Tariff Index Theory

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Abstract

Tariff indexes such as trade weighted means, variances and coefficients of variation are commonly used to compare the overall restrictiveness of trade policy over time and across countries despite their lack of a theoretical foundation. Anderson and Neary (1991) define a welfare–consistent tariff index, the trade restrictiveness index. This paper compares the various concepts and shows how the trade restrictiveness index may be expressed in terms of 'marginal trade weighted moments' of the tariff schedule. In the Cobb–Douglas case, the trade restrictiveness index collapses to a function of the trade weighted moments of the tariff schedule. For this case, mean–preserving reductions in the trade–weighted variance of tariffs are efficient.

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Tariff index numbers are widely used for policy evaluation, despite the deficiencies which stem from their lack of theoretical roots. The restrictiveness of tariff structures is routinely compared over space and time with import–weighted mean tariffs.[1] Another practically important tariff index is the import–weighted coefficient of variation of tariffs (the square root of the variance divided by the mean). The World Bank operations staff use the coefficient of variation along with the mean tariff to measure progress in trade liberalization as a condition for structural adjustment loans.

For a single tariff, the height of the tariff is an unambiguous measure of the restrictiveness of policy. With more than one tariff, theory has not provided an extension capturing the idea of the 'height' of the tariff, hence analysts have used the mean and coefficient of variation. For uniform tariffs, a higher mean tariff is trivially more restrictive, which is the intuitive sense behind the import weighted mean tariff index. But with a differentiated tariff structure, high tariffs will get low weight and low tariffs will get high weight, due to the substitution effect. This seems to be wrong. Concerning the variance, on the one hand, the reasoning behind the World Bank's promotion of lower dispersion is that uniform ad valorem tariffs do not distort relative prices among tariff–ridden goods. This appears to suggest that if two tariff schedules have equal import–weighted mean tariffs, hence in some sense equal distortion relative to other goods, the schedule having less variance is more efficient. On the other hand, the theoretical literature on piecemeal reform of tariffs (Fukushima (1979), Hatta (1977), and Bertrand–Vanek (1971)) has very stringent sufficient conditions for efficient reform which warn that this is a dangerous procedure. Because the conditions are over–sufficient, and not

[1] See for example any undergraduate text in international economics.

directly related to tariff moments, they provide little practical guidance. That is the subject of this paper.

The problem of finding a single number analogous to the 'height' of tariffs is the tariff index number problem. Anderson and Neary (1991) develop a solution, the trade restrictiveness index. It is defined to be the *uniform* tariff factor which is equivalent in trade restrictiveness (equivalent in the balance of trade) to the actual differentiated tariff structure. In rates of change it is equal to the familiar marginal dead–weight loss of changes in tariffs normalized by the 'shadow value of distorted trade'. The normalization is based on the distance function formalization of index number theory. In contrast, the standard assessment of the welfare effect of tariff changes uses other normalizations, such as national income. For the purpose of comparing changes in the restrictiveness of trade policy internationally, normalizing by national income has the fatal defect that 'natural' openness and economy size interact with the 'height' of the tariff.[2]

This paper develops the trade restrictiveness index in terms of mean and variance–covariance indices of the tariff schedule. There are two payoffs. First, the trade restrictiveness index can be decomposed into expressions which rescue the common sense idea that lower mean and lower variance of tariffs are both efficient. Second, a special case is offered in which the proper weights in the mean and variance of tariffs are the observed trade weights.

Section I reviews the theory of the trade restrictiveness index developed in more detail in Anderson and Neary (1991). Its interpretation and relation to the standard measure of efficiency are set out. Section II develops rate of change of the trade restrictiveness index in terms of changes in the mean and 'generalized variance–

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[2]Consider the uniform tariff case, where the height of the tariff is unambiguous. The welfare measure normalized by national income is the Harberger triangle divided by national income, which works out to $\tau^2 \varepsilon \frac{pZ}{I}$ for the move to free trade, where τ is the height of the ad valorem tariff (on the domestic price base), pZ/I is the ratio of the value of tariff–ridden imports to national income, and ε is the elasticity of composite import demand. Natural openness and economy size affect both pZ/I and ε, hence make this index useless for the purpose of comparing the height of the tariff across nations.

covariance' functions of the tariff schedule. Efficiency is indeed decreasing in both of these 'moments'. The coefficient of variation index used by the World Bank varies monotonically with the trade restrictiveness index when: (i) the only form of distortion is tariffs, (ii) the two indices are used to compare a given tariff schedule with the uniform mean–equivalent tariff schedule, (iii) the import–weights in the index are "marginal" import weights, and (iv) the variance concept used in the coefficient of variation is a 'generalized variance'.

Section III shows that when the trade preferences for imported final goods are CES, the ordinary variance concept may be used in place of the generalized variance, and marginal trade weights may be replaced with the ordinary trade weights. Thus mean preserving eliminations of trade weighted variance in tariffs are efficiency–increasing. The rate of change of the trade restrictiveness index in the Cobb–Douglas case is generally a function of the trade weighted first and second moments of the tariff schedule, although it is not the coefficient of variation function. This form of the trade restrictiveness index highlights an intuitive sense in which the trade weighted mean and variance of tariffs were both 'partially' legitimate. Finally, a Cobb–Douglas example is offered in which variance reductions are efficiency improving subject to the constraint that the trade–weighted mean tariff be the same. That is, comparisons can be made in which variance need not be eliminated. However, even in the Cobb–Douglas model, when the mean and variance both change there are always welfare–improving tariff reforms which coincide with a rise in the import weighted coefficient of variation.

Section IV concludes with some suggestions for future practice. Future work (underway at the World Bank as part of the project on trade policy evaluation) will reveal whether the refinement of the trade restrictiveness index yields different results in practice. However, in related work Anderson and Bannister (1991) show that the trade restrictiveness index for Mexican agriculture yields results very different from the standard producer subsidy and consumer subsidy equivalent indices.

I. Index Numbers for Tariffs

Changes in index numbers are generally weighted averages of changes in the components of the index. To be consistent with economic theory, the weights must arise from a fundamental economic structure. It is helpful to begin with reviewing the consumer price index, or CPI where the weights are familiar. Subsection I.1 derives the consumer price index based on the consumer's expenditure function and relates it to the average tariff. In Subsection I.2, the trade restrictiveness index, or TRI, is derived based on the economy's trade balance function. The latter is defined in Anderson and Neary. Here, the weights are less familiar, but the same logic girds the construction of the index. Subsection I.3 relates this welfare–consistent index to the trade–weighted mean and variance of the tariff structure.

I.1 The Consumer Price Index and the Average Tariff

The basis for the CPI is the consumer's expenditure function, e(q,u), where q is the vector of prices and u is the reference level of utility. e is the minimum level of income required to achieve u when the consumer faces q. The derivative of e with respect to q is equal to the vector of the consumer's demands, X, owing to the minimum value property of e(.).

The vector of consumer prices changes by dq. The consumer price index (CPI) in rates of change measures the (hypothetical) *uniform* rate of change in all prices which produces an equivalent change in the expenditure required to maintain welfare. The effect of an arbitrary set of price changes on the level of income required to support u is X'dq. The effect of a *uniform proportional* set of changes is $X'q^0 d\alpha$, where a is the (scalar) proportionality factor ($q = \alpha q^0$, where q^0 is the initial level of prices). Solving for the uniform proportional change in q which creates the same rise in required expenditure as the arbitrary change in q implies $d\alpha = X'dq/X'q$, or

$$(1.1)\quad d\alpha \quad = \quad \Sigma\left(\frac{X_i q_i}{X'q}\right)\hat{q}_i.$$

Initially α is equal to one, so $d\alpha$ is a percentage change. This familiar expression weights the proportionate change (denoted by a ^) in each q_i by the consumption share of good i.

