OPTIMAL BUFFERING POLICY ON FIXPRICE MARKETS: GENERAL EQUILIBRIUM

James E. Anderson

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Introduction

Fixed prices ordinarily prevent private intertemporal arbitrage of supply and demand shifts. For storable goods, government arbitrage can ameliorate the distortion created by the fixprice in the presence of fluctuations over time. Publically provided electric power, water and sewerage are a few familiar U.S. examples where such policies are relevant. The sale of timber and grazing rights from the public lands has exactly the characteristics noted. Some aspects of the provision of health care also seem to fit with a bit of stretching. Socialist countries have a much wider set of fixprice/storable goods markets, prominently including food and shelter.

This paper will characterize optimal buffering policy in a simple general equilibrium model with "risk aversion", or finite intertemporal substitution elasticity. The optimal quantity policy can have surprising properties, involving de-stabilization of consumption or production. The second best policy of no buffering then dominates any stabilizing policy. Arbitrage considerations alone imply at least partial stabilization of the quantity consumed (produced) under a supply (demand) constraint. Deeper considerations of general equilibrium structure and risk aversion fundamentally modify and enrich this insight.

This richness of structure arises because securities markets are "incomplete". The analysis is thus of second-best type. For simplicity, we assume there are no securities markets. The qualitative properties of our analysis survive the introduction of securities so long as markets are incomplete; do not allow private agents to completely offset the effect of fluctuations on real income. Government buffering policy then inevitably has the sorts of effects we study.

Optimal buffering policy in general involves complex structures of intertemporal utility and constraints. The key elements can be seen in a much more streamlined model with severe assumptions. Following standard practice we impose additive-over-time utility. To focus on pure buffering activity, we assume that storage is productive with constant marginal return equal to the consumer's rate of time preference. In the absence of fluctuations, this implies stationary consumption is optimal, with no frustrated desire for short sales. The buffering authority is assumed to have on hand at the beginning of the analysis a sufficient stock to allow the optimal sequence of buffering sales and purchases, subject to maintaining the present discounted value of the initial stock. Private storage is ruled out by assuming a fixed cost sufficient to deter household activity, but irrelevant to the government's decisions.

The introduction of randomness complicates the cost of buffering considerably (see Newbery-Stiglitz) without adding appreciably to the insights presented here on intertemporal smoothing. The structure of the state and time-contingent buffer schedule will, however, be qualitatively the same in the dynamic stochastic programming model where stock carried forward creates expected future utility. The main complication is that the feasibility constraints limit the amount of buffering, effectively making it more costly. The model presented here is thus a parable about optimal intertemporal quantity policy in economies where temporarily rigid prices create persistent opportunities for buffering between periods in order to even out the impact of the fixprice distortion. We should note that our structure implies no intervention when the distortion is absent--quantity policy is not used to smooth out variations in real income save as they arise in conjunction with fixprice distortion. Newbery-Stiglitz(1981) have argued that quantity policy would be dominated by income policy for this purpose on intertemporal cost grounds.

Section I contains an intuitive discussion of the structure and results of the paper. Section II sets out the basic planner's optimization program, and shows that the optimal policy never incorporates a regime switch. Sections III, IV, and V consider the effects of traded goods prices, non-traded good price, and production fluctuations on optimal quantities, respectively. Having characterized optimal policies, it is straightforward to rank second best alternatives such as fixed quantity or fixed shadow price. Formal conditions for this are developed in Appendix B. Section VI concludes with some observations on future work.

I. Intuitive Discussion

Most economists' instincts are that with market prices fixed it will still pay to arbitrage shadow prices. Anderson(1983) has has shown that this is indeed appropriate in a partial equilibrium setting--quantity stabilization, cet. par., will generally benefit a planner while maintaining producer profits or consumer utility. A deeper treatment of the issue involves specification of the source of the disturbance to the supply or demand constraint, making allowance for it in the link of consumer income to production income, and conducting the welfare evaluation with an uncompensated representative consumer.

New effects appear. In the supply constrained region, a rise in export price will (assuming non-traded goods substitute for exportables in supply) reduce non-tradeable supply, raise its shadow price, hence the shadow price margin over the fixprice, and on arbitrage grounds suggest selling more buffer stocks. On the other hand, real income is raised by the terms of trade improvement (the real income effect of the non-traded good shadow price change is zero in a non-monetary economy). With risk aversion, the marginal utility of income is lowered, which suggests reducing the non-traded goods supply still more to lower real income. Thus risk aversion can imply de-stabilizing the quantity consumed in order to stabilize real income. For rises in import prices the terms of trade deteriorate and the two influences run in the same direction.

In the demand constrained region, a rise in export price will (with Hicksian substitutes and normality) increase demand for the non-traded good and raise its shadow price, hence reduce the gap between fixprice and shadow price. Arbitrage considerations imply less purchase for stock. Risk aversion reinforces this, since the terms of trade improvement raises real income and lowers the marginal utility of income. A rise in import price, on the other hand, implies that risk aversion offsets the arbitrage consideration. Optimal policy could imply increasing the fluctuation in non-traded goods production.

Non-traded goods price increases will close the gap between shadow and market prices in the supply constrained region. Arbitrage considerations imply a smaller sale of buffer stocks. The fall in the shadow value of the stock sale, however, lowers income and raises the marginal utility of income. Thus arbitrage and risk aversion act in opposite directions. In the demand constrained region, on the other hand, the rise in price reduces demand, lowers the shadow price, increases the gap between fixprice and shadow price, and on arbitrage grounds implies an increase in buffer stock purchases. The demand decrease also lowers real income on "Keynesian" grounds and implies a rise in the marginal utility of income. Thus the two effects run in the same direction.

Production fluctuations can occur in either traded or non-traded goods sectors, or both. Traded goods production fluctuations simply affect income and non-traded goods demand. With an increase in traded goods production, the rise in income increases demand and lowers the marginal utility of income. In the supply constrained region, the increase in demand raises the shadow price, hence arbitrage implies a rise in stock sales while risk aversion implies a fall. In the demand constrained region, on the other hand, the two influences go in the same direction. Non-traded production fluctuations affect income and non-traded goods supply. The rise in supply and income in the supply constrained region will lower the shadow price and the marginal utility of income, hence the two effects reinforce. In the demand constrained region, the increase in supply lowers the shadow price; however, the supply shift is unable to become effective so income does not change. Risk aversion is thus irrelevant. Arbitrage considerations indicate lowering the stock purchase.

