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Abstract

Recent success in introducing road pricing, as well as recent polls, suggest that road pricing schemes are politically viable if a large majority of drivers benefit. In this paper we analyze the welfare effects of an optimal time-varying toll imposed during the morning commute. The toll tends to benefit drivers with high unit values of travel time and schedule delay. Other drivers can become worse off even with an equal per-capita rebate. A capacity expansion benefits drivers in proportion to their trip costs. If initial capacity is sufficiently small, a toll-financed expansion leaves all drivers better off.

The Welfare Effects of Congestion Tolls with Heterogeneous Commuters*

Economists have been recommending congestion tolls as a means of regulating urban traffic congestion for many years (Vickrey (1963), U.K. Ministry of Transport (1964) - the Smeed Report). Until recently, however, the opposition to congestion tolling from politicans and the public appeared overwhelming. An overriding objection was the cost of collection; the queuing cost at tollbooths would have exceeded the benefit from the toll. The famous Hong Kong experiment persuasively demonstrated that electronic road pricing (ERP) is technically feasible. But it was not introduced in Hong Kong due to political opposition. A major concern was that the form of ERP employed would have permitted government monitoring of individuals' travel (Borins (1986)). That concern has now been addressed through the development of 'smart cards', which are completely anonymous and permit instantaneous payment of tolls by making deductions from a prepaid credit balance.

As a result, the focus of discussion on urban road pricing has been shifting from technological considerations towards political acceptability. Until recently, it seemed that no amount of reasoned argument would persuade most drivers to give up their 'right' to free travel on urban roads, even under conditions of extreme congestion. But late developments suggest that this pessimism was excessive. Three cities in Norway - Bergen, Oslo, and Trondheim - have successfully implemented cordon tolls around the C.B.D. for inbound traffic, based on the need to supplement grants from the central

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government to finance expansion of the urban road network (Ramjerdi (1992)). Countries such as the Netherlands and Singapore are seriously considering implementing ERP. Private toll roads, already common in Southern Europe, are being constructed in the U.S. And a recent public attitude survey in London found that "Road charges would be acceptable to the majority if the revenues were reinvested in improving roads and public transport, or reducing taxes" (Bayliss (1992, p.7)).

These developments are very encouraging since they suggest that urban road pricing will be politically viable when the majority is persuaded that pricing is in its self-interest. This in turn suggests that economists and transport planners should design tolling schemes that benefit all, or almost all, urban travellers. The design of such tolls is the motivation of this paper. What we accomplish is more limited. We consider a population of heterogeneous commuters, who differ in terms of unit costs of travel time and schedule delay (time early and time late) as well as desired arrival time at work, and who commute by car along a common congested urban traffic corridor. We ask: Who benefits and who is hurt by an optimal tolling system when: i) toll revenues are not returned in cash or services to drivers; ii) toll revenues are rebated as an equal lump-sum payment to all drivers; and iii) toll revenues are used to finance expansion of the road? Answers to these questions should provide insight into the design of tolling schemes, including the disposition of toll revenue.

The welfare effects of tolls have been examined in other papers, both theoretical (Layard (1977), Wigan (1977) and Glazer (1981)) and empirical (Kulash (1974), Segal and Steinmeier (1980) and Small (1983)). The majority have concluded that the incidence of congestion tolls is probably regressive, and that distributional effects can dominate efficiency gains. Foster (1974, 1975) reached the tentative conclusion that road pricing is progressive; a dissenting opinion was voiced by Richardson (1974).

The results of these papers must be interpreted carefully because different authors use different definitions of progressivity; see Section 3.3 for a discussion. Moreover, they all employ static models of congestion. They therefore ignore important dynamic features of congestion and congestion pricing, notably that different types of individuals will tend to travel at different times during the rush hour[1] and will therefore be differently affected by tolls. For example, an assembly-line worker required to start work at 8:30 a.m. will have to pay a large toll since he travels in a highly congested period. If his value of travel time is low, he may derive little benefit from the toll. In contrast, a professional with a high value of travel time may benefit greatly from the reduction in travel time induced by the toll.

The first dynamic model of automobile congestion with endogenous driver behaviour was developed by Vickrey (1969) for the morning rush hour. In his model, each commuter decides when to depart from home so as to minimize the sum of her travel time and schedule delay. Congestion takes the form of queuing behind a bottleneck. Henderson (1974, 1977, 1981) was the first to incorporate driver heterogeneity into a dynamic model of rush-hour congestion. He treated two groups, differing in the ratio of unit travel time to unit schedule delay costs. Unlike Vickrey, he assumed flow rather than queuing

[1]Differences in commuting behaviour across socioeconomic groups have been documented in a number of empirical studies (*e.g.* Ott *et al.* (1980), Small (1982), and Moore *et al.* (1984)).

congestion. A more general analysis, using the bottleneck model, was provided by Newell (1987), who dealt with continuous distributions of commuters differing in work start time, costs of travel time and costs of schedule delay. In previous work (Arnott, de Palma and Lindsey (1988, 1992)) we also incorporated heterogeneous commuters into Vickrey's model to examine such issues as cost-benefit analysis of road investments and route choice.

The only study we know of to examine the welfare effects of tolls in a dynamic setting is that of Cohen (1987). Like Henderson, Cohen considers two groups of commuters differing in unit schedule delay and travel time costs. Like Vickrey, he treats queuing congestion and assumes that the desired arrival times of drivers are distributed uniformly over a one-hour interval. In this paper we extend Cohen's analysis in several directions. First, we allow for differences across commuters in the relative costs of late to early arrival. Second, we consider at a general level differences in commuters' desired arrival times. Third, we analyze the welfare effects of tolls when toll revenue is returned as a lump sum to drivers. And fourth, we consider the welfare effects of road investments, both exogenous investments and those financed through toll revenue.

The paper is organized as follows. In Section 2 we review briefly the Vickrey model, as treated in Arnott et al. (1988, 1990). Then we proceed by examining the welfare effects of tolls, capacity expansions, and capacity expansions funded by tolls, for different dimensions of commuter heterogeneity: first with respect to the ratio of unit travel time to unit schedule delay costs (Section 3), then with respect to the ratio of unit time late to time early costs (Section 4), and finally with respect to desired arrival time at work (Section 5). Section 6 contains some concluding remarks.

2. Review of the Model with Identical Commuters

In this section we briefly review the Vickrey model, as adapted in Arnott *et al.* (1990). Details are found in Arnott *et al.* (1988, 1990) *inter alios*. Here we provide only the background necessary to understand subsequent sections.

In the model, N identical commuters travel each morning, one per car, from home to work along a single road. N is assumed to be fixed; trip demand is completely inelastic.[2] Travel is uncongested except at a bottleneck with a capacity of s cars per unit time. If the arrival rate at the bottleneck exceeds s, a queue develops. Travel time is

$$T(t) = T^f + T^v(t), \tag{1}$$

where T^f is free-flow travel time, $T^v(t)$ is waiting time at the bottleneck, and t is departure time from home. It is assumed without loss of generality that $T^f = 0$, so that an individual reaches the queue at the bottleneck as soon as he leaves home, and arrives at work immediately upon exiting the bottleneck. If Q(t) is the number of vehicles in the queue at time t, then

$$T^v(t) = \frac{Q(t)}{s}. \tag{2}$$

Individuals are assumed to have a preferred arrival time at work, t^*. Their travel cost is given by the linear function

$$C(t) = \alpha T^v(t) + \beta(\text{time early}) + \gamma(\text{time late}) + \text{toll}(t). \tag{3}$$

To assure that the equilibrium departure rate of drivers is finite, it is assumed that $\alpha > \beta$. For individuals who arrive before t^*, time late = 0, and

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[2]Generalizing the model to incorporate elastic demand is not conceptually difficult, but makes the algebra more tedious. Furthermore, empirical studies (McFadden (1974), Pucher and Rothenberg (1976) and Small (1983)) have found that the demand for commuting trips by car, at least in most U.S. cities, is highly inelastic.

for those who arrive after t^*, time early = 0. Henceforth, we take 'depart early' to mean arrive at work early, and 'depart late' to mean arrive at work late.

