Disclosing Effort in Dynamic Team Contests with Effort Complementarity

Maria Arbatskaya and Hideo Konishi*

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Abstract

This paper studies strategic effort disclosure in dynamic team contests. Two teams of two players compete in a Tullock contest with teammates' sequential efforts aggregated across two periods using a Cobb-Douglas production function. We examine how equilibrium efforts and winning probabilities are affected by the teams' communication policies: no communication, private communication (efforts shared internally with stage-2 teammates), and public communication (efforts disclosed to stage-2 rivals as well. To describe asymmetric information generated by privately observable efforts for each communication policy profile, we use perfect Bayesian equilibrium with an appropriate belief refinement for multi-stage complete information games. In the unique positive-effort equilibrium, the optimal choice of a communication strategy differs for the favorite (the strong team) and the underdog (the weak team). Private communication only benefits the underdog team, fostering effort complementarity and improving their chances of winning and payoffs. In contrast, the favorite team prefers either public or no communication to deter rival efforts or avoid intra-team free-riding. Importantly, endogenous communication policies reshape competitive dynamics, with private disclosure of efforts serving as a strategic equalizer for weaker teams.

Keywords: team contest, complementarity in efforts, communication policy, commitment, information.
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Maria Arbatskaya, Department of Economics, Emory University, Atlanta, GA 30322. Email: marbats@emory.edu.
Hideo Konishi, Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467. Email: hideo.konishi@bc.edu.

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1 Introduction

Competitions in business, sports, and legal battles have teams engaged in contests to win prizes. In these and similar settings, team members are assigned specific complementary tasks to fulfill the team's project. For instance, consider a market for a new product with multiple entrants striving to be a dominant firm. Each firm has product development and marketing divisions, and cooperation between the divisions is important in competing with the rivals. Industries that heavily rely on research and development (R&D) and marketing efforts often require a number of sequential efforts by separate units within the firm before a new product is developed, patented, and successfully marketed. Similarly, in sports or legal battles, teams may have tasks that must be executed in a specific order – such as hiring agents, training or going through discovery, and then performing on the field or in a courtroom. Questions arise for such team contests: How would the choice of communication policy affect the equilibrium efforts of all players when players' efforts in earlier stages can be disclosed privately (internally within teams) or publicly (openly across teams)? How should team managers choose a communication policy for their team to maximize the team's winning probability in the contest?

Our study focuses on the strategic management of private and public communication among competing teams. Private communication encompasses the flow of information between members of the same team, utilizing channels like emails, memorandums, and meetings to enhance coordination, potentially leading to higher efforts and chances of winning for a team. Public communication involves interactions outside the organization including press conferences, company websites, and social media posts. From a strategic standpoint, public communication could be viewed as a team "staking their claim" or showing their strengths to disincentivize other teams from exerting more effort.

To illustrate the importance of the strategic choice of private versus public information policies by a company, consider the technological development of smartphones. As cell phones became commonplace in the early 2000s, there was a race within the industry to incorporate cutting-edge features and capture a larger share of the market. Although Apple was a dominant member in the computer industry, it had not yet joined the cellphone fight with Blackberry, Nokia, and Motorola (among others). Before the first iPhone was released in 2007, Apple's CEO, Steve Jobs, worked to keep development under wraps.[1] For five years, he (and Apple more broadly) systematically denied all public speculation about what a phone release from Apple could entail. Although this information did eventually go public in January of 2007, prior to that, there was certainly private communication shared between engineers, developers, and managers about prototypes and progress related to the creation of the iPhone. This case study brings us to our central question: Was it strategically advantageous for Apple to keep the information private until the iPhone was developed? As the "underdogs" (weak teams) within the cellphone space, would they have benefited from sharing information publicly about their progress?

Apple's strategy runs counter to other examples where established players use public information disclosure to discourage rivals from investing in certain technological paths or products. For example, Tesla's CEO, Elon Musk, often communicates openly through social media, press releases, and public statements about the company's progress on its projects. This transparency may serve as a commitment device, potentially deterring or discouraging competitors. There is evidence that large pharmaceutical companies use public disclosures of their progress in drug development to strategically discourage rivals' investments in areas where they advance in their pipelines. For example, Kao (2024) finds that the approval of a competitor's drug reduces the likelihood of a firm reporting its own clinical trial results by 13%, suggesting that selective disclosure is used strategically in dynamic patent races. Our paper shows that for the underdog (or a new entrant), keeping information private, as Apple did,

[1] An article from the New York Times reads: "Certainly, Apple's push into the market for a hand-held communicator would be an abrupt departure for Mr. Jobs, who continues publicly to disavow talk of such a move. However, analysts and people close to the company say that the plan is underway and that the evidence is manifest in the features and elements of the new version of the Macintosh operating system." (Markoff, 2002).

is the optimal information strategy, whereas the favorite (an industry leader) would rather use public communication.

To understand the incentives for teams to engage in different types of communication policies, we develop a model of team competition with complementary efforts and a Tullock contest success function. Team members' efforts are complementary in the sense that the effort of one player on a team enhances the impact of their teammate's effort. We build on the models of Lu and Lu (2020) and Arbatskaya and Konishi (2023), who explore two-period team contests with a Cobb-Douglas effort-aggregation function and publicly observed efforts.[2] We restrict effort to be unobserved as a default case unless team(s) choose to reveal effort levels exerted by earlier teammates through internal or external communication. This allows us to investigate the impact of different communication policies on a team's effort levels and probability of winning. In addition, team members are allowed to have heterogeneous marginal costs of effort, and teams' strengths could be different. This allows for situations where teams have similar power, as well as for lopsided competitions with a clear underdog and a favorite. We explore whether the communication policy chosen can change that power imbalance, allowing the start-up company a greater probability of winning.

In order to analyze the impact of different communication policies, we need to have the correct equilibrium concept to describe players' beliefs and equilibrium behaviors. Although our game is not a game of incomplete information, we adopt the Perfect Bayesian Equilibrium (PBE) to describe players' beliefs about other players' unobservable actions. Using the suitable refinement of beliefs (see below), we can characterize the equilibrium strategy profile consistent with what players can observe.

We find that a weak team (underdog) benefits from private communication among its members because it fosters collaboration. Conversely, a strong (favorite) team may not benefit from private (internal) communication because the high effort from early players creates an adverse strategic effect (free-riding in-

[2] A Cobb-Douglas aggregation function allows us to utilize elasticity formulas developed in Arbatskaya and Konishi (2023), which help us to obtain clear-cut results that can be compared with the past literature.

centives) for team members in the second period. Stronger teams benefit more from maintaining external communication channels, such as organizing press conferences or sharing progress updates publicly. This insight suggests that industry leaders will likely benefit from broad public communication strategies, whereas underdogs may find a strategic advantage in focusing on private communications.

The following section reviews related literature. Section 3 presents the dynamic team contest framework we employ – a two-period, two-team, two-player contest model. Section 4 describes the equilibrium concept used in our multi-stage complete information game – the Perfect Bayesian Equilibrium (PBE) with "no-signaling-past-actions-you-do-not-know" (NSPAYDK) that is an appropriate modification of the "no-signaling-what-you-do-not-know" (NSWYDK) condition for PBEs in incomplete information games (see Fudenberg and Tirole, 1991). Section 5 provides a preliminary analysis, offering useful tools for investigating the contest outcomes for each communication policy configuration. Section 6 describes the outcomes in equilibrium for contests with public and private communication of efforts by teams. In Section 7, we endogenize the choice of communication policy by team managers seeking to maximize the team's odds of winning and show that it is a dominant strategy for the underdog team's manager to choose private communication and for the favorite team's manager to choose either public or no communication. Some extensions to our benchmark model are offered in Section 8, and Section 9 concludes. Appendix collects the proofs not found in the main text.

2 Literature Review

There are three branches of the literature that are most closely related to our paper: dynamic contests with observable and unobservable actions, team (group) contests, and player communication in contests.

Harris and Vickers (1987) considered an R&D race as a dynamic contest by individual players, analyzing how an initial lead by a team affects the subsequent race. Klumpp and Polborn (2006) asked the same question using the Tullock contests in the context of the US presidential primary races. Konrad and Kovenock (2009) analyzed the dynamic race more generally using all-pay auctions. All of these studies assumed that the earlier results were *publicly* observable and found that the race favors the player who gained a lead in the initial stages. Hinnosaar (2023) studied an n-player contest design problem in which each player moves once in an arbitrary sequence, allowing for some synchronous moves (i.e., simultaneous moves of a subset of players). Assuming that earlier players' efforts are observable by later players, he investigated the sequential contest that maximizes the total effort. He found that the optimal contest is fully sequential with no synchronous move (and every player's move being observable).

Lu and Lu (2020) is the first paper to analyze a dynamic team contest with complementary efforts. The main result from Lu and Lu (2020) is that the order in which tasks are performed in team contests does not change the equilibrium efforts as long as the tasks are chosen synchronously.[3] Arbatskaya and Konishi (2023) took the model by Lu and Lu (2020) and allowed for various sequential moves by players in two teams.[4] They showed that in contrast with synchronous contests in Lu and Lu (2020), in asynchronous contests, there are strategic incentives for a player who moves in stage 1 to commit to a higher (lower) effort if that player is part of a stronger (weaker) team. In Lu and Lu (2020) and Arbatskaya and Konishi (2023), earlier players' moves are assumed to be completely observable by subsequent players. The current paper takes the model of Lu and Lu (2020) but allows for privately observable players' moves. The introduction of private information complicates players' reactions, and to solve the model, we adopted Perfect Bayesian Equilibrium with NSPAYDK belief refinement as our solution concept.

[3] In contrast, in contests where players choose efforts in multiple tasks, Arbatskaya and Mialon (2012) showed that when some tasks are chosen before others, players will choose them in a way that relaxes the competition in the second stage. This cost-saving strategic incentive is absent when different team members perform tasks, each maximizing their payoff.

[4] Arbatskaya and Konishi (2023) also extended the result by Lu and Lu (2020) to any arbitrary number of synchronous moves.

The literature on team contests uses a variety of team production functions in order to describe the complementarity of team members' efforts. Chowdhury et al. (2016) assumed that a team's output is determined by the minimum contributions made by its members (the weakest link).[5] Kolmar and Rommswinkel (2013), Choi at al. (2016), and Konishi and Pan (2020, 2021) assumed a CES production function to describe complementarity in team production.[6] As was mentioned before, Cobb-Douglas production functions were used by Arbatskaya and Mialon (2010, 2012) and Lu and Lu (2020). Unlike other papers, these papers analyze dynamic team contests in which team members may exert efforts sequentially. Although players' optimization problems in such problems are generally highly complicated, a Cobb-Douglas production function provides tractable analytical results that are interpretable.

There is literature on strategic communication in contests, but the literature mainly offers models with players having privately known types that they can communicate. Also, apart from experimental papers on team contests, the literature is on contests by individual players.[7] Our paper is unique because we analyze team contests that involve communicating verifiable information about earlier players' moves to subsequently moving players.[8]

Fu et al. (2013) explore the strategic effects of a "message of confidence" communication by a player in a two-player Tullock contest. Sending a (publicly observable) message is assumed to incur a cost if the sender loses the contest,

[5] Assuming that team members' privately informed valuations of the prize (or marginal costs of effort), Barbieri and Malueg (2016) and Barbieri et al. (2019) analyzed how group size differences affect the competitiveness of contests assuming "best-shot" (the maximum contribution) and "weakest link" technologies, respectively. Barbieri and Topolyan (2021) considered a weighted sum of best-shot and weakest-link to analyze the degree of effort complementarity and competitiveness of teams of different sizes.

[6] For more comprehensive surveys of effort aggregation functions, see Konrad (2009) and Fu and Wu (2019).

[7] There are experimental studies on group contests (Sutter and Strassmair 2009, and Leibbrandt and Sääksvuori 2012). However, they ask whether or not non-binding communication (cheap talk) can help players coordinate their actions (and show that it does). In contrast, in our paper, players reveal their earlier efforts to subsequent movers, and these messages are assumed to be trustworthy (verifiable). We are interested in how private and public communication policies affect players' incentives to exert effort.

[8] Kempf and Graziosi (2010) consider leadership in a public good provision game. Players' actions are verifiable in their model, too, but their model does not have competing teams, and therefore, the logic behind the results is very different.

and it works as a commitment device, affecting the behavior of competitors. The strategic discouragement effect on the rival benefits the stronger player. Our paper has a similar discouragement effect on a strong team's rival. However, since our model is that of team contests, there is an additional strategic effect on teammates. In fact, internal communication within teams allows the underdog team to benefit from strategic interaction between teammates.

