Modeling Aggregate Productivity Growth: The Importance of Intersectoral Transfer Prices and International Trade

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A measure of aggregate productivity growth should serve as a barometer, sensitive to the economy's success in solving its economic problem. Economists long have reasoned that goods destined for final demand are the ultimate objective of economic production. These are the goods, as Kendrick (1973) argues, "that satisfy current consumer wants or that add to stocks of productive capacity for satisfying future wants."[1] The sum of sectoral deliveries to final demand, therefore, are the appropriate output criterion for evaluating aggregate productivity growth.

Sectoral deliveries to intermediate demand typically are excluded from studies of aggregate economic performance. Intersectoral deliveries are viewed as wholly internal transactions, self-canceling in both real and money terms. Moreover, transfer prices are assumed to reflect marginal production costs. Under these conditions, deliveries to final demand can be shown to equal national net output or, as conventionally described, aggregate value added.[2] The result is that studies of aggregate economic performance typically define productivity at the economy-wide level as the efficiency with which labor and capital inputs are converted into aggregate value added. For all their important differences, Solow (1957), Kendrick (1961, 1973), Denison (1962, 1974, 1979), Jorgenson and Griliches (1967), Christensen and Jorgenson (1973), and Kendrick and Grossman (1980) adopt this conventional approach to aggregate growth accounting.

The primary focus of this paper is the characterization of microeconomic activity implicit in models of aggregate productivity growth. The self-canceling properties of intersectoral transactions are not economic truisms. They follow, instead, from two assumptions:

(1) the economy is closed to trade in foreign-produced inputs and (2) intersectoral transfer prices equal marginal production costs.

Neither assumption need hold even in a competitive economy. In the presence of imported inputs, material inputs consumed in the domestic economy exceed the quantity supplied by domestic producers. Furthermore, given taxes and/or subsidies on intersectoral transactions, producer and consumer prices for intermediate goods are not equal. Transfer prices no longer reflect marginal production costs. Transactions in material inputs neither are purely internal transfers nor are they offsetting in either real or money terms.

The implications for modeling and measuring aggregate productivity growth are direct. For a competitive economy both closed to trade in imported inputs and having marginal cost pricing in all intermediate input markets, value-added and delivery-to-final-demand models can be shown to produce equivalent measures of aggregate productivity growth. Once international trade, taxes, and subsidies are introduced, however, modeling aggregate productivity in terms of value added or deliveries to final demand does make a difference. The final measure of aggregate productivity growth can be shown to depend importantly on the initial description of the economy's macroeconomic objectives for production, the process of intersectoral transfers, and the technical properties of microeconomic production.

Sections 1 and 2 of this paper contrast measures of aggregate productivity growth derived, respectively, from value-added and delivery-to-final-demand models of economic activity. Though the models produce substantively different measures of aggregate productivity growth, they can be related through terms adjusting for the productivity consequences of intersectoral and international trade.

Sections 3 and 4 investigate the properties of defining aggregate productivity growth for an open economy without marginal cost transfer prices. The value-added and delivery-to-final-demand rates of aggregate productivity growth are decomposed into their microeconomic source components. The decompositions not only demonstrate the unique productivity contributions of sectoral productivity growth, its intersectoral transmission, transfer prices, imported inputs, and the changing distribution of intermediate inputs but also make explicit the microeconomic assumptions underlying the alternative models of economic growth. Section 4 completes the analysis with a discussion of the practical consequences of these findings both for intertemporal and international productivity comparisons as well as for the design of regional models of productivity growth.

1. Value Added

Studies defining aggregate productivity growth in terms of value added implicitly view the macro economy as a set of horizontally independent sectors, each producing value added from labor and capital inputs. The maximum value of aggregate value added (λ) is expressed as a function of all quantities of sectoral value added (V_j), aggregate labor (L) and capital (K) inputs, and time (t)

(1) $\lambda = F(V_1, V_2, \ldots, V_n, L, K, t)$

Society's economic problem is to maximize λ given linearly homogeneous value-added functions

(2) $V_j = V^j(L_j, K_j, t)$ $(j = 1, 2,\ldots,n)$,

market equilibrium conditions, and aggregate supplies of labor and capital, where $L = \sum L_j$ and $K = \sum K_j$.[3] The function F is homogeneous of degree minus one in quantities of sectoral value added, degree one in factor supplies, and degree zero in quantities of value added and inputs.[4]

The particular characterization of macroeconomic activity embodied in equation (1) has important technical implications for sectoral production. The existence of sectoral value-added functions V^j implies that sectoral production of gross output is characterized by value-added separability:

(3) $Z_j = f^j[V^j(L_j, K_j, t), X_{1j}, X_{2j},\ldots,X_{nj}]$ $(j = 1, 2,\ldots,n)$,

where $Z_j \equiv$ gross output of the jth sector and

$X_{ij} \equiv$ ith intermediate input used in the jth sector.

Maintaining the separability of value added and intermediate inputs implies that the technical properties of V^j, including all the properties of productivity growth, can be analyzed in isolation from intermediate inputs. These inputs cannot be sources or mediums of productivity growth. Productivity improvements can affect output only through V^j. Stated formally, intermediate inputs are excluded from a value-added model of macroeconomic activity not only because they are viewed as self-canceling transactions but also because they are considered unimportant to the analysis of underlying sectoral productivity growth.

The derivation of the value added productivity formula, though not unfamiliar in the productivity literature, sets the stage for the comparative analysis which follows. That derivation begins by fixing the level of aggregate value added at unity:

(4) $1 = F(V_1, V_2, \ldots, V_n, L, K, t)$.