Succeeding sections make these insights rigorous and present the less intuitive arguments which are also involved. The formal analysis allows us to stipulate conditions on measurable parameters which lead to the surprising results that consumption or production should be de-stabilized.

II. The Basic Model

The technique we shall use to analyze optimal quantity policy involves specifying the utility function of the representative consumer, the production revenue function, the price constraint and the allocation method on the constrained market, and the link between government action and consumer income.

The income of the representative consumer is the production revenue plus the effect of government action. The latter enters in two ways potentially. First, for the supply constrained case it is convenient to run the analysis in terms of shadow prices, in which case consumer income must include the value of the ration coupons implicitly given to consumers. Second, we must account for the revenue needed to run the buffer activity. The method of financing the buffer fund is to some extent arbitrary in our stripped down model with no securities. We could assume that all purchases or sales are taken from or added to the income of the consumer in lump-sum fashion. It seems somewhat more realistic to assume that the buffer agency has access to borrowing externally. Thus we assume that government buffer stock purchases and sales are not taken from or added to current income, being financed from an external fund which charges interest equal to the representative agent's rate of time preference. Private agents are forbidden access to this market by assumption, which can be rationalized on transactions cost grounds. The analysis presented below is not sensitive to the assumption, in the sense that the controlling expressions are nearly identical when the other polar case of all-current-financing is analyzed. We can rationalize the structure used by appeal to the example of public utilities. Their stocks or bonds are sold in national markets to agents widely dispersed, while their activities are concentrated. Another example is proposals for externally financed stabilization policies of developing countries.[1]

To keep matters simple we specify an import good and an export good, both tradeable at exogenous world prices π_m and π_x. The vector of tradeable good prices is π. The non-tradeable price

P is institutionally fixed, but possibly fluctuating. Since fixprice models lose the usual homogeneity properties, it is necessary to consider nominal prices.

Following Young-Anderson (1982) we conduct welfare analysis in terms of the "general equilibrium" indirect utility function, with arguments being the time contingent variables and the quantity control instrument. We shall build this function using the elements outlined above. At the end of this section we show the derivation of optimal quantity policy which maximizes general equilibrium indirect utility subject to the zero present value stock change constraint. An immediate result is that it can never pay to switch regimes.

We begin by specifying the basic element of constrained markets; the solution to the implied allocation problem. A standard specification is to suppose that the amount consumed equals the lesser of notional demand and notional supply, where "notional" means amount desired if unconstrained. Denote the amount consumed as Z, and we have:

(1) $Z=\min[C(\pi,P,Z,b),D(\pi,P)-b]$

C( ) is the notional demand function for the non-traded good, where π is the traded goods price vector, and b is the government purchase (sale if negative). We will restrict the form of C( ) presently. D( ) is the notional supply function.

Supply is assumed to occur as a result of maximizing production income subject to a convex technology and possibly also a quantity constraint in the non-traded goods sector. The assumption is consistent with a competitive economy in all sectors but in light of the public sector motivation above, it might better be thought of as an efficient planned or mixed economy. When the demand constraint is not effective, production income is given by the revenue function $r(\pi,P)$ and supply of non-tradeables by $r_P(\pi,P)=D(\pi,P)$. When the constraint is effective, production income is $R(\pi,Z+b)+P(Z+b)$, where $R(\ )$ is the production revenue from the traded group of goods $= \pi_x Y+\pi_m M$ with Y and M denoting production of exportables and importables respectively. We note that $R_{Z+b}=-P^{**}$, so that total production revenue changes by $P-P^{**}$ when the constraint is relaxed.

The representative consumer's welfare is assumed to be the expectation of a concave function of his consumption bundle. In any period, his demand functions have as arguments P, π, income, and the supply quantity constraint. When the latter constraint binds, it is convenient to work with shadow price P^*, which is the price which would induce the consumer to freely buy the available supply. This decision must be based on income which includes the value of the implicit ration coupons. In the demand constrained case consumer income includes production income which of course constrained by the level of demand. (1) can be used in the two cases to implicitly solve either the shadow price P^* or the effective quantity supplied to consumers, Z. Formally,

$$(2)\quad C(\pi,P^*,r(\pi,P)+(P^*-P)(D(\pi,P)-b))=D(\pi,P)-b$$

is solved for $P^*(\pi,P,b)$ in the supply constrained case.[2]

In the demand constrained case we solve for $Z(\pi,P,b)$ from:

(3) $C(\pi,P,R(\pi,Z+b)+P(Z+b))=Z$.

We are now ready to organize the general equilibrium indirect utility function. We substitute the general equilibrium demand functions into the concave (risk averse) direct utility function and report the general equilibrium indirect utility function:

(4) $V=V(\pi,P^*(\pi,P,b),I(\pi,P,b))=v(\pi,P,b)$

where: $P^*=P$

$I(\pi,P,b)=R(\pi,Z+b)+P(Z(\pi,P,b)+b)$ for demand constraint

or

$P^*=P^*(\pi,P,b)$

$I(\pi,P,b)=r(\pi,P)+(P^*(\pi,P,b)-P)[D(\pi,P)-b]$ for supply constraint.

The planner's problem is to maximize the present discounted value of indirect utility over the set of time-contingent b's, subject to the stock change equal zero in present value terms. This constraint arises naturally from the stock rule of accumulation:

$$B_t=B_{t-1}\alpha+b_t,$$

where B_t is the stock at time t, α is the growth factor (set equal to the discount factor), and b_t is the buffer purchase at t. This is solved for $B_t=B_0\alpha^t+\sum_{i=0}^{t} b_{t-i}\alpha^i$

and dividing by α^t we get

$$B_t\alpha^{-t}=B_0+\sum_{s=0}^{t} b_s\alpha^{-s}.$$

The constraint is $B_0-B_T\alpha^{-T} = -\sum_{s=0}^{T} b_s\alpha^{-s}=0$, or $\sum_{s=0}^{T} b_s\alpha^{-s}=0$.

