ENTRY DETERRENCE IN THE COMMONS

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Abstract: In this paper, we analyze a model in which initially there is a single firm that harvests from a common property resource. The firm faces potential entry of a rival in the future. The cost of harvest from the resource is a function of the stock size. By drawing down the initial population, the monopolist credibly commits to a smaller future stock. Because this increases costs, in particular the entrant's, it makes the post–entry equilibrium less profitable for the entrant. By drawing down current stock sufficiently, the incumbent can make entry unprofitable. Of course this raises the monopolist's costs as well; if the requisite first period harvest is sufficiently great, deterrence will prove unattractive. We analyze the conditions under which the incumbent firm would deter entry and when entry would be allowed. Further, we analyze the effect that potential entry has on the harvest rate both before and after the date of potential entry and whether or not potential entry is welfare improving.

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I. Introduction

On the issue of entry into a common property resource, the literature to date has focused on two extreme cases. The commons has been assumed to be either open–access (no barriers to entry) or open to a fixed exogenously specified number of participants (infinite barriers to entry). For many common property resources, there are significant, but not insurmountable, setup costs. To produce from an oil or gas pool or an aquifer, a well must be drilled; to harvest fish or lobster, a boat must be purchased. In this paper, we study a model with positive, but finite, costs of entry into the commons. Inclusion of costly entry raises two important issues not previously considered in a common property context. First, a firm that is an early entrant, either because of historical accident or because of some special skill or foresight, may have a strategic advantage as an incumbent. The threat of future entry may alter the behavior of incumbents and of common property resource exploitation in important ways. Second, equilibrium market structure in exploiting the commons can be related to the height of entry barriers.

An example of strategic behavior by an incumbent faced with the threat of entry, or of expansion by rivals, is that of the Hudson's Bay Company in furtrading in the 18th and 19th centuries. In the 19th century, the Company faced a threat from small traders who were pushing into Company territory. The Hudson's Bay Company is alleged to have responded to these threats in the following manner:

> "In certain parts of the country, it is the Company's policy to destroy [furbearing animals] along the whole frontier; and our general instructions recommend that every effort be made to lay waste the country, so as to offer no inducement to petty traders to encroach on the Company's limits." [1]

In the 18th century, the Hudson's Bay Company faced a threat from French traders. French furtraders began to encroach on the area surrounding the trading post at York Factory in the 1730's. Either in reaction to the presence of the French, or to dissuade further expansion by the French, Hudson's Bay Company beaver harvests increased each year from 1736 to 1740 after having been generally declining in the early 1730's.

[1] McLean [1849], pp. 261–263.

The estimated population fell from 208,000 to 173,000 in the time period 1736 to 1740. The French, however, were not dissuaded and built an outpost in the area in 1741.[2]

Elements of this type of strategy exist in other examples as well. Johnson and Libecap [1980] indicate that Navajo tribesmen would graze a sufficiently large number of cattle on communal meadows so as to deter other individuals from also using the grazing area. Similarly, one can imagine firms with newly–acquired leases for oil extracting at a very fast rate so as to dissuade others from obtaining leases on adjoining lands (Wiggins and Libecap [1985]).

We analyze a model in which there is an incumbent harvesting from a common property resource that faces potential entry of a rival in the future. We assume that the cost of harvesting the resource is a function of the current stock size and harvest level. By choosing a high level of pre–entry harvest, which lowers future stock size, the incumbent can credibly commit to entry deterrence. Of course this strategy raises the incumbent's costs as well; if the requisite first period harvest is sufficiently great, deterrence will prove unattractive. We solve the model for subgame perfect equilibria and analyze the conditions under which the incumbent would deter or allow entry. We show that the threat of entry increases the pre–entry equilibrium harvest of the incumbent over the equilibrium harvest without threat of entry. Even when the incumbent chooses not to deter, future competition causes the incumbent to internalize less of the future costs of depleting the stock.

The result that the threat of entry increases the harvest rate contrasts with the results of Gilbert and Goldman [1978] and highlights the central importance of property rights. In their model, a monopoly exploiter of an exhaustible resource faces entry from a single rival. Each of the two firms has well established property rights to separate and distinct reserve pools. Entry is forestalled as long as the incumbent's reserves are above a cutoff level (or market price is below a cutoff level) because profits for the entrant in the post–entry game decrease with higher levels of reserves held by the incumbent. In contrast, if instead the potential rival entered by sinking a well into a common pool of oil initially exploited solely by the incumbent, then entry can only be forestalled by holding reserves below some cutoff level.

In addition to analyzing the positive aspects of the model, we also analyze the welfare consequences of potential entry and entry deterrence. As is well known, unless the resource is exploited by a single firm, any one firm's actions induce a negative

[2] Harvest statistics and population estimates are from Carlos and Lewis [1990].

externality on other firms and the market outcome will be inefficient.[3] We also assume that firms may have market power in the output market.[4] In this setting, the welfare consequences of potential entry are ambiguous. The ambiguity results from having both a cost externality and market power in the model. Potential entry is good in the sense that it reduces the market power of the incumbent, but bad in the sense that it increases the rent dissipation from the common property resource. When the demand–side distortions associated with an unthreatened monopolist are sufficiently important relative to the cost–side externalities, the increased output associated with potential entry can lead to a net increase in social welfare. On the other hand, if the cost externalities are of paramount importance, or if entry deterrence entails a sufficiently large expansion in output, then the threat of potential entry can reduce social welfare. The result that the threat of entry can be welfare–enhancing is likely to be of surprise to resource economists; that the threat of entry can lower welfare is likely to prove counter–intuitive to industrial organization economists.

One counter–intuitive result we obtain in this context is that when the demand–side distortions are sufficiently unimportant relative to the dynamic cost effects, social welfare could be increased by having an unthreatened monopolist reduce output. The explanation for this phenomenon is that the monopolist will underproduce in the later periods relative to the social optimum and so does not take proper account from a social perspective of the effect that increased harvests have on future harvest costs.

In section II, we analyze a general two period model with static and dynamic externalities and market power. We show that entry deterrence can be a subgame perfect equilibrium in this context and that the threat of entry leads the incumbent to expand his harvests, whether or not he deters entry. In section III, we use a linear quadratic framework to analyze a continuous time infinite horizon model. The essential insights shown in section II continue to hold in a continuous time infinite horizon model. In section IV, we study the welfare implications of potential entry and of deterring versus

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[3]This externality can be static in nature, in that each firm's activities reduce its rivals' productivity (Cornes and Sandler [1983], Dasgupta and Heal [1979], Dorfman [1974], Haveman [1973]), or intertemporal, in that industry actions today reduce all firms' productivity tomorrow (Berck and Perloff [1984], Kemp and Long [1980], Levhari and Mirman [1980], McMillan and Sinn [1980]). Our model incorporates both static and dynamic externalities. Examples of market failure with common property resources include oil exploration and oil drilling in a common–pool (e.g., Siegel [1985], Wiggins and Libecap [1985]), grazing on communal land (e.g., Johnson and Libecap [1980]), fishing and hunting (e.g., Henderson and Tugwell [1979], Paterson and Wilen [1977]).

[4]The possibility that over–production due to the common property externality is exactly offset by under–production due to market power in the output market has been analyzed in a static framework by Cornes et al. [1986] and Mason et al. [1988].

allowing entry. In selecting a strategy (deterrence vs. acquiescence), the incumbent usually, but not always, selects the socially preferred action. Further, the threat of entry is likely to be welfare enhancing (reducing) relative to an unthreatened monopolist when the demand–side distortions are relatively more (less) important than the dynamic cost effects. Section V contains concluding remarks.

II. The Two Period Model

We analyze a two period game in which an incumbent firm is the sole harvester of a common property resource in the first period but faces potential entry from a rival firm in the second period. In the first period, the incumbent chooses a harvest level and thereby chooses a level of stock available in the second period. After the initial harvest decision by the incumbent, the entrant decides whether or not to enter. The entrant must sink costs of $\$\delta F$ in period one dollars to enter the market in period two, where δ is the discount factor between periods. Once the entrant is in the market it is identical to the incumbent and the two firms compete in the second period by simultaneously choosing levels of harvest and sales. We solve the model for subgame perfect equilibria. Throughout the paper we will use i subscripts to denote the incumbent's strategy and e subscripts to denote the potential entrant's strategy.

Let S_t represent the stock size of the resource in period t, t =1,2. Initial stock size at the beginning of period one is given as S_1. Let H_t equal the total harvest in period t and let h_{it} and h_{et} equal the period t harvest of the incumbent and entrant respectively, $H_t = h_{it} + h_{et}$. The amount harvested in period t can never exceed the stock size at the beginning of the period, $H_t \leq S_t$.[5] The harvest is assumed to be perishable so that it cannot be stored from period to period. The resource stock is linked intertemporally by the growth function g(.): $S_{t+1} = g(S_t - H_t)$. In the case of a non–renewable resource $g(x) = x$ and in the case of a renewable resource $g(x) > x$, $g'(.) > 0$, for all x.

