Consider the example in the proof of Proposition 2 such that teacher t_2 prefers school s most. Then, the French mechanism assigns t_1 and t_2 to s' and s, respectively. This matching is not status-quo improving for school s.

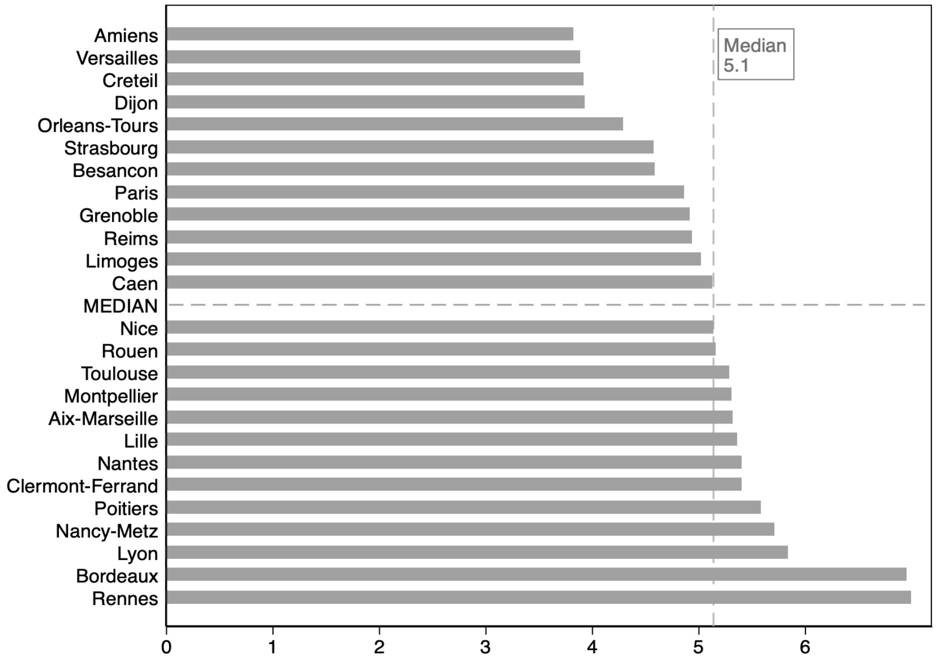
F Additional Figures and Tables

Figure A.4: Distribution of Teacher Experience Types



Notes: This figure shows the number of teachers with each experience type. We define a teacher type as her experience and we classify teachers into 12 experience bins, where the first bin corresponds to teachers with one or two years of experience, the second bin to teachers with three to four years of experience, and so on. Because a large number of teachers belong to the first bin, we further use a tie-breaker for the first bin by ordering new teachers above tenured teachers, hence effectively generating 13 experience bins. The first bin corresponds to new teachers with one or two years of experience, the second bin corresponds to tenured teachers with one or two years of experience, the third bin to new and tenured teachers with three to four years of experience, and so on.

Figure A.5: Average Teacher Experience Types at Status Quo

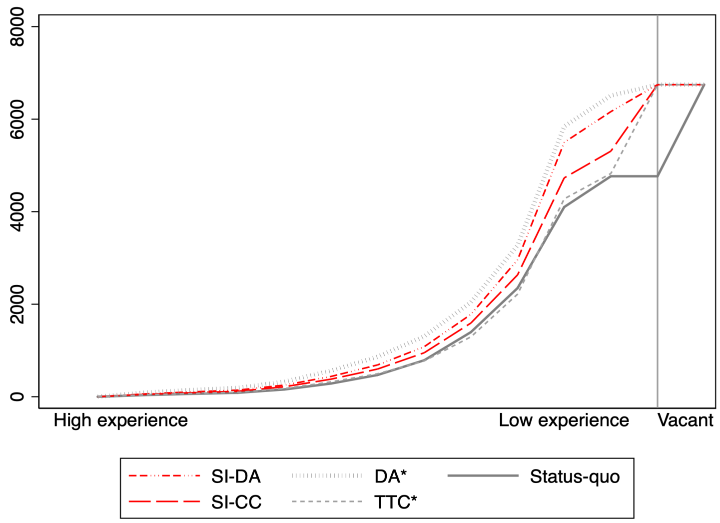


Notes: This figure shows the average teacher experience type at status quo (lower types correspond to lower experience levels). The *younger regions* are listed above the median and the *older regions* are listed below the median.