We may now define the normalized present discounted value operator Δ:

$$\Delta(b)=\sum_{s=0}^{T} b_s\alpha^{-s}/\sum_{s=0}^{T}\alpha^{-s}.$$

Δ converts the standard present discounted value formula into an average, which eases subsequent proofs considerably. The constraint $\Delta(b)=0$ rules out indefinite accumulation or decumulation. The notation Δ_{π} signifies that the Δ operation is performed over the variable π which varies over time.

We should note what the structure implies about payments balance. Non-traded goods market clearance is imposed in (2) and (3). This plus the budget constraint implies a private balance of trade deficit of Pb in each period in domestic currency terms. For fixed P the constraint on b implies present value payments balance. For variable P, we need not have payments balance for any arbitrary π, but we assume away the problem by supposing that the government imposes a constant lump sum tax such that payments balance.

We can now set out the planner's maximization problem. For completeness we also introduce a production disturbance q into the model.

$$(5)\quad \underset{b}{\text{Max}}\ \underset{\pi,P,q}{\Delta}\ [v(\pi,P,b)]+\lambda\{\Delta[b]\}$$

The first order (necessary) conditions for a solution to the classical maximization problem (5) require for each time period:

(6a) $v_{b^s}=V_I(P-P^*)=-\lambda$

(6b) $v_{b^d}=V_I(P-P^{**})(1+Z_b)=-\lambda$

(6c) $\Delta(b)=0$

where b^s,b^d denote supply and demand constrained regimes, respectively.

In interpreting (6a),(6b) we note that the middle expression of (6a) must be non-positive and the middle expression of (6b) must be non-negative. λ can have either sign. We immediately have the implication that if in any time period we have $V_I(P-P^*)<0$, we are in the supply constrained regime, then we must be in the supply constrained regime in all periods. A symmetric argument applies to the demand constrained regime, thus:

Theorem 1

The optimal policy cannot include regime switches.

Optimal policy requires efficient management of generalized excess demand or supply. This can imply buffer policy converting excess demand into excess supply in given periods. Appendix A shows that sufficiency is guaranteed when utility is concave ($V_{II}<0$) and the "multiplier" is well-behaved.

The remainder of this paper will investigate the qualitative nature of the optimum in (6). The essential structure involves formation of the implicit derivatives:

(7) $b^i_a = -v_{b^i_a}/v_{b^i b^i}$ $\qquad$ $i=s,d$; $a=\pi_x,\pi_m,P,q$

The derivative will have the sign of $v_{b^i_a}$, since the denominator is negative. Of course, it must be examined in both the demand and supply constrained regions. (7) will typically reveal elements of both arbitrage (implying quantity stabilization) and risk aversion, as well as some less intuitive elements.

We should note that in one special case, the policy is quite simple. If asset markets are such that consumers are able to equate the marginal utility of income across states, then the optimal policy consists purely of arbitrage such that the shadow price differential is eliminated. This is the case examined in Anderson (1983).

III. Traded Goods Price Fluctuations

We examine the qualitative nature of the optimal response to traded goods price fluctuations in this section. Thus we report the expressions which sign the derivatives of b with respect to import and export price fluctuations in the supply and demand constrained regions, using equation (7) and results from the Appendix. In the supply constrained region, when traded goods are both gross substitutes for non-traded goods, a rise in π_x or π_m will increase (notional) nontraded goods excess demand. Stabilization of excess demand, indicated on arbitrage grounds, will imply b (the stock purchase) must fall, hence the derivative is negative. In the demand constrained region, the gross substitutes assumption implies a rise in π_x or π_m will increase demand. Stabilization of (notional) excess supply, indicated on arbitrage grounds, will involve a reduction in b, hence the derivative is negative. Provided the gross substitutes assumption is carried through to the demand and supply derivatives, the above statements apply also to consumption and production stabilization. Other factors, as noted in the intuitive discussion, can act to de-stabilize consumption or production. These other factors can also imply "ultra-stabilization" in which the change in b more than offsets the change in constraining consumption or production. Conditions sufficient for this possibility can easily be developed based on the analysis of the Appendix. Here we report:

Theorem 2

The optimal response to export price fluctuations will destabilize (stabilize) excess demand in the supply constrained region whenever:

$$(20)\quad -\left(1-\frac{P^*-P}{P^*}\mu_C\right)\varepsilon_X+\frac{P^*-P}{P^*}\mu_X+\frac{P^*-P}{P^*}\quad\frac{\pi_X(X-H)}{I}\rho \geqslant(\leqslant)\ 0,$$

where $\varepsilon_X = \dfrac{P^*}{\pi_X} P^*_{\pi_X}$

$\mu_C = P^*C_I$ the marginal propensity to consume c

$\mu_X = \pi_X H_I$ the marginal propensity to consume the export good

$\rho = -\dfrac{V_{II}}{V_I} I$, the coefficient of relative risk aversion.

Proof: see Appendix.

$P^*_{\pi_X} \geqslant 0$ with gross substitutes. (See (8) in the Appendix.)

$-P^*_{\pi_X}$ is the pure arbitrage effect term. The terms involving

C_I, H_I represent the influence of π_x directly and through the shadow price of non-traded goods as it acts on the compensated marginal utility of income. With normality these influences run against arbitrage. $Y-H>0$ for exportables so the influence of risk aversion runs against arbitrage. A rise in the price of exports raises real income while increasing excess demand for non-tradeables. An increase in b will lower real income and exacerbate excess demand, hence the arbitrage and risk aversion motives conflict. (20) also informs us that for small distortions (P^*-P close to zero), arbitrage considerations dominate. In fact, in the limit as P^* approaches P, optimal b shifts to fully offset the shift in excess demand induced by π_x. For finite distortions and low elasticity of P^* with respect to π_x, destabilization of excess demand will be optimal, especially as export share of national income and risk aversion grow.

