FRACTIONAL DIFFERENCING MODELING AND FORECASTING OF EUROCURRENCY DEPOSIT RATES

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Abstract

Using the spectral regression method, we test for long-term stochastic memory in three- and six-month daily returns series of Eurocurrency deposits denominated in major currencies. Significant evidence of positive long-term dependence is found in several Eurocurrency returns series. Compared to benchmark linear models, the estimated fractional models result in dramatic out-of-sample forecasting improvements over longer horizons for the Eurocurrency deposits denominated in German marks, Swiss francs, and Japanese yen.

I. Introduction

Many economic and financial time series exhibit considerable persistence. Using standard unit root tests of the $I(1)/I(0)$ variety, most of these series are best characterized as integrated processes of order one, denoted by $I(1)$. The assumption of an integer integration order is arbitrary, and relaxing it allows for a wider range of subtle mean reverting dynamics to be captured. Allowing for the integration order of a series to take any value on the real line (fractional integration) leads to the development of long-memory models.

The long-memory, or long-term dependence, property describes the high-order correlation structure of a series. If a series exhibits long memory, persistent temporal dependence exists even between distant observations. Such series are characterized by distinct but nonperiodic cyclical patterns. The presence of long memory dynamics gives nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics. On the other hand, the short-memory, or short-term dependence, property describes the low-order correlation structure of a series. For short-memory series, observations separated by a long time span are nearly independent. Standard autoregressive moving average processes cannot exhibit long-term (low-frequency) dependence as they can only describe the short-run (high-frequency) behavior of a time series.

The presence of fractional structure in asset prices raises issues regarding theoretical and econometric modeling of asset prices, statistical testing of pricing models, and pricing efficiency and rationality. Applications of long-memory analysis include Greene and Fielitz (1977), Lo (1991), and Barkoulas and Baum (1996) for U.S. stock prices; Cheung (1993a) for spot exchange rates; and Fang, Lai, and Lai (1994), and Barkoulas, Labys, and Onochie (1996) for futures prices. The overall evidence suggests that stochastic long memory is absent in stock market returns but it may be a feature of some spot and futures foreign currency rates.

In the present study we extend the aforementioned literature by investigating the presence of fractional dynamics in the returns series (yield changes) of three- and six-month Eurocurrency deposits denominated in eight major currencies: the U.S. dollar, Canadian dollar, German mark, British pound, French franc, Swiss franc, Italian lira, and Japanese yen. We emphasize the implications of long memory for predictability and market efficiency. According to the market efficiency hypothesis in its weak form, asset prices incorporate all relevant information, rendering asset returns unpredictable. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. If the Eurocurrency returns series exhibit long memory, they display significant autocorrelations between observations widely separated in time. Since the series realizations are not independent over time, past returns can help predict future returns, calling into question the validity of the efficient capital market hypothesis.

Using the spectral regression method of estimating the fractional integration parameter, evidence of long memory dynamics is obtained in the Eurocurrency returns series for the Canadian dollar (three-month maturity only), German mark, Swiss franc, and Japanese yen. With the exception of the three-month Eurocanadian dollar returns series, long memory forecasts are superior to linear predictors over longer forecasting horizons, thus establishing significant nonlinear mean predictability in these series.

II. The Spectral Regression Method

The model of an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA(p,d,q), with mean μ, may be written using operator notation as

$$\Phi(L)(1-L)^d\left(y_t-\mu\right)=\Theta(L)u_t, \qquad u_t \sim \text{i.i.d.}(0,\ \sigma_u^2) \tag{1}$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + \ldots + \vartheta_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty}\frac{\Gamma(k-d)\,L^k}{\Gamma(-d)\Gamma(k+1)} \tag{2}$$

