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BORROWING CONSTRAINTS UNDER SECURED LOAN CONTRACTS*

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ABSTRACT

The effect of capital market imperfections on consumption choice is studied in an environment where secured loan contracts are written. When individuals cannot borrow against the full value of their durable (collateralizable) assets, intertemporal constraints cause intratemporal distortions in consumption choice; constrained agents consume fewer durables relative to nondurables than do unconstrained agents. This distortion allows the shadow prices associated with borrowing constaints to be estimated over the lifecycle using cross–section data. The estimation procedure assumes that whether an agent is constrained cannot be observed directly and estimates the probability that an agent is constrained as a function of observables and unobservables.

Using data from the Consumer Expenditure Survey, the results indicate that borrowing constraints are significant for households under the age of 40. Approximately 60% of households under the age of 30 and more than 35% of households between the ages of 30 and 40 face binding constraints. Although the percentage of constrained households falls with age, point estimates indicate that the severity of constraints increases for those households that remain constrained: between the ages of 30 and 40, family size increases dramatically and the shadow price associated with binding constraints increases by about 30%. Overall, the total percentage of households that are constrained is estimated to be 22% of the population, which confirms estimates from panel and time series data studies that focus on intertemporal distortions.

Although the focus of this paper is on consumption, the methodology logically extends to constrained firms; a constrained firm that cannot borrow against the full value of its capital will optimally distort its factors of production toward labor. The intratemporal distortion in factors of production, arising as a consequence of constraints on borrowing, is an important topic for future research.

I. INTRODUCTION

Since Hall's 1978 paper on the time series properties of consumption, a significant body of literature has developed that has tried to document the existence and magnitude of distortions in consumption arising from capital market imperfections. Recent empirical work shows a growing concensus that approximately 20% of households face binding constraints on borrowing. Hall and Mishkin (1982) find that 20% of households are constrained. Mariger (1986) estimates that 19.6% of U.S. families face binding constraints. Hayashi (1985b) analyzes Japanese data and estimates the proportion of constrained households to be 16 percent. However, Altonji and Siow (1987) cast doubt on these findings, arguing that measurement error in panel data studies accounts for the observed correlation in income and consumption. Once accounting for error in measurement, they find little evidence against the assumption of perfect capital markets. In his 1987 review article, Hayashi argues that the failure of orthogonality tests can be explained by inappropriate treatment of durable expenditures. Hayashi (1985b) finds a substantial degree of durability in Japanese consumption data on nondurables and services. Bernanke (1984, 1985) also stresses the importance of treating durable expenditures correctly. Using panel data, Bernanke (1984) constructs a partial adjustment model of automobile expenditures and cannot reject the PIH. However, in a fully optimizing model of durable and nondurable consumption with adjustment costs, Bernanke (1985) rejects the PIH in aggregate time series data. Although there is little debate whether capital markets are imperfect, substantial disagreement remains as to the actual impact of these imperfections on consumption behavior.

Utilizing the distinction between durable and nondurable consumption goods, I estimate the effect of borrowing constraints on optimal consumption choice using cross–section data. This approach has the benefit of being able to estimate directly the shadow prices associated with borrowing constraints at different stages of the lifecycle without having to confront the aggregation and measurement error problems associated with panel and aggregate time series data.

Previous studies have focused on the intertemporal distortion in consumption that results from an individual's inability to obtain simple consumption loans. In contrast to this approach, I analyze the effect of borrowing constraints on intratemporal consumption decisions in an environment where secured loan contracts are written. In this context, if an agent cannot borrow sufficiently to finance current consumption, he will optimally distort his consumption away from those items that necessitate any degree of saving.

Generally, whether an individual faces a binding borrowing constraint is not known. Previous studies, Runkle (1983), Bernanke (1984), Hayashi (1985a) and Zeldes (1989), impose some exogenous criterion, such as a critical level of observed holdings of liquid assets, to identify constrained households. As Zeldes points out, the choice of splitting criterion is crucial for these tests. Alternatively, I treat an agent's type, constrained or not constrained, as an unobserved variable and estimate the probability that a particular individual is constrained. Since the criterion for 'splitting' the sample is endogenous, this approach allows for reliable estimates of the percentage of the population that is constrained and the resulting distortions. In addition, these probablity estimates can be used to critique splitting rules used in previous analyses.

Since borrowing constraints arise from an agent's inability to transfer wealth over his lifetime, the timing of income is crucial when capital markets are imperfect. Using detailed expenditure data from the Survey of Consumer Expenditures, I estimate the magnitude of shadow prices associated with constraints at different stages of the lifecycle as well as the percentage of the population constrained at each stage. The results indicate that approximately 49% of households under the age of 40 face significant constraints. Constraints are more widespread for younger households, with 60% of households under the age of 30 being constrained, while point estimates indicate that the magnitude of constraints increases for older households that remain constrained. In total, approximately 22% of all households are constrained.

The outline of the paper is as follows. Section II analyzes the agent's optimization problem under secured loan contracts. Section III describes the empirical methodology and derives the explicit functional form to be estimated. Section IV describes the dataset and analyzes the empirical results.

II. THE MODEL

Environment

Consider the following pure exchange economy. Agents live for T periods and are identical except for the timing of their income. In every period of life, each agent is endowed with income, some positive amount of the nondurable good. Income is stochastic and exogenous to an agent's decisions. An agent's type is defined by the shape of his age–income profile. Each agent maximizes utility over two composite consumption goods: durables and nondurables. Nondurables depreciate completely with one period, so that they either must be traded or consumed. Durables, however, depreciate only in part and may be consumed over several periods; an agent is able to use durable stocks as a form of saving. The utility derived from durable services is a function of the stock held and is independent of the rate of depreciation, which is exogenous. There are perfect resale markets for existing durable stocks and no adjustment costs, such that rental markets are not necessary.

An agent's expected income is a function of his age and type. Since the timing of the agent's income will not, in most cases, coincide with his optimal consumption plan, he borrows or lends in order to smooth consumption over his lifetime; agents buy and sell one period bonds in order to achieve their optimal consumption plan. This is the basic premise of the Life Cycle

Hypothesis (LCH). The Permanent Income Hypothesis (PIH) applies to stochastic movements of and around the age–income profile. Survey data from a variety of countries show that age–earnings profiles are single humped and peak between the ages of 40 and 50 (Japelli 1989, p. 1099).