2.1 No-toll Equilibrium

In choosing when to leave home, individuals face a trade-off between trip duration and schedule delay. They are assumed to have full information about the departure time distribution. Equilibrium obtains when no driver can reduce her travel costs by altering her departure time. With identical individuals, this means that costs are constant over the rush hour.

The equilibrium is depicted in Figure 1. The number of vehicles in the queue, Q(t), is measured by the vertical distance between the cumulative departures and cumulative arrivals schedules. Travel time is measured by the horizontal distance. The queue builds up at a constant rate from t_q until t_n, the departure time for which an individual arrives on time. The queue then dissipates, again at a constant rate, reaching zero at $t_{q'}$. t_q and $t_{q'}$ are solved by equating the costs of the first and last individuals to depart, both of whom incur only schedule delay costs:

$$\beta(t^*-t_q) = \gamma(t_{q'}-t^*). \tag{4}$$

Since the bottleneck operates at capacity throughout the rush hour

$$t_{q'} = t_q + \frac{N}{s}. \tag{5}$$

Combining (4) and (5) one obtains

$$t_q = t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s}, \quad t_{q'} = t^* + \frac{\beta}{\beta+\gamma}\frac{N}{s}. \tag{6a,b}$$

Using the equal-cost condition, it is easily shown that

$$t_n = t^* - \frac{\delta}{\alpha}\frac{N}{s},$$

where $\delta \equiv \beta\gamma/(\beta+\gamma)$ is half the harmonic mean of β and γ. Total travel time, total time early and total time late are identified as areas in Figure 1. Total Travel Time Costs, TTC, total Schedule Delay Costs, SDC, and total Travel Costs (net of free-flow travel time costs), TC, are:

$$TTC^e = SDC^e = \frac{\delta}{2}\frac{N^2}{s}, \tag{7}$$

$$TC^e = TTC^e + SDC^e = \delta\,\frac{N^2}{s}, \tag{8}$$

where the superscript e denotes no-toll equilibrium.

2.2 The Social Optimum

The social optimum is determined by minimizing the sum of travel time and schedule delay costs. Since queuing is wasteful the departure rate is held at s throughout the rush hour. The time of first departure is chosen so that the first and last commuters incur equal costs, since otherwise costs could be reduced by moving an individual from the beginning of the rush hour to the end, or *vice versa*. Since this condition is also true of the no-toll equilibrium, the timing of the rush hour and the arrival distribution are the same as in equilibrium.

With variables corresponding to the social optimum denoted by a superscript o, aggregate costs are:

$$TTC^o = 0, \tag{9}$$

$$SDC^o = TC^o = \frac{\delta}{2}\frac{N^2}{s}. \tag{10}$$

Comparing (10) with (8), one sees that total costs (net of free-flow travel time costs) are just half their level in the no-toll equilibrium.

2.3 Equilibrium with Tolls

Since the number of commuters is fixed and there is only one route, a uniform (time-independent) toll has no effect on equilibrium; it merely extracts income from drivers. However, a time-varying toll does affect queuing. The social optimum can be decentralized by a toll that increases linearly from zero at time t_q to a maximum at t^*, and subsequently decreases linearly to zero at $t_{q'}$. The toll replaces queuing time as the rationing device for desired arrival time slots, leaving drivers' private costs inclusive of the toll unchanged, but overall welfare higher by the amount of toll revenue. The social optimum can also be decentralized by the above time-varying toll combined with a uniform toll. We assume hereafter that the uniform component is zero.

3. Individuals with Different α and β, but the Same γ/β

In this section, we allow the unit costs of travel time and schedule delay to differ across commuters, but assume commuters have the same relative cost of late to early arrival, γ/β, and the same t^*. Cohen's (1987) specification (and Vickrey's (1969)) differs in that he assumes t^* to be uniformly distributed in the driving population over a one-hour time interval.

Suppose that there are G groups of commuters indexed in order of increasing β/α, so that group G dislikes schedule delay most highly relative to travel time, while group 1 has the greatest relative aversion to travel time:

$$\beta_1/\alpha_1 \leq \beta_2/\alpha_2 \leq \ldots \leq \beta_G/\alpha_G. \qquad (11)$$

Let N_i be the number of individuals in group i, and N the number in all groups.

3.1 No-Toll Equilibrium

A derivation of the no-toll equilibrium is given in Arnott *et al.* (1987); descriptions are found in Cohen (1987) and Arnott *et al.* (1988). Figure 2 shows equilibrium for three groups. The intuition for the timing and shape of the queuing pattern is as follows. Since departures are continuous over the rush hour, $t_q{}' - t_q = N/s$. The first and last individuals to depart are members of group 1, with the lowest relative cost of schedule delay to travel time. These individuals must have the same cost in equilibrium, and since neither faces a queue this is entirely schedule delay cost: $\beta_1(t^* - t_q) = \gamma_1(t_q{}' - t^*)$. t_q and $t_q{}'$ are therefore given by (6a,b) above, with β_1 and γ_1 in place of β and γ. Let $\delta_i \equiv \beta_i\gamma_i/(\beta_i+\gamma_i)$, i = 1,2,3. Then the equilibrium cost per driver of the first group is

$$C_1 = \delta_1 \frac{N}{s}. \tag{12}$$

The locus of points in t-Q space along which group 1 drivers incur cost C_1 is labelled C_1 in Figure 2. Cohen (1987) refers to this as an 'isocost' contour. It is defined by the condition

$$C_1(t) = C_1 = \begin{cases} \beta_1(t^*-t) + (\alpha_1-\beta_1)Q(t)/s & \text{for early departures,} \\ \gamma_1(t-t^*) + (\alpha_1+\gamma_1)Q(t)/s & \text{for late departures.} \end{cases}$$

Q(t) evolves so as to satisfy this condition during the time intervals when group 1 is departing.

Group 2, with the second-lowest relative cost of schedule delay to travel time, is the second group to depart early and the second-last to depart late. The boundary times between group 1 and group 2 departures, t_{12} and t_{21}, are determined simultaneously by the conditions that $C_2(t_{12}) = C_2(t_{21})$, and that the number of group 1 departures between t_q and t_{12}, plus the number between t_{21} and $t_{q'}$, equals the group 1 population. From this calculation, one obtains

$$C_2 = \delta_2 \frac{N_2+N_3}{s} + \left(\frac{\alpha_2}{\alpha_1}\right) \delta_1 \frac{N_1}{s}. \tag{13}$$

The departure interval for group 3 and its equilibrium cost are determined sequentially in a similar manner:

$$C_3 = \delta_3 \frac{N_3}{s} + \left(\frac{\alpha_3}{\alpha_2}\right) \delta_2 \frac{N_2}{s} + \left(\frac{\alpha_3}{\alpha_1}\right) \delta_1 \frac{N_1}{s}. \tag{14}$$

Figure 2 displays the equilibrium isocost contours for the three groups. To the left of the locus Ot^* with slope $-s$, along which individuals arrive on time, the slope of C_i is $\beta_i s/(\alpha_i-\beta_i)$; to the right it is $-\gamma_i s/(\alpha_i+\gamma_i)$. The slopes are greater for higher-numbered groups, who are more willing to queue in order to reduce schedule delay.

The equilibrium queue, or Travel Equilibrium Frontier (TEF), is the upper envelope of the various groups' equilibrium isocost contours. Commuters may be viewed as choosing departure times to minimize their cost on the TEF. The equilibrium departure times for a group are given by points of tangency of its equilibrium isocost curve with the TEF. Viewed alternatively, each departure slot is allocated to that group which is willing to pay the most, in terms of queuing time, for it. Group 2, for example, departs between t_{12} and t_{23}, and again between t_{32} and t_{21}. It is evident that the higher a group's β/α, the closer to the center of the rush hour it travels.