One final step is required to link the development of the CPI firmly to the trade restrictiveness index developed below. Suppose that the means of compensation is through (hypothetical) price reductions rather than through (hypothetical) income transfers. The compensating change in expenditure is replaced by a (hypothetical) uniform proportionate change in prices which compensates for (i.e., offsets) the actual change in prices dq. This procedure defines a version of (1.1) which has opposite sign:

$$(1.1')\quad d\alpha' = -\sum\left(\frac{X_i q_i}{X'q}\right)\hat{q}_i.$$

The opposite sign denotes the direction of the compensatory change in hypothetical prices. This latter form is the one used in the trade restrictiveness index.

The import-weighted average tariff index is based on the false analogy of the small trading country with the price taking consumer model which is the basis of (1.1), with X being the import quantity. A set of tariffs raises expenditure of the consumer relative to free trade, and the index computes the uniform tariff which raises expenditure by an equivalent amount. Thus the proportionate change in q_i is set equal to the ad valorem tariff rate and the value of trade in each category is used as the weight:

$$(1.2)\quad \bar{\tau} = \sum\left(\frac{X_i q_i}{X'q}\right)\tau_i.$$

The mean tariff defined in (1.2) appears to have the index number problem that the current level of imports reflects the tariff–ridden level of prices. An appropriate adjustment might seem to involve using the structure of substitution effects to obtain the 'right' weights.

It is well-known, however, that the economic structure of a tariff-ridden small trading economy is not equivalent to that of the price-taking consumer. The external budget constraint links consumer expenditure requirements to the level of production income and the government revenue. A change in the tariff vector causes a change in the expenditure required to support u equal to X'dq. It also causes a change in government tariff revenue equal to X'dq + (q-p*)'dX, where p* is the external price vector. Assuming redistribution, the net effect is equal to (q-p*)'dX, the marginal dead weight loss. This suggest that the weights in a properly based tariff index must somehow be linked to marginal dead weight loss. The trade restrictiveness index formalizes this insight.

I.2 The Trade Restrictiveness Index

Depending on the analyst's objective, there are a number of possible ways to define an index of tariffs. Govenrment bureaus and international agencies which use tariff indices for comparison are interested in potential welfare, so welfare-equivalent indices are useful. Mercantilists are interested in the trade balance, so trade-balance equivalent indices are useful. The trade restrictiveness index is the uniform tariff which is equivalent in its (utility–constant) trade balance impact to the differentiated tariff structure. Thus the trade restrictiveness index is both equivalent in welfare and in the trade balance.

The first step of this section is to develop a counterpart to the expenditure function in the general equilibrium context of a small tariff-ridden trading economy. Based on it, the trade restrictiveness index may be defined.

The Balance of Payments Function

The *balance of payments function* gives the net foreign exchange required to maintain the utility of a representative consumer facing given levels of tariffs. The balance of payments function is built up from the consumer's expenditure function and the gross domestic product function.

The gross domestic product function g(p,v) is the maximum national value added (output less imported inputs) when the economy faces a vector p of prices of final output or imported inputs and a vector v of primary factor supplies. The vectors p and v are assumed to be fixed, the former by the small country assumption combined with a tax policy; the latter by institutional constraints on employment. The derivative of g with respect to p, g_p (in what follows, a subscript denotes differentiation save for the index i or j), is equal to the output supply or input demand vector, due to the maximum value property.

The trade balance is equal under the balance of payments constraint to the net external borrowing of the economy. This means that the value of trade in terms of external prices q* and p* is equal to the value of expenditure evaluated at prices q, less national product evaluated at prices p, less $(q-q^*)'e_q$ plus $(p-p^*)'g_p$:

$$q^{*\prime}e_q - p^{*\prime}g_p = q'e_q - p'g_p - (q-q^*)'e_q + (p-p^*)'g_p.$$

Government revenue is equal to the final consumption tax revenue $(q-q^*)'e_q$ plus the net revenue from taxation of final output or imported inputs, $-(p-p^*)'g_p$. In balance of payments equilibrium, the trade balance is equal to the amount of external borrowing, β. The balance of payments function is equal to

$$(1.3)\quad B(q,p,u;q^*,p^*,v,\beta) = e(q,u) - g(p,v) - \beta - (q-q^*)'e_q(q,u) + (p-p^*)'g_p(p,v).$$

B(.) gives the net foreign exchange required to support the initial level of utility u when the vector of final consumption taxes is equal to q-q*, the vector of output or input taxes is equal to -(p-p*), external prices are equal to (q*,p*), and the net external borrowing is equal to β. The standard case of international trade theory is when imported inputs are not taxed and when domestic output is a perfect substitute for imports. Then consumers and producers face the same price vector p, and an import tariff (p-p*) is equivalent to a tax on consumption plus a subsidy on production at the same rate. The balance of payments function may be reduced to:

$$(1.3')\quad B(p,u;p^*,v,\beta) = e(p,u) - g(p,v) - \beta - (p-p^*)'\big(e_p(p,u) - g_p(p,v)\big).$$

The last term on the right hand side of (1.3) is equal to the tariff revenue, since Z, the import vector, is equal to $e_p - g_p$ (negative for exports).

The marginal dead weight loss from a change in the tariff vector is equal to minus the change in the foreign exchange required to support u, or $-B_p'dp$. Here, the marginal cost of tariffs $-B_p$, may be obtained from differentiating (1.3') as:

$$(1.4)\quad -B_p' = (p-p^*)'\big(e_{pp} - g_{pp}\big) = (p-p^*)'Z_p.$$

The marginal dead weight loss from a change in the tariff vector is equal to minus the change in the foreign exchange required to support u, or $-B_p'dp$. Here, the marginal cost of tariffs $-B_p$, may be obtained from differentiating (1.3') as:

(1.4) $$-B_p' = (p-p^*)'(e_{pp} - g_{pp}) = (p-p^*)'Z_p.$$

In obtaining (1.4) the envelope theorem is used along with cancellation of like terms. Combining (1.4) with the vector of tariff changes, the marginal dead weight loss from the tariff change is equal to

$$-B_p'dp = (p-p^*)'(e_{pp} - g_{pp})'dp$$

$$= (p-p^*)'Z_p dp = (p-p^*)'dZ,$$

where dZ is the change in the import demand vector. See Anderson and Neary (1991) for more on the properties of B(.).

