Growth Accounting in an Open Economy

Frank M. Gollop

Boston College

Working Paper # 107

March 1981

Prepared for

Conference on Current Issues in Productivity

Rutgers University

April 1981

The proper economic characterization of aggregate productivity growth depends on society's economic objective for production. Kuznets, Solow, Denison and others analyzing aggregate productivity growth reason that goods destined for final demand are the ultimate objective of economic production. Kendrick (1973) presents the argument clearly:

> It is the final products included in national product that are the objective of production. These are the goods that satisfy current consumer wants or that add to stocks of productive capacity for satisfying future wants, to use the definition of Kuznets and many earlier national income theorists.[1]

The appropriate measure of aggregate output is the sum of sectoral deliveries to final demand.

Sectoral deliveries to intermediate demand are excluded from present aggregate productivity research because they are viewed as self-canceling transactions. Kendrick (1973) states the argument best:

> Inclusion of intermediate inputs obviously involves double counting, since such inputs have already been included in the final products and the factor services required to produce them are likewise included in total factor input.[2]

Given this characterization of economic activity, deliveries to final demand can be shown to equal national net output or, as conventionally described, aggregate value added.[3] The result is that studies of aggregate economic performance typically define productivity at the economy-wide level as the efficiency with which labor and capital inputs are converted into aggregate value added. For all their important differences, Solow (1957), Kendrick (1961, 1973), Denison (1962, 1974, 1979), Jorgenson and

Griliches (1967), Jorgenson and Christensen (1973), and Kendrick and Grossman (1980) adopt this conventional approach to aggregate growth accounting.

This paper does not take issue with the above description of the economy's macroeconomic objective. Deliveries to final demand are certainly an economy's ultimate economic concern for production.[4] What this paper does focus on is the implicit characterization of microeconomic activity, particularly the treatment of intermediate input and deliveries to intermediate demand. Their self-canceling property is neither an economic truism nor the result of any particular characterization of society's economic objective. It follows, instead, from the assumption that the economy is closed to trade in foreign-produced inputs.

The intuition underlying this proposition is straightforward. The domestic economy's intermediate input equals the sum of domestically produced intermediate goods <u>and</u> foreign-supplied materials. Total intermediate input purchases and domestic deliveries to intermediate demand are self-canceling transactions only in an economy importing no intermediate inputs.

The implications for modeling and measuring aggregate productivity growth are equally direct. For an economy closed to trade in imported inputs, intermediate input transactions can be viewed as internal, offsetting transfers. Incorporating them into a model of aggregate production unnecessarily involves double-counting. Consequently, macroeconomic models can suppress the vertically interdependent structure of macroeconomic activity. The important result demonstrated in this paper is that, for an economy closed to foreign-produced materials, value-added and delivery-to-final-demand models produce equivalent measures of aggregate productivity growth. The models' distinctly different descriptions of macroeconomic objectives and characterizations of microeconomic activity and sectoral production simply are of no consequence. Once trade in foreign-supplied inputs is introduced, however, domestic deliveries to intermediate demand and intermediate input purchases by domestic sectors are neither internal nor offsetting transfers. The aggregate economy can be viewed neither as a composite of horizontally independent sectors nor as an entity independent of foreign producing sectors. Modeling aggregate productivity in terms of value added or deliveries to final demand now makes a difference. The important conclusion is that, for an economy importing material inputs, the final measure of aggregate productivity growth depends importantly on the initial descriptions of both the economy's macroeconomic objectives and the technical properties of microeconomic production.

Sections 1 and 2 of this paper contrast measures of aggregate productivity growth derived from value-added and delivery-to-final-demand models of economic activity. The value-added model is the subject of section 1. Its specific assumptions for sectoral production and microeconomic activity, implicit in most productivity research, are introduced explicitly into the model of aggregate production. Section 2 develops an analogous framework for a model of aggregate productivity based on deliveries to final demand. Again, the model's underlying assumptions regarding sectoral production and microeconomic transactions are introduced explicitly. The most important finding is that, for an economy open to trade in foreign-supplied materials, the value-added model results in a higher measure of aggregate productivity growth than does the model defined in terms of deliveries to final demand.

The remainder of the paper addresses the important implications of redefining aggregate productivity growth in an "open" economy framework. Section 3 derives the link between sectoral measures of productivity growth and the measure of aggregate productivity growth. Both intersectoral and international transactions are relevant. Section 4 completes the analysis with a discussion of the practical consequences of these findings for intertemporal and international productivity comparisons.

1. Value Added

The aggregate economy is a collection of vertically related, interdependent sectors. Each sector uses labor, capital, and intermediate inputs to produce products delivered to final and/or intermediate demands. There is certainly no requirement that intermediate input purchases by an individual sector exactly offset its deliveries to intermediate demand. Stated equivalently, there is no requirement that the net output (or value added) of any sector equal its deliveries to final demand. Still, for an economy closed to trade in foreign material inputs, domestic deliveries to intermediate demand and intermediate input purchases can be viewed as internal transfers that at the economy-wide level are self-canceling transactions. Labor and capital are the only primary inputs. Aggregate deliveries to final demand equal aggregate value added. The macro economy then can be viewed as a set of horizontally independent sectors, each producing value added from labor and capital inputs. This line of reasoning forms the basis for those studies of aggregate productivity which describe the economy's objectives in terms of deliveries to final demand yet model aggregate productivity growth and microeconomic production in terms of value added.

Modeling macroeconomic activity and aggregate productivity growth in terms of value added begins with defining aggregate value added as a proportion of all quantities of sectoral value added. The maximum value of aggregate value added (λ) then can be expressed as a function of all quantities of sectoral value added, the supplies of labor and capital inputs, and time:

$$(1) \quad \lambda = F(V_1, V_2, \ldots, V_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn}, t)$$

where $V_j \equiv$ quantity of the jth sector's value added;

$L_{\ell j} \equiv \ell$th labor input used in the jth sector;

$K_{kj} \equiv k$th capital input used in the jth sector; and

$t \equiv$ an index of time.

Society's economic problem is to maximize λ given linearly homogeneous value-added functions

$$(2) \quad V_j = V^j(L_{1j}, L_{2j}, \ldots, L_{rj}, K_{1j}, K_{2j}, \ldots, K_{sj}, t) \quad (j = 1,2,\ldots,n),$$

market equilibrium conditions, and supplies of labor and capital inputs.

The model of aggregate value added is characterized by constant returns to scale. This is required by the sectoral value-added functions which constrain the macroeconomic maximization problem. The function F is therefore homogeneous of degree minus one in the quantities of sectoral value added, homogeneous of degree one in the factor supplies, and homogeneous of degree zero in the quantities of value added and inputs. Taken together, these conditions imply:

$$\sum_j \frac{\partial \ln F}{\partial \ln V_j} = -1$$

$$(3) \quad \sum_{j}\sum_{\ell} \frac{\partial \ln F}{\partial \ln L_{\ell j}} + \sum_{j}\sum_{k} \frac{\partial \ln F}{\partial \ln K_{kj}} = 1$$

$$\sum_j \frac{\partial \ln F}{\partial \ln V_j} + \sum_{j}\sum_{\ell} \frac{\partial \ln F}{\partial \ln L_{\ell j}} + \sum_{j}\sum_{k} \frac{\partial \ln F}{\partial \ln K_{kj}} = 0.$$

It is important to emphasize that the particular characterization of macroeconomic activity embodied in equation (1) has important technical implications for sectoral production. The existence of sectoral value-added functions V^j implies that sectoral production of gross output is characterized by value-added separability:

$$(4) \quad Z_j = f^j[V^j(L_{1j}, L_{2j}, \ldots, L_{rj}, K_{1j}, K_{2j}, \ldots, K_{sj}, t), X_{1j}, X_{2j}, \ldots, X_{nj}] \qquad (j = 1,2,\ldots,n),$$

where $Z_j \equiv$ gross output of the jth sector and

$X_{ij} \equiv$ ith intermediate input used in the jth sector.