Table A.1: Number of teachers and vacant positions

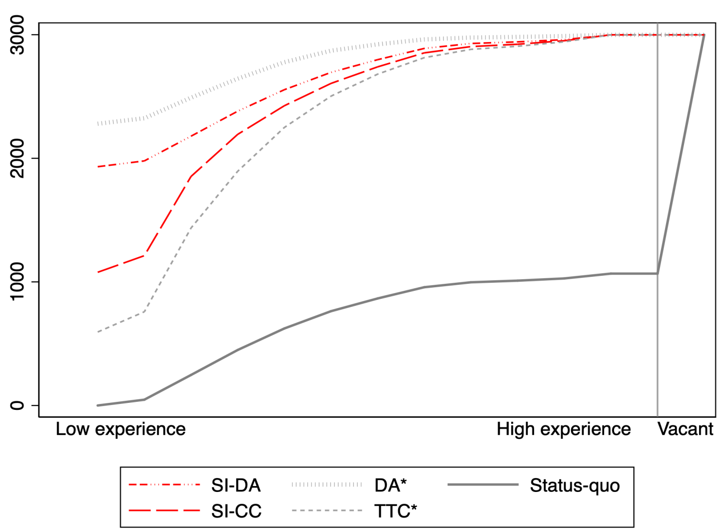
Subjects	All teachers (1)	New teachers (2)	Tenured teachers (3)	Vacant positions (4)
All subjects	10,460	4,627	5,833	3,912
Sports	2,066	568	1,498	475
French	1,645	786	859	663
Math	1,563	958	605	824
English	1,374	746	628	640
History-Geography	1,230	657	573	562
Spanish	999	316	683	248
Physics-Chemistry	837	310	527	254
Biology	746	286	460	246

Figure A.6: Cumulative Distribution of Teacher Experience Types in the *Younger Regions*



Notes: This figure shows the cumulative distribution of teacher experience types in younger regions of France. These are the regions whose average teacher type is strictly lower than the median of average teacher type distribution at status quo. The horizontal axis reports the 13 types of teachers, ordered from the most experienced to the least experienced. The 14th type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in gray. The thick dark gray line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.7: Cumulative Distribution of Teacher Experience Types in the *Older Regions*



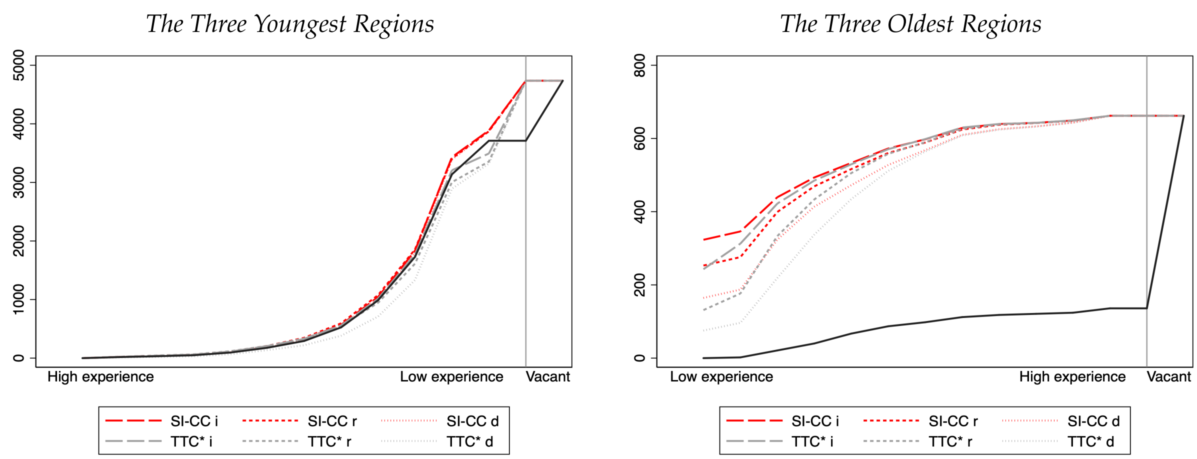
Notes: This figure shows the cumulative distribution of teacher experience types in older regions of France. These are the regions whose average teacher type is higher than the median of average teacher type distribution at status quo. The horizontal axis reports the 13 types of teachers, ordered from the least experienced to the most experienced. The 14th type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in gray. The thick dark gray line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Table A.2: Statistics on Regions