Derivation of the Trade Restrictiveness Index

The utility-constant change in the foreign exchange requirement (the trade deficit measured in terms of foreign prices) due to a change in p is equal to $B_p'dp$. All vectors dp which result in equal changes in $B_p'dp$ are termed equivalent in trade restrictiveness. $-B_p'dp$ is also a compensating variation measure of welfare change. The trade restrictiveness index is based on first noting that the *uniform proportional* change in p which offsets the change in B is $-B_p'p^0 d\Delta$. Solving for the uniform proportional change in p which is equivalent in trade restrictiveness to the arbitrary change dp,

(1.5) $$d\Delta = -\Sigma\left(\frac{B_{pj}p_j}{B_p'p}\right)\hat{p}_j.$$

Note the similarity of structure between (1.1') for the consumer price index and (1.5) for the tariff index. Index number construction has two steps. The first is to uncover the correct marginal real income effect (X_i in the case of the consumer problem, B_{pi} in the case of the trading economy). The second step is to find the appropriate normalization of a set of marginal changes ($q'X$ in the consumer problem and $B_p'p$ in the trading economy). The normalization $B_p'p$ in (1.5) is called 'the shadow value of distorted trade'

A more general definition is used in index theory to define an index even when the values of p^1 are far from the initial point p^0. Thus:[2]

(1.6) $\Delta(p^1,u^0,\beta^0) \quad = \quad \{\Delta \mid B(p^1\Delta,u^0) = \beta^0\}.$

Here, β^0 is the initial foreign exchange requirement (trade deficit), and the constraint requires Δ to change as p changes so as to maintain a constant trade deficit at the given u^0. The most intuitive interpretation arises when the new value of p is equal to p*. Then Δ is equal to one plus the uniform ad valorem tariff rate which is equivalent in restrictiveness to the initial tariff structure. Elsewhere, Δ is equal to the uniform tariff factor *surcharge* which compensates for (offsets the change in trade restrictiveness implied by) the move to a new tariff structure. Note that under definition (1.6), the compensating change in foreign exchange induced by Δ is just sufficient to maintain u^0 when prices shift to p^1. Thus Δ is a compensating variation measure of the welfare effect of the change, and Δ is interpreted as the uniform tariff surcharge which is equivalent in welfare to the change to the new prices.

The rate of change of Δ is obtained from implicit differentiation of (1.6):

(1.5') $$\hat{\Delta} = \frac{d\Delta}{\Delta} = -\sum\left(\frac{B_{pj}p_j}{B_p'p}\right)\hat{p}_j.$$

The structure of the TRI combines the standard welfare economics of the marginal dead weight loss due to a tax change, which is the numerator of (1.5'), with a normalization (division) by $B_p'p$, the 'shadow value of distorted trade'. In contrast to other normalizations which have little or no theoretical foundation, scaling by the shadow value of distorted trade leads to a well–founded and intuitive index.

Equation (1.5') is made operational using the partial derivatives B_{pj} in (1.4) and information on substitution effects. The elements of B_p' are expected to be positive, but cross effects can make some elements negative. This index can be integrated to obtain levels of Δ, using an initial level of Δ equal to one to tie down the constant of integration. Alternatively, a computable general equilibrium model can solve Δ in (1.6).

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[2]The analog for the CPI is: $\alpha(q,u,\beta) = \{\alpha \mid e(q\alpha,u) = \beta\}$.

I.3 A Diagrammatic Exposition

The balance of payments function and the trade restrictiveness index are both defined above in terms of the domestic price vector p. The instruments of protection are tariffs, however, and Δ is interpreted as a uniform tariff factor surcharge. To build intuition it is helpful to rewrite the balance of payments and trade restrictiveness index functions in terms of the tariff factor vector T such that $p_i = p^*_i T_i$. Based on this,

$$(1.7) \quad B_T = B_p \mathrm{diag}(p^*) = (T-1)'\mathrm{diag}(p^*)Z_p \mathrm{diag}(p^*),$$

where diag(p*) is a diagonal matrix with the elements of the vector p* on the principal diagonal, and 1 is the vector of ones. Then:

$$B_T'dT = B_p'dp.$$

Figure 1 plots balance of payments contours in tariff factor space (T_1,T_2) for a level of utility u^0. The level of the trade balance is constant along the contours, and if u^0 is an equilibrium utility associated with a tariff setting on the contour, the balance of trade value B must be zero. For a given u, B, the foreign exchange requirement of consumption, increases as tariffs rise. The curves are drawn as convex, and will ordinarily but not necessarily be so.

Suppose that the initial equilibrium setting of tariffs is at A with equilibrium utility u^0. The purpose of any compensating variation[3] index number of tariffs is to consistently map some alternative setting of the tariffs such as point B into a tariff setting which supports the base level of utility u^0. Consistency means that the index should be monotonically related to efficiency for all tariff comparisons.

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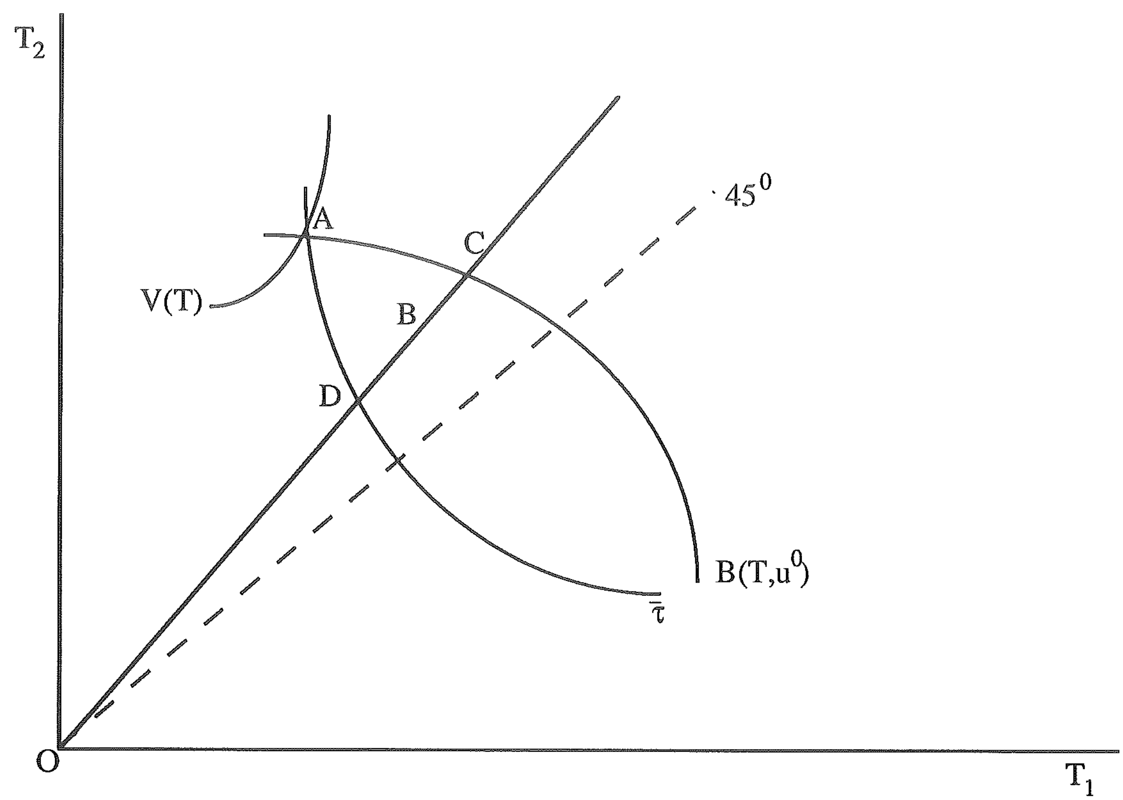
[3]It is also possible to define equivalent variation indices. The present development focusses on compensating variation measures since they are more practical. The equivalent variation concept is illustrated with a plot of isoutility contours in tariff factor space. Set B equal to zero and solve (1.3') for the equilibrium level of utility. The isoutility contours will look like trade balance contours of Figure 1, with lower tariff settings required to achieve higher levels of utility.