3.2 The Social Optimum

There is no queuing in the social optimum; thus, groups are allocated to departure time slots so as to minimize schedule delay costs. The group with the largest β travels closest to t^*, the group with the second-highest β departs on adjacent time intervals earlier and later, and so on. The first and last commuters to depart, who belong to the same group, incur the same schedule delay costs. Because all groups have the same γ/β, the rush hour begins and ends at the same time as in equilibrium. However, the order in which groups depart differs unless the ranking of groups according to β is the same as the ranking according to β/α, which need not be the case. For example, assembly-line workers may have a relatively higher β than stockbrockers because there is little for assembly-line workers to do before the shift starts, but have a lower absolute β than stockbrokers, whose hourly wage is significantly higher. If these are the only two groups of commuters, then assembly-line workers travel at the peak in the no-toll equilibrium, whereas stockbrokers travel at the peak in the social optimum. Thus, travel order in the no-toll equilibrium is determined by relative schedule delay and travel time costs, while in the social optimum it follows absolute schedule delay costs. If the orders differ, aggregate schedule delay costs in the no-toll equilibrium are not minimized.

The social optimum can be decentralized with a time-varying toll. A toll for 3 groups is drawn in Figure 3, where it is assumed that $\beta_2 < \beta_1 < \beta_3$, so that groups 1 and 2 are reversed in the departure sequence relative to the no-toll equilibrium. The toll coincides with the upper envelope of the toll-equilibrium isocost curves of each group. Since there is no queuing, departure and arrival times coincide, and each group's curve peaks at t^*

(rather than before t^* as in Figure 2). Group i's curve rises with slope β_i for $t < t^*$, since its schedule delay cost falls at this rate. For $t > t^*$, the isocost curve has slope $-\gamma_i$.

In the toll equilibrium, group i departs during the time intervals when its isocost curve is tangent to the toll: early while the toll is increasing at rate β_i, and late while it is decreasing at rate γ_i. Since commuters in a given group prefer their existing departure interval to departure at any other times, the toll induces commuters to self-select into the efficient departure time intervals. Furthermore, because the toll changes at minus the rate of schedule delay costs of the group departing, there is no queuing.

Since calculation in the general case is tedious, for the remainder of the section we shall focus on two groups, 1 and 2. Substituting $N_3 = 0$ into (12) and (13) one has

$$C_1 = \delta_1 \frac{N_1+N_2}{s}, \quad C_2 = \delta_2 \frac{N_2}{s} + \left(\frac{\alpha_2}{\alpha_1}\right) \delta_1 \frac{N_1}{s}.$$

Total costs are simply $TC = N_1C_1 + N_2C_2$. Total schedule delay and total travel time costs are straightforward to compute. Let $f_2 \equiv N_2/N$ be the fraction of commuters in group 2. Further, define $\alpha_r \equiv \alpha_2/\alpha_1$ and $\beta_r \equiv \beta_2/\beta_1$. Aggregate costs in the no-toll equilibrium work out to be (see Arnott *et al.* (1987)):

$$SDC^e = \frac{\delta_1}{2} \frac{N^2}{s} [1 + (\beta_r - 1)f_2^2], \tag{15}$$

$$TTC^e = \frac{\delta_1}{2} \frac{N^2}{s} [1 - 2(1-\alpha_r)f_2 + (1+\beta_r-2\alpha_r)f_2^2], \tag{16}$$

$$TC^e = \frac{\delta_1}{2} \frac{N^2}{s} [1 + (\alpha_r - 1)f_2 + (\beta_r - \alpha_r)f_2^2]. \tag{17}$$

In the social optimum, group 2 travels in the middle of the rush hour if $\beta_r > 1$, and on the tails if $\beta_r < 1$. Total costs can be derived geometrically from Figure 3 using a procedure analogous to that dicussed above. The result is

$$SDC^o = TC^o = \begin{cases} \dfrac{\delta_1 N^2}{2\,s}\,[1 + (\beta_r - 1) f_2^2], & \text{if } \beta_r \geq 1, \quad (18a) \\[2ex] \dfrac{\delta_1 N^2}{2\,s}\,[1 - 2(1-\beta_r) f_2 + (1-\beta_r) f_2^2], & \text{if } \beta_r \leq 1. \quad (18b) \end{cases}$$

3.3 Welfare

3.3.1 Welfare Effects of the Optimal Time-Varying Toll

In Section 2.2 we saw that if commuters are identical their welfare is unaffected by the optimal time-varying toll because it substitutes perfectly for queuing time. If commuters differ, intuition suggests that those who value congestion relief highly benefit most from tolls. If there is no rebate of toll revenue, some may gain and others lose. With a uniform per capita lump-sum rebate, it is more likely that everyone benefits.

The toll considered is that of Figure 3. Table 1 shows that its welfare distributional effects depend not only on whether revenue is rebated, but also on whether departure order gets reversed by the toll.

Consider panel (a), where there is no toll rebate, and the case $\beta_2 > \beta_1$ for which the order of departure is unaffected. Group 1 is equally well off with the toll; instead of incurring a queuing cost of $\alpha_1 Q(t)$ in the no-toll equilibrium, a member departing at t pays a toll of the same amount. Group 2's costs, however, are affected. To see this, consider the group 2 individual who departs at t_{12}. He pays a toll equal to $\alpha_1 Q(t_{12})$ rather than face a queue of length $Q(t_{12})$, the cost of which is $\alpha_2 Q(t_{12})$ to him. Group 2 ends up worse off if $\alpha_2 < \alpha_1$, and better off if $\alpha_2 > \alpha_1$. If group 1 comprises professionals, and group 2 assembly-line workers, $\alpha_2 < \alpha_1$ is likely. But if group 1 comprises office workers, $\alpha_2 > \alpha_1$ may be true.

If $\beta_2 < \beta_1$, group 1 travels in the middle of the rush hour. Any member is free to depart at the beginning of the rush hour, and to incur the same travel cost as in the no-toll equilibrium. But the toll rises only at a rate β_2, while schedule delay costs for group 1 fall at rate $\beta_1 > \beta_2$. Delaying departure therefore makes group 1 strictly better off. Group 2 though is worse off. In the no-toll equilibrium, it 'outbids' group 1 for the choice departure time slots because of its relatively low value of travel time and hence tolerance to queuing. But with the toll, willingness to pay becomes the rationing device, and group 2 is forced to the tails of the rush hour.

Suppose now, as in panel (b) of Table 1, that toll revenues are returned as an equal lump sum to all drivers. While group 1 is always better off with the toll, group 2 is not. In particular, if group 2 is a minority (f_2 is small) and relatively insensitive to travel time costs (α_2/α_1 is small), its travel costs are so low when traveling near the peak in the no-toll equilibrium that it is worse off with the toll even with a rebate.

Overall, Table 1 suggests that <u>the group with a lower β/α fares better with a toll than the group with a higher β/α</u>. Whether or not the toll is progressive or regressive depends on the definition of progressivity. There are three commonly-employed notions of progressivity — absolute progression, average progression, and marginal progression. A tax (a bad) is said to be absolute progressive if tax payable rises with income, average progressive if the average tax rate rises with income, and marginal progressive if the marginal tax rate rises with income. A benefit is said to be absolute progressive if the benefit falls with income, *etc*.

Because our analysis makes no reference to income, we must employ an alternative definition. We shall say that a toll is progressive if the net benefit falls with the (relevant) unit shadow value of time. (If the shadow value varies positively with income, then our definition of progressivity is consistent with the standard definition.) According to this definition, if group 1 comprises highly-paid white-collar workers and group 2 blue-collar workers, so that $\alpha_1 > \alpha_2$, the toll considered here tends to be regressive.

This is consistent with the results of Small (1983) who finds that, exclusive of redistributed revenues, tolls favour higher-income groups. Lower-income users are disadvantaged by having to share roads with those who value congestion relief highly. With revenue redistribution, Small finds that only the poorest individuals still end up worse off. His results are not directly comparable with those here because trip scheduling is not considered in his model, and because his commuters can travel by bus as well as automobile.