Maintaining the separability of value added and intermediate inputs implies that the technical properties of V^j, including all the properties of technical change, can be analyzed in isolation from intermediate inputs. In particular, the marginal rate of substitution between any pair of labor and/or capital inputs is independent of the level and mix of intermediate inputs. In addition, intermediate inputs cannot be sources or mediums of productivity growth. If technical change occurs, it can affect output only through V^j. If technical change is of the factor-augmenting type, it can augment only capital and/or labor inputs. Neither Hicks-neutral nor Leontief-neutral technical change is possible in a sector characterized by value-added separability. Stated formally, intermediate inputs are excluded from a value-added model of macroeconomic activity not only because they are viewed as self-canceling transactions but also because they are considered unimportant to the analysis of underlying sectoral productivity growth.[5]

The expression for aggregate productivity growth follows directly both from the technical properties of macroeconomic and microeconomic production and from the necessary conditions for producer equilibrium. The measure of aggregate productivity growth is derived by first fixing the level of aggregate value added at unity:

$$(5) \quad 1 = F(V_1, V_2, \ldots, V_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn}, t).$$

This expression defines a production-possibilities frontier for the macro economy. Given this frontier, equilibrium conditions in a competitive economy require that the elasticities described in (3) equal value shares in aggregate value added:

$$\frac{\partial \ln F}{\partial \ln V_j} = \frac{-q_j^V V_j}{\sum_j q_j^V V_j} \qquad (j = 1,2,\ldots,n)$$

$$(6) \qquad \frac{\partial \ln F}{\partial \ln L_{\ell j}} = \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^V V_j} \qquad (\ell = 1,2,\ldots,r;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln F}{\partial \ln K_{kj}} = \frac{p_{kj} K_{kj}}{\sum_j q_j^V V_j} \qquad (k = 1,2,\ldots,s;\ j = 1,2,\ldots,n)\ ,$$

where q_j^V is the price of the quantity of value added in the jth sector, and $p_{\ell j}$ and p_{kj} are the prices corresponding to the ℓth labor input and the kth capital input, respectively, in the jth sector.

The homogeneity conditions (3) and the equilibrium conditions (6) together require that the value shares satisfy the summation conditions:

$$(7a) \qquad \sum_j \frac{q_j^V V_j}{\sum_j q_j^V V_j} = 1$$

$$(7b) \qquad \sum_j \sum_\ell \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^V V_j} + \sum_j \sum_k \frac{p_{kj} K_{kj}}{\sum_j q_j^V V_j} = 1.$$

Condition (7a) is true by construction. The value of aggregate value added equals the sum of all sectoral value added. Condition (7b) follows from sectoral and aggregate accounting identities. Verifying (7b) begins with the sectoral accounting identity relating the value of total output and payments to all factors of production:

$$(8)\quad q_jZ_j = \sum_{\ell} p_{\ell j}L_{\ell j} + \sum_{k} p_{kj}K_{kj} + \sum_{i} p_iX_{ij} \qquad (j = 1,2,\ldots,n),$$

where p_i is the price of the ith intermediate input.[6] The value of sectoral value added equals the value of output less the value of intermediate input purchases:

$$(9)\quad q_j^VV_j = q_jZ_j - \sum_{i} p_iX_{ij} \qquad (j = 1,2,\ldots,n).$$

Substituting (8) into (9) and summing over all n sectors yields:

$$(10)\quad \sum_{j} q_j^VV_j = \sum_{j\ell}\sum p_{\ell j}L_{\ell j} + \sum_{jk}\sum p_{kj}K_{kj} ,$$

establishing the result required by (7b).

An expression for the rate of aggregate productivity growth now can be derived from the production-possibilities frontier (5) and the conditions for producer equilibrium (6). The total logarithmic derivative of F with respect to time can be expressed in the form:

$$(11)\quad 0 = \sum_{j} \frac{\partial \ln F}{\partial \ln V_j} \frac{d\ln V_j}{dt} + \sum_{j\ell}\sum \frac{\partial \ln F}{\partial \ln L_{\ell j}} \frac{d\ln L_{\ell j}}{dt} + \sum_{jk}\sum \frac{\partial \ln F}{\partial \ln K_{kj}} \frac{d\ln K_{kj}}{dt}$$

$$+ \frac{\partial \ln F}{\partial t} ,$$

where $\frac{\partial \ln F}{\partial t}$ is the rate of aggregate productivity growth. Using the equilibrium conditions (6), the aggregate rate of productivity growth (E_V) can be expressed in the following form:

$$(12)\quad E_V \equiv \frac{\partial \ln F}{\partial t} = \sum_j \frac{q_j^V V_j}{\sum_j q_j^V V_j} \frac{d\ln V_j}{dt} - \sum_j \sum_\ell \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^V V_j} \frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_j \sum_k \frac{p_{kj} K_{kj}}{\sum_j q_j^V V_j} \frac{d\ln K_{kj}}{dt} .$$

This result can be summarized as a familiar proposition.

> Proposition 1: The rate of aggregate productivity growth for an economy maximizing net output (value added) is defined as the weighted average of rates of growth of sectoral value added (with weights equal to sectoral value added shares in aggregate net output) less the weighted average of rates of growth of labor and capital inputs (with weights equal to the value shares of input payments in aggregate net output). Both sets of weights sum to unity.

The expression (12) applies to all economies, whether open or closed to trade in foreign-produced inputs. Intermediate inputs, whether purchased from domestic or foreign producers, do not enter the production-possibilities frontier (5) or the sectoral value-added functions (2), nor do they influence the equilibrium conditions (6). The sectoral value-added functions (2) maintain that microeconomic and therefore macroeconomic production are characterized by value-added separability. This assumption maintains that all the properties of productivity growth, including its measurement, can be analyzed in isolation from the level and mix of intermediate inputs.

What is important to note is that the model of aggregate productivity stated in terms of value added does not depend on the assumption that domestic deliveries to intermediate demand and total intermediate input purchases are self-canceling transactions. It depends solely on the assumption of value-added separability. Not surprisingly, the resulting measure of aggregate productivity growth (12) is insensitive to the economy's external trade in foreign-produced intermediate goods.

2. Deliveries to Final Demand

A model of aggregate productivity growth stated in terms of deliveries to final demand is constrained by sectoral production functions, not sectoral value-added functions. Value-added separability is not a maintained hypothesis. Consequently, intersectoral *and* international transactions in material inputs are not suppressed. The vertical structure of the aggregate economy is recognized explicitly.