Equilibrium in a competitive economy and the homogeneity restrictions underlying the frontier F together imply the following conditions:

$$(5)\quad \frac{\partial \ln F}{\partial \ln V_j} = \frac{-q_j^v V_j}{\sum_j q_j^v V_j} \qquad \frac{\partial \ln F}{\partial \ln L} = \frac{p_L L}{\sum_j q_j^v V_j} \qquad \frac{\partial \ln F}{\partial \ln K} = \frac{p_K K}{\sum_j q_j^v V_j}$$

$$(6a)\quad \sum_j \frac{q_j^v V_j}{\sum_j q_j^v V_j} = 1$$

and

$$(6b)\quad \frac{p_L L}{\sum_j q_j^v V_j} + \frac{p_K K}{\sum_j q_j^v V_j} = 1,$$

where q_j^v is the producer price of value added produced in the jth sector and p_L and p_K are the economy-wide average prices paid by producers for labor and capital inputs, respectively; that is $p_L = \sum p_{L_j} L_j / L$ and $p_K = \sum p_{K_j} K_j / K$.

Condition (6a) is true by construction. The value of aggregate value added equals the sum of all sectoral value added. Verifying (6b) begins with the sectoral accounting identity relating the value of total output and payments to all factors of production:

$$(7) \quad q_j Z_j = p_{L_j} L_j + p_{K_j} K_j + \sum_i p_i X_{ij} \qquad (j = 1, 2, \ldots, n),$$

where q_j is the producer price of gross output in the jth sector and p_i is the price paid for intermediate input produced in the ith sector.[5] The value of sectoral value added equals the value of output less the value of intermediate input purchases:

$$(8) \quad \overset{v}{q}_j V_j = q_j Z_j - \sum p_i X_{ij} \qquad (j = 1, 2, \ldots, n).$$

Substituting (7) into (8) and summing over all n sectors yields:

$$(9) \quad \sum_j \overset{v}{q}_j V_j = \sum_j p_{L_j} L_j + \sum_j p_{K_j} K_j$$
$$= p_L L + p_K K,$$

establishing the result required by (6b).

The expression for the rate of aggregate productivity growth E_v now can be derived by substituting (5) into the total logarithmic derivative of F with respect to time:

$$(10) \quad E_v \equiv \frac{\partial \ln F}{\partial t} = \sum_j \frac{\overset{v}{q}_j V_j}{\sum_j \overset{v}{q}_j V_j} \frac{d\ln V_j}{dt} - \frac{p_L L}{\sum_j \overset{v}{q}_j V_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j \overset{v}{q}_j V_j} \frac{d\ln K}{dt}.$$

This result can be summarized as a familiar proposition.

> Proposition 1: The rate of aggregate productivity growth for an economy maximizing net output (value added) is defined as the weighted average of rates of growth of sectoral value added (with weights equal to sectoral value added shares in aggregate net output) less the weighted average rates of growth of aggregate labor and capital inputs (with weights equal to the value shares of input payments in aggregate net output). Both sets of weights sum to unity.

Note that the expression for E_v defined in (10) applies to all economies, whether open or closed to trade in foreign-produced inputs and regardless of the structure of taxes on intersectoral transactions. Intermediate inputs, whether purchased from domestic or foreign producers, do not enter either the production-possibilities frontier (4) or the sectoral value-added functions (2). That intersectoral transfer prices might not reflect true marginal production costs is of no consequence to the equilibrium and homogeneity conditions (5) and (6). The macro economy is viewed as if it were a set of horizontally independent sectors.

The important observation is that the model of aggregate productivity stated in terms of value added does not depend on the assumptions that the economy imports no material inputs and that internal transfers are allocatively efficient. Though these assumptions are used to rationalize the value-added model, nowhere are they introduced in the derivation of E_V. That derivation, instead, depends solely on the assumption of value-added separability. Not surprisingly, the resulting measure of aggregate productivity growth (10) is insensitive to both the economy's external trade in foreign-produced goods as well as the efficiency of intermediate input markets.

2. Deliveries to Final Demand

The delivery-to-final-demand model of aggregate productivity growth is constrained by sectoral production functions, not sectoral value-added functions. Value-added separability is not a maintained hypothesis. Intersectoral and international transactions in material inputs are not suppressed. The vertical structure of the aggregate economy is recognized explicitly. Consequently, the model must be sensitive both to the absence of marginal cost transfer pricing and to the degree of the economy's openness to trade in foreign-produced inputs.

The formal specifications of the delivery-to-final demand models appropriate to a "closed and untaxed" economy and to an "open and taxed" economy are developed separately below. The results are contrasted with the value-added model described in section 1.

The closed economy with marginal cost transfer prices. Society's objective is to maximize aggregate output defined as deliveries to final demand. The maximum value of aggregate deliveries to final demand (μ) can be expressed as a function of all quantities of sectoral deliveries to final demand (Y_j), all primary inputs, and time:

(11) $\mu = G(Y_1, Y_2, \ldots, Y_n, L, K, t)$.

The economy-wide maximization problem is constrained by the supplies of primary inputs and by linearly homogeneous sectoral production functions:

(12) $Z_j = g^j (L_j, K_j, X_{1j}, X_{2j}, \ldots, X_{nj}, t) \qquad (j = 1, 2, \ldots, n)$.

It necessarily follows that the aggregate function G is homogeneous of degree minus one in sectoral deliveries to final demand, degree one in labor and capital inputs, and degree zero in final demand deliveries and all inputs. Though intermediate inputs do not appear explicitly in the macro model (11), they enter the problem through the constraints (12). Microeconomic production for delivery to intermediate demand is an important part of the solution to the macroeconomic problem maximizing deliveries to final demand.

Given the homogeneity properties of the production-possibilities frontier

(13) $1 = G(Y_1, Y_2, \ldots, Y_n, L, K, t)$,

necessary conditions for producer equilibrium in a competitive economy require the following relations:

$$(14)\quad \frac{\partial \ln G}{\partial \ln Y_j} = \frac{-q_j^y Y_j}{\sum_j q_j^y Y_j} \qquad \frac{\partial \ln G}{\partial \ln L} = \frac{p_L L}{\sum_j q_j^y Y_j} \qquad \frac{\partial \ln G}{\partial \ln K} = \frac{p_K K}{\sum_j q_j^y Y_j}$$

$$(15a)\quad \sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} = 1$$

and

$$(15b)\quad \frac{p_L L}{\sum_j q_j^y Y_j} + \frac{p_K K}{\sum_j q_j^y Y_j} = 1,$$

where q_j^y is the price of goods delivered to final demand in the jth sector.