There are studies that address strategic incentives to disclose private information in all-pay auctions. Ewerhart and Lasreida (2024) examine two-sided incomplete information contests. They show that full disclosure of private information by contestants is generally the unique perfect Bayesian equilibrium outcome. In contrast, Kovenock et al. (2015) show that sharing information is strictly dominated in the context of an independent private value auction. In their model, information sharing happens before the contest, and they show that in the case of independent values, information sharing reduces efforts. Kubitz (2023) shows that in a perfectly discriminating contest, weak players should choose to appear stronger, and strong players should appear weaker when sharing private values. These results are for players with private information about their types. In contrast, in our model, the player's type (marginal cost of effort) is publicly known (common knowledge).[9]

3 Model

Consider a contest among two teams, $i = 1, 2$, each having two members $j = 1, 2$ responsible for performing task j. We will refer to team i's player j as player ij. Team members independently choose their effort levels e_{ij}; $i = 1, 2$ and $j = 1, 2$. Throughout the paper, we will assume that the first player on each team (player $i1$) moves in stage 1, and the second player on each team (player $i2$) moves in stage 2. Players' efforts contribute to their team's chances of winning a prize,

[9] There is literature on information disclosure and unraveling, e.g., Dranove and Jin (2010) and Okuno-Fujiwara et al. (1990). However, our setting differs in that players' types are common knowledge, and the strategic uncertainty is about actions, not types. Full unraveling does not occur because sender payoffs are not monotonic in receiver beliefs, and the contest structure introduces strategic interaction among receivers.

which is the public good with a value normalized to 1. Team i members' efforts are aggregated using production (effort-aggregation) function f to produce an impact X_i, which is assumed to be a Cobb-Douglas function

$$X_i = f(e_{i1}, e_{i2}) = e_{i1}^{\alpha_1} e_{i2}^{\alpha_2}, \tag{1}$$

where $\alpha_j \in (0,1)$ is the discriminatory power of task j that captures the sensitivity of the team's impact to individual player effort e_{ij}. With this Cobb-Douglas specification of each team's impact, we will obtain clear-cut results in our model.

We denote by X the aggregate impact of both teams, $X = X_1 + X_2$, and by $X_{-i} = X - X_i$ the impact of the rival team. The winning probability of team i is described as a Tullock's (logit) contest success function with a standard tie-breaking rule:

$$P_i = \begin{cases} \frac{X_i}{X_i + X_{-i}} & \text{if } X_i + X_{-i} > 0 \\ \frac{1}{2} & \text{if } X_i = X_{-i} = 0 \end{cases} \tag{2}$$

thus, when $X_i + X_{-i} > 0$, we can rewrite the formula as

$$P_i = \frac{X_i}{X_i + X_{-i}} = \frac{\Theta_i}{1 + \Theta_i}, \tag{3}$$

where $\Theta_i = \frac{X_i}{X_{-i}}$ is the *relative power* of team i with $\Theta_2 = 1/\Theta_1$. Player ij has the cost of effort described by $c_{ij}e_{ij}$, with $c_{ij} > 0$. The expected payoff of player ij is

$$U_{ij} = P_i - c_{ij}e_{ij} = \frac{\Theta_i}{1 + \Theta_i} - c_{ij}e_{ij}, \tag{4}$$

and each player ij chooses their effort level e_{ij} to maximize U_{ij}. We assume that all of the above is common knowledge among all players.

Both players on a team are subject to one of three possible team *communication policies*: no communication, private communication, and public communication. When team $i = 1, 2$ has the "no communication" policy ($a_i = 0$), then the effort level by the first player of team i (player $i1$) will not be known by any other players. When team i has the "private communication" policy ($a_i = 1$), then player $i1$'s effort e_{i1} will be observed only by the teammate, player $i2$. When team i has "public communication" policy ($a_i = 2$), then player $i1$'s effort e_{i1} will be observed by both stage-2 players $i2$ and $k2$ but cannot be observed by player $k1$, who moves at stage 1 ($k \neq i$). Let a communication policy profile of the teams be (a_1, a_2). That is, $a = (a_1, a_2) = (0, 0)$ corresponds to no communication, which makes the contest a one-stage game, while $a = (2, 2)$ corresponds to the case of public communication of stage-1 efforts in both teams.

We will compare the equilibrium efforts of players and chances of winning for the teams under nine possible communication policy profiles: $a \in \{(0, 0),$ $(0, 1),\ (1, 0),\ (1, 1),\ (2, 0),\ (0, 2),\ (1, 2),\ (2, 1),\ (2, 2)\}$. Throughout the paper, we refer to a contest by its communication policy profile - as contest $C(a_1, a_2)$. After comparing nine contests, we introduce manager i for each team $i = 1, 2$. A team manager is neither player $i1$ nor $i2$; they are fictitious players whose whole purpose is to choose a communication policy for their team to maximize their team's winning probability (as they are rewarded only when their team wins). Managers $i = 1, 2$ simultaneously choose communication policies $a_i \in \{0, 1, 2\}$ before players $ij \in \{11, 12, 21, 22\}$ participate in the contest. The model assumes that team managers can credibly commit to a communication policy before players engage in the contest. This is justified in settings where communication channels (e.g., internal messaging systems) are observable and verifiable by all parties or where organizational rules and monitoring ensure compliance (e.g., with disclosure regulations). After discussing the equilibrium concept and analyzing equilibria in the next three sections, we will investigate the selection of communication policies Section 7.

4 Equilibrium Concept

Here, we will explain the equilibrium concept we employ in this paper. Unlike Lu and Lu (2020) and Arbatskaya and Konishi (2023), we will analyze multi-stage games where players are imperfectly informed about other players' actions due to private communication policies. To address such asymmetric information games, we employ the concept of perfect Bayesian equilibrium in pure strategies (PBE) with an *appropriate* belief refinement. Although there is no incomplete information in the game, we will use PBE to describe imperfect information over e_{11} and e_{21}. This is because the information structure can be very complicated under different communication policies, and PBE is the appropriate equilibrium concept to describe how players make decisions, as it appropriately defines what their beliefs depend on.

In the standard PBE under incomplete information, each player has beliefs over other players' type space. In our application of PBE to a game under complete information, players do not have "types." However, we can apply PBEs to describe the equilibrium allocations with beliefs over "actions" under different communication structures. We use PBE with an appropriate refinement of beliefs over *past* actions, which is an adaptation of the "no-signaling-what-you-do-not-know" condition for incomplete information games (NSWYDK: see Fudenberg and Tirole 1991 page 332) to multi-stage complete information games.[10] We call this belief refinement "no-signaling-past-actions-you-do-not-know" (NSPAYDK).[11] The NSPAYDK belief refinement requires that stage-1 player $i1$'s action e_{i1} does not affect a stage-2 player's belief over the other stage-1 player's action $e_{i'1}$ $(i \neq i')$ by being informed about e_{i1}, since players $i1$ and $i'1$ play simultaneously and player $i1$ does not have a chance to learn $e_{i'1}$. This condition shuts off all communication channels except those specified in the communication policy profile. In the next section, we will represent communication policy profile with a directed graph and describe how PBEs with NSPAYDK (in pure strategies) are defined for two specific illustrative communication policy profiles.

[10] Fudenberg and Tirole (1991) introduced NSWYDK as one of the requirements for PBE in their definition of PBE for incomplete information games (pages 332-333). Nitzan and Ueda (2011) refined PBEs by applying NSWYDK to their complete information games. See also Crutzen et al. (2024a), Crutzen et al. (2024b), and Kobayashi et al. (2025). However, our games allow for more complex information flows, and the NSWYDK condition may not be appropriate in some cases, e.g., in contest $C(1, 2)$ below.

[11] We thank Mehmet Ekmekci for his valuable suggestions on reasonable belief refinement conditions of PBEs for multi-stage complete information games.

4.1 Directed Graph Representations of Communication Policies and NSPAYDK

In this subsection, we will consider the implications of NSPAYDK belief refinements in contests $C(1,1)$ and $C(1,2)$. To characterize player beliefs, we need to describe the information flow about players' actions to other players moving later. For this purpose, it is helpful to use directed graphs on the set of players.

First, consider contest $C(1,1)$. The actions of players 12 and 22 are affected by the actions of players 11 and 21, respectively. Thus, in contest $C(1,1)$, the information flow graph is described as:

$$\begin{array}{r|l} \text{team } 1 & \text{player } 11 \quad \longrightarrow \quad \text{player } 12 \\ & \\ \text{team } 2 & \text{player } 21 \quad \longrightarrow \quad \text{player } 22 \end{array} \qquad (5)$$
$$\text{Contest } C(1,1)$$

Player 12 does not know e_{22}, so player 12 will have a belief about X_2. Player 11 knows neither e_{21} nor e_{22} and so has a belief about X_2, too. Players 11 and 12 have beliefs $X_2^{\mu_{11}}$ and $X_2^{\mu_{12}}$ about X_2, and $X_2^{\mu_{12}}$ does not depend on e_{11} due to NSPAYDK, since player 11 does not know e_{21} simultaneously chosen by player 21. In addition, for beliefs to be consistent with the actual actions in equilibrium, $X_2^{\mu_{11}} = X_2^{\mu_{12}} = X_2$ must hold in equilibrium. Player 11's choice of e_{11} can only affect player 12's choice of e_{12}, who assumes $X_2^{\mu_{12}} = X_2$ as given when she responds to any e_{11}. Let $\beta_{12}(e_{11}; X_2^{\mu_{12}}) \equiv \arg\max_{e_{12}} U_{12}(e_{12}; e_{11}, X_2^{\mu_{12}})$. Player 11's belief over player 12's action $e_{12}^{\mu_{11}} : e_{11} \longmapsto e_{12}$ given $X_2^{\mu_{11}}$ should satisfy $e_{12}^{\mu_{11}}(e_{11}; X_2^{\mu_{11}}) = \beta_{12}(e_{11}; X_2^{\mu_{12}})$ for all e_{11}. Player 11 rationally expects player 12's reaction when choosing e_{11}.

Similarly, player 22 can observe e_{21}, but player 22's belief about e_{11} is not affected by e_{21} due to NSPAYDK. By the same argument, $X_1^{\mu_{21}} = X_1^{\mu_{22}} = X_1$ must hold in equilibrium. Let $\beta_{22}(e_{21}; X_1^{\mu_{22}}) \equiv \arg\max_{e_{22}} U_{22}(e_{22}; e_{21}, X_1^{\mu_{22}})$. Player 21's belief over player 22's reaction to e_{21}, $e_{22}^{\mu_{21}} : e_{21} \longmapsto e_{22}$ given $X_1^{\mu_{22}}$, should satisfy $e_{22}^{\mu_{21}}(e_{21}; X_1^{\mu_{21}}) = \beta_{22}(e_{21}; X_1^{\mu_{22}})$ for all e_{21}. Player 21 rationally expects player 22's reaction when choosing e_{21}. Finally, for consistency in equilibrium, $X_1 = e_{11}^{\alpha_1} e_{12}^{\alpha_2}$ and $X_2 = e_{21}^{\alpha_1} e_{22}^{\alpha_2}$ must hold.

What if the NSPAYDK condition is violated in this case? We may discuss this using the following example of contest $C(1,1)$. Suppose that the contest prize equal to 1, $\alpha_1 = \alpha_2 = \frac{1}{2}$, and $c_{11} = c_{12} = c_{21} = c_{22} = 1$. Under NSPAYDK, in each team i, player $i2$'s belief about $e_{i'1}$ does not depend on e_{i1}, and her belief about $e_{i'2}$ also cannot depend on e_{i1} ($i' \neq i$). This means that player $i2$ takes $X_{i'}$ as given, and player $i1$ also takes $X_{i'}$ as given. As we will see in the next sections, we have $e_{ij} = 0.125$ and $U_{ij} = 0.375$ for all $i, j = 1, 2$ as the equilibrium outcome under NSPAYDK in each contest $C(a_1, a_2)$. In contrast, if NSPAYDK is not imposed, we can allow the following kind of unreasonable beliefs in contest $C(1,1)$: for each team i and i' with $i' \neq i$, player $i2$'s belief $e^{\mu_{i2}}_{i'1} : e_{i1} \longmapsto e_{i'1}$ is (i) $e^{\mu_{i2}}_{i'1}(e_{i1}) = a$ if $e_{i1} = a$, and (ii) $e^{\mu_{i2}}_{i'1}(e_{i1}) = 10000$, otherwise. In this case, unless player $i1$ chooses $e_{i1} = a$, player $i2$ believes $e^{\mu_{i2}}_{i'1}(e_{i1}) = 10000$, and thinks that there is almost no chance for team i to win, making her extend very limited effort. Thus, player $i1$ would choose $e_{i1} = a$ securing winning probability $\frac{1}{2}$. This scenario occurs as long as a is a reasonable positive number (with Pareto improvement over the NSPAYDK equilibrium occurring if $a < 0.125$). Thus, without the NSPAYDK condition, we cannot exclude multiplicity of equilibria.