The cost of harvesting the resource for a firm in period t can depend upon its own level of harvest, the total level of harvest of both firms in the period and stock size. We assume costs can be written as: $C(h_{jt},H_t,S_t) = c(H_t,S_t)h_{jt}$, for j = i, e. We assume

[5]Because of rising cost or falling price (or both), the optimizing choice of harvest may be less than stock size even in the final period. Unless otherwise stated, we will assume that the optimal harvest choice is less than the stock so that the stock constraint is not binding.

that $c_H(.) \geq 0$, and $c_S(.) \leq 0$. There is a static cost externality if and only if $c_H(.) > 0$. There is a dynamic cost externality from depletion if and only if $c_S(.) < 0$.

Let Q_t equal total sales by all firms in period t and let q_{it} and q_{et} equal the market sales of the incumbent and the entrant respectively in period t, $Q_t = q_{it} + q_{et}$. The per period market inverse demand function for the sales is given by $P(Q_t)$, with $P'(.) \leq 0$. In a competitive market, where the sales of the firms in this commons make up a negligible share of total resource sales, $P'(.) = 0$, otherwise $P'(.) < 0$. When $P'(.) < 0$, we assume that marginal revenue for a firm falls as a rival increases sales, $P'(.) + P''(.)q < 0$, which implies that revenue is concave in the firm's sales, $2P'(.) + P''(.)q < 0$. When $P'(.) = 0$, we assume $c_H > 0$ to insure that the profit function is concave. These assumptions insure that firms' harvests are strategic substitutes, i.e., that reaction functions are negatively sloped (Bulow et. al [1985]). We allow for the possibility that not all of the harvest will be marketed so that $Q_t \leq H_t$. It is easy to show that the profit maximizing strategy for every firm in the final period (period two) is to set harvest and sales equal and so we set $h_{j2} = q_{j2}$, for j = i, e.

We can write the incumbent's present value of profit as:

$$\Pi_i = P(Q_1)q_{i1} - c(H_1,S_1)h_{i1} + \delta[P(Q_2) - c(Q_2,S_2)]q_{i2}. \tag{1}$$

The entrant's present value of profit is:

$$\Pi_e = \begin{cases} \delta\{[P(Q_2) - c(Q_2,S_2)]q_{e2} - F\} & \text{if the firm enters;} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Given second period stock size, the entrant forecasts the equilibrium profits it will earn if it enters. Entry will occur if and only if equilibrium profit for the entrant in the duopoly game is greater than the sunk cost of entry. If entry occurs, the two firms play a Cournot game in which each firm simultaneously chooses a level of harvest and sales. Let $q_{e2}{}^d(S_2)$ and $q_{i2}{}^d(S_2)$ be the Cournot equilibrium level of harvest and sales for the entrant and the incumbent respectively in the second stage game given stock size S_2. The entrant will decide not to enter if and only if:

$$[P(q_{i2}{}^d(S_2)+q_{e2}{}^d(S_2)) - c(q_{e2}{}^d(S_2)+q_{i2}{}^d(S_2),S_2)]q_{e2}{}^d(S_2) \leq F. \tag{3}$$

The way in which the incumbent can deter entry is by increasing first period harvest in order to lower second period stock. If $c_S < 0$, lower stock values increase marginal cost and decrease equilibrium profit in the second period duopoly game. It is also possible that lower stock levels will constrain the amount of second period production since harvest cannot be more than stock. We focus attention on the case where $c_S < 0$ and do not consider stock constraints. There will exist a critical value of stock size, $\bar{S}_2$, that makes profit for the entrant upon entry just equal to zero. Let $h_{i1}(\bar{S}_2)$ be the level of first period harvest that results in second period stock size equal to $\bar{S}_2$. The incumbent will compare the profits made by harvesting at least $h_{i1}(\bar{S}_2)$ in period one and deterring entry versus harvesting less in period one and allowing entry to occur.

Let us first analyze the case of entry deterrence. Let $q_{i2}^m(\tilde{S}_2)$ be the equilibrium level of harvest for the incumbent when it is a sole harvester in period two and has stock size $\tilde{S}_2$. The maximization problem for the incumbent that deters entry by harvesting h_{i1}, which yields a stock in period two of $\tilde{S}_2 \le \bar{S}_2$, is:

$$\begin{aligned} \text{Max}_{q_{i1},h_{i1}}\ & P(q_{i1})q_{i1} - c(h_{i1},S_1)h_{i1} + \delta[P(q_{i2}^m(\tilde{S}_2)) - c(q_{i2}^m(\tilde{S}_2),\tilde{S}_2)]q_{i2}^m(\tilde{S}_2) \\ \text{s.t.}\quad & q_{i1} \le h_{i1}, \\ & \tilde{S}_2 \le \bar{S}_2 \\ & \tilde{S}_2 = g(S_1 - h_{i1}). \end{aligned} \tag{4}$$

Except in instances of blockaded entry, where the profit maximizing harvest of a sole exploiter is greater than $h_{i1}(\bar{S}_2)$, the first period harvest level will be set equal to $h_{i1}(\bar{S}_2)$.

The first thing to note is that for high enough values of F, deterring entry will be the profitable equilibrium strategy for the incumbent.

<u>Proposition 1</u>: Given that $c_S < 0$, there exist values of the fixed cost, F, for which the subgame perfect equilibrium outcome involves entry deterrence.

Proof: Let $h_{i1}{}^a$ equal the equilibrium first period harvest of the incumbent if it expects to play a duopoly game with the entrant in period two. In other words, $h_{i1}{}^a$ is the first period "acquiescence" level of harvest. Period two stock size will be $S_2 = g(S_1 - h_{i1}{}^a)$. Given S_2, the Cournot duopoly harvest can be found, which defines the left hand side of equation (3). There exists a value of F such that equation (3) is satisfied with equality, i.e., the entrant is just indifferent between entering and staying out when the incumbent harvests $h_{i1}{}^a$. At this critical value of F, if the incumbent increases first period harvest infinitesimally above $h_{i1}{}^a$, thereby decreasing S_2 and increasing cost, it will deter entry and be a monopolist in period two rather than a duopolist. Doing so would yield a discontinuous jump in profit so that deterring entry is more profitable than allowing entry. QED

Clearly, the deterrence first period harvest is decreasing in F and so deterrence profits for the incumbent are monotonically increasing in F (up until the point where entry is deterred by following the monopoly harvest strategy). On the other hand, acquiescence profits do not change with F. It follows that for levels of F above some threshold value, deterrence is more profitable than acquiescence.

In some cases, the level of first period harvest required to deter entry may be so high that first period marginal revenue from selling the entire harvest is negative. If there is free disposal, the incumbent would then prefer to destroy some of the harvest and not bring it to market.[6]

Proposition 2: An incumbent wishing to deter entry will engage in excess harvest if and only if demand is inelastic at $h_{i1}(\bar{S}_2)$.

Proof: Let λ_1 represent the Lagrange multiplier for the constraint that $q_{i1} \leq h_{i1}$ in the incumbent's maximization problem as stated in equation (4). Then, $\lambda_1 \geq 0$, $(h_{i1}-q_{i1}) \geq 0$ and $\lambda_1(h_{i1}-q_{i1}) = 0$. The constrained first order conditions with respect to q_{i1} is:

$$P(.) + P'(.)q_{i1} - \lambda_1 = 0. \tag{5}$$

If demand is inelastic at $h_{i1}(\bar{S}_2)$, then setting $q_{i1} = h_{i1}(\bar{S}_2)$ will yield negative marginal revenue, $P(.) + P'(.)q_{i1} < 0$. Since $\lambda_1 \geq 0$, equation (5) cannot be satisfied. To satisfy equation (5), $q_{i1} < h_{i1}(\bar{S}_2)$ must be true.

Conversely, suppose that $q_{i1} < h_{i1}(\bar{S}_2)$. Then $\lambda_1 = 0$ and q_{i1} is the level of production that maximizes revenue. Since the revenue function concave in own output, it follows that demand is inelastic at $h_{i1}(\bar{S}_2)$. QED

The incumbent engages in excess harvest much as an incumbent monopolist can benefit from holding excess capacity (Spence [1977]). A key distinction, though, is that deterrence through excess harvest can be credible in our model, since an entrant would fail to earn positive economic profit in the post–entry equilibrium, even when, as assumed in this model, firms' harvests are strategic substitutes. By contrast, in a traditional industrial organization model, it is not credible for an incumbent to deter entry by holding excess capacity when firms' outputs are strategic substitutes (Bulow et al. [1985]).

If first period harvest is less than $h_{i1}(\bar{S}_2)$, there will be entry resulting in a Cournot duopoly game in the second period. It is easy to show that the incumbent will

[6]If storage were possible, the incumbent would not destroy harvest but would store excess harvest for future sale. Storage possibilities increase the profitability of entry deterrence in the case of excess harvest. The inclusion of storage possibilities involves a straight forward extension of the current model.

set first period sales equal to first period harvest when it knows that entry will occur so we let $h_{i1} = q_{i1}$. Write $Q_2^d(S_2) = q_{i2}^d(S_2) + q_{e2}^d(S_2)$. Differentiating the profit function for the incumbent, given in equation (1), yields the first order condition for the optimal first period harvest:

$$[P(q_{i1}) - c(q_{i1},S_1) + (P'(q_{i1}) - c_H(q_{i1},S_1))q_{i1}]$$
$$+ \delta g' \frac{\partial q_{e2}}{\partial S_2} q_{i2}^d(S_2)[c_H(Q_2^d(S_2),S_2) - P'(Q_2^d(S_2))] + \delta g' c_S(Q_2^d(S_2),S_2)q_{i2}^d(S_2) = 0. \quad (6)$$

The first term represents marginal profit from harvest and sales in period one. The second term represents the change in second period profit due to a change in harvest and sales by the entrant caused by a change in stock level. There are two effects: i) a demand side effect from price changes caused by different levels of sales by the entrant and ii) a cost effect through the changes in the entrant's harvest (static externality). The third term represents the change in harvest cost in period two caused by a change in stock.