Information

An agent's type is assumed to be private information, known only to the agent himself. Thus, any individual is able to misrepresent his expected future income, presenting himself as a high expected future income type. Lenders know this and must assume that any potential borrower is a low expected future income type. In this environment of extreme adverse selection, unsecured loans cannot be made.

Because income is in the form of the nondurable good and depreciates completely within the period, it is not possible for lenders to seize future income. However, secured loan contracts can be written since lenders are able to tie the availability of credit to the purchase of durables.[1] Durable stocks depreciate only in part and can be repossessed at a later date in order to settle debt contracts.

Loan Contracts

As a result of extreme adverse selection, only secured loan contracts are available; all loans must be backed by the purchase of a durable asset that may be repossessed should the borrower default. I assume that each loan contract is settled on a case by case basis; lenders require that each loan be backed by a particular durable asset. In order to guarantee the future value of their funds, banks lend subject to the following constraint:

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[1]A example of this type of contract is an ordinary automobile loan; the bank holds the lien to the automobile until the loan is completely paid off.

$$L_{it}(1+r_t) \leq q^e_{t+1}(1-\delta)d_{it} - K_{it+1}, \tag{1}$$

where: L_{it} = nominal value of a loan made to agent i at time t,

K_{it+1} = nominal repossession cost associated with L_{it},

d_{it} = stock of durables held by agent i at time t,

q^e_{t+1} = the expectation at time t of the price of a unit of durables at t+1,

r_t = one period nominal interest rate at time t,

δ = depreciation rate for durable stocks.

The bank's lending constraint requires that for each loan contract, the future value of the collateralizing asset, less any costs of repossession, must be sufficient to cover principle and interest on the loan. Defining the net value of a loan to be

$$NV_{it} = L_{it}(1+r_t) - (q^e_{t+1}(1-\delta)d_{it} - K_{it+1}) \leq 0, \tag{2}$$

the bank's rule for issuing funds requires that the net value of each loan be nonpositive. With this rule, the bank escapes all default risk. Should the borrower default, the bank liquidates the securing asset and uses the proceeds to settle the debt contract and finance any associated costs. However, from the borrower's perspective, the rule requires that he save an amount greater than or equal to $K_{it+1}/(1+r_t)$ to cover potential costs incurred by the lender. Assuming repossession costs are proportional to the future value of the securing asset, such that

$$K_{it+1} = kq^e_{t+1}(1-\delta)d_{it}, \qquad 0 \leq k \leq 1, \tag{3}$$

the maximum amount that can be lent to an agent is proportional to the value of the stock of durables that the agent chooses to hold:

$$L_{it} \leq \alpha_t q_t d_{it}, \tag{4}$$

where: $\alpha_t = \dfrac{(1-k)(1-\delta)}{R^e_{t+1}}$,

$$R^e_{t+1} = \frac{1+r_t}{q^e_{t+1}/q_t}.$$

The proportion of a durable purchase financed by lenders is a function of the depreciation rate, the real rate of return on the loan in terms of the securing asset and the cost of repossession. Increases in the rate of depreciation, the cost of repossession or real interest rates reduce the fraction of a purchase that can be financed. The expected future price of durables is crucial in the bank's lending decision. If lenders expect the future price of durables to fall, they will reduce the fraction of a durable purchase that they will finance.[2] However, at the time the contract is written, expectations are fixed. In addition, to make the contracting problem simple, I assume that expectations are uniform between borrowers and lenders.[3] The lender, having formulated an expectation on future durable prices, offers to finance some fraction of the agent's durable purchase. Thus, each agent maximizes utility, taking α_t as given. Since there is no risk associated with loans for which the net value is nonpositive, I assume that profit motives force condition (4) to hold as an equality.

<u>The Agent's Planning Problem</u>

I assume that utility is time separable and that the within period sub–utility function is quasiconcave in nondurable consumption and the stock of durables held by the agent.[4] The

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[2]This contraction of credit can be thought of as a calling in of loans and a subsequent dumping of durable assets on the market. The resulting collapse of durable prices fulfills the lender's expectations; this is Fisher's debt–deflation theory. Mayfield (1989) studies intertemporal aspects of secured loan contracts and real economic activity.

[3]The rational expectations assumption is that expectations are uniform across individuals at a point in time and that errors over time are not serially correlated.

[4]The assumption of time separability is not crucial. It is straightforward to solve a perfect foresight version of the problem with completely general preferences.

agent's planning horizon is finite, ending at time T, and he discounts future utility by the discount factor β. In each period t, the agent solves the following planning problem:

$$\max_{\{c_i, d_i\}} E_t \sum_{s=0}^{T-t} \beta^s U(c_{it+s}, d_{it+s}) \tag{5.1}$$

$$\text{s.t.} \quad -\alpha_{t+s} q_{t+s} d_{it+s} \le -L_{it+s} = Y_{it+s} - p_{t+s} c_{it+s} - q_{t+s}(d_{it+s} - (1-\delta) d_{it+s-1}) - (1+r_{t+s-1}) L_{it+s-1}, \; s = 0, \ldots, T-t, \tag{5.2}$$

$$c_{it+s} \ge 0, \; s=0, T-t,$$

$$d_{it+s} \ge 0, \; s=0, T-t,$$

where: c_{it+s} = real nondurable consumption by agent i at time t+s,

p_{t+s} = price of nondurable consumption at time t+s,

Y_{it+s} = nominal income received by agent i at time t+s.

If information were perfect, the agent would face a single lifetime budget constraint. However, as a consequence of extreme adverse selection, the agent maximizes lifetime utility subject to a sequence of budget constraints, one constraint for each period in the planning problem. The timing of income is crucial to the solution of this problem. If the agent receives his income relatively early in his lifetime, he will save and be unconstrained. However, if his income arrives relatively late in life, he will need to borrow in order to smooth consumption and constraint (5.2) may bind. Given the shape of age–income profiles, it is likely that an agent will face binding constraints on the amount of debt he incurs when young.