Second, consider contest $C(1,2)$. In this contest, the information flow graph can be described as:

$$\begin{array}{r||lcl} \text{team 1} & \text{player 11} & \longrightarrow & \text{player 12} \\ & & \nearrow & \\ \text{team 2} & \text{player 21} & \longrightarrow & \text{player 22} \\ \multicolumn{4}{c}{\text{Contest } C(1,2)} \end{array} \tag{6}$$

This case can be analyzed in the same way as the previous case. Let us start with player 22's belief on e_{11}. Since player 21 does not know e_{11}, player 21's communication with player 22 on e_{21} does not alter player 22's belief $e^{\mu_{22}}_{11}$ on e_{11} by NSPAYDK. Player 22's belief over e_{12} is written as a function $e^{\mu_{22}}_{12} : e_{21} \longmapsto e_{12}$ given $e^{\mu_{22}}_{11}$, and player 12's belief over e_{22} is a function $e^{\mu_{12}}_{22} : e_{21} \longmapsto e_{22}$ given e_{11}.[12] Stage-2 players' reaction functions to stage 1 actions take these

[12] If we apply the NSWYDK condition of Fudenberg and Tirole (1991) *literally* to this multi-stage game by replacing "types" by "actions," then player 12's future action cannot be

beliefs into account, and they are written as

$$\beta_{22}(e_{11}^{\mu_{22}}, e_{21}) \equiv \arg\max_{e_{22}} U_{22}(e_{22}, e_{12}^{\mu_{22}}(e_{21}; e_{11}^{\mu_{22}}); e_{11}^{\mu_{22}}, e_{21})$$

and

$$\beta_{12}(e_{11}, e_{21}) \equiv \arg\max_{e_{12}} U_{12}(e_{12}, e_{22}^{\mu_{12}}(e_{21}; e_{11}); e_{11}, e_{21}).$$

Thus, stage-2 outcome (e_{12}, e_{22}) satisfies $(e_{12}, e_{22}) = (\beta_{12}(e_{11}, e_{21}), \beta_{22}(e_{11}^{\mu_{22}}, e_{21}))$ for each (e_{11}, e_{21}). In stage 1, player 21 expects the stage-2 players' reaction functions β_{22} and β_{12} correctly. Thus, player 21 chooses e_{21} to maximize $U_{21}(e_{21}; e_{11}^{\mu_{21}}, \beta_{12}(e_{11}^{\mu_{21}}, e_{21}), \beta_{22}(e_{11}^{\mu_{21}}, e_{21}))$. Player 11 also expects the stage 2 players' reaction functions β_{22} and β_{12} correctly, and she chooses e_{11} to maximize $U_{11}(e_{11}; e_{21}, \beta_{12}(e_{11}, e_{21}), \beta_{22}(e_{11}^{\mu_{22}}, e_{21}))$, where $e_{11}^{\mu_{22}}$ is treated as a constant due to missing communication channel. To be consistent, $e_{11} = e_{11}^{\mu_{22}}$ must hold in equilibrium. Throughout the paper, we will only consider PBEs with NSPAYDK. We will show that there always exists a unique positive-effort pure strategy PBE with NSPAYDK in each contest $C(a_1, a_2)$.[13]

5 Preliminary Analysis using Elasticity Formulas

This section shows how the first-order conditions for equilibrium efforts can be written in terms of elasticities. Our elasticity formulas prove helpful for understanding players' strategic incentives in dynamic team contests with private and public communication policies.

In the dynamic game under a communication policy profile $a = (a_1, a_2)$, team i's member j chooses an effort level e_{ij} to maximize U_{ij} by taking her follower's reaction (if any) into account. Recalling $U_{ij} = P_i - c_{ij}e_{ij} = \frac{\Theta_i}{1+\Theta_i} - c_{ij}e_{ij}$, the

inferred by player 22 from the observation of player 21's past action even if e_{21} is known by both players 12 and 22, since player 21 would not know player 12's action and e_{21} cannot transmit information to player 22. This is the reason that we replaced NSWYDK (what you do not know) with NSPAYDK (past actions you do not know).

[13] We thank one of the referees for pointing out that the zero-effort equilibrium is (i) superior to a positive-effort equilibrium and (ii) robust in the sense that the NSPAYDK condition is satisfied.

first-order condition can be written as follows:

$$\frac{dU_{ij}}{de_{ij}} = \frac{1}{(1+\Theta_i)^2}\frac{d\Theta_i}{de_{ij}} - c_{ij} = 0. \tag{7}$$

Here, we introduce a key concept of our analysis, the *balance of power* of the teams in the contest:

$$\Lambda \equiv \frac{\partial P_i}{\partial \Theta_i}\Theta_i = P_1 P_2 = \frac{X_1 X_2}{(X_1+X_2)^2} = \frac{\Theta_i}{(1+\Theta_i)^2} \tag{8}$$

for $i = 1, 2$. The balance of power Λ is low in lopsided contests, and it reaches its maximum of $1/4$ in a symmetric contest, where the two teams have an equal chance of winning, $P_1 = P_2 = 1/2$.

Following Arbatskaya and Konishi (2023), we will analyze the games with different communication policies by using the first-order conditions in terms of elasticities. This approach is particularly helpful when using Cobb-Douglas production functions. Let $E_{\Theta_i,e_{ij}} = \frac{d\Theta_i}{de_{ij}}\frac{e_{ij}}{\Theta_i}$ be the elasticity of team i's (relative) power Θ_i with respect to effort e_{ij}. The capitalized $E_{\Theta_i,e_{ij}}$ signifies the *total elasticity*, where we are taking the *total* (not partial) derivative: $E_{\Theta_i,e_{ij}} = \frac{d\Theta_i}{de_{ij}}\frac{e_{ij}}{\Theta_i}$: that is, *player ij evaluates the effect of her effort choice e_{ij} by taking the reactions by successive movers (followers) into account, if there are any.* Thus, the first-order condition for player ij's effort in an elasticity form is as follows:

$$\frac{dU_{ij}}{de_{ij}} = E_{\Theta_i,e_{ij}}\Lambda\frac{1}{e_{ij}} - c_{ij} = 0 \tag{9}$$

or

$$E_{\Theta_i,e_{ij}}\Lambda = c_{ij}e_{ij}. \tag{10}$$

The expenditures (cost of effort) by player ij are equal to the total elasticity of team i's (relative) power with respect to effort e_{ij} times the balance of power. The expenditures $c_{ij}e_{ij}$ are higher when the contest is more balanced and when the power of team i is more responsive to changes in player ij's effort level. Equation (10) holds for any player ij, with $E_{\Theta_i,e_{ij}}$ and Λ being functions of the (relative) power of team 1, Θ_1, that depends a communication policy profile. We make comparisons in terms of Θ_1 since $\Theta_2 = 1/\Theta_1$.

Notice that for stage-2 efforts, $E_{\Theta_i,e_{ij}} = \varepsilon_{\Theta_i,e_{ij}} = \alpha_j$ always holds, and this implies that

$$\alpha_2 \Lambda = c_{i2} e_{i2} \tag{11}$$

for $i = 1, 2$. Additionally,

$$\alpha_1 \Lambda = c_{i1} e_{i1} \tag{12}$$

for player i moving at stage 1 if that player's effort is not observable. The first-order conditions would not simplify this way for player $i1$ if some player(s) in stage 2 can observe e_{i1} before they choose their actions.

From equation (11), the followers' responses depend on the extent of the change in the balance of power, Λ, that the observed effort of a leader (say, player 11) brings about, and the responses to e_{11} are proportional for stage-2 players: $\varepsilon_{e_{12},e_{11}} = \varepsilon_{e_{22},e_{11}} = E_{\Lambda,e_{11}}$. Recall that $\Lambda = \Theta_1(1+\Theta_1)^{-2}$, so

$$\varepsilon_{e_{12},e_{11}} = E_{\Lambda,e_{11}} = \varepsilon_{\Lambda,\Theta_1} E_{\Theta_1,e_{11}} = \frac{1-\Theta_1}{1+\Theta_1} E_{\Theta_1,e_{11}} \tag{13}$$

where $\varepsilon_{\Lambda,\Theta_1} \in (-1, 1)$ for $\Theta_1 > 0$. We will have $E_{\Theta_1,e_{11}} > 0$ (the effort of player 11 is increasing team 1's power), and therefore a contest is balancing as effort e_{11} increases ($E_{\Lambda,e_{11}} > 0$) whenever team one is the underdog, $\Theta_1 < 1$.

To find $E_{\Theta_1,e_{11}}$, recall that $\Theta_1 = \left(\frac{e_{11}}{e_{21}}\right)^{\alpha_1} \left(\frac{e_{12}(e_{11})}{e_{22}(e_{11})}\right)^{\alpha_2}$ and notice that Θ_1 is a power function of e_{11}, e_{12}, e_{21}, and e_{22} with partial elasticities $\varepsilon_{\Theta_1,e_{12}} = \alpha_2$ and $\varepsilon_{\Theta_1,e_{22}} = -\alpha_2$. We find the following:

$$\begin{aligned} &E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2 \varepsilon_{e_{12},e_{11}} - \alpha_2 \varepsilon_{e_{22},e_{11}} \text{ if } a_1 = 2 && (14)\\ &E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2 \varepsilon_{e_{12},e_{11}} \text{ if } a_1 = 1 \\ &E_{\Theta_1,e_{11}} = \alpha_1 \text{ if } a_1 = 0. \end{aligned}$$

Using an indicator function $I(s) = 1$ when statement s is true and $I(s) = 0$ otherwise, we can write these equations compactly as

$$E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2 \varepsilon_{e_{12},e_{11}} I(a_1 \neq 0) - \alpha_2 \varepsilon_{e_{22},e_{11}} I(a_1 = 2) \tag{15}$$

and by similar arguments,

$$E_{\Theta_2,e_{21}} = \alpha_1 + \alpha_2 \varepsilon_{e_{22},e_{21}} I(a_2 \neq 0) - \alpha_2 \varepsilon_{e_{12},e_{21}} I(a_2 = 2). \tag{16}$$

where the first term in these expressions corresponds to the direct effect of an effort change, the second term is due to the reaction of the stage-2 teammate, and the third term is due to the reaction of the stage-2 rival.

To summarize, the first-order conditions for the stage-2 efforts, e_{12} and e_{22}, are $c_{12}e_{12} = \alpha_2\Lambda$ and $c_{22}e_{22} = \alpha_2\Lambda$, respectively. The ratio of stage-2-activity levels does not depend on e_{11} and e_{12} in equilibrium; it is just the ratio of the marginal costs:

$$\frac{e_{12}}{e_{22}} = \frac{c_{22}}{c_{12}}. \tag{17}$$

The first-order conditions for the stage-1 efforts e_{11} and e_{21} are

$$\begin{aligned} c_{11}e_{11} &= E_{\Theta_1,e_{11}}\Lambda \\ c_{21}e_{21} &= E_{\Theta_2,e_{21}}\Lambda, \end{aligned} \tag{18}$$

where $E_{\Theta_1,e_{11}}$ and $E_{\Theta_2,e_{21}}$ are the total elasticities of team i's (relative) power Θ_i with respect to the team's stage-one effort e_{i1}, as in equations (15) and (16). The ratio of stage-1 efforts e_{11} and e_{21} is then

$$\frac{e_{11}}{e_{21}} = \frac{c_{22}}{c_{12}}\frac{E_{\Theta_1,e_{11}}}{E_{\Theta_2,e_{21}}} \tag{19}$$

and the (relative) power of team 1 is

$$\Theta_1 = \left(\frac{e_{11}}{e_{21}}\right)^{\alpha_1}\left(\frac{e_{12}}{e_{22}}\right)^{\alpha_2} = \bar{\Theta}_1\left(\frac{E_{\Theta_1,e_{11}}}{E_{\Theta_2,e_{21}}}\right)^{\alpha_1} \tag{20}$$

with $\bar{\Theta}_1 \equiv \left(\frac{c_{21}}{c_{11}}\right)^{\alpha_1}\left(\frac{c_{22}}{c_{12}}\right)^{\alpha_2} > 0$.