Segal and Steinmeier (1980) also find tolls regressive. They assume that toll revenue is rebated in proportion to income, rather than on a uniform per-capita basis. A proportional rebate would reinforce our conclusion that tolls tend to be regressive. Henderson (1974) assumes that high-income workers have a higher β, higher α <u>and</u> higher β/α than low-income workers. High-income workers then travel in the middle of the rush hour in both the no-toll and with-toll equilibria. The welfare of low-income workers is unaffected by a toll without rebate, while high-income workers are better off.[3]

3.3.2 Welfare Effects of Capacity Expansion

In both the no-toll equilibrium and the social optimum, total costs are inversely proportional to capacity (*viz.* equations (17) and (18)). The same is true for individual costs in each group. Thus, the benefits to individuals from capacity expansion are proportional to their travel costs. For conciseness we limit attention here to the no-toll equilibrium. It is straightforward to show that the costs of a group 2 commuter exceed those of a group 1 commuter if and only if:

$$f_2\beta_r + (1-f_2)\alpha_r > 1. \tag{19}$$

To see this, consider first the limit $f_2 \uparrow 1$. The first member of group 2 then departs immediately after the last early member of group 1, just after t_q. Both incur only a schedule delay cost: $\beta_1(t^*-t_q)$ for the group 1 member, $\beta_2(t^*-t_q)$ for the group 2 member. Group 2's cost thus exceeds group 1's iff $\beta_r > 1$. This is the limiting condition of (19) as $f_2 \uparrow 1$. Now let $f_2 \downarrow 0$. Members of both groups then arrive at t^*, and incur only travel time costs. Group 2's cost exceeds group 1's iff $\alpha_r > 1$, which is the limiting condition of (19) as $f_2 \downarrow 0$.

The general case of (19) is simply a weighted average of the limit cases, with the fractions of individuals in each group as the weights. If group 1

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[3]Since the toll generates efficiency gains, if the groups were identifiable it would be possible to design a rebate scheme that would result in everyone being made better off. Though individuals' α, β and γ are not directly observable, if they were strongly correlated with income there would be some scope for differentiating the rebate via the income tax system. An alternative approach to designing a tax-cum-rebate scheme that benefits everyone would be to provide a uniform rebate along with a Pareto-improving time-varying toll.

comprises highly paid white-collar workers and group 2 blue-collar workers, then $\alpha_r < 1$. For downtown areas in which white-collar workers predominate (f_2 is small), condition (19) is likely to fail, so that road investments benefit higher-paid workers more. Before concluding that road investments have regressive welfare effects, however, it is necessary to consider how capacity expansions are financed. One possibility is via general tax revenue; if the tax system is strongly progressive at the margin, the progressive tax effects might offset the regressivity of benefits. Investments could also be financed by a toll, a possibility that we now examine.

3.3.3 Welfare Effects of a Toll-Financed Capacity Expansion

In the introduction we mentioned recent developments which suggest that tolling is more politically acceptable when toll revenues are used to finance transport improvements. Thus, it is of practical interest to consider the welfare effects of a toll when the revenue is used to finance capacity expansion. To simplify, we consider only the case where departure order is not changed by the toll. Let s^b and s^a denote capacity before and after expansion respectively. Then, given (12),

$$C_1^b - C_1^a = \delta_1 \frac{N}{s^b} - \delta_1 \frac{N}{s^a}.$$

It is straightforward to show (Arnott *et al.* (1987, Appendixes A,B)) that with capacity s^b and no toll

$$C_2^b = \delta_1 \frac{N}{s^b} [\alpha_r(1-f_2) + \beta_r f_2],$$

whereas with capacity s^a and the optimal toll without rebate

$$C_2^a = \delta_1 \frac{N}{s^a} [1 - f_2 + \beta_r f_2].$$

Thus

$$C_2^b - C_2^a = \delta_1 \frac{N}{s^b} [\alpha_r(1-f_2) + \beta_r f_2] - \delta_1 \frac{N}{s^a} [1 - f_2 + \beta_r f_2].$$

Revenue raised from the optimal toll can be calculated as $R = N_1 C_1^a + N_2 C_2^a - TC^o$ (TC^o is given by (18)). If unit capacity costs are k, $s^a - s^b = R/k$. Combining the results yields a complicated condition under which group 2 is made better off when the optimal toll is used to finance capacity expansion. If initial capacity is sufficiently suboptimal from group 2's perspective, the toll/capacity expansion scheme unambiguously benefits group 2, even if $\alpha_2 < \alpha_1$. Of course, group 1 always benefits because a toll with neither rebate nor capacity expansion leaves its costs unchanged.

4. Individuals Differing with respect to γ

4.1 No-Toll Equilibrium and Social Optimum

In this section we allow the cost of late arrival, γ, to differ across commuters. For simplicity, all other parameters are assumed to be the same for everyone. Let there be G groups of commuters, indexed in order of decreasing γ so that:

$$\gamma_1 > \gamma_2 \ldots > \gamma_G.$$

The derivation and intuition behind this case is similar to that for heterogeneity with respect to α and β; accordingly we will be brief.

4.1.1 No-Toll Equilibrium

In the no-toll equilibrium, groups with the lowest γ depart late, in strict sequence of decreasing γ, with group G the last to depart. Groups with higher γ depart early. The order of departure of these groups is indeterminate. At most one group departs both early and late.

4.1.2 The Social Optimum

In the social optimum, the departure rate is held at s to prevent queuing. To minimize schedule delay costs, groups depart in strict sequence of decreasing γ during late departure, just as in the no-toll equilibrium. The timing of the rush hour is also the same as in equilibrium.[4] The optimum can be decentralized with a time-varying toll. Since schedule delay costs are minimized in the no-toll equilibrium, toll benefits equal travel time saved.

4.2 Welfare

4.2.1 Welfare Effects of the Optimal Time-Varying Toll

Since the order of departure and the timing of the rush hour are unaffected by imposing an optimal time-varying toll, and since toll revenue equals travel time costs without a toll, a toll without rebate is welfare neutral. With a rebate, all commuters are better off. These results apply to any number of groups.

4.2.2 Welfare Effects of Capacity Expansion

The welfare effects of capacity expansion are not as immediate as those for the toll, and we simplify as in Section 3 by considering two groups.[5] Group 1, with relatively high penalties for late arrival at work, can be thought of as assembly-line workers. Group 2, with the smaller γ, can be

[4] This result, which is not intuitive, is proved in Arnott *et al.* (1987, Appendix D).

[5] The two-group case incorporates the solution for any number of groups as long as no more than two groups depart late.

thought of as clerks or office workers, provided they work largely independently of each other.

The welfare effects of capacity expansion depend on the relative sizes of the two groups. If group 2 is sufficiently large that only it departs late, then the equilibrium is the same as if all individuals belonged to group 2. (All of group 1 arrives early, and thus is indistinguishable from group 2.) This is the case when

$$f_2 > \frac{\beta}{\beta+\gamma_2}, \tag{20}$$

where $\beta/(\beta+\gamma_2)$ is the fraction of drivers who depart late when all belong to group 2 (see equation (6b)). Individuals in the two groups incur the same travel costs, and road investment is welfare neutral.

If condition (20) fails, so that some members of group 1 depart late, group 1 incurs the higher travel costs. To see this, note that the last member of group 1 to depart and the first member of group 2 have the same travel time and schedule delay, but the group 1 individual incurs a greater cost for time late. Group 1 thus benefits more from road investment. If group 1 comprises blue-collar workers and group 2 white-collar workers, road investment favours the commuting 'poor'.

4.2.3 Welfare Effects of a Toll-Financed Capacity Expansion

The welfare effects of a toll-financed investment follow immediately from the results of the two preceding subsections. Because a toll without rebate leaves the welfare of both groups unchanged, whereas a capacity expansion benefits them both, a toll-financed capacity expansion leaves both groups better off. If condition (20) is met, then expansion by itself is welfare neutral, as is a toll-financed expansion. If (20) is not satisfied then group 1 (tentatively identified as the commuting 'poor') benefits more.

