"A Model of Hyperinflation"

by

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A Model of Hyperinflation

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(First draft; comments solicited)

Abstract

Standard models of hyperinflation use a money demand function based on asset-market considerations: households adjust their real balances according to expected inflation, which is the negative of the real rate of return to money. But these models yield inaccurate and sometimes counterintuitive predictions. One is that if a hyperinflation is a price-level bubble, then hyperinflation is possible at any rate of money growth. Another is that, for some equilibria, an increase in a government's reliance on seigniorage reduces rather than raises the steady-state inflation rate.

This paper proposes an alternative way to look at hyperinflation based on a careful description of the microeconomics of monetary exchange. Money is primarily an institution required to finance consumption and only incidentally a financial asset. The decision to accept money is a decision to engage in monetary exchange, and a hyperinflation occurs when most households choose to abstain from monetary exchange. The macroeconomic implications of this model are more appealing than those of the traditional models. First, while a hyperinflation in my model may have the same properties as a price-level bubble, this "bubble" is very unlikely when money growth is low and inevitable when money grows too quickly. Second, a greater reliance on seigniorage increases the rate of inflation, and can ultimately cause a hyperinflation. The model also mimics the non-monotonic path of real balances as inflation accelerates. Finally, the model suggest that it may be very difficult to restore a currency's place in exchange after a hyperinflation.

Introduction

Standard rational-expectations models of hyperinflation[1] suffer from two important shortcomings. First, they imply that hyperinflation, if it is a price-level bubble, is possible at any rate of money growth. This is because monetary equilibria tend to be unstable, so small perturbations can send the economy along explosive hyperinflationary (or hyperdeflationary) paths. The evidence is mixed on whether hyperinflations are bubbles. For example, Flood and Garber (1980) reject the hypothesis that the German hyperinflation of the early 1920s was a bubble, whereas Burmeister and Wall (1987) come the opposite conclusion. What is clear, however, is that hyperinflations are associated with high rates of money growth[2] and that countries with low and stable money growth rates do not experience hyperinflations.

One could rule out hyperinflationary bubbles by looking only at stable monetary equilibria. But this brings on a second problem with the standard models: their stable equilibria often imply that a greater seigniorage requirement lowers inflation and raises real balances. In fact, however, hyperinflation is generally associated with, and usually caused by, a ballooning fiscal deficit.[3] As for real balances, the historical record suggests that as inflation accelerates, real balances at first rise but later fall to very low levels.[4]

At the heart of these problems is that money demand is based on asset-market considerations: the demand for real balances is determined by the expected future return of this asset, that is, the negative of expected inflation. If there were excess demand for real balances because the money supply dropped or the price level jumped without warning, the expected return to real balances would have to fall to restore equilibrium—expected inflation would have to rise. But higher inflation would make real balances fall even further (if money growth is constant), exacerbating the excess demand and

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[1] e.g., Sidrauski (1967), Brock (1975), and the many papers following Cagan (1956) but assuming rational expectations.

[2] Consider the money supply data reported in Sargent (1982).

[3] See Sargent (1982).

[4] See Bernholz, Gartner and Heri (1985) and data on real balances reported in Barro (1970).

requiring even higher expected inflation. Any small perturbation that throws the money market out of equilibrium can ignite this process, leading to a hyperinflation (or hyperdeflation).[5]

It is unlikely, however, that asset-market dynamics dominate during a hyperinflation. Most people are trying hard to keep their cash balances to a minimum, and have long stopped thinking of money as an asset worthy of the cost-benefit analysis they apply to other assets in their portfolio. Money is losing value so quickly that the only reason people hold it is because of an institutional constraint requiring them to use cash to purchase consumption goods. Were this constraint to disappear (because of, say, currency substitution), people would no longer hold the depreciating currency, and it would lose all value.

The present paper captures this logic formally. An agent's decision to hold money is based on money's expected usefulness financing consumption, not on its expected return as an asset. Thus, the decision is binary: use or do not use money. A hyperinflation, in the simple set-up presented below, occurs when agents decide to eschew money. Its value falls to zero and the price level tends to infinity.

The model I build, though highly stylized, generates what I think are more plausible predictions than those of the standard models. First, a hyperinflationary bubble is unlikely when money grows modestly and inevitable when money grows too quickly, whereas the standard models say that a bubble is possible at any rate of money growth. Second, a larger seigniorage requirement leads to higher rather than lower inflation when the economy is at its stable monetary equilibrium. Third, accelerating inflation leads first to rising and then falling real balances, a path consistent with many hyperinflation episodes.[6] The model also predicts that it may be very difficult to resurrect a fallen currency after a hyperinflation has occurred.