Elasticities $E_{\Theta_1,e_{11}}$ and $E_{\Theta_2,e_{21}}$ depend on the communication policies of the two teams. When player i's effort is not observed by any other player, $E_{\Theta_i,e_{i1}} = \varepsilon_{\Theta_i,e_{i1}} = \alpha_1$. In the following sections, we compare the (relative) power of team 1 across contests, and since $\Theta_1 = 1/\Theta_2$, we can use $E_{\Theta_1,e_{i1}} = -E_{\Theta_2,e_{i1}}$ to make all the comparisons in terms of Θ_1 only.

In the next section, we will formally prove that for any costs $c_{ij} > 0$ and weights $\alpha_j \in (0,1)$ for $i,j = 1,2$, there exists a unique positive-effort PBE with

NSPAYDK under any communication policy profile $a = (a_1, a_2)$; $a_i \in \{0, 1, 2\}$. This solution uniquely determines team 1's equilibrium power Θ_1 (by equation (20)) and balance of power Λ in each contest; substituting the equilibrium Λ back to the first-order conditions, we can determine players' equilibrium effort levels e^*_{ij} uniquely, for any communication policy profile a. Therefore, we will be able to conclude that there is a unique positive-effort PBE with NSPAYDK under any communication policy profile $a = (a_1, a_2)$.

6 Perfect Bayesian Equilibria for All Communication Policies Profiles

This section compares team 1's equilibrium power (and probability of winning) across communication policy profiles. We start by examining contests with public communication of stage-1 efforts to show the equivalence of public and no communication policies for Cobb-Douglas production function and then show that the equivalence breaks down when private communication is allowed.

6.1 Equivalence Between Public Communication and No Communication

A two-stage contest with public information $C(2, 2)$ was examined by Lu and Lu (2020). They show the equivalence of efforts in $C(2, 2)$ and in a one-period (simultaneous-move) contest when effort-aggregation functions are the same Cobb-Douglas function. Since a one-period contest is equivalent to a two-period synchronous contest with no communication, $C(0, 0)$, we know that contests $C(2, 2)$ and $C(0, 0)$ are equivalent. In addition to contests $C(2, 2)$ and $C(0, 0)$, we consider contests $C(0, 2)$ and $C(2, 0)$ in which only one team communicates publicly. We can, therefore, prove a stronger equivalence result.

Proposition 1 (Public Communication and No Communication). *Under any communication policy profile $a = (a_1, a_2) \in \{(0, 0), (2, 0), (0, 2), (2, 2)\}$, we have the same outcome in the unique positive-effort PBE with NSPAYDK. The equilibrium power of team 1 equals team 1's relative cost advantage $\bar{\Theta}_1$.*

Equilibrium efforts are $e^*_{ij} = \frac{\alpha_j}{c_{ij}}\Lambda(\bar{\Theta}_1)$ *for all* $i, j = 1, 2$. *Team 1's equilibrium winning probability is* $P^*_1 = \frac{\bar{\Theta}_1}{1+\bar{\Theta}_1}$.

Proof. Consider contest $C(2,0)$, in which only team 1 communicates by publicly announcing stage-1 effort e_{11}. To find $E_{\Theta_1,e_{11}}$, recall that $\Theta_1 = \frac{X_1}{X_2} = \left(\frac{e_{11}}{e_{21}}\right)^{\alpha_1}\left(\frac{e_{12}}{e_{22}}\right)^{\alpha_2}$ and notice that Θ_1 is a function of e_{11}, e_{12}, e_{21}, and e_{22} with $\varepsilon_{\Theta_1,e_{11}} = \alpha_1$, $\varepsilon_{\Theta_1,e_{12}} = \alpha_2$, and $\varepsilon_{\Theta_1,e_{22}} = -\alpha_2$. From (15), we find that

$$E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2\varepsilon_{e_{12},e_{11}} - \alpha_2\varepsilon_{e_{22},e_{11}}. \tag{21}$$

From equations (11) and (17), $\varepsilon_{e_{12},e_{11}} = \varepsilon_{e_{22},e_{11}}$ because stage-2 efforts are proportional, and both elasticities are equal to $E_{\Lambda,e_{11}}$. Hence, the second and the third term in (21) cancel out, and $E_{\Theta_1,e_{11}} = \alpha_1$. We conclude that in contest $C(2,0)$, $\alpha_j\Lambda = c_{ij}e_{ij}$ for all players, and therefore the equilibrium in contest $C(2,0)$ is the same as in contests $C(0,0)$ and $C(2,2)$. A similar argument shows that contest $C(0,2)$ is equivalent to contests $C(0,0)$ and $C(2,2)$. We conclude that contests $C(0,0)$, $C(0,2)$, $C(2,0)$, and $C(2,2)$ result in identical PBEs with NSDK. Q.E.D.

The equivalence of contests $C(0,0)$ and $C(2,0)$ under the Cobb-Douglas production function is somewhat intuitive. Stage-2 efforts e_{12} and e_{22} always satisfy the same first-order conditions as in $C(0,0)$ because players 12 and 22 are followers. In contest $C(2,0)$, player 21 moves at stage 1, but e_{21} is not observable and, therefore, cannot strategically affect other players' efforts. Only player 11's effort can strategically change other players' efforts, but the impact of e_{11} on stage-2 teammate and stage-2 rival cancel out (again due to the proportionality of stage-2 efforts).

Next, we compare equilibria in contests where one team has a no-communication or public-communication policy while the other has a private one. The information flow graphs for $C(1,2)$ and $C(1,0)$ are:

$$\begin{array}{r||lcl} \text{team 1} & \text{player 11} & \longrightarrow & \text{player 12} \\ & & \nearrow & \\ \text{team 2} & \text{player 21} & \longrightarrow & \text{player 22} \end{array}$$

Contest $C(1,2)$

and

$$\begin{array}{r||lcl} \text{team 1} & \text{player 11} & \longrightarrow & \text{player 12} \\ & & & \\ \text{team 2} & \text{player 21} & & \text{player 22} \end{array} \tag{22}$$
$$\text{Contest } C(1,0)$$

As in the case of public information only, we can show that public information does not change the equilibrium outcome in a contest.

Proposition 2 (Contests $C(1,2)$ and $C(2,1)$)**.** *The PBE with NSPAYDK in contest $C(1,2)$ is the same as in contest $C(1,0)$, and the PBE with NSPAYDK in contest $C(2,1)$ is the same as in contest $C(0,1)$.*

Proof. Consider contest $C(1,2)$, in which team 1 communicates privately stage-1 effort e_{11}, while team 2 publicly announces stage-1 effort e_{21}. The first-order conditions for the stage-2 efforts, e_{12} and e_{22}, are $c_{12}e_{12} = \alpha_2\Lambda$ and $c_{22}e_{22} = \alpha_2\Lambda$. The ratio of stage-2 efforts depends on neither e_{11} nor e_{12} in equilibrium; it is just the ratio of the marginal costs $\left(\frac{e_{12}}{e_{22}} = \frac{c_{22}}{c_{12}}\right)$ and $\varepsilon_{e_{22},e_{21}} = \varepsilon_{e_{12},e_{21}}$ because stage-2 efforts are proportional, and both elasticities are equal to $E_{\Lambda,e_{21}}$.

From equation (10), the first-order conditions for the stage-1 efforts e_{11} and e_{21} are $c_{11}e_{11} = E_{\Theta_1,e_{11}}\Lambda$ and $c_{21}e_{21} = E_{\Theta_2,e_{21}}\Lambda$, where

$$E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2\varepsilon_{e_{12},e_{11}} \tag{23}$$

and

$$E_{\Theta_2,e_{21}} = \alpha_1 + \alpha_2\varepsilon_{e_{22},e_{21}} - \alpha_2\varepsilon_{e_{12},e_{21}} \tag{24}$$

since e_{11} could be only observed by player 12 and e_{21} is observed by players 12 and 22. Since stage-2 efforts are proportional and $\varepsilon_{e_{12},e_{11}} = \varepsilon_{e_{22},e_{11}}$, the second and the third term in (24) cancel out for player 21 whose effort is publicly observable, and $E_{\Theta_2,e_{21}} = \alpha_1$. The first-order conditions in contest $C(1,2)$ are then equivalent to those in $C(1,0)$. The same is true about contests $C(2,1)$ and $C(0,1)$. Q.E.D.

Having established the equivalence of public and no communication in our model, we next analyze how, in contrast, private communication matters for the equilibrium outcome and can, therefore, be used strategically by a team.

6.2 Perfect Bayesian Equilibria under Private Communication

Let us next look at contests with private or no communication only: $a_i \in \{0,1\}$. Using a directed graph representation of information flows (as in (5) and (22)), we can describe imperfect information caused by private communications (See Appendix B for details on that for contest $C(1,1)$). The following proposition describes the unique positive-effort PBE with NSPAYDK in each case. The proof is provided in Appendix.

Proposition 3 (Contests $C(0,0)$, $C(0,1)$, $C(1,0)$, and $C(1,1)$)**.** *There exists a unique positive-effort PBE with NSPAYDK characterized by the equilibrium power of team 1, $\Theta_1 = \Theta_1^*$, implicitly defined by*

$$\Theta_1 \left(\frac{1 - \alpha_2 \frac{1-\Theta_1}{1+\Theta_1} I(a_1 = 1)}{1 + \alpha_2 \frac{1-\Theta_1}{1+\Theta_1} I(a_2 = 1)} \right)^{\alpha_1} = \bar{\Theta}_1, \tag{25}$$

where $\bar{\Theta}_1 \equiv \left(\frac{c_{21}}{c_{11}}\right)^{\alpha_1} \left(\frac{c_{22}}{c_{12}}\right)^{\alpha_2}$ is team 1's relative cost advantage. The underdog team becomes stronger while the favorite team becomes weaker in contests with private communication: i.e. if $\bar{\Theta}_1 < 1$ (team 1 is the underdog), then $\bar{\Theta}_1 = \Theta_1^{00} < \Theta_1^{01} < \Theta_1^{10} < \Theta_1^{11} < 1$; and if $\bar{\Theta}_1 > 1$ (team 1 is the favorite), then the inequalities are reversed: $\bar{\Theta}_1 = \Theta_1^{00} > \Theta_1^{01} > \Theta_1^{10} > \Theta_1^{11} > 1$.

Proposition 3 defines the equilibrium power of team 1 in contests $C(0,0)$, $C(0,1)$, $C(1,0)$, and $C(1,1)$. The ranking of the power of the underdog team 1 shows that private communication systematically boosts the underdog team's equilibrium power. Depending on whether players on teams 1 and 2 use private communication or not, we have the following implicitly defined equilibrium values of the power of team 1:

a_1/a_2	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	$\bar{\Theta}_1$	$\Theta_1^{01} : \Theta_1 \left(\frac{1}{1+\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}} \right)^{\alpha_1} = \bar{\Theta}_1.$
$a_1 = 1$	$\Theta_1^{10} : \Theta_1 \left(1 - \alpha_2 \frac{1-\Theta_1}{1+\Theta_1}\right)^{\alpha_1} = \bar{\Theta}_1$	$\Theta_1^{11} : \Theta_1 \left(\frac{1-\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}}{1+\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}} \right)^{\alpha_1} = \bar{\Theta}_1$

(26)

Combining Propositions 1, 2, and 3, we will analyze communication policy choices in the next section.

7 Communication Policy Choices

Suppose that each team has a manager whose sole role is to choose communication policy for their team whether to communicate stage-1 efforts privately, publicly, or not at all, before the game starts. We assume that managers are rewarded if their teams win the contest, and so, they maximize the winning probability of their teams by simultaneously choosing $a_i \in \{0, 1, 2\}$; $i = 1, 2$. This stage of the game played by the two managers is called *stage 0*.