The model of economic activity defined in terms of deliveries to final demand not only accounts for material inputs but depends importantly on the sources of those inputs. Total deliveries to an economy's intermediate demand equal the sum of domestic and foreign deliveries. *Domestic* deliveries to intermediate demand are truly intermediate goods. They are produced from primary labor and capital inputs supplied to the domestic economy. *Foreign* deliveries to domestic intermediate demand are not intermediate goods from the perspective of the domestic economy. They are primary inputs to be treated symmetrically with the domestic economy's labor and capital inputs. Domestic deliveries to intermediate demand are purely internal, offsetting transfers in *intermediate* inputs. Foreign deliveries, in contrast, are wholly external transactions in primary inputs. It follows that the formal specification of an aggregate model defined in terms of the domestic economy's deliveries to final demand and its primary inputs differs depending on whether the economy is "open" or "closed" to trade in foreign-produced inputs. The two models are developed separately below. Their results are contrasted with the value-added model described in section 1.

The closed economy. Society's objective is to maximize aggregate output defined as deliveries to final demand. The maximum value of aggregate deliveries to final demand (μ) can be expressed as a function of all quantities of sectoral deliveries to final demand (Y_j), labor and capital inputs, and time:

(13) $$\mu = G(Y_1, Y_2, \ldots, Y_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn}, t).$$

The model abstracts from intermediate inputs and sectoral deliveries to intermediate demand since, given the absence of any imported inputs, all domestic sales and purchases of intermediate inputs are self-canceling transactions.

The economy-wide maximization problem is constrained by the supplies of the primary inputs, labor and capital, and by linearly homogeneous sectoral production functions:

(14) $$Z_j = g^j(L_{1j}, L_{2j}, \ldots, L_{rj}, K_{1j}, K_{2j}, \ldots, K_{sj}, X_{1j}, X_{2j}, \ldots, X_{nj}, t) \qquad (j = 1,2,\ldots,n).$$

It is important to note that all inputs are treated symmetrically in the sectoral production functions g^j, unlike the sectoral production functions (4) constraining the value-added model described in section 1. Value-added separability is not a maintained hypothesis. There are no restrictions on either the marginal rate of substitution between any pair of inputs or on the properties of technical change. Though intermediate inputs do not appear explicitly in the macro model (13), they enter the problem through the constraints (14). Microeconomic production for delivery to intermediate demand is an important part of the solution to the macroeconomic problem maximizing deliveries to final demand.

The aggregate function G exhibits constant returns to scale. It is homogeneous of degree minus one in sectoral deliveries to final demand, homogeneous of degree one in labor and capital inputs, and homogeneous of degree zero in final demand deliveries and all inputs:

$$\sum_j \frac{\partial \ln G}{\partial \ln Y_j} = -1$$

$$(15) \qquad \sum_j \sum_\ell \frac{\partial \ln G}{\partial \ln L_{\ell j}} + \sum_j \sum_k \frac{\partial \ln G}{\partial \ln K_{kj}} = 1$$

$$\sum_j \frac{\partial \ln G}{\partial \ln Y_j} + \sum_j \sum_\ell \frac{\partial \ln G}{\partial \ln L_{\ell j}} + \sum_j \sum_k \frac{\partial \ln G}{\partial \ln K_{kj}} = 0.$$

Forming the production-possibilities frontier

$$(16) \quad 1 = G(Y_1, Y_2, \ldots, Y_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn}, t),$$

necessary conditions for producer equilibrium require the following relations:

$$\frac{\partial \ln G}{\partial \ln Y_j} = \frac{-q_j^Y Y_j}{\sum_j q_j^Y Y_j} \qquad (j = 1,2,\ldots,n)$$

$$(17) \quad \frac{\partial \ln G}{\partial \ln L_{\ell j}} = \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} \qquad (\ell = 1,2,\ldots,r;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln G}{\partial \ln K_{kj}} = \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} \qquad (k = 1,2,\ldots,s;\ j = 1,2,\ldots,n),.$$

where q_j^Y is the price of goods delivered to final demand in the jth sector.

The homogeneity restrictions (15) and the equilibrium conditions (17) together require:

$$(18a) \quad \sum_j \frac{q_j^Y Y_j}{\sum_j q_j^Y Y_j} = 1$$

$$(18b) \quad \sum_{j\ell}\sum \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} + \sum_{jk}\sum \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} = 1.$$

Condition (18a) is true by construction. Condition (18b) follows directly from accounting identities relating the value of a sector's total production both to the sum of the values of that sector's deliveries to intermediate and final demands and to the sum of that sector's payments to all factors of production:[7]

$$(19) \quad q_j Z_j = \sum_i p_j X_{ji} + q_j^Y Y_j \qquad (j = 1,2,\ldots,n)$$

$$(20)\quad q_j Z_j = \sum_{\ell} p_{\ell j} L_{\ell j} + \sum_{k} p_{kj} K_{kj} + \sum_{i} p_i X_{ij} \qquad (j = 1,2,\ldots,n).$$

Summing (19) and (20) over all sectors and equating the results produces the condition required by (18b):

$$(21)\quad \sum_j q_j^Y Y_j = \sum_j \sum_{\ell} p_{\ell j} L_{\ell j} + \sum_j \sum_{k} p_{kj} K_{kj},$$

since $\sum_j \sum_i p_j X_{ji} = \sum_j \sum_i p_i X_{ij}$.

The rate of aggregate productivity growth (E_Y) now can be solved from the total logarithmic derivative of G with respect to time:

$$(22)\quad E_Y \equiv \frac{\partial \ln G}{\partial t} = -\sum_j \frac{\partial \ln G}{\partial \ln Y_j} \frac{d\ln Y_j}{dt} - \sum_j \sum_{\ell} \frac{\partial \ln G}{\partial \ln L_{\ell j}} \frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_j \sum_{k} \frac{\partial \ln G}{\partial \ln K_{kj}} \frac{d\ln K_{kj}}{dt}$$

so that, using equilibrium conditions (17),

$$(23)\quad E_Y = \sum_j \frac{q_j^Y Y_j}{\sum_j q_j^Y Y_j} \frac{d\ln Y_j}{dt} - \sum_j \sum_{\ell} \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} \frac{d\ln L_{\ell j}}{dt} - \sum_j \sum_{k} \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} \frac{d\ln K_{kj}}{dt}.$$

The rate of aggregate productivity growth for a model stated in terms of deliveries to final demand can be expressed as the weighted average of the rates of growth of sectoral deliveries to final demand less a weighted average of rates of growth of labor and capital inputs.[8]

Distinctly different characterizations of macroeconomic objectives and microeconomic activity are modeled by the value-added and delivery-to-final-demand production-possibilities frontiers F and G defined, respectively, in (5) and (16). The frontier F is concerned with maximizing aggregate net output, is defined in terms of sectoral value added, and is based on value-added functions for each sector. The macro economy is viewed as a set of horizontally independent sectors. Intersectoral transactions are ignored. Value-added separability, with all its implications for sectoral and aggregate productivity growth, is a maintained assumption. In contrast, the frontier G is concerned with maximizing deliveries to economy-wide final demand, is stated in terms of sectoral deliveries to final demand, and is based on constant returns to scale production functions for each producing sector. The macro economy is modeled as a collection of vertically interdependent sectors. An active market in intersectoral transactions is recognized explicitly.

In spite of these fundamental differences, the following proposition holds:

> Proposition 2: For an economy closed to trade in foreign-produced inputs, value-added and delivery-to-final-demand models of macroeconomic production lead to identical measures of aggregate productivity growth.

The proposition asserts that the rate of aggregate productivity growth E_V defined in (12) equals the aggregate rate E_Y defined in (23). Verifying this proposition for a closed economy is a straightforward exercise.