Since loans are tied to the purchase of durable goods individuals, by their choice of durable stocks, are able to influence the amount that banks will lend. However, since lenders finance only a fraction of the value of any durable purchase, the borrower is forced to finance the remainder. The fraction $(1-\alpha_t)$ is the percentage downpayment required in the loan contract. If the amount that the individual is forced to self–finance is greater than the value of the current period services provided by the durable good, then he is forced to finance the future consumption of durable services in the current period; the agent is forced to save in the form of the durable asset. This is the mechanism by which intertemporal borrowing constraints distort intratemporal consumption choice. When the consumption of durable services requires that an agent have a positive net asset position in the form of a durable asset to cover potential losses on the part of the lending institution, the desire to consume durable services acts to worsen further the severity of intertemporal constraints. At the margin, constrained agents can lessen the magnitude of the constraints they face by consuming fewer durables relative to nondurables. A constrained agent is forced to trade off the utility derived from the consumption of durable services with that derived from a lower net wealth position.

The optimality condition for intratemporal consumption choice is found by rewriting (5.1) as a dynamic programming problem. Defining

W_{it} = net total wealth at the beginning of period t, and

A_{it} = net financial assets at the end of period t,

the following relationships are identically true:

$$W_{it+1} = (1+r_t)A_{it} + Y_{it+1} + q_{t+1}(1-\delta)d_{it}, \tag{6.1}$$

$$A_{i,t} = W_{it} - p_t c_{it} - q_t d_{it}. \tag{6.2}$$

Given (6.1) and (6.2), an individual's net total wealth evolves subject to the following differenceequation:

$$W_{it+1} = (1+r_t)(W_{it} - p_t c_{it} - q_t d_{it}) + Y_{it+1} + q_{t+1}(1-\delta)d_{it}. \tag{7}$$

At each time t, the agent solves the following problem:

$$V_{it}(W_{it}) = \max_{\{c_{it},d_{it}\}} U(c_{it},d_{it}) + \beta E_t V_{it+1}(W_{it+1}), \tag{8.1}$$

$$\text{s.t.} \quad -\alpha_t q_t d_{it} \leq A_{it} = W_{it} - p_t c_{it} - q_t d_{it}. \tag{8.2}$$

where: V_{it} = the optimal value function at time t.

Dropping agent superscripts for simplicity, the following first–order conditions must hold for each agent at each point in time:

$$U_c + \beta E_t V'_{t+1}(W_{t+1})(-p_t(1+r_t)) - \lambda_t p_t \leq 0, = \text{if } c_t > 0, \tag{9.1}$$

$$U_d + \beta E_t V'_{t+1}(W_{t+1})(-q_t(1+r_t) + q^e_{t+1}(1-\delta)) - \lambda_t q_t(1-\alpha) \leq 0, = \text{if } d_t > 0. \tag{9.2}$$

Assuming an interior solution, condition (9.1) yields

$$E_t V'_{t+1}(W_{t+1}) = \frac{U_c - p_t\lambda_t}{\beta p_t(1+r_t)}. \tag{10}$$

The condition describing the optimal holdings of durables relative to nondurables at a point in time is found by substituting (10) into (9.2) and collecting terms. Condition (11) shows the factors that affect the optimal choice of durables relative to nondurables at a point in time:

$$\frac{U_d}{U_c} = \frac{q_t}{p_t}\left(1 - \frac{1-\delta}{R^e_{t+1}}\right) + \frac{\lambda_t q_t}{U_c}\left(\frac{1-\delta}{R^e_{t+1}} - \alpha_t\right). \tag{11}$$

Condition (11) consists of two terms: a relative price term and a constraint term. λ_t is the Lagrange multiplier associated with the intertemporal constraint in period t. For an unconstrained individual, $\lambda_t = 0$ and only the first term is relevant. The relative price term captures the price of current durable services relative to nondurables by deducting the present discounted value of the remaining durable asset from the current purchase price.

The second term captures the effect of borrowing constraints on intratemporal consumption choice. If no repossession costs exist, such that an agent can borrow against the full value of his durable assets, then $\alpha_t = \frac{1-\delta}{R^e_{t+1}}$, and intertemporal constraints do not affect an agent's intratemporal consumption choice. In this case, the individual finances only current durable services. However, if repossession costs exist, such that k > 0, then $\alpha_t < \frac{1-\delta}{R^e_{t+1}}$, and a constrained agent optimally distorts consumption away from durables. By shifting consumption away from durables, the agent avoids saving and lowers his net asset position.

<u>The Effect of Constraints on Intratemporal Consumption</u>

By defining $\lambda_t' = \frac{\lambda_t}{U_c/p_t}$, and substituting for α_t, condition (11) can be rewritten as:

$$\frac{U_d}{U_c} = \frac{q_t}{p_t}\left((1 - \frac{1-\delta}{R^e_{t+t}}) + \lambda_t' k \frac{1-\delta}{R^e_{t+1}}\right). \tag{12}$$

The multiplier on the current period constraint interacts with the degree of forced saving to determine the optimal current period consumption choice. The following propositions formally establish the effect of intertemporal contraints on intratemporal consumption choice.

<u>Proposition 1</u>: *In the last period of life, an individual consumes such that his intratemporal marginal rate of substitution is equal to the per unit required downpayment on durables in terms of nondurables.*

Proof: If t = T, then the agent faces a single constraint: $\lambda_t = \frac{U_c}{p_t}$ and $\lambda_t' = 1$. Thus, his MRS is such that $\frac{U_d}{U_c} = \frac{q_t}{p_t}(1 - \alpha_t)$. In the last period of life, an agent who desires the consumption of durable services is forced to save an amount equal to $\frac{q_t k(1-\delta)}{R^e_{t+1}} d_t$, which he cannot recover. Thus, the old age consumption of durables is very costly.[5] Since the individual is guaranteed to die before repaying the loan, in old age, he must finance both his current consumption as

[5]In an old–age individual has an operative bequest motive, then consumption in the last period of the plan need not be distorted. The agent can simply leave part of his bequest in the form of durable goods that the child can incorporate into his own durable stock.

well as recovery costs incured by the lending institution. The forced savings implied by the consumption of durables is the motivating force behind the distortion of intratemporal consumption of a constrained agent.