with $\Gamma(\cdot)$ denoting the gamma, or generalized factorial, function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process y_t is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d|<0.5$. The process is nonstationary for $d\geq 0.5$, as it possesses infinite variance, i.e. see Granger and Joyeux (1980). Assuming that d $(0,0.5)$ and $d\neq 0$, Hosking (1981) showed that the correlation function, $\rho(\,)$, of an ARFIMA process is proportional to k^{2d-1} as $k\to\infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $k\to\infty$ which is contrary to the faster, geometric decay of a stationary ARMA process. For d $(0,0.5)$, $\sum_{j=-n}^{n}|\rho(j)|$ diverges as $n\to\infty$, and the ARFIMA process is said to exhibit long memory, or long-range positive dependence. The process is said to exhibit intermediate memory (anti-persistence), or long-range negative dependence, for $d \in (-0.5,0)$. The process exhibits short memory for $d = 0$, corresponding to stationary and invertible ARMA modeling. For $d \in [0.5,1)$ the process is mean reverting, even though it is not covariance stationary, as there is no long run impact of an innovation on future values of the process.

Geweke and Porter-Hudak (1983) suggest a semi-parametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency $\xi = 0$. More specifically, let $I(\xi)$ be the periodogram of **y** at frequency ξ defined by

$$I(\xi) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\xi}(y_t - \bar{y})\right|^2 . \tag{3}$$

Then the spectral regression is defined by

$$\ln\{I(\xi_\lambda)\} = \beta_0 + \beta_1 \ln\left\{\sin^2\left(\frac{\xi_\lambda}{2}\right)\right\} + \eta_\lambda , \qquad \lambda = 1,\ldots,\nu \tag{4}$$

where $\xi_\lambda = \frac{2\pi\lambda}{T}$ $(\lambda = 0,\ldots, T-1)$ denotes the Fourier frequencies of the sample, T is the number of observations, and $\nu = g(T) << T$ is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty}\left\{ g(T)/T \right\} = 0$, and $\lim_{T\to\infty} \ln(T)^2 / g(T) = 0$, the negative of the OLS estimate of the slope coefficient in (4) provides an estimate of d. Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for $d < 0$, while Robinson (1990) proves consistency for $d \in (0,0.5)$. Hassler (1993) proves consistency and asymptotic normality in the case of Gaussian ARMA innovations in (1). The spectral regression estimator is not $T^{1/2}$ consistent and will converge at a slower rate. The theoretical asymptotic variance of the spectral regression error term is known to be $\pi^2/6$.

To ensure that stationarity and invertibility conditions are met, we apply the spectral regression test to the returns series (yield changes) of the Eurocurrency deposit rates.

III. Data and Empirical Estimates

The data set consists of daily rates for Eurocurrency deposits denominated in U.S. dollars (US), Canadian dollars (CD), German marks (GM), British pounds (BP), French francs (FF), Swiss francs (SF), Italian lira (IL), and Japanese yen (JY) for three- and six-month term maturities. These rates represent bid rates at the close of trading in the London market and were obtained from Data Resources, Inc. The total sample spans the period from January 2, 1985 to February 8, 1994 for a total of 2303 observations for the US, FF, and IL, 2305 observations for the CD, 2300 observations for the GM and JY, and 2302 for the BP and SF. The last 347 observations of each series (roughly 15 months) are reserved for out-of-sample forecasting while the remainder is used for in-sample estimation.

Table 1 presents the spectral regression estimates of the fractional differencing parameter d for the Eurocurrency deposit returns series. The number of low frequency periodogram ordinates used in the spectral regression must be chosen carefully. Improper inclusion of medium- or high-frequency periodogram ordinates will bias the estimate of d; at the same time too small a regression sample will increase the sampling variability of the estimates. To check the sensitivity of results to the choice of the sample size of the spectral regression, we report fractional differencing estimates for

$\nu = T^{0.55}, T^{0.575}$, and $T^{0.60}$. The statistical significance of the d estimates is tested by performing two-sided ($d = 0$ versus $d \neq 0$) as well as one-sided ($d = 0$ versus $d > 0$) tests. The known theoretical variance of the regression error $\pi^2/6$ is imposed in the construction of the $t-$statistic for d.

