

International R & D Rivalry and Industrial Strategy

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This paper presents a theory of government intervention which provides an explanation for "industrial strategy" policies such as R & D or export subsidies in imperfectly competitive international markets. Domestic net welfare is improved by the capture of a greater share of the output of rent earning industries, although the subsidy-ridden noncooperative international equilibrium is jointly suboptimal. Behaviour of governments and firms is modelled as a three stage subgame perfect Nash equilibrium. The assumption that the government is the first player in this game allows it to influence equilibrium outcomes by altering the set of credible actions open to firms.

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1. Introduction

National governments in a number of countries subsidize research and development (R & D) activities of domestic firms, particularly in industries in which foreign and domesticly owned firms are in competition for international markets. This paper presents a positive theory to explain such industrial strategy policies in the context of an imperfectly competitive world where the R & D rivalry between firms plays an important role. It is shown that a government which has the objective of maximizing domestic welfare has an incentive to introduce such subsidies.

We focus on possible subsidization of cost-reducing or "process" R & D such as, for example, the recent Japanese and French subsidization of robotics in automobile assembly. Export subsidies are also considered. We show that with export subsidies available, countries would not choose to subsidize R & D. Nevertheless, because GATT codes effectively restrict direct export subsidies, we view the setting in which only R & D subsidies are available as the most relevant case.

R & D is assumed to be undertaken before the associated output is produced, with firms anticipating the effect of R & D on the resolution of output shares. Thus R & D serves as a commitment or credible threat, along the lines considered in Spence (1977, 1979), Friedman (1979), Dixit (1980) and Eaton and Lipsey (1980, 1981). In contrast to these papers, however, where an established firm acts first¹, we use a model developed in Brander and Spencer (1983) where firms have equal opportunity² in setting R & D levels.

The efficacy of government policy in this paper arises from the assumption that a government can credibly commit itself to R & D (or output) subsidies before the R & D decisions are made by private firms. We would like to emphasize that this is an example of a more general principle in understanding government policy: the government becomes the first player in a multi-stage game and can influence the equilibrium outcome of the game played by private agents by altering the set of credible actions open to them.

The motivation for government policy in the paper is that it is to the advantage of a country to "capture" a larger share of the production of imperfectly competitive rent earning industries operating in international markets. It is well known that monopoly power in international trade provides incentives for governments to carry out policies designed to extract rent from foreign trade. This includes the monopoly tariff argument (as in Johnson (1953)), the use of export taxes and encouragement of export cartels to exploit the monopoly power of domestic firms

(as in, for example, Auquier and Caves (1979)), or the use of tariffs to extract rent from foreign imperfectly competitive firms (as, for example, in Brander and Spencer (1981)). The capture idea is somewhat different and is more closely related to the classic infant industry argument in trade theory. The difference here is that national incentives do not arise from positive externalities or because capital markets are imperfect, but simply from obtaining a larger domestic share of internationally profitable industries.

2. Overview

The model of firm behaviour is based on a two stage game played by two competing firms, which we imagine to be located in different countries. In the first stage firms choose R & D levels, and in the second stage, output levels. The second stage equilibrium is a Nash equilibrium in outputs, taking R & D levels as given by the preceeding stage. Using this second stage solution we can then write down the payoff functions for the game played at the preceding stage: the profits of each firm are written as a function of the pair of R & D levels chosen. We seek a Nash equilibrium in that game. This gives rise to a subgame perfect equilibrium in the two stage game³, which is characterized by inefficiently high levels of R & D for the output levels chosen.

This basic game is subsequently extended in several ways. First, the government of one country is allowed to make a prior commitment to subsidize R & D. Then, both governments are allowed to simultaneously

set such R & D subsidies. Finally the domestic government is allowed to announce an export subsidy simultaneously with its announcement of a subsidy (or tax) on R & D. In each case we examine the subgame perfect equilibrium for the extended three stage game.

The principle results are as follows. With a single government allowed to subsidize R & D we get an outcome equivalent to that which would obtain if a leader-follower equilibrium were used in the basic game at the "choice of R & D" stage. If both governments can subsidize R & D both will do so. Finally, if R & D and output taxes or subsidies are available, both governments will tax R & D, thereby achieving production efficiency, and will accompany this by a subsidy on exports.

Section 3 contains the basic model, the government is introduced and the main results are developed in section 4, and section 5 contains concluding remarks.

3. The Basic Model

We begin by analysing the last stage (the choice of output stage) in the firms' rivalry. Each firm i produces output y^i at variable cost C^i , which includes all costs except R & D, and earns revenue R^i . The R & D level of firm i is denoted x^i and costs v^i per unit. Profit π^i of firm i is then

$$\pi^i(y^1, y^2; x^i) = R^i(y^1, y^2) - C^i(y^i; x^i) - v^i x^i \quad (1)$$

Outputs y^1 and y^2 are substitutes and we also assume that increasing the output of one good decreases the marginal revenue of the other. Using subscripts to denote derivatives, this implies

$$R_j^i < 0 \quad ; \quad R_{ij}^i < 0 \quad (2)$$

The effect of an increase in cost-reducing R & D is, of course, to reduce C^i given y^i , and the rate of decrease declines as x^i increases. Marginal cost $\partial C^i / \partial y^i$ is denoted c^i and is assumed to fall as x^i increases:

$$C_x^i < 0 \quad ; \quad C_{xx}^i > 0 \quad ; \quad c^i > 0 \quad ; \quad c_x^i < 0 \quad (3)$$

The Nash equilibrium in outputs is characterized by first order conditions

$$\pi_i^i = R_i^i(y^1, y^2) - c^i(y^i; x^i) = 0 \quad (4)$$

and second order conditions

$$\pi_{ii}^i = R_{ii}^i - c_y^i < 0 \quad (5)$$

We also assume that own effects of output on marginal profit dominate cross effects, giving rise to the following condition:

$$A = \pi_{11}^1 \pi_{22}^2 - \pi_{12}^1 \pi_{21}^2 > 0 \quad (6)$$

This conditions holds for most demand and cost structures, and is a fairly standard condition in noncooperative models because, if it holds globally, it insures uniqueness and global stability of the equilibrium.⁴

The solutions, y^1 and y^2 to (4) depend on x^1 and x^2 and can be written as

$$y^1 = q^1(x^1, x^2) \quad ; \quad y^2 = q^2(x^1, x^2) \quad (7)$$

Output (and market share) depend on marginal cost, which depends on x^i . An increase in cost-reducing R & D by firm 1 will lower c^1 , shift its

reaction function outward and increase its output and market share, as illustrated in Figure 1 by the move from A to B. The reaction functions in the diagram are downward sloping. This follows from total differentiation of (4) with respect to y^1 and y^2 holding x^1 and x^2 constant, which yields the slope of the reaction function, $dy^i/dy^j = -R_{ij}^i/\pi_{ji}^i$ which is negative from (2) and (5).