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[5]See Evans and Yarrow (1981) and Bruno and Fischer (1990) for a more exhaustive discussion of the points in this paragraph.
[6]See Bernholz, Gartner and Heri (1985).

A Review of the Standard Model

Before presenting my model, it is worth demonstrated the attributes of the standard monetary models I have mentioned. These models are typically built on a stable money demand function of the form

$$m_t^d \equiv \left(\frac{M_t}{P_t}\right)^d = L(\pi_t^e) \tag{1}$$

where M_t is the nominal money supply, P_t is the general price level and π_t^e is expected inflation. The standard assumption is that $L'(\cdot) < 0$. For simplicity, assume that the money market is always in equilibrium:

$$m_t = L(\pi_t^e) \tag{2}$$

The assumption of rational expectations in this non-stochastic setting amounts to assuming perfect foresight, that is, that $\pi_t^e = \pi_t$. Combining money-market equilibrium and perfect foresight and differentiating with respect to time yields the condition (suppressing time subscripts)

$$\dot{\pi} = \frac{\mu - \pi}{L'(\cdot)/L(\cdot)} \tag{3}$$

A money supply rule closes the model. Suppose the central bank follows a simple rule specifying a constant money growth rate: $\mu = \bar{\mu}$. In steady state $\dot{\pi} = 0$, which means that $\pi = \bar{\mu}$. This yields a unique steady-state equilibrium level of real money balances, $m^* = L(\bar{\mu})$.

While the equilibrium may be unique, one can see from equation (3) that the equilibrium is locally unstable, for if $\pi > \bar{\mu}$ then $\dot{\pi} > 0$. A small perturbation away from $\pi = \bar{\mu}$ would send the economy along a divergent hyperinflationary (or hyperdeflationary) path. Notice, moreover, that such a speculative bubble is possible regardless of the rate of money growth. There is nothing in this model that suggests that a hyperinflationary bubble is more likely for high values of $\bar{\mu}$ than for low ones. Thus, if one is to insist that a hyperinflation is in fact a price-level bubble, one must contend with the prediction from the same set of models that such a hyperinflation is possible at any rate of money growth.[7]

Suppose that the government operates a different money supply rule, one that specifies that the government raises a fixed amount of real seigniorage. That is, assume that $\mu \cdot m = S$, where S represents the government's real seigniorage requirement. Taking into account money demand, this means that $\mu \cdot L(\mu) = S$. This admits the possibility of multiple equilibria, since an increase in the seigniorage requirement could be met by raising or lowering money growth (depending on the functional form of money demand).

Under what conditions would a larger seigniorage requirement increase steady-state money growth (and inflation)? The condition is

(4) $$\frac{d}{d\mu}[\mu L(\mu)] = L(\mu) + \mu L'(\mu) > 0$$

which can be rewritten as a restriction on the interest-elasticity of money demand:

(5) $$\eta = -\frac{\mu L'(\mu)}{L(\mu)} < 1$$

In other words, the steady-state equilibrium at which the elasticity of money demand is less than unity is the one that displays the plausible feature that a higher seigniorage requirement generates a higher rate of inflation.

To analyze the stability of equilibria in this version of the model, incorporate the money supply rule into equation (3) to get

(6) $$\dot{\pi} = \frac{S - \pi \cdot L(\pi)}{L'(\pi)}$$

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[7]A hyperdeflationary bubble can be ruled out by appealing to explicit utility maximization, as Obstfeld and Rogoff (1991) do, and noting that a hyperdeflation must at some point violate a transversality condition.

Stability requires that $d\dot{\pi}/d\pi < 0$. Carrying out the differentiation, this in turn implies that $\eta > 1$. That is, any equilibrium at which a greater seigniorage requirement raises inflation is a locally unstable equilibrium. Locally stable equilibria display a relationship between seigniorage and inflation that seems to contradict reality.

A final point to note is that the assumption that real balances are falling in expected inflation (i.e., that $L'(\cdot) < 0$) guarantees that real balances will fall as a hyperinflation approaches. Absent some unusual assumptions about the form of the money demand function, these models cannot generate the non-monotonic time path of money apparent from many hyperinflationary episodes, namely that real balances first rise and then fall to very low levels.