For comparison purposes, consider the case where the incumbent faces no threat of entry in period two. Let $q_{i2}^m(S_2)$ represent the incumbent's optimal second period harvest and sales when it is the sole harvester in the second period given stock size S_2. Taking the first order condition of the incumbent's profit function (equation (1)), we find that the unthreatened incumbent's optimal first period harvest and sales is characterized by:

$$[P(q_{i1}) - c(q_{i1},S_1) + (P'(q_{i1}) - c_H(q_{i1},S_1))q_{i1}] + \delta g' c_S(q_{i2}^m(S_2),S_2)q_{i2}^m(S_2) = 0. \quad (7)$$

The interpretation of this condition is similar to equation (6) above where the first term represents the marginal profit from first period harvest and sales and the last term represents the change in second period profitability from an increase in harvest cost caused by lower stock.

Comparing the harvest and sales of an incumbent that faces no potential entry versus an incumbent that will allow entry in the following period yields the following result.

Proposition 3: Given that $c_S < 0$, an incumbent facing entry will expand first period equilibrium harvest as compared to an incumbent that faces no threat of entry.

Proof: The difference between equations (6) and (7) comes from the presence of an additional strategic term in equation (6) and from having $q_{i2}{}^d$ instead of $q_{i2}{}^m$ in the last term. If $c_S(.) < 0$, then the last term in both equation (6) and equation (7) will be negative. Also, because firms' harvests are strategic substitutes, $q_{i2}{}^d(S_2) \leq q_{i2}{}^m(S_2)$. Therefore, for equal S_2, the third term will be greater (less negative) in equation (6) than in equation (7). Further, with $c_S(.) < 0$, $\frac{\partial q_{e2}{}^d(S_2)}{\partial S_2} > 0$. The sign of the additional strategic term in equation (6), $\delta g' \frac{\partial q_{e2}}{\partial S_2} q_{i2}{}^d[c_H(.) - P'(.)]$, is strictly positive since $c_H(.) \geq 0$ or $-P'(.) \geq 0$, with one of the two being a strict inequality. Because profit is a concave function in q_{i1}, it follows that the value of first period harvest which satisfies equation (7) must be smaller than the value of first period harvest which satisfies equation (6). QED

The incumbent facing entry will expand first period harvest and sales for two reasons. First, the increase in harvest costs in the second period will be partially shifted to a rival firm and will not be borne entirely by the incumbent. Secondly, a decrease in stock will cause the entrant to harvest and sell less in the second period.

It will also be useful to characterize the first period harvest and sales level when there is actual (duopoly) competition in period one as well as in period two. With actual competition in period one, the first order condition characterizing optimal first period harvest is:

$$[P(q_{i1}+q_{e1}) - c(q_{i1}+q_{e1},S_1) + (P'(q_{i1}+q_{e1}) - c_H(q_{i1}+q_{e1},S_1))q_{i1}]$$
$$+ \delta g' \frac{\partial q_{e2}}{\partial S_2} q_{i2}{}^d(S_2)[c_H(Q_2{}^d(S_2),S_2) - P'(Q_2{}^d(S_2))] + \delta g' c_S(Q_2{}^d(S_2),S_2) q_{i2}{}^d(S_2) = 0. \quad (8)$$

A similar expression exists for the other firm. Equation (8) is very similar to equation (6). The only change from equation (6) comes in the first term where total harvest and sales in period one is: $Q_1 = q_{i1} + q_{e1} > q_{i1}$.

Proposition 4: Equilibrium harvest is greater when two firms exploit the commons in each period, as compared to the case where there is one harvester in the first period who allows entry.

Proof: Let $q_{i1}{}^a = Q_1{}^a$ be the equilibrium level of first period harvest in the case where there is a single harvester in period one and two harvesters in period two, i.e., $q_{i1}{}^a = Q_1{}^a$ solves equation (6). Let $Q_1{}^d = q_{i1}{}^d + q_{e1}{}^d$, $q_{i1}{}^d > 0$, $q_{e1}{}^d > 0$, be the equilibrium level of first period harvest in the case where there are two harvesters throughout, i.e., $q_{i1}{}^d$ and $q_{e1}{}^d$ solve equation (8) and the analogous equation for the other firm.

We evaluate the firms' incentives if $Q_1^d = Q_1^a$. In this case, second period stock, S_2, is equal in equations (6) and (8). Since both cases involve two harvesters in the second period, it follows that the final two terms in equation (6) are equal to the final two terms in equation (8). Further, with $Q_1^d = Q_1^a$, first period price and average cost are equal in the two cases. The difference between the left hand side of equation (8) and the left hand side of equation (6) is simply:

$$(P'(.) - c_H(.))(q_{i1}^d - q_{i1}^a).$$

This expression is positive since $P'(.) < 0$ or $c_H(.) > 0$ and $q_{i1}^d < Q_1^d = Q_1^a = q_{i1}^a$. Therefore, a duopolist has positive marginal profit from first period harvest at the level of harvest that satisfies equation (6). Since the profit function is concave in first period harvest, Q_1^d must be greater than Q_1^a. QED

The level of first period harvest necessary to deter entry depends upon sunk cost and can be either greater or less than the quantities defined by equations (6) through (8). We will generally be interested only in cases where $h_{i1}(\bar{S}_2)$ is greater than the desired first period harvest when entry is allowed (characterized in equation (6)). If this is not true, then equilibrium in the model always involves deterring entry. Therefore, we can rank first period harvest in the following order. First period harvest with a single incumbent facing no threat of entry is less than with an incumbent that will allow entry in period two, which is less than with either an incumbent that will deter entry or with two harvesters in both periods. The only ranking that is ambiguous is between the case with an incumbent that will deter entry and the case with two harvesters in both periods.

In the second period, harvest size is an increasing function of the number of harvesting firms and of the level of second period stock. Deterrence will yield smaller second period harvests than either allowing entry or with a single incumbent facing no threat of entry.

III. Extension to an Infinite Horizon Continuous Time Model

It is, perhaps, more natural to think about an infinite horizon continuous time model rather than the two period model of the previous section. In this section, we analyze an infinite horizon continuous time model and show that the important results about entry and deterrence are unchanged from the two period model. In the continuous time infinite horizon model there is an explicit characterization of the lag until the rival can enter as distinct from the discount rate. In the two period model, the entry lag is implicit in the discount rate, which weights the relative importance of the first versus the second period. Otherwise, the two period model is strictly analogous to the infinite horizon continuous time model.

Consider a linear quadratic infinite horizon continuous time version of the model given previously. The sole incumbent is insulated from the threat of entry for $t < T$, where $0 < T < \infty$. A potential rival may enter at any date $t \geq T$ by paying a sunk cost of F (in period t dollars).[7] Let the single known discount rate be $r > 0$. Assume that no firm engages in excess harvest so harvest and sales are the same. The instantaneous harvest cost function for firm j, j = i or e, is $C(q_{jt},Q_t,S_t) = (\alpha+\eta Q_t-\beta S_t)q_{jt}$. Let the instantaneous inverse demand curve for the resource be $P(Q_t) = 1 - Q_t$. The initial resource stock level at time 0 is S_0. For historical relevance and to focus the discussion, we assume that S_0 is "large" relative to steady–state and deterrence stock levels. Stock evolves according to the growth function: $\dot{S}_t = \gamma S_t - Q_t$.

In a game with an infinite horizon, many outcomes can be supported as subgame perfect equilibria through the use of history dependent strategies.[8] Here, we restrict firm's strategies to Markov strategies, i.e., the strategy taken at time t is only a function of the state at time t and is not a function of past actions independent of their influence on the current state. In the appendix, we derive a Markov perfect equilibrium for the complete game by deriving the equilibrium for the game after the entrant either enters or is deterred and then, using these results, derive the equilibrium in the initial phase.

If the entrant decides to enter, there will be a post–entry duopoly game. Suppose the entry occurs at date τ when stock is S_τ. After solving for a Markov perfect equilibrium for this game, we can write the equilibrium value of profit for a firm from this game measured in period τ dollars as $V_2^A(S_\tau)$.

If the incumbent wishes to deter entry it must keep $V_2^A(S_t) \leq F$ for all $t \geq T$. Define $\bar{S}$ as the entry deterring level of stock: $V_2^A(\bar{S}) = F$.[9] The flow level of profits is completely determined by $\bar{S}$, as we can define the harvest level that keeps stock fixed at

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[7]Without changing the substantive conclusions of the paper, one could also model a more complex situation in which the sunk costs of entry are a function of time. Gilbert and Newbery [1982] consider such a model in an R&D context. For example, as potential rivals gain information over time, entry costs may decline. In this case, the constraint on stock size in order to deter entry is a function of time rather than being constant.

[8]For a summary of the literature on history dependent strategies in repeated games see Fudenberg and Tirole [1991].

[9]If there is no discounting, $r = 0$, then $\bar{S}$ is not defined. A positive level of stock can lead to an infinite value function while zero stock yields a value function of 0. With $r > 0$, however, $\bar{S}$ is well defined.