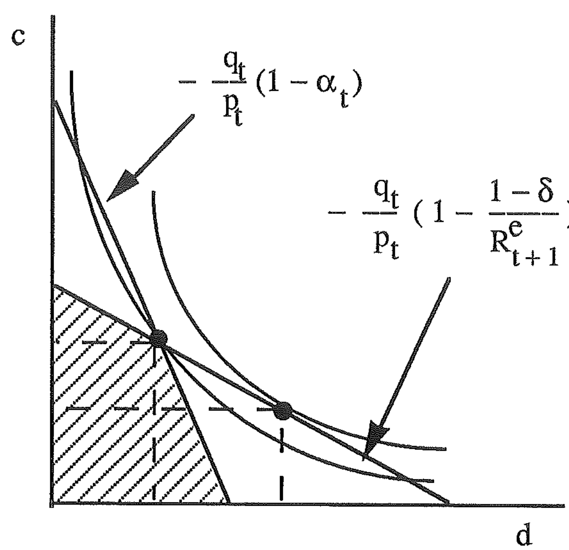
<u>Proposition 2:</u> *A constrained individual, not in the last period of life, consumes such that his marginal rate of substitution is greater than that of an unconstrained agent and less than that of an agent in the last period of life.*

Proof: For a constrained agent who is not in the last period of life, $\lambda_t = \frac{U_c}{p_t} - \beta(1+r_t)E_tV'_{t+1}$. Therefore, $\lambda'_t < 1$ and $\left((1 - \frac{1-\delta}{R^e_{t+t}}) + \lambda_t'k\,\frac{1-\delta}{R^e_{t+1}}\right) < 1 - \alpha_t$. Since the agent is constrained, $\lambda'_t > 0$ and $(1 - \frac{1-\delta}{R^e_{t+t}}) < \left((1 - \frac{1-\delta}{R^e_{t+t}}) + \lambda_t'k\,\frac{1-\delta}{R^e_{t+1}}\right)$.

<u>Corollary:</u> *A constrained agent, not in the last period of life, will consume at the 'kink' in his intratemporal budget set.*

This follows directly from Proposition 2 and is shown in Figure 1.

Figure 1
The Intratemporal Distorion of Consumption



Proposition 3: *If preferences are homothetic, binding intertemporal borrowing constraints distort intratemporal consumption away from durables toward nondurables.*

Proof: If preferences are homothetic, then the income effects associated with the current period constraint do not affect the consumption of durables relative to nondurables. Thus, the substitution effect is guaranteed to dominate. The assumption of homotheticity is a stronger assumption than necessary, but is sufficient to motivate the effect of constraints on consumption.

III. EMPIRICAL ANALYSIS

Methodology

From condition (12), the magnitude of the shadow prices associated with borrowing constraints at different stages of the lifecycle can be estimated. The outline of my approach is as follows: (1) specify preferences and derive the allocation rule for durables relative to nondurables when consumption is measured perfectly, (2) control preferences for age, household income and family size, (3) given that whether a household is constrained is unobserved, estimate the probability that a particular household is constrained, and (4) estimate preferences and shadow prices by maximum likelihood, assuming that a household's consumption of durables is measured with error.

Functional Form

Assuming that an agent's within–period preferences are Cobb–Douglas, such that

$$U_i(c_{it},d_{it}) = c_{it}^{\omega_i} d_{it}^{1-\omega_i},$$

the marginal rate of substitution between durables and nondurables is

$$\frac{U_d}{U_c} = \frac{1-\omega_i}{\omega_i}\frac{c_{it}}{d_{it}}. \tag{13}$$

Substituting (13) into (12) and rearranging, the optimal value of durables relative to nondurable expenditures yields:

$$\Delta_{it} = \left\{\frac{\omega_i}{1-\omega_i}\left((1-\frac{1-\delta}{R^e_{t+1}}) + \lambda'_i k\,(\frac{1-\delta}{R^e_{t+1}})\right)\right\}^{-1}, \tag{14}$$

where: $\Delta_i = \dfrac{qd_{it}}{pc_{it}}$.

The nondurable consumption share, ω_i, is given by:

$$\omega_i = \frac{\exp(\beta X_i)}{1+\exp(\beta X_i)}, \tag{15}$$

which ensures that ω_i is bounded on the (0,1) interval. X_i is a vector of observable characteristics that includes the age of the head of household, family size and total household after–tax income. Age is included since the nature of a family's consumption activities may change over the lifecycle. The size of the family is important because there are economies of scale to the production of durable services; the use of durable items may be shared by members of the family; thus, as family size increases, one expects the consumption of nondurables, such as food, to increase at a faster rate than durables. Finally, income controls for nonlinear income expansion paths.

Since an agent's stock of durables is unobserved, d_i is measured with error. Since data on durable expenditures are available, (14) is reformulated in terms of durable expenditures. At time t, a household's durable stock is given by the following difference equation:

$$d_{it} = (1-\delta)d_{it-1} + \frac{x_{it}^*}{q_t}, \tag{16}$$

where: x_{it}^* = nominal durable expenditure required to maintain an agent's stock of durables.

Under the assumption that the agent's current durable stock is stationary, $d_{it} = d_{it-1}$ and

$$x_{it}^* = \delta q_t d_{it}. \tag{17}$$

Durable expenditures are a proportion of the value of an agent's durable stock and yield information on the unobserved consumption of durables. However, because of the lumpy nature of durable purchases, a household interviewed at a particular point in time may be between durable purchases and x_{it}^* will be measured with error. Therefore, actual expenditures are modelled as measuring required purchases imperfectly, such that

$$x_{it} = x_{it}^* + \varepsilon_i \tag{18}$$

Combining (17) and (18) and substituting into (14) yields

$$\frac{x_{it} - \varepsilon_{it}}{p_t c_{it}} = \delta\Delta_{it} = \delta\left\{\frac{\omega_i}{1-\omega_i}\left((1 - \frac{1-\delta}{R^e_{t+1}}) + \lambda'_i\, k\, (\frac{1-\delta}{R^e_{t+1}})\right)\right\}^{-1}. \tag{19}$$

Given Cobb–Douglas preferences, consumption shares are constant and it is appropriate to scale the error in measurement by nondurable expenditures. Equation (19) is rewritten as

$$\frac{x_{it}}{p_t c_{it}} = \delta\Delta_{it} + \nu_{it}. \tag{20}$$

The lumpy nature of durable purchases has two implications: (1) purchases will be made at discrete points in time and (2) when made, purchases will reflect both current and future consumption. Consequently, even though an individual is currently consuming durable services, he may be between purchases and report too few durable expenditures. In addition, when durable expenditures are made, they will overstate current consumption.[6] Thus, the ratio of durable to nondurable expenditures will be skewed. I assume that $\frac{x_{it}}{p_t c_{it}}$ is lognormally distributed; consequently, the conditional error in measurement, v_i, has the displaced lognormal distribution.