Consider first the accounting identities expressed in equations (9) and (19). The former defines the value of a sector's value added as the value of that sector's output less the value of its intermediate input purchases. The latter states that the value of each sector's output equals the sum of the values of its products delivered to intermediate and final demands. Totally differentiating (9) and (19) with respect to time leads to expressions that, respectively, can be written in the forms:

$$(24)\quad \frac{d\ln V_j}{dt} = \frac{q_j Z_j}{q_j^v V_j}\frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^v V_j}\frac{d\ln X_{ij}}{dt} \qquad (j = 1,2,\ldots,n)$$

$$(25)\quad \frac{d\ln Z_j}{dt} = \sum_i \frac{p_j X_{ji}}{q_j Z_j}\frac{d\ln X_{ji}}{dt} + \frac{q_j^y Y_j}{q_j Z_j}\frac{d\ln Y_j}{dt} \qquad (j = 1,2,\ldots,n)\,.$$

Substituting (25) into (24) and substituting the resulting expression for the growth of sectoral value added into equation (12), the expression for E_v, yields

$$(26)\quad E_v = \sum_j \frac{q_j^v V_j}{\sum_j q_j^v V_j}\left[\frac{q_j Z_j}{q_j^v V_j}\left(\sum_i \frac{p_j X_{ji}}{q_j Z_j}\frac{d\ln X_{ji}}{dt} + \frac{q_j^y Y_j}{q_j Z_j}\frac{d\ln Y_j}{dt}\right)\right.$$

$$\left. - \sum_i \frac{p_i X_{ij}}{q_j^v V_j}\frac{d\ln X_{ij}}{dt}\right] - \sum_{j\ell}\sum \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^v V_j}\frac{d\ln L_{\ell j}}{dt} - \sum_{jk}\sum \frac{p_{kj} K_{kj}}{\sum_j q_j^v V_j}\frac{d\ln K_{kj}}{dt}$$

$$= \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j}\frac{d\ln Y_j}{dt} - \sum_{j\ell}\sum \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^v V_j}\frac{d\ln L_{\ell j}}{dt} - \sum_{jk}\sum \frac{p_{kj} K_{kj}}{\sum_j q_j^v V_j}\frac{d\ln K_{kj}}{dt}$$

since $\sum\limits_{j}\sum\limits_{i} \frac{p_j X_{ji}}{\sum\limits_j q_j^V V_j} \frac{d\ln X_{ji}}{dt} = \sum\limits_{j}\sum\limits_{i} \frac{p_i X_{ij}}{\sum\limits_j q_j^V V_j} \frac{d\ln X_{ij}}{dt}$.

Multiplying the expression for E_V in (26) by the ratio of the value of aggregate value added to the value of aggregate deliveries to final demand produces an expression identical to the definition of E_Y in (23):

$$(27) \quad \frac{\sum\limits_j q_j^V V_j}{\sum\limits_j q_j^Y Y_j} E_V = \sum_j \frac{q_j^Y Y_j}{\sum\limits_j q_j^Y Y_j} \frac{d\ln Y_j}{dt} - \sum_{j}\sum_{\ell} \frac{p_{\ell j} L_{\ell j}}{\sum\limits_j q_j^Y Y_j} \frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_{j}\sum_{k} \frac{p_{kj} K_{kj}}{\sum\limits_j q_j^Y Y_j} \frac{d\ln K_{kj}}{dt}$$

so that

$$(28) \quad E_V = \frac{\sum\limits_j q_j^Y Y_j}{\sum\limits_j q_j^V V_j} E_Y .$$

Finally, consider the accounting relation in a closed economy between the values of aggregate value added and aggregate deliveries to final demand. As defined in (9), the value of sectoral value added equals the value of sectoral output less that sector's purchases of intermediate input. Substituting the value sum of deliveries to intermediate and final demands, equation (19), for the value of total output in (9) yields:

$$(29) \quad q_j^V V_j = \sum_i p_j X_{ji} + q_j^Y Y_j - \sum_i p_i X_{ij} \qquad (j = 1,2,\ldots,n).$$

Summing (29) over all sectors produces the expected result:

(30) $\sum_j q_j^v V_j = \sum_j q_j^y Y_j$

since $\sum_j \sum_i p_j X_{ji} = \sum_j \sum_i p_i X_{ij}$, the value of total deliveries to intermediate demand equals the value of total intermediate input purchases.

Proposition 2 follows directly from (28) and (30):

(31) $E_v = E_y$.

The different initial descriptions of macroeconomic objectives and microeconomic production offered by value-added and delivery-to-final-demand models are of no consequence to productivity accounting in a closed economy. The models produce identical measures of aggregate productivity growth.

The open economy. The derivation of the open economy model parallels the development of its closed economy counterpart. The only important difference is that imported inputs (M_{mj}) enter both the aggregate production-possibilities frontier and the sectoral production functions.[9] Society's production problem is to maximize aggregate deliveries to final demand (μ)

(32) $\mu = H(Y_1, Y_2, \ldots, Y_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn}, M_{11}, \ldots, M_{un}, t)$

subject to fixed supplies of domestic labor and capital inputs, market equilibrium conditions, and linearly homogeneous sectoral production functions:

$$(33)\quad Z_j = h^j(L_{1j}, L_{2j}, \ldots, L_{rj}, K_{1j}, K_{2j}, \ldots, K_{sj}, X_{1j}, X_{2j}, \ldots, X_{nj},$$

$$M_{1j}, M_{2j}, \ldots, M_{uj}, t) \qquad (j = 1,2,\ldots,n).$$

Like the production function g^j defined in (14), h^j describes an unconstrained model of sectoral production.[10]

Setting the aggregate value of μ equal to unity, the constant returns to scale function H can be expressed as a production-possibilities frontier:

$$(34)\quad 1 = H(Y_1, Y_2, \ldots, Y_n, L_{11}, L_{12}, \ldots, L_{rn}, K_{11}, K_{12}, \ldots, K_{sn},$$

$$M_{11}, M_{12}, \ldots, M_{un}, t).$$

Homogeneity and market equilibrium conditions together imply:

$$(35)\quad \begin{aligned}
\frac{\partial \ln H}{\partial \ln Y_j} &= \frac{-q_j^Y Y_j}{\sum_j q_j^Y Y_j} && (j = 1,2,\ldots,n) \\
\frac{\partial \ln H}{\partial \ln L_{\ell j}} &= \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} && (j = 1,2,\ldots,n) \\
\frac{\partial \ln H}{\partial \ln K_{kj}} &= \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} && (j = 1,2,\ldots,n) \\
\frac{\partial \ln H}{\partial \ln M_{mj}} &= \frac{p_m M_{mj}}{\sum_j q_j^Y Y_j} && (j = 1,2,\ldots,n)
\end{aligned}$$

and

$$\sum_j \frac{\partial \ln H}{\partial \ln Y_j} = -\sum_j \frac{q_j^Y Y_j}{\sum_j q_j^Y Y_j} = -1$$

(36)

$$\sum_j\sum_\ell \frac{\partial \ln H}{\partial \ln L_{\ell j}} + \sum_j\sum_k \frac{\partial \ln H}{\partial \ln K_{kj}} + \sum_j\sum_m \frac{\partial \ln H}{\partial \ln M_{mj}}$$

$$= \sum_j\sum_\ell \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} + \sum_j\sum_k \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} + \sum_j\sum_m \frac{p_m M_{mj}}{\sum_j q_j^Y Y_j} = 1,$$

where p_m is the price of the mth imported input.[11]

