Technological Progess in a Model
of the Housing–Land Cycle

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by

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I. Introduction

In a pair of seminal papers, Sweeney [1974 a,b] provided a new way of conceptualizing a competitive housing market -- as a hierarchy of quality submarkets, in which durable housing is constructed at high quality and filters down the quality hierarchy at a speed depending on the level of maintenance. Sweeney's model was restrictive in four important respects : i) it solved only for the stationary state; ii) it ignored land; iii) it assumed housing unit size to be fixed; and iv) it assumed that housing is doomed to deteriorate in quality. In the culminating paper of a series, Arnott, Braid, Davidson and Pines [1988] relaxed ii), iii) and iv), but at the cost of treating only a single household group, in contrast to Sweeney who allowed for a continuum of households by income. The complexity of that paper suggests that full <u>analytical</u> development of the Sweeney model will be very difficult. Sweeney's vision of the housing market (without the restrictive assumptions) is appealing and intuitive. Durability and quality differentiation of the housing stock are essential features of the market, and have together been treated satisfactorily only in models in the Sweeney tradition.

It was with these considerations in mind that, several years ago, we embarked on the development of a series of <u>simulation</u> models that would move development of the Sweeney model forward to the point where it would be a useful policy tool. The first of these models is presented in Anas and Arnott [1991]. That model improves on previous analytical work in two important respects. It not amortized construction costs, but amortized construction plus land costs.

Intuition concerning the paradox can be gleaned from the following model, which though non-stochastic is similar to the one we shall present. Consider a stationary housing market with a fixed supply of homogeneous land, a fixed population, and a single type of housing (of fixed quality and structural density). The housing unit size chosen by households is inversely related to rent, and the lifetime of a building is directly proportional to the level of maintenance. If construction costs are high relative to maintenance, the rate of demolition and reconstruction is low -- it is most profitable to maintain buildings so that they last a long time and to demolish and reconstruct only infrequently. Assume, as is realistic, that demolition and reconstruction take time. If construction costs are decreased, the optimal level of maintenance falls so that the rates of demolition and reconstruction increase. But this results in a higher proportion of land being vacant and consequently a reduced housing stock, which, with a fixed population, implies a higher market-clearing rent. Put alternatively, a fall in construction costs may increase land prices sufficiently that even though amortized construction costs fall, the long-run rent of housing, which equals the sum of amortized construction plus land costs, rises. In view of these results, we must change the "back-of-the-envelope" way we think about the operation of the housing market to incorporate endogenous land price.

accordingly. On the demand side, households have idiosyncratic tastes and the city is open -- the marginal resident is indifferent between living in the city or outside, where an exogenous outside utility is obtained.[2] The higher is housing rent, the smaller is the city's population. Housing unit size is fixed. In each time period, rent adjusts so that the population equals the stock of housing. Land and housing values evolve such that investor-builders are indifferent between investing in housing or land and a riskless outside asset yielding the interest rate.[3]

We now turn to the detailed description of the model. We describe the model as non-stationary, and then we impose the stationarity conditions. We assume that in a stationary environment of the exogenous parameters, the stationary state is approached asymptotically.

II.1 Supply

Investors face uncertainty in construction costs. The idiosyncratic costs affecting an investor's construction or demolition options are i.i.d. across investors of the same type and serially uncorrelated. These assumptions imply that an investor's decisions in period t are unaffected by his investment history. Each period's asset markets are open at the beginning of the period, <u>before</u> that period's uncertainty is realized. Housing is

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[2] The "outside utility" need not be interpreted as the utility level obtained at alternative geographic locations. It can also be interpreted as : i) the exogenous utility of homelessness, or ii) the exogenous utility of non-householdship.

[3] In the Anas-Arnott model, investors' expected returns equal the after-tax interest rate. In the stripped-down version the income and property tax rates have been set to zero. Hence, investors'expected returns equal the real rate of interest.