Figure 1

The algebra corresponding to Figure 1 comes from total differentiation of (4) with respect to y^1 , y^2 and x^1 . Using (5) and (6) it follows that a firm's Nash equilibrium level of output is increasing in own R & D and decreasing in the other firm's R & D.

$$q_i^i = dy^i/dx^i = c_x^i \pi_{jj}^j / A > 0 \quad (8)$$

$$q_i^j = dy^j/dx^i = -c_x^i \pi_{ji}^j / A < 0 \quad (9)$$

This completes the analysis of the output stage in which R & D levels are treated as exogenous. We now analyse the preceeding stage, in which firms choose R & D levels. Firms are aware of the dependence (via (7)) of output on R & D levels. Therefore, profits can be written as functions of x^1 and x^2 . Let g^i (for gain) represent the profit function for firm i.

$$g^i \equiv \pi^i(q^1(x^1, x^2), q^2(x^1, x^2) ; x^i) \quad (10)$$

The Nash equilibrium in R & D levels using (1), (4), and (10) is characterized by the first order conditions for each firm

$$\begin{aligned} g_i^1 &= \pi_i^1 q_i^1 + \pi_j^1 q_i^j - C_x^1 - v^1 = 0 \\ &= R_j^1 q_i^j - C_x^1 - v^1 = 0 \end{aligned} \quad (11)$$

since $\pi_i^1 = 0$ and $\pi_j^1 = R_j^1$. The second order condition is

$$g_{ii}^1 = R_j^1 q_{ii}^j + q_i^j dR_j^1/dx^1 - c_{xx}^1 q_i^1 - C_{xx}^1 < 0 \quad (12)$$

and we also assume the condition analogous to (6)

$$D = g_{11}^1 g_{22}^2 - g_{12}^1 g_{21}^2 > 0 \quad (13)$$

$$\text{where } g_{ij}^1 = R_j^1 q_{ij}^j + q_i^j dR_j^1/dx^j - c_{xx}^1 q_i^1$$

Condition (13) implies that own effects of R & D on marginal profit dominate cross effects. As was the case with condition (6), (13) ensures stability with the standard adjustment mechanism and ensures uniqueness, provided (13) holds globally.

The sign of g_{ij}^1 is important for some of the comparative static results. From examination of its terms (see (13)), g_{ij}^1 would normally be negative⁵; an increase in the other firm's R & D normally reduces the effect of own R & D on own profit. Unless otherwise indicated we will assume this is the case.

$$g_{ij}^1 < 0 \quad (14)$$

Also inspection of the expressions for g_{ii}^1 and g_{ij}^1 (12) and (13) shows that existence and uniqueness of equilibrium can be a problem in two-stage models. At least one of the terms of g_{ii}^1 is positive, making it difficult to ensure that the second order condition for an interior profit maximum holds. Nevertheless (12) and (13) will hold if C_{xx}^1 is relatively

large, that is, if the marginal cost reducing effect of R & D declines relatively rapidly. The study of existence, uniqueness and stability of perfect equilibrium is an interesting and nontrivial matter, but is not something which we wish to pursue here given our emphasis on the comparative static properties of well behaved cases.

We conclude this section by noting that firms do not minimize costs in this model. The condition for overall cost minimization is

$$C_x^1 + v = 0 \quad (15)$$

but these firms set

$$C_x^1 + v = R_j^1 q_i^j > 0 \quad (16)$$

by (11), (2) and (9). Indeed, since $C_{xx}^1 > 0$ (which is the second order condition for cost minimization), R & D is overused in that more R & D is used than required to minimize total costs for the output chosen.⁶

4. Industrial Strategy

In this section we demonstrate that industrial strategy, in the form of R & D subsidies or export subsidies, can enable a domestic firm to capture a larger share of the world market so as to increase profits and rent, net of subsidy, to the domestic country. We should also point out that the variable x could be interpreted as capital, leading to a theory of investment subsidies. In any case, our approach is to characterize the subsidies that would maximize rent. In practise, of course, governments have neither the information nor the singlemindedness necessary to implement such finely tuned policies.

Nevertheless, the arguments presented here seem to capture one significant aspect of real government incentives and behaviour.

The government is introduced as an agent that can set subsidy rates on R & D expenditure at a period before the firms spend on R & D. The assumption that the government can pre-commit itself to such subsidies is essential to the analysis, and our conclusions are tied to it. We do feel, however, that the assumption that the government can act in this leadership role is fairly natural.

There are at least two major reasons for such an assumption. One is simple bureaucratic sluggishness. The government's technology for changing tax rates, once set, is relatively slow: the inflexibility of government gives it its strength, placing it naturally in a leadership role. A second point is that the government has a large number of taxes and subsidies to set (and, for that matter, many other policies as well) and is concerned with maintaining its reputation for future policies.⁷ The model would not, in the words of a referee, apply to a crude Marxian image of a banana republic, where the government follows the directions of a corporation.

The setting is as follows. There are two firms in the industry, one located in the "domestic" country and one located in a foreign country. In order to focus on the purely rent-seeking rationale for industrial strategy we assume that all output is for export to other countries. The possibility of domestic consumption would generally strengthen any incentives for subsidization of the domestic firm

since it tends to increase quantities and decrease prices, but we do not analyze that issue here. The domestic government wishes to maximize net rent accruing to the domestic country, which in this simple context is just the profit of the domestic firm minus the cost of any policies carried out.