The rate of aggregate productivity growth (E_Y') appropriate for an open economy is derived by substituting the necessary conditions for producer equilibrium (35) into the total time derivative of the aggregate frontier H. The resulting expression is:

$$(37) \quad E_Y' \equiv \frac{\partial \ln H}{\partial t} = \sum_j \frac{q_j^Y Y_j}{\sum_j q_j^Y Y_j} \frac{d\ln Y_j}{dt} - \sum_j\sum_\ell \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^Y Y_j} \frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_j\sum_k \frac{p_{kj} K_{kj}}{\sum_j q_j^Y Y_j} \frac{d\ln K_{kj}}{dt} - \sum_j\sum_m \frac{p_m M_{mj}}{\sum_j q_j^Y Y_j} \frac{d\ln M_{mj}}{dt} .$$

Comparing the expressions for E_Y and E_Y' defined in (23) and (37), respectively, leads to the following proposition:

> Proposition 3: The rate of aggregate productivity growth for an economy maximizing aggregate output (deliveries to final demand) is defined as the weighted average of rates of growth of sectoral deliveries to final demand (with weights equal to sectoral delivery-to-final-demand shares in aggregate output) less the weighted average of rates of growth of primary inputs (with weights equal to the value shares of input payments in aggregate output). Both sets of weights sum to unity. For a closed economy, primary inputs include domestic supplies of labor and capital. For an open economy, primary inputs include domestic labor and capital and imported materials.

The most important finding of this paper follows from Propositions 1 and 3. It concerns the relation between E_V and E_Y', the measures of aggregate productivity growth respectively derived from value-added and delivery-to-final-demand models for an open economy. The result can be stated as a proposition.

> Proposition 4: For an economy open to trade in foreign-produced inputs, the rate of aggregate productivity growth derived from a value-added model of aggregate production is greater than the corresponding rate resulting from a delivery-to-final-demand model. The two rates differ by an amount that is proportional to the value share of the economy's imported materials in total deliveries to final demand.

Proving this proposition begins with both the sectoral accounting identity (19), equating the value of each sector's output to the sum of the values of its deliveries to intermediate and final demands, and the accounting identity defining the value of sectoral value added in an open economy:

$$(38) \quad q_j^V V_j = q_j Z_j - \sum_i p_i X_{ij} - \sum_m p_m M_{mj} \qquad (j = 1,2,\ldots,n).$$

The total derivative of (19) with respect to time is expressed in equation (25). The time derivative of (38) is:

$$(39) \quad \frac{d\ln V_j}{dt} = \frac{q_j Z_j}{q_j^V V_j} \frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^V V_j} \frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_m M_{mj}}{q_j^V V_j} \frac{d\ln M_{mj}}{dt}$$

$$(j = 1,2,\ldots,n).$$

Substituting (25) into (39) and substituting the resulting expression for the growth of sectoral value added into equation (12), the expression for E_V, yields:

$$(40)\quad E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j} \frac{d\ln Y_j}{dt} - \sum_j \sum_\ell \frac{p_{\ell j} L_{\ell j}}{\sum_j q_j^v V_j} \frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_j \sum_k \frac{p_{kj} K_{kj}}{\sum_j q_j^v V_j} \frac{d\ln K_{kj}}{dt} - \sum_j \sum_m \frac{p_m M_{mj}}{\sum_j q_j^v V_j} \frac{d\ln M_{mj}}{dt} .$$

Multiplying (40) by the ratio $\sum_j q_j^v V_j / \sum_j q_j^y Y_j$ produces an expression equal to the definition of E_y' in (37) so that

$$(41)\quad E_v = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^v V_j} E_y' .$$

Equation (41) has precisely the same form as equation (28), the expression relating the aggregate productivity rates E_v and E_y for an economy closed to imported inputs. The difference is that the ratio $\sum_j q_j^y Y_j / \sum_j q_j^v V_j$ equals unity for a closed economy, as demonstrated above, but is greater than unity for an open economy. Substituting (19) for the value of total output, $q_j Z_j$, in (38) and summing over all n sectors yields:

$$(42)\quad \sum_j q_j^v V_j = \sum_j q_j^y Y_j - \sum_j \sum_m p_m M_{mj} .$$

The value of aggregate deliveries to final demand (aggregate output) exceeds the value of aggregate value added (net output) by an amount equal to the value of imported inputs.

Proposition 4 follows directly. Substituting (42) into (41) yields:

$$(43)\quad E_v = \left[\frac{\sum_j q_j^y Y_j}{\sum_j q_j^y Y_j - \sum_{jm}\sum p_m M_{mj}}\right] E_y' .$$

It necessarily follows that $E_v > E_y'$ and, verifying Proposition 4,

$$(44)\quad E_y' = \left(1 - \frac{\sum_{jm}\sum p_m M_{mj}}{\sum_j q_j^y Y_j}\right) E_v ;$$

the two rates differ by an amount that is proportional to the value share of the economy's imported materials in total deliveries to final demand. Value-added and delivery-to-final-demand models of aggregate production produce identical measures of aggregate productivity growth <u>if and only if</u> the economy is closed to trade in foreign-produced inputs ($\sum_m p_m M_{mj} = 0$).

3. Aggregation Over Sectors in an Open Economy

The value-added and delivery-to-final-demand models of aggregate production are each constrained by different microeconomic models of production. These micro models describe sectoral productivity growth which, in turn, determines aggregate productivity growth. The objective of this section is to derive the economic relation between sectoral and aggregate measures of productivity growth for an economy open to trade in foreign-produced inputs. The value-added and delivery-to-final-demand models are contrasted.

<u>Sectoral productivity growth</u>. The value-added separable production functions f^j defined in (4) describe microeconomic production

in the aggregate value-added model. Given value-added separability, the subfunctions V^j defined in (2) become the effective constraints for the aggregate value-added maximization problem (1). These functions contain all the important properties describing sectoral productivity growth. Expressing the logarithmic derivative of V^j with respect to time in terms of sectoral conditions for producer equilibrium results in an expression that can be solved for the measured rate of sectoral productivity growth (e_v^j):[12]

$$(45) \quad e_v^j \equiv \frac{\partial \ln V^j}{\partial t} = \frac{d\ln V_j}{dt} - \sum_{\ell} \frac{p_{\ell j} L_{\ell j}}{q_j^v V_j} \frac{d\ln L_{\ell j}}{dt} - \sum_{k} \frac{p_{kj} K_{kj}}{q_j^v V_j} \frac{d\ln K_{kj}}{dt}$$

$$(j = 1,2,\ldots,n).$$

Neither foreign-produced inputs nor domestically-produced intermediate inputs enter this description of sectoral productivity growth.