-----------------------------INSERT Table 1 AROUND HERE-------------------------

As Table 1 indicates, robust evidence of fractional dynamics with long-memory features is obtained for the three- and six-month GM, SF, and JY returns series and the three-month CD returns series.[1] The fractional differencing parameters are similar in value across the two maturities considered for the GM, SF, and JY returns series. These series are not short memory processes, which would exhibit a rapid exponential decay in their impulse response weights. However, they are clearly covariance stationary as their d estimates lie below the stationarity boundary of 0.5. The presence of long memory is stronger, in terms of magnitude of the estimated fractional differencing parameters, for the SF and JY returns series while it is milder for the GM and CD series. The implications of the long-memory evidence in these Eurocurrency returns series can be seen in both the time and frequency domains. In the time domain, long memory is indicated by the fact that the returns series eventually exhibit positive dependence between distant observations. A shock to the series persists for a long time span even though

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[1] We also applied the Phillips-Perron (PP) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) unit-root tests to the returns series of Eurocurrency deposits. The combined use of these unit-root tests offers contradictory inference regarding the low-frequency behavior of several Eurocurrency returns series, which provides motivation for testing for fractional roots in the series. The long-memory evidence to follow reconciles the conflicting inference derived from the PP and KPSS tests. To conserve space, these results are not reported here but are available upon request from the authors.

it eventually dissipates. In the frequency domain, long memory is indicated by the fact that the spectral density becomes unbounded as the frequency approaches zero; the series has power at low frequencies.

The evidence of fractional structure in these returns series may not be robust to nonstationarities in the mean and short-term dependencies. Through extensive Monte Carlo simulations, Cheung (1993b) shows that the spectral regression test is robust to moderate ARMA components, ARCH effects, and shifts in the variance. However, possible biases of the spectral regression test against the no long memory null hypothesis may be caused by infrequent shifts in the mean of the process and large AR parameters (0.7 and higher), both of which bias the test toward detecting long memory. A similar point is made by Agiakloglou, Newbold, and Wohar (1993). We now investigate the potential presence of these bias-inducing features in our sample series.

Graphs of the fractal Eurocurrency returns series do not indicate that the data generating process for the series in question underwent a shift in the mean.[2] Therefore the evidence of long memory for these series should not be a spurious artifact of changes in the mean of the series. To examine the possibility of spurious inference in favor of long-term persistence due to strong dependencies in the data, an autoregressive (AR) model is fit to each of the series in question according to the Schwarz information criterion. An AR(1) model is found to adequately describe dependence in the conditional mean of the three- and six-month GM, three-month SF, and three- and six-month JY returns series while an AR(2) representation is chosen for the three-month CD and six-month SF returns series. All AR coefficient

[2] These graphs are not presented here to conserve space but they are available upon request from the authors.

estimates are very small in value suggesting the absence of strong short-term dependencies.[3] Therefore, neither a shift in mean nor strong short-term dynamics appears to be responsible for finding long memory in the Eurocurrency returns series.

IV. A Forecasting Experiment

The discovery of fractional orders of integration suggests possibilities for constructing nonlinear econometric models for improved price forecasting performance, especially over longer forecasting horizons. An ARFIMA process incorporates this specific nonlinearity and represents a flexible and parsimonious way to model both the short- and long-term dynamical properties of the series. Granger and Joyeux (1980) discuss the forecasting potential of such nonlinear models and Geweke and Porter-Hudak (1983) confirm this by showing that ARFIMA models provide more reliable out-of-sample forecasts than do traditional procedures. The possibility of consistent speculative profits due to superior long-memory forecasts would cast serious doubt on the basic tenet of market efficiency, which states unpredictability of future returns. In this section the out-of-sample forecasting performance of an ARFIMA model is compared to that of benchmark linear models.