Investor utilities (U) are measured at the beginning of each year, and are the sum of two parts : i) the deterministic profit (Π) received from renting and conversion, and ii) the random idiosyncratic nonfinancial cost (c) associated with the activity.[4] The activity set for conversions is (k,k') ={(L,L),(H,H),(L,H),(H,L)}. Hence,

$$U_{kk',t} = \Pi_{kk',t} - c_{kk',t} \tag{1}$$

The c.d.f. of each of the idiosyncratic nonfinancial costs is the following double exponential distribution :

$$G_{kk',t}(c_{kk',t}<z) = \exp\{-\exp[-\Phi_{kk',t}(z + g/\Phi_{kk',t})]\} \tag{2}$$

where $c_{kk',t}$ stands for any random realization of an idiosyncratic cost, $E[c_{kk',t}]=0$, $Var[c_{kk',t}]=\pi^2/6(\Phi_{kk',t})^2$, $Cov[c_{kk',t}, c_{ss',z}]= 0$ for all $kk',t \neq ss',z$. g=0.5772 is Euler's constant, and $\Phi_{kk',t}$, which is inversely related to the variance of costs, is the "dispersion parameter" or "heterogeneity coefficient". As $\Phi_{kk',t} \to \infty$, cost uncertainty vanishes and all investors who purchased k choose the k' with the highest level of non-idiosyncratic utility. Thus, heterogeneity in investors' choices also vanishes. As, $\Phi_{kk',t} \to 0$, cost outcomes are perceived as extremely uncertain, all activities become equiprobable and choices are extremely heterogeneous. Under the assumption that the idiosyncratic costs are i.i.d. for all construction options and are serially uncorrelated, $\Phi_{Hk',t} = \Phi_H$ for k'=H,L and for all t. Thus, Φ_H is the

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[4] In the absence of taxes, it makes no essential difference whether the idiosyncratic component of costs is financial or nonfinancial. Both the idiosyncratic and systematic component of costs are incurred at the end of the year. We measure the idiosyncratic costs in beginning-of-period (discounted) dollars and the systematic costs in end-of-period (undiscounted) dollars. These measurement conventions are arbitrary.

realizing each activity.

We now specify the deterministic part of the investor's conversion utility :

$$\Pi_{kk',t} = a(V_{k',t+1}-C_{kk',t}) - V_{k,t} , \qquad (7)$$

where r is the real rate of interest (inflation is zero) and $a \equiv 1/(1+r)$ is the discount factor. The first term in (7) is the year-end sale price of the converted asset $V_{k',t+1}$, less the year-end systematic financial conversion cost, $C_{kk',t}$, discounted to the beginning of the year. The second term is the purchase price and equals the sunk cost associated with an asset and does not influence the conversion decision on that asset. Appropriately then, the purchase price cancels out of the multinomial logit choice probabilities given by (6). These become

$$Q_{kk',t} = \exp a\Phi_k(V_{k',t+1}-C_{kk',t}) \;/\; \Sigma_{s\epsilon(L,H)}\exp a\Phi_k(V_{s,t+1}-C_{ks,t}) \qquad (8)^5$$

II.2 Asset Price Determination

In a competitive asset market, the price of each asset is bid up until the expected rate of return generated by that asset equals the real interest rate. Because investors owning each asset of the same type are ex ante identical at the beginning of each year, this bidding process results in identical asset prices for each asset in the same class. To find the equilibrium asset bid-prices for land and housing, we substitute (7) for land and housing into (4) and (4') respectively. We then set these equal to zero and

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[5] If the idiosyncratic costs were financial, they would have to be discounted by the interest rate r. The variance of a financial idiosyncratic cost would be a times the variance of a non-financial idiosyncratic cost and the cost dispersion parameters would be $(1/a)\Phi_H$ and $(1/a)\Phi_L$ respectively. The conversion probabilities would be as given in (8) but without the coefficient a.

interpretation is that housing is subject to random "breakdown". If the breakdown is sufficiently costly to repair, the (expected) least-cost alternative is to demolish the housing unit and build anew.