In sum, standard models used to analyze hyperinflation have a variety of unappealing traits. Locally unstable equilibria are susceptible to perturbations that could set off a hyperinflationary bubble, and these bubbles can arise at any money growth rate, though one might expect them at high rates of monetary expansion and consider them impossible at low ones. Locally stable equilibria, on the other hand, display counterintuitive comparative static results, especially the prediction that a larger fiscal deficit reduces the steady-state inflation rate, though one might expect the opposite. Finally, standard models predict a steady decline in real balances as inflation rises to hyperinflation, while history suggests that real balances at first rise and then fall.

The Model

Building a model around an agent's choice between using and not using money requires a microeconomic perspective and some assumptions about why people would want to hold money. I make two important assumptions. Firsts, I assume that economic institutions are such that money is needed to pay for most consumption goods. A cash-in-advance constraint captures this assumption. This constraint does not force people to use money regardless of its merits, however. They are free to eschew money if they please. It simply means that a decision to forego the use of money is equivalent to a decision to forego consumption. The decision to forego consumption is a painful one, but it need not be fatal if some consumption is possible without monetary exchange. This assumption is implicit in the model.

The second important assumptions is that, with many people buying from and selling to each other, one person's decision to accept money is based on his or her guess about how many others will do the same. It would be senseless for anyone to accept cash if no one else is doing so, for the cash would be useless in exchange. By the same logic, money is more valuable to an individual the more prevalently it is used by others.

These two assumptions are the building blocks of the model that follows.

There is a continuum of households who live infinitely and who are indexed by i. Each household is made up of a consumer and a producer. The producer's responsibility is to make a good, at a linear production cost of ε_i, and sell it. The consumer takes the cash proceeds from the sale and buys consumption goods for the household. As in similar models (e.g., Diamond, 1982, 1984), the household cannot consume the good it produces. This simplification captures how unappealing self-sufficiency is.

A cash-in-advance constraint mandates that goods be sold for cash. This assumption represents the costliness of barter. As mentioned above, it also captures the assumption that at high rates of inflation, agents try to hold cash balances to a minimum, that is, at the level necessary to finance consumption. The model assumes that this minimum is determined exogenously by institutional constraints. A model allowing the possibility of institutional change would better capture reality, but is left out of the present paper.

Interaction between buyers and sellers is marked by a trading friction. Households meet according to a Poisson matching process: at any instant a buyer will meet a seller with probability s, the mean arrival rate. While the model will determine this rate endogenously, physical constraints on trading place an upper bound on it at $\bar{s} < 1$.

Producers face no individual constraint on the amount they can produce when they meet a buyer. However, an aggregate constraint limits total production at any moment to $\overline{Y}_t \equiv 1$. Think of this as a limit on the factors of production available at any moment.

When a buyer and seller meet, the buyer offers his cash holdings to the seller. Utility is linear, so there is no benefit to the buyer of offering anything less then his entire stock of cash if he offers anything at all. If the seller accepts the offer, she then produces the good instantaneously and makes the exchange. This differs from Diamond (1982, 1984) because the production decision need not be made until after a buyer has been found. The buyer consumes the good immediately.

Assume for simplicity that a seller must decide whether or not to accept money before she enters the market, and that this decision is tantamount to a commitment to stay in the market to consummate at least one transaction.[8] Once she has entered the market she will meet buyers randomly. When she meets a buyer, she submits herself to a random drawing from $f(p_{i,t})$, the density function for prices at time t. This is equivalent to a drawing from $h(y_i|M_{i,t})$, where y_i is the individual output level and $M_{i,t}$ is the total cash holdings of the buyer.

At each moment, all potential output is produced and all cash brought to market is spent. This assumption guarantees that nominal output always equals the cash brought to market, which in turn guarantees that $E(p_{i,t}) = E(\tilde{M}_t) = s \cdot M_t$ for all t, where $E(p_{i,t})$ is the expected price level and $\tilde{M}_t$ is total cash brought to market at moment t. (Recall that total output is normalized to unity.)

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[8]This assumption allows me to avoid a rather complicated derivation of optimal search strategies. Both buyers and sellers would operate optimal strategies based on an assumption about the distribution of prices, but this distribution would be endogenous and a function of search strategies on the buy and sell side of the market.

Suppose that the total stock of money grows at rate μ, and is expected to continue growing at this rate indefinitely. By the assumptions made so far, this means that $E(p_{i,t})$ also grows at rate μ. This imposes a restriction on the density function that generates price drawings. In order to satisfy this restriction (that the mean of the distribution grows at rate μ), assume that $p_{j,t} = p_{j,0}e^{\mu t}$ for all possible realizations j of the random variable.