For the import price fluctuation case we have:

Theorem 3

The optimal response to import price fluctuations will destabilize (stabilize) excess demand whenever:

$$(21)\quad -\left(1-\frac{P^*-P}{P^*}\mu_c\right)\varepsilon_m+\frac{P^*-P}{P^*}\mu_m-\frac{P^*-P}{P^*}\;\frac{\pi_m(G-M)}{I}\rho \geq (\leq) 0. \|$$

ε_m and μ_m are defined analogously to ε_x, μ_x.

Proof: see Appendix.

The only difference between (20) and (21) qualitatively lies in the reversal of the influence of risk aversion. The real income effect of the rise in import price is negative, this increases the marginal utility of income and thus the desirability of raising -b to offset the decline in supply of the non-traded good. Even so, low risk aversion and low elasticity of P* with respect to π_m can result in the middle term dominating, so it becomes optimal to destabilize excess demand.

Turning to the demand constrained cases, we have:

Theorem 4

The optimal response to export price fluctuations will destabilize (stabilize) production whenever:

$$(22)\quad -P^{**}_{\pi_x} - P^{**}_Z Z_{\pi_x} - (P-P^{**})H_I - \rho\left[(P-P^{**})\frac{X-H}{I} + (P-P^{**})^2 \frac{Z_{\pi_x}}{I}\right] + (P-P^{**})\frac{Z_{b\pi_x}}{1+Z_b} \geqslant (\leqslant) 0.$$

Proof: see Appendix.

Normality and substitutability assumptions (see (11), (12), and (16) of the Appendix) imply all but the last term of (22) are negative. P^{**}_z is positive for convex technologies, Z_{π_x} is positive for gross sustitiutes and normality, and $P^{**}_{\pi_x}$ is positive with exports and non-tradeables being quantity substitutes in production. The last term in (22) cannot be signed by economic theory. If we disregard its influence, (22) tells us that arbitrage considerations alone do not go far enough. Again we note that in the limit as the fixprice distortion disappears it can be shown that the optimal policy exactly offsets the shifts in excess supply due to changes in π_x.

For import price fluctuations the only difference is that the terms of trade effect now goes in the opposite direction. This can, however, imply destabilization of production is optimal.

Theorem 5

The optimal response to import price fluctuations will destabilize (stabilize) production whenever:

$$(23)\quad -P^{**}_{\pi_m} - P^{**}_{Z} X_{\pi_m} - (P-P^{**})G_I + \rho[(P-P^{**})\frac{G-M}{I} - (P-P^{**})^2 Z_{\pi_m}/I](1+Z_b)$$

$$+(P-P^{**})\frac{Z_{b\pi_m}}{1+Z_b} \geqslant (\leqslant) 0.$$

Proof: see Appendix

Theorems 2-5 can be used for useful second best analysis. Following the methods of Young-Anderson (1982), whenever the optimal policy involves destabilization we can show that a zero buffer policy will dominate a pure arbitrage (i.e., stabilization) policy, or indeed any policy which stabilizes. See Appendix B. If as a practical matter buffer stock agencies will always stabilize, (20)-(23) provide guidance on whether to establish them.

IV. Non-traded Goods Price Fluctuations

"Fixed" prices do of course change. We allow for unsystematic (i.e. exogenous) change here. Considering the quirks in the political processes which usually create the fixprices of interest here, this may not be entirely unrealistic. Deeper consideration requires endogeneity and intertemporal structure which has so far defied reasonable solution by fixprice theorists.

One complication which should be noted is that with non-traded goods price fluctuations, the government's external debt is not automatically zero over the planning period, as previously. This can be finessed by assuming that the optimal buffer policy is accompanied by a lump-sum tax scheme which internally finances the debt. Its operation can be tucked away in the revenue function and need not be considered further.

Our task is to consider the comparative static derivatives of b with respect to P in the supply and demand constrained regions. In the supply constrained region, we expect that a rise in P will lower the shadow price P* via income effects. See (9) of the Appendix for details. Thus P*-P must fall and arbitrage considerations require smaller sales out of stock: -b falls. Since the rise in P lowers excess demand, the optimal policy stabilizes excess demand. In the demand constrained region, the shadow price P** is lowered by a rise in P. Demand falls, hence production, and with connvex technology this lowers supply shadow price. See (19) of the Appendix. Thus the rise in P raises P-P** and requires greater stock purchases on arbitrage grounds. This acts to stabilize excess supply, which has increased due to the rise in P. The other elements influencing optimal quantity policy are set out in the next two theorems.

Theorem 6

The optimal response to nontraded goods price fluctuations will stabilize (destabilize) excess demand whenever:

$$(24)\quad 1-(1-\frac{P^*-P}{P^*}\mu_C)P^*_P+\rho[\frac{P^*-P}{P^*}\frac{P^*b}{I}+(\frac{P^*-P}{P})^2\theta\,\eta_P] \gtrless 0,$$

where $\theta = \frac{PD}{I}$, the share of non-traded goods production in national income

$\eta_P = D_P \frac{P}{D}$, the general equilibrium supply elasticity of non-traded goods output.

Proof: see Appendix.

The first two terms of (24) are positive. Risk aversion can act in either direction, since the first term in square brackets is negative with buffer sales (b<0). Again we note that it can be shown that in the limit as P* approaches P, the optimal policy is to fully offset the shift in excess demand.

In the demand constrained region we have:

Theorem 7

The optimal response to non-traded goods price fluctuations will stabilize (destabilize) production whenever:

$$(25)\quad 1-P^{**}_Z Z_P-\frac{P-P^{**}}{P}\mu_C-(P-P^{**})^2\frac{Z_P}{I}\rho-(P-P^{**})\frac{b}{I}\rho$$

$$+(P-P^{**})\frac{Z_{bP}}{1+Z_b} \gtrless 0.$$

Proof: see Appendix.

The first two terms represent the arbitrage effect and are positive. Since z_P<0 with normality (see (13) in the Appendix) while b>0 sometimes, the influence of risk aversion can be in either direction. The term involving C_I represents the effect of P on the marginal utility of income at constant real income, and with normality this influence counters the arbitrage influence. Again, it can be shown that in the limit as P^{**} approaches P, the optimal policy fully offsets the shift in production.

We should also note that as in Part III we can develop second best arguments to show that a zero buffer policy dominates any excess supply or demand stabilization policy whenever the optimal policy is to destabilize. See Appendix B for a formal treatment.