II.3 Demand

The (aggregate) demand for housing is a function of the rent for housing and the utility level, U_o, that can be obtained by renting housing elsewhere. Let this demand function be $D(R_H,U_o)$ with $\partial D/\partial R_H < 0$ and $\partial D/\partial U_o < 0$. Then, the demand relation satisfies strict gross substitution. In the Anas-Arnott model, adapted to our context, housing units are of fixed unit size and the probability that a household rents a unit is [6]

$$P = \exp \alpha(y- R_H)/[\exp \alpha(y-R_H) + \exp U_o], \tag{12}$$

where y is household income. Demand is D = NP, where N is the exogenous number of households in the relevant population. This model is derived from the assumption that households' utilities contain additive idiosyncratic utilities u_o and u respectively, such that each household chooses the maximum of U_o+u_o and $\alpha(y-R_H)+u$ depending on the realized values of u_o and u for that household. The expected realized utility level is [7]

$$W = E[\ \max\ (U_o+u_o,\ \alpha(y-R_H)+u)\ ]. \tag{12'}$$

Imposing a distributional assumption analogous to (2), W becomes

[6] Although we use the logit model to represent demand side behavior, all of the analytical results of this paper are not specific to the logit model but require only that the demand function satisfy strict gross substitution.

[7] We do not review the derivation of the logit choice probabilities, since this is well known. See, for example, Domencich and McFadden (1974).

is conditional on the following exogenous variables and parameters : the outside utility level, U_o; the land rent, R_L; the total land supply, L; the population of households, N; conversion costs, C_{LH} and C_{HL}; the cost heterogeneity coefficients, Φ_L and Φ_H; the interest rate, r; the utility coefficient, α; and household income, y.

The equilibrium system of equations can be simplified somewhat by simultaneous solution of (14) and (15). This gives the supply function of housing :

$$S_H = L\ (1 + Q_{HL}/Q_{LH})^{-1}\ . \tag{16}$$

Since the demolition and construction probabilities are functions of the asset prices according to (15') and (15'') and since, from the asset bid-price equations (10) and (11), the asset prices are implicit functions of the housing rent, equation (16) can be viewed as the supply of housing as a function of rent. We note that since the probabilities are each strictly positive, the stock of housing is always strictly positive as well, and from (15), so is the stock of vacant land. It will be shown in the course of proving Theorem 2 below that housing supply is an upward sloping function of rent.

III. Existence and Uniqueness of Equilibrium

In proving the existence of an equilibrium, we will first show that an equilibrium exists with rent and the two asset prices unrestricted in sign. We will then show that, with a sufficiently high population, the positivity of the rent and asset prices can be ensured. This is done in two steps. Theorem 1 proves that unique asset prices exist given any value for rent. Theorem 2 proves the existence and uniqueness of a stationary equilibrium in which the

To prove existence of the equilibrium R_H, we note that : (i) as $R_H \to -\infty$ (for any finite U_o), $P \to 1$ and $D(R_H,U_o) \to N$. Since $N > L$, as $R_H \to -\infty$, the left side of (17) is positive. (ii) as $R_H \to +\infty$, $P \to 0$ and since $1 + Q_{HL}/Q_{LH}$ remains finite, the left side of (17) is negative. Hence, because the left side of (17) changes sign in $-\infty < R_H < +\infty$, an equilibrium exists. Uniqueness is proved by showing that the left side of (17), denoted by LS, is negatively sloped :

$$d(LS)/dR_H = \partial D/\partial R_H + L[(1+(Q_{HL}/Q_{LH})]^{-2}[-Q_{HL}Q_{LH}^{-2}(dQ_{LH}/dR_H)+Q_{LH}^{-1}(dQ_{HL}/dR_H)].$$