$\bar{S}$ as $\bar{h} = \gamma\bar{S}$. The value of profit for the incumbent from deterrence from date T on, $V_2^N(\bar{S})$, is given in the appendix as equation (A–7).

At the first stage, the incumbent will choose a harvest plan that will either lead to acquiescence or deterrence. In the latter event, the incumbent will choose the profit maximizing harvest from 0 to T given initial stock S_0 and terminal stock condition $S_T = \bar{S}$, which yields a present value of profit prior to T of $V_1^N(S_0)$.

In the case where the incumbent will allow entry, the first stage problem for the incumbent differs from the deterrence case only in the terminal condition. Instead of setting $S_T = \bar{S}$, the incumbent will set S_τ, where τ is the date of entry, such that the marginal value of a unit of stock is the same in both the initial sole harvester phase and the post–entry duopoly game. As we show in the appendix, the entrant will find it most profitable to enter at the first possible date, T, or not at all, because the equilibrium level of stock falls as the date of potential entry approaches. Given the transversality condition at T, one can solve for the profit maximizing level of stock to have to enter the duopoly game and the harvest path from 0 to T. We let the present value of profits along the optimal path from 0 to T when entry is allowed at T to be $V_1^A(S_0)$.

The decision on whether to allow or to deter entry comes down to a simple comparison that can be written in a form that is analogous to the two period model. The incumbent opts to deter entry if and only if:

$$V_1^N(S_0) + e^{-rT}V_2^N(\bar{S}) \geq V_1^A(S_0) + e^{-rT}V_2^A(S_T). \tag{9}$$

The essence of the problem is captured by dividing the game into pre– and post–entry time periods just as in the two period model. We now have equilibrium harvest paths in each phase rather than equilibrium harvests, but otherwise the features of the equilibrium are unchanged from the two period model.

IV. Welfare

A. Analysis of the Two Period Model

The welfare consequences of potential entry in this model are complex. There is a cost externality (both static and dynamic) generated by the common property resource in addition to a pricing distortion. Increases in production in response to the threat of entry may lower price and increase current consumer welfare but may also cause rent dissipation from a static cost externality and may increase future harvest costs through stock depletion. The welfare results of potential entry versus no possibility of entry are only clear in cases where there is just a cost externality or where there is just a price distortion. When there is only a cost externality present, potential entry causes much of the rents that could have been attained from harvest of the resource over time to be dissipated. With only a demand side distortion, however, potential entry forces the incumbent to expand output and increase welfare.

Comparing welfare under deterrence and acquiescence, we find that welfare is unambiguously lower under deterrence than under acquiescence when there is only a dynamic cost externality (with no static externality and no price externality). Here, the extra period one harvest associated with deterrence only serves to raise period two costs, which lowers welfare. Comparisons of welfare effects under deterrence and acquiescence are not clear even when there is only a static cost externality or a price externality. Relative to acquiescence, deterrence yields larger harvests in the first period but lower stock size and less competition in the second period. The first effect is beneficial (harmful) if there is only a price (static cost) externality; the second effect has the reverse sign of the first. Thus, the net effect is ambiguous.

In the welfare analysis of this section, we will use the two period model as presented in section II. The discounted flow of social welfare with harvests and sales in period one (H_1,Q_1) and in period two (H_2,Q_2) can be written as:

$$\Omega = \int_0^{Q_1} P(x)\,dx - c(H_1,S_1)H_1 + \delta\Big\{ \int_0^{Q_2} P(x)\,dx - c(H_2,g(S_1-H_1))H_2 \Big\}. \qquad (10)$$

To facilitate further analysis, let us assume that harvest costs are additively separable: $c(Q_t,S_t,q_{jt}) =[c_1(Q_t) + c_2(S_t)]q_{jt}$. Let $\phi(Q_t) = P(Q_t) - c_1(Q_t) - c_1'(Q_t)Q_t$. Then the effect of a marginal increase in period one harvest can be expressed as:

$$\frac{\partial\Omega}{\partial Q_1} = \phi(Q_1) - c_2(S_1) - \delta g'(.)c_2'(S_2)Q_2^*\Big\{\frac{\phi(Q_2^*) - c_2(S_2)}{c_2(S_2)}\,\frac{\partial Q_2^*}{\partial c_2}\,\frac{c_2(S_2)}{Q_2^*} - 1\Big\}. \quad (11)$$

Further, let λ reflect the shadow cost of period one output and let ξ_s measure the responsiveness of period two supply to changes in $c_2(S_2)$:

$$\lambda = \frac{\partial c_2(S_2)}{\partial Q_1} Q_2^* = g'(S_1 - Q_1) c_2'(S_2) Q_2^* \tag{12a}$$

$$\xi_s = \frac{\partial Q_2^*}{\partial c_2(S_2)} \frac{c_2(S_2)}{Q_2^*} . \tag{12b}$$

Combining equations (11), (12a), and (12b) yields:

$$\frac{\partial \Omega}{\partial Q_1} = \phi(Q_1) - c_2(S_1) - \delta\lambda[\frac{\phi(Q_2^*) - c_2(S_2)}{c_2(S_2)} \xi_s - 1] \tag{13}$$

where Q_2^* is the equilibrium choice of harvest in period two given period two conditions. Using the above derivations we can draw several conclusions, which are given in the following propositions. We begin with a well known and useful benchmark result.

Proposition 5: A single incumbent without threat of entry will set the socially optimal level of harvest in both periods if and only if $P'(.) = 0$.

Proof: The welfare maximizing level of second period harvest will satisfy:

$$\phi(Q_2^*) - c_2(S_2) = 0. \tag{14}$$

Using this fact and equation (13), the welfare maximizing level of first period harvest is Q_1 that solves:

$$\frac{\partial \Omega}{\partial Q_1} = \phi(Q_1^*) - c_2(S_1) + \delta\lambda = 0. \tag{15}$$

A sole harvester will set second period harvest such that:

$$\phi(Q_2) - P'(Q_2)Q_2 - c_2(S_2) = 0. \tag{16}$$

First period harvest will be set to satisfy equation (7), which can be rewritten as:

$$\phi(Q_1) + P'(Q_1)Q_1 - c_2(S_1) + \delta\lambda = 0. \tag{17}$$

Comparing equations (14) and (16) shows that if $P'(.) = 0$, second period harvest, conditional on stock size, will be set in the optimal fashion by the sole harvesting firm. Given that second period harvest is chosen optimally, comparing equations (15) and (17) shows that if $P'(.) = 0$, first period harvest will be set in an optimal fashion.

To prove the converse, note that conditional on second period stock, the firm setting the profit maximizing level of harvest equal to the optimal harvest level implies that $P'(.) = 0$. QED

When the incumbent firm has some market power, $P'(.) < 0$, it follows that $\phi(Q_t) - c_2(S_t) > 0$, for both $t = 1$ and 2. The first–best socially efficient level will not be harvested in either period. Hence, the following welfare analysis of the threat of entry is second–best in nature.

One counter–intuitive result follows from the above analysis. An unthreatened monopolist may harvest more than is socially optimal in the first period. Using equations (13) and (17), we see that the difference between the change in the incumbent's profits and social welfare with a change in first period harvest evaluated at the incumbent's profit maximizing level of first period harvest is:

$$\frac{\partial \Omega}{\partial Q_1} = -P'(Q_1)Q_1 + \delta\lambda\xi_s \frac{\phi(Q_2) - c_2(S_2)}{c_2} . \tag{18}$$

When this expression is negative, the monopolist harvests more than the socially optimal amount in the first period. The first term is positive because of the first period market power distortion. However, the second term, which captures the dynamic impact of period one harvest, is negative. The dynamic benefit of lower period one harvest is related to increased period two stocks, which yield lower period two costs, but this is weighted by period two production. Because the monopolist produces less than the socially preferred level in period two (where price equals marginal cost), the dynamic effect is given less weight than is socially optimal. If the dynamic cost effect dominates, the unthreatened monopolist may harvest more than the optimum in the first period. This result can occur when the product of dynamic externality (c_S), discount factor (δ) and the derivative of the growth function (g') have large absolute value. When this is true, there is a large effect on second period costs from changes in the first period harvest and second period surplus is given a relatively large weight.

We now turn to the question of whether the threat of entry increases or decreases welfare. In cases where the unthreatened monopolist harvests the optimal amount, as when $P'(.) = 0$, or when it harvests more than the socially optimal amount in the first period and deters entry, it follows that any threat of entry must be welfare–reducing. Whenever $P'(.)$ is close to zero, the threat of entry will cause the incumbent to increase the harvest and dissipate some of the potential rent that could be gained by harvesting the resource in an optimal manner. When the unthreatened monopolist harvests more than is socially optimal in the first period, the threat of entry causes an increase in first period harvest, which lowers social welfare. If the monopolist deters entry, then it remains a monopolist in the second period so that there is no reduction in the second period distortion. We summarize this discussion in the following proposition.

Proposition 6: There exists a set of parameters under which the threat of competition must lower social welfare.

