Erratum: The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov Switching Model of GNP

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There was an error in Hansen (1992). I am very grateful to James Hamilton for pointing out the error.

Equations (2)-(3) in the original read

$$\frac{1}{\sqrt{n}}Q_n(\alpha) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} q_i(\alpha) \Rightarrow Q(\alpha) \tag{2}$$

where $Q(\alpha)$ is a mean zero Gaussian process with covariance function

$$K(\alpha_1, \alpha_2) = E\left(q_i(\alpha_1)q_i(\alpha_2)\right). \tag{3}$$

While (2) is correct, (3) is not. Instead, the correct expression is

$$K(\alpha_1, \alpha_2) = \sum_{k=-\infty}^{\infty} E\left(q_i(\alpha_1)q_{i+k}(\alpha_2)\right). \tag{3'}$$

The reason is that the likelihood components $q_i(\alpha)$ will be serially correlated for some values of α. This will be the case even when the original data are iid, since the likelihood $q_i(\alpha)$ is a function of all data up to time i. It should be noted that this problem does not apply to the testing methods of Hansen (1994), which involve application of empirical process theory to specific likelihood scores which are serially uncorrelated.

This error implies that the method of calculating the asymptotic distribution in section 3.2 is incorrect. Instead, set $\hat{q}_i(\alpha) = q_i(\alpha, \hat{\theta}(\alpha))$ and

$$\hat{K}_n(\alpha_1, \alpha_2) = \sum_{i=1}^{n} \hat{q}_i(\alpha_1)\hat{q}_i(\alpha_2) + \sum_{k=1}^{M} w_{kM}\left[\sum_{1\le i\le n-k} \hat{q}_i(\alpha_1)\hat{q}_{i+k}(\alpha_2) + \sum_{1+k\le i\le n} \hat{q}_i(\alpha_1)\hat{q}_{i-k}(\alpha_2)\right],$$

where $w_{kM} = 1 - |k|/(M+1)$ is the Bartlett kernel and M is a bandwidth number (selected to grow to infinity slowly with sample size). Then a consistent estimate of

$$K^*(\alpha_1, \alpha_2) = \frac{K(\alpha_1, \alpha_2)}{V(\alpha_1)^{1/2}V(\alpha_2)^{1/2}}$$

is given by

$$K_n^*(\alpha_1, \alpha_2) = \frac{\hat{K}_n(\alpha_1, \alpha_2)}{V_n(\alpha_1)^{1/2}V_n(\alpha_2)^{1/2}}.$$

Sample draws from this process can be obtained by constructing

$$\widetilde{LR}^*(\alpha) = \frac{\sum_{k=0}^{M}\sum_{i=1}^{n} q_i(\alpha, \hat{\theta}(\alpha))u_{i+k}}{\sqrt{1+M}\ V_n(\alpha)^{1/2}}$$

where $\{u_i\}_{i=1}^{n+M}$ is a sample of random $N(0,1)$ variables. The reader may verify that conditional on the data, $\widetilde{LR}^*(\alpha)$ is a mean zero Gaussian process with exact covariance function $K_n^*(\alpha_1, \alpha_2)$, and the latter is an asymptotic approximation to $K^*(\alpha_1, \alpha_2)$.

The theory does not give any particular guidance for choice of M. It therefore seems prudent to calculate the tests for several choices to assess sensitivity.

The original paper reported estimates of $K_n^*(\alpha_1, \alpha_2)$ and $\widetilde{LR}^*(\alpha)$ effectively with $M = 0$. Thus all test statistics and Monte Carlo evidence were presented with $M = 0$. All the numerical work was recalculated for $M = 1, ..., 4$. Other than the change discussed above, the methods were essentially identical to those outlined in Hansen (1992). The corrected results are presented in the following tables. It it interesting to note that the results are not very sensitive to M. None of the conclusions drawn are affected. The category "CPU hours" refered to the time required for the programs to run on a 486/66 computer.

A GAUSS program which produces the empirical results reported here is available on request from the author.

References

Hansen, B.E. (1992), 'The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP,' *Journal of Applied Econometrics*, 7, S61-S82.

Hansen, B.E. (1994), 'Inference when a nuisance parameter is not identified under the null hypothesis,' unpublished manuscript, Department of Economics, Boston College.

Table III: Standardized LR statistics for Hamilton model

	LR_n^*	p-value					CPU Hours
		$M=0$	$M=1$	$M=2$	$M=3$	$M=4$	
Grid 1	1.24	.77	.75	.75	.76	.73	1.2
Grid 2	1.56	.73	.68	.67	.66	.62	2.5
Grid 3	1.55	.74	.70	.69	.69	.65	4.6

Table IV: Monte-Carlo size and power, no autoregressive component (percentages)

M	Null			Alternative		
Nominal Size:	20	10	5	20	10	5
0	12	4	0	86	80	74
1	14	8	0	86	80	74
2	16	8	2	86	76	74
3	14	8	2	86	76	74
4	14	6	2	86	76	74

Table VI: Standardized LR statistics for Markov switching intercept model

	LR_n^*	p-value					CPU Hours
		$M=0$	$M=1$	$M=2$	$M=3$	$M=4$	
Grid 1	2.20	.29	.26	.30	.27	.28	0.3
Grid 2	1.95	.49	.51	.50	.46	.48	0.6
Grid 3	2.09	.44	.42	.43	.41	.43	1.1
Grid 4	2.21	.41	.38	.38	.37	.38	2.6
Grid 5	2.23	.39	.38	.36	.34	.34	5.2

Table VII: Monte-Carlo size and power with AR components (percentages)

	M	Null			Alternative		
Nominal Size:		20	10	5	20	10	5
AR(1)	0	14	10	6	52	40	30
	1	14	10	6	52	40	30
	2	14	10	6	52	38	28
	3	16	10	4	52	38	28
	4	16	10	4	52	36	26
AR(4)	0	26	18	14	44	36	24
	1	30	20	16	44	40	24
	2	28	20	16	44	40	22
	3	28	22	16	44	38	24
	4	30	22	16	44	33	20

Table X: Standardized LR statistics for switching parameters model

	LR_n^*	p-value					CPU Hours
		$M=0$	$M=1$	$M=2$	$M=3$	$M=4$	
Model A: (μ, ϕ_2) vary between states, $q = 1 - p$							
Grid 1	3.50	.01	.01	.01	.02	.01	0.4
Grid 2	3.53	.01	.01	.01	.02	.02	0.6
Grid 3	3.61	.01	.02	.01	.01	.01	0.8
Grid 4	3.59	.01	.01	.01	.01	.01	1.2
Model B: (μ, ϕ_2) vary between states, q unconstrained							
Grid 1	3.50	.02	.02	.02	.02	.03	1.8
Grid 2	3.55	.03	.03	.03	.03	.04	4.0
Grid 3	3.61	.02	.03	.03	.04	.03	7.1
Grid 4	3.61	.04	.04	.02	.03	.04	15.9