From total differentiation of the choice probabilities (15) and (15') we find that

$$dQ_{LH}/dR_H = a\Phi_L\ Q_{LH}\ Q_{LL}\ [dV_H/dR_H - dV_L/dR_H],$$

and

$$dQ_{HL}/dR_H = -a\Phi_H\ Q_{HL}\ Q_{HH}\ [dV_H/dR_H - dV_L/dR_H].$$

Totally differentiating (10) and (11) we can find

$$dV_H/dR_H - dV_L/dR_H = (1+r)/(Q_{HL} + Q_{LH}) > 0.$$

Hence, $dQ_{LH}/dR_H > 0$ and $dQ_{HL}/dR_H < 0$. Therefore, $d(LS)/dR_H < 0$, throughout the range $-\infty < R_H < +\infty$. It follows that the equilibrium rent is unique and, by Theorem 1, the two asset prices are uniquely determined from the equilibrium rent. Hence, the equilibrium housing stock is uniquely determined from (16) and the stock of land is uniquely determined from (15).

b) Next, to prove that the rent and the asset prices can be made positive by choice of sufficiently large N/L, we note from (17) that by varying N while keeping L constant, any level of market-clearing rent can be obtained. Similarly, V_H and V_L can be made arbitrarily large by raising rent. It follows that with

$$\underset{\{R_{H,t},V_{L,t},V_{H,t},S_{L,t},S_{H,t}\}}{\text{Maximize}} \quad Z= \Sigma_{t=0,\infty}\ N\ E\ \{\max[U_o+u_o, \alpha(y-R_{H,t})+u_1]\}a_t$$

$$+ \Sigma_{t=0,\infty}\ S_{H,t}\ \{\ E[\ \text{Max}(U_{HL,t},\ U_{HH,t})\ ]+R_{H,t}\ -V_{H,t}-T_t\}a_t$$

$$+ \Sigma_{t=0,\infty}\ S_{L,t}\ \{\ E[\ \text{Max}(U_{LH,t},\ U_{LL,t})\ ]+R_{L,t}\ -V_{L,t}-T_t\}a_t$$

subject to :

$$S_{L,t}+S_{H,t}-L \leq 0 \text{ for } t=0,1,2,\ldots,\infty.$$

The coefficient $a_t \equiv (1+r)^{-t}$ is the discount factor for $t > 0$ (a_0 =1). The first summation gives the present value of aggregate expected household welfare. The second summation is the value of aggregate expected profits accruing to housing investors, net of an annual rental per unit of housing (or land), T_t, to be paid to the original owners of land. The third summation is equivalent to the second, but for a land investor. The constraints state that the amount of vacant and built-up land in each period cannot exceed the total available amount, L. The planner's decision variables are the rent $R_{H,t}$, the asset prices $V_{H,t}$ and $V_{L,t}$, and the stocks $S_{H,t}$ and $S_{L,t}$ for each time t. We form the Lagrangean of the above problem with Lagrangean multipliers, $\lambda_t a_t$, for the marginal social benefit of land, and we substitute in the appropriate expressions for the expected values. We obtain

$$Z' = \Sigma_{t=0,\infty}\ N\ (1/\alpha)\ln[\exp U_o + \exp \alpha(y-R_{H,t})]a_t$$
$$+ \Sigma_{t=0,\infty}\ S_{H,t}\{(1/\Phi_H)\ln[\exp a\Phi_H V_{H,t+1} + \exp a\Phi_{H,t}(V_{L,t+1}-C_{HL})]+R_{H,t}\ -V_{H,t}-T_t\}a_t$$
$$+ \Sigma_{t=0,\infty}\ S_{L,t}\{(1/\Phi_L)\ln[\exp a\Phi_L V_{L,t+1} + \exp a\Phi_{L,t}(V_{H,t+1}-C_{LH})]+R_{L,t}\ -V_{L,t}-T_t\}a_t$$
$$+ \Sigma_{t=0,\infty}\ \lambda_t(S_{H,t}+S_{L,t}-L)a_t.$$

Maximizing with respect to $R_{H,t}$ we get $-NP_t + S_{H,t} = 0$ which is the market-clearing equilibrium condition given by equation (13) the two adjustment equations is reduced to (14) which states that, at stationary equilibrium, the conversions from housing to land must equal the conversions from land to housing.