Figure 1. The Trade Restrictiveness Index



The curve labelled $\bar{\tau}$ illustrates the locus of tariff factors along which the imported-weighted average tariff remains constant. Its shape depends on the substitution properties of the economy, but it is likely to be downward sloping. The trade restrictiveness index associated with the move from A to B is equal to OC/OB. The interpretation is that the tariff reform to point B reduces the foreign exchange required to support u^0. The compensating uniform rise in tariff factors is equal to OC/OB. In contrast, the mean tariff index $\bar{\tau}$ registers a rise in protection in the move from A to B, and is thus an inconsistent index. In either its level form or its rate of change form, Δ is a tariff index which consistently ranks values of T relative to u. If the transitivity qualification of compensating variation measures is met[4], Δ ranks values of T relative to each other.

Now consider the application of these concepts to the evaluation of a proposed tariff reform which preserves the trade–weighted mean tariff, such as point D at the intersection of the constant–mean tariff locus and the ray OC. Is it possible that the coefficient of variation of tariff rates will properly register an increase in efficiency? The coefficient of variation of tariff factors is equal to the square root of the variance divided by the mean. Isovariance contours are positively sloped and increasing in variance with distance from the 45^0 line.[5] As drawn, the move from A to D is a move toward the 45^0 line and is associated with a reduction in dispersion, as mandated by the World Bank. However, point D could lie northeast rather than southwest of point C. Thus it appears that the dispersion index cannot safely be used even when the mean does not shift. The remainder of this paper is an attempt to be more specific.

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[4]It is well–known that compensating variation measures need not yield transitive comparisons. A sufficient condition for transitivity is homothetic preferences.

[5]For given weights w_i, the partial derivative of the variance with respect to tariff rate i is equal to $2w_i(\tau_i - \bar{\tau})$. The slope of the isovariance contour is equal to $-w_2(\tau_2 - \bar{\tau})/w_1(\tau_1 - \bar{\tau})$. One of the partial derivatives must be positive and the other negative; hence the slope is positive.

II. The TRI and the Standard Indices

The relation of the the trade restrictiveness index to the mean and coefficient of variation of tariffs indices has two separate elements. The first is the issue of the proper weights, and the second is the issue of the proper formula given the proper weights. In this section, the proper marginal trade weights are defined, based on the concept of the marginal cost of tariffs. The trade restrictiveness index is then expressed as a function of the mean and 'generalized variance' of the tariff schedule using these weights. Next, the trade restrictiveness index is used to restate the two main theorems on piecemeal tariff reform in terms of tariff 'moments'. Finally, assuming the use of the proper weights, and also the generalized variance concept in the coefficient of variation function, changes in the mean or coefficient of variation are related to change in the trade restrictiveness index.

First, define the usual tariff indices, the mean, the variance and the coefficient of variation of tariffs for some set of weights $\{w_i\}$[6]. The mean ad valorem tariff is:

$$(2.1) \quad M(T) = \sum_i w_i(T_i - 1) = 1'\mathrm{diag}(w)(T-1).$$

The variance of the tariff schedule is equal to

$$(2.2) \quad V(T) = \sum_i w_i\big(T_i - M(T)\big)^2 = (T - M(T)1)'\mathrm{diag}(w)(T - M(T)1).$$

The coefficient of variation of the tariff schedule is

$$(2.3) \quad W(T) = \frac{V(T)^{1/2}}{M(T)}.$$

Here, diag(w) is the diagonal matrix with w_i as the ith diagonal element, and 1 is the vector of ones. The mean tariff and the coefficient of variation of tariffs are usually measured with weights w_i equal to the observed relative trade weights, $w_i = p_i Z_i / \Sigma p_j Z_j$.

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[6]Normally, $w_i \geq 0$ and $\Sigma w_i = 1$ are imposed. The first restriction is relaxed here.

Two tariff vectors are to be compared with the various index functions, the initial tariff factor vector being T^0 and the new tariff factor vector being T^1. The difference in tariff factors equals the difference in ad valorem tariffs.

The TRI in Terms of Tariff Moments

The relative change form of the trade restrictiveness index is given by (1.5'). It is valid only for local changes. For discrete changes, using the intermediate value theorem and evaluating at $\tilde{T}$, an intermediate value of T, (1.5') becomes:

$$(2.4) \quad \hat{\Delta} = -\sum \frac{B_{Ti}(\tilde{T})}{\sum B_T(\tilde{T})\tilde{T}} \left(T_i^1 - T_i^0\right).$$

Here,

$$(2.5) \quad \tilde{T} = \lambda T^0 + (1-\lambda)T^1 \text{ for } 1 \geq \lambda \geq 0.$$

The Divisia index approximation to (2.4), often used as a practical expedient, replaces the weights evaluated at $\tilde{T}$ in (2.4) with the arithmetic average of the weights evaluated at T^1 and at T^0: the Divisia index.[7]

Using (1.4) and (1.7) in (2.4), the numerator of (2.4) is equal to

$$(2.6) \quad \sum B_{Ti}(\tilde{T})\left(T_i^1 - T_i^0\right) = \sum_i \sum_j -(\tilde{T}_j - 1)\text{diag}(p^*)Z_{jpi}(\tilde{T})\text{diag}(p^*)\left(T_i^1 - T_i^0\right).$$

The denominator of (2.4) is equal to:

$$(2.7) \quad \sum B_{Ti}(\tilde{T})\tilde{T} = \sum_i \sum_j -(\tilde{T}_j - 1)\text{diag}(p^*)Z_{jpi}(\tilde{T})\text{diag}(p^*)\tilde{T}$$

Next, divide both the numerator and the denominator of $\hat{\Delta}$ by the double sum of the elements $p^*_j Z_{jpi} p^*_i$. The resulting weights will be termed *marginal trade weights.* Formally, the marginal trade weights (evaluated at the intermediate value $\tilde{T}$) are defined as:

$$(2.8) \quad z_{ij} = \frac{p^*_j Z_{jpi} p^*_i}{\sum_i \sum_j p^*_j Z_{jpi} p^*_i}.$$

The marginal trade weights are symmetric and add to one. Then $\hat{\Delta}$ becomes:

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[7] The justification is that with a convex function Δ, the weights must be intermediate between the endpoint weights. Convex Δ is assumed here by imposing convexity on b(T,u) in T.

$$(2.4') \quad \hat{\Delta} = -\frac{-(\tilde{T}-1)'\{z_{ij}\}(T^1 - T^0)}{-(\tilde{T}-1)'\{z_{ij}\}\tilde{T}}.$$

The quadratic expressions in the numerator and denominator of (2.4') resemble the generalized variance expressions which arise in generalized least squares.[8] The mean $M(T)$ is equal to $1'\{z_{ij}\}T = z_{.j}'T$, which is the standard form for a mean with the 'probabilities' w_i being the column sums of $\{z_{ij}\}$. (Note, however, that while $1'z_{.j}$ is equal to 1, not all elements of $z_{.j}$ are necessarily positive.) The generalized variance $V^*(T)$ is equal to $(T-M(T)1)'\{z_{ij}\}(T-M(T)1)$. In the trade restrictiveness index structure, the matrix $\{z_{ij}\}$ is positive definite and plays the role of the positive definite matrix Ω^{-1} in generalized least squares. The quadratic expression in the denominator of (2.4'), using the algebra of covariance, is equal to $-\left(V^*(\tilde{T}) + M(\tilde{T})^2 - M(\tilde{T})\right)$.