Propositions 1 and 2 show the equivalence of public communication and no communication policy for any team in a synchronous contest. Thus, we have the following equivalence table under the Cobb-Douglas production function:

a_1/a_2	$\mathbf{a_2 = 0}$	$\mathbf{a_2 = 1}$	$a_2 = 2$
$\mathbf{a_1 = 0}$	$\boldsymbol{\Theta}_1^{00}$	$\boldsymbol{\Theta}_1^{01}$	Θ_1^{00}
$\mathbf{a_1 = 1}$	$\boldsymbol{\Theta}_1^{10}$	$\boldsymbol{\Theta}_1^{11}$	Θ_1^{10}
$a_1 = 2$	Θ_1^{00}	Θ_1^{01}	Θ_1^{00}

(27)

This table shows that we can order two teams' winning probabilities under all communication policy profiles by assuming for $a_i \in \{0, 1\}$ and comparing only four of them: Θ_1^{00}, Θ_1^{10}, Θ_1^{01}, and Θ_1^{11}.

Proposition 3 provides a complete ranking of team 1's (relative) power in all these cases. In particular, the ranking is $\bar{\Theta}_1 = \Theta_1^{00} < \Theta_1^{01} < \Theta_1^{10} < \Theta_1^{11} < 1$ if team 1 is the underdog ($\bar{\Theta}_1 < 1$), and so team 1's chances of winning are the highest when both teams communicate privately and the second-highest when only team 1 does that. No communication leads to the lowest chances of winning for the underdog team 1 (and the highest for the favorite team 2 because the ranking is reversed for team 2). For the favorite team 2, the chances of winning are higher when only team 2 communicates by privately sharing information than when only its weaker opponent does that. Importantly, we have the following result about managers' stage-0 preferred choices of communication policies for their teams.

Theorem 1. *The manager of the underdog team (who maximizes the chances of their team winning) prefers private communication for their team. In contrast, the manager of the favorite team prefers either a public or no communication policy.*

Proof. The proof follows directly from Proposition 3. When picking a communication policy, team 1's manager (who maximizes the team's chances of winning) prefers higher equilibrium values of Θ_1 in (27), while team 2's manager prefers the lower values. Suppose team 1 is the favorite ($\bar{\Theta}_1 > 1$). Then, the equilibrium ranking of team 1's power across contests is $\bar{\Theta}_1 = \Theta_1^{00} > \Theta_1^{01} > \Theta_1^{10} > \Theta_1^{11} > 1$, and it is the same for the probability of winning $P_1 = \left(1 + (\Theta_1^*)^{-1}\right)^{-1}$. Since $\Theta_2 = 1/\Theta_1$, the ranking of team 2's power and probability of winning are reversed. When team 2 is the favorite ($\bar{\Theta}_1 < 1$), then the ranking is reversed $\bar{\Theta}_1 = \Theta_1^{00} < \Theta_1^{01} < \Theta_1^{10} < \Theta_1^{11} < 1$. Hence, the favorite team's manager is indifferent between no communication and public communication, while the underdog team's manager prefers private communication. Q.E.D.

From the proof of Theorem 1, it is clear that it does not matter whether team managers choose communication policies simultaneously or sequentially prior to the contest. Either way, we find that the underdog team's stage-1 player $i1$ would communicate with their stage-2 teammate (player $i2$), keeping the communication private. Since an increase in e_{i1} balances the contest, private communication motivates the underdog team's follower to respond with a higher effort e_{i2}. Making a higher e_{i1} known publicly would not be beneficial for the underdog team i because it would generate an unfavorable response from the favorite team's second mover, who would react by increasing e_{k2} ($k \neq i$).

In contrast, the favorite team's chances of winning decrease with private communication because it discourages the effort of the stage-2 teammate. The favorite team has a higher chance of winning under public communication or no communication, and the choice between these policies does not affect the competitive position of the favorite firm. Other non-strategic benefits may need to be considered. Benefits from public communication include reputation effects, regulatory compliance, or investor relations, which may motivate public disclosure even when it is not strictly beneficial strategically. For example, firms may publicly announce progress to attract investment, lock in consumers, or comply with disclosure regulations, regardless of strategic contest considerations.

Next, we consider a scenario where an arbitrary team member decides on their team's communication policy prior to a contest. The following proposition shows that for each team, the payoff ranking of any individual team member over all possible communication policy profiles perfectly aligns with the team's winning probability ranking under a Cobb-Douglas production function.

Proposition 4. *For each player ij and any communication policy profiles (a_1, a_2) and $(\tilde{a}_1, \tilde{a}_2)$, $U_{ij}^{a_1 a_2} \gtreqless U_{ij}^{\tilde{a}_1 \tilde{a}_2}$ holds if and only if $\Theta_i^{a_1 a_2} \gtreqless \Theta_i^{\tilde{a}_1 \tilde{a}_2}$.*

With Theorem 1 and Proposition 4, the following statement is straightforward.

Theorem 2. *Any member of the underdog team prefers their team to communicate privately. Any member of the favorite team prefers their team to have either a public or no communication policy.*

Corollary 1. *For the underdog team, aggregate payoffs are maximized with private communication. For the favorite team, aggregate payoffs are maximized with public or no communication.*

Theorem 2 and Corollary 1 say that the outcomes of endogenous communication policy choices are very robust, at least with a Cobb-Douglas production function. It does not matter whether a team's communication policy is chosen to maximize the team's chances of winning, individual payoffs of team members, or aggregate team payoffs.

8 Extensions

The analysis of this section demonstrates that the paper's results extend to a broader class of contests and to a larger number of players. We also explore the nature of strategic interaction between players in a contest with general effort-aggregation functions and examine CES production functions in particular.

8.1 Generalized Tullock Model

The results of the paper extend to a generalized Tullock model and additional asymmetries between players. First, notice that when each player ij has a distinct value $v_{ij} > 0$ from their team's winning, the expected payoff for player ij is

$$U_{ij} = P_i v_{ij} - c_{ij} e_{ij}, \tag{28}$$

but an equivalent objective for player ij is $U_{ij} \frac{1}{v_{ij}} = P_i - \frac{c_{ij}}{v_{ij}} e_{ij}$, and our previous analysis follows by redefining player ij costs as $\frac{c_{ij}}{v_{ij}}$.

Team i's winning probability P_i can be extended to a generalized Tullock contest success function:

$$P_i = \frac{b_i X_i^r}{b_i X_i^r + b_{-i} X_{-i}^r} = \frac{1}{1 + (b_{-i}/b_i)\, \Theta_i^{-r}}. \tag{29}$$

with parameter r that corresponds to returns to scale to teams' impacts ($0 < r \leq 1$, with $r = 1$ corresponding to the standard Tullock contest) and parameters $b_i > 0$ capturing potential bias (asymmetry) in treating teams when allocating the contest prize. As before, the first-order condition characterizing player ij's optimal effort is $E_{\Theta_i, e_{ij}} \Lambda = c_{ij} e_{ij}$ with $\Lambda \equiv \left(\frac{\partial P_i}{\partial \Theta_i} \Theta_i \right) = r \frac{(b_{-i}/b_i)\Theta_i^{-r}}{\left(1+(b_{-i}/b_i)\Theta_i^{-r}\right)^2} = r P_i \left(1 - P_i\right)$, and so $\alpha_2 \Lambda = c_{i2} e_{i2}$ and $\frac{e_{12}}{e_{22}} = \frac{c_{22}}{c_{12}}$ are still true for stage-2 efforts, and all the analysis extends to generalized Tullock contests.

8.2 $M \geq 2$ Team Members

The analysis also extends to contests among two teams with an arbitrary number $M \geq 2$ of players each, who independently decide on their efforts e_{ij} ($i = 1, 2$

and $j = 1, ..., M$), with efforts aggregated according to a Cobb-Douglas function:

$$X_i = \prod_{m=1}^{M} e_{im}^{\alpha_m} \tag{30}$$

where $\alpha_m \in (0, 1)$. Let the first M_1 players on each team choose their efforts at stage 1 and the remaining $M_2 = M - M_1 \geq 1$ players at stage 2. Let $\alpha_{t=1} = \sum_{m=1}^{M_1} \alpha_m$ and $\alpha_{t=2} = \sum_{m=M_1+1}^{M} \alpha_m$ be the stage-1 and stage-2 aggregate discriminatory powers of stage-1 and stage-2 tasks. Private information policy in this setting ($a_i = 1$) means that a stage-1 mover privately informs all the successive teammates, while $a_i = 2$ assumes that a stage-1 mover informs all rival stage-2 movers as well.

As in the benchmark contest $C(2, 2)$, public information about stage-1 efforts proportionally increases or decreases all stage-2 efforts through balancing or unbalancing the contest (the effect on $\Lambda = P_i (1 - P_i)$). Propositions 1, 2, and 3 generalize to the case of $M > 2$ team members. To see that note that from proportionality of stage-2 efforts $\frac{e_{1m}}{e_{2m}} = \frac{c_{2m}}{c_{1m}}$, the effect of stage-1 effort e_{1m} on Θ_1 is $E_{\Theta_1,e_{1m}} = \alpha_m$ for $m = 1, ..., M_1$ because the effects on stage-2 rivals and teammates cancel out. Similarly, $E_{\Theta_2,e_{2m}} = \alpha_m$ for $m = 1, ..., M_1$. Hence, $c_{im}e_{im} = \alpha_i\Lambda$ for all $i = 1, 2$ and $m = 1, ..., M$, and therefore contests $C(0, 0)$, $C(0, 2)$, $C(2, 0)$, and $C(2, 2)$ are resulting in identical PBE with NSPAYDK with team 1's (relative) power equal to $\bar{\Theta}_1 \equiv \prod_{m=1}^{M} \left(\frac{c_{1m}}{c_{2m}}\right)^{\alpha_m}$.

In the case of private communication of stage-1 efforts, we have $E_{\Theta_1,e_{11}} = \alpha_1 + \sum_{m=M_1+1}^{M} \alpha_m \varepsilon_{e_{1m},e_{11}}$ and $E_{\Theta_2,e_{21}} = \alpha_1 + \sum_{m=M_1+1}^{M} \alpha_m \varepsilon_{e_{2m},e_{21}}$. So, a variant of Proposition 3 holds when each team has $M \geq 2$ players and only internal communication is considered, and Theorem 1, Proposition 4, and Theorem 2 also hold. We obtain the following Theorem 3, which is true for any information policy profile $a = (a_1, a_2)$; $a_i \in \{0, 1, 2\}$. Values of $a_i = 2$ do not enter the formula because, in the case of public information, the strategic effects of stage-1 efforts on teammates and their stage-2 rivals cancel out (since they change by the same proportion and it is only a ratio of efforts that influences the chances of winning for a team).

Theorem 3. *There exists a unique positive-effort PBE with NSPAYDK characterized by the equilibrium power of team 1, $\Theta_1 = \Theta_1^*$, implicitly defined by*

$$\Theta_1 \left(\frac{1 - \alpha_{t=2} I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}}{1 + \alpha_{t=2} I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}} \right)^{\alpha_{t=1}} = \bar{\Theta}_1 \tag{31}$$

where $\bar{\Theta}_1 \equiv \prod_{m=1}^{M} \left(\frac{c_{2m}}{c_{1m}}\right)^{\alpha_m}$ is team 1's relative cost advantage. The underdog team becomes stronger while the favorite team becomes weaker in contests with private communication: i.e., if $\bar{\Theta}_1 < 1$ (team 1 is the underdog), then $\bar{\Theta}_1 = \Theta_1^{00} = \Theta_1^{02} = \Theta_1^{20} = \Theta_1^{22} < \Theta_1^{01} = \Theta_1^{21} < \Theta_1^{10} = \Theta_1^{12} < \Theta_1^{11} < 1$; and if $\bar{\Theta}_1 > 1$ (team 1 is the favorite), then the inequalities are reversed: $\bar{\Theta}_1 = \Theta_1^{00} = \Theta_1^{02} = \Theta_1^{20} = \Theta_1^{22} > \Theta_1^{01} = \Theta_1^{21} > \Theta_1^{10} = \Theta_1^{12} > \Theta_1^{11} > 1$. When communication policies are chosen endogenously, the underdog team's manager (or any player on the team) chooses private communication for their team, while the favorite team's manager (or any player on the team) chooses either public or no communication.

The strategic benefit of private communication is always present for a weak team but not for a strong one. A strong team faced with a choice between private and public communication will prefer public communication.