Whether or not money is used in equilibrium depends on whether sellers are willing to accept it. They make this decision before they submit to the market. The seller will accept an offer of M_i if doing so brings an expectation of positive utility. If household utility is linear in consumption and production, she will accept the offer if

$$E\left(\frac{M_i}{p_{j,\tau}}\right) - \varepsilon_i E\left(\frac{M_i}{p_{j,\sigma}}\right) > 0 \tag{7}$$

where $t = 0$ is the moment of entry into the market, $t = \sigma$ is the moment the producer finds a buyer, and $t = \tau$ is the moment the cash from the sale is used to buy the consumption good. Given how the price distribution evolves through time, this condition can be rewritten as

$$E\left(\frac{M_i}{p_{j,0}}e^{-\mu\tau}\right) - \varepsilon_i E\left(\frac{M_i}{p_{j,0}}e^{-\mu\sigma}\right) > 0 \tag{8}$$

The inter-arrival time between meetings is an exponential random variable. Thus, the criterion for accepting money becomes

$$\begin{aligned} E\left(e^{-\mu\tau}\right) - \varepsilon_i E\left(e^{-\mu\sigma}\right) &= \left(\frac{s_i}{s_i + \mu}\right)^2 - \varepsilon_i \frac{s_i}{s_i + \mu} > 0 \\ &\Rightarrow \frac{s_i}{s_i + \mu} - \varepsilon_i > 0 \end{aligned} \tag{9}$$

where $s_i \in S = [0, \bar{s}]$ is household i's conjecture of the mean arrival rate of trading opportunities, a rate that households expect to remain constant into the indefinite future. The mean arrival rate of trading opportunities can be thought of quite naturally as a function of the proportion of all households willing to accept money (and thus looking for trading opportunities). I assume the simplest form for such a function, namely that s is proportional to the fraction of households seeking trades. Thus,

$$(10) \qquad s_i = \bar{s} \cdot \Pr\left[\varepsilon_i < \frac{s_i}{s_i + \mu}\right] = \phi(s_i)$$

which is a mapping $\phi:[0,\bar{s}] \to [0,\bar{s}]$. If one assumes that ε_i is distributed uniformly on $[\varepsilon^{\min},\varepsilon^{\max}]$ with $0 < \varepsilon^{\min} \le \varepsilon^{\max} < 1$, then the mapping is

$$(11) \qquad \phi(s_i) = \begin{cases} \bar{s} \quad \text{if} \quad \dfrac{s_i}{s_i+\mu} \ge \varepsilon^{\max} \\ \dfrac{\bar{s}}{\varepsilon^{\max} - \varepsilon^{\min}}\left(\dfrac{s_i}{s_i+\mu} - \varepsilon^{\min}\right) \quad \text{if} \quad \varepsilon^{\min} \le \dfrac{s_i}{s_i+\mu} \le \varepsilon^{\max} \\ 0 \quad \text{if} \quad \dfrac{s_i}{s_i+\mu} < \varepsilon^{\min} \end{cases}$$

Equation (11) states simply that the mean arrival rate ranges from 0 to $\bar{s}$. It is equal to $\bar{s}$ when all households are willing to accept money. It is equal to 0 when no households are willing to do so. In the intermediate case, the mean arrival rate is determined by the intermediate segment of this mapping, $\sigma(s_i)$. That is,

$$(12) \qquad \sigma(s_i) = \frac{\bar{s}}{\varepsilon^{\max} - \varepsilon^{\min}}\left(\frac{s_i}{s_i+\mu} - \varepsilon^{\min}\right) \quad .$$

One possible version of $\sigma(s_i)$ is shown in Figure 1 (suppressing the subscript i).