Similarly, the numerator expression is equal to

$$-\left(Cov^*(\tilde{T},T^1 - T^0) + M(\tilde{T})\left(M(T^1) - M(T^0)\right)\right) + \left(M(T^1) - M(T^0)\right)$$

$$= \lambda V^*(T^0) - (1-\lambda)V^*(T^1) + (2\lambda - 1)Cov^*(T^1,T^0) + \left(1 - M(\tilde{T})\right)\left(M(T^1) - M(T^0)\right).$$

Here, Cov* stands for the generalized covariance.[9] The second line follows from the algebra of covariance, using equation (2.5) for $\tilde{T}$.

Now $\hat{\Delta}$ may be written in terms of the 'moments' of the 'distribution' of tariffs by collecting terms and substituting back into (2.4'). Thus:

<u>Proposition 1</u>

$$(2.9) \quad \hat{\Delta} = \frac{\lambda V^*(T^0) - (1-\lambda)V^*(T^1)}{V^*(\tilde{T}) + \left(1 - M(\tilde{T})\right)M(\tilde{T})} + \frac{(2\lambda-1)Cov(T^1,T^0)}{V^*(\tilde{T}) + \left(1 - M(\tilde{T})\right)M(\tilde{T})}$$

$$+ \frac{\left(1 - M(\tilde{T})\right)}{V^*(\tilde{T}) + \left(1 - M(\tilde{T})\right)M(\tilde{T})}\left(M(T^1) - M(T^0)\right).$$

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[8]The standard linear regression model is $Y = X\beta + e$. If the variance–covariance matrix of e is equal to Ω, where Ω is positive definite, then the generalized least squares estimator of β, β^*, is equal to $(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y$. The matrix $(X'\Omega^{-1}X)$ is a generalized variance–covariance matrix of X, $(X'\Omega^{-1}X)^{-1}$ is the variance–covariance matrix of the estimator β^*, and so forth .

[9]$Cov^*(T^0,T^1 \quad = \quad (T^0-M(T^0)'\{z_{ij}\}(T^1-M(T^1)).$

The TRI and Piecemeal Reform Theorems

Several intuitive propositions on tariff reform can be stated using Proposition 1. Note from the first term on the right hand side of (2.9) that (i) mean–tariff–preserving generalized variance reduction is efficiency improving provided the influence of the covariance term is small. The influence of the covariance term goes to zero (and λ is equal to 1/2) as tariff changes are small. Intuitively, mean–preserving variance reduction implies cutting high tariffs and raising low tariffs. In the case of generalized variance, however, this implies cutting highly weighted tariffs and raising lowly weighted tariffs. Formally, mean–preserving tariff reform is efficient if

$$\sum_i \left(T_i - M(T)\right) z_{ij} dT_j < 0, \quad \text{for each } j, \text{ and}$$

$$\sum_{i\&j} z_{ij} dT_i = 0.$$

Next, (ii) mean tariff reduction with constant variance is efficiency improving. Thus the rule $T_i^1 = T_i^0 - a$ for all tariffs i is efficient. That is, a constant ad valorem rate cut is efficient. For tariff reforms of this type, the mean tariff is a valid index.

The three main piecemeal tariff reform theorems in the literature may be stated in terms of (2.9):

Uniform radial cut $dT = -\alpha T^0$ is efficient because it lowers both the mean and the generalized variance[10],

Concertina cut If all goods are net substitutes, cutting the highest tariff will reduce both the generalized variance and the mean, and[11]

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[10] This theorem is most thoroughly developed in Hatta (1977a,b).

[11]Under the condition $\{z_{ij}\}$ has negative off–diagonal and positive diagonal elements. Moreover, due to the homogeneity property of excess demand functions combined with substitutability of taxed with untaxed goods, $T'\{z_{ij}\}>0$. This means that $T_j z_{jj} > -\sum_{i \neq j} T_i z_{ij}$. If tariff j is cut,

$$(T-M(T))'\{z_{.j}\}dT_j = \left(\sum_i \frac{T_i - M(T)}{T_i} T_i z_{ij}\right) dT_j.$$

The term in brackets is positive if T_j is the maximal element of the vector T, noting that (T–M)/T is increasing in T. QED. This proposition is due to Bertrand and Vanek (1971).

Dispersion cut $dT = -\alpha\left(T^0 - M(T^0)1\right)$ is efficient because it lowers variance.[12]

Thus both the marginal trade weighted mean and variance of the tariff structure have the influence on efficiency anticipated from simple intuition based on the trade weighted mean and variance. For large changes in tariffs, the influence of the covariance in (2.9) is always zero if λ is equal to 1/2. While this is a 'natural' value, the intermediate value theorem does not guarantee it for large changes unless import demand is linear.

Notice that the normalization term in the denominator of (2.9) is not simply the square of the mean tariff, as in the (square of) the coefficient of variation. The normalization of the trade balance effect of tariff changes in (2.9) is in fact the main contribution of Anderson and Neary. Using it to form the rate of change of the TRI yields the proportional change in the uniform tariff factor surcharge which compensates for the tariff changes.

Finally, what can be said of the coefficient of variation index relative to (2.9)? If the mean is preserved and in addition the schedule T^1 is uniform, when the coefficient of variation is formed with the same marginal trade weights, the difference in the coefficient of variation is equal to

$$V^*(T^0)^{1/2}/(M(T^0)-1).$$

Then:

Corollary

Let all indices be based on marginal trade weights. Then for the mean–preserving changes in the schedule of tariffs which eliminates variance, both $\hat{\Delta}$ and the change in the coefficient of variation index correctly sign efficiency changes.

The germ of reasonable intuition behind analysts' use of the coefficient of variation is revealed in Proposition 1 and its corollary. The marginal dead weight loss of tariff changes is equal to $(p-p^*)'Z_p dp$, which is essentially quadratic in p. Thus

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[12] This rather under–appreciated proposition is is due to Fukushima (1979).

something like a variance must be involved in correctly signing the efficiency change due to a tariff shift. But even with marginal trade weights, the (change in the) generalized variance alone is not a consistent measure. The trade restrictiveness index, which is consistent, is not monotonically related to the change in the coefficient of variation of tariffs, even when the latter is uses generalized variance based on marginal trade weights. The change in the coefficient of variation is equal to

$$(2.10)\quad W(T^1) - W(T^0) = \frac{V^*(T^1)^{1/2}}{M(T^1)} - \frac{V^*(T^0)^{1/2}}{M(T^0)} .$$

The first problem is that the numerator of (2.9), which signs $\hat{\Delta}$, may differ in sign from $V^*(T^1)^{1/2} - V^*(T^0)^{1/2}$, which signs the change in W under a mean–preserving reduction in variation. Covariation of T^0 and T^1 matters to $\hat{\Delta}$, as does the magnitude of λ. However, in Section III a Cobb–Douglas example is presented in which covariation and the size of λ do not interfere with the validity of inference from variance reduction subject to a constant mean. The second problem is that when the mean shifts, $\hat{\Delta}$ is shifted through both the numerator and denominator. In contrast, it acts through the denominator alone in W. No definite relation can be established between $\hat{\Delta}$ and the change in W given by (2.10).