R & D Subsidies

The first type of policy to consider is a subsidy (or tax) on R & D by itself. This subsidy affects the levels of R & D committed by firms but not the resolution of the output game given R & D levels which is represented by $y^i = q^i(x^1, x^2)$.⁸ Therefore, with a subsidy, s , per unit of R & D, the profit of the domestic firm is (from (1) and (10)),

$$g^1(x^1, x^2; s) = R^1(y^1, y^2) - C^1(y^1; x^1) - (v^1 - s)x^1 \quad (17)$$

where $y^1 = q^1(x^1, x^2)$; $y^2 = q^2(x^1, x^2)$

The first point of interest concerns the effects of the subsidy on R & D levels. The subsidy shifts out the R & D reaction function of the domestic firm, increasing its equilibrium R & D and reducing the R & D undertaken by the foreign firm, provided reaction functions are downward sloping. These results are obtained by total differentiation of the first order conditions $g_1^1(x_1, x_2; s) = 0$ (from 17)) and $g_2^2 = 0$ to obtain the

following comparative static matrix equation.

$$\begin{bmatrix} g_{11}^1 & g_{12}^1 \\ g_{21}^2 & g_{22}^2 \end{bmatrix} \begin{bmatrix} x_s^1 \\ x_s^2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (18)$$

This implies $x_s^1 = -g_{22}^2/D > 0$ since $D > 0$ by (13) and $g_{22}^2 < 0$ by (12). Also $x_s^2 = g_{21}^2/D$ is negative if (14) holds and positive if $g_{21}^2 > 0$.

Proposition 1

A domestic R & D subsidy increases domestic R & D and, provided $g_{21}^2 < 0$, reduces foreign R & D. If $g_{21}^2 > 0$, foreign R & D rises.

We might note that the relationship between the comparative static effects x_s^1 and x_s^2 is just $x_s^2 = -x_s^1(g_{21}^2/g_{22}^2)$. Since the slope of the foreign firm's reaction function is given by $dx^2/dx^1 = -g_{21}^2/g_{22}^2$ it follows that

$$x_s^2 = x_s^1 dx^2/dx^1 \quad (19)$$

The Optimal Domestic R & D Subsidy

The optimal subsidy is found by maximizing net domestic benefit B^1 , which is the profit of the domestic firm less the cost of the subsidy.

$$B^1(s) = g^1(x^1, x^2; s) - sx^1 \quad (20)$$

From (17), the domestic benefit (20), with a subsidy, is just the profit of the domestic firm (earned from exports) when there is no subsidy. The question arises as to why subsidization could ever be called for, since, after all, the firm wishes to maximize its own profit. What is it that the government can do by subsidizing R & D that the firm cannot do for itself?

The answer lies in the nature of the firm actions which are compatible with the subgame perfect equilibrium. The level of R & D chosen by the domestic firm is the level which maximizes its profit within the confines of the behaviour which characterizes the two-stage Nash equilibrium. If a firm violates this equilibrium it risks the possibility it will earn lower profit in the unstable situation which follows. By providing a subsidy to firms, the government alters the perceived cost structure and thus changes the set of actions which are compatible with the two-stage Nash equilibrium. This allows the domestic firm to earn higher profit net of the subsidy. In essence the government perceives an advantage from introducing an earlier stage of pre-commitment. The government commits itself to lowering the cost of pre-commitment by domestic firms.

It is simply not credible for the firm to do this by itself. For example, just as a firm cannot deter entry by threatening to produce a large output should entry occur, it is not credible for the firm to persuade other firms not to expand, by announcing that it is subsidizing, in some sense, its own R & D. Without an actual subsidy, this is not profit maximizing behaviour given the level of costs and the nature of the equilibrium. On the other hand, the government is assumed able to offer a credible R & D subsidy, and can therefore influence the final equilibrium.⁹

There is, of course, nothing to stop the government in the other country from acting in a similar manner. Later in the paper we discuss

the noncooperative equilibrium that would arise with both governments involved. For the present, however, we examine the incentives facing a single government. We might also mention that the use of benefit function (20) involves the usual assumptions necessary for partial equilibrium surplus analysis. In particular if R & D is not subsidized the private cost of R & D reflects its full social opportunity cost.

From (20) the first order condition for the welfare maximizing subsidy is

$$dB^1/ds = g_1^1 x_s^1 + g_2^1 x_s^2 + g_s^1 - x^1 - s x_s^1 = 0 \quad (21)$$

The partial derivative $g_s^1 = x^1$ from (17). Also $g_1^1 = 0$ from (11) and $x_s^2 = x_s^1 dx^2/dx^1$ from (19) so (21) reduces to

$$dB^1/ds = (g_2^1 (dx^2/dx^1) - s) x_s^1 = 0$$

$$\text{or} \quad s = g_2^1 dx^2/dx^1 \quad (22)$$

The optimal R & D subsidy is equal to the increase in own profit from a reduction in the foreign firm's R & D brought about by an increase in own R & D.

Proposition 2

The optimal subsidy is positive.

Proof:

$g_2^1 = \pi_1^1 q_2^2 + \pi_2^1 q_2^2$ from (10) (or (17)). Since $\pi_1^1 = 0$ (by (4)), $\pi_2^1 = R_2^1 < 0$. by (2), and $q_2^2 > 0$ by (8), it follows that $g_2^1 = R_2^1 q_2^2 < 0$. Since by (12) and (14) $dx^2/dx^1 = -g_{12}^2/g_{22}^2 < 0$, s is positive by (22).

Another way of looking at this point is simply to observe that dB^1/ds is positive at $s = 0$, indicating that there is a rent-capturing incentive for the government to subsidize R & D, despite the fact that the subsidy increases the bias toward excess use of R & D for any given level of output.¹⁰ Of course, in the unusual but not impossible case in which the foreign firm's reaction function is positively sloped ($g_{21}^2 > 0$), then the domestic country has an incentive to tax domestic R & D.

Proposition 3

The optimal R & D subsidy maximizes domestic rent earned from exports by moving the domestic firm to what would have been the Stackelberg leader-follower point in R & D space with no subsidy.