5. Individuals with Different t^*

5.1 No-Toll Equilibrium and Social Optimum

Suppose now that commuters differ only in their desired arrival times at work. Let N_i be the number of commuters with desired arrival time t_i^*, i = 1...G, with $t_1^* < t_2^* < \ldots < t_G^*$, so that group 1 has the earliest desired arrival time, *etc.*[6] It is assumed that desired arrival times are sufficiently close that the bottleneck operates at capacity throughout the rush hour. Groups can comprise worker teams with different work hours, or teams with identical work hours but employed at varying distances from the bottleneck (teams with further to travel wish to pass through the bottleneck earlier).

5.1.1 No-Toll Equilibrium

An example of equilibrium with 2 groups is shown in Figure 4. Group 1 wishes to arrive at t_1^*, and group 2 at t_2^*. The cumulative desired arrival time distribution, W(t), is shown by the heavy line OFGIKD. Cumulative departures are OABCD and cumulative arrivals OD. The equilibrium isocost curves of the two groups, C_1 and C_2, intersect at t_{12} when group 1 stops departing and group 2 starts. $t_{n,i}$ denotes the departure time for which an individual arrives at t_i^*. For departures between t_q and $t_{n,1}$, cumulative arrivals exceed cumulative desired arrivals, and the departure rate exceeds capacity. Between $t_{n,1}$ and t_{12}, cumulative arrivals lag desired arrivals, and

[6]Hendrickson and Kocur (1981) and Newell (1987) treat continuous distributions of t^*, but do not analyze their welfare implications.

the departure rate falls below capacity, *etc.* The queue peaks at $t_{n,1}$ and $t_{n,2}$, and has a local minimum at t_{12}. Aggregate travel time, early arrival time and late arrival time correspond to the areas indicated.

Figure 5 shows another possible equilibrium for two groups in which the queue has a single peak. All of group 2 arrives late. The equilibrium isocost curves C_1 and C_2 coincide over the interval $[t_{n,2}, t_{q'}]$. Members of group 1 and group 2 may depart at any time in this interval; the order of departure is indeterminate. Because group 1 is indifferent about departing at any time in the interval $[t_q, t_{q'}]$, the timing of the rush hour and the costs of group 1 are the same as if all travellers belonged to group 1. For group 2, late arrival (and total travel) costs are smaller than for the same number of group 1 individuals by the area EFGH. Per capita travel costs are smaller by $\gamma(t_2^* - t_1^*)$. A single-peaked queue also arises when all of group 1 arrives early. The timing of the rush hour and the costs of group 2 are the same as if all travellers belonged to group 2.

It can be shown (Arnott *et al.* (1987, Appendix E)) that the queue is single-peaked iff

$$s(t_2^* - t_1^*) \le \left\| \frac{\beta}{\beta+\gamma} N_1 - \frac{\gamma}{\beta+\gamma} N_2 \right\|, \tag{21}$$

and double-peaked iff

$$\left\| \frac{\beta}{\beta+\gamma} N_1 - \frac{\gamma}{\beta+\gamma} N_2 \right\| < s(t_2^* - t_1^*) < \frac{\beta}{\beta+\gamma} N_1 + \frac{\gamma}{\beta+\gamma} N_2. \tag{22}$$

The rush hours of the two groups are separate if

$$s(t_2^* - t_1^*) \ge \frac{\beta}{\beta+\gamma} N_1 + \frac{\gamma}{\beta+\gamma} N_2. \tag{23}$$

5.1.2 The Social Optimum

Because groups share the same parameters except t^*, having groups depart in strict sequence is optimal, although not necessarily the unique optimum. The timing of the rush hour turns out to be the same as in equilibrium.[7] So is the indeterminacy in the order of departure. To see this, note that if two individuals in different groups who are both arriving early remain early if interchanged in the departure sequence, or if they are both arriving late and remain late if interchanged, then total schedule delay costs are unaffected. These are the conditions under which indeterminacy arises in equilibrium.

5.2 Welfare

5.2.1 Welfare Effects of the Optimal Time-Varying Toll

Since the timing of the rush hour and order of departure in equilibrium are socially optimal, and α, β and γ are the same for everyone, a toll simply replaces queuing time cost with money, leaving the costs of both groups unchanged. If toll revenue is rebated, both groups are better off. These results apply to any number of groups. Thus, tolls are welfare neutral.

5.2.2 Welfare Effects of Capacity Expansion

As in Sections 3 and 4, the welfare effects of capacity expansion are not transparent, and so we shall again limit consideration to two groups. If the queue is single-peaked (condition (21) is satisfied) before and after expansion the two groups benefit equally because the difference in their travel costs is independent of capacity. If the queue is double-peaked

[7] See Arnott *et al.* (1987, Appendix E).

(condition (22) is satisfied) before and after expansion, group 2 turns out to benefit more than group 1 if

$$f_2 > \frac{\beta}{\beta+\gamma}. \tag{24}$$

Group 1 benefits more if $f_2 < \beta/(\beta+\gamma)$, and the groups benefit equally if $f_2 = \beta/(\beta+\gamma)$. Because conditions (21) - (23) involve s, a capacity expansion may change the shape of the queuing pattern. A single-peaked queue may be transformed into one with two peaks, or two separate rush hours. Similarly, a double-peaked queue may evolve to separate rush hours. The welfare rankings are readily generalized to allow for such changes:

1. If the queue is single-peaked <u>after</u> capacity expansion, then the two groups benefit equally.
2. If, <u>after</u> expansion, the queue is double-peaked or there are separate rush hours, then group 2 benefits more than group 1 if condition (24) holds. Group 1 benefits more if $f_2 < \beta/(\beta+\gamma)$, and the groups benefit equally if $f_2 = \beta/(\beta+\gamma)$.

Since γ/β is typically large this suggests that individuals with later work start times (*viz.* group 2) benefit more if they comprise more than a modest fraction of the commuting population. Whether this means that the welfare effect of investment is progressive depends on how the income distribution of commuters varies with work hours, an empirical issue to be resolved on a city-specific basis.

5.2.3 <u>Welfare Effects of a Toll-Financed Capacity Expansion</u>

As in Section 4 the welfare effects of a toll-investment package can be decomposed. The toll without rebate by itself is welfare-neutral. The welfare effect of investment depends on the shape of the queuing pattern after investment, as described in the preceding subsection. A toll-financed capacity expansion affects welfare in the same way.

5.3 Generalizations

Table 2 presents a summary of the qualitative results derived in the previous three sections. We have treated only cases in which groups differ in one or two parameters. In reality, groups generally differ in all parameters. Investigating this hybrid case would generate little additional insight but is necessary for practical application. Analytical solution is feasible up to a point, but tedious, as Newell (1987) has shown.[8] Furthermore, it is evident that the qualitative characteristics of the equilibrium and optimal rush hours are sensitive to the cost parameters of different commuter groups, and to the proportion of commuters in each group. Further empirical work along the lines of Small (1982, 1983) is needed to refine parameter estimates. And city-specific studies are required to measure the composition of the commuting population.

6. Concluding Remarks

We have analyzed the welfare effects of tolls, capacity investments, and toll-financed capacity investments on commuters who differ in their cost of travel time, their preferred arrival time at work, and the costs they incur from early and late arrival. Absent tolling, departure times are rationed by

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[8]Whereas in all cases exposited here the equilibrium and optimal arrival time intervals coincide, this is not true in general, a complication that adds to the analytical complexity. Richter *et al.* (1990) have developed fixed point algorithms for computing equilibria numerically.

queuing. The order in which different groups of drivers depart is based on the ratio of unit schedule delay costs to unit travel time costs, the relative costs of late to early arrival, and work start times.