As noted above, one scenario where the threat of entry is welfare reducing occurs when the unthreatened monopolist harvests more than the socially preferred level. In this case, the increase in harvest that occurs with deterrence exacerbates the tendency towards over exploitation and so lowers welfare. It is interesting to contrast this result with the corresponding result in Gilbert and Goldman [1978]. They also find that the threat of entry is welfare–reducing but for very different reasons. Because entry is forestalled in their framework by holding reserves above a certain level, the threat of entry tends to reduce the incumbent's rate of activity. It is well–known in models of non–renewable resources, however, that monopolists tend to under exploit the resource relative to a social planner. Hence, the threat of entry is welfare–reducing because it further retards the pace of extraction. As we noted in the introduction, the key distinction between the Gilbert and Goldman model and ours is the nature of property rights: in their model, the firms exploit different deposits of the resource; in ours, the firms exploit the same deposit. In some sense then, incentives in the two models are reversed. Hence, our Proposition 6 might be viewed as a common–property analog of Gilbert and Goldman's welfare results.

One key difference between Gilbert and Goldman's welfare results and ours is that our results are not unambiguous. If the price distortions are sufficiently important, then for sufficiently large values of F, the threat of entry must be welfare–enhancing.

Proposition 7: If $P'(.) < 0$, then there exists a set of parameters under which the threat of competition must raise social welfare.

Proof: For sufficiently large values of F, entry is blockaded. Take the smallest such value; call it $\bar{F}$. For values of F just a bit smaller than $\bar{F}$, the deterrence harvest is a bit larger than the monopoly harvest, and entry will surely be deterred. Hence, this gives the same number of firms in period two as the unthreatened single–firm exploitation, with slightly larger first period output. When equation (18) is positive, this effect will cause an increase in social welfare. QED

In the next section we explore the question of whether the threat of entry is likely to increase or decrease welfare.

B. Linear Quadratic Example

Using a linear quadratic model we compare the welfare consequences of potential entry and of the incumbent's decision to deter or acquiesce. We use the same demand and costs function as given in section III. The growth function is modified to discrete time ($S_2 = \gamma(S_1 - H_1)$). We solve for the incumbent's discounted flow of profit when it allows entry and when it deters entry to determine when entry deterrence is profitable and compare the incumbent's decision to deter or acquiesce with the decision that yields highest social welfare. We also compare the welfare results with potential entry to the case where there is no threat of entry (monopoly). These results may provide some useful insights into the welfare effects of regulating a common property resource through limiting or unleashing competitive forces.

In figures 1 through 3, we compare the incumbent's profit under deterrence (Π_N) and acquiescence (Π_A) and social welfare under deterrence (SW_N) and acquiescence (SW_A) over a range of initial stock sizes (S_1). The values of S_1 shown in the figures ranges from approximately 1.35 to 1.75. As shown in all three figures, increases in initial stock size increase the attractiveness of acquiescence both in terms of profit and social welfare. Acquiescence profits and welfare are increasing in the level of initial stock as increases in stock size lower harvesting cost. For deterrence, the incumbent must drive stock down to $\overline{S}_2$, which fixes second period profit, regardless of initial stock size. Thus, larger values of S_1 require larger amounts of period one harvest to deter entry, without yielding second period gains. At some point, the requisite harvest forces static marginal revenue below marginal cost in period one; past this point, the discounted flow of profits accruing from deterrence is declining in S_1. At some larger value of initial stock, period one price is driven below marginal cost, past this point the discounted flow of social welfare under deterrence is also declining in S_1.[10]

In the figures, S_1^N is the level of stock at which profits under deterrence and acquiescence are equal. Further increases in stock size above S_1^N make acquiescence more profitable than deterrence. S_1^* is the level of stock at which social welfare is identical under deterrence and acquiescence. Again, increases in initial stock size above

[10]While neither of these results depends on excess harvest, they are both reinforced by it. Since excess harvest implies constant period one sales (at the revenue maximizing level), increases in S_1 do not change period one consumer surplus. Increases in S_1 do raise harvest costs and so lower profits and net surplus.

this level of stock make acquiescence socially more desirable than deterrence. By comparing S_1^N and S_1^* we can see if the decision of the incumbent is second–best optimal or not. In figure 1, $S_1^N < S_1^*$ and there exists a range of initial stock for which the firm will choose to acquiesce when from a welfare perspective it would have been better to deter entry. This result may occur when there are relatively severe period one demand–side output distortions and future cost externalities are less important. The result can go the other way, i.e., there may be some range of initial stock for which the incumbent will choose to deter when it is second–best optimal to acquiesce. This result is shown in figure 2. This result can occur when period two cost externalities are relatively more important than period one demand–side distortions.[11]

In figure 3, we show the somewhat surprising result that the incumbent may find it profitable to acquiesce when it is socially preferable for the incumbent to deter entry even though deterrence involves excess harvest. In figures 1 through 3, we label the initial stock at which the firm will choose to engage in excess harvest in order to deter entry as S_1^k. In figure 3, S_1^k lies to the right of S_1^N but to the left of S_1^*. For initial stock values lying between S_1^k and S_1^*, the firm will find it optimal to acquiesce; in this range welfare would be greater with deterrence even though it entails excess harvest. This result can occur when large period one harvests are desirable, while at the same time deterrence is relatively unattractive to the incumbent. The former effect can emerge when the static cost externality is relatively less significant so that the demand–side distortion is more important. Increasing the magnitude of the dynamic externality (the degree to which increases in stock lower harvest costs) makes deterrence less attractive in two ways: it raises requisite period one harvest, which lowers deterrence profits; at the same time it raises acquiescence profits.[12]

In figures 4 and 5, we show the welfare effects of having potential entry as compared to the case with no threat of entry. In figures 4 and 5, we label $\tilde{S}_1$ as the

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[11]The parameterizations used for these diagrams confirms the intuition given in the text. Figure 1 used $\delta = 0.5$, $\eta = 0.4$, $\beta = 0.4$, $\gamma = 1.25$, $\alpha = 0.75$, $F = 0.05$. By contrast, in figure 2, $\gamma = 1.5$, $\eta = 0.5$, and $\delta = 1.5$; all other parameters are as in figure 1. The increase in η raises period one marginal cost and so blunts the output distortion, while the increase in γ enlarges the dynamic cost externality. Raising δ reinforces this effect by placing a larger weight on the second period effect.

[12]In figure 3, $\beta = 0.5$ and $\eta = 0.3$; all other parameters are as in figure 1. Relative to figure 1, figure 3 assumes a higher value of β and a lower value of η.

initial stock level at which welfare is equal in the two cases. Figure 4 illustrates that the threat of entry can have positive or negative welfare consequences. The parameters used in figure 4 are exactly the same as used in figure 1. As in figure 1, S_1^N is less than S_1^* so there is a downward jump in social welfare when the incumbent changes strategy to acquiescence. The solid portions of the SW_A and SW_N curves illustrate the relevant portions of the social welfare function with potential entry. To the left of $\tilde{S}_1$, potential entry leads to welfare improvements, while the reverse is true for high levels of stock. In this example, monopoly harvest throughout is preferred in welfare terms to acquiescence, largely due to the sunk cost of entry. At large levels of stock, the comparison that matters is between SW_M and SW_A. For a wide range of parameter values we find $SW_M > SW_A$. Correspondingly, it seems likely that no entry is preferred to the threat of entry when stocks are large but not when stocks are small.

In figure 5, we show the effects of a small increase in entry costs (F increases from 0.05 to 0.0525). The curves SW_A' and SW_N' are the relevant curves after the increase in F. An increase in F reduces the level of period one harvest required to deter entry. This has two effects on welfare: it raises period one price, which in this example is welfare–reducing; and it yields a larger period two stock, which lowers period two costs and so is welfare–increasing. Beyond some critical value of initial stock, the total effect increases SW_N. Thus, social welfare is higher with larger F for most of the range of initial stocks for which the incumbent chooses to deter. Further, there is a wider range of values of initial stock size for which the threat of entry yields increases in social welfare. However, an increase in F also causes the divergence between SW_M and SW_A to increase. For large initial stocks where the incumbent will choose to acquiesce, there will be an increased loss in welfare from the threat of entry.

Entry costs can increase either because of exogenous events, like weather–related factors that make it harder for the entrant to travel to the commons, or because of government intervention, as with the imposition of a tax on entry. Although economists are likely to recoil from such a tax, figure 5 suggests that it may be welfare–enhancing. In particular, so long as price distortions are not too small (i.e., P'(.) is of sufficient magnitude), a regulating authority can guarantee an improvement in social welfare from the threat of entry by taxing entry to a point where entry is almost, but not quite, blockaded.

V. Concluding Remarks

Our purpose in this paper has been to develop a simple model in which an incumbent sole–exploiter of a common–property resource can credibly deter entry by increasing the harvest in early periods. By drawing down the stock early on, the incumbent ensures that future populations will be too small to attract entry. This incentive is reinforced if the incumbent can let part of his catch go to waste, since he may then strategically reduce the population without flooding the market. Regardless of whether or not the incumbent chooses to deter entry in equilibrium, the threat of entry increases the incumbent's initial harvest rate.

We used the model to discuss possible welfare effects of the threat of entry and of acquiescence versus deterrence. Some of the welfare results are surprising. For example, a firm may find it profitable to acquiesce when it is socially preferable to deter, even though deterrence may involve excess harvest. Further, it is possible for an unthreatened monopolist to harvest more than is socially optimal in early periods. In general, the welfare results of the threat of entry and of acquiescence versus deterrence are ambiguous. There are two conflicting effects influencing harvest decisions: cost externalities (both static and dynamic) and market power in the output market. That the threat of entry can raise welfare may prove surprising to resource economists, as the threat entails increased harvesting in early periods. That the threat of entry can lower welfare may prove surprising to industrial organization economists, since increased output tends to reduce the output distortions which commonly arise when only one firm is present.