Proof:

Suppose the domestic firm were a Stackelberg leader in R & D space and $s = 0$. The first order condition for a profit maximum, taking account of the reaction of the foreign firm, is

$$g_1^1(x^1, x^2; 0) + g_2^1(x^1, x^2; 0) (dx^2/dx^1) = 0 \quad (23)$$

With a subsidy the firm's profit is increased by sx^1 so $g^1(x^1, x^2; s) = g^1(x^1, x^2; 0) + sx^1$. The first order condition for the domestic firm in the first stage Nash R & D game, taking subsidy $s = g_2^1 dx^2/dx^1$ as given is $g_1^1(x^1, x^2; 0) + s = 0$, or $g_1^1(x^1, x^2; 0) + g_2^1(x^1, x^2; s)(dx^2/dx^1) = 0$, which is the same as (23) since from the proof of Proposition 2, $g_2^1 = R_2^1 q_2^2$ which is independent of s . That is, Stackelberg leader-follower behaviour without a subsidy gives rise to the same situation as Nash behaviour with the optimal subsidy, which proves the result.

The optimal R & D subsidy is illustrated in Figure 2. R & D reaction functions are drawn sloping downward. Without a subsidy, both reaction functions are satisfied at point N. Isoprofit contours show combinations of x^1 and x^2 that would yield equal profit to firm 1. The most profitable position for firm 1, given the reaction function of firm 2, is S, the Stackelberg leader-follower point. The subsidy shifts the reaction function of firm 1 so that it intersects the reaction function of firm 2 at the Stackelberg point.

Figure 2

The government does for the domestic firm what the firm cannot do for itself.¹¹ The domestic firm might have the same understanding of industry structure as the government but the subsidy can allow the firm to achieve the profit maximizing Stackelberg leader point without disturbing the Nash equilibrium behaviour of the private firms. In addition, the domestic firm would naturally prefer to be moved to the Stackelberg point by a government subsidy since, without offsetting taxes, the subsidy results in a redistribution of income from taxpayers to the shareholders of the firm. Persuading the government that subsidization of R & D will lead to a domestic benefit from a greater share of the world market allows firms to enjoy a higher level of profits without the risk of the price wars which might result from the breakdown of the perfect Nash equilibrium.

An alternative interpretation is that the government knows the structure of industry behaviour while the naive firms do not, or simply that the

government has a different, possibly incorrect, view than do the firms. This interpretation is not without interest or relevance but is not really in the spirit of the paper.

The incentive for the government to subsidize R & D would still remain (and perhaps be enhanced) if we had not assumed firms use R & D strategically. The subsidy would still induce the domestic firm to use additional R & D shifting out its output reaction function, and increasing its market share. Furthermore the incentive to subsidize R & D remains if the foreign government has announced credible R & D subsidies.

Noncooperative International Equilibrium

This R & D rivalry does have a beggar-thy-neighbour aspect. By imposing a subsidy, country 1 gains at the expense of country 2. (This follows easily since $dg^2/ds = g^2_{1s} < 0$ from (18) and the proof of Proposition 2). Nevertheless, the international non-cooperative equilibrium which would occur if each country acted independently given the subsidy imposed by the other, does involve positive subsidies.

To show this consider the net benefit

$$B^1(s^1, s^2) = g^1(x^1, x^2; s^1) - s^1 x^1 \quad (24)$$

earned by each country when both have subsidized R & D. This is the same form as (20) since the subsidy of country 2, s^2 , affects the profit of firm 1 only indirectly through its impact on R & D levels. The non-cooperative equilibrium occurs where $\partial B^1/\partial s^1 = 0$ and $\partial B^2/\partial s^2 = 0$ and implies positive subsidies (see the proof of Proposition 2).

$$s^i = g_j^i(dx^j/dx^i) \quad (25)$$

Although (25) has the same form as the single country subsidy (22), the values of g_j^i and dx^j/dx^i will be different at the noncooperative equilibrium.

If each firm had attempted to be a Stackelberg leader in R & D space the outcome would not be informationally consistent and could not be a reasonable candidate for an equilibrium structure (hence the term Stackelberg disequilibrium). However, if we think of governments setting credible subsidy levels, then firms setting R & D levels given subsidy levels, and finally setting output given R & D levels, the new equilibrium is essentially an informationally consistent 3 stage perfect equilibrium. This should make clear the role of the government in being able to credibly set subsidies.

It should be emphasized that this structure is suboptimal for the two countries taken together. If we define $B(s^1, s^2) = B^1 + B^2$ and maximize B by setting $\partial B/\partial s^1 = 0$, $\partial B/\partial s^2 = 0$ it follows (see Appendix A) that

$$s^1 = g_1^2 < 0 ; s^2 = g_2^1 < 0 \quad (26)$$

The jointly optimal policy is to tax R & D so as to just offset the negative effect of own R & D on the other firm's profit.

If the two firms are similar, total rent is lower and both countries earn less rent at the noncooperative equilibrium than they would if they had been able to come to an agreement not to subsidize R & D (see Appendix A).

Both producing countries are then worse off by their subsidization of R & D rivalry. Consuming countries, of course, gain from the fall in prices which results from greater production.

Attention is restricted to symmetric cases. However three kinds of asymmetry should be mentioned. First, of course, there may be exogenous asymmetry in demand for the two products or in costs, and it is possible that in a very asymmetric case one country could be better off in the noncooperative subsidy equilibrium than at the subsidy-free position. Secondly, a less restricted version of the model would admit multiple equilibria, including asymmetric equilibria, even if the exogenous structure were symmetric (along the lines of Flaherty (1980)). This raises the possibility of one country obtaining an ex post advantage by admissible strategic moves. Finally, of course, one country might have a timing advantage and be able to gain an advantage by acting first.

Export Subsidies

The second tool of industrial strategy to consider is the export subsidy. The export subsidy could be announced either before or after R & D has occurred and could be used in conjunction with an R & D tax or subsidy, or by itself. This creates a fairly large taxonomy of possible cases. The case we wish to examine in some detail is the case in which the export subsidy is announced before R & D is in place and is used simultaneously with an R & D tax or subsidy.