Given the spectral regression d estimates, we approximate the short-run series dynamics by fitting an AR model to the fractionally differenced series using Box-Jenkins methods. An AR representation of generally low order appears to be an adequate description of short-term dependence in the

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[3] Full details of the AR representations of the fractionally integrated Eurocurrency deposit returns series are available upon request from the authors.

data. The AR orders are selected on the basis of statistical significance of the coefficient estimates and Q statistics for serial dependence (the AR order chosen in each case is given in subsequent tables). A question arises as to the asymptotic properties of the AR parameter estimates in the second stage. Conditioning on the d estimate obtained in the first stage, Wright (1995) shows that the $\mathrm{AR}(p)$ fitted by the Yule-Walker procedure to the d-differenced series inherit the T^{δ}-consistency of the semiparametric estimate of d. We forecast the Eurocurrency deposit rates by casting the fitted fractional-AR model in infinite autoregressive form, truncating the infinite autoregression at the beginning of the sample, and applying Wold's chain rule. Ray (1993) uses a similar procedure to forecast IBM product revenues.

The long-memory forecasts are compared to those generated by two standard linear models: an autoregressive model (AR), described earlier, and a random-walk-with-drift model (RW). We reserve the last 347 observations from each series for forecasting purposes. We analyze out-of-sample forecasting horizons of 1-, 5-, 10-, 24-, 48-, 72-, 96-, 120-, 144-, 168-, 192-, 216-, 240-, 264-, and 288-steps ahead, corresponding approximately to one-day, one-week, two-week, one-, two-, three-, four-, five-, six-, seven-, eight-, nine-, ten-, eleven-, and twelve-month forecasting horizons. These forecasts are truly ex ante, or dynamic, as they are generated recursively conditioning only on information available at the time the forecast is being made. Forecasting performance is judged by root mean square error (RMSE) and mean absolute deviation (MAD) criteria.

Tables 2 through 8 report the out-of-sample forecasting performance of the competing modeling strategies for our fractal Eurocurrency returns series. Comparing the linear models first, the AR and RW forecasts are very similar for all series across the various forecasting horizons. Looking at Table 2, we see that the long-memory forecasts for the three-month Eurocanadian dollar returns series are inferior to linear forecasts across all forecasting horizons (with a few exceptions based on the MAD metric). The long-memory model may fail to improve upon its linear counterparts if the impact of long memory is considerably further into the future, and the adjustment to equilibrium takes considerable time to complete. Therefore, any improvement in forecasting accuracy over the benchmark models may only be apparent in the very long run.

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A different picture is evident for the remaining fractal Eurocurrency returns series. As Tables 3 through 8 report, the long-memory forecasts for the GM, SF, and JY returns series significantly outperform the linear forecasts on the basis of both RMSE and MAD forecasting measures. The percentage reductions in the forecasting criteria attained by the long-memory models appear at very short horizons, they are dramatic, and they generally increase with the length of the forecasting horizon. The superior performance of the long-memory fits holds true across the various estimates of d for each returns series, suggesting robustness. It appears that the higher (lower) d estimates provide superior forecasting performance over longer (shorter) horizons.

To better judge the relative forecasting performance of the alternative modeling strategies, Table 9 reports ratios of the forecasting criteria values (RMSE and MAD) attained by the long-memory model with the highest d estimate for each series to that obtained from the RW model. For the GM, SF, and JY series, the improvements in forecasting accuracy are sizable, while the poor performance of the long-memory forecasts of the Eurocanadian dollar returns series is obvious. The largest forecasting improvements occur for the SF and JY series with smaller, yet significant improvements for the GM series.