One could imagine a variety of extensions of our analysis. With only a small number of exploiters, it is possible that the exploiters may be able to collude. As is shown in Mason and Nowell [1992], there is a range of sunk entry costs for which it is in the incumbent's best interest to allow entry and collude rather than to deter entry, as would have been the case without collusion. They also find, however, that if entry costs are above a particular threshold level, entry will still be deterred in equilibrium. A second extension involves allowing for the possibility of a threshold stock size below which growth is negative. Clark [1973] shows that open–access equilibrium may yield extinction. In our context, it is possible that deterrence, or the threat of entry, will cause extinction of the resource when extinction would otherwise have been avoided.

Our model has potential policy implications for management of limited access, common property resources. Imagine a regulatory authority charged with managing this commons to maximize social welfare. The authority has three options at its disposal: (i) mandate acquiescence; (ii) prohibit entry ; or (iii) laissez faire. The regulatory authority can influence entry costs, by taxing or subsidizing entry (which alters F), or by changing the time lag necessary to obtain a license (which has the effect of altering δ). This decision problem is trivial to solve if the regulatory authority knows all the underlying parameters. The regulator's problem then can be reduced to one of selecting the optimal level of F and the optimal delay added onto time to entry. When the regulator is uncertain of the value of some key parameter(s), the expected welfare loss that occurs when the incumbent chooses the less–efficient alternative tends to be fairly small (cf. Figures 1 – 3). By contrast, there is a risk that large inefficiencies will occur when mandating acquiescence if deterrence is optimal or prohibiting entry if acquiescence is optimal. It seems doubtful, therefore, that a regulator would be able to improve social welfare through regulation of entry conditions unless the regulator is well–informed about the true conditions in the commons.

Appendix: Continuous Time Infinite Horizon Linear–Quadratic Model

A. Analysis of the Second Stage.

1. Acquiescence

Suppose the entrant has chosen to enter at date τ with stock size S_τ. The profit maximization problem for a firm j, j = i or e, is:

$$\text{Max} \int_T^\infty [(1-H_t) - (\alpha+\eta H_t-\beta S_t)]h_{jt}e^{-rt}\,dt$$

$$\text{subject to:} \quad \begin{aligned} &\dot{S}_t = \gamma S_t - H_t \\ &S(\tau) = S_\tau \\ &h_{it} \geq 0,\ h_{et} \geq 0 \quad S_t \geq 0 \quad \text{all } t. \end{aligned} \tag{A–1}$$

Let (h_{it}^*, h_{et}^*) be a Markov perfect equilibrium. Then, the value function for a firm in the duopoly game beginning at τ with stock S_τ is:

$$V_2^A(S_\tau) = \int_\tau^\infty [(1-h_{it}^*-h_{jt}^*) - (\alpha + \eta(h_{it}^*+h_{jt}^*) - \beta S_t)]h_{jt}^*e^{-rt}\,dt\ . \tag{A–2}$$

One can solve for the equilibrium by solving the dynamic programming equations for each firm:

$$rV_2^A(S_t) = \text{Max}\ [(1-H_t) - (\alpha+\eta H_t-\beta S_t)]h_{jt} + \lambda(\gamma S_t - H_t) \tag{A–3}$$

where $\lambda = \dfrac{\partial V_2^A(S_t)}{\partial S_t}$. Solving for h_{it}^* and h_{et}^* yields:

$$h_{it}^* = h_{et}^* = \frac{1-\alpha+\beta S_t-\lambda}{3(1+\eta)} . \tag{A-4}$$

Given the structure of the model, we "guess" that the value function is quadratic:

$$V_2^A(S_t) = aS_t^2/2 + bS_t + c \tag{A-5}$$

where a, b, and c are to be determined. Let $\bar{\eta} = 9(1+\eta)$ and $\bar{\alpha} = 1 - \alpha$. Using the equilibrium strategies in equation (A-4) and noting that the equation must hold for all values of S_t, we find:

$$a = \frac{1}{8}\left\{\frac{r\bar{\eta}}{2} + 5\beta - \gamma\bar{\eta} - \left[\left(\frac{r\bar{\eta}}{2} + 5\beta - \gamma\bar{\eta}\right)^2 - 16\beta^2\right]^{0.5}\right\}$$
$$b = \frac{\bar{\alpha}(2\beta - 5a)}{r\bar{\eta} + 5\beta - \gamma\bar{\eta} - 8a}$$
$$c = \frac{\alpha^2 - 5\bar{\alpha}b + 4b^2}{r\bar{\eta}} .$$

2. Deterrence

The entrant may enter at any date $t \geq T$. In order to deter entry, the incumbent must keep $V_2^A(S_t) \leq F$ for all $t \geq T$. We can define the highest level of stock which deters entry, $\bar{S}$, as: $V_2^A(\bar{S}) = F$. We assume that the deterrence constraint is binding so that the incumbent will set $S_t = \bar{S}$ for all $t \geq T$. In order to keep stock fixed at $\bar{S}$, the incumbent will set harvest, $\bar{h} = \gamma\bar{S}$. The value for the incumbent is then:

$$\begin{aligned} V_2^N(\bar{S}) &= \int_T^{\infty} e^{-rt}(1-\alpha-\eta-[\gamma(1+\eta)-\beta]\bar{S})\gamma\bar{S}\, dt \\ &= \frac{(1-\alpha-\eta-[\gamma(1+\eta)-\beta]\bar{S})\gamma\bar{S}}{re^{rT}} . \end{aligned} \tag{A-7}$$

B. Analysis of the First Stage.

1. Deterrence

In the first stage the incumbent is the sole harvester and chooses a harvest path in order to maximize profits with endpoint conditions defined by the second stage derived above. We can define the first stage value function for the incumbent who plans to deter entry as:

$$V_1(S_0) = \text{Max} \int_0^T e^{-rt}[(1-h_{it}) - (\alpha+\eta h_{it}-\beta S_t)]h_{it}\,dt$$

$$\begin{aligned} \text{subject to:} \quad & \dot{S}_t = \gamma S_t - H_t \\ & S(0) = S_0 \\ & S_T = \overline{S} \\ & h_{it} \geq 0 \; S_t \geq 0 \quad \text{all } t . \end{aligned} \qquad \text{(A–8)}$$

This problem can be transformed into a standard calculus of variations problem by substituting $h_{it} = \dot{S}_t + \gamma S_t$ into the instantaneous profit function:

$$\Pi(\dot{S}_t,S_t,t) = [1-\alpha+\beta S_t-(1+\eta)(\gamma S_t-\dot{S}_t)][\gamma S_t-\dot{S}_t] . \qquad \text{(A–9)}$$

Dropping the time subscripts and letting subscripts represent partial derivatives, the Euler equation is:

$$e^{-rt}\Pi_S = -re^{-rt}\Pi_{\dot{S}} + e^{-rt}\dot{S}\Pi_{S\dot{S}} + e^{-rt}\ddot{S}\,\Pi_{\dot{S}\dot{S}} . \qquad \text{(A–10)}$$

By substituting the values of the first and second derivative functions and simplifying, we obtain a linear second order differential equation:

$$2(1+\eta)\ddot{S} - 2r(1+\eta)\dot{S} + [2\gamma(1+\eta)(r-\gamma) + \beta(2\gamma+r)]S + (1-\alpha)(\gamma-r) = 0. \qquad \text{(A–11)}$$

The solution to this differential equation can be found by adding together a particular solution of the (non–homogeneous) equation with the general solution of the reduced (homogeneous) equation where the constant term $(1-\alpha)(\gamma-r)$ is set to zero. A particular solution, S_P, is:

$$S_P = \frac{(1-\alpha)(r-\gamma)}{\beta(2\gamma-r)+2\gamma(1+\eta)(r-\gamma)} . \qquad \text{(A–12)}$$

The general solution to the homogeneous equation yields a solution of the form:

$$S_t = A_1e^{d_1t} + A_2e^{d_2t} \qquad \text{(A–13)}$$

where d_1 and d_2 are the two roots of the equation:

$$d^2 - rd + \gamma(r-\gamma) + \frac{\beta(2\gamma-r)}{2(1+\eta)} = 0$$

$$d_1 = \frac{r}{2} - \frac{1}{2}\,[r^2 - \frac{2\beta(2\gamma-r)}{1+\eta} - 4\gamma(r-\gamma)]^{0.5} \qquad \text{(A–14a)}$$

$$d_2 = \frac{r}{2} + \frac{1}{2}\left[r^2 - \frac{2\beta(2\gamma-r)}{1+\eta} - 4\gamma(\gamma+r)\right]^{0.5} . \qquad \text{(A–14b)}$$

We assume $r^2 - \frac{2\beta(2\gamma-r)}{1+\eta} - 4\gamma(\gamma+r) > 0$, so that d_1 and d_2 are real and $0 < d_1 < r < d_2$. Combining the particular solution and the general solution to the homogeneous equation gives the full solution to equation (A–11): $S_t = A_1e^{d_1t} + A_2e^{d_2t} + S_P$. The constants A_1 and A_2 are defined by specifying the initial and terminal level of stock $(S_0, \bar{S})$:

$$A_1 = \frac{S_0e^{d_1T} - \bar{S} + S_P(1-e^{d_2T})}{e^{d_2T}-e^{d_1T}} \qquad \text{(A–15a)}$$

$$A_2 = \frac{-S_0e^{d_1T} + \bar{S} - S_P(1-e^{d_1T})}{e^{d_2T}-e^{d_1T}} . \qquad \text{(A–15b)}$$

2. Acquiescence

The conditions characterizing a profit maximizing harvest path for the incumbent when the incumbent plans to acquiesce are the same as defined above except for the terminal condition. In the deterrence case, the terminal condition is: $S_T = \bar{S}$. In the acquiescence case, the terminal stock condition is replaced by a transversality condition that the shadow value of a unit of stock to the incumbent is the same in the initial sole harvesting phase as it is in the post–entry duopoly game: $\Pi_S(\dot{S}_\tau, S_\tau, \tau) = aS_\tau + b$, where a and b are defined above in the second stage acquiescence solution.