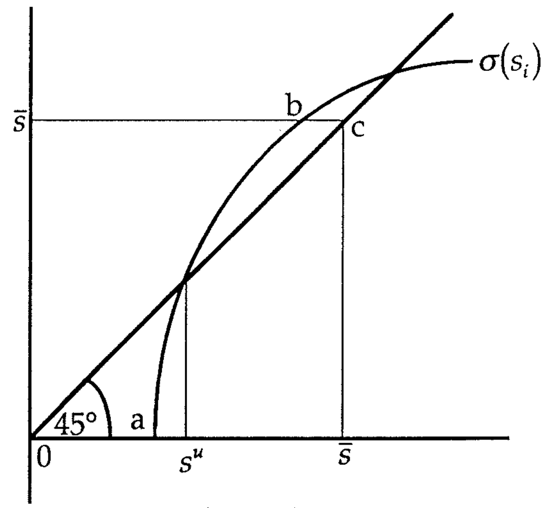

Figure 1

This mapping $\phi(s_i)$ has a few important features. As drawn, $\phi(s_i)$ is the segment **abc** plus the origin. There is no reason, however, why both intersections of $\sigma(s_i)$ and the 45°-line cannot occur below the horizontal line $\bar{s}$. The mapping $\phi(s_i)$ intersects the horizontal axis (i.e., at **a**) at

(13) $$s_i = \mu \cdot \frac{\varepsilon^{\min}}{1-\varepsilon^{\min}}$$

so the intersection is strictly positive as long as $\mu, \varepsilon^{\min} > 0$.

Suppose μ were to increase. Note that $\sigma(s_i)$ is a continuous function and strictly decreasing in μ given any strictly positive value of s_i. When $s_i = 0$, $\sigma(s_i)$ is independent of μ. This means that any point where $\sigma(s_i)$ intersects the 45°-line from above will move closer to the origin as μ rises, whereas any point where the intersection is from below will move away from the origin. The function $\sigma(s_i)$ remains anchored at its y-intercept but becomes straighter and flatter in the right-hand quadrants. Once μ is high enough, $\sigma(s_i)$ no longer crosses the 45°-line inside the interval $[0,\bar{s}]$, if at all.

Assumptions about Conjectures

Identifying equilibria in this economy depends very delicately on the definition of equilibrium. At the heart of this problem is the fact that s_i is a *conjecture* about how prevalently money is being used, so the equilibria in this model depend upon whether and how the conjectures are coordinated across households. One could make several different assumptions at this point. One would be to consider any rational expectations equilibrium path. The set of equilibria would include any steady-state equilibria (be they dynamically stable or unstable) as well as any rational expectations paths that converge to one of the steady states.[9] But this assumption would beg two questions: First, how quickly does the economy converge to a steady state? Second, how does the collective of agents select from among more than one steady state? Alternatively, one could focus only on steady-state equilibria. Yet this still leaves the problem of equilibrium selection.

I limit my attention to fixed points of $\phi(s_i)$, and assume that, absent a hyperinflation, the economy is at the fixed point furthest from the origin. Although this assumption may seem ad hoc, it can be justified with the following argument. Assume that individuals are rational, they all know the true structure of the economy, and these two assumptions are common knowledge. In game theory, these are the assumptions associated with rationalizable equilibria. With these assumptions one can imagine each agent's conjecture converging immediately to a stationary equilibrium as if this convergence were taking place in real time. Each agent reasons as follows: if the mean arrival rate is $s_{i,0}$ then it will induce either entry or exit by other households, so the mean arrival rate adjusts to $s_{i,1} = \phi(s_{i,0})$. But $s_{i,1}$ will, in turn, elicit entry or exit, causing a further adjustment to $s_{i,2} = \phi(s_{i,1}) = \phi^2(s_{i,0})$. This process proceeds in what Guesnerie (1993) calls virtual time until a fixed point $s_{i,\infty}$ of the mapping is reached.

This argument is not enough to guarantee that the conjectures of all agents coincide. The problem is that the mapping $\phi(s_i)$ may have more than one fixed point, so agents initiating their thought process with different values of $s_{i,0}$ may arrive at different conjectures $s_{i,\infty}$. Absent information about the starting conjectures of other agents, each individual household cannot predict how many households, having reached their conclusions

[9]Other rational expectations equilibria exist. See Guesnerie (1993) for a more exhaustive catalogue.

about $s_{i,\infty}$, will decide to accept money and how many will abstain. Therefore, no household can predict what the equilibrium mean arrival rate will be. As a result, each individual considers any value of $s_i \in S = [0, \bar{s}]$ to be a possible stationary state.

One must, therefore, make an additional assumption about individual behavior to resolve this problem: each household builds its conjecture on an initial value for $s_{i,0}$ equal to the actual current value, which households can observe. Furthermore, that households behave this way is common knowledge. One can justify this by thinking of the status quo as a focal point that serves to coordinate expectations in the absence of any other device. Under this assumption, not only will each household arrive at the same conjecture $s_{i,\infty}$, but each household will know that every other household has arrived at the same conjecture about the mean arrival rate. Thus, if at any moment t it happens that $s^u < s_t \leq \bar{s}$ then conjectures will converge immediately to the highest fixed point of $\phi(s_i)$, which I call the stable monetary equilibrium. If $s_t = s^u$, then conjectures will remain at s^u. This is the unstable monetary equilibrium. If $s_t < s^u$ then conjectures about the mean arrival rate will converge to zero, the stable non-monetary equilibrium.