V. Production Fluctuations

Production fluctuations can be generally represented as a shift in the supply functions. It is convenient to consider them concentrated in either non-traded or traded goods to spell out the intuition. A rise in non-traded goods supply will decrease excess demand in the supply constrained region normally. P^* will fall, less stock will be sold on arbitrage grounds, hence excess demand is stabilized. A rise in traded goods output increases excess demand for non-tradeables with normality. P^* rises indicating greater stock sales on arbitrage grounds, thus stabilizing excess demand.

In the demand constrained case, non-traded goods capacity increases are ineffective. Traded goods output increases increase demand for non-tradeables with normality, thus reduce $P-P^*$ and indicate smaller stock purchases on arbitrage grounds. In general we have the following pair of theorems:

Theorem 8

The optimal response to production fluctuations will stabilize (destabilize) excess demand whenever:

$$(26)\quad -(P_q^*)^2[1-\frac{P^*-P}{P^*}\mu_c]+\frac{P_q^*}{I}\{(P^*-P)^2D_q-(P^*-P)(\pi_x X_q+\pi_m M_q)\}\rho\leq(\geq)0.$$

Proof: see Appendix.

We note from (26) and (10) of the Appendix that with negative correlation of outputs, $P_q^* \leq 0$, and risk aversion can never overturn arbitrage influences. Negative correlation arises naturally in Rybczynski effects of primary factor supply disturbances. Stabilization (possibly "ultra-stabilization") is then always indicated. With positive correlation it is possible for the two terms in (26) to differ in sign. This is more likely as disturbances are concentrated in the traded goods sector. We note again that as the limiting case of $P=P^*$ is approached the optimal policy fully offsets the effect of production disturbances on excess demand. Interestingly, if disturbances are entirely concentrated on the nontraded good production, the limiting case holds for all $P^*>P$.

For the demand constrained case we have:

Theorem 9

The optimal response to production fluctuations will stabilize (destabilize) production whenever:

$$(27a)\quad (P_q^{**}+P_Z^{**}Z_q)^2+(P_q^{**}+P_Z^{**}Z_q)\rho[(P-P^{**})^2\frac{Z_q}{I}+(P-P^{**})\frac{R_q}{I}]$$

$$-(P_q^{**}+P_Z^{**}Z_q)(P-P^{**})\ \frac{Z_{bq}}{1+Z_b}\ \gtrless 0.$$

Noting $Z_b \geq 0$, optimal policy will stabilize (destabilize) production whenever b_q^d and Z_q have opposite signs, or

$$(27b)\quad P_q^{**}Z_q+P_Z^{**}Z_q^2+Z_q\rho[(P-P^{**})^2\ \frac{Z_q}{I}+(P-P^{**})\ \frac{R_q}{I}]$$

$$+Z_q(P-P^{**})\ \frac{Z_{bq}}{1+Z_b}\ \gtrless 0.$$

Proof: see Appendix.

(27b) is likely to be positive. P_q^{**} is unsignable, as is the last term involving z_{bq}. Disregarding these influences (27b) should be positive. From (14), z_q is positively proportional to R_q; this makes the risk aversion term positive and reinforces the first , arbitrage, term. If non-traded and traded goods production fluctuations are negatively correlated, Theorem 9 also implies that optimal policy will stabilize excess supply, the usual form we have used. Again the limiting case of P* approaching P has optimal policy which fully offsets the shifts in excess supply.

Finally, we note that where optimal policy involves destabilization, the zero buffer policy dominates any stabilizing policy. Our consideration of (26) and (27) indicates this is unlikely with negative correlation or with excess supply. See Appendix B for a formal treatment.

VI. Conclusion

Buffering policy on fixprice markets involves elements of shadow price arbitrage interacting with risk aversion and general equilibrium structure in ways which can produce surprising results. De-stabilization of quantities can sometimes be indicated. On second best grounds, in this case, no buffering is preferred to stabilizing buffering. While developed in a very special model, the results should be robust to a good deal of generalization. One extension which should be enlightening is to introduce asset markets. Perfect asset markets allow smoothing the variation in marginal utility of income, and hence reduce buffering policy to arbitraging shadow price variation. More limited forms of asset exchange will produce an interaction of the two. This should be particularly relevant to fixprice economies running buffering policies and external borrowing simultaneously.

Appendix A. Proof of Theorems 2-9

The basic structure of the model is given in equations (2)-(4) of Section I. Repeating them here for convenience:

(2) $C(\pi,P^*,r(\pi,P,q)+[P^*-P][D(\pi,P,q)-b])=D(\pi,P,q)-b$ supply constraint

(3) $C(\pi,P,R(\pi,Z+b,q)+P[Z+b])=Z$ demand constraint

(4) $v(\pi,P,q,b)=V(\pi,P^*(\pi,P,q,b),I(\pi,P,q,b))$ general equilibrium indirect utility

where: $P^*=P$

$I=R(\pi,P,Z+b,q)+P[Z+b]$ demand constraint

$Z=Z(\pi,P,b,q)$ from (3)

$P^*=P^*(\pi,P,b,q)$ from (2) supply constraint

$I(\pi,P,b,q)=r(\pi,P,q)+[P^*-P][D(\pi,P,q)-b]$

In Section II, a discounted sum of utilities (4) is maximized subject to the discounted sum of b's equal zero. The first order conditions (6) characterize the optimal policy. Theorems 2-9 follow from the comparative static derivatives (7):

(7) $b^i_a = -v_{b^i_a} / v_{b^i b^i}$; $i=s,d; a=\pi,P,q.$

Our task in this appendix is to evaluate these derivatives using (2)-(4).

A preliminary issue is the sign of the denominator of (7). For well-behaved optima, which meet the sufficiency condition, it must be negative. Under concave utility (risk aversion), this condition is met in the supply constrained region. In the demand constrained region we must additionally impose that the marginal propensity to consume non-tradeables is nonincreasing in income for sufficiency. Under these conditions, (7) is signed strictly by the numerator.[3]

Turning to consideration of the numerator expressions, we note immediately that we will need a collection of comparative static derivatives for P*, P** and Z (in the demand constrained region) with respect to π_x, π_m, P, q. These follow from differentiation of (3) and (4). We report these derivatives and conditions for signing them in (8)-(19).