As a result of borrowing constraints, constrained agents face a higher effective real interest rate than unconstrained agents. Previous studies identify this distortion by exogenously splitting the sample.[7] This approach is equivalent to assuming that an indicator variable, such as stock of liquid assets, is perfectly correlated with an individual facing a binding constraint.[8] I depart from this tradition by modeling the probability that a particular household is constrained as a function of both observables and unobservables.

Formally, the problem of identifying constrained households is set up in the following manner. Dropping time subscripts for simplicity, I define a latent variable, W_i^*, such that:

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[6]A good example of this effect is the purchase of an automobile. Assume that an automobile costs $10,000 and depreciates in straight line fashion, such that at the end of 10 years a car will have depreciated completely. In addition, assume that automobile services remain constant over the 10 year period. In this case, an individual will make an automobile purchase once every 10 years. In a cross–section dataset, we will observe 9 out of 10 individuals with no durable purchases and one individual with a single purchase of $10,000. The mean durable expenditure, $1000, will reflect the amount of the durable good that depreciates each year, however the distribution of expenditures will be skewed; a majority of the individuals will report no purchases, while a few respondents will report large expenditures.

[7]Mariger is an exception to this; his estimate of constrained agents' horizons allows the percentage of constrained agents to be determined endogenously.

[8]Of course this is recognized to be false. As Zeldes points out, for consistency, it is crucial to have no constrained households in the unconstrained subsample. However, if the purpose of the test is to detect distortions in consumption, unconstrained households may be put in the constrained subsample; their inclusion simply lowers the power of the test and the estimated magnitude of the distortion.

$\lambda_i = 0$ if $W_i \geq W_i^*$,

$\lambda_i > 0$ if $W_i < W_i^*$,

where: W_i = observed holdings of liquid assets,

W_i^* = minimum level of liquid assets required to facilitate unconstrained consumption.

Writing W_i^* as a function of a household's observed characteristics, such that:

$$W_i^* = \Gamma X_i \varepsilon_i^*, \tag{21}$$

where: X_i = vector of characteristics,

Γ = vector of parameters,

ε_i^* = unit mean random error term defined on $[0,\infty)$,

the probability that a particular household is constrained is written as:

$$\begin{aligned} \Pr\,[\lambda_i > 0] &= \Pr\,[W_i < W_i^*], \\ &= \Pr\,[W_i < \Gamma X_i \varepsilon_i^*], \\ &= \begin{cases} \Pr\,[\varepsilon_i^* > \dfrac{W_i}{\Gamma X_i}], & \text{if } \Gamma X_i > 0, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{22}$$

Under the assumption that ε_i^* has the exponential distribution, then

$$\Pr\,[\lambda_i > 0] = \begin{cases} \mathrm{Exp}[-\dfrac{W_i}{\Gamma X_i}], & \text{if } \Gamma X_i > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

The minimum level of liquid assets required to be unconstrained is parameterized as:

$$\Gamma X_i = \gamma_0 + \gamma_1 \text{Income}_i + \gamma_2 \text{Retire}_i. \qquad (24)$$

For a household earning no income, the parameter γ_0 is the estimated amount of assets required for transactions and precautionary purposes. Since higher income families can afford to hold more of their wealth in a nonliquid form, γ_1 captures income effects. γ_2 adjusts the base level of assets for households that are in retirement, since the nature of their decisions is likely to be substantially different than that of younger households.

Since I am interested in estimating the magnitude of distortions over the lifecycle, I define five age groups and estimate the mean distortion in consumption for each group. Thus, for each observation, I define four dummy variables, such that

$$\begin{aligned} D1_i &= 1, && \text{if } Age_i < 30, \\ &= 0, && \text{otherwise,} \end{aligned}$$

$$\begin{aligned} D2_i &= 1, && \text{if } 30 \leq Age_i < 40, \\ &= 0, && \text{otherwise,} \end{aligned}$$

$$\begin{aligned} D3_i &= 1, && \text{if } 40 \leq Age_i < 50, \\ &= 0, && \text{otherwise,} \end{aligned}$$

$$\begin{aligned} D4_i &= 1, && \text{if } 50 \leq Age_i < 60, \\ &= 0, && \text{otherwise.} \end{aligned}$$

The residual category is the age group 60 and over. In the estimation section of the paper, Table 2 gives the number of observations in each group.

Given (23), the probability that a particular household is constrained is jointly estimated with (20). The likelihood function is a weighted average of the likelihoods conditioned on whether the household is constrained or unconstrained; the weights are equal to the probabilities of being constrained and unconstrained. Thus, the following composite likelihood function is maximized:

$$\text{Max} \quad \prod_{i}^{N} L_i = \prod_{i}^{N} \left(\Pr[\lambda_i > 0]\, \tilde{L}_i + (1 - \Pr[\lambda_i > 0])\, L_i^* \right) \tag{25}$$

where: $\tilde{L}_i = L_i(\tilde{v}_i)$, $\tilde{v}_i = \frac{x_i}{pc_i} - \delta \left\{ \frac{\omega_i}{1-\omega_i} \left((1 - \frac{1-\delta}{R^e}) + (\lambda_{60}\, k\, (\frac{1-\delta}{R^e}) + D_i) \right) \right\}^{-1}$,

$$L_i^* = L_i(v_i^*),\ v_i^* = \frac{x_i}{pc_i} - \delta \left\{ \frac{\omega_i}{1-\omega_i} \left((1 - \frac{1-\delta}{R^e}) \right) \right\}^{-1},$$

$$\Pr\,[\lambda_i > 0] = \begin{cases} \text{Exp}[-\frac{W_i}{\Gamma X_i}], & \text{if } \Gamma X_i > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$D_i = D1_i + D2_i + D3_i + D4_i.$$