Given the characterization of the profit maximizing path in the initial phase derived above, we can show that if entry will occur, it will occur at the first possible date, T, and not later. In order to satisfy the transversality condition, the stock size at the date of entry must be below the steady–state value for the sole harvester, S_P. Recall that along the profit maximizing path S_t is given by: $S_t = A_1e^{d_1t} + A_2e^{d_2t} + S_P$, where d_1 and d_2 are defined above and A_1 and A_2 are defined by:

$$A_1 = \frac{S_0e^{d_1\tau} - \hat{S}_\tau + S_P(1-e^{d_2\tau})}{e^{d_2\tau}-e^{d_1\tau}}$$

$$A_2 = \frac{-S_0e^{d_1\tau} + \hat{S}_\tau - S_P(1-e^{d_1\tau})}{e^{d_2\tau}-e^{d_1\tau}}$$

where τ is the date of entry and $\hat{S}_\tau$ satisfies the transversality condition. The time derivative of stock, $\dot{S}_t$, is:

$$\begin{aligned}\dot{S}_t &= A_1d_1e^{d_1t} + A_2d_2e^{d_2t} \\ &= e^{d_1t}(A_1d_1 + A_2d_2e^{(d_2-d_1)t}) .\end{aligned} \qquad \text{(A–16)}$$

Given that $S_0 > S_P > \hat{S}_\tau$, $A_2 < 0$. Also, $\dot{S}_t$ must be negative over some time interval. If $\dot{S}_t < 0$, then from equation (A–16) it follows that $\dot{S}_{t'} < 0$ for all $t' > t$. Since stocks are declining over time, the entrant will choose to enter as soon as possible or not at all.

Since the characterization of the paths is identical except for the terminal condition, and since the level of stock at T is higher in acquiescence than in deterrence (otherwise entry would never occur), we can conclude that the incumbent harvests less at each date $t \in (0,T)$ when acquiescing than when deterring.

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Figure 1: Social vs. Private Optima