Conditions (15a) and (15b) follow directly from accounting identities. Condition (15a) is true by construction. Condition (15b) follows from the equality of the value of a sector's total production both to the sum of the values of that sector's deliveries to intermediate and final demands and to the sum of that sector's payments to all factors of production:

$$(16)\quad q_j Z_j = \sum_i q_j^x X_{ji} + q_j^y Y_j \qquad (j = 1, 2, \ldots, n)$$

$$(17)\quad q_j Z_j = p_{L_j} L_j + p_{K_j} K_j + \sum_i p_i X_{ij} \qquad (j = 1, 2, \ldots, n),$$

where q_j^x is the price of goods delivered to intermediate demand in the jth sector.[6] Summing (16) and (17) over all sectors and equating the results produces the condition required by (15b):

$$(18)\quad \sum_j q_j^y Y_j = \sum_j p_{L_j} L_j + \sum_j p_{K_j} K_j$$
$$= p_L L + p_K K$$

since, given the equivalence of producer and consumer prices in markets for intermediate goods ($q_j^x = p_j, \forall j$), $\sum_j \sum_i q_j^x X_{ji} = \sum_j \sum_i p_i X_{ij}$.

The rate of aggregate productivity growth E_y can be derived by substituting the equilibrium conditions (14) into the total logarithmic derivative of G taken with respect to time:

$$(19)\quad E_y = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^y Y_j} \frac{d\ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^y Y_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j q_j^y Y_j} \frac{d\ln K}{dt}.$$

The rate of aggregate productivity growth for a model stated in terms of deliveries to final demand can be expressed as the weighted average of the rates of growth of sectoral deliveries to final demand less a weighted average of rates of growth of labor and capital inputs.

Distinctly different characterizations of macroeconomic objectives and microeconomic activity are modeled by the value-added and delivery-to-final demand production-possibilities frontiers F and G defined, respectively, in (4) and (13). The frontier F is concerned with maximizing aggregate net output, is defined in terms of sectoral value added, and is based on value-added functions for each sector. The macro economy is viewed as if it were a set of horizontally independent sectors. Intersectoral transactions are ignored. Value-added separability, with all its implications for sectoral and aggregate productivity growth, is a maintained assumption. In contrast, the frontier G is concerned with maximizing deliveries to economy-wide final demand, is stated in terms of sectoral deliveries to final demand, and is based on gross output production functions for each producing sector. The macro economy is modeled as a collection of vertically interdependent sectors. An active market in intersectoral transactions is recognized explicitly.

In spite of these fundamental differences, the following proposition holds:

> <u>Proposition 2</u>: For an economy both closed to trade in foreign-produced inputs and with marginal cost pricing on intersectoral transfers, value-added and delivery-to-final-demand models of macroeconomic production lead to identical measures of aggregate productivity growth.

Verifying this proposition is a straightforward exercise. Consider first the value-added separable production function (3) and the functional expression for gross output implicit in (16):

$$(20)\quad Z_j = Z^j\,(Y_j,\ X_{j1},\ X_{j2},\ldots,X_{jn}) \qquad (j = 1,\ 2,\ldots,n).$$

Totally differentiating (3) and (20) with respect to time and applying equilibrium conditions derived above leads to expressions that, respectively, can be written in the forms:

$$(21)\quad \frac{d\ln V_j}{dt} = \frac{q_j Z_j}{q^v_j V_j}\,\frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q^v_j V_j}\,\frac{d\ln X_{ij}}{dt} \qquad (j = 1,\ 2,\ldots,n)$$

$$(22)\quad \frac{d\ln Z_j}{dt} = \sum_i \frac{q^x_j X_{ji}}{q_j Z_j}\,\frac{d\ln X_{ji}}{dt} + \frac{q^y_j Y_j}{q_j Z_j}\,\frac{d\ln Y_j}{dt} \qquad (j = 1,\ 2,\ldots,n).$$

Substituting (22) into (21) and substituting the resulting expression for the growth of sectoral value added into equation (10) yields

$$(23)\quad E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j} \frac{d\ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^v V_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j q_j^v V_j} \frac{d\ln K}{dt}.$$

Multiplying this expression by the ratio of the value of aggregate value added to the value of aggregate deliveries to final demand produces an expression identical to the definition of E_y in (19):

$$(24)\quad \frac{\sum_j q_j^v V_j}{\sum_j q_j^y Y_j} E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} \frac{d\ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^y Y_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j q_j^y Y_j} \frac{d\ln K}{dt}$$

so that

$$(25)\quad E_v = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^v V_j} E_y .$$

Finally, the ratio premultiplying E_y in (25) equals unity. As defined in (8), the value of sectoral value added equals the value of sectoral output less that sector's purchases of intermediate input. Substituting the value sum of deliveries to intermediate and final demands, equation (16), for the value of total output in (8) yields:

$$(26)\quad q_j^v V_j = \sum_i q_j^x X_{ji} + q_j^y Y_j - \sum_i p_i X_{ij} \qquad (j = 1, 2, \ldots, n).$$

Summing (26) over all sectors produces the expected result:

(27) $\sum_j q_j^v V_j = \sum_j q_j^y Y_j$

since

$$\sum_j \sum_i q_j^x X_{ji} = \sum_j \sum_i p_i X_{ij} \ .$$

Proposition 2 follows directly from (25) and (27): $E_V = E_Y$. The different initial descriptions of macroeconomic objectives and microeconomic production offered by value-added and delivery-to-final-demand models are of no consequence to productivity accounting in a closed economy with marginal cost transfer prices. The models produce identical measures of aggregate productivity growth.