8.3 Strategic Interaction and Effort-Aggregation Function

To gain a deeper understanding of the strategic incentives of players in contests $C(a_1, a_2)$, it is insightful to look at the nature of the underlying payoffs of players in a static setting. We examine the pairwise strategic interaction between players within a team and across teams, keeping all the other efforts constant, and then show that the communication policy choice by team managers can be explained using the concepts of strategic substitutes and complements between pairs of players in a static setting. We derive the conditions for the efforts of players on the same and opposing team to be strategic complements or substitutes and link them to the incentives to overcommit and undercommit stage-1 effort in the contest and the choice of making that effort visible to teammates and opponents.

Assume that the impact of team i, X_i, is determined by a twice continuously differentiated increasing effort-aggregation function $X_i = f_i(e_{i1}, e_{i2})$.

The first-order condition for e_{ij} (keeping all the other efforts constant) is

$$\frac{\partial U_{ij}}{\partial e_{ij}} = MB_{ij} - c_{ij} = 0 \tag{32}$$

where

$$MB_{ij} \equiv \frac{dP_i}{de_{ij}} = \left(\frac{\partial P_i}{\partial \Theta_i}\Theta_i\right)\left(\frac{\partial \Theta_i}{\partial e_{ij}}\frac{e_{ij}}{\Theta_i}\right)\frac{1}{e_{ij}} = \Lambda \varepsilon_{\Theta_i, e_{ij}}\frac{1}{e_{ij}} \tag{33}$$

with $\Lambda = \left(\frac{\partial P_i}{\partial \Theta_i}\Theta_i\right) = \frac{\Theta_i}{(1+\Theta_i)^2}$. In an interior equilibrium, we have $MB_{ij} > 0$ and $e_{km} > 0$.

Efforts are strategic complements if $\frac{\partial MB_{ij}}{\partial e_{km}} > 0$, or equivalently $\varepsilon_{MB_{ij}, e_{km}} > 0$. Using (33), the second-order cross-partial derivative of player ij's utility is

$$\varepsilon_{MB_{ij}, e_{km}} = \frac{1-\Theta_i}{1+\Theta_i}\varepsilon_{\Theta_i, e_{km}} + \varepsilon_{\varepsilon_{\Theta_i, e_{ij}}, e_{km}} \tag{34}$$

because $\varepsilon_{\Lambda, \Theta_i} = \frac{\partial \Lambda}{\partial \Theta_i}\frac{\Theta_i}{\Lambda} = \frac{1-\Theta_i}{1+\Theta_i}$. The first term is due to the effect of e_{km} on the balance of power. The second term is due to the effect of e_{km} on the responsiveness of team i's power Θ_i to effort e_{ij}.

For Cobb-Douglas production function, $\varepsilon_{\varepsilon_{\Theta_i, e_{ij}}, e_{km}} = 0$, and so

$$\varepsilon_{MB_{ij}, e_{km}} = \varepsilon_{\Lambda, e_{km}} = \frac{1-\Theta_i}{1+\Theta_i}\varepsilon_{\Theta_i, e_{km}} \tag{35}$$

where $\varepsilon_{\Theta_i, e_{km}} > 0$ if $k = i$ and $\varepsilon_{\Theta_i, e_{km}} < 0$ if $k \neq i$ because $\varepsilon_{\Theta_i, e_{km}} = \alpha_{km} > 0$ for $k = i$ and $\varepsilon_{\Theta_i, e_{km}} = -\alpha_{km} < 0$ for $k \neq i$. Note that the power of team i increases when one of its players increases their effort $\varepsilon_{\Theta_i, e_{im}} > 0$ and decreases when a rival player increases their effort $\varepsilon_{\Theta_i, e_{km}} > 0$ for $k \neq i$. Then, for players on the same team ($k = i$), $\varepsilon_{\Theta_i, e_{km}} > 0$ and efforts are strategic complements $\left(\frac{\partial MB_{ij}}{\partial e_{km}} > 0\right)$ iff $\Theta_i < 1$. The opposite condition $\Theta_i > 1$ is necessary and sufficient for the efforts of rival players ($k \neq i$) to be strategic complements. The conditions are reversed for strategic substitutes. The following table summarizes the nature of the strategic effect of e_{km} on e_{ij}.

Effect of e_{km} on e_{ij}	$\Theta_i < 1$	$\Theta_i > 1$	$\Theta_i = 1$
for teammates ($k = i$)	strategic complements	strategic substitutes	no effect
for rivals ($k \neq i$)	strategic substitutes	strategic complements	no effect

Player ij considers efforts strategic e_{ij} and e_{km} to be strategic complements (substitutes) if $(1-\Theta_i)(2I(k=i)-1)>0$ (<0). Note that $\frac{\partial MB_{ij}}{\partial e_{km}}=0$ when $\Theta_i=1$, and no strategic effect is present because e_{km} does not affect the marginal benefit MB_{ij} of player ij's effort when the contest is completely balanced $(\Theta_i=1)$.

More generally, as long as the first term (the direct effect) dominates the second term, and $\varepsilon_{\Theta_i,e_{km}}>0$ for teammates $(k=i)$ and $\varepsilon_{\Theta_i,e_{km}}<0$ for rivals $(k\neq i)$, we have the same direction of the strategic interaction between player efforts on the same team and across the teams as in the case of Cobb-Douglas production function.

As a special case, consider the CES production function

$$X_i=\left(\alpha_1 e_{i1}^{\rho}+\alpha_2 e_{i2}^{\rho}\right)^{\frac{1}{\rho}} \tag{36}$$

with $\rho\leq 1$ and $\sigma=\frac{1}{1-\rho}\in(0,1)$ being an elasticity of substitution between teammate efforts. Proposition 5 provides a condition for player efforts to be strategic complements and substitutes in this case.

Proposition 5 (CES). *Efforts of members on team i are strategic complements if $\sigma<\frac{1}{2}+\frac{1}{2\Theta_i}$ and strategic substitutes if $\sigma>\frac{1}{2}+\frac{1}{2\Theta_i}$. A player on a favorite (underdog) team views rival efforts as strategic complements (substitutes).*

We find that strategic complementarity or substitutability of rival efforts does not depend on the intra-team degree of effort substitution σ. However, this is not the case for teammate efforts. There is a range of values of the elasticity of substitution σ such that teammate efforts are strategic complements for the underdog team and strategic substitutes for the favorite team. Assuming $\Theta_1<1<\Theta_2$, this will be the case for $\sigma\in\left(\frac{1}{2}+\frac{1}{2\Theta_2},\frac{1}{2}+\frac{1}{2\Theta_1}\right)$, where the interval includes $\sigma=1$. In general, the CES production function preserves the strategic complementarity/substitutability patterns for player efforts on underdog and favorite teams for $\sigma\in\left(\frac{1}{2}\left(1+\min(\Theta_1,\Theta_2)\right),\frac{1}{2}\left(1+\max(\Theta_1,\Theta_2)\right)\right)$.

We can state the conditions of Proposition 5 in terms of the elasticity of substitution σ. When $\sigma\leq\frac{1}{2}$, then teammate efforts are always strategic complements, regardless of the power of their team. For $\sigma > \frac{1}{2}$, teammate efforts are strategic complements for sufficiently low team power ($\Theta_i < \frac{1}{2\sigma-1}$) and strategic substitutes otherwise. In the case of the Cobb-Douglas production function ($\sigma = 1$), the condition for strategic complements is $\Theta_i < 1$ (team i is the underdog). Therefore, in the Cobb-Douglas case, we have the following corollary.

Corollary 2 (Cobb-Douglas). *(i) Efforts of members on the favorite team are strategic substitutes. Efforts of members on the underdog team are strategic complements. (ii) Players on the favorite team play in strategic complements vis-a-vis their direct or cross opponents. Players on the underdog team play in strategic substitutes vis-a-vis their direct or cross opponents.*

The results we obtain in this paper follow from the strategic incentives outlined in the corollary. The underdog team benefits from private information because the teammates' efforts are strategic complements, and so a greater stage-1 effort by a player on such a team will produce a desirable increase in stage-2 effort by the player's teammate. This effect is reversed for players on the favorite team – the leading player would prefer not to communicate their greater effort just to their teammate because such communication reduces the effort of the stage-2 teammate. On the other hand, public communication has a favorable strategic effect for the favorite team because it allows the team to discourage the rival team's stage-2 effort. These results hold for any effort-aggregation function that supports the same strategic complementarity/substitutability patterns for player efforts, for example, for the CES production function with elasticity of substitution σ close to 1.

9 Conclusion

In this paper, we study the management of private and public communication by teams competing in a contest with effort complementarity. We assume that two teams competing for a prize in a Tullock contest each have two teammates who exert effort sequentially. Players differ in their marginal costs of exerting effort, which makes two teams generally asymmetric, with a favorite and underdog teams in the contest. This asymmetry plays an essential role in deciding which communication policy each team would adopt to maximize its winning probability. We find that the underdog team benefits from establishing private communication between its members (for example, a meeting behind closed doors). On the other hand, the favorite team benefits from using public communication channels (for example, organizing press conferences to share the news about the team's progress broadly).

Although our game is a complete information game, we propose using the Perfect Bayesian Equilibrium (PBE) to analyze team players' strategic moves to deal with asymmetric information between teams resulting from their choices of communication policies. With an appropriate belief refinement, called "no-signaling-past-actions-you-do-not-know" (NSPAYDK) condition, which is an adaptation of NSWYDK condition (Fudenberg and Tirole, 1991), we can formalize players' equilibrium behavior consistent with privately known earlier players' effort levels. Throughout the paper, we only consider positive-effort PBEs with NSPAYDK, which are uniquely determined for any communication policy profile.

Understanding the strategic effects of information flows within and across organizations can enhance the effective management of organizational behavior. The framework applies broadly, including to product development settings where firms must decide whether to reveal early-stage design investments privately or publicly, influencing both internal coordination and external competition. The current model could potentially address additional questions in the field, such as team formation, employee development, and compensation design. Although adding more teams and/or production stages will complicate the analysis, the use of the Cobb-Douglas effort-aggregation function can make it manageable in some cases. The equilibrium concept here also seems applicable to more general asymmetric information problems under complete information. We hope to move further in this direction of research in the future.

References

[1] Arbatskaya, M., & Konishi, H. (2023). Dynamic team contests with complementary efforts. *Review of Economic Design.* https://doi.org/10.1007/s10058-023-00332-y

[2] Arbatskaya, M., & Mialon, H. M. (2010). Multi-activity contests. *Economic Theory, 43*(1), 23-43.

[3] Arbatskaya, M., & Mialon, H. M. (2012). Dynamic multi-activity contests. *Scandinavian Journal of Economics, 114*(2), 520-538.

[4] Barbieri, S., Kovenock, D., Malueg, D. A., & Topolyan, I. (2019). Group contests with private information and the "Weakest Link." *Games and Economic Behavior, 118*, 382-411.

[5] Barbieri, S., & Malueg, D. A. (2016). Private-information group contests: Best-shot competition. *Games and Economic Behavior, 98*, 219-234.

[6] Barbieri, S., & Topolyan, I. (2021). Private-information group contests with complementarities. *Journal of Public Economic Theory, 23*, 772-800.

[7] Choi, J. P., Chowdhury, S. M., & Kim, J. (2016). Group contests with internal conflict and power asymmetry. *Scandinavian Journal of Economics, 118*(4), 816-840. https://doi.org/10.1111/sjoe.12152

[8] Chowdhury, S. M., Lee, D., & Topolyan, I. (2016). The max-min group contest: Weakest-link (group) all-pay auction. *Southern Economic Journal, 83*(1), 105-125.

[9] Crutzen, B., Konishi, H., & Sahuguet, N. (2024a), The Best at the Top? Candidate Ranking Strategies Under Closed List Proportional Representation, Political Science Research and Methods 12, 706 - 728.

[10] Crutzen, B., Konishi, H., & Sahuguet, N. (2024b), Allocation Rules of Indivisible Prizes in Team Contests, Economic Theory 78, 69-100.

[11] Dranove, D., & Jin, G. Z. (2010). Quality Disclosure and Certification: Theory and Practice. *Journal of Economic Literature* 48 (4): 935–63.

[12] Ewerhart, C., & Lareida, J. (2024). Voluntary disclosure in asymmetric contests. *The Review of Economic Studies.* https://doi.org/10.1093/restud/rdae001

[13] Fu, Q., Gürtler, O., & Münster, J. (2013). Communication and commitment in contests. *Journal of Economic Behavior & Organization, 95*, 1-19.