The problem with the mean shift is worth emphasizing. The coefficient of variation normalizes the standard deviation of a random variable by the mean, which is a useful procedure in descriptive statistics. In the context of an index of trade policy, however, the proper normalization of the efficiency change, the numerator of (2.4), is the 'shadow value of distorted trade' (see Anderson and Neary (1991) for more discussion) which appears in the denominator of (2.4).

III.The Cobb-Douglas Example

Under Cobb–Douglas preferences for imported final goods, the trade restrictiveness index can be written as a function of the trade–weighted moments of the tariff schedule. Mean–preserving eliminations of variance are then efficiency improving. The sufficient condition can be relaxed to the CES case, but it does not appear possible to go beyond this: for example, strong separability is not sufficient. Subsection III.1 gives the details. Subsection III.2 further develops the Cobb–Douglas structure to show that all mean preserving reductions in tariff variance are efficiency improving.

Import preferences which are Cobb–Douglas or CES are very restrictive, of course, but perhaps not quite as restrictive as they appear at first. If the Armington assumption is made, then no import is a perfect substitute for a domestically supplied good. The reasoning is that packaging, advertising, and safety design segment all national markets. Moreover, undistorted nontraded goods consumer prices are invariant to trade taxes if a constant returns technology has more traded goods than nontraded goods and factors. Then imports are based on the representative consumer's expenditure function.

In contrast, it is not possible to extend the Cobb–Douglas model to tax–ridden exports and imported inputs. A Cobb–Douglas gross domestic product function arises only under joint production of a very special kind. Moreover, Lopez and Panagariya (1990) show that for tariffs on imported inputs, complementarity necessarily arises somewhere in the substitution effects structure. Thus the special case is offered mainly to illuminate how restrictive the circumstances are which yield the validity of the World Bank dispersion index. A second use of this section is that in offering a fully worked out example of the trade restrictiveness index, it provides more intuition as to the general structure.

III.1 Marginal Trade Weights and Trade Weights

The relevant portion of the trade expenditure function is assumed to be the consumer's expenditure function e(q,u). With Cobb–Douglas preferences, this implies a demand system:

$$(3.1)\quad e_{q_i}(q,u) \quad = \quad \alpha_i \frac{\prod_j q_j^{\alpha_j} u}{q_i} \quad = \quad e\alpha_i/q_i \quad = \quad Z_i.$$

Each internal price q_i is equal to the external price q^*_i times the tariff factor T_i, possibly different from unity.

The marginal trade weights are based on relative substitution effects. The substitution effects system is:

$$(3.2)\quad q^*_j e_{q_i q_j} q^*_i \quad = \quad e(-\delta_{ij} + \alpha_j)\alpha_i/T_iT_j,$$

where δ_{ij} is the Kronecker delta. The z_{ij} weights are formed by the division of the elements of (3.2) by $\Sigma\Sigma q^*_j e_{q_i q_j} q^*_i$.

The trade restrictiveness index in rate of change form is

$$(3.3)\quad \hat{\Delta} \quad = \quad -\frac{B_T'(T^1 - T^0)}{B_T'T} \quad = \quad -\frac{-(T-1)'\{z_{ij}\}(T^1 - T^0)}{-(T-1)'\{z_{ij}\}T}$$

For discrete changes, $\hat{\Delta}$ is formed by substituting $\tilde{T}$ for T. The simplification of this expression begins by removing the normalization of the z_{ij} weights, multiplying numerator and denominator by $\Sigma\Sigma q^*_j e_{q_i q_j} q^*_i$. Now substitute the elements of (3.2) evaluated at $\tilde{T}$ for z_{ij} on the right hand side of (3.3). The structure of (3.2) substituted into (3.3) suggests defining tariffs on the domestic price base:

$$\tilde{\tau}_i = (\tilde{T}_i - 1)/\tilde{T}_i$$
$$\tilde{\tau}_i^1 = (T_i^1 - 1)/\tilde{T}_i$$
$$\tilde{\tau}_i^0 = (T_i^0 - 1)/\tilde{T}_i.$$

The numerator is then equal to

$$(3.4)\quad \Sigma B_{Ti}(\tilde{T})\left(T_i^1 - T_i^0\right) = -e\sum_i\sum_j \tilde{\tau}_j(-\delta_{ij} + \alpha_j)\alpha_i\left(\tilde{\tau}_i^1 - \tilde{\tau}_i^0\right)$$

$$= e\sum_i \alpha_i\tilde{\tau}_i(\tilde{\tau}_i^1 - \tilde{\tau}_i^0) - e\left(\Sigma\alpha_i\tilde{\tau}_i\right)\sum_i \alpha_i(\tilde{\tau}_i^1 - \tilde{\tau}_i^0).$$

The denominator of (3.3) is evaluated at $\tilde{T}$ using (3.2). Note that post–multiplying by $\tilde{T}$ cancels one of the denominator terms in (3.2). Thus:

$$(3.5)\quad B_T'\tilde{T} = e\sum_i \alpha_i\tilde{\tau}_i - e(\sum_i \alpha_i)\sum_i \alpha_i\tilde{\tau}_i = \left(1 - \Sigma\alpha_i\right)\sum_i \alpha_i\tilde{\tau}_i.$$

Finally, form a simplified expression for $\hat{\Delta}$ by substituting (3.5) and (3.4) into (3.3), and dividing numerator and denominator by $\Sigma\alpha_i$ to form weights $w_i = \alpha_i/\Sigma\alpha_i$. The weights w_i are the domestic value of (tariff–ridden) trade share weights, $q_iZ_i/\Sigma q_iZ_i$ (based on (3.1)). The numerator term (3.4) becomes

$$Cov(\tilde{\tau},\tilde{\tau}^1-\tilde{\tau}^0) + M(\tilde{\tau})\left(1-\Sigma\alpha_i\right)\left(M(\tilde{\tau}^1) - M(\tilde{\tau}^0)\right)$$

Now the ratio in (3.3) can be simplified to yield:

<u>Proposition 2</u>

$$(3.6)\quad \hat{\Delta} = \frac{\lambda V(\tilde{\tau}^0) - (1-\lambda)V(\tilde{\tau}^1) + (2\lambda-1)Cov(\tilde{\tau}^0,\tilde{\tau}^1)}{\left(1-\Sigma\alpha_i\right)M(\tilde{\tau})} - \left(M(\tilde{\tau}^1) - M(\tilde{\tau}^0)\right).$$

For the Cobb–Douglas case, Proposition 2 shows that the *variance and covariance terms of the TRI are the usual trade weighted forms.* The term $(1-\Sigma\alpha_i)$ is equal to one minus the tariff–ridden–import share of total expenditure. Subject to a given interpolation procedure, the trade restrictiveness index can be expressed in terms of trade weighted moments of the interpolated tariff schedule. The problem of properly interpolating between τ^0 and τ^1 is generically the same for any discrete approximation, but it does mean that the covariance may matter. Thus the coefficient of variation of tariffs can be a consistent index when variance is eliminated (so covariance is zero) and the mean tariff does not rise. Formally

Corollary

Let final good import preferences be Cobb–Douglas. Then for mean–preserving changes in the schedule of tariffs which eliminate variance, both $\hat{\Delta}$ *and the change in the trade–weighted coefficient of variation of tariffs index correctly sign welfare.*

For CES expenditure functions, the shares, the α's, are endogenous, and z_{ij} is equal to the elasticity of substitution times expression (3.2). Since this is a constant (which moreover appears in numerator and denominator, hence cancels) it does not alter the results. Thus Proposition 2 can be extended to the CES case.