<u>The open economy with tax distorted transfer prices</u>. The development of this model differs from its untaxed closed economy counterpart in two important ways. First, imported inputs enter both the aggregate production-possibilities frontier and the sectoral production functions. Second, taxes and/or subsidies on intersectoral transactions enter the analysis through market equilibrium conditions.

The model, though focusing on imported inputs and taxes on intersectoral transfers, places no restrictions on wider aspects of the economy's international trade or tax structure. Imported final products and exports may exist but need not be considered. Imports that are delivered directly to domestic final demand are not included in the model since they bear no influence on domestic productivity measurement. Since exports are goods and services produced domestically and no longer consumed internally, exports simply are part of deliveries to final demand. Similarly, final demand deliveries may be subject to transaction taxes yet, since productivity is modeled in terms of prices faced by producers, prices paid by consumers of products delivered to final demand never enter the model. In contrast, prices paid by consumers of intermediate inputs do enter the model. Those taxes or subsidies differentiating consumer and producer prices for goods delivered to intermediate demand must be considered explicitly.[7]

One assumption, however, is introduced. All taxes collected on intermediate demand deliveries are assumed to be disbursed as subsidies to other intersectoral transactions:

$$(28) \quad \sum_i\sum_j p_i X_{ij} = \sum_j\sum_i q_j^x X_{ji},$$

where $p_i \gtreqless q_i^x$ for all i. This simplifies the comparison between delivery-to-final-demand and value-added models without compromising the importance of accounting for intersectoral transfer prices.

Given this setting, the economy's production problem is to maximize aggregate deliveries to final demand (μ)

$$(29) \quad \mu = H(Y_1, Y_2, \ldots, Y_n, L, K, M_1, M_2, \ldots, M_u, t),$$

where M_m is the quantity of the mth imported input, subject to fixed supplies of domestic labor and capital inputs, market equilibrium conditions, and linearly homogeneous sectoral production functions:

$$(30) \quad Z_j = h^j\,(L_j, K_j, X_{1j}, X_{2j}, \ldots, X_{nj}, M_{ij}, M_{2j}, \ldots, M_{uj}, t) \qquad (j = 1, 2, \ldots, n).$$

Like the production function g^j defined in (12), h^j describes an unconstrained model of sectoral production.

Setting the aggregate value of μ equal to unity, the constant returns to scale function H can be expressed as a production-possibilities frontier:

(31) $1 = H(Y_1, Y_2, \ldots, Y_n, L, K, M_1, M_2, \ldots, M_u, t).$

Homogeneity and market equilibrium conditions together imply:

$$\frac{\partial \ln H}{\partial \ln Y_j} = \frac{-q_j^y Y_j}{\sum_j q_j^y Y_j} \qquad (j = 1, 2, \ldots, n)$$

(32) $$\frac{\partial \ln H}{\partial \ln L} = \frac{p_L L}{\sum_j q_j^y Y_j} \quad \frac{\partial \ln H}{\partial \ln K} = \frac{p_K K}{\sum_j q_j^y Y_j} \quad \frac{\partial \ln H}{\partial \ln M_m} = \frac{p_{Mm} M_m}{\sum q_j^y Y_j} \qquad (m = 1, 2, \ldots, u)$$

(33) $$\sum_j \frac{\partial \ln H}{\partial \ln Y_j} = -\sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} = -1$$

and

(34) $$\frac{\partial \ln H}{\partial \ln L} + \frac{\partial \ln H}{\partial \ln K} + \sum_m \frac{\partial \ln H}{\partial \ln M_m} = \frac{p_L L}{\sum_j q_j^y Y_j} + \frac{p_K K}{\sum_j q_j^y Y_j} + \sum_m \frac{p_{Mm} M_m}{\sum_j q_j^y Y_j} = 1,$$

where p_{Mm} is the price of the mth imported input.[8]

The rate of aggregate productivity growth (E_y') appropriate for an open economy is derived by substituting the necessary conditions for producer equilibrium (32) into the total time derivative of the aggregate frontier H. The resulting expression is:

(35)
$$E_y^{'} \equiv \frac{\partial \ln H}{\partial t} = \sum_j \frac{q_j Y_j}{\sum_j q_j Y_j} \frac{d\ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j Y_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j q_j Y_j} \frac{d\ln K}{dt} - \sum_m \frac{p_{Mm} M_m}{\sum_j q_j Y_j} \frac{d\ln M_m}{dt} .$$

Comparing the expressions for E_y and $E_y^{'}$ defined in (19) and (35), respectively, leads to the following proposition:

> Proposition 3: The rate of aggregate productivity growth for an economy maximizing aggregate output (deliveries to final demand) is defined as the weighted average of rates of growth of sectoral deliveries to final demand (with weights equal to sectoral delivery-to-final-demand shares in aggregate output) less the weighted average of rates of growth of primary inputs (with weights equal to the value shares of input payments in aggregate output). Both sets of weights sum to unity. For a closed economy primary inputs include domestic supplies of labor and capital. For an open economy, primary inputs include domestic labor and capital and imported materials.

This, together with Proposition 1, leads to the following result regarding the relation between E_v and $E_y^{'}$:

> Proposition 4: For an economy open to trade in foreign-produced inputs and having taxes and/or subsidies on deliveries to intermediate demand, the rate of aggregate productivity growth derived from a value-added model of aggregate production differs from the corresponding rate resulting from a delivery-to-final-demand model. The two rates differ by an amount that depends on (1) the relative importance of imported inputs in domestic production and (2) changes in the sectoral distribution of domestically produced intermediate inputs.