[14] Fu, Q., & Wu, Z. (2019). Contests: Theory and topics. *Oxford Research Encyclopedia of Economics.*

[15] Fudenberg, D., & Tirole, J. (1991). *Game theory.* MIT Press.

[16] Harris, C., & Vickers, J. (1987). Racing with uncertainty. *Review of Economic Studies, 54*(1), 1-21.

[17] Hinnosaar, T. (2024). Optimal sequential contests. *Theoretical Economics, 19*, 207-244. https://doi.org/10.3982/TE5536

[18] Lu, J., & Lu, D. (2020). Task arrangement in team competitions. *Economics Letters, 193.*

[19] Kao, J. (2024). Information Disclosure and Competitive Dynamics: Evidence from the Pharmaceutical Industry. *Management Science*, 70(10).

[20] Kempf, H., & Graziosi, G.R. (2010). Leadership in Public Good Provision: A Timing Game Perspective. *Journal of Public Economic Theory*, 12, 763-787.

[21] Klumpp, T., & Polborn, M. (2006). Primaries and the New Hampshire effect. *Journal of Public Economics, 90*(6-7), 1073-1114.

[22] Kobayashi, K., Konishi, H., and Ueda, K. (2025). Prize allocation Rules in Generalized Contests, Economic Theory 79, 151-179.

[23] Kolmar, M., & Rommeswinkel, H. (2013). Contests with group-specific public goods and complementarities in efforts. *Journal of Economic Behavior & Organization, 89*, 9-22.

[24] Konishi, H., & Pan, C.-Y. (2020). Sequential formation of alliances in survival contests. *International Journal of Economic Theory, 16*, 95-105.

[25] Konishi, H., & Pan, C.-Y. (2021). Endogenous alliances in survival contests. *Journal of Economic Behavior & Organization, 189*, 337-358.

[26] Konrad, K. A. (2009). *Strategy and dynamics in contests.* Oxford University Press.

[27] Konrad, K. A., & Kovenock, D. (2009). Multi-battle contests. *Games and Economic Behavior, 66*(1), 256-274.

[28] Kovenock, D., Morath, F., & Münster, J. (2015). Information sharing in contests. *Journal of Economics & Management Strategy, 24*, 570-596.

[29] Kubitz, G. (2023). Two-stage contests with private information. *American Economic Journal: Microeconomics, 15*(1), 239-287.

[30] Leibbrandt, A., & Sääksvuori, L. (2012). Communication in intergroup conflicts. *European Economic Review, 56*(6), 1136-1147.

[31] Nitzan, S., & Ueda, K. (2011). Prize sharing in collective contests. *European Economic Review, 55*, 678-687.

[32] Okuno-Fujiwara, M., Postlewaite, A., & Suzumura, K. (1990). Strategic Information Revelation. *The Review of Economic Studies*, 57(1), 25–47.

[33] Sutter, M., & Strassmair, C. (2009). Communication, cooperation and collusion in team tournaments: An experimental study. *Games and Economic Behavior, 66*(1), 506-525.

Appendix: Proofs

Proof of Proposition 3. We start with the second stage. The first-order conditions for the stage-2 efforts, e_{12} and e_{22}, are $c_{12}e_{12} = \alpha_2\Lambda$ and $c_{22}e_{22} = \alpha_2\Lambda$, respectively (equation (11)). The ratio of stage-2-activity levels does not depend on e_{11} and e_{21} in equilibrium; it is just the ratio of the marginal costs $\frac{e_{12}}{e_{22}} = \frac{c_{22}}{c_{12}}$ (equation (17)). Here, each team either has private (internal) communication ($a_i = 1$) or no communication ($a_i = 0$).

From equation (10), the first-order conditions for stage-1 efforts e_{11} and e_{21} are

$$\begin{aligned} c_{11}e_{11} &= E_{\Theta_1,e_{11}}\Lambda \\ c_{21}e_{21} &= E_{\Theta_2,e_{21}}\Lambda, \end{aligned} \tag{37}$$

where $E_{\Theta_1,e_{11}} = \alpha_1 + \alpha_2\varepsilon_{e_{12},e_{11}}I(a_1 = 1)$ and $E_{\Theta_2,e_{21}} = \alpha_1 + \alpha_2\varepsilon_{e_{22},e_{21}}I(a_2 = 1)$ since e_{11} could be only observed by player 12 (when $a_1 = 1$) and e_{21} could be only observed by player 22 (when $a_2 = 1$).

From the first-order conditions for stage-2 efforts, we find that $\varepsilon_{e_{12},e_{11}} = E_{\Lambda,e_{11}} = \frac{1-\Theta_1}{1+\Theta_1}E_{\Theta_1,e_{11}}$ and $\varepsilon_{e_{22},e_{21}} = E_{\Lambda,e_{21}} = \frac{1-\Theta_2}{1+\Theta_2}E_{\Theta_2,e_{21}}$. Since $\Theta_2 = 1/\Theta_1$, we have $\frac{1-\Theta_2}{1+\Theta_2} = -\frac{1-\Theta_1}{1+\Theta_1}$ and

$$\begin{aligned} E_{\Theta_1,e_{11}} &= \alpha_1 + \alpha_2 I(a_1 = 1)\frac{1-\Theta_1}{1+\Theta_1}E_{\Theta_1,e_{11}} \\ E_{\Theta_2,e_{21}} &= \alpha_1 + \alpha_2 I(a_2 = 1)\frac{1-\Theta_2}{1+\Theta_2}E_{\Theta_2,e_{21}}, \end{aligned} \tag{38}$$

and solving for $E_{\Theta_1,e_{11}}$ and $E_{\Theta_2,e_{21}}$ we obtain

$$\begin{aligned} E_{\Theta_1,e_{11}} &= \alpha_1\left(1 - \alpha_2 I(a_1 = 1)\frac{1-\Theta_1}{1+\Theta_1}\right)^{-1} \\ E_{\Theta_2,e_{21}} &= \alpha_1\left(1 + \alpha_2 I(a_2 = 1)\frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}. \end{aligned} \tag{39}$$

Substituting equations (39) back into the first-order conditions for state-1 efforts, we have

$$c_{11}e_{11} = \alpha_1 \left(1 - \alpha_2 I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1} \Lambda \tag{40}$$
$$c_{21}e_{21} = \alpha_1 \left(1 + \alpha_2 I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1} \Lambda,$$

and

$$\frac{e_{11}}{e_{21}} = \frac{\left(1 - \alpha_2 I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}}{\left(1 + \alpha_2 I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}} \times \frac{c_{21}}{c_{11}}. \tag{41}$$

Substituting (17) and (41) into Θ_1 (as in equation (20)), we obtain

$$\Theta_1 = \left(\frac{e_{11}}{e_{21}}\right)^{\alpha_1} \left(\frac{e_{12}}{e_{22}}\right)^{\alpha_2} = \bar{\Theta}_1 \left(\frac{\left(1 - \alpha_2 I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}}{\left(1 + \alpha_2 I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}} \right)^{\alpha_1}. \tag{42}$$

Thus, $\Theta_1 = \Theta_1^*$ is implicitly defined by

$$\Theta_1 \left(\frac{1 - \alpha_2 I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}}{1 + \alpha_2 I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}} \right)^{\alpha_1} = \bar{\Theta}_1. \tag{43}$$

To prove the existence and uniqueness of the solution to this equation, we use Lemma 1.

Lemma 1. *Let $g : [0, \infty) \to [0, \infty)$ be such that $g(x) = x \left(\frac{1-\alpha_2 \frac{1-x}{1+x} I(a_1=1)}{1+\alpha_2 \frac{1-x}{1+x} I(a_2=1)} \right)^{\alpha_1}$ with $\alpha_1, \alpha_2 \in (0, 1)$. Then, equation $g(x) = \bar{x}$ has a unique solution: if $0 < \bar{x} < 1$ then $\bar{x} < x^* < 1$; if $\bar{x} > 1$ then $\bar{x} > x^* > 1$.*

Proof of Lemma 1. Let $g(x) \equiv x \left(\frac{1-\alpha_2 \frac{1-x}{1+x} I(a_1=1)}{1+\alpha_2 \frac{1-x}{1+x} I(a_2=1)} \right)^{\alpha_1}$. We have $g(0) = 0$, $g(1) = 1$, and $\lim_{x\to\infty} g(x) = \infty$ (because $\lim_{x\to\infty} \frac{1-x}{1+x} = -1$). Moreover, $g(x)$ is increasing in x ($g'(x) > 0$) because $\frac{1-x}{1+x}$ is decreasing in x and $\alpha_1, \alpha_2 \in (0, 1)$. If $I(a_1 = 1) = I(a_2 = 1) = 0$, then $g(x) = x$. If $I(a_1 = 1)I(a_2 = 1) \neq 0$ then we have: if $\bar{x} > 1$, $g(\bar{x}) = \bar{x} \left(\frac{1+\bar{x}-\alpha_2(1-\bar{x})I(a_1=1)}{1+\bar{x}+\alpha_2(1-\bar{x})I(a_2=1)} \right)^{\alpha_1} > \bar{x}$, and if $\bar{x} < 1$, $g(\bar{x}) = \bar{x} \left(\frac{1+\bar{x}-\alpha_2(1-\bar{x})I(a_1=1)}{1+\bar{x}+\alpha_2(1-\bar{x})I(a_2=1)} \right)^{\alpha_1} < \bar{x}$. Therefore, by the intermediate value theorem and $g'(x) > 0$, there exists a unique solution x^* for any $\alpha_1, \alpha_2 \in (0, 1)$; for $a_1 = a_2 = 0$, we have $x^* = \bar{x}$; for $a_1 a_2 \neq 0$, we have: if $\bar{x} < 1$ then $\bar{x} < x^* < 1$; if $\bar{x} > 1$, then $\bar{x} > x^* > 1$. Q.E.D.

Lemma 1 assures that since $g(1) = 1$, $\lim_{x\to\infty} g(x) = \infty$, and $g'(\Theta_1) > 0$, by the intermediate value theorem, there exists a unique Θ_1^*. Lemma 1 also proves that $\Theta_1^* = \bar{\Theta}_1$ if $a_1 = a_2 = 0$ or $\bar{\Theta}_1 = 1$, and otherwise, $\bar{\Theta}_1 > \Theta_1^* > 1$ or $\bar{\Theta}_1 < \Theta_1^* < 1$ depending on the value of $\bar{\Theta} \gtrless 1$.

Finally, we rank the contests in terms of the equilibrium power of team 1 (or the probability of team 1 winning, $P_1 = \frac{1}{1+\Theta_1^{-1}}$). Let $U \equiv 1 - \alpha_2 \frac{1-\Theta_1}{1+\Theta_1} I(a_1 = 1)$ and $D \equiv 1 + \alpha_2 \frac{1-\Theta_1}{1+\Theta_1} I(a_2 = 1)$. For $\bar{\Theta}_1 < 1$, we want to show that $\bar{\Theta}_1 = \Theta_1^{00} < \Theta_1^{01} < \Theta_1^{10} < \Theta_1^{11} < 1$. For $\Theta_1 < 1$, we have $0 < \frac{1-\Theta_1}{1+\Theta_1} < 1$, and $U \leq 1$, $D \geq 1$, and $\frac{U}{D} \leq U \leq \frac{1}{D} \leq 1$ for any $\Theta_1 < 1$. In contest $C(0,0)$, $\frac{U}{D} = 1$; in contest $C(0,1)$, $\frac{U}{D} = \frac{1}{D} = \frac{1}{1+\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}}$; in contest $C(1,0)$, $\frac{U}{D} = U = \frac{1-\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}}{1}$; and in contest $C(1,1)$, $\frac{U}{D} = \frac{1-\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}}{1+\alpha_2 \frac{1-\Theta_1}{1+\Theta_1}}$. Hence, solutions to $\Theta_1 \left(\frac{U}{D}\right)^{\alpha_1} = \bar{\Theta}_1$ for different a_1 and a_2 are ranked as follows: $\Theta_1^{00} < \Theta_1^{01} < \Theta_1^{10} < \Theta_1^{11}$. By similar arguments, the ranking is reversed when $\bar{\Theta}_1 > 1$. Q.E.D.