IV. ESTIMATION

Data

I use cross–section data from the 1980–81 Consumer Expenditure Survey (CES) to estimate the parameters of (25). The CES collects detailed information on the expenditure patterns of U.S. households. During 1980–81, approximately 17,000 households were surveyed. Each household is interviewed once every three months over a twelve month period; demographic information is collected in the first interview and detailed expenditure data are collected in the second through the fifth interviews. Since financial information is collected only in the final interview, I drop those observations that do not stay in the sample for the entire year. In addition, to preserve anonymity, the BLS topcoded a number of observations; these households are also dropped from the analysis. The final size of the sample is reduced to 9,304 households. For each household, I aggregate annual expenditures into two composite expenditure categories: durables and nondurables. Table 1 gives a list of the categories used to define durable expenditures. With the exception of interest payments, items not listed in Table 1 are treated as nondurable. Since interest payments reflect past expenditure decisions, they are removed from the analysis. The one exception to this rule is a household's mortgage payment. In order to treat payments for shelter consistently across households, mortgage payments are categorized as nondurable expenditures.[9]

TABLE 1
Durable Expenditures

1. Automobiles, Motorcycles, Boats, Aircraft and other Recreational Equipment
2. Furniture
3. Wall and Floor Coverings
4. Appliances
5. Television, Stereo, Photographic and other Entertainment Equipment
6. Yard Machinery
7. Power Tools
8. Equipment and Material for Painting, Wallpapering, Repairing and Remodelling
9. Medical Equipment
10. Sports Equipment
11. China, Crystal, Glassware, Silverware and Flatware
12. Jewelry.

Table 2 lists sample means for the entire sample in addition to low and high liquid asset subsamples. Liquid assets are defined to be the sum of balances in checking and saving accounts, stocks, bonds and money owed to the household. The low liquid asset subsample

[9]Another approach would be to exclude payments for shelter from the analysis. However, since wealthy, unconstrained households are most likely to be homeowners, mortgage payments are treated as a nondurables in order to offset some of the required durable expenditures associated with homeownership.

consists of those households with liquid assets less than one month's after tax income. 54% of the observations fall into the low liquid asset group. 60% of households below the age of 50 are in the low group while only 46% of older households are in the low group. For the sample as a whole, after tax income peaks in the 40 to 50 year old age group, and falls off dramatically in old age, while holdings of liquid assets grow throughout the life cycle. Family size peaks between the ages of 40 and 50, with the largest absolute increase occuring between the ages of 30 and 40. Given the definition of durables in Table 1, the ratio of durable to nondurable expenditures is approximately 0.19 for the sample as a whole. The ratio is greatest for households aged less than 30, who are just beginning their consumption plans, and is smallest for oldest households, who are reaching the end of their consumption plans. Across all age groups, the raw means indicate that the low liquid asset group has smaller relative durable expenditures than the high liquid asset group.

Estimation

In Table 3, I report the results from estimating (25) on the sample described above. The first column reports parameter estimates controlling preferences for current after–tax household income. The second column controls preferences for estimated household permanent income. I calculate an estimate of the family's permanent income using the procedure in King and Dicks–Mireaux (1982). These estimates are given in the appendix.

Preference parameter estimates are similar for both estimations, except that the family size effect is significant when controlling for permanent income. The negative sign of the coefficient on family size confirms the hypothesis of increasing returns to household production of durable services. Substituting these estimates into (15) yields an average nondurable consumption share of approximately 0.5. This is well below that implied by simply looking at the raw expenditure data. This is a consequence of the skewed nature of the durable expenditure data.

TABLE 2
Sample Means

Age Group	Number of Observations	Family Size	After–Tax Income	Liquid Assets	Durables/ Nondurables
Total Sample					
< 30	2,257	2.12	11,580	1,799	0.222
30 – 40	1,907	3.46	17,810	3,942	0.185
40 – 50	1,330	3.83	18,380	5,096	0.187
50 – 60	1,380	2.87	16,070	6,387	0.193
60 ≤	2,430	1.87	9,150	9,091	0.156
All Ages	9,304	2.68	13,860	5,295	0.188
Low Liquid Assets					
< 30	1420	2.32	12,150	282.8	0.220
30 – 40	1089	3.66	16,720	417.6	0.163
40 – 50	739	4.01	18,140	369.2	0.177
50 – 60	705	3.03	15,070	317.3	0.195
60 ≤	1055	1.96	8,470	85.5	0.124
All Ages	5,008	2.88	13,660	288.2	0.177
High Liquid Assets					
< 30	837	1.77	10,610	4,372	0.226
30 – 40	818	3.19	19,260	8,634	0.215
40 – 50	591	3.61	18,690	11,010	0.199
50 – 60	675	2.71	17,110	12,730	0.190
60 ≤	1,375	1.80	9,661	16,000	0.181
All Ages	4,296	2.45	14,090	11,130	0.200

The estimate of $\frac{1-(1-\delta)/R}{\delta}$ is significantly less that 1, implying that real interest rates in terms of durable goods were negative at the time of the survey. This is indicative of the high inflation rates of the latter 1970s and the popularity of household durables, such as oriental rugs, as investment goods. The most interesting difference between the two estimations is the estimate of the critical wealth level, W^*. When preferences are controlled for by permanent income, the estimate of γ_0 indicates that a household earning no income needs to hold approximately $5,000 in liquid assets for transactions and precautionary purposes. In addition, the estimate of γ_1 indicates that for every dollar in after–tax income, 25 cents can be transfered to a less liquid form. Thus, a household earning $20,000 after–tax income can afford to hold most of their wealth in higher return, less liquid assets. However, when controlling for current income, the estimate of γ_0 is not significant and γ_1 is extremely small.

Figure 2 shows the estimated probability, conditional on holdings of liquid assets, of a mean income household facing a binding borrowing constraint. Zeldes (1989) first splits his sample at liquid assets less than two months income. My estimates indicate that there is approximately a 0.4 probability that a household with liquid assets equal to one months income is constrained and that at two months income the probability falls to about 0.2. These results indicate that his more severe split — zero liquid assets versus holdings greater than six months of income — is much more appropriate. An unfortunate consequence of this split is the loss of information associated with excluded observations. I would argue that, if at all possible, the procedure used in this paper should adopted in future studies of this nature; information is not lost, the 'splitting' criterion is endogenous and the methodology matches the actual problem being faced.