Proving this proposition begins with both the sectoral output function (20) defining each sector's output in terms of its delivery to final and intermediate demand components and the value-added separable production function for any sector in an open economy:

$$(36)\quad Z_j = f^j[V^j(L_j, K_j, t), X_{1j}, X_{2j}, \ldots, X_{nj}, M_{1j}, M_{2j}, \ldots, M_{uj}] \qquad (j = 1, 2, \ldots, n).$$

The time derivative of (20) is presented in equation (22); the growth rate of value added can be derived from the logarithmic time derivative of (36):

$$(37)\quad \frac{d\ln V_j}{dt} = \frac{q_j Z_j}{q_j^v V_j} \frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^v V_j} \frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_{Mm} M_{mj}}{q_j^v V_j} \frac{d\ln M_{mj}}{dt}$$

$$(j = 1, 2, \ldots, n).$$

Substituting (22) into (37) and substituting the resulting expression for the growth of sectoral value added into equation (10), the expression for E_v, yields:

$$(38)\quad E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j} \frac{d\ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^v V_j} \frac{d\ln L}{dt} - \frac{p_K K}{\sum_j q_j^v V_j} \frac{d\ln K}{dt}$$

$$- \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^v V_j} \frac{d\ln X_{ij}}{dt} - \sum_j \sum_m \frac{p_{Mm} M_{mj}}{\sum_j q_j^v V_j} \frac{d\ln M_{mj}}{dt},$$

where $\Gamma_i \equiv (p_i - q_i^x) \gtrless 0$ is the tax imposed on the transfer price of the ith commodity delivered to intermediate demand. Multiplying (38) by the ratio $\sum q_j^v V_j / \sum q_j^y Y_j$ and substituting the definition of E_y' in (35) into the resulting expression produces the following relation:

$$(39)\quad E_v = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^v V_j} E_y' - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^v V_j} \frac{d\ln X_{ij}}{dt}.$$

The importance of taxes on intersectoral transactions is explicit in equation (39). The role of trade in foreign-produced inputs is implicit in the ratio $\sum q_j^y Y_j / \sum q_j^v V_j$. While that ratio equals unity for a closed economy, it is greater than unity for an open economy. Substituting (16) for the value of total output, $q_j Z_j$, into the accounting identity

$$(40)\quad q_j^{v}V_j = q_jZ_j - \sum_i p_iX_{ij} - \sum_m p_{Mm}M_{mj}$$

and summing over all n sectors yields:

$$(41)\quad \sum_j q_j^{v}V_j = \sum_j q_j^{y}Y_j - \sum_{jm}\sum p_{Mm}M_{mj}.$$

The value of aggregate deliveries to final demand exceeds the value of aggregate value added by an amount equal to the value of imported inputs. Substituting (41) into (39) makes explicit the importance of imported inputs and Proposition 4 follows directly:

$$(42)\quad E_v = E_y' + \left[\frac{\sum\limits_{jm}\sum p_{Mm}M_{mj}}{\sum\limits_j q_j^{y}Y_j - \sum\limits_{jm}\sum p_{Mm}M_{mj}}\right]E_y' - \sum_{ji}\sum \frac{\Gamma_iX_{ij}}{\sum\limits_j q_j^{y}Y_j - \sum\limits_{jm}\sum p_{Mm}M_{mj}}\ \frac{d\ln X_{ij}}{dt}.$$

The two rates of productivity growth E_v and E_y' differ by an amount that depends on (1) the relative importance of imported inputs in domestic production and (2) changes in the sectoral distribution of domestically produced intermediate inputs.

If either taxes and subsidies equal zero ($\Gamma_i = 0, \forall_i$) or the distribution of intermediate inputs is unchanging ($d\ln X_{ij}/dt = d\ln X_j/dt, \forall_i$), equation (39) reduces to

$$(43)\quad E_v = E_y' + \left[\frac{\sum\limits_{jm}\sum p_{Mm}M_{mj}}{\sum\limits_j q_j^{y}Y_j - \sum\limits_{jm}\sum p_{Mm}M_{mj}}\right]E_y'.$$

It necessarily follows that $E_v > E_y'$.

Conversely, if the economy is closed to trade in foreign-produced inputs ($M_m = 0, \forall m$), equation (39) reduces to

$$(44) \quad E_v = E_y' - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\dfrac{y}{\sum_j q_j Y_j}} \frac{d\ln X_{ij}}{dt}.$$

The relation between E_v and E_y' is no longer unambiguous but depends on the changing composition of intermediate inputs. For any period during which the distribution of intermediate inputs shifts to relatively higher taxed products, $E_y' > E_v$. Conversely, $E_v > E_y'$ during any period when the composition of intermediate inputs shifts to more heavily subsidized transactions.

The important summary conclusion is that value-added and delivery-to-final-demand models of aggregate production produce identical measures of aggregate productivity growth if and only if the economy is closed to trade in foreign-produced inputs and has either no taxes on deliveries to intermediate demand or an unchanging distribution of intermediate inputs. In the absence of these limiting conditions, $E_v \neq E_y'$.

3. Sources of Growth

The value-added and delivery-to-final-demand models of aggregate production are constrained by different models of microeconomic production and intersectoral transfers. One objective of this section is to make these differences explicit by decomposing the alternative measures of aggregate productivity growth, E_v and E_y', into their microeconomic source components.

Sectoral productivity growth. The economy's value-added maximization problem described in (1) is constrained by the value-added subfunctions V^j defined in (2). The rate of sectoral productivity growth e_V is derived by substituting conditions for producer equilibrium into the logarithmic time derivative of V^j:[9]

$$(45)\quad e_V^j \equiv \frac{\partial \ln V^j}{\partial t} = \frac{d\ln V_j}{dt} - \frac{p_{L_j}L_j}{q_j^{v}V_j}\frac{d\ln L_j}{dt} - \frac{p_{K_j}K_j}{q_j^{v}V_j}\frac{d\ln K_j}{dt}. \qquad (j = 1, 2, \ldots, n).$$

Given value-added separability, neither foreign-produced inputs nor domestically-produced intermediate inputs enter the sectoral functions V_j or the resulting expression for sectoral productivity growth.