Regions	Ratio: # of tenured teachers asking to enter / exit the region	% of teachers asking for a new assignment coming from each region	Ratio: # of teachers aged more than 50 / less than 30	% of students enrolled in priority education	% of students whose reference parent has no diploma	% of students obtaining their baccalaureate
	(1)	(2)	(3)	(4)	(5)	(6)
Rennes	15.55	0.5	8.10	7.9	14.18	91.54
Bordeaux	8.95	0.8	6.56	14.6	19.22	86.25
Toulouse	6.56	1.5	5.29	8.9	17.38	88.57
Paris	3.02	2.8	6.90	25.5	21.54	85.48
Aix-Marseille	2.54	1.9	5.08	30.1	27.20	81.77
Grenoble	1.74	2.3	3.91	16.5	19.80	88.17
Amiens	0.08	6.2	1.89	23.9	27.71	82.41
Créteil	0.03	22.7	1.14	35.5	31.62	83.94
Versailles	0.05	25.7	1.62	24.9	21.88	87.92

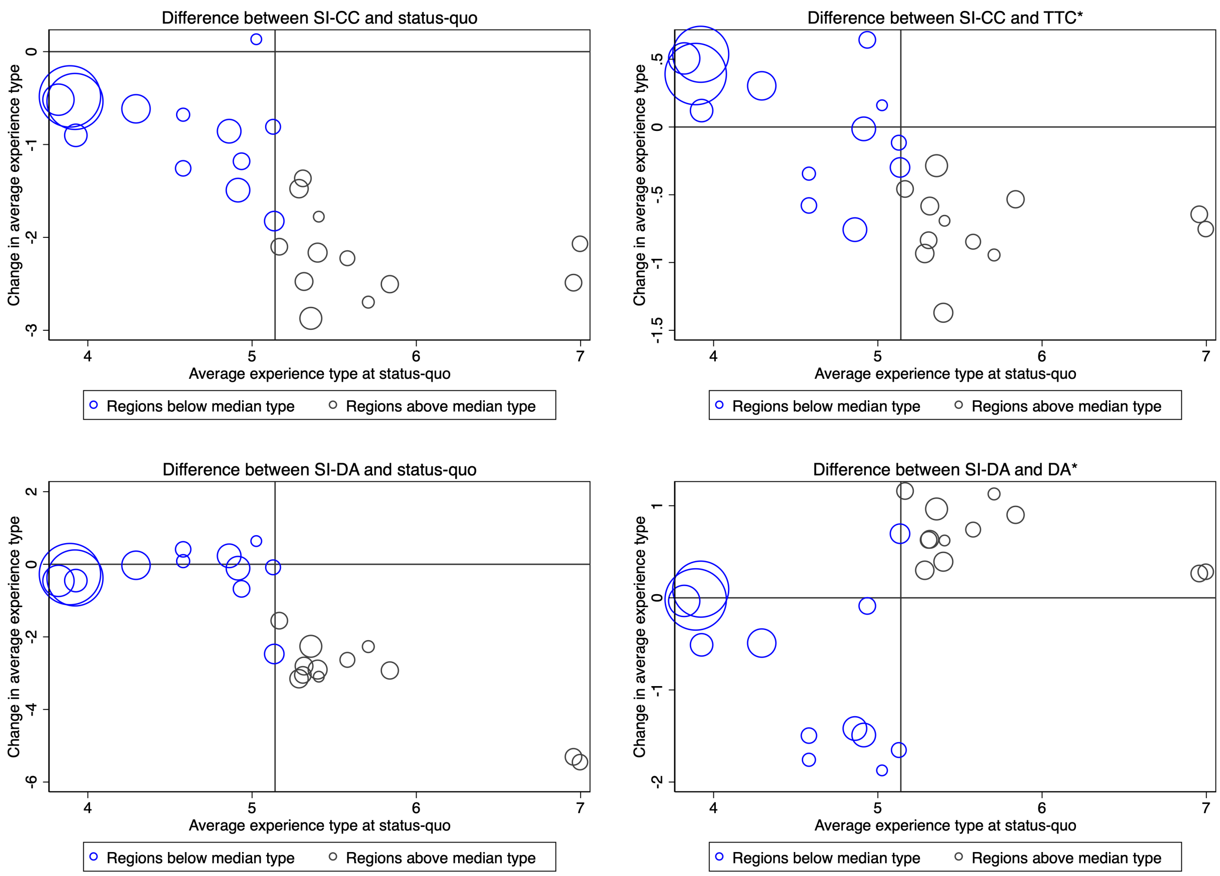
Notes: This table reports descriptive statistics for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Amiens, Créteil, and Versailles), and three intermediate regions (Paris, Aix-Marseille, and Grenoble). Attractiveness is measured by the ratio of the number of tenured teachers asking to enter a region to the number of teachers asking to leave the region (reported in column 2). All statistics reported in this table come from the following reference: Direction de l'Evaluation de la Prospective et de la Performance (2014). In column (1), the number of teachers asking to enter the region corresponds to the number of teachers who rank the region as their first choice in their preference list, while the number of teachers asking to leave the region corresponds to the number of teachers who are initially assigned the region and submit a preference list to move to another region.