A time-varying toll can be designed that eliminates queuing. With such a toll, the order of departure is based on absolute unit schedule delay cost. Groups who value on-time arrival most incur the least schedule delay. The elimination of queuing benefits drivers with a higher unit value of travel time, and hurts those with lower values. If the toll changes the order of departure, it benefits drivers with higher unit schedule delay costs. The toll is neutral with respect to differences in relative late arrival to early arrival costs, and differences in work start times. Overall, therefore, a toll without rebate benefits drivers with high unit travel time and schedule delay costs, and harms those with low values. Since unit travel time costs are typically strongly positively correlated with income, a toll without rebate tends to benefit the rich on average, and hurt the poor.

An equal per-capita rebate of toll revenue is (average) progressive since a fixed payment results in a larger proportional increase in income for the poor than for the rich. The progressivity of a toll with a uniform rebate is therefore determined by the balance between the typical regressivity of the toll without rebate, and the progressivity of the rebate. Which effect dominates depends in a complex way on the distribution of the population by work start time, the shadow value of queuing time and the unit costs of schedule delay.

The distributional effects of capacity expansion in our model are simple. Because each group's travel costs (net of fixed travel time) are inversely proportional to capacity, a capacity expansion reduces everyone's costs by the same proportion. Hence those with higher absolute travel costs enjoy higher absolute benefits. In the case of a toll-financed capacity expansion, the analytics of the general case are complex but the intuition is clear. The effects are the same as those for a toll with an effective rebate proportional to (net of fixed) travel costs.

In the introduction we mentioned some encouraging recent developments suggesting that road pricing schemes may well be politically acceptable if they can be designed in such a way that a large majority of the population benefits. This points to the need for research on the design of such schemes, which are defined by a time-varying toll, a cash rebate formula, and a size of capacity expansion. Our analysis was positive. But it can be adapted for normative analysis to solve for the road pricing scheme that is optimal when considerations of acceptability are taken into account. The objective could be to maximize a social welfare function subject to a political acceptability constraint, or to maximize political acceptability.

A number of practical complications absent from our model will need to be considered in the design of an actual road pricing scheme — public transit, carpooling[9], discretionary travel[10], and differences in origins and destinations, *inter alia*. Furthermore, more sophisticated and detailed estimation of travel cost functions for different groups in the population will be needed[11], in order to determine accurately the distributional effects of a particular road pricing scheme. Finally, more research is needed on the determinants of the perceived benefits of alternative road pricing schemes, and hence of their political acceptability.

Economists are largely to blame for the political unreceptiveness to road pricing. They have done a poor job of educating politicians and the public about the theoretical virtues of road pricing. But, at least as importantly,

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[9]To analyze carpooling properly, it will be necessary to model which individuals travel together, and how differences in desired arrival time are resolved. Carpooling can affect both the physical and psychic costs of travel time and schedule delay. For example, Small (1982) found that carpoolers value schedule delay more relative to travel time than do single drivers. Poorer workers tend to carpool more than affluent workers, and to take transit in greater proportions. Carpoolers and bus users benefit from tolls because they occupy less road space than solo drivers, and hence pay lower charges. Bus users also benefit more to the extent that congestion slows buses more than cars. These factors tend to make road pricing less regressive.

[10]Non-commuting travel was ignored. Since distance driven tends to rise with income, the rich would pay more with road tolls, particularly for non-work related trips, if charges are levied at off-peak hours. However, the rich benefit more from road investments. And tolls tend to reduce travel by those with low values of time, and encourage travel by those who value congestion relief highly, making the welfare impact more likely to be regressive. The effect of demand elasticity has been considered in static models by Layard (1977), Glazer (1981) and Niskanen (1987).

[11]The linear travel cost function employed here and elsewhere in the literature is not based on a production technology. A model incorporating workplace interactions is needed to explain why employers prefer employees to keep the same work hours, and to calculate the benefits of flextime or staggering of work hours. As Henderson (1981) noted, employers prefer their most productive and interactive workers to travel in the middle of the rush hour. Neglect of this consideration could lead to poor predictions as to which groups of commuters travel at the rush-hour peak.

they have paid little attention to considerations of political acceptability and have been content to let others worry about the practical details of implementation. If they pursue the research program sketched above with diligence, intelligence and sensitivity, they should be able to mount a far stronger case.[12]

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[12]We have limited consideration throughout to road pricing on public roads. Privately-operated toll roads are common in Europe and the Pacific Rim, and a number are at the planning or construction stages in the U.S. The efficiency and welfare distributional effects of private-sector road pricing have been the subject of some study (*e.g.* Vickrey (1968), Edelson (1971), Nichols *et al* (1971), DeVany and Saving (1980), Poole (1988)) but have yet to be considered in a dynamic equilibrium framework.

Table 1

Welfare Effects of the Optimal Time-Varying Toll
on Two Groups of Commuters
with Different α and β but the Same γ/β and t^*

+ means better off, − means worse off, 0: no change

(a) No rebate of toll revenue		
	Order of departure preserved ($\beta_2>\beta_1$)	Order of departure reversed ($\beta_2<\beta_1$)
Group 1	0	+
Group 2	− if $\alpha_2<\alpha_1$ + if $\alpha_2>\alpha_1$	−

(b) Equal per-capita lump-sum rebate of toll revenue		
	Order of departure preserved ($\beta_2>\beta_1$)	Order of departure reversed ($\beta_2<\beta_1$)
Group 1	+	+
Group 2	+ unless f_2 and α_2/α_1 both small	+ unless f_2 and α_2/α_1 both small

Table 2

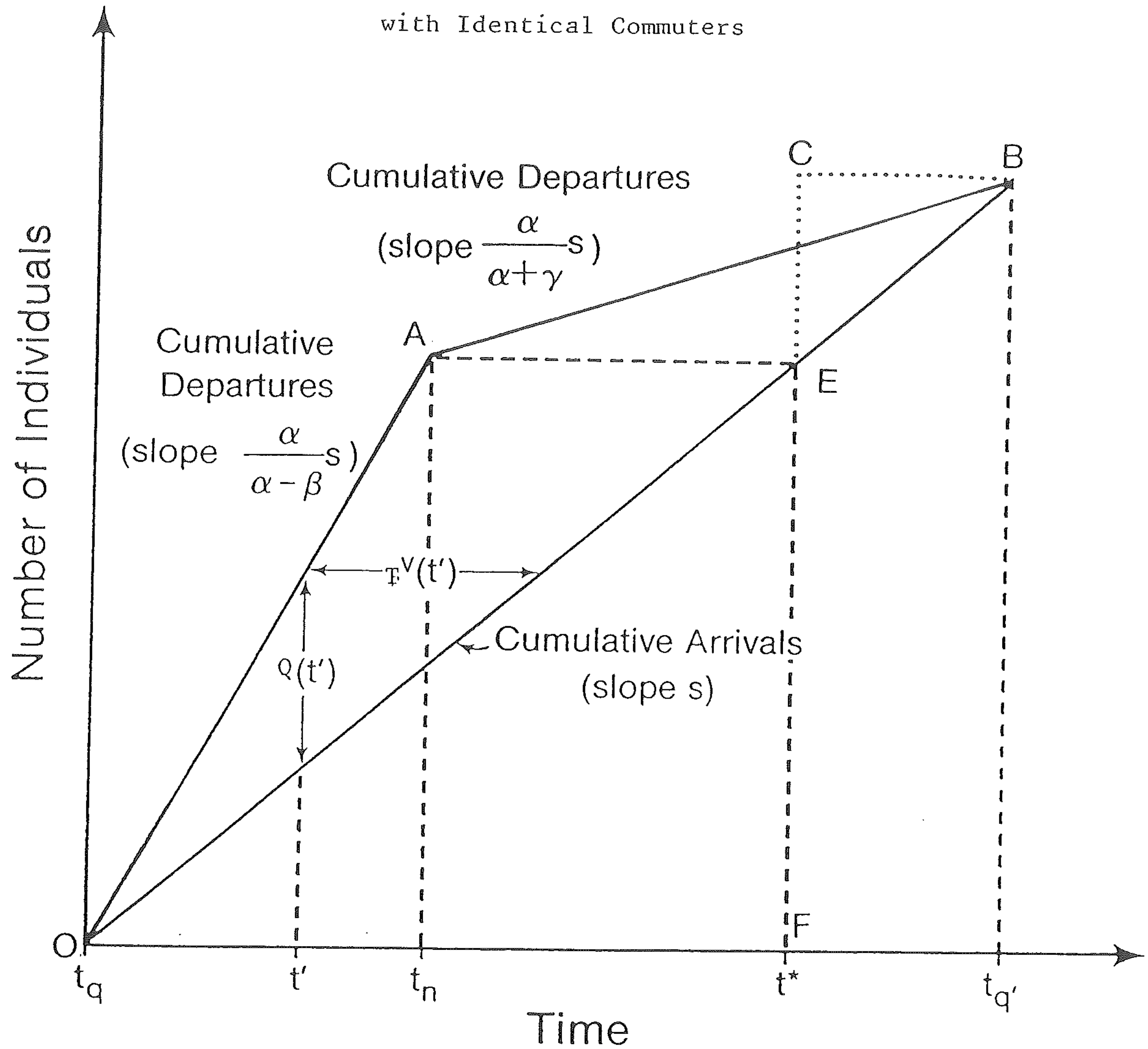
Distributional Effects of a Time-Varying Toll