Increase in the Growth Rate of Money

Suppose that the economy is currently at the stable monetary equilibrium, and consider the case in which the government runs a constant money supply growth rate. What is the effect of an unexpected increase in μ, the rate of money growth, in this model?

An increase in the money supply growth rate will push the function $\sigma(s)$ down and make it flatter, as shown in Figure 2. As the figure makes clear, an increase in μ will reduce the proportion of households accepting money if $\sigma(s)$ intersects the 45°-line below $\bar{s}$ (if the intersection is above $\bar{s}$ then the rise in μ will have no effect on the mean arrival rate).

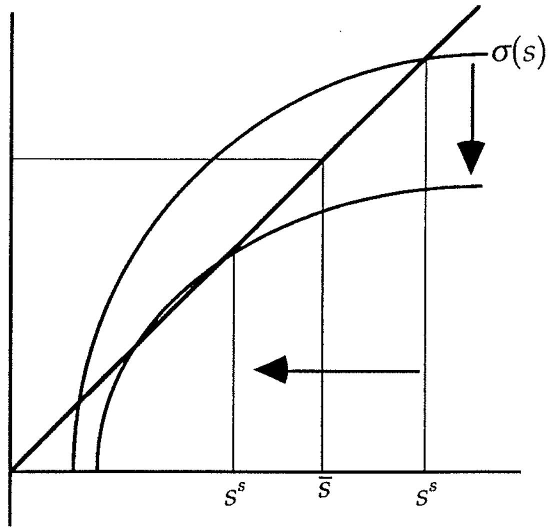

Figure 2

As long as $\sigma(s)$ intersects the 45°-line, however, money still has value. With the simple assumptions I have made, total output remains equal to unity, although fewer households are participating in production and monetary exchange.

Once the growth rate of money is high enough, $\sigma(s)$ no longer intersects the 45°-line; it either lies everywhere below the 45°-line or intersects the line outside the interval $[0, \bar{s}]$. Only one equilibrium remains, the non-monetary equilibrium, which is stable. Money is no longer accepted by anyone in the economy, so it loses all value. This can be thought of as a hyperinflation.

The threshold value of μ is where $\sigma(s)$ either is tangent to the 45°-line or intersects it from below at $\bar{s}$. Tangency gives the condition[10]

(14) $$\mu^* = \frac{\bar{s}}{\varepsilon^{\max} - \varepsilon^{\min}}\left(\sqrt{\varepsilon^{\min}} - 1\right)^2$$

[10]This is calculated by solving for the fixed points of $\sigma(s) = s$ and finding the value of μ at which there is only one such fixed point.

Defining s^u and s^s as the values of s that are, respectively, the unstable and stable monetary steady states, then the threshold value of money growth μ^{max} is given by

$$(15) \qquad \mu^{max} = \min\left\{\mu^*, \left(\mu \middle| s^u = \bar{s}\right)\right\}$$

At any rate of money growth greater than μ^{max} money has no value.

Money's loss of value in this model, which I am equating to a hyperinflation, resembles a hyperinflationary bubble in the models surveyed at the outset. The speed at which real balances converge to zero depends on the assumptions one makes about conjectures. If convergence to a non-monetary steady state occurs in virtual time, then the value of real balances goes to zero instantaneously—the price level effectively goes to infinity at once. Convergence that takes place in real time according to some Bayesian adjustment process might yield a more realistic time path for prices. Either way, however, prices are rising very quickly and at a pace seemingly detached from nominal money expansion, which during more stable times is the fundamental force driving the price level. The price level would appear to be on a bubble path.

Yet if the economy is originally at the stable monetary equilibrium, then a hyperinflationary "bubble" occurs only when the money supply growth rate exceeds μ^{max}. This is in contrast to the models surveyed above, in which a bubble can occur at any rate of money growth. The model predicts a hyperinflation that is observationally similar to a bubble in other models but which is unlikely at low rates of money growth and inevitable at high ones.