Figure A.8: Cumulative Distribution of Teacher Experience Types with Different Chain Selection Rules in SI-CC and TTC*



Notes: This figure shows the cumulative distribution of teacher experience types. For each mechanism (SI-CC and TTC*), we report results for three different chain selection rules. The suffixes "i", "r", and "d" respectively stand for *increasing*, *random*, and *decreasing*. These orderings mean that the teachers starting a chain are respectively selected by increasing, random, and decreasing order of their maximum Ministry-mandated priority points. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14[th] type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in gray. The thick dark gray line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.9: Change in Average Experience Type for All Teachers Across Regions: SI-CC and SI-DA



Notes: This figure shows the difference in the average experience of teachers (among both tenured and new teachers) between the matching obtained with SI-CC and the status-quo matching (top left figure) and between SI-CC and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA and the status-quo matching (bottom left figure) and between SI-DA and its benchmark DA* (bottom right figure). Each circle represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by the average experience of their teachers at status quo (the status-quo matching). The vertical line represents the median type. All regions on the left of the vertical line have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on the right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teacher average experience post reassignment is larger than at the status-quo matching. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

G Variables Used for Teacher Preference Estimations and Goodness of Fit Measures

Variables used for teacher preference estimation. The way they are abbreviated in Table 2 is written in parentheses.

We use the following region characteristics:

- Share of students classified as disadvantaged.
- Share of students living in an urban area as % (labeled as "% stud. urban").
- Share of students who attend a school classified as priority education (labeled as "% stud. in priority ed."). Priority education is a label given to the most disadvantaged schools in France.
- Share of students who attend a private school (labeled as "% stud. in private sch.").
- Share of teachers who are younger than 30 (labeled as "% teach. younger than 30")
- Region is in South of France (labeled as "Region in South of France"). The following 5 regions are classified as being in the South of France: Aix-Marseille, Bordeaux, Montpellier, Toulouse, and Nice.

We use the following teacher characteristics:

- Current region of the teacher (labeled as "Status-quo region"). This is the region a teacher is initially assigned to.
- Region where a teacher was born (labeled as "Birth region").
- Distance between the region ranked and the status-quo region of a teacher (labeled as "Distance to status-quo region").
- Teacher's current region is Créteil or Versailles, which are the two least attractive regions (labeled as "Teach. from CV"). The attractiveness of a region is measured by the ratio of the number of teachers who rank the region as their first choice divided by the number of teachers who ask to leave the region.
- Teacher is married (labeled as "Married").
- Teacher has spent at least 5 years in a school labelled as priority education (labeled as "Teach. in priority education").
- Teacher has an advanced teaching qualification (labeled as "Advanced qualif.").