<u>Comparative Static Derivatives: Preliminaries</u>

$$(8)\quad P^*_{\pi_i} = -\frac{C_{\pi_i} - D_{\pi_i}}{C_{P^*}\big|_{comp}} > 0 \text{ with gross substitutes; } i=x,m.$$

$$(9)\quad P^*_P = \frac{-bC_I + D_P\left[1 - P^* C_I \frac{P^*-P}{P^*}\right]}{C_{P^*}\big|_{comp}} \gtrless 0.$$

$$(10)\quad P^*_q = \frac{(1-P^*C_I)D_q - C_I(\pi_x X_q + \pi_m M_q)}{C_{P^*}\big|_{comp}} \gtrless 0.$$

(9) is negative under b>0 and all-goods-normal, since in this case the square bracket expression must lie in the unit interval. In (10), $P^*_q<0(>0)$ as q raises D alone or traded goods production alone. If traded and non-traded goods production is negatively correlated, $P^*_q<0$ under the convention $D_q>0$.

In the demand constrained region we report:

(11) $P^{**}_{Z+b} = P^{**}_{Z} = P^{**}_{b} = -R_{ZZ} \geq 0$ for convex technology

(12) $Z_{\pi_i} = \dfrac{C_{\pi_i}}{1 - PC_I \dfrac{P-P^{**}}{P}} \geq 0$ for gross substitutes and all-goods normal, $i=x,m$.

The expression $\dfrac{1}{1-PC_I \dfrac{P-P^{**}}{P}}$ is equal to $1+Z_b$, and is effectively the Keynesian multiplier. Common sense use of the model implies analyzing the region where $Z_b \geq 0$. All-goods-normal is a sufficient condition for this.

(13) $Z_P = \dfrac{C_P \big|_{comp}}{1-PC_I \dfrac{P-P^{**}}{P}} \leq 0$ for all-goods-normal

(14) $Z_q = \dfrac{C_I(\pi_x X_q + \pi_m M_q)}{1-PC_I \dfrac{P-P^{**}}{P}}$ has sign of $R_q = \pi_x X_q + \pi_m M_q$ for all-goods-normal

(15) $P^{**}_{q} = - R_{Zq} \gtrless 0$

(16) $P^{**}_{\pi_i} = - R_{Z\pi_i} = - R_{\pi_i Z} \geq 0$ for non-traded and traded goods being "quantity substitutes" in production[4]

(17) $\dfrac{dP^{**}}{dP} = P^{**}_{Z} Z_P \leq 0$ with non-Giffen non-traded goods consumption

(18) $\dfrac{dP^{**}}{d\pi_i} = P^{**}_{\pi_i} + P^{**}_{Z} Z_{\pi_i} \geq 0$ with quantity substitutes and all-goods-normal

(19) $\dfrac{dP^{**}}{dq} = P^{**}_{q} + P^{**}_{Z} Z_q \gtrless 0.$

Comparative Static Derivatives: Proofs of Theorems 2-9

We now turn to evaluation of (7), which yields Theorems 2-9. In all cases, by previous steps, we need consider only the numerator of (7).

Proof of Theorem 2

$b^s_{\pi_x}$ has the sign of $v_{b^s_{\pi_x}} = V_I(-P^*_{\pi_x}) + (P-P^*)(V_{I\pi_x} + V_{IP^*}P^*_{\pi_x})$.

Using standard properties of the indirect utility function $V_{I\pi} = -V_I H_I + (X-H)V_{II}$. Further, since C=D, the same operation yields $V_{IP^*} = -V_I C_I \cdot P^*_{\pi_x} \geqslant 0$ under gross substitutes, by (9). Substituting into the expression for $v_{b^s_{\pi_x}}$ dividing by $V_I P^*/\pi_x$ and rationalizing terms we have:

$b^s_{\pi_x} \geqslant (\leqslant 0)$ whenever:

$$(20)\quad -\left(1-\frac{P^*-P}{P^*}\mu_c\right)\varepsilon_x + \frac{P^*-P}{P^*}\mu_x + \frac{P^*-P}{P^*}\ \frac{\pi_x(X-H)}{I}\rho \geqslant (\leqslant)\ 0,$$

where $\varepsilon_x = \dfrac{P^*}{\pi_x} P^*_{\pi_x}$

$\mu_c = P^* C_I$ the marginal propensity to consume c

$\mu_x = \pi_x H_I$ the marginal propensity to consume the export good

$\rho = -\dfrac{V_{II}}{V_I} I$, the coefficient of relative risk aversion.

Excess demand rises with π_x under the gross substitutes assumption, hence (20) is also the condition for optimal policy to destabilize (stabilize) excess demand.‖

Proof of Theorem 3

$b^s_{\pi_m}$ has the sign of $v_{b^s_{\pi_m}} = V_I(-P^*_{\pi_m}) + (P-P^*)(V_{I\pi_m} + V_{IP^*} P^*_{\pi_m})$.

The same steps as in the preceding proof yield that $b^s_{\pi_m} \gtrless 0$ and policy is destabilizing (stabilizing) whenever:

$$(21)\quad -\left(1-\frac{P^*-P}{P^*}\mu_c\right)\varepsilon_m + \frac{P^*-P}{P^*}\mu_m - \frac{P^*-P}{P^*}\frac{\pi_m(G-M)}{I}\rho \gtrless 0. \|$$

ε_m and μ_m are defined analogously to ε_x, μ_x.