Microeconomic production in the delivery-to-final-demand model is characterized by the sectoral production functions h^j defined in (33). They constrain the aggregate production problem (32) directly. The measured rate of sectoral productivity growth (e_z^j)[13] can be solved from the logarithmic time derivative of (33) with conditions for producer equilibrium imposed:[14]

$$(46) \quad e_z^j \equiv \frac{\partial \ln h^j}{\partial t} = \frac{d\ln Z_j}{dt} - \sum_{\ell} \frac{p_{\ell j} L_{\ell j}}{q_j Z_j} \frac{d\ln L_{\ell j}}{dt} - \sum_{k} \frac{p_{kj} K_{kj}}{q_j Z_j} \frac{d\ln K_{kj}}{dt}$$

$$- \sum_{i} \frac{p_i X_{ij}}{q_j Z_j} \frac{d\ln X_{ij}}{dt} - \sum_{m} \frac{p_m M_{mj}}{q_j Z_j} \frac{d\ln M_{mj}}{dt} \qquad (j = 1,2,\ldots,n).$$

The rates of sectoral productivity growth depend on both inter-sectoral and international transactions.

It is important to emphasize that the variables e_v^j and e_z^j describe different rates of sectoral productivity growth. The following proposition holds for an "open" economy.

> Proposition 5: For an economy open to trade in foreign-produced inputs, the rate of sectoral productivity growth derived from a sectoral net output (value added) production function is greater than the corresponding rate resulting from a sectoral gross output production function. The two rates differ by an amount that is proportional to the share of the sector's purchases of domestic intermediate and foreign-produced inputs in the value of sectoral gross output.

Formally demonstrating this proposition begins with substituting equation (39), the logarithmic time derivative of value added, into equation (45) defining e_v^j:

$$(47) \quad e_v^j = \left(\frac{q_j Z_j}{q_j^v V_j} \frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^v V_j} \frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_m M_{mj}}{q_j^v V_j} \frac{d\ln M_{mj}}{dt} \right)$$

$$- \sum_\ell \frac{p_{\ell j} L_{\ell j}}{q_j^v V_j} \frac{d\ln L_{\ell j}}{dt} - \sum_k \frac{p_{kj} K_{kj}}{q_j^v V_j} \frac{d\ln K_{kj}}{dt} \qquad (j = 1,2,\ldots,n).$$

Substituting (46) into (47) yields:

$$(48) \quad e_v^j = \left(\frac{q_j Z_j}{q_j Z_j - \sum_i p_i X_{ij} - \sum_m p_m M_{mj}} \right) e_z^j \qquad (j = 1,2,\ldots,n).$$

It necessarily follows that $e_v^j > e_z^j$ and

$$(49) \quad e_z^j = \left(1 - \frac{\sum_i p_i X_{ij} + \sum_m p_m M_{mj}}{q_j Z_j} \right) e_v^j \qquad (j = 1,2,\ldots,n);$$

the two rates differ by an amount that is proportional to the share of sectoral purchases of domestically-produced and foreign-produced inputs in the value of sectoral gross output.[15]

<u>Aggregation</u>. This paper has emphasized the fundamentally different characterizations of economic activity offered by value-added and delivery-to-final-demand models of economic growth. These differences are nowhere more apparent than in the formulas defining E_y' and E_v as aggregate rates of productivity growth formed from the sectoral measures e_z^j and e_v^j, respectively. Since e_z^j is generated from a sectoral model incorporating interindustry and international transactions, the aggregating algorithm must capture the contributions of each sector's productivity growth through deliveries of its output to both final <u>and</u> intermediate demand. In contrast, since the sectoral model maintaining value-added separability abstracts from all intersectoral and international transactions, aggregating over e_v^j must capture sectoral advances in productivity transmitted only through value-added contributions to aggregate net output.

Both aggregate and sectoral value-added models view the economy as a set of horizontally independent sectors. The relevant measure of sectoral output is the quantity of value added. The aggregate economy's production-possibilities frontier is defined in terms of the sectoral quantities of value added. Both models consider only labor and capital inputs. The formal link between the aggregate and sectoral rates E_v and e_v^j is simple and direct.

<u>Proposition 6</u>. The rate of aggregate productivity growth E_v for a value-added model of aggregate production equals a weighted <u>average</u> of the sectoral rates of productivity growth e_v^j, with weights equal to sectoral value shares of value added in total value added.

This result is derived by substituting the expression for e_v^j, equation (45), into equation (12), the expression for E_v:

$$(50) \quad E_v = \sum_j \frac{q_j^v V_j}{\sum_j q_j^v V_j} e_v^j .$$

The delivery-to-final-demand model of aggregate production and its microeconomic production constraints view the economy as consisting of vertically interrelated sectors depending not only on each other but on international trade as well. Aggregate and sectoral models each define output in terms of <u>its</u> particular final product. While deliveries to final demand define output at the aggregate level, individual sectors produce output delivered to both final and intermediate demands. Similarly, each model considers all inputs primary to <u>its</u> particular production process. The economy's primary inputs include labor, capital, and imported inputs. Sectors employ labor, capital, imported inputs, and domestically-produced intermediate inputs. Given this vertical model of economic activity, advances in productivity in an individual industry contribute to aggregate economic growth both directly through deliveries to final demand and indirectly through increased deliveries to sectors dependent on its output as intermediate input. The aggregation formula must capture both direct and indirect transmissions of sectoral productivity growth.

<u>Proposition 7</u>. The rate of aggregate productivity growth E_Y' for a delivery-to-final-demand model of aggregate production equals a weighted <u>sum</u> of the sectoral rates of productivity growth e_Z^j, with weights equal to sectoral ratios of the value of gross output to the value of total deliveries to final demand.

Demonstrating this proposition begins with equation (19), the expression equating the value of each sector's total output to the sum of the values of intermediate demand for that output by all n sectors and the value of final demand for that output. Substituting the logarithmic derivative of (19) with respect to time, equation (25), into (46) permits the measure of sectoral productivity to be expressed in terms identifying the immediate uses of the sector's output:

$$(51)\quad e_Z^j = \sum_i \frac{p_j X_{ji}}{q_j Z_j}\frac{d\ln X_{ji}}{dt} + \frac{q_j^Y Y_j}{q_j Z_j}\frac{d\ln Y_j}{dt} - \sum_\ell \frac{p_{\ell j} L_{\ell j}}{q_j Z_j}\frac{d\ln L_{\ell j}}{dt}$$

$$- \sum_k \frac{p_{kj} K_{kj}}{q_j Z_j}\frac{d\ln K_{kj}}{dt} - \sum_i \frac{p_i X_{ij}}{q_j Z_j}\frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_m M_{mj}}{q_j Z_j}\frac{d\ln M_{mj}}{dt}$$

$$(j = 1,2,\ldots,n).$$

Multiplying (51) by the ratio $q_j Z_j / \sum_j q_j^Y Y_j$ and summing over all n sectors produces the economic relation between sectoral and aggregate measures of productivity growth:

$$(52)\quad E_Y' = \sum_j \frac{q_j Z_j}{\sum_j q_j^Y Y_j} e_Z^j .$$

The sum of the weights in (52) exceeds unity by an amount equal to the ratio of the aggregate value of deliveries to intermediate demand to the aggregate value of deliveries to final demand:

$$(53)\quad \sum_j \frac{q_j Z_j}{\sum_j q_j^Y Y_j} = \sum_j \frac{(\sum_i p_j X_{ji} + q_j^Y Y_j)}{\sum_j q_j^Y Y_j} = \sum_j \frac{\sum_i p_j X_{ji}}{\sum_j q_j^Y Y_j} + 1 \,.$$

The weights sum to a value larger than unity because each sector contributes to the rate of productivity growth for the aggregate economy through its deliveries to both final and intermediate demand. Gollop (1979) presents an heuristic interpretation of this weighting structure:

> Consider an individual sector that experiences an advance in its rate of technical change. Holding constant all primary and intermediate inputs, the sector can provide the economy with increased output. The objective of creating the appropriate weight for this sector's technological advance is to correctly assign causal responsibility to this sector for any effect on aggregate technical change. Since the ultimate macroeconomic concern is the effect of this technical change on aggregate output, the appropriate denominator in the weight is the sum of all n sector contributions to aggregate output. Since the individual sector transmits the benefits of its productivity growth to final consumers both directly through deliveries to final demand and indirectly through deliveries to intermediate demand, the appropriate numerator in the weight is the sector's total output, i.e., the sum of its deliveries to final and intermediate demand.[16]

Gollop (1979) concludes that the appropriately weighted sum contributions of sectoral productivity growth to aggregate productivity growth are identical whether the sectoral rates are derived from gross-output or value-added models of sectoral production. This implies:

$$(54)\quad \sum_j \frac{q_j Z_j}{\sum_j q_j^Y Y_j}\, e_Z^j = \sum_j \frac{q_j^V V_j}{\sum_j q_j^V V_j}\, e_V^j \,.$$

It is important to recognize that this result holds only for a closed economy. Given (50) and (52), proposition 4 suggests:

$$(55)\quad \sum_j \frac{q_j Z_j}{\sum_j q_j^Y Y_j} e_z^j = E_Y' = \left(1 - \frac{\sum\limits_{jm}\sum p_m M_{mj}}{\sum\limits_j q_j^Y Y_j}\right) E_V$$

$$= \left(1 - \frac{\sum\limits_{jm}\sum p_m M_{mj}}{\sum\limits_j q_j^Y Y_j}\right) \sum_j \frac{q_j^V V_j}{\sum\limits_j q_j^V V_j} e_V^j \,.$$

Equation (54) holds <u>if and only if</u> $\sum\limits_{jm}\sum p_m M_{mj} = 0$, the economy is closed to trade in foreign-produced inputs.

4. Conclusion

Value-added and delivery-to-final-demand models of aggregate production offer fundamentally different descriptions of macroeconomic and microeconomic activity. For an economy open to trade in foreign-produced inputs, those differences have important implications for the measured rate of aggregate productivity growth. This paper demonstrates that the two models produce measures that differ in direct proportion to the extent of the economy's dependence on imported inputs. The important remaining question is: Which model forms a more appropriate basis for measuring aggregate productivity growth?

The answer follows directly from the arguments developed throughout this paper. First, productivity growth is a measure of economic performance. Performance measures should serve as barometers, sensitive to the economy's success in solving its economic problem. The choice of productivity measure therefore depends on society's objectives. Economists long have recognized that society's economic objective is to satisfy human wants through deliveries to final demand.[17] Second, the solution to society's economic problem is constrained by scarce resources. Productivity growth measures how well the economy allocates its scarce resources among its producing agents. Primary inputs for the macro economy include labor, capital, and imported inputs. These plus domestically-produced intermediate inputs form the primary inputs for each micro sector. Productivity growth views all primary inputs symmetrically. Unless affirmed by economic evidence, value-added separability is an inappropriate as well as unnecessary assumption. Third, the economy does not consist of horizontally independent sectors. It is, instead, a set of vertically interdependent sectors dependent on both intersectoral and international transactions. Only the delivery-to-final-demand model recognizes these transactions. Only it describes how an individual sector transmits the benefits of its productivity growth to final consumers both directly through deliveries to final demand and indirectly through deliveries to intermediate demand. Fourth, a measure of aggregate productivity growth is useful only as a comparative measure of economic performance over time or across countries. Only the delivery-to-final-demand model is sensitive to the intertemporal and international variation in imported input requirements.

Accepting or rejecting these arguments and their implicit conclusion is much more than an academic exercise. It has important practical consequences for both intertemporal and international comparisons of productivity growth. Table 1 reports the current price ratios of imported inputs to deliveries to final demand for selected years for eight industrial nations. Not surprisingly, most of the ratios not only increase over time and increase at different rates but also vary considerably across countries. It follows from equation (44) that the rates E_v and E_y' maintain no constant relation either across countries or over time.

The practical implications for intertemporal productivity comparisons follow directly from the trends observed in Table 1. The U.S. ratio doubled between 1969 and 1977. During the same period, the ratios for Italy and France increased by 50 percent and 33 percent, respectively, while the ratio increased by only 5 percent in the Netherlands. If one accepts the premise that aggregate economic growth should be evaluated in a delivery-to-final-demand framework, then one concludes that the value-added model produces a not only upward biased but also increasingly upward biased measure of postwar aggregate economic performance. The extent of the intertemporal bias depends on the change over time in a nation's dependence on imported inputs.[18]

The consequences for international productivity comparisons are equally important. Table 2 reports both rates of aggregate productivity growth for the eight nations. As expected, the differential is small for the United States while large for most other nations--most notably, the Netherlands. Conclusions regarding relative productivity growth across nations may be

Table 1

Current Price Ratios of Imported Inputs to Deliveries-to-Final-Demand[a]

	1950[b]	1953	1957	1960	1966	1969	1973	1977[c]
Canada	.196	.187	.198	.184	.202	.184	.186	.194
France	.140	.136	.139	.132	.131	.136	.153	.181
Germany	.109	.115	.159	.151	.162	.151	.147	.181
Italy	.119	.116	.128	.135	.138	.148	.181	.222
Japan	.114	.118	.131	.107	.101	.093	.097	.121
Netherlands	.326	.303	.329	.318	.308	.296	.300	.311
United Kingdom	.185	.184	.178	.178	.160	.174	.195	.229
United States	.036	.033	.035	.034	.037	.040	.053	.081

[a]Current price imported inputs are defined as the product of gross imports reported in OECD (1970, 1979) and the ratio of imported inputs to total inputs calculated from data reported in U.N. Department of International Economic and Trade Affairs (1978). Deliveries to final demand equal the sum of national income (including subsidies but excluding indirect business taxes) and imported inputs. All data are stated in national currencies.

[b]The ratios for Italy and Japan refer to 1951 and 1952, respectively.

[c]The ratio for Japan refers to 1976.

Table 2

Average Annual Growth Rates
of Aggregate Productivity Growth: 1957-1973

	Value-Added Model[a] E_v	Delivery-to-Final-Demand Model E'_y
Canada	.0158	.0127
France	.0306	.0261
Germany	.0308	.0260
Italy	.0326	.0275
Japan	.0410	.0363
Netherlands	.0233	.0159
United Kingdom	.0199	.0162
United States	.0123	.0117

[a]The average annual growth rates reported for the value-added model are calculated from the growth rates reported in Christensen, Cummings, and Jorgenson (1980), Table 11.10.

biased by a value-added model, especially comparisons involving nations with very different imported input requirements.

The summary conclusion of this paper is that the delivery-to-final-demand model provides the appropriate framework in which to measure and analyze aggregate productivity performance. It produces measures which offer the most meaningful intertemporal and international productivity comparisons. Looking to future research, only it offers a framework for evaluating regional productivity growth among trading nations.[19] Stated alternatively, just as <u>intersectoral</u> transactions are an important component of sectoral models of productivity growth, so too <u>international</u> transactions are an important component in an aggregate model of productivity growth. Excluding trade at the aggregate level is analogous to excluding intermediate input at the sectoral level.

Notes

1. Kendrick (1973), p. 16

2. *Ibid*.