Microeconomic production in the delivery-to-final-demand model, in contrast, is characterized by the sectoral production functions h^j defined in (30). The rate of sectoral productivity growth e_Z^j is solved by imposing equilibrium conditions on the logarithmic time derivative of h^j:[10]

$$(46)\quad e_Z^j \equiv \frac{\partial \ln h^j}{\partial t} = \frac{d\ln Z_j}{dt} - \frac{p_{L_j}L_j}{q_jZ_j}\frac{d\ln L_j}{dt} - \frac{p_{K_j}K_j}{q_jZ_j}\frac{d\ln K_j}{dt}$$

$$- \sum_i \frac{p_iX_{ij}}{q_jZ_j}\frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_{Mm}M_{mj}}{q_jZ_j}\frac{d\ln M_{mj}}{dt} \qquad (j = 1, 2, \ldots, n).$$

The variables e_V^j and e_Z^j provide different descriptions of sectoral productivity growth, leading to the following proposition:

> Proposition 5: For an economy open to trade in foreign-produced inputs and subject to taxes on deliveries to intermediate demand, the rate of sectoral productivity growth derived from a sectoral value added production function is greater than the corresponding rate resulting from a sectoral gross output production function. The two rates differ by an amount that is proportional to the share of the sector's pur-

chases of domestic intermediate and foreign-produced inputs in the value of sectoral gross output.

Formally demonstrating this proposition begins with substituting equation (37), the logarithmic time derivative of value added, into equation (45):

$$(47)\quad e_V^j = \frac{q_j Z_j}{q_j^v V_j}\frac{d\ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^v V_j}\frac{d\ln X_{ij}}{dt} - \sum_m \frac{p_{Mm} M_{mj}}{q_j^v V_j}\frac{d\ln M_{mj}}{dt}$$

$$- \frac{p_{Lj} L_j}{q_j^v V_j}\frac{d\ln L_j}{dt} - \frac{p_{Kj} K_j}{q_j^v V_j}\frac{d\ln K_j}{dt} \qquad (j = 1, 2, \ldots, n).$$

Substituting (46) into (47) and rearranging terms yields:

$$(48)\quad e_Z^j = \left[1 - \frac{\sum_i p_i X_{ij} + \sum_m p_{Mm} M_{mj}}{q_j Z_j}\right] e_V^j \qquad (j = 1, 2, \ldots, n);$$

It necessarily follows that $e_V^j > e_Z^j$. The two rates differ by an amount that is proportional to the share of sectoral purchases of domestically-produced and foreign-produced inputs in the value of sectoral gross output.[11]

Note that the relationship between e_Z^j and e_V^j is unaffected by the presence or absence of taxes on intersectoral transfers. Though the tax created differentials between consumer and producer prices of goods delivered to intermediate demand may influence aggregate productivity growth, those differentials do not influence the measures of sectoral productivity growth. Both measures e_Z^j and e_V^j are sector specific and depend on equilibrium conditions unique to the jth sector.

Only prices <u>paid</u> by the jth sector for intermediate inputs matter. Prices <u>received</u> by producers of intermediate inputs delivered to the jth sector do not enter the formulas defining e_Z^j or e_V^j.

<u>Sources of productivity growth</u>. The formal decomposition of E_V into its source components is simple and direct.

> <u>Proposition 6</u>: The rate of aggregate productivity growth E_V for a value-added model of aggregate production equals the sum of two components: (1) a weighted <u>average</u> of the sectoral rates of productivity growth e_V^j, with weights equal to sectoral value shares of value added in total value added; and (2) reallocation terms reflecting the aggregate productivity gains resulting from the economy's recomposition of labor and capital inputs among sectors.

This result is derived by substituting the expression for e_V^j, equation (45), into equation (10), the expression for E_V:

$$(49) \quad E_V = \sum_j \frac{q_j^V V_j}{\sum_j q_j^V V_j} e_V^j + \sum_j \frac{(p_{Lj}-p_L)L_j}{\sum_j q_j^V V_j} \frac{d\ln L_j}{dt} + \sum_j \frac{(p_{Kj}-p_K)K_j}{\sum_j q_j^V V_j} \frac{d\ln K_j}{dt}.$$

The labor and capital reallocation terms identify the productivity consequences resulting from any sectoral recomposition of the economy's primary inputs. Each term has a straightforward interpretation. Consider the one corresponding to capital. The variable p_K, recall, refers to the average price of capital input in the aggregate economy; p_{Kj} represents the return to capital input in the jth sector. Aggregate productivity growth is affected by the reallocation of capital among sectors with varying rates of return. If, for example, capital input moves from a sector with a relatively low rate of return ($p_{Kj} < p_K$) to a sector with a high rate of return ($p_{Ki} > p_K$), the quantity of capital input for the economy as a whole is unchanged, but the level of output is increased--that is, aggregate productivity is enhanced. In this example, the capital reallocation component would have a positive value. Similarly, sectoral shifts in labor resources are monitored by the labor reallocation term. If labor input moves from a sector where it has relatively low marginal productivity to a sector where it has relatively high marginal productivity, economy-wide labor input is unchanged but aggregate productivity is improved.

The important but not surprising observation that follows from (49) is that neither international nor intersectoral transactions, whether stated in nominal or real terms, enters the source decomposition of E_V. Imported inputs enter neither the aggregate nor sectoral production accounts. Since sectors are viewed as horizontally independent producers, sectoral reallocations of deliveries to intermediate demand and their transfer prices are ignored.

The delivery-to-final-demand model of aggregate production and its microeconomic production constraints, in contrast, view the economy as consisting of sectors depending not only on each other but on international trade as well. Given this vertical model of economic activity, advances in productivity in an individual industry contribute to aggregate economic growth both directly through deliveries to final demand and indirectly through increased deliveries to sectors dependent on its output as intermediate input. The source decomposition of E_Y' must capture not only both direct and indirect transmissions of sectoral productivity growth but also changes in the efficiency of markets linking buyers and sellers of intermediate inputs.