First, however, it is useful to consider the simplest case: a subsidy per unit of exports after R & D has taken place. In general the Nash equilibrium levels of production are a function of the levels of R & D and the export subsidy, denoted z : $y^1 = q^1(x^1, x^2, z)$ and $y^2 = q^2(x^1, x^2, z)$. In this case x^1 and x^2 are fixed and the export subsidy by reducing marginal cost serves to increase the domestic firm's share of the export market. This holds since the partial derivatives q_z^1 and q_z^2 are respectively positive and negative holding x^1 and x^2 fixed (see Appendix B(i)). The net domestic benefit now depends on x as follows

$$B^1(x^1, x^2; z) = g^1(x^1, x^2; z) - zy^1 \quad (27)$$

where

$$g^1 = R^1(y^1, y^2) - C^1(y^1; x^1) - v^1 x^1 + zy^1$$

If R & D expenditures are already sunk, the subsidy cannot affect them, and the first order condition for maximization of (27) is

$$\partial B^1 / \partial z = g_z^1 - y^1 - zq_z^1 = 0 \quad (28)$$

From (27) it is easily seen that $g_z^1 = \pi_2^1 q_z^2 + \partial \pi^1 / \partial z$ since $\pi_1^1 = 0$. Then, since $\pi_2^1 = R_2^1$ and $\partial \pi^1 / \partial z = y^1$ we have $z = R_2^1 q_z^2 / q_z^1$. The export subsidy shifts out the reaction function of the domestic firm in output space.

It does not affect the reaction function of the foreign firm. Therefore, $q_z^2 = q_z^1 (dy^2 / dy^1)$ so the rent maximizing subsidy

$$z = R_2^1 (dy^2 / dy^1) > 0 \quad (29)$$

is positive by (2) and (5). Given any R & D level, this subsidy moves the domestic firm to what would be the Stackelberg position in output space, much as the R & D subsidy moves the domestic firm to the Stackelberg point in R & D space.

Now consider the more difficult case in which R & D and export subsidies can be imposed before R & D take place. This is more complicated because the export subsidy affects both the choice of output given R & D and R & D directly. The profit function of the domestic firm is

$$g^1(x^1, x^2; s, z) = R^1(y^1, y^2) - c^1(y^1; x^1) + zy^1 - (v^1 - s)x^1 \quad (30)$$

where $y^1 = q^1(x^1, x^2; z)$, $y^2 = q^2(x^1, x^2; z)$ and x^1 and x^2 themselves depend on z and s . The benefit function is

$$B^1(s, z) = g^1(x^1, x^2; s, z) - zy^1 - sx^1 \quad (31)$$

Taking the derivative of B^1 with respect to s and solving for s yields (See Appendix B (iii))

$$s = g_2^1(dx^2/dx^1) - zdy^1/dx^1 \quad (32)$$

Looking at (22), which gives the R & D subsidy in isolation, we see that this naturally coincides with (32) if $z = 0$. However if $z > 0$, the optimum subsidy on R & D is reduced by the term zdy^1/dx^1 where dy^1/dx^1 , the total derivative of $y^1 = q^1(x^1, x^2, z)$ with respect to x^1 is positive since $dx^2/dx^1 < 0$ by (12) and (14). This term reflects the effect of a unit increase in R & D on the cost of the export subsidy to the government. Similarly, the first order condition for the export subsidy z yields

$$z = (g_2^1x_z^2 + R_2^1q_z^2)(dy^1/dz) - sx_z^1/(dy^1/dz) \quad (33)$$

Expressions (32) and (33) must be solved simultaneously. It is no longer clear that both s and z will be positive. The solution is derived in appendix B.

$$s = -R_2^1q_1^2 < 0 \quad (34)$$

$$z = (g_2^1(dx^2/dx^1) + R_2^1q_1^2)/(dy^1/dx^1) > 0 \quad (35)$$

Proposition 4

The optimum export subsidy, z , is positive, but the optimal R & D subsidy is negative. R & D should be taxed. The tax on R & D is exactly as required to undo the R & D bias and induce the domestic firm to minimize costs.

Proof: The sign of s follows from (2) and (9). z can be signed using (2), (8), (9), (12), (13) and (14). That s exactly offsets the R & D bias is clear from (16) since the firm now sets $C_x^1 + (v-s) = R_2^1 q_1^2$ or $C_x^1 + v = 0$, as required to minimize cost for a given level of output.
* * *

Because the export subsidy and the R & D tax can be credibly imposed before investment in R & D takes place, they can be used to move the domestic firm to the best position given that the foreign government does not retaliate. The export subsidy increases domestic exports while the R & D tax restores production efficiency. The noncooperative international equilibrium would have both countries imposing export subsidies and R & D taxes, and overall production efficiency would result. As in the case of an R & D subsidy alone, noncooperative behaviour reduces the total rent to be divided relative to the rent at the collusive output level. Nevertheless the additional output from subsidization allows the consuming nations to gain.

5. Concluding Remarks

National governments clearly play a major role in certain international industries, particularly high technology and high investment industries. Such industries often have only a few firms, or at least, are characterized by some national cooperation so that there are few competitive units in the market. These firms themselves normally have a clear understanding that they are involved in a strategic game in which foreign firms and national governments are players.

This is a view of the world that is displayed prominently, perhaps too prominently, in business and financial magazines. This paper is a step toward understanding such a world. Our approach is to assume that governments can credibly commit themselves to policies which alter the set of credible actions open to private players in subsequent strategic games. In this setting we show that there are national incentives to subsidize R & D if export subsidies are not available.

The strategic game played by firms leads them to overuse R & D in the absence of government policy. If the government can use both export subsidies and R & D subsidies (or taxes), a fairly striking result is obtained: the government has an incentive tax R & D to restore domestic production efficiency, and to use an export subsidy to enable the domestic firm to capture a larger share of the industry than it would unaided. Thus a prediction of the model is that a relaxation of GATT restrictions on export subsidies would cause an increase in the use of export subsidies and a decrease in subsidies to investment and to process R & D.

We have not examined entry deterrence but it is fairly clear that the policies and structure outlined here makes it possible to deter entry while leaving current participants with strictly positive profits. Also; symmetrically expanding the number of firms in each country to more than 1 is a conceptually minor (although algebraically major) extension. Similarly, one could modify the precise nature of final stage competition. In particular moving to price-Nash rather than quantity-Nash does not change the nature of the results, provided products are slightly differentiated. (The choice of quantity versus price Nash for the final stage is, of course, not arbitrary in any specific case; rather it arises naturally from the type of production process involved. At the level of modelling, however, the quantity version is slightly more convenient).