and Capacity Expansion on Two Groups

Variable Parameters	Time-varying toll without rebate	Time-varying toll with lump-sum rebate	Capacity expansion
None (Homogeneous commuters)	Neutral	Everyone benefits equally	Everyone benefits equally
α and β (γ/β fixed)	Typically favours drivers travelling in tails of rush hour	Typically favours drivers travelling in tails of rush hour	Typically favours drivers travelling in tails of rush hour
γ/β	Neutral	Everyone benefits equally	Everyone benefits equally or favours drivers travelling at peak
t^*	Neutral	Everyone benefits equally	Everyone benefits equally or favours drivers travelling at peak

FIGURE 1

Cumulative Departures and Arrivals, Queue Length, Total Travel Time, Total Time Early and Total Time Late in No-Toll Equilibrium with Identical Commuters



Total Travel Time = Area OABO
Total Time Early = Area OEFO
Total Time Late = Area BECB

FIGURE 2

No-Toll Equilibrium with Three Groups of Commuters
with Different α and β but the Same γ/β and t^*

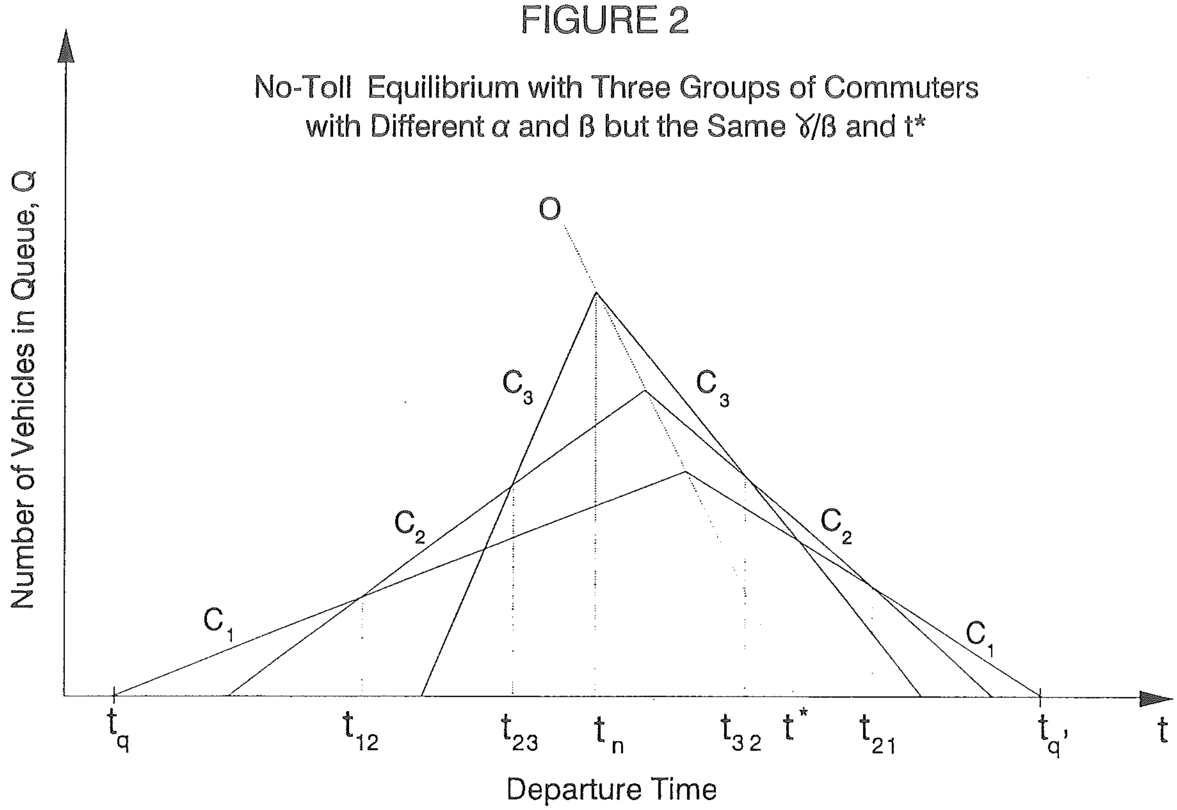


FIGURE 3

Optimal Time-Varying Toll for Three Groups of Commuters
With Different α and β but the Same γ/β and t*