The main conclusion which should be drawn from (3.6) is that if the preferences for imports really are CES, so that the coefficient of variation is valid for mean–preserving eliminations of tariff variance, it is also feasible to calculate $\hat{\Delta}$ from (3.6) subject to the interpolation error. There was some intuitive sense to the earlier procedure, but the trade restrictiveness index makes optimal use of the moments of the tariff schedule. More generally, using any available information on substitution effects, it is possible to measure $\hat{\Delta}$ using the marginal trade weights identified in Section II. The trade restrictiveness index can then consistently measure all tariff changes. Moreover, it has the interpretation of a uniform tariff surcharge which compensates for the tariff change.

III.2 Mean-preserving Reductions in Variance

A full development of the Cobb–Douglas example leads to a proposition in which variance need only be reduced, not eliminated. It also further illustrates the the properties of the trade restrictiveness index and the issues of Section II.

Let the representative consumer's expenditure function with two tariff–ridden goods be

$$e(T_1,T_2,u) \quad = \quad T_1^{\alpha_1}T_2^{\alpha_2}u,$$

where $\alpha_1+\alpha_2<1$, $0<(\alpha_1,\alpha_2)<1$ and the remaining goods (nontraded goods and other imports and exportables), are untaxed, hence have prices equal to unity.

Under the Armington assumption, there is no production of the tariff–ridden goods and with no taxation of exports, or nontraded goods and a constant returns technology with more exportables than nontraded goods and factors, the value of production is fixed in world prices at y. This becomes a parameter with respect to the trade policy analysis.

Under these circumstances, the balance of trade function is

$$(3.7) \quad B(T,u) = T_1^{\alpha_1}T_2^{\alpha_2}u - y - (T_1-1)\alpha_1\frac{T_1^{\alpha_1}T_2^{\alpha_2}u}{T_1} - (T_2-1)\alpha_2\frac{T_1^{\alpha_1}T_2^{\alpha_2}u}{T_2}.$$

Assuming no external borrowing for simplicity and setting $B(T^0,u^0)$ equal to zero, the trade balance equation may be solved for the initial level of utility u^0. The solution utility level u is related to T by the *distorted trade utility function:*[13]

$$(3.8) \quad v(T) \quad = \quad \frac{y}{1-\alpha_1-\alpha_2+\alpha_1/T_1+\alpha_2/T_2}\,T_1^{-\alpha_1}T_2^{-\alpha_2}.$$

The trade restrictiveness index is implicitly defined as

$$(3.9) \quad \Delta(T,u^0) \quad = \quad \{\Delta \mid B(T\Delta,u^0) = 0\}.$$

With u^0 defined by $v(T^0)$ in (3.8), this implies that Δ solves:

[13]The distorted trade utility function is the basis for an equivalent variation version of the trade restrictiveness index.

$$(3.10)\quad \frac{\left(\frac{T_1}{T_1^0}\right)^{\alpha_1}\left(\frac{T_2}{T_2^0}\right)^{\alpha_2}}{1-\alpha_1-\alpha_2+\alpha_1/T^0_1+\alpha_2/T^0_2} - \frac{\Delta^{1-\alpha_1-\alpha_2}}{\Delta(1-\alpha_1-\alpha_2)+\alpha_1/T_1+\alpha_2/T_2} = 0.$$

Now consider the standard measures, the mean and variance of the tariff schedule. The import–weighted mean tariff is:

$$(3.11)\quad M(T) = \frac{\alpha_1}{\alpha_1+\alpha_2}T_1 + \frac{\alpha_2}{\alpha_1+\alpha_2}T_2 - 1.$$

The import–weighted variance of the tariff is:

$$(3.12)\quad V(T) = \frac{\alpha_1}{\alpha_1+\alpha_2}(T_1-1)^2 + \frac{\alpha_1}{\alpha_1+\alpha_2}(T_2-1)^2 - M(T)^2.$$

The purpose of this subsection is to show that variance reducing reforms which preserve the mean tariff are also welfare–improving. Mean–preserving tariff changes imply, from (3.11), that

$$\frac{dT_2}{dT_1} = -\frac{\alpha_1}{\alpha_2}.$$

Under the constant mean–tariff constraint, the variance changes with a rise in T_1 according to

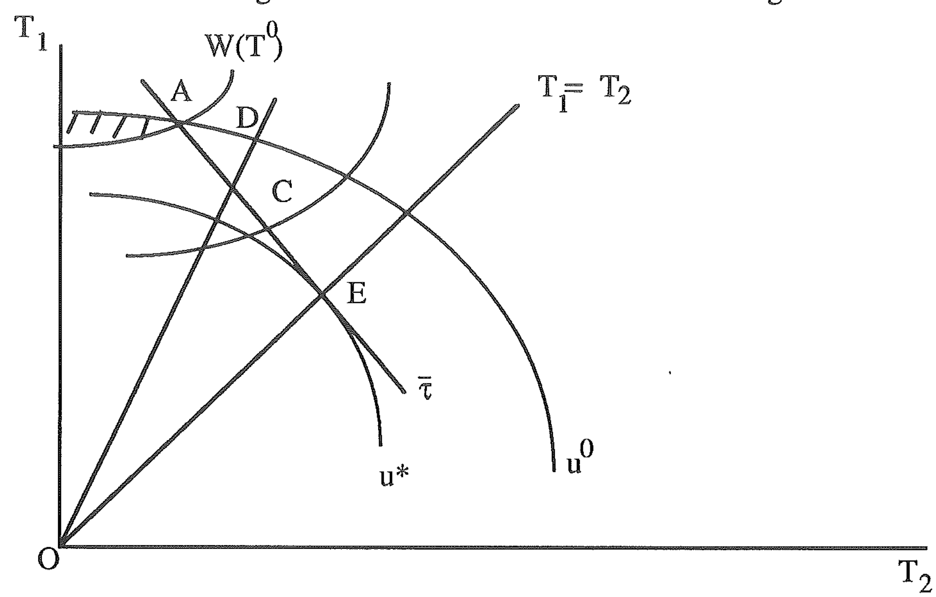
$$(3.13)\quad \frac{dV(\tau)}{dT_1} = 2\frac{\alpha_1}{\alpha_1+\alpha_2}\left(T_1-T_2\right).$$

The analysis is completed by showing that $d\Delta/dT_1$ under the constraint has the opposite sign to (3.13); i.e., if the variance decreases, efficiency measured by Δ increases. See the Appendix for the technical details. This means that

Proposition 3 *In the Cobb–Douglas case, mean–preserving local reductions in the trade–weighted variance of tariffs are welfare–improving.*

Discrete changes in the tariff structure appear to allow for a possible reversal of implications. But if the distorted trade utility function is quasi–concave (the tariff factor indifference curve bounds a convex set of tariffs preferred to the initial tariff), local superiority implies superiority over a range of changes on the mean tariff line. The Cobb–Douglas model has a concave distorted trade utility function for non–negative tariffs. Moreover, for the Cobb–Douglas model, for a constant mean tariff, it is readily shown that the *welfare–maximizing tariff structure is a uniform tariff*, which agrees with the variance–minimizing solution. This means that each step of variance reduction is welfare–improving, so that subject to a constant mean, the coefficient of variation of tariffs is a legitimate welfare index. Figure 2 illustrates.