<u>Proposition 7</u>: The rate of aggregate productivity growth E_y for a delivery-to-final-demand model of aggregate production equals the sum of three components: (1) a weighted <u>sum</u> of the sectoral rates of productivity growth e_Z^j, with weights equal to sectoral ratios of the value of gross output to the value of total deliveries to final demand; (2) reallocation terms for labor and capital inputs; and (3) a term measuring the net productivity effect of changes in allocative efficiency across all intermediate demand markets.

Demonstrating this proposition begins with equation (22), the logarithmic time derivative expressing the growth in sectoral output as the sum of its final and intermediate demand components. Substituting (22) into (46) permits the measure of sectoral productivity growth to be expressed in terms identifying the immediate uses of the sector's output:

$$(50)\quad e_Z^j = \frac{\sum_i q_j^x X_{ji}}{q_j Z_j}\,\frac{d\ln X_{ji}}{dt} + \frac{q_j^y Y_j}{q_j Z_j}\,\frac{d\ln Y_j}{dt} - \frac{p_{L_j} L_j}{q_j Z_j}\,\frac{d\ln L_j}{dt}$$

$$- \frac{p_{K_j} K_j}{q_j Z_j}\,\frac{d\ln K_j}{dt} - \frac{\sum_i p_i X_{ij}}{q_j Z_j}\,\frac{d\ln X_{ij}}{dt} - \frac{\sum_m p_{Mm} M_{mj}}{q_j Z_j}\,\frac{d\ln M_{mj}}{dt}$$

$$(j = 1, 2, \ldots, n).$$

Multiplying (50) by the ratio $q_j Z_j / \sum q_j^y Y_j$, summing over all n sectors, and substituting the resulting expression into (35) identifies the unique source components contributing to aggregate productivity growth:

(51)

$$\dot{E}_y = \sum_j \frac{q_j Z_j}{\sum_j q_j^y Y_j} e_z^j + \sum_j \frac{(p_{Lj}-p_L)L_j}{\sum_j q_j^y Y_j}\frac{d\ln L_j}{dt} + \sum_j \frac{(p_{Kj}-p_K)K_j}{\sum_j q_j^y Y_j}\frac{d\ln K_j}{dt}$$

$$+ \sum_j \sum_i \frac{(p_i-q_i^x)X_{ij}}{\sum_j q_j^y Y_j}\frac{d\ln X_{ij}}{dt}.$$

The first term in (51) reflects the sum contribution of productivity growth across individual producing sectors. This term is formed as a weighted sum, not weighted average. The sum of the weights in (51) exceeds unity by an amount equal to the ratio of the aggregate value of deliveries to intermediate demand to the aggregate value of deliveries to final demand:

$$(52) \qquad \sum_j \frac{q_j Z_j}{\sum_j q_j^y Y_j} = \sum_j \frac{(q_j^y Y_j + \sum q_j^x X_{ji})}{\sum_j q_j^y Y_j} = 1 + \sum_j \sum_i \frac{q_j^x X_{ji}}{\sum_j q_j^y Y_j}.$$

The weights sum to a value larger than unity because each sector contributes to the rate of productivity growth for the aggregate economy through its deliveries to both final and intermediate demands.[12]

The remaining three terms in (51) measure the aggregate productivity consequences of the changing composition of economic activity. The labor and capital reallocation terms in (51) have exactly the same interpretation as their respective counterparts in equation (49), the source decomposition of E_y. Shifts in either primary input to higher (lower) marginal productivity uses enhances (diminishes) aggregate productivity performance. The final term in (51) likewise depends on input shifts but has a uniquely different interpretation. Whereas the labor and capital reallocation effects focus on primary input price differences among sectors, the market term concentrates on the differentials between consumer and producer prices for each good delivered to intermediate demand markets.

The productivity effect represented in the market term depends on the tax-induced differentials $(p_i - q_i^x)$ and the growth rates of products delivered to intermediate demand $(d\ln X_{ij}/dt)$. Market equilibrium conditions in a competitive economy require that p_i equal the marginal value product (MVP_i) of the ith intermediate input to consuming sectors while q_i^x equals the marginal cost (MC_i) incurred by the sector producing that input. If there is a tax on the ith product $(p_i > q_i)$, then $MVP_i > MC_i$ and aggregate productivity growth will increase (decrease) if deliveries of X_i to intermediate demand are increased (decreased). Subsidized transactions have exactly the opposite effect since $\Gamma_i < 0$ necessarily implies $MVP_i < MC_i$. Finally, should there be neither taxes nor subsidies on the economy's internal transactions, then $MVP_i = MC_i$ for all i and the market terms equal zero. In this case, proper transfer prices guide transactions among vertically related sectors. When transfer prices reflect marginal costs, changes in the composition of deliveries to intermediate demand have no independent effect on aggregate productivity growth.

The decomposition presented in equation (51) makes clear that the measure E_Y' is sensitive to the economy's internal transfers among interdependent sectors and to its external relations with other economies. The role of imported inputs is captured in the sectoral productivity measures e_z^j. The transmission of each sector's productivity growth throughout the economy is reflected in the sectoral productivity weights $(q_j Z_j / \sum q_j^Y Y_j)$. The market term models the efficiency of intermediate demand markets. The assumption of value-added separability assures that none of these effects appears in the source decomposition of E_V presented in equation (49). In short, only the aggregate performance measure E_Y' is sensitive to international trade, deliveries to intermediate demand, and intersectoral transfer prices.

4. Conclusion

A measure of aggregate productivity growth is useful only as a comparative measure of economic performance over time or across countries. The source decompositions derived above demonstrate that the delivery-to-final-demand model is sensitive to intertemporal and international variations in both imported input requirements and the allocative efficiency of domestic markets. The value-added model is not. As equation (42) makes clear, E_V can be made equal to E_Y' only after adjustments for the extent of the economy's trade in foreign-produced inputs, the structure of taxes on intersectoral transfers, and the changing distribution of deliveries to intermediate demand.

This result has important practical consequences for both intertemporal and international comparisons of productivity growth. Transactions in intermediate inputs can be suppressed if and only if the composition of intermediate input purchases is unchanging or transfer prices equal marginal costs. Neither condition is true.