Non-monotonic Real Balances

The experience of many countries suggests that during the period of hyperinflation real balances tend to first rise and then fall.[11] The model can

[11]See Bernholz, et al. (1985).

replicate this time path. Recall that at every moment while money is valued $E(p_{i,t}) = E(\tilde{M}_t) = s \cdot M_t$. This implies that

(16) $$m_t = \frac{M_t}{E(p_{i,t})} = \frac{1}{s}$$

where $E(p_{i,t})$ is an unbiased estimator of the price level and m_t is, therefore, a good approximation of real balances. Thus, if an increase in the rate of money growth reduces s it will increase real balances. Indeed, noting that the equilibrium value of s is a non-increasing function of μ, the equation can be relabelled as

(17) $$m_t = \frac{1}{s(\mu)} = \lambda(\mu)$$

with $\lambda'(\mu) \geq 0$. Obviously, if a higher rate of money growth leaves s unchanged at $\bar{s}$ then real balances will remain constant.

Real balances rise when a higher money growth rate leads some households to abandon monetary exchange. Their exit makes search more difficult for those still accepting money, so a smaller fraction of the total money supply will be exchanged for goods each instant. This, in turn, puts downward pressure on the price level, raising the level of real balances.

Real balances will continue to rise with higher money growth rates until the hyperinflation takes hold, at which point money is no longer valued, the price level goes to infinity, and real balances fall to zero. This collapse may occur instantaneously or over a brief interval, depending on the assumptions made about conjectures.

Increase in the Seigniorage Requirement

Up to now I have considered a policy of constant money growth. The common alternative policy assumption, that the government must finance a fixed seigniorage target, leaves the model virtually unchanged, however.

Suppose monetary policy must satisfy $\mu \cdot m = S$, with the seigniorage requirement S fixed in real terms. Substituting for real balances gives

(18) $$S = \mu \cdot \lambda(\mu) = \delta(\mu)$$

Since $\delta'(\mu) \geq 0$ so long as money is valued, the government can finance a greater seigniorage requirement by increasing the rate of money growth. If the government tries to raise too much revenue this way, however, it will cause a hyperinflation. The threshold above which hyperinflation is inevitable is simply

(19) $$S^{\max} = \delta(\mu^{\max})$$

Thus, not only does this model predict that a greater reliance on seigniorage raises the inflation rate (confirming the empirical correlation between fiscal deficits and inflation), it also predicts that if the need for seigniorage is too high a hyperinflation is inevitable.

Restoring a Monetary Equilibrium

Suppose that a hyperinflation has occurred, so that the currency has lost all value. What would it take for the government to restore a monetary equilibrium? The model I have described suggests that the government would have to resort to a dramatic shift in policy: the rate of money growth (or the seigniorage requirement) would have to fall not simply below $\mu^{\max}$ (or $S^{\max}$) but perhaps close to zero. The reason the government faces such resistance is that restoring a currency's value poses a coordination problem: no household will accept money again unless it is persuaded that other households will do the same.

The logic is as follows. Having experienced a hyperinflation, the economy is now at the non-monetary steady state, which in Figure 3 is the origin. This steady state is stable, as I have noted above; after any small perturbation away from the origin (in the positive quadrant, since s is strictly positive), the economy would converge back to the origin.

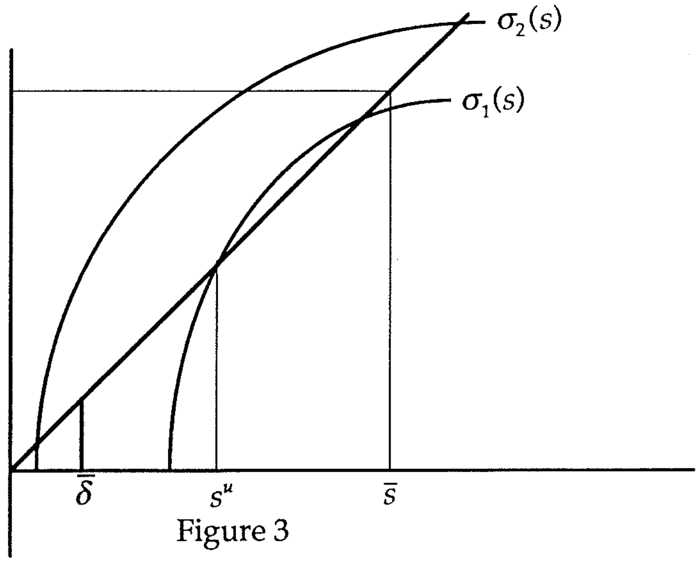

Figure 3

Now suppose that the government reduced the rate of money growth to μ_1, just below μ^{max}, which in Figure 3 corresponds to placing the function $\sigma(s)$ at σ_1 (assume that s^u, the intersection of $\sigma(s)$ and the 45°-line that is closest to the origin, is always interior to $[0,\bar{s}]$). The economy will not move to a monetary equilibrium unless agents use a mean arrival rate greater than or equal to s^u as a focal point around which to coordinate their conjectures. Absent any perturbations, agents will go on using $s=0$ as their focal point and the economy will remain in a non-monetary state indefinitely.