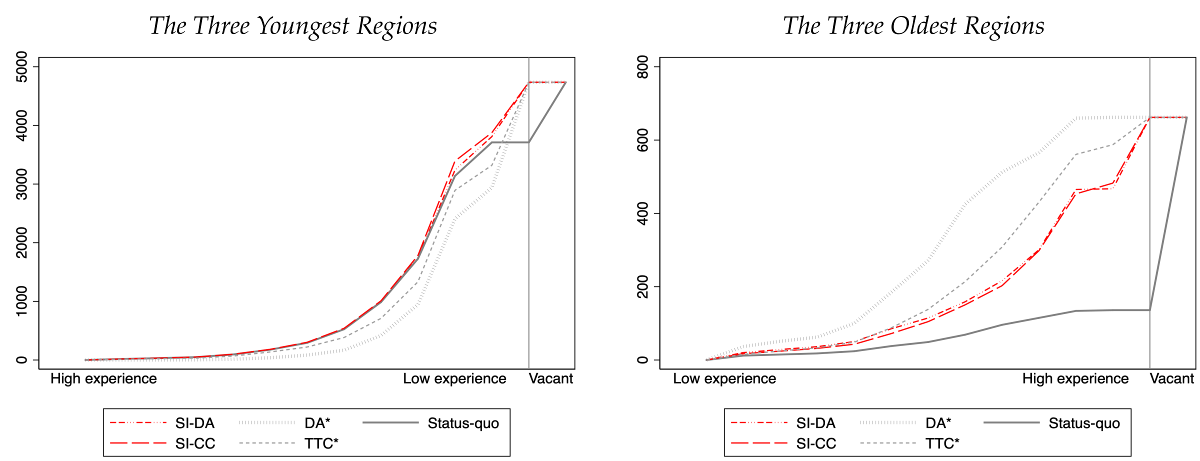
Goodness of fit measures. Our main fit measure (also reported in Table 2) considers the top two regions that a teacher has included in her submitted preference list. We then compute the probability of observing this particular relative ordering in the preference list predicted by our estimations. This fit measure based on relative ranking (instead of the characteristics of the school ranked first, for instance) is particularly suitable for our environment in which some teachers might not rank regions that they consider as infeasible.[86] In addition to the overall fit quality, we also compute fit measures for the tenured teachers who are employed in the two least attractive regions, namely, Créteil and Versailles, at the status quo. Inspecting the fit quality for this sub-group of teachers is particularly important because teachers from Créteil and Versailles represent a large share of the tenured teachers who submit a transfer request every year and they are more likely to stay in their positions. These two facts could affect the preference estimation for these teachers under our fairness assumption. Across the 8 subjects, our fit measures range from 0.62 to 0.72 for tenured teachers and from 0.56 to 0.69 for new teachers, which compare favorably to those obtained by Fack et al. (2019) (between 0.553 and 0.615).

[86] When teachers skip regions perceived as infeasible, the first region they report might not be their most preferred region — and indeed, the tests we perform reject truth telling — but conditional on ranking schools, the order in which a teacher ranks the schools might correspond to teacher true relative preference. This is why we prefer to use a fit measure that is based on relative ranking rather than on the characteristics of the school ranked first.