Finally, we should emphasize that the analysis presented here is not in any sense a recommendation that "industrial strategy" policies be used. In the first place, any policy advocating subsidies should be viewed with suspicion because the opportunity cost of government revenue may be much higher than value of unity assumed in simple surplus analysis. Also, of course, we have been characterizing the incentives for noncooperative international behaviour. Such behaviour will naturally be welfare inferior to the joint-maximizing optimum. Indeed the positive prediction for government behaviour is that a government will undertake industrial strategy if no other government does, but be willing to negotiate limitations on such policies in the event of retaliation.

Appendix A

R & D taxes which jointly maximize rent

Total rent is $B = B^1 + B^2$. From (24)

$$B(s^1, s^2) = g^1(x^1, x^2; s^1) - s^1 x^1 + g^2(x^1, x^2; s^2) - s^2 x^2 \quad (1A)$$

using $g_i^i = 0$, $\partial g^i / \partial s^i = x^i$ and $dx^j / ds^i = (dx^j / dx^i)(\partial x^i / \partial s^i)$ we obtain the first order conditions for a maximum of B with respect to s^1 and s^2 .

$$\partial B / \partial s^1 = [g_2^1(dx^2/dx^1) - s^1 + g_1^2 - s^2(dx^2/dx^1)](dx^1/ds^1) = 0 \quad (2A)$$

$$\partial B / \partial s^2 = [g_2^1 - s^1(dx^1/dx^2) + g_1^2(dx^1/dx^2) - s^2](dx^2/ds^2) = 0 \quad (3A)$$

From (2A) and (3A), s^1 and s^2 satisfy

$$s^1 + s^2(dx^2/dx^1) = g_1^2 + g_2^1(dx^2/dx^1) \quad (4A)$$

$$s^1(dx^1/dx^2) + s^2 = g_1^2(dx^1/dx^2) + g_2^1 \quad (5A)$$

Solving (4A) and (5A) simultaneously and cancelling terms we obtain (26)

$s^1 = g_1^2$ and $s^2 = g_2^1$, which implies both s^1 and s^2 are negative.

Response of total rent to positive subsidies

We show here that if firms are identical total rent $B = B^1 + B^2$ falls for any small increase in the equal positive R & D subsidies. Since the subsidies (see(25)) are positive at the noncooperative equilibrium this implies that both countries earn less rent at the noncooperative equilibrium than if neither had subsidized R & D.

For any small changes in s^1 and s^2 ,

$$dB = (\partial B / \partial s^1) ds^1 + (\partial B / \partial s^2) ds^2 \quad (6A)$$

Suppose the two firms have the same profit functions, and that $s^1 = s^2$ and $ds^1 = ds^2$. Substituting (2A) and (3A) into (6A), using $dx^1/ds^1 = dx^2/ds^2$ and gathering terms we obtain

$$dB = [(g_2^1 - s^2)(1 + (dx^2/dx^1)) + (g_1^2 - s^1)(1 + dx^1/dx^2)](dx^1/ds^1) ds^1 \quad (7A)$$

Since $g_j^i < 0$ and from (13), $|dx^j/dx^i| = |-g_{ij}^i/g_{jj}^i| < 1$, $dB < 0$ if $s^1, s^2 \geq 0$.
* * *

The intuition of this result is that, with zero or positive subsidies, duopoly firms already produce more than the collusive output and use more than the collusive amount of R & D. Further subsidies just push them further away from the joint profit maximum.

Appendix B

Comparative Static effects of the Export Subsidy

(i) Effects of the Export Subsidy on Output

In order to obtain the solutions for s and z , (34) and (35) respectively, we require the partial derivatives $q_z^1 = \partial q^1(x^1, x^2; z)/\partial z$. By total differentiation of the first order conditions, $\pi_1^1 = R_1^1 - c^1 + z = 0$ and $\pi_2^2 = R_2^2 - c^2 = 0$ with respect to y^1 , y^2 and z , holding x^1 and x^2 fixed we obtain

$$q_z^1 = -\pi_{22}^2/A \text{ and } q_z^2 = \pi_{21}^2/A \quad (1B)$$

Since $\pi_{22}^2 < 0$ by (5), $A > 0$ by (6) and $\pi_{21}^2 = R_{21}^2 < 0$ by (2), $q_z^1 > 0$ and $q_z^2 < 0$. An increase in the export subsidy increases own output and reduces the output of the foreign firm. Also from $dy^2/dy^1 = -\pi_{21}^2/\pi_{22}^2$ we have

$$q_z^2 = q_z^1 dy^2/dy^1 \quad (2B)$$

Further from the expressions (8) and (9) of the text for q_1^2 and q_2^2 , and from (1B),

$$q_z^1 = -q_1^1/c_x^1 \text{ and } q_z^2 = -q_1^2/c_x^1 \quad (3B)$$

(ii) Effects of the Export Subsidy on R & D

For the purposes of the simultaneous solution for s and z , we also require the relationship,

$$x_z^2 = (dx^2/dx^1)(x_z^1 - (1/c_x^1)) \quad (4B)$$

The effect of the export subsidy on the R & D level of firm 2, x_z^2 , depends not only on its effect on the R & D of firm 1, but on changes in the resolution of the output game, q_z^1 and q_z^2 (as in Appendix B (i)). We now prove expression (4 B).

The subsidies, s and z , affect the profit of firm 2 only indirectly through effects on y^1 and x^1 , so

$$g_2^2(x^1, x^2, s, z) = R_1^2(y^1, y^2)q_2^1(x^1, x^2, z) - (c_x^2(y^2, x^2) + v^2) = 0 \quad (5B)$$

where $y^1 = q^1(x^1, x^2, z)$ and $x^1 = x^1(s, z)$.