Proof of Theorem 4

$b^d_{\pi_x}$ has the sign of $v_{b^d_{\pi_x}} = V_I(-P^{**}_{\pi_x} - P^{**}_Z Z_{\pi_x})(1+Z_b)$

$$+ V_I(P-P^{**})(V_{I\pi_x} + V_{IZ} Z_{\pi_x})(1+Z_b)$$

$$+ V_I(P-P^{**}) Z_{b\pi_x}.$$

Noting $V_{IZ} = V_{II}(P-P^{**})$ and $V_{I\pi_x} = -V_I H_I + (X-H)V_{II}$ we substitute into the expression for $v_{b^d_{\pi_x}}$, divide by $V_I(1+Z_b)$, and rationalize to form $b^d_{\pi_x} \gtrless 0$ whenever:

$$(22)\quad -P^{**}_{\pi_x} - P^{**}_Z Z_{\pi_x} - (P-P^{**})H_I - \rho\left[(P-P^{**})\frac{X-H}{I} + (P-P^{**})^2\frac{Z_{\pi_x}}{I}\right]$$

$$+ (P-P^{**})\frac{Z_{b\pi_x}}{1+Z_b} \gtrless 0. \|$$

Z_{π_x} is positive under the gross substitutes assumption. Thus (22) is also the condition for optimal policy to destabilize (stabilize) production.

Proof of Theorem 5

$b^d_{\pi_m}$ has the sign of $v_b d_{\pi_m} = V_I(-P^{**}_{\pi_m}-P^{**}_Z Z_{\pi_m})(1+Z_b)$

$$+ V_I(P-P^{**})(V_{I\pi_m}+V_{IZ}Z_{\pi_m})(1+Z_b)$$

$$+V_I(P-P^{**})Z_{b\pi_m}.$$

From steps taken in the preceding proof we have:

$b^d_{\pi_m} \geq (\leq) 0$ and optimal policy is destabilizing (stabilizing) to production whenever:

$$(23)\quad -P^{**}_{\pi_m}-P^{**}_Z X_{\pi_m}-(P-P^{**})G_I+\rho[(P-P^{**})\frac{G-M}{I}-(P-P^{**})^2 Z_{\pi_m}/I](1+Z_b)$$

$$+(P-P^{**})\frac{Z_{b\pi_m}}{1+Z_b} \geq (\leq) 0. \|$$

Proof of Theorem 6

b^s_p has the sign of $v_b s_p = V_I(1-P^*_P)+(P-P^*)(V_{IP^*}P^*_P+V_{IP})$.

Using standard properties of the indirect utility function we note $V_{IP^*} = - V_I C_I$ and $V_{IP}=V_{II}(b+(P^*-P)D_P)$. Substituting into the expression for $v_b s_p$, dividing through by $V_I P^*$ and rationalizing we have b^s_p is $\geq (\leq) 0$ whenever:

$$(24)\quad 1-(1-\frac{P^*-P}{P^*}\mu_C)P^*_P+\rho[\frac{P^*-P}{P^*}\frac{P^*b}{I}+(\frac{P^*-P}{P})^2\theta\,\eta_P] \geq (\leq) 0,$$

where $\theta = \frac{PD}{I}$, the share of non-traded goods production in national income

$\eta_P = D_P \frac{P}{D}$, the general equilibrium supply elasticity of non-traded goods output.

With gross substitutes, excess demand falls with a rise in P. Thus (24) is also the condition for optimal policy to stabilize (destabilize) excess demand.‖

Proof of Theorem 7

b_P^d has the sign of $v_b d_P = [V_I(1-P_Z^{**}Z_P)+(P-P^{**})(V_{IP}+V_{IZ}\ Z_P)](1+Z_b)$
$+V_I(P-P^{**})Z_{bP}$.

Using standard properties of the indirect utility function we obtain $V_{IP} = -V_I C_I + V_{II} b$ and $V_{IZ} = V_{II}(P-P^{**})$. Substituting into the expression for $v_b d_P$, dividing by $V_I(1+Z_b)$ and rationalizing we have $b_P^d \gtrless 0$ (and with $Z_P \gtrless 0$, optimal policy stabilizes (destabilizes) production) whenever:

$$(25)\quad 1-P_Z^{**}Z_P-\frac{P-P^{**}}{P}\mu_C-(P-P^{**})^2\frac{Z_P}{I}\rho-(P-P^{**})\frac{b}{I}\rho$$

$$+(P-P^{**})\frac{Z_{bP}}{1+Z_b}\gtrless 0.‖$$

Proof of Theorem 8

b_q^s has the sign of $v_b s_q = -P_q^* V_I + (P-P^*)V_{Iq}$. Using standard properties of the indirect utility function, $V_{Iq} = -V_I C_I P^*_q + V_{II}(P^*-P)D_q + V_{II} r_q$. Substituting into the expression for $v_b s_q$, dividing by V_I and rationalizing the expression we have $b_q^s \gtrless 0$ whenever:

$$(26')\quad -(P^*_q)[1-\mu_c \frac{P^*-P}{P^*}]+\frac{1}{I}\{(P^*-P)^2 D_q-(P^*-P)(\pi_x X_q+\pi_m M_q)\}\rho \geq (\leq) 0.$$

Note that the cet. par. effect of q on excess demand for non-tradeables, $\frac{\partial(C-D)}{\partial q}$, has the negative of the sign of P^*_q, from (10). We have $b^s_q \cdot \frac{\partial(C-D)}{\partial q}$ positive for stabilization policy, hence policy is stabilizing (destabilizing) whenever:

$$(26)\quad -(P^*_q)^2[1-\frac{P^*-P}{P^*}\mu_c]+\frac{P^*_q}{I}\{(P^*-P)^2 D_q-(P^*-P)(\pi_x X_q+\pi_m M_q)\}\rho \leq (\geq) 0. \|$$

Proof of Theorem 9

b^d_q has the sign of $v_b d_q = \{-[P^{**}_q+P^{**}_Z Z_q]V_I+(P-P^{**})V_{Iq}\}(1+Z_b)$
$+V_I(P-P^{**})Z_{bq}.$

$V_{Iq}=V_{II}(R_q+(P-P^{**})Z_q)$ from differentiating (4) twice. Substituting into the expression for $v_b d_q$, dividing by $V_I(1+Z_b)$ and rationalizing we obtain $b^d_q \geq (\leq) 0$ whenever:

$$(27')\quad -P^{**}_q-P^{**}_Z Z_q-\rho[(P-P^{**})^2\frac{Z_q}{I}+(P-P^{**})\frac{R_q}{I}]+(P-P^{**})\frac{Z_{bq}}{1+Z_b} \geq (\leq) 0.$$