The role of initial stock size

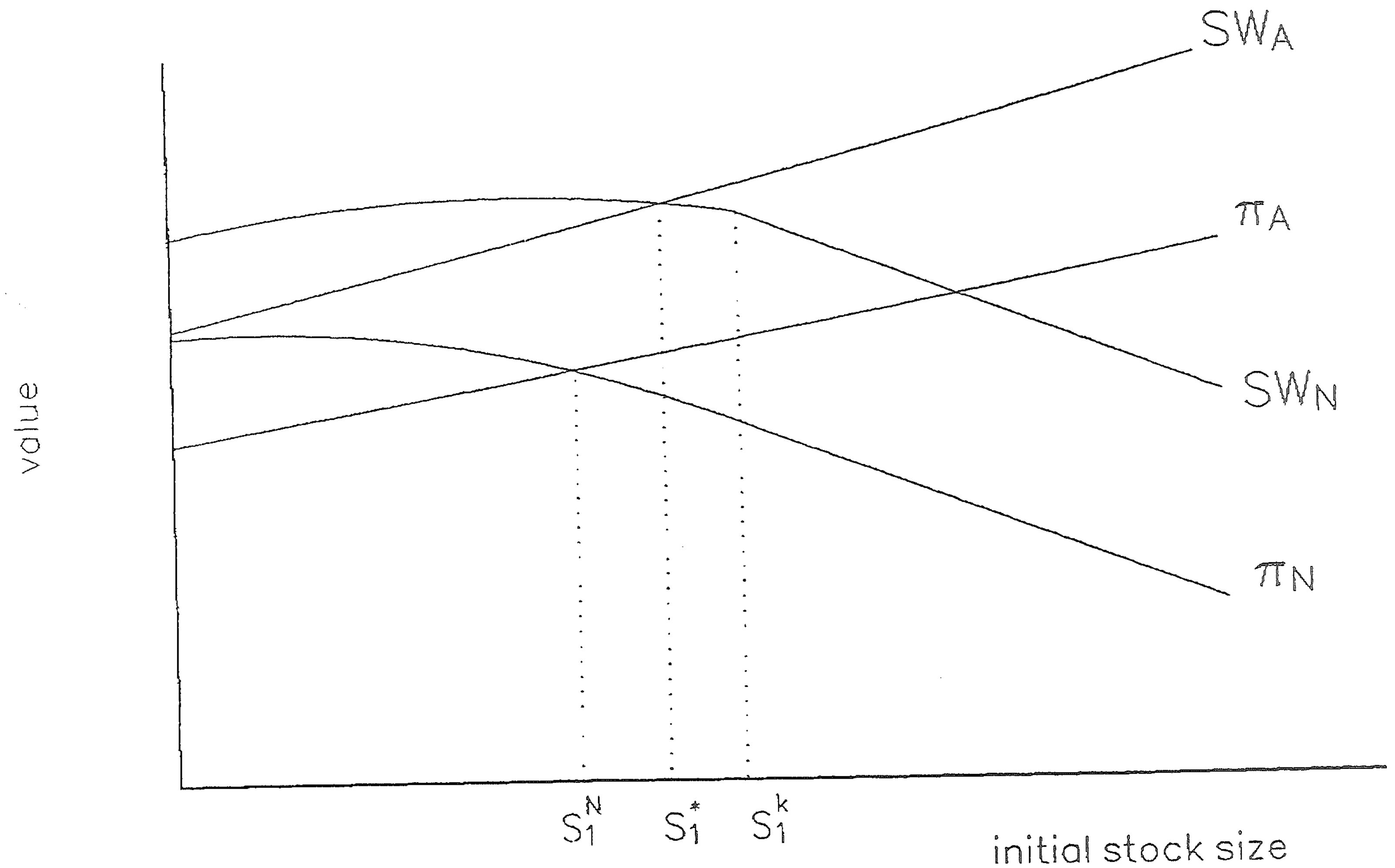

Figure 2: Privately optimal deterrence
can be socially inefficient

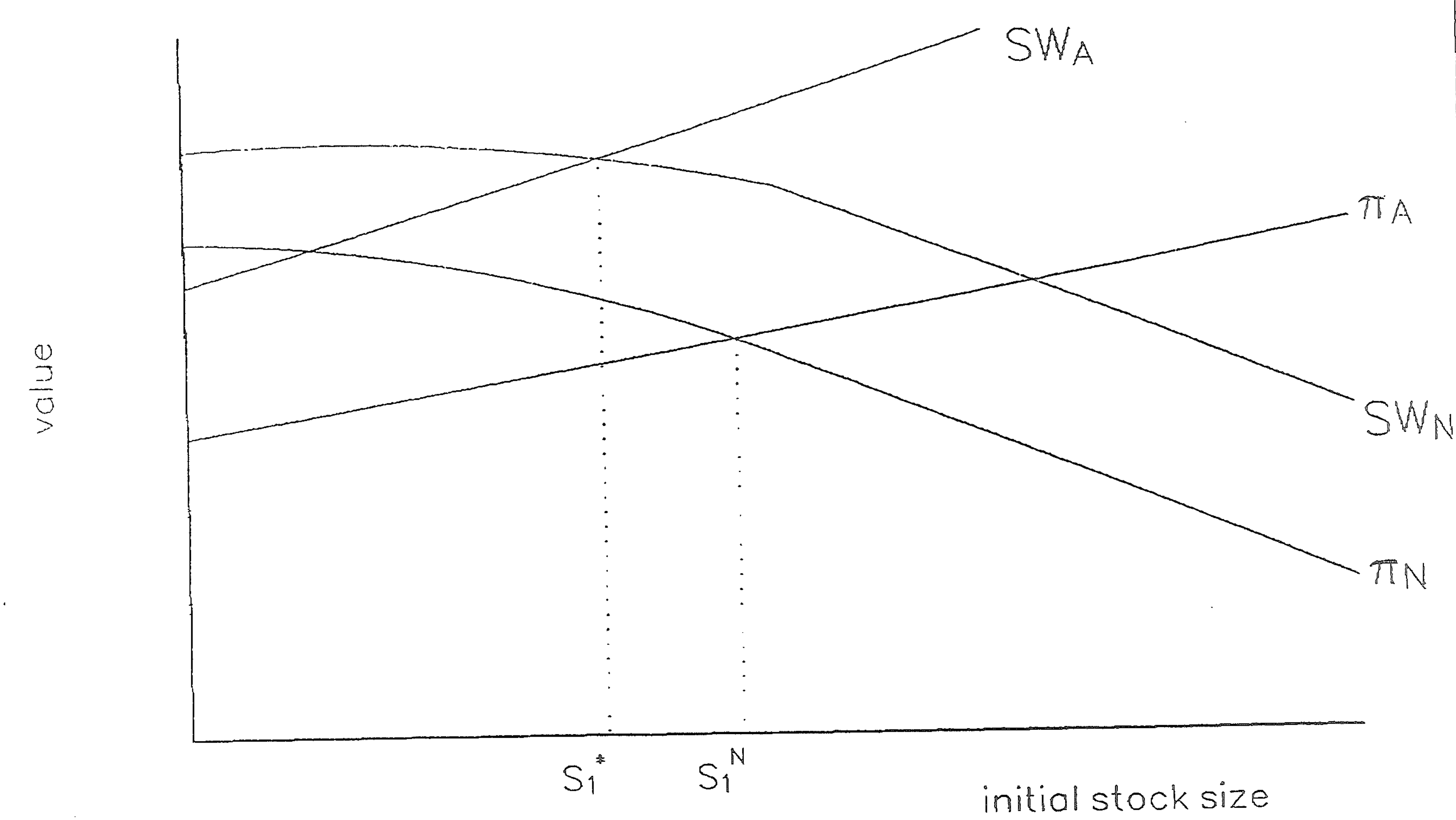

Figure 3: Excess harvest is privately suboptimal
and second-best socially optimal

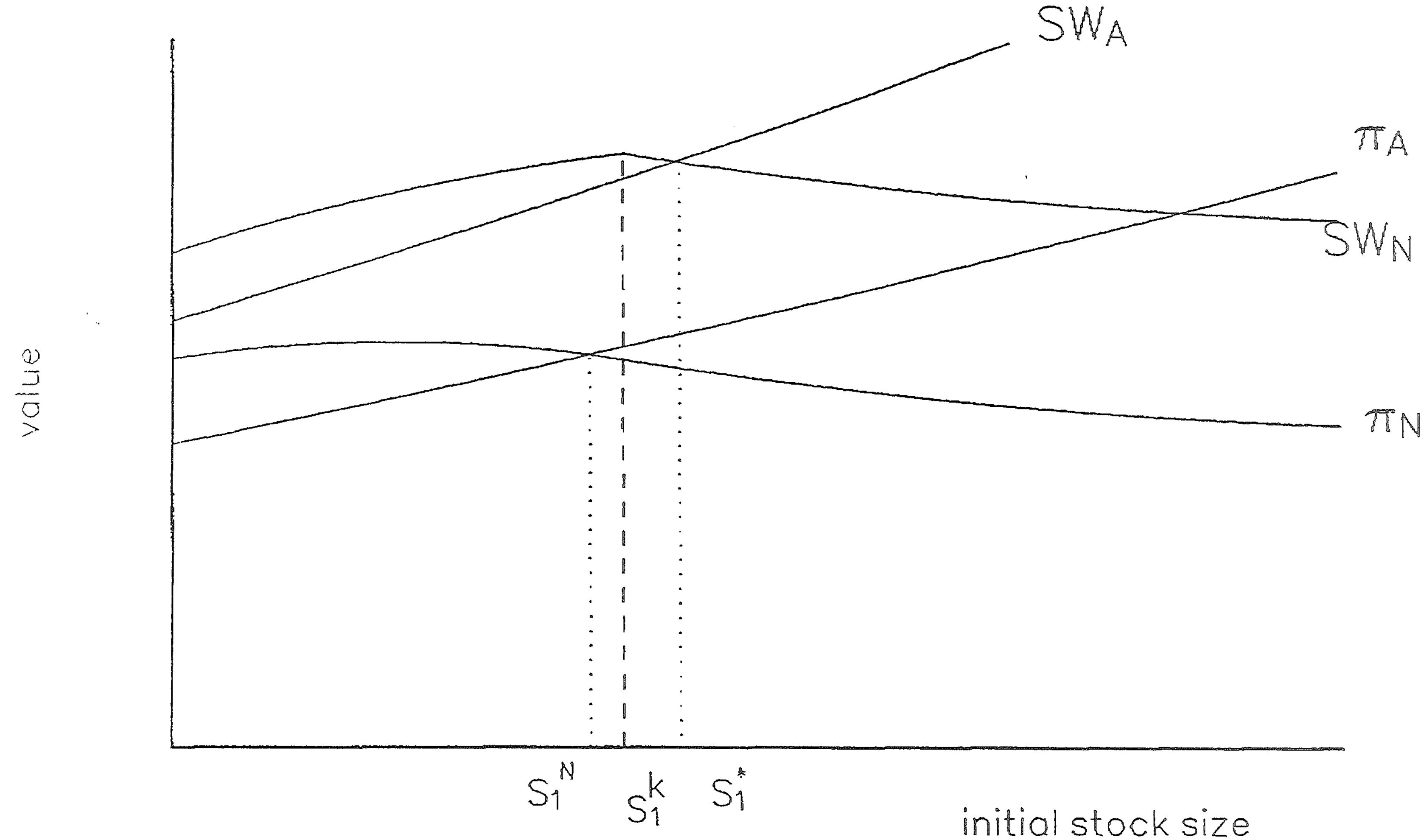

Figure 4: The Welfare Effects of Potential Entry

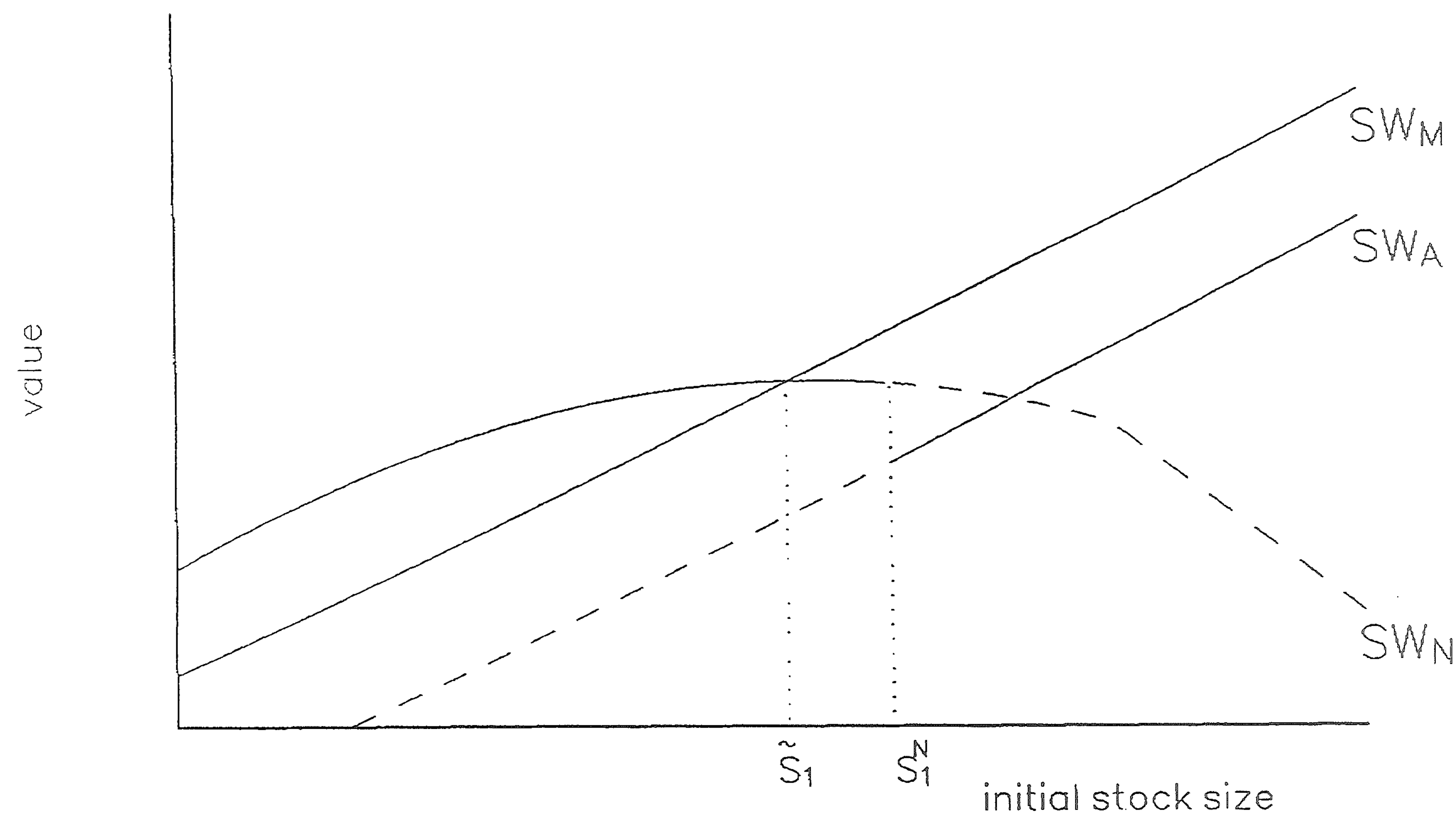

Figure 5: The Welfare Effects of an Increase in Entry Cost