3. This condition will be demonstrated formally in section 1.

4. Deliveries to final demand include conventional goods and services as well as less tangible but no less important products like health, safety, and environmental quality.

5. For a detailed discussion of value-added separability, see Gollop (1979), pp. 320-22.

6. Note that the intermediate input prices p_i do not have j subscripts. It is assumed that there is no price discrimination across sectors in the market for intermediate inputs. All sectors pay the same f.o.b. price for the same intermediate input.

7. Note that there is no requirement that the price of the jth sector's deliveries to final demand equal the price of that sector's deliveries to intermediate demand.

8. This result will be stated formally as part of Proposition 3 below.

9. Imports that are delivered directly to domestic final demand are not included in the model since they bear no influence on domestic productivity measurement.

   All exports are considered deliveries to final demand. Exports are goods and services produced domestically and no longer consumed internally. In a regional economic model, national exports delivered to final and intermediate demands would be distinguished. In a national model, there is no more need to make this distinction than there is to distinguish sectoral output by destination in a sectoral model.

10. Note that domestically produced intermediate inputs enter the problem through the production constraints (33). At the economy-wide level, however, all sales and purchases of domestically produced intermediate inputs are offsetting transactions.

11. The fact that the input shares in aggregate output sum to unity can be verified from the sectoral accounting identity:

$$\sum_i p_j X_{ji} + q_j^Y Y_j = \sum_\ell p_{\ell j} L_{\ell j} + \sum_k p_{kj} K_{kj} + \sum_i p_i X_{ij} + \sum_m p_m M_{mj}.$$

   Summing the identity over all n sectors produces the result required by (36):

$$\sum_j q_j^y Y_j = \sum_j \sum_\ell p_{\ell j} L_{\ell j} + \sum_j \sum_k p_{kj} K_{kj} + \sum_j \sum_m p_m M_{mj} .$$

Note that import prices p_m do not have j subscripts. It is assumed that all domestic sectors pay the same f.o.b. price for identical imported materials.

12. Competitive equilibrium conditions at the sectoral level require that output elasticities with respect to inputs equal the corresponding value shares of input payments in sectoral output:

$$\frac{\partial \ln L_{\ell j}}{\partial t} = \frac{p_{\ell j} L_{\ell j}}{q_j^v V_j} \qquad (\ell = 1,2,\ldots,r;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln K_{kj}}{\partial t} = \frac{p_{kj} K_{kj}}{q_j^v V_j} \qquad (k = 1,2,\ldots,s;\ j = 1,2,\ldots,n).$$

Since the function V^j exhibits constant returns to scale, the output elasticities and the value shares both sum to unity.

13. Note that the variable defining the sectoral rate of productivity growth is subscripted with "z," not "y." Subject to production and market constraints, sectoral producers maximize gross output Z_j, not deliveries to final demand Y_j.

14. Market conditions for producer equilibrium require:

$$\frac{\partial \ln L_{\ell j}}{\partial t} = \frac{p_{\ell j} L_{\ell j}}{q_j Z_j} \qquad (\ell = 1,2,\ldots,r;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln K_{kj}}{\partial t} = \frac{p_{kj} K_{kj}}{q_j Z_j} \qquad (k = 1,2,\ldots,s;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln X_{ij}}{\partial t} = \frac{p_i X_{ij}}{q_j Z_j} \qquad (i = 1,2,\ldots,n;\ j = 1,2,\ldots,n)$$

$$\frac{\partial \ln M_{mj}}{\partial t} = \frac{p_m M_{mj}}{q_j Z_j} \qquad (m = 1,2,\ldots,u;\ j = 1,2,\ldots,n).$$

Linear homogeneity requires that both the output elasticities and the value shares each sum to unity.

15. Note that even if the jth sector imports no foreign-produced inputs ($\sum_m p_m M_{mj} = 0$), the measure e_v^j is still greater than e_z^j. The relation between e_v^j and e_z^j in a "closed" economy is discussed in Gollop (1979).

16. Gollop (1979), pp. 330-31.

17. See note 4.

18. The increasing U.S. dependence on imported inputs is neither restricted to a few producing sectors nor is it limited to the post-1973 era. Gollop and Roberts (1981) study the importance of imported inputs to 21 two-digit manufacturing industries over the 1948-1973 period. They find that the cost share of imported materials in total production cost increased in 17 of 21 sectors. More importantly, they find that this shift in cost shares is the result of real input usage, not simple price effects. The average annual rate of growth in real imported materials across all manufacturing industries is 6.7 percent. The average among durable goods industries is 8.8 percent. The rate of growth in imported input exceeds the growth rate for capital and labor inputs and even domestic materials in sixteen of twenty-one manufacturing sectors.

19. A well-designed model of regional productivity would examine how efficiently the regional economy uses inputs under its control to produce output for regional final demand and foreign markets. The model would consider not only labor, capital, and nationally produced and consumed intermediate inputs but also all intra-regional imports and exports and all regional imports of foreign-produced material inputs.

References

Christensen, Laurits R., Dianne Cummings, and Dale W. Jorgenson (1980). "Economic Growth, 1947-73: An International Comparison," in J.W. Kendrick and B.N. Vaccara (eds.), New Developments in Productivity Measurement and Analysis. Chicago: University of Chicago Press for the NBER.

Christensen, Laurits R. and Dale W. Jorgenson (1973). "Measuring the Performance of the Private Sector of the U.S. Economy, 1929-1969," in M. Moss (ed.), Measuring Economic and Social Performance. New York: National Bureau of Economic Research, 233-338.

Denison, Edward F.(1962). Sources of Economic Growth in the United States and the Alternatives Before Us. New York: Committee for Economic Development.

_______________ (1974). Accounting for United States Economic Growth, 1929-1969. Washington, D.C.: Brookings Institution.

_______________ (1979). Accounting for Slower Economic Growth: The United States in the 1970s. Washington, D.C.: Brookings Institution.

Gollop, Frank M. (1979). "Accounting for Intermediate Input: The Link Between Sectoral and Aggregate Measures of Productivity Growth," in A. Rees and J. Kendrick (eds.), The Measurement and Interpretation of Productivity. Washington, D.C.: National Academy of Sciences.

Gollop, Frank M. and Mark J. Roberts (1981). "Imported Intermediate Input: Impact on Sectoral Productivity in U.S. Manufacturing," in N. Adam and A. Dogramaci (eds.), Aggregate and Industry Level Productivity Analysis. Boston: Martinus Nijhoff.

Jorgenson, Dale W. and Zvi Griliches (1967). "The Explanation of Productivity Change," The Review of Economic Studies, 34 (July), 249-83.

Kendrick, John W. (1961). Productivity Trends in the United States. National Bureau of Economic Research. Princeton: Princeton University Press.

_______________ (1973). Postwar Productivity Trends in the United States, 1948-1969. New York: National Bureau of Economic Research.

Kendrick, John W. and Elliot S. Grossman (1980). Productivity in the United States: Trends and Cycles. Baltimore and London: The Johns Hopkins University Press.

Organization for Economic Cooperation and Development (1970). National Accounts of OECD Countries, 1950-1968. Paris: OECD.

____________ (1979). National Accounts of OECD Countries, 1952-1977. Paris: OECD.

Solow, Robert M. (1957). "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39 (3), 312-320.

United Nations Department of International Economic and Social Affairs (1978). Yearbook of International Trade Statistics 1977. New York: United Nations.