Figure 2. Tariff Reform in the Cobb–Douglas Model



The positively sloped curves are coefficient of variation isoquants, more variation implied by more distance from the equal tariff factors ray. For the Cobb–Douglas model, the slope of the coefficient of variation function is

$$\frac{dT_1}{dT_2} = -\frac{\alpha_2}{\alpha_1}\frac{T_2 - \bar{T} - \frac{V(T)^{1/2}}{\bar{T}}}{T_1 - \bar{T} - \frac{V(T)^{1/2}}{\bar{T}}}.$$

The "isovariation" curves, labelled W (for the World Bank) necessarily have positive slope in the two good case. The local properties of $\Delta(T,u)$ at A show that Δ rises as variance falls in the move along the constant tariff line $\bar{\tau}$ from A to C. Both imply a rise in welfare. The global properties of v(T) show that variance falls all along AE to the maximal utility under the constraint, so that the variance measure is globally valid under the constant mean restriction. In contrast, when the mean and variance can both shift, the diagram shows that there is a set of tariff reforms which are welfare–superior, but which the coefficient of variation would measure as welfare–inferior. This is depicted as the cross–hatched region to the left of point A. When the mean shifts, the intuitive scaling of the standard deviation by the square of the mean does not provide a legitimate welfare index.

It would be nice if more general preference structures, which would require different weights, still had the property that variance reduction subject to a constant mean was welfare increasing. If so, (i) the optimal tariff would be uniform and (ii) each reduction in marginally weighted variance would improve welfare. The results of Section II show that (ii) cannot be guaranteed. Regarding (i), uniformity of the optimal tariff subject to a mean constraint unfortunately cannot be guaranteed for more general preference structures. The constant–mean tariff constraint is similar to the revenue constraint of public finance theory, which shows that a uniform tax on the set of goods available for taxation is possible only under very strong assumptions.

IV. Lessons

Section II lays out the foundation of the trade restrictiveness index in relation to standard tariff indices. The trade restrictiveness index is linked to the marginal trade weighted mean and the generalized variance of the tariff schedule, which fulfills the intuition behind the World Bank's use of trade weighted mean and coefficient of variation indices. Section III gives a strong sufficient condition for marginal trade weights to be equal to trade weights, and it does not appear to be possible to weaken the substitution conditions beyond the CES. For example, strong separability does not suffice. A correct version generally requires the use of marginal rather than average trade weights, but even so, covariance and "curvature" also matter outside the Cobb–Douglas case.

It is simple to construct examples where welfare moves in the opposite direction from either the mean or the variance or coefficient of variation, so long as these move in the opposite direction. Appendix 2 presents four such simulations for the Cobb–Douglas case. Thus, the trade restrictiveness index should be used to replace tariff averages or coefficients of variation, while recognizing the limits of any such measure due to having to identify or assume a substitution effects structure. This problem is met with severe substitution structure assumptions in Computable General Equilibrium (CGE) models, and sensitivity analysis on substitution parameters. A similar procedure in calculation of the trade restrictiveness index should be usable in principle. Here, a potentially great advantage is that the TRI can feasibly aggregate consistently from the fine structure of protection, a task which is currently not possible with CGE models.

Future work should experiment with various substitution effects assumptions and approximative vs. numerical integration, in order to see how much difference the refinements make, and how the trade restrictiveness index compares with the standard indices. A current World Bank project is attempting to implement the index in a pilot study of a dozen developing countries. Early results[14] indicate that (i) the trade restrictiveness index often gives results opposite to the standard methods, and (ii) the trade restrictiveness index is not very sensitive to the substitution elasticity values.

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[14]See Anderson and Bannister (1991) and Anderson (1991).

Appendix 1. Proof of Proposition 3.

Under the constant mean–tariff constraint, the index Δ changes with a rise in T_1 in a rather complex fashion. Fortunately, the issue of the relation of tariff variance to Δ can be illustrated by evaluating in the neighborhood of T^0, where Δ is equal to one. Let (3.10) be written as:

$$(3.10') \qquad \frac{A(T)}{B} - \frac{\Delta^{1-\alpha_1-\alpha_2}}{C(\Delta,T)} = 0.$$

At T equal to T^0, Δ is equal to one, B is equal to C, and A is equal to one. Differentiating (3.10') under the constant–mean–tariff constraint,

$$\left(\alpha_1\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\frac{A}{B} - \alpha_1\left(\frac{1}{T_1^2}-\frac{1}{T_2^2}\right)\frac{\Delta^{1-\alpha_1-\alpha_2}}{C^2}\right)dT_1$$

$$= (1-\alpha_1-\alpha_2)\left(\frac{1}{\Delta}-\frac{1}{C}\right)\frac{\Delta^{1-\alpha_1-\alpha_2}}{C}\,d\Delta.$$

Using the local properties at T equal to T^0, this implies:

$$(3.14) \qquad \frac{d\Delta}{dT_1} = \frac{\alpha_1}{(1-\alpha_1-\alpha_2)\frac{C-1}{C}}\left(\left(\frac{1}{T_1}-\frac{1}{T_2}\right)-\frac{1}{C}\left(\frac{1}{T_1^2}-\frac{1}{T_2^2}\right)\right).$$

At T equal to T^0, C is equal to $1-\alpha_1\frac{T_1-1}{T_1}-\alpha_2\frac{T_2-1}{T_2}$, which is less than one for non–negative tariffs. Thus the first ratio is negative. The large bracketed term factors into

$$\left(\frac{1}{T_1}-\frac{1}{T_2}\right)\left(1-\frac{\frac{1}{T_1}+\frac{1}{T_2}}{1-\alpha_1\frac{T_1-1}{T_1}-\alpha_2\frac{T_2-1}{T_2}}\right)$$

$$= \left(\frac{1}{T_1}-\frac{1}{T_2}\right)\left(\frac{(1-\alpha_1)\frac{T_1-1}{T_1}+(1-\alpha_2)\frac{T_2-1}{T_2}-1}{C}\right),$$

where the right bracket is negative. Then:

$$\text{sign } d\Delta/dT1 = \text{sign }\left(\frac{1}{T_1}-\frac{1}{T_2}\right) \text{ for non–negative tariffs.}$$

Appendix 2. Cobb-Douglas Simulations

This appendix presents the results of the simulation of the Cobb–Douglas model. The two tariff case is chosen with a tariff initially equal to .5 for good 2 with 10% of expenditure on it and .2 for good 1 with 5% of expenditure on it. The initial trade–weighted mean tariff is equal to 40%. In the Cobb–Douglas model, so long as the mean and variance of tariffs move in the same direction, the TRI must also move in that direction. The interest is in the cases where this does not hold. The simulations show four cases of tariff changes for which the mean and variance (and coefficient of variation) move in opposite directions. The first two rows show the relative tariff changes (from the initial base of $\tau_1 = .2$ and $\tau_2 = .5$). The next four rows of the table illustrate that any possible combination of TRI change with mean and offsetting variance or coefficient of variation change may be found. For tariff changes of the type used here, the mean or variance (or coefficient of variation) index would be wrong half the time.

cases	I	II	III	IV
relative tariff 1	1.458333333	1.125	0.91666667	1.18333333
relative tariff 2	0.8	0.96666667	0.96666667	0.93333333
relative mean tariff	0.958333333	1.04166667	0.83333333	1.01666667
relative variance	1.007631258	0.11111111	1.36111111	0.00444444
rel coeff var	1.047452219	0.32	1.4	0.06557377
TRI	0.99192921	0.99836569	1.04294631	1.00648972

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