Whereas the hyperinflation may have been ignited by a small increase in μ that pushed it just beyond μ^{max}, a reduction of μ by the same magnitude would not be enough to restore a monetary equilibrium. The only chance of accomplishing such a goal would be to reduce μ to zero.

Allowing for small perturbations in the observed value of s, perhaps because it cannot be observed noiselessly, the government may be able to restore a monetary equilibrium with slightly less effort. Assume that households cannot observe the mean arrival rate perfectly, but instead observe $\tilde{s}=s+\delta_j$, where δ_j is some random variable drawn from $[0,\bar{\delta}]$. In this case, if the economy is currently at the non-monetary steady state, the government stands a chance of restoring a monetary equilibrium if it can reduce the rate of money growth sufficiently to move $\sigma(s)$ to a position such as σ_2. A realization of $\tilde{s}$ that boosts it above s^u may now lead the economy back to a stable monetary equilibrium.

Other authors have insisted that ending a hyperinflation requires decisive government action to shock expectations out of their inflationary mode. Sargent (1982), echoing the rational expectations view, says that ending a hyperinflation requires "an abrupt change in the continuing government policy... that is sufficiently binding as to be widely believed." People will conclude that the hyperinflation is over only if they conclude that government policy now and into the indefinite future is consistent with an end to hyperinflation.

The model I have presented suggests that the conventional rational expectations view of Sargent and others captures only part of the story, however. It is not enough that each household believes that the central bank is once again committed to a non-hyperinflationary monetary policy. Each household must also be convinced that most other households have the same view and are prepared to act on it by accepting money once again. That is, government policy has the added burden of coordinating expectations so that the economy can converge to a monetary equilibrium.

Conclusion

The model I have presented is very simple. My task was not to develop a sophisticated analysis of hyperinflation but to suggest that it may be more fruitful to analyze hyperinflations using a set a premises different from those found in the traditional models of hyperinflation.

The main premise found in the existing literature on hyperinflation is that agents demand money based on its expected rate of return. While this may be a valid assumption in times of modest inflation, it leads to counterintuitive propositions about the dynamics of hyperinflation. In particular, it proposes that a hyperinflation may be a price-level bubble, and one that could occur regardless of how loose or austere is government policy.

I have developed an alternative model based on the assumption that when inflation is very high, agents essentially face a binary choice: accept or reject money when it is offered in exchange for consumption goods. Increasing or decreasing real balances based on expected inflation is a calculus almost absent from a household's consideration. Using this microeconomic assumption about monetary exchange, as well as the assumption that money is more valuable to a household the more prevalently it is used by other households, I have built a model that I think generates more plausible macroeconomic predictions about hyperinflation.

The model reaches four conclusions:

1) Hyperinflations resemble price-level bubbles, as in standard models, but these "bubbles" are unlikely when money growth rates and seigniorage requirements are low and inevitable when they are high. In the standard models, the likelihood of hyperinflationary bubbles is independent of money growth rates and seigniorage requirements.

2) As inflation approaches hyperinflation, real balances first rise and then fall precipitously, a pattern common to many hyperinflationary episodes. In the standard models, real balances fall steadily as inflation rises.

3) A large seigniorage requirement increases the inflation rate (or at best has no effect on it) when the economy is at a stable equilibrium. In standard models, a larger seigniorage requirement reduces the inflation rate at stable equilibria.

4) It may be very difficult to restore a currency's value after a hyperinflation, since it requires that households overcome a potentially serious coordination problem. The standard models, which are representative agent models, cannot accommodate coordination problems.

The model's simplicity means that it can be improved in many directions. One would be to use a more general utility function and derive more realistic search strategies for consumers and producers. Another would be to include a more sophisticated cash-in-advance constraint and variable search intensity. Finally, the model does not admit the possibility of currency substitution, even though substitution has taken place in almost all countries that have experienced hyperinflation. A more complete model would include another currency.

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