Noting that (19) $P^{**}_q+P^{**}_Z Z_q=\frac{dP^{**}}{dq}$ and $P^{**}_b \geq 0$, we have that optimal policy stabilizes (destabilizes) P^{**} whenever $-b^d_q \cdot \frac{dP^{**}}{dq} \geq (\leq) 0$, or

$$(27a)\quad (P^{**}_q+P^{**}_Z Z_q)^2+(P^{**}_q+P^{**}_Z Z_q)\rho[(P-P^{**})^2\frac{Z_q}{I}+(P-P^{**})\frac{R_q}{I}]$$
$$-(P^{**}_q+P^{**}_Z Z_q)(P-P^{**})\frac{Z_{bq}}{1+Z_b} \geq (\leq) 0.$$

Noting $Z_b \geq 0$, optimal policy will stabilize (destabilize) production whenever b_q^d and Z_q have opposite signs, or

$$\text{(27b)}\quad P_q^{**}Z_q+P_Z^{**}Z_q^2+Z_q\rho[(P-P^{**})^2\frac{Z_q}{I}+(P-P^{**})\frac{R_q}{I}]$$

$$+Z_q(P-P^{**})\frac{Z_{bq}}{1+Z_b} \geq(\leq)0. \|$$

Appendix B. Formal Second Best Rankings

Theorems 2-9 essentially permit second best rankings of interesting classes of policies. In particular, no buffering can be better than stabilizing buffering when conditions are such that destabilizing buffer activity is optimal. This appendix formalizes the intuitive argument.

The analysis follows Young-Anderson (1982). Let α be the exogenous variable of interest, either π, P, or q. We compare $\Delta_\alpha(v(\alpha,0))$ the level of utility under b set at 0 with $\Delta_\alpha(\alpha,b^0(\alpha))$, where b^0 is some other policy obeying $\Delta(b^0)=0$.

<u>Lemma</u>: If the optimal policy involves $b^*_\alpha \geq 0$ and the reference policy $b^0_\alpha \leq 0$, the second best $b_\alpha=0$ does better than the reference policy.

<u>Proof</u>:

With concave utility, $v_{bb}\leq 0$, so the second mean value theorem implies that: (i) $v(\alpha,0)-v(\alpha,b^0(\alpha))>v_b(\alpha,0)[0-b^0(\alpha)]$.

The Δ operator can be applied to both sides of the inequality (i). From (7) the sign of b^*_α is also the sign of $v_{b\alpha}$. Then for $b^*_\alpha \geq 0$ and $b^0_\alpha \leq 0$ we have $v_b(\alpha,0)$ and $[0-b^0(\alpha)]$ non-negatively correlated. Δ is equivalent to an expectation, and the "expectation" of the right hand side of (i) is:

$$\text{(ii)}\quad \Delta[v_b(\alpha,0)[0-b^0(\alpha)]]\geq\Delta[v_b(\alpha,0)]\Delta[0-b^0(\alpha)]=0,$$

where the right equality follows from the constraint on b policy.‖

The lemma supplies the formal structure needed to complete the second best rankings stipulated in the text.

FOOTNOTES

1. Ideally, we would like a complete specification of the transactions costs and the imperfect capital markets. This is a complex and ambitious undertaking, and much can be learned from simple polar models.

One awkward consequence of the structure chosen is that the government could use its implicit ability to borrow to achieve a first best solution, compensating for the absence of domestic securities markets. The appropriate external borrowing would smooth away all variation in the marginal utility of income, and the role of the buffer agency would be reduced to arbitraging away shadow price differentials. We rule this out on the ground that external borrowing is in practice limited too tightly to permit such a solution.

2. The form of income used in (2) becomes clear with an expanded treatment of the consumer's original choice problem. Let G=consumption of importables, H=consumption of exportables, and U(G,H,C) be the direct utility function. The consumer's program is:

$$\max_{G,H,C} U(G,H,C)$$

subject to:

$$\pi_m G+\pi_x H+PC \leq r(\pi,P)$$

$$C \leq D(\pi,P)-b.$$

When the shadow price P* is formed, the constraint system becomes (use the second constraint with equality, subtract PC and

PD-Pb from both sides of the first equality, and add P*C and P*D-P*b to each side):

$$\pi_m G+\pi_x H+P^*C\leq r(\pi,P)+(P^*-P)(D-b)$$

3. Recall from (6) that the first derivative v_b is

$$V_I(P^*-P) \quad \text{supply constraint}$$

$$V_I(P^{**}-P)(1+Z_b) \quad \text{demand constraint}$$

In the supply constrained region the second derivative is:

(i) $v_{b^s b^s}= -V_I P_b^*+V_{IP^*}\big|_{comp}(P-P^*)P_b^*+V_{II}(P-P^*)^2$

To evaluate (i), first note that $V_{IP^*}\big|_{comp} = -V_I C_I$ (from the symmetry of the Slutsky matrix). Second, note that P_b^* follows from differentiating (3):

(ii) $P_b^*= \dfrac{-[1-C_I(P^*-P)]}{C_{P^*}\big|_{comp}}$

Substituting into (i) we have

(iii) $v_{b^s b^s}= V_I[1-C_I(P^*-P)]^2/C_{P^*}\big|_{comp}+V_{II}(P^*-P)^2$

(iii) is always negative in the first term and negative in the second for $V_{II}\leq 0$, the case of risk aversion.

In the demand constraint region we have the second derivative:

(iv) $v_{b^d b^d}= [-V_I P_b^{**}+V_{II}(P-P^{**})^2](1+Z_b)^2+V_I(P-P^{**})Z_{bb}$.

To evaluate (iv) we first note that convex technology implies $P_b^{**}\geq 0$; the non-traded good has increasing marginal cost. Then the first term is negative. The second term is negative if and only if $C_{II}\leq 0$; the marginal propensity to consume is non-increasing in income.

4. This is not equivalent to the usual substitutes definition: $\frac{\partial M}{\partial P^{**}} \leqslant 0$, $\frac{\partial X}{\partial P^{**}} \leqslant 0$. Instead we stipulate $\frac{\partial M}{\partial Z} \leqslant 0$, $\frac{\partial X}{\partial Z} \leqslant 0$ the definition of quantity substitutes.

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