H Empirical Results when All Regions Have the Same Preferences over Teachers

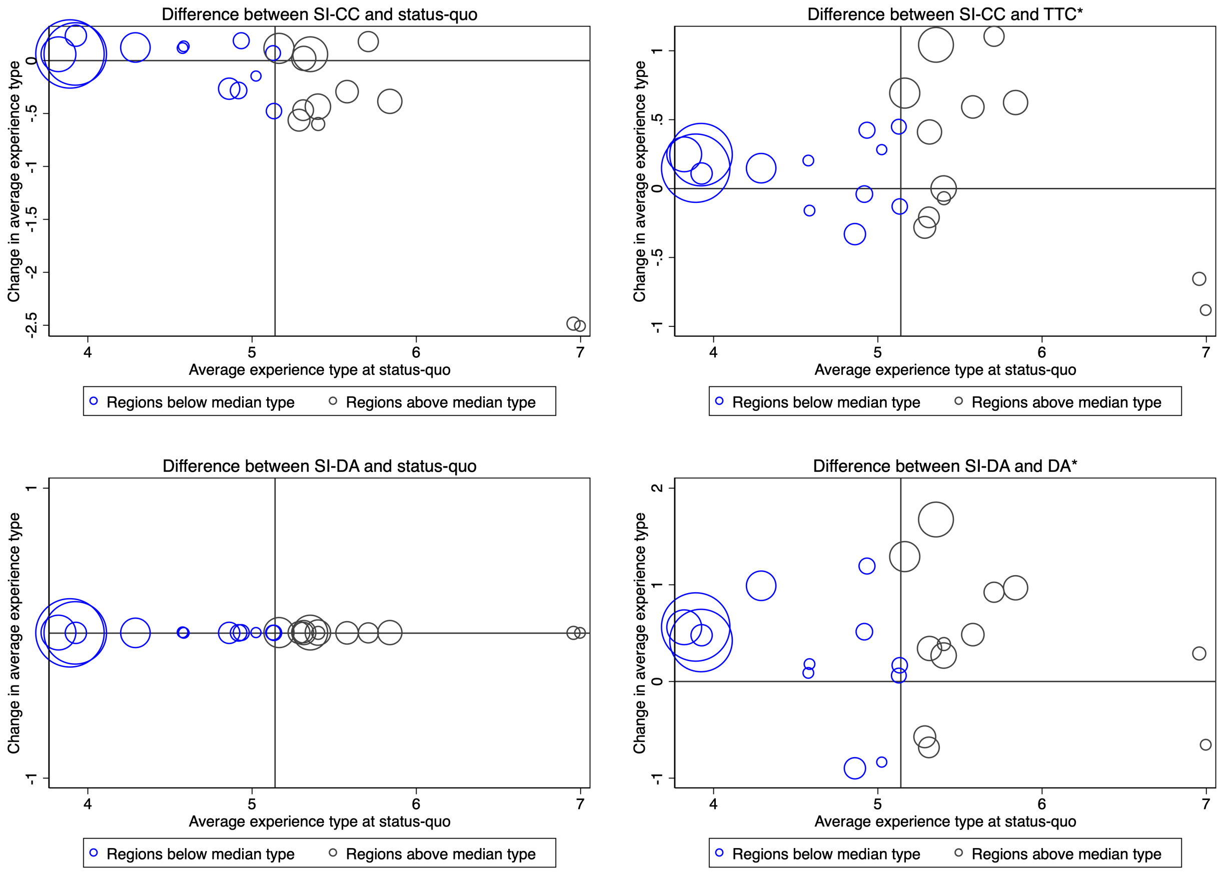
This appendix reports results in which all regions have the same preferences over teachers: Each region ranks types by decreasing level of experience, i.e., the most experienced teachers are always preferred to the least experienced teachers.

Figure A.10: Cumulative Distribution of Teacher Experience Types - Same Preferences for all Regions



Notes: This figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14th type corresponds to the vacant positions. The mechanisms that are status-quo improving are plotted in red. Those that do not are in gray. The thick dark gray line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.11: Change in Tenured Teacher Experience Type Across Regions - Same Preferences for all Regions



Notes: This figure shows the difference in the average experience of teachers between the matching obtained with SI-CC and the status-quo matching (top left figure) and between SI-CC and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA and the status-quo matching (bottom left figure) and between SI-DA and its benchmark DA* (bottom right figure). Each circle represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by the average experience of their teachers at status quo. The vertical line represents the median type. All regions on the left of the vertical line have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on the right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teacher average experience post reassignment is larger than that of the benchmark matchings to which it is compared. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.