V. Comparative Statics : Construction Cost

We now turn to the comparative statics analysis of the stationary equilibrium. In this section, we report on the effects of changes in construction cost, C_{LH}, and in the next section, on the effects of changes in demolition cost, C_{HL}. The Jacobian matrix of the equation system (10), (11) and (13)-(15), the partial derivatives with respect to exogenous variables and the signing of the Jacobian's determinant are in Appendix 1. The determinant (hereafter, A) is always negative.

Differentiation with respect to the construction cost gives

$$dV_L/dC_{LH} = A^{-1}a^2[-(\partial D/\partial R_H)Q_{LH}(Q_{HL}+Q_{LH})(Q_{HL}+r) + S_H\Phi_H Q_{HL}Q_{HH}Q_{LH}] < 0 \quad (18)$$

$$dV_H/dC_{LH} = A^{-1}a^2\{-(\partial D/\partial R_H)Q_{LH}Q_{HL}(Q_{HL}+Q_{LH}) + S_H Q_{HL}[\Phi_H Q_{LH}Q_{HH} - \Phi_L Q_{LL} r]\} \quad (19)$$

$$dR_H/dC_{LH} = A^{-1}a^3 r S_H Q_{HL}\ [\Phi_H Q_{HH} - (1+r)\Phi_L Q_{LL} + Q_{HH}Q_{LL}(\Phi_L - \Phi_H)] \quad (20)$$

$$dS_H/dC_{LH} = A^{-1}a^3 r(\partial D/\partial R_H)S_H Q_{HL}\ [\Phi_H Q_{HH} - (1+r)\Phi_L Q_{LL} + Q_{HH}Q_{LL}(\Phi_L - \Phi_H)] \quad (21)$$

$$dS_L/dC_{LH} = -dS_H/dC_{LH}. \quad (22)$$

The last four expressions are unrestricted in sign. Hence, a reduction in construction cost always increases land price but can either increase or decrease rent, housing price and the housing stock. Housing rent and stock move in opposite directions, since

$$dR_H/dC_{LH} = (\partial D/\partial R_H)^{-1}\ dS_H/dC_{LH}\ ,$$

where $(\partial D/\partial R_H) < 0$. Three distinct cases are implied from the above conditions and these are shown in Table 1(a).

Cases 1 and 2 correspond to the intuitively expected results

Second, consider the Sweeney model [1974 a,b] with a distribution of household types by income. It too ignores land but it incorporates quality differentiation and provides a rigorous treatment of maintenance and durability. The analysis is of a stationary state and the maintenance technology is such that housing always deteriorates. Assume the normal case in which construction occurs only at higher qualities. Then there are two quality intervals. Over the higher quality interval where construction occurs, rent is supply-determined, while over the lower quality interval it is demand-determined. Over the higher quality interval, housing value falls by the reduction in construction costs, and housing rents fall accordingly. Over the lower quality interval, the reduced construction costs typically result in lower rents.[11] With identical households, utility must either rise, fall or remain the same. If it were to remain the same, the equilibrium bid-rent curve (by quality) would be unaltered, and if it were to fall, the equilibrium bid-rent curve would shift up. In either case, landlords would make a profit by following the pre-reduction maintenance program, which is inconsistent with equilibrium. Hence, rents must fall at all quality levels, and therefore housing values fall too.