From total differentiation of (5B) with respect to x^1 , x^2 and z , we have

$$dg_2^2 = g_{21}^2 dx^1 + g_{22}^2 dx^2 + g_{2z}^2 dz = 0 \quad (6B)$$

Rearranging we obtain

$$x_z^2 = -(g_{21}^2/g_{22}^2)x_z^1 - g_{2z}^2/g_{22}^2 \quad (7B)$$

where from differentiation of (5B)

$$g_{2z}^2 = \partial g_2^2 / \partial z = R_1^2 q_{2z}^1 + q_2^1 (dR_1^2 / dz) - c_x^2 q_z^2 \quad (8B)$$

To simplify (7B) we need to express g_{2z}^2 in terms of g_{21}^2 . Since $dR_1^2 / dz = R_{11}^2 q_z^1 + R_{12}^2 q_z^2$, and from (3B),

$$\begin{aligned} dR_1^2 / dz &= (R_{11}^2 q_z^1 + R_{12}^2 q_z^2) / c_x^1 \\ &= -(dR_1^2 / dx^1) / c_x^1 \end{aligned} \quad (9B)$$

Also since from (3B), $q_z^1 = -q_1^1 / c_x^1$, we have $q_{z2}^1 = -q_{12}^1 / c_x^1$.

Substituting (9B), q_z^2 from (3B) and $q_{z2}^1 = q_{12}^1 / c_x^1$ into (8B), we obtain

$$\begin{aligned} g_{2z}^2 &= -(R_{11}^2 q_{21}^1 + q_2^1 (dR_1^2 / dx^1) - c_x^2 q_1^2) / c_x^1 \\ &= -g_{21}^2 / c_x^1 \end{aligned} \quad (10B)$$

Substituting (10B) into (7B) and using $dx^2 / dx^1 = -g_{21}^2 / g_{22}^2$

we obtain expression (4B).

(iii) Solution for the Optimal Combined R & D and Export Subsidies

We first obtain expression (32) for s . The first order conditions for the optimal choice of s and z are $\partial B^1/\partial s = 0$ and $\partial B^1/\partial z = 0$. From differentiation of (31),

$$\partial B^1/\partial s = (g_1^1 - s)x_s^1 + g_2^1 x_s^2 + g_s^1 - x^1 - z dy^1/ds = 0 \quad (11B)$$

$$\text{where } dy^1/ds = q_1^1 x_s^1 + q_2^1 x_s^2$$

$$\text{Using (19), } x_s^2 = (dx^2/dx^1)x_s^1 \text{ we have } dy^1/ds = (dy^1/dx^1)x_s^1$$

where $dy^1/dx^1 = q_1^1 + q_2^1(dx^2/dx^1)$. Since $g_1^1 = 0$ and $g_s^1 = x^1$ from (17), (11B) becomes

$$\partial B^1/\partial s = -sx_s^1 + (g_2^1(dx^2/dx^1) - z dy^1/dx^1)x_s^1 \quad (12B)$$

which implies expression (32),

$$s = g_2^1(dx^2/dx^1) - z dy^1/dx^1 \quad (13B)$$

We obtain expression (33) for z in a similar manner. From differentiation of (31),

$$\partial B^1/\partial z = (g_1^1 - s)x_z^1 + g_2^1 x_z^2 + g_z^1 - y^1 - z dy^1/dz = 0 \quad (14B)$$

Recognizing that $\pi_1^1 = 0$, from partial differentiation of (30), we have $g_z^1 = R_2^1 q_z^2 + y^1$.

Substituting this into (14B) and recognizing that $g_1^1 = 0$, we obtain

$$\partial B/\partial z = -sx_z^1 + g_2^1 x_z^2 + R_2^1 q_z^2 - z dy^1/dz = 0 \quad (15B)$$

which rearranged yields (33).

We now derive expressions (34) and (35) for s and z . Substituting (32) or (13B) for s , (4B) for x_z^2 and (3B) for q_z^2 into (15B) and cancelling, we obtain

$$\partial B / \partial z = z((dy^1/dx^1)x_z^1 - dy^1/dz) - (g_2^1 dx^2/dx^1 + R_2^1 q_1^2)/c_x^1 = 0 \quad (16B)$$

Now $dy^1/dz = q_1^1 x_z^1 + q_2^1 x_z^2 + q_z^1$ and $dy^1/dx^1 = q_1^1 + q_2^1 dx^2/dx^1$

so the first term of (16B)

$$(dy^1/dx^1)x_z^1 - dy^1/dz = -q_2^1(x_z^2 - (dx^2/dx^1)x_z^1) - q_z^1 \quad (17B)$$

Using (4B) and (3B), (17B) reduces to $(dy^1/dx^1)/c_x^1$. Substituting this into (16B) and rearranging we obtain

$$z = (g_2^1 dx^2/dx^1 + R_2^1 q_1^2)/(dy^1/dx^1) \quad (18B)$$

which is expression (35). The expression (34), $s = -R_2^1 q_1^2$ is then obtained by substituting (35) into (32) of the text.

Footnotes

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In particular, James Friedman, Sandy Moroz, Robert Willig, and the referees were very helpful. Shortcomings, of course, remain our responsibility. The paper has benefitted considerably from exposure at conferences on "Trade Theory Post Heckscher-Ohlin" at the University of Western Ontario, on "Monopolistic Competition in International Trade" at the Graduate Institute of International Studies in Geneva and at the 1982 NBER Summer Institute. J. Brander wishes to acknowledge gratefully financial support from a Social Sciences and Humanities Research Council of Canada post-doctoral fellowship.

1. Prescott and Visscher (1977) have a similar structure except that it is the product type or location decision which constitutes a commitment rather than investment or R & D.
2. Equal opportunity product choice commitment models have been analyzed by Shaked and Sutton (1982, 1983). Attention should also be drawn to Flaherty (1980), where distinct capital and output decisions are made by firms with equal opportunity in a dynamic setting.
3. The subgame perfect Nash equilibrium has been fairly widely used in the literature and is an example of a "closed-loop" solution for the players involved. It is an attractive concept because the equilibrium is self-enforcing at each stage in that there are no incentives to cheat, and secondly because the equilibrium is not characterized by systematic errors about the levels of strategy variables to be chosen by rivals: the model confirms expectations in equilibrium.