TABLE 3
Maximum Likelihood Estimates*

	CURRENT INCOME	PERMANENT INCOME
PREFERENCES		
Family Size	–0.001995 (–1.4787)	–0.002312 (–2.0493)
Age	0.0007573 (5.3963)	0.0007325 (3.8758)
After–Tax Income (1000s)	–0.001007 (–5.2502)	
Permanent Income (1000s)		–0.001334 (–2.6408)
RELATIVE PRICES		
$\frac{1-(1-\delta)/R}{\delta}$	0.7896 (46.143)	0.7821 (24.898)
$\lambda_{60}\frac{k}{\delta}\left(\frac{1-\delta}{R^e}\right)$	0.005981 (1.2498)	0.01035 (1.5744)
D1	0.004122 (0.5750)	0.007152 (0.8079)
D2	0.01180 (1.4103)	0.01338 (1.7519)
D3	–0.001413 (–0.2231)	0.003221 (0.3683)
D4	–0.003635 (–0.5830)	0.0004838 (0.07067)

* Asymptotic t–ratios are in parentheses.

TABLE 3 (Cont.)
Maximum Likelihood Estimates

	CURRENT INCOME	PERMANENT INCOME
PROBABILITY DISTRIBUTION		
γ_0	1134.99	4968.6
	(0.6681)	(28.531)
γ_1	–0.01670	–0.2657
	(–3.1056)	(–23.060)
γ_2	–1026.39	11957.1
	(–0.6038)	(1.0130)
LOG LIKELIHOOD PARAMETERS		
Variance	0.06270	0.06269
	(70.358)	(72.204)
Displacement Factor	2.1422	2.1482
	(111.19)	(43.104)
Observations	9304	9304
Log Likelihood	–320.32	–320.74

FIGURE 2
Probability of Binding Constraint for Mean Income Households
as a Function of Observed Holdings of Liquid Assets
(Liquid Assets in Months of Income)

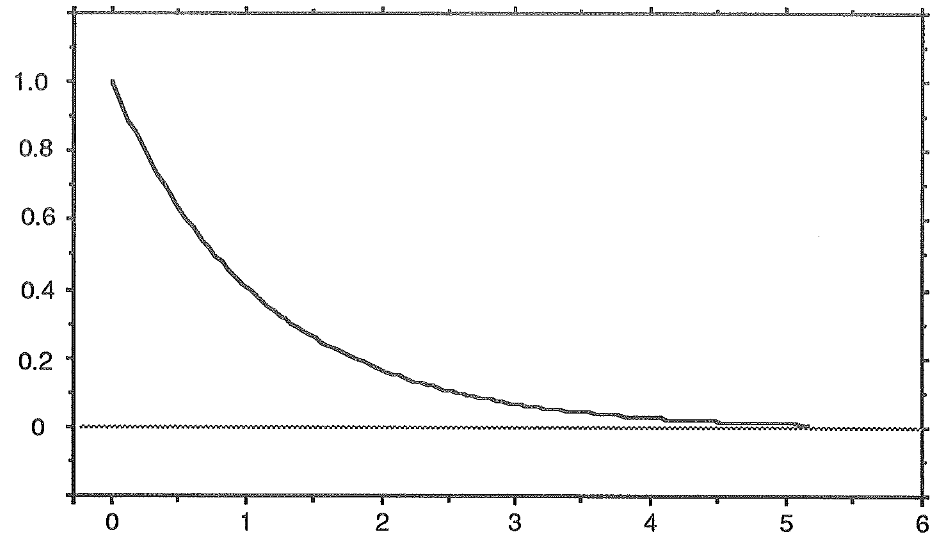


Table 4 shows the estimated distortion in consumption by age group and the p–value for each constraint. Households under the age of 40 face distortions that are significant at the 0.05 level; and when controlling for permanent income, the distortions are significant at the 0.01 level. Although the estimates are not statistically different, the point estimate of the distortion in consumption of households aged 30–40 is about 30% greater than that of younger households. One explanation is that the increase in family size occuring between the ages of 30 and 40 places financial burdens on families that cause distortions to become more severe; even though household income increases over this stage of the lifecycle, consumption demands grow at a faster rate. If individuals have any degree of foresight, they should recognize that they will have greater consumption demands as family size increases and save in advance; clearly family size and the timing of children is endogenous. However, if these families were previously constrained, then they may not have had the ability to prepare in advance for the added financial burden associated with having a family.

TABLE 4
Wald Test of Distortion by Age Group
$$H_0: \lambda_i' \frac{k}{\delta}\left(\frac{1-\delta}{R^e}\right) = 0$$

Age Group	Estimated Shadow Price	Chi Square $\chi^2(1)$	P–Value ($H_1 \neq H_0$)
CURRENT INCOME			
< 30	0.010103	4.4768	0.9656
30 – 40	0.017780	8.6315	0.9967
40 – 50	0.004568	0.7641	0.6179
50 – 60	0.002345	0.2613	0.3908
60 <	0.005981	1.5621	0.7886
PERMANENT INCOME			
< 30	0.01750	9.0436	0.9974
30 – 40	0.02373	16.334	0.9999
40 – 50	0.01357	3.0365	0.9186
50 – 60	0.01083	2.7295	0.9015
60 <	0.01035	2.4789	0.8846

Table 5 reports the estimated percent of households that face binding constraints. Approximately 60% of households under the age of 30 and 37% of households in the 30 to 40 year old age group are constrained. Young constrained households make up about 14.5% of all households and approximately 12% of total consumption, while older constrained households are only 7.6% of all households and account for slightly more that 9% of total consumption in the sample. Combining these results with those of from Table 4, it is apparent that binding constraints are more widespread in the youngest age group, but that the magnitude of the distortions may not be as severe as for older households who remain constrained. In total, approximately 22% of households are estimated to face significant constraints.