Between 1967 and 1972, for example, the U.S. motor vehicle and equipment sector increased its deliveries to intermediate demand by 40 percent. During this same period, the construction and mining machinery sector decreased its intermediate demand deliveries by 19 percent.[13] The pattern in each country depends on each economy's cyclical behavior.

The allocative efficiency implications of these redistributions depend on the structure of intersectoral transfer prices. Excise taxes are just one example of how transfer prices may fail to reflect marginal costs. More importantly, the structure of these taxes differs significantly across countries. Between 1973 and 1979, for example, excise taxes increased by 13 percent in the United States while incresing by 50 percent in both Canada and the Netherlands.[14]

The important conclusion is that domestic transactions in intermediate inputs do affect aggregate productivity growth and cannot be judged _a priori_ to be unimportant. After all, intermediate inputs account for nearly 65 percent of production costs in U.S. manufacturing.[15] Deliveries to intermediate demand account for approximately 45 percent of total output in the U.S. private domestic economy.[16] Accounting for intertemporal and international differences in both the structure of transfer prices and the changing distribution of intermediate inputs cannot be ignored.

The analysis of imported inputs leads to a similar conclusion. The current price ratio of imported inputs to deliveries to final demand in 1977 was 8.1 percent in the United States but was 18.1 and 22.2 percents in France and Italy, respectively, and as high as 31.1

percent in the Netherlands.[17] Moreover, these ratios change at differing rates over time. The U.S. ratio doubled between 1969 and 1977. During the same period, the ratios for Italy and France increased by 50 percent and 33 percent, respectively, while the ratio increased by only 5 percent in the Netherlands.[18] The important conclusion is that these trade ratios not only change over time but also vary considerably across countries.

It necessarily follows from equation (42) that the rates E_v and E_y' maintain no constant relation either across countries or over time. In short, conclusions regarding relative rates of productivity growth over time and/or across nations can be biased by a value-added model.

The delivery-to-final-demand model forms the appropriate basis for measuring aggregate productivity growth. It treats all inputs symmetrically. It monitors changes in the allocative efficiency of intermediate demand markets. It views the economy as a set of vertically interdependent sectors. It models how an individual sector transmits the benefits of its productivity growth to final consumers both directly through deliveries to final demand and indirectly through deliveries to intermediate demand. Unless affirmed by economic evidence, value-added separability is an inappropriate as well as unnecessary assumption.

The flexibility of the delivery-to-final-demand model provides an opportunity to generalize productivity modeling significantly. It offers a framework for evaluating the productivity consequences of imperfect competition in domestic markets. It provides a formal structure for analyzing regional productivity growth among trading nations. The effects of tariffs and quotas on international transfer prices can be modeled explicitly. Proper measurement is the first step toward identifying the true sources of aggregate productivity growth.

Notes

1. Kendrick (1973), p. 16.

2. This proposition will be demonstrated formally in section 1.

3. Important differences in the demographic and occuptional composition of labor input and the asset mix of capital input are suppressed to reduce notation. Distinct categories of labor and capital inputs can be introduced without changing any of the propositions derived in this paper.

4. These conditions are required by the linearly homogeneous sectoral value-added functions which constrain the macroeconomic maximization problem.

5. Intermediate input prices p_i do not have j subscripts. It is assumed that there is no price discrimination across sectors in the markets for intermediate inputs. All sectors purchasing the ith intermediate input (X_{ij}, $\forall j$) pay the identical f.o.b. price p_i.

6. Note that there is no requirement that the price of the jth sector's deliveries to final demand equal the price of that sector's deliveries to intermediate demand.

7. These arguments suggest only that imported final products and taxes on final deliveries do not enter the formula for productivity measurement. They do not argue that these variables have no effect on aggregate productivity growth. Clearly, they do by influencing the economy's primary input requirements and its level of final production.

8. The fact that the input shares in aggregate output sum to unity can be verified from the sectoral accounting identity:

$$\sum_i q_j^x X_{ji} + q_j^y Y_j = p_{Lj} L_j + p_{Kj} K_j + \sum_i p_i X_{ij} + \sum_m p_{Mm} M_{mj}.$$

Summing the identity over all n sectors produces the result required by (34). Note also that import prices p_{Mm} do not have j subscripts. It is assumed that all domestic sectors pay the same f.o.b. price for identical imported materials.

9. Competitive equilibrium conditions at the sectoral level require that output elasticities with respect to inputs equal the corresponding value shares of input payments in sectoral output. Linear homogeneity requires that both the output elasticities and the value shares each sum to unity.

10. See note 9.

11. Note that even if the jth sector imports no foreign-produced inputs ($M_{mj} = 0, \forall m$), the measure e_V^j still will be greater than e_Z^j.

12. Gollop (1979), pp. 330-31, presents an heuristic interpretation of this weighting structure:

> Consider an individual sector that experiences an advance in its rate of technical change. Holding constant all primary and intermediate inputs, the sector can provide the economy with increased output. The objective of creating the appropriate weight for this sector's technological advance is to correctly assign causal responsibility to this sector for any effect on aggregate productivity growth. Since the ultimate macroeconomic concern is the effect of this technical change on aggregate output, the appropriate denominator in the weight is the sum of all n sector contributions to aggregate output. Since the individual sector transmits the benefits of its productivity growth to final consumers both directly through deliveries to final demand and indirectly through deliveries to intermediate demand, the appropriate numerator in the weight is the sector's total output, i.e., the sum of its deliveries to final and intermediate demand.

13. These percentages were calculated from the current price 1967 and 1972 input-output transaction matrices reported respectively in Bureau of Economic Analysis (1974) and Ritz, Roberts, and Young (1979).

14. United Nations Department of International Economic and Social Affairs (1977-80).

15. This percentage is based on 1972 input-output transactions data reported in Ritz, Roberts, and Young (1979).

16. <u>Ibid.</u>

17. Gollop (1981), pp. 33-34.

18. <u>Ibid.</u>

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