4. Uniqueness follows from direct application of Gale-Nikaido global univalence, provided (6) holds globally. (See Nikaido (1968), ch. 7, p. 371). This is also the Routh-Hurwitz condition for stability of the standard adjustment mechanism. That is, (6) is the condition for the reaction function diagram to be stable.
5. Letting $i = 1$ and $j = 2$, the term $dr_2^1/dx^2 = R_{21}^1 q_2^1 + R_{22}^1 q_2^2$ is positive from (2), (8) and (9) in the normal case in which $R_{22}^1 \geq 0$; an increase in y^2 reduces R^1 but at a diminishing rate. This ensures that the second term of g_{12}^1 is negative. Since the third term is negative from (3) and (9), $g_{12}^1 < 0$ if the first term $R_{21}^1 q_2^1$ is not too positive. From differentiation of (9), $q_{12}^2 = 0$ if demand is linear and marginal cost c^i is constant with respect to output.
6. This result can be regarded as an extension of Dixit (1980). It is reported in Brander and Spencer (1983) along with a number of other results concerning the comparison with a corresponding one-stage (or non-strategic) model. Output in the industry is higher and prices and profits are lower in the two-stage model. Although R & D is not used efficiently, for many cases, overall welfare actually is higher.
7. This discussion follows some helpful comments by a referee who also pointed out that the sluggishness idea appears in Schelling (1960), and that reputation effects can give rise to credible threats even in finite horizon models as in, for example, Kreps and Wilson (1982).

The sluggishness point seems particularly relevant to the R & D subsidy since the R & D phase is presumably relatively short in real time so the subsidy would not have to be maintained for very long. This is a second reason, in addition to GATT restrictions, for taking the R & D subsidy as more relevant than the export subsidy.

8. This fact permits a fairly straightforward generalization of the analysis of R & D subsidies: any second stage game giving rise to q^i functions with appropriate reasonable properties could be incorporated in the analysis.
9. A recent paper by Paul Krugman (1983) has the government engaging in similar behaviour. By guaranteeing a "home market" for domestic firms, those firms believe, as do their rivals, that they are guaranteed some kind of learning economy (or other economy of scale) which gives domestic firms an advantage in export markets as well.
10. With an R & D subsidy, the first order conditions for profit maximization reduce to $c_x^1 + v^1 = R_2^1 q_1^2 + s$. Since, without an R & D subsidy, $c_x^1 + v^1 = R_2^1 q_1^2$ (see (16)), the effect of a positive s is to increase the positive value of $c_x^1 + v^1$ which (for a given output) implies a greater deviation from the cost-minimizing solution: $c_x^1 + v^1 = 0$.
11. This Stackelberg leader-follower equilibrium is the equilibrium that would emerge if the Prescott-Visscher (1977) first stage sequential entry with foresight concept were used. In this case, since the domestic firm could undertake its R & D (or install its capital) first, its position would be credible. If, on the other hand, it was the foreign firm who had the opportunity to act first then the domestic government would have an incentive to announce R & D subsidies before the R & D chosen by the foreign firm was determined.

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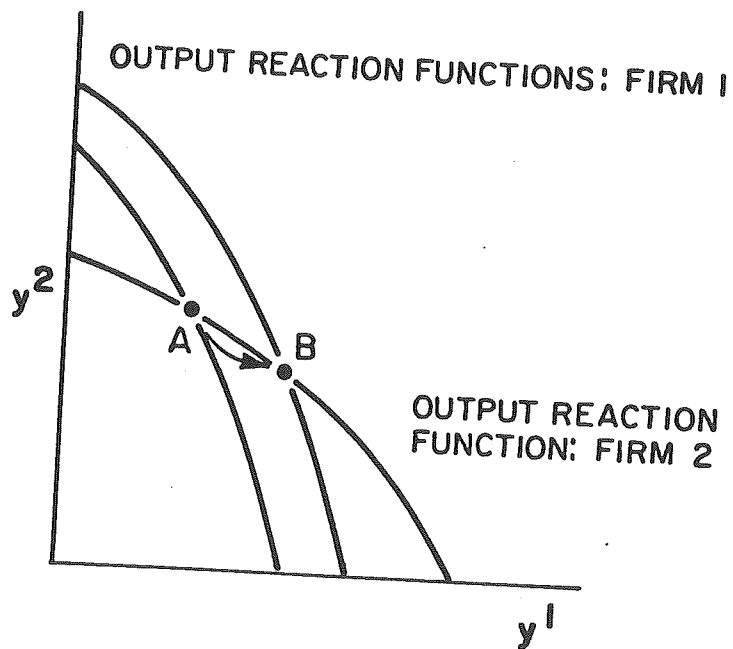


FIGURE 1

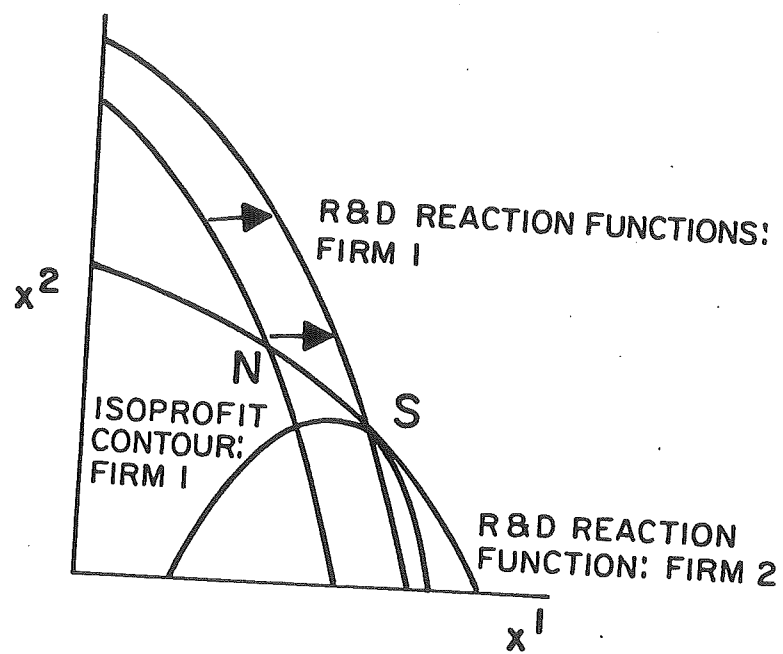


FIGURE 2