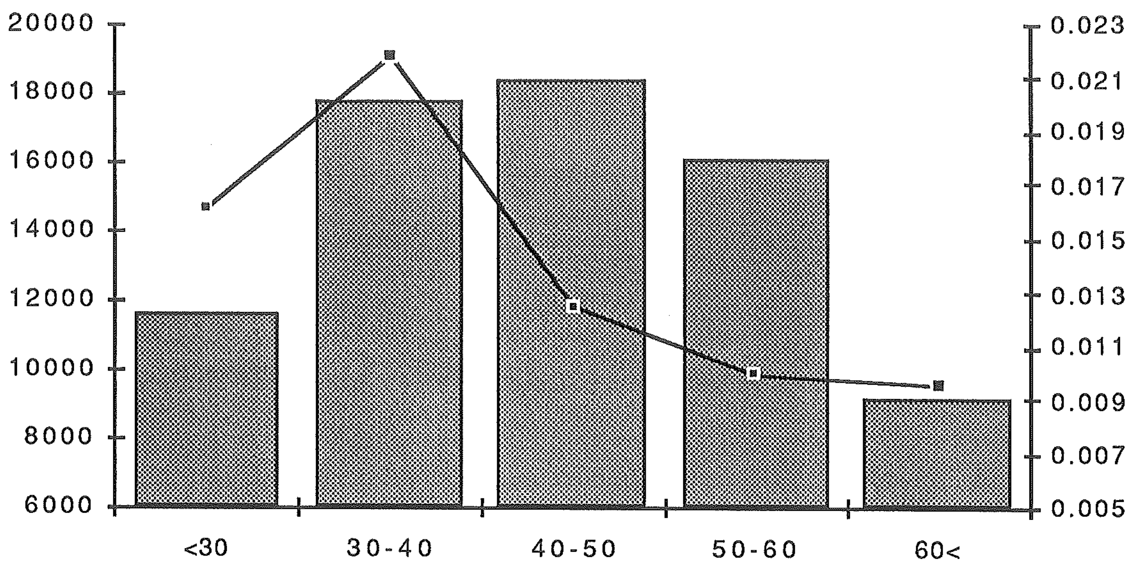
TABLE 5
Estimated Percent of Sample that Faces Significant Constraints

Age Group	Percent of Age Group	Percent of Total Households	Percent of Total Consumption
CURRENT INCOME			
< 30*	59.1	14.3	12.1
30 – 40*	45.8	9.4	11.5
40 – 50	48.2	6.9	9.2
50 – 60	48.5	7.2	8.3
60 ≤	50.0	13.1	9.1
Total Sample		23.7**	23.6**
PERMANENT INCOME			
< 30*	59.4	14.4	12.2
30 – 40*	37.0	7.6	9.3
40 – 50	36.6	5.2	6.9
50 – 60	42.2	6.3	7.2
60 ≤	53.4	14.0	9.7
Total Sample		22.0**	21.5**

*Group is significant at the 95% level.
**Includes only those groups that are significant at the 95% level.

Since the primary focus of this analysis is to determine the nature of borrowing constraints over the lifecycle, I compare point estimates of Lagrange multipliers with the distribution of income in Figure 3. The shaded bars represent the average after–tax household income for each age group and solid line indicates the estimated value of the Lagrange multiplier associated with constraints at each stage of the lifecycle.[10] The distortion in consumption of households under the age of 40 is significantly larger than that of older households who are past the peak in the age–income profile.

Figure 3
Estimated Shadow Price of Borrowing Constraints over the Lifecycle
and the Age–Income Profile



V. CONCLUSION

Utilizing the distinction between durable and nondurable goods, the magnitude and extent of borrowing constraints over the lifecycle is estimated using cross section data. When

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[10]Assumes $\delta = k = 0.1$.

constrained agents cannot borrow against the full value of their collateralizable assets, they optimally distort consumption away from goods that necessitate any degree of saving; a constrained agent is forced to trade–off the utility derived from the consumption of durable services with that derived from a lower net wealth position. Using data from the 1980–81 Consumer Expenditure Survey, I find that approximately half of the households under the age of 40 face significant constraints. Overall, approximately 22% of households are constrained. This estimate is in agreement with studies, using panel and time–series data, that focus on distortions in intertemporal consumption.

Although I focus on distortions in consumption, the methodology presented in this paper logically extends to firms that face imperfections in capital markets. If a firm cannot borrow against the full value of its capital, it will optimally distort its factors of production toward labor. The intratemporal distortion in factors of production, arising as a consequence of constraints on borrowing, is an important topic for future research.

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APPENDIX
Construction of Esimated Permanent Income

Estimates of household permanent income are calculated using the procedure in King and Dicks–Mireaux (1982). Households with less than $4,000 in after–tax income are excluded from the calculations. Adjusting for selection bias, age–income function estimates made for observations above this threshold. Permanent income, using age 45 as the reference age, is imputed with an intraclass correlation coefficient equal to 0.5 and a cohort effect of 0.75 percent.

TABLE A1
Probit Analysis
Dependent Variable: Household Income Less than $4,000

Variable	Estimated Coefficient	Asymptotic t–statistic
Constant	–0.6371	–15.737
Low Education	0.1344	2.920
Age ≤ 22	0.7012	12.901
Age ≥ 63	0.1926	3.837
Family Size	–0.1696	–13.990
Nonworker	0.5986	13.424
Asset Holdings (1000s)	–0.03431	–14.340
DependentVariable Count	0 1	7468 1836
Observations	9304	
Log Likelihood	–3920.6	

TABLE A2
Least Squares Adjusted for Selection Bias
Dependent Variable: Log of Household Income

Variable	Estimated Coefficient	t–statistic
Age–Income Profile		
Constant	8.9650	48.992
Age	0.04315	3.935
Age Squared	-6.35×10^{-4}	–2.863
Age Cubed	2.43×10^{-6}	1.741
Education		
Some High School	0.02157	0.951
High School Graduate	0.1686	6.558
Some College	0.2157	7.710
College Graduate	0.2749	8.066
Graduate/Professional School	0.3071	8.616
Occupation		
Professional/Technical	0.08692	2.567
Managerial	0.1031	3.016
Sales	0.03984	1.271
Crafts	0.0650	2.003
Operative	–0.01284	–0.382
Laborer	–0.07997	–1.677
Services	–0.1093	–3.109
Farm	–0.1448	–1.432
Demographics		
Race	–0.07685	–3.524
Sex	–0.2671	–15.144
Inverse Mill's Ratio	–1.0369	–18.955
Adj. R–Squared	0.3655	
F–Statistic	227.35	
Observations	7468	