Proof of Proposition 4. Note that $U_{ij} = P_i - c_{ij}e_{ij}$, and recall equilibrium P_i and e_{ij} are written as: $P_i = \frac{1}{1+\Theta_i^{-1}} = \frac{\Theta_i}{1+\Theta_i}$ and $c_{ij}e_{ij} = E_{\Theta_i,e_{ij}}\Lambda$ (equation (10)), with $\Lambda = \frac{\Theta_i}{(1+\Theta_i)^2}$. At stage 2, $E_{\Theta_i,e_{i2}} = \alpha_2$ and player $i2$'s equilibrium payoff is

$$U_{i2} = \left(\frac{1}{1+\Theta_i^{-1}}\right)\left(1 - \frac{\alpha_2}{1+\Theta_i}\right), \tag{44}$$

which is an increasing function of Θ_i (because each term is an increasing function of Θ_i) with

$$\frac{dU_{ij}}{d\Theta_i} = \frac{1}{(1+\Theta_i)^3}\left[1 - \alpha_j + \Theta_i\left(1+\alpha_j\right)\right] > 0. \tag{45}$$

For stage-1 efforts,

$$\begin{aligned} E_{\Theta_1,e_{11}} &= \alpha_1 \left(1 - \alpha_2 I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1} \\ E_{\Theta_2,e_{21}} &= \alpha_1 \left(1 + \alpha_2 I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}\right)^{-1}. \end{aligned} \tag{46}$$

as shown in the proof of Proposition 3 (equation (39)). Then,

$$U_{11} = P_1 - c_{11}e_{11} = \frac{\Theta_1}{1+\Theta_1}\left(1 - \frac{\alpha_1}{1+\Theta_1 - \alpha_2 I\left(a_1 = 1\right)\left(1-\Theta_1\right)}\right) \tag{47}$$
$$= \frac{1}{1+\Theta_1^{-1}}\left(1 - \frac{\alpha_1}{1 - \alpha_2 I\left(a_1 = 1\right) + \left(1 + \alpha_2 I\left(a_1 = 1\right)\right)\Theta_1}\right)$$

which is an increasing function of Θ_1 (because each term is an increasing function of Θ_1).

Similarly, U_{21} can be written as

$$U_{21} = \frac{1}{1+\Theta_2^{-1}}\left(1 - \frac{\alpha_1}{1 - \alpha_2 I\left(a_2 = 1\right) + \left(1 + \alpha_2 I\left(a_2 = 1\right)\right)\Theta_2}\right), \tag{48}$$

and it is an increasing function of Θ_2. Moreover, when $\bar{\Theta}_1 < 1$, in any equilibrium, we have $\Theta_1 < 1$, and U_{11} is higher when $a_1 = 1$, while U_{21} is higher when $a_1 \neq 1$. In contrast, when $\bar{\Theta}_1 > 1$, in any equilibrium, we have $\Theta_1 > 1$, and U_{11} is higher when $a_1 \neq 1$, while U_{21} is higher when $a_1 = 1$.

For the underdog team i, stage-2 player payoff is not affected by the communication policy profile (given Θ_i), and because Θ_i increases with private communication for the underdog, that will be the best for the underdog team. Private communication is even more beneficial for stage-1 player because their payoff is improved by the policy change directly as well as by an increase in Θ_i. For the favorite team i, stage-2 player payoff is not affected by the communication policy profile (given Θ_i), and she prefers public or no communication because private communication reduces her payoff further (in addition to the lower Θ_i). Hence, player ij's payoff ranking over all possible communication policy profiles is completely aligned with her team's winning probability ranking. We have completed the proof. Q.E.D.

Proof of Theorem 3. Stage-2 efforts are proportional: $\frac{e_{1m}}{e_{2m}} = \frac{c_{2m}}{c_{1m}}$ because they are characterized by $c_{ij}e_{ij} = \alpha_j \Lambda$ for $i = 1, 2$ and $j = M_1 + 1, ..., M$. The first-order condition for stage-1 effort e_{ij} is

$$c_{ij}e_{ij} = E_{\Theta_i, e_{ij}}\Lambda \tag{49}$$

for $i = 1, 2$ and $j = 1, ..., M_1$, where

$$E_{\Theta_i, e_{ij}} = \alpha_j + \sum_{m=M_1+1}^{M} \alpha_m \varepsilon_{e_{im}, e_{ij}} I\left(a_i = 1\right) \tag{50}$$

with $a_i = 1$ if team i communicates privately, and $a_i = 0$ if it has no communication.

The ratio of stage-1 efforts by the mth players on teams 1 and 2 is

$$\frac{e_{1m}}{e_{2m}} = \frac{c_{2m}}{c_{1m}} \frac{E_{\Theta_1, e_{1m}}}{E_{\Theta_2, e_{2m}}} \tag{51}$$

for $m = 1, ..., M_1$. The (relative) power of team 1 is

$$\Theta_1 = \bar{\Theta}_1 \prod_{m=1}^{M_1} \left(\frac{E_{\Theta_1, e_{1m}}}{E_{\Theta_2, e_{2m}}} \right)^{\alpha_m} \tag{52}$$

with $\bar{\Theta}_1 \equiv \prod_{m=1}^{M} \left(\frac{c_{1m}}{c_{2m}} \right)^{\alpha_m} > 0$.

From the first-order conditions for stage-2 efforts, same-team elasticities of response are $\varepsilon_{e_{1m}, e_{1j}} = E_{\Lambda, e_{1j}} = \frac{1-\Theta_1}{1+\Theta_1} E_{\Theta_1, e_{1j}}$ and $\varepsilon_{e_{2m}, e_{2j}} = E_{\Lambda, e_{2j}} = \frac{1-\Theta_2}{1+\Theta_2} E_{\Theta_2, e_{2j}}$ for $j = 1, ..., M_1$ and $m = M_1 + 1, ..., M$. Then, $E_{\Theta_i, e_{ij}} = \alpha_j + \sum_{m=M_1+1}^{M} \alpha_m \varepsilon_{e_{im}, e_{ij}} I\left(a_i = 1\right) = \alpha_j + \alpha_{t=2} \frac{1-\Theta_i}{1+\Theta_i} I\left(a_i\right) E_{\Theta_i, e_{ij}}$, where $\alpha_{t=2} = \sum_{m=M_1+1}^{M} \alpha_m$. Solving for $E_{\Theta_i, e_{ij}}$, we obtain

$$E_{\Theta_i, e_{ij}} = \alpha_j \left(1 - \alpha_{t=2} I\left(a_i = 1\right) \frac{1 - \Theta_i}{1 + \Theta_i} \right)^{-1}. \tag{53}$$

Substituting this elasticity back into the first-order conditions for state-1 efforts, we have

$$c_{ij} e_{ij} = \alpha_j \left(1 - \alpha_{t=2} I\left(a_i = 1\right) \frac{1 - \Theta_i}{1 + \Theta_i} \right)^{-1} \Lambda \tag{54}$$

and

$$\frac{e_{1j}}{e_{2j}} = \frac{\left(1 - \alpha_{t=2} I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1} \right)^{-1}}{\left(1 + \alpha_{t=2} I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1} \right)^{-1}} \frac{c_{2j}}{c_{1j}}. \tag{55}$$

for $j = 1, ..., M_1$ since $\Theta_2 = 1/\Theta_1$ implies $\frac{1-\Theta_2}{1+\Theta_2} = -\frac{1-\Theta_1}{1+\Theta_1}$. We find that

$$\Theta_1 = \prod_{m=1}^{M} \left(\frac{e_{1m}}{e_{2m}} \right)^{\alpha_m} = \bar{\Theta}_1 \left(\frac{1 - \alpha_{t=2} I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}}{1 + \alpha_{t=2} I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}} \right)^{-\alpha_{t=1}} \tag{56}$$

where $\alpha_{t=1} = \sum_{m=1}^{M_1} \alpha_m$. We can then show that equation

$$\Theta_1 \left(\frac{1 - \alpha_{t=2} I\left(a_1 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}}{1 + \alpha_{t=2} I\left(a_2 = 1\right) \frac{1-\Theta_1}{1+\Theta_1}} \right)^{\alpha_{t=1}} = \bar{\Theta}_1 \tag{57}$$

defines the unique equilibrium balance of power Θ_1^* with properties stated in Theorem 3. The proof is the same as that of Proposition 3, with α_1 replaced with $\alpha_{t=1}$ and α_2 replaced with $\alpha_{t=2}$. Q.E.D.

Proof of Proposition 5. Efforts of team i's members are strategic complements if and only if

$$\varepsilon_{MB_{ij},e_{km}} = \frac{1-\Theta_i}{1+\Theta_i} \varepsilon_{\Theta_i,e_{km}} + \varepsilon_{\varepsilon_{\Theta_i,e_{ij}},e_{km}} > 0. \tag{58}$$

Let us focus on the second term, $\varepsilon_{\varepsilon_{\Theta_i,e_{ij}},e_{km}}$ by first finding $\varepsilon_{\Theta_i,e_{ij}}$.

Since $\Theta_i = \frac{X_i}{X_{-i}}$,

$$\frac{d\Theta_i}{de_{ij}} = \frac{1}{X_{-i}} \times \frac{1}{\rho} \left(\alpha_1 e_{i1}^{\rho} + \alpha_2 e_{i2}^{\rho}\right)^{\frac{1}{\rho}-1} \rho e_{ij}^{\rho-1}, \tag{59}$$

thus we have

$$\begin{aligned} \varepsilon_{\Theta_i,e_{ij}} &= \frac{e_{ij}}{\Theta_i} \frac{d\Theta_i}{de_{ij}} \\ &= \frac{e_{ij} X_{-i}}{X_i} \frac{1}{X_{-i}} \times \frac{1}{\rho} \left(\alpha_1 e_{i1}^{\rho} + \alpha_2 e_{i2}^{\rho}\right)^{\frac{1}{\rho}-1} \rho \alpha_j e_{ij}^{\rho-1} \\ &= \frac{\alpha_j e_{ij}^{\rho}}{\alpha_1 e_{i1}^{\rho} + \alpha_2 e_{i2}^{\rho}}. \end{aligned} \tag{60}$$

This implies that

$$\varepsilon_{\varepsilon_{\Theta_i,e_{ij}},e_{km}} = \frac{e_{km}}{\varepsilon_{\Theta_i,e_{ij}}} \times \frac{\partial \varepsilon_{\Theta_i,e_{ij}}}{\partial e_{km}} = \begin{cases} 0 & \text{if } k \neq i \\ -\rho \varepsilon_{\Theta_i,e_{im}} & \text{if } k = i \text{ and } j \neq m, \end{cases} \tag{61}$$

since in the second case, we have

$$\varepsilon_{\varepsilon_{\Theta_i,e_{ij}},e_{im}} = \frac{e_{im}}{\varepsilon_{\Theta_i,e_{ij}}} \times \frac{\partial \varepsilon_{\Theta_i,e_{ij}}}{\partial e_{im}} = \tag{62}$$

$$= \frac{e_{im}\left(\alpha_1 e_{i1}^{\rho} + \alpha_2 e_{i2}^{\rho}\right)}{\alpha_j e_{ij}^{\rho}} \times \frac{-\rho \alpha_j e_{ij}^{\rho} \alpha_m e_{im}^{\rho-1}}{\left(\alpha_1 e_{i1}^{\rho} + \alpha_2 e_{i2}^{\rho}\right)^2} = -\rho \varepsilon_{\Theta_i,e_{im}}. \tag{63}$$

The elasticity of substitution of this CES function is described by $\sigma = \frac{1}{1-\rho}$ (or $\rho = \frac{\sigma-1}{\sigma}$). The Cobb-Douglas case occurs when $\sigma = 1$. Thus, we have

$$\varepsilon_{MB_{ij},e_{km}} = \begin{cases} \frac{1-\Theta_i}{1+\Theta_i}\varepsilon_{\Theta_i,e_{km}} & \text{if } k \neq i \\ \left(\frac{1-\Theta_i}{1+\Theta_i} - \frac{\sigma-1}{\sigma}\right)\varepsilon_{\Theta_i,e_{im}} & \text{if } k = i \text{ and } j \neq m, \end{cases} \tag{64}$$

thus, when $k = i$, the efforts of members in team i are strategic complements if $\frac{1-\Theta_i}{1+\Theta_i} > \frac{\sigma-1}{\sigma}$ (or $\sigma < \frac{1}{2} + \frac{1}{2\Theta_i}$) and strategic complements when the inequalities are reserved. Q.E.D.