There is another branch of the literature which is relevant--that on general equilibrium incidence in the Harberger-Mieszkowski-Jones tradition, particularly the subset of that literature dealing

[11] What Sweeney [1974,Therem 10] shows is that there exists a construction subsidy program (which corresponds to a particular form of technological improvement in construction) that reduces rent at all quality levels.

decline as in case 2). (24) says that the decrease in the differential price is always less than the exogenous drop in the cost of construction. Condition (24) implies that the construction probability (15') increases. (23) implies that the demolition probability (15'') increases. From the supply function given by (16), what happens to the stock of housing depends on which probability increases proportionally more. If the demolition probability increases proportionally more than the construction probability, then the stock of housing decreases. If the construction probability increases more, then the stock of housing increases.

The paradox that a fall in construction costs may lead to a rise in housing rents is now resolved. Since rents are negatively related to the stock of housing, a fall in construction costs leads to a rise in rents when it leads to a fall in the stock of housing. The stock of housing, in turn, is directly related to the ratio Q_{LH}/Q_{HL} ; the faster the conversion of land to housing and the slower the conversion of housing to land, the larger the stock of housing. Thus, <u>the stock of housing falls if Q_{HL} increases proportionally more than Q_{LH}</u>. From (20) or (21) this condition reduces to

$$\Phi_H Q_{HH} -(1+r)\Phi_L Q_{LL} +Q_{HH}Q_{LL}(\Phi_L -\Phi_H) > 0.$$

This is not an intuitive condition. It does, however, permit identification of a limiting case where the paradox occurs. Suppose that $\Phi_L=0$. Then, $Q_{LL}=Q_{LH}=0.5$; the dispersion among housing investors in the costs of converting from land to housing is so large that in each period one-half of all land is converted to housing. Now, by shown in Table 1(b).

A fall in demolition costs widens the difference between housing and land asset prices, but by less than the fall in demolition costs :

$$d(V_L - V_H)/dC_{HL} = Q_{HL} ra^2 A^{-1} [(\partial D/\partial R_H)(Q_{LH}+Q_{HL}) - \Phi_H S_H Q_{HH}] > 0; \qquad (30)$$

$$d(V_L - V_H - C_{HL})/dC_{HL} = - Q_{LH} ra^2 A^{-1} [(\partial D/\partial R_H)(Q_{LH}+Q_{HL}) - \Phi_L S_L Q_{LL}] < 0. \qquad (31)$$

A fall in demolition cost therefore has effects which are similar to the fall in construction cost; from the conversion rates given by (15) and (15'), the rates of demolition and construction are both increased. The direction of shift in the supply function again depends on which rate is increased proportionally more.

VII. Concluding Comments

In this paper, we have provided a rather exhaustive analysis of a simple housing model on which our simulation model [Anas-Arnott, 1991] is based. The model has a single type of durable housing, a single type of land, and a single household type.

Our most interesting findings concerned the comparative statics of the model with respect to the average (or systematic) components of construction and demolition costs. Much of the conventional wisdom concerning the effects of housing policies is implicitly or explicitly based on models of the housing market which ignore land. In such models, a technological improvement in housing construction results in a fall in housing rents equal to the amortized reduction in construction costs. In our model, which incorporates a fixed supply of land, such a technological improvement may result in rents falling but by less than the model with several housing quality levels.

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(a)

	dV_L/dC_{LH}	dV_H/dC_{LH}	dR_H/dC_{LH}	dS_H/dC_{LH}	dS_L/dC_{LH}
Case 1	−	−	+	−	+
Case 2	−	+	+	−	+
Case 3	−	−	−	+	−

(b)

	dV_L/dC_{HL}	dV_H/dC_{HL}	dR_H/dC_{HL}	dS_H/dC_{HL}	dS_L/dC_{HL}
Case 1	−	−	+	−	+
Case 2	−	−	−	+	−

<u>TABLE</u> <u>1</u> : The cases established by the comparative statics analysis. (a: the effect of construction cost; b: the effect of demolition cost)