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The forecasting performance of the long-memory model for the GM, SF, and JY series is consistent with theory. As the effect of the short-memory (AR) parameters dominates over short horizons, the forecasting performance of the long-memory and linear models is rather similar in the short run. In the long run, however, the dynamic effects of the short-memory parameters are dominated by the fractional differencing parameter d, which captures the long-term correlation structure of the series, thus resulting in superior long-memory forecasts. This evidence accentuates the usefulness of long-memory models as forecast generating mechanisms for some Eurocurrency returns series, and casts doubt on the hypothesis of the weak form of market efficiency for longer horizons. It also contrasts with the failure of ARFIMA models to improve on the random walk model in out-of-sample forecasts of foreign exchange rates (Cheung (1993a)).

V. Conclusions

Using the spectral regression method, we find significant evidence of long-term stochastic memory in the returns series (yield changes) of three- and six-month Eurodeposits denominated in German marks, Swiss francs, and Japanese yen, as well as three-month Eurodeposits denominated in Canadian dollars. These series appear to be characterized by irregular cyclic fluctuations with long-term persistence. With the exception of the Canadian dollar returns series, the out-of-sample long-memory forecasts result in dramatic improvements in forecasting accuracy especially over longer horizons compared to benchmark linear forecasts. Price movements in these markets appear to be influenced not only by their recent history but also by realizations from the distant past. This is strong evidence against the martingale model, which states that, conditioning on historical returns, future returns are unpredictable.

We have established the practical usefulness of developing long-memory models for some Eurocurrency returns series. These results could potentially be improved in future research via estimation of ARFIMA models based on maximum likelihood methods (e.g. Sowell (1992). These procedures avoid the two-stage estimation process followed in this paper by allowing for the simultaneous estimation of the long and short memory components of the series. Given the sample size of our series, however, implementing these procedures will be very computationally burdensome, as closed-form solutions for these one-stage estimators do not exist. Additionally, in some cases the ML estimates of the fractional-differencing parameter appear to be sensitive to the parameterization of the high-frequency components of the series. Future research should investigate why certain Eurocurrency returns series exhibit long memory while others do not.

References

Agiakloglou, C., P. Newbold, and M. Wohar, 1993, Bias in an estimator of the fractional difference parameter, *Journal of Time Series Analysis*, 14, 235-246.

Barkoulas, J. T. and C. F. Baum, 1996, Long term dependence in stock returns, *Economic Letters*, forthcoming.

Barkoulas, J. T., W. C. Labys, and J. Onochie, 1996, Long memory in futures prices, *Financial Review*, forthcoming.

Cheung, Y. W., 1993a, Long memory in foreign-exchange rates, *Journal of Business and Economic Statistics*, 11, 93-101.

Cheung, Y. W., 1993b, Tests for fractional integration: A Monte Carlo investigation, *Journal of Time Series Analysis*, 14, 331-345.

Fang, H., K. S. Lai and M. Lai, 1994, Fractal structure in currency futures prices, *Journal of Futures Markets*, 14, 169-181.

Geweke J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, *Journal of Time Series Analysis*, 4, 221-238.

Granger, C. W. J. and R. Joyeux, 1980, An introduction to long-memory time series models and fractional differencing, *Journal of Time Series Analysis*, 1, 15-39.

Greene, M. T. and B. D. Fielitz, 1977, Long-term dependence in common stock returns, *Journal of Financial Economics*, 5, 339-349.

Hassler, U., 1993, Regression of spectral estimators with fractionally integrated time series, *Journal of Time Series Analysis*, 14, 369-380.

Hosking, J. R. M., 1981, Fractional differencing, *Biometrika*, 68, 165-176.

Lo, A. W., 1991, Long-term memory in stock market prices, *Econometrica*, 59, 1279-1313.

Ray, B., 1993, Long range forecasting of IBM product revenues using a seasonal fractionally differenced ARMA model, *International Journal of Forecasting*, 9, 255-269.

Robinson, P., 1990, Time series with strong dependence, *Advances in Econometrics*, 6th World Congress (Cambridge: Cambridge University Press).

Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally-integrated time-series models, *Journal of Econometrics*, 53, 165-188.

Wright, J. H., 1995, Stochastic orders of magnitude associated with two-stage estimators of fractional ARIMA systems, *Journal of Time Series Analysis*, 16, 119-125.