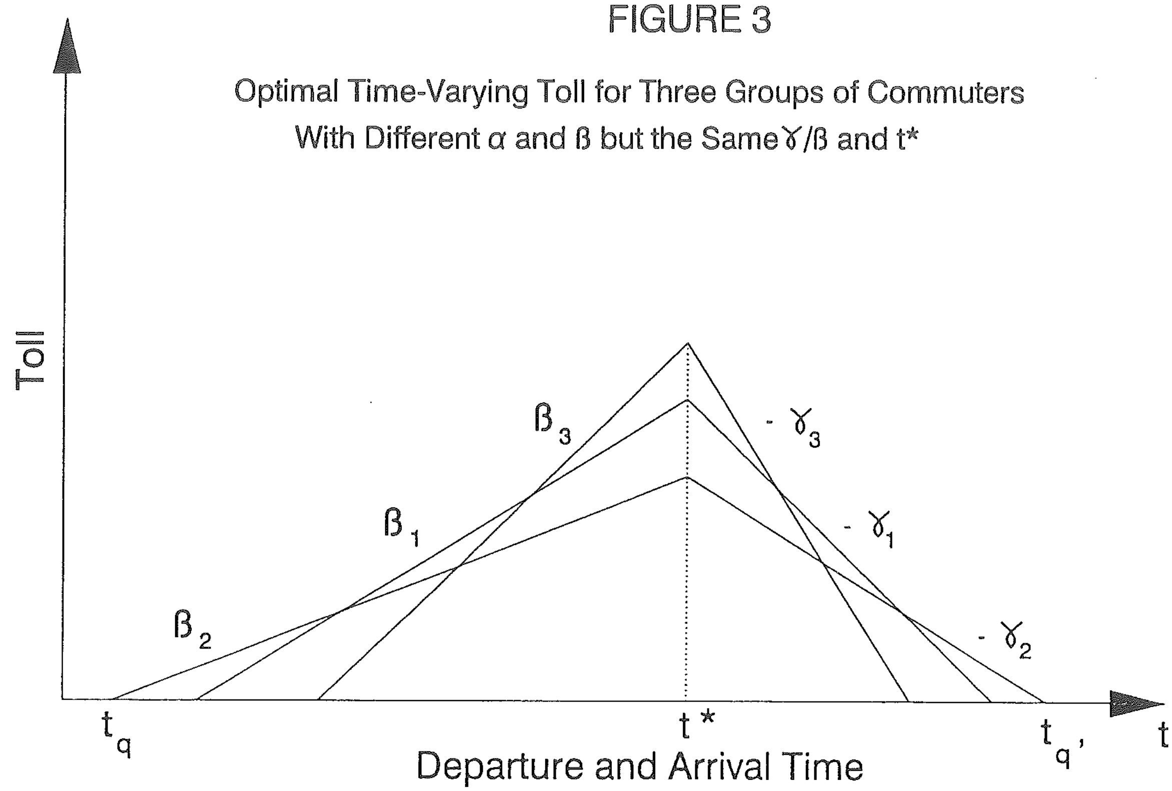
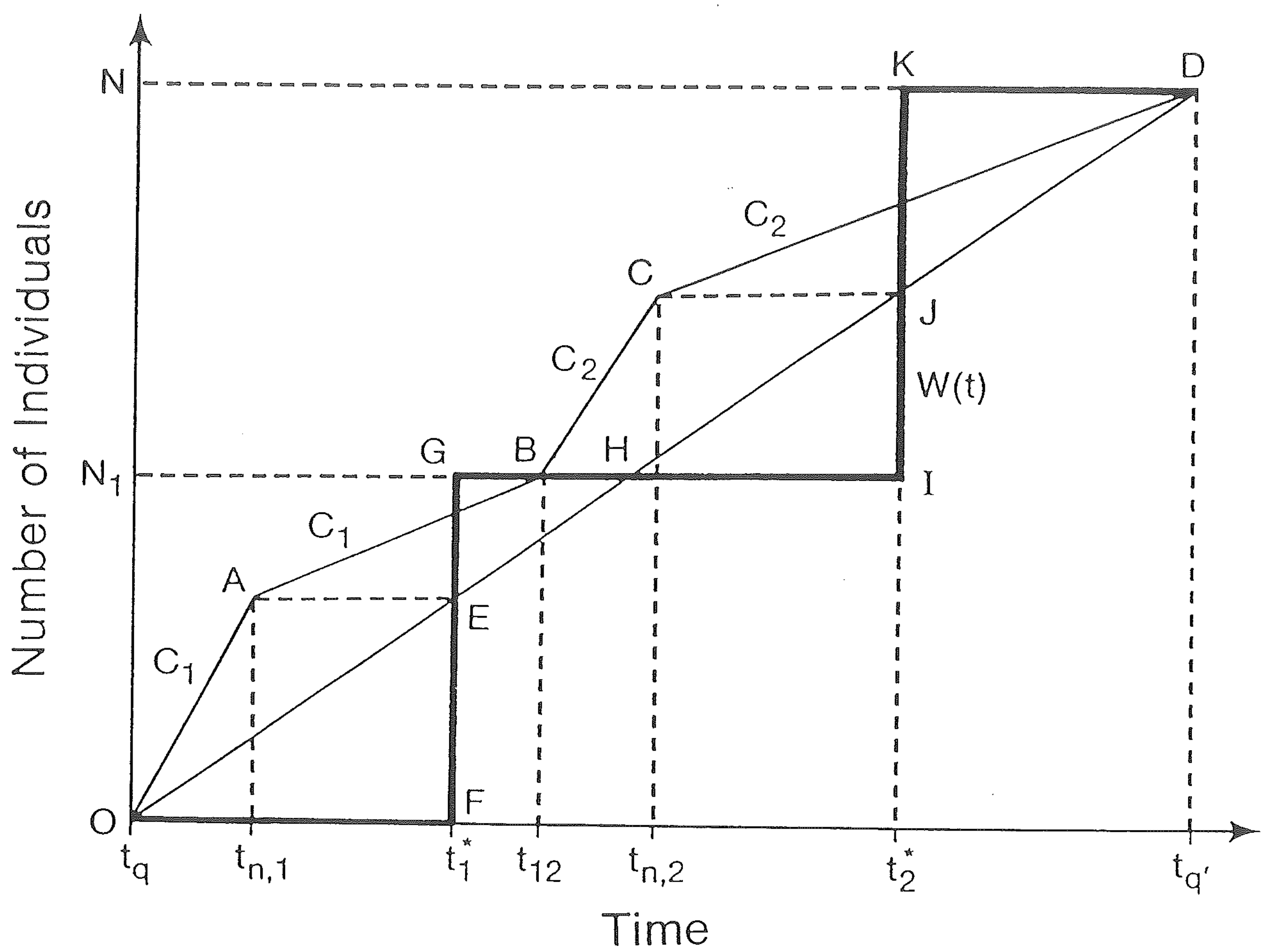


FIGURE 4

Equilibrium for Two Groups of Commuters
with Different Desired Arrival Times
Queue Double-Peaked



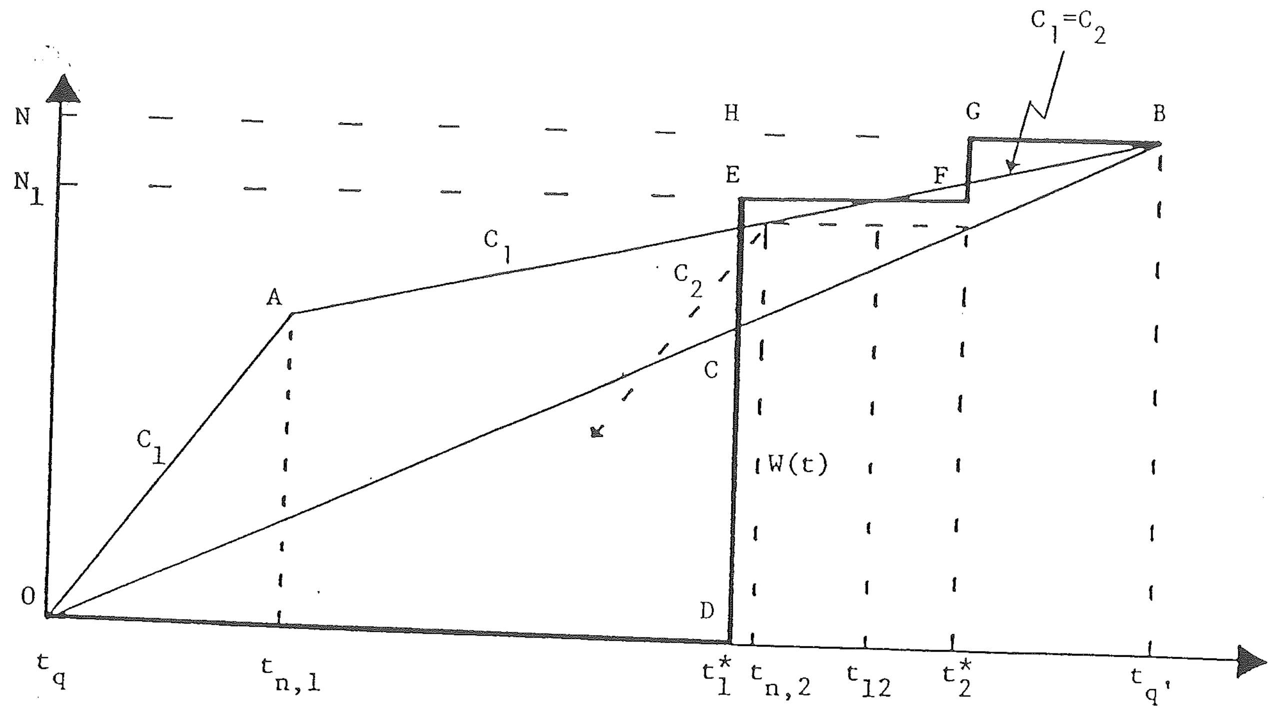
Total Travel Time = Area OABCDO
Total Time Early = Area OEFO + HJIH
Total Time Late = Area EGHE + JKDJ

FIGURE 5

Equilibrium for Two Groups of Commuters

with Different Desired Arrival Times

Single Peak with all Group 2 Arriving Late



Total Travel Time = OABO

Total Time Early = OCDO

Total Time Late = BCEFGB

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