Table 1: Estimates of the Fractional-Differencing Parameter d for the Eurocurrency Deposit Returns Series

Series	$d(0.55)$	$d(0.575)$	$d(0.60)$
3-Month			
US Dollar	0.092	0.106	0.037
	(1.137)	(0.106)‡	(0.526)
Canadian Dollar	0.181	0.213	0.180
	(2.219)**,‡‡	(2.679)***,‡‡‡	(2.504)**,‡‡‡
German Mark	0.149	0.181	0.172
	(1.826)*,‡‡	(2.262)**,‡‡	(2.404)**,‡‡‡
British Pound	0.059	0.041	0.060
	(0.730)	(0.515)	(0.848)
French Franc	-0.042	0.013	0.006
	(-0.516)	(0.171)	(0.093)
Swiss Franc	0.215	0.210	0.152
	(2.635)***,‡‡‡	(2.635)***,‡‡‡	(2.114)**,‡‡
Italian Lira	0.054	0.064	0.069
	(0.669)	(0.805)	(0.962)
Japanese Yen	0.244	0.250	0.190
	(2.996)***,‡‡‡	(3.113)***,‡‡‡	(2.645)***,‡‡‡
6-Month			
US Dollar	0.101	0.106	0.041
	(1.241)	(1.333)‡	(0.570)
Canadian Dollar	0.086	0.108	0.053
	(1.059)	(1.358)	(0.739)***,‡‡‡
German Mark	0.109	0.148	0.130
	(1.335)‡	(1.842)**,‡‡	(1.821)**,‡‡
British Pound	0.028	0.025	0.031
	(0.344)	(0.325)	(0.435)
French Franc	0.052	0.110	0.093
	(0.637)	(1.383)‡	(1.299)‡
Swiss Franc	0.285	0.308	0.211
	(3.500)***,‡‡‡	(3.862)***,‡‡‡	(2.944)***,‡‡‡
Italian Lira	0.039	0.059	0.059
	(0.481)	(0.743)	(0.824)
Japanese Yen	0.290	0.298	0.237
	(3.563)***,‡‡‡	(3.707)***,‡‡‡	(3.308)***,‡‡‡

Notes: The sample corresponds to the in-sample number of observations (total number of observations minus 347, which are reserved for out-of-sample forecasting). $d(0.55)$, $d(0.575)$, and $d(0.60)$ give the d estimates corresponding to the spectral regression of sample size $\nu = T^{0.55}$, $\nu = T^{0.575}$, and $\nu = T^{0.60}$. The $t-$statistics are given in parentheses and are constructed imposing the known theoretical error variance of $\pi^2/6$.

*** Statistical significance at the 1 percent level (two-tailed, $d = 0$ versus $d \neq 0$).

** Statistical significance at the 5 percent level (two-tailed).

* Statistical significance at the 10 percent level (two-tailed).

‡‡‡ Statistical significance at the 1 percent level (right-tailed, $d = 0$ versus $d > 0$).

‡‡ Statistical significance at the 5 percent level (right-tailed).

‡ Statistical significance at the 10 percent level (right-tailed).

Table 2: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 3-Month Eurocanadian Dollar Rate

Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
d = 0.181, AR(6)	0.1939	0.3731	0.4995	0.6725	0.8967	1.1485	1.3511	1.5557	1.7288	1.9032	1.9885	2.1110	2.3994	2.8483	3.3485
	0.1184	0.2161	0.3066	0.4803	0.6882	0.8608	1.0067	1.1110	1.1883	1.3088	1.4638	1.6622	2.0837	2.5637	2.9942
d = 0.213, AR(6)	0.1940	0.3747	0.5055	0.6927	0.9216	1.1942	1.4131	1.6326	1.8166	2.0060	2.1031	2.2363	2.5307	3.0153	3.5864
	0.1185	0.2177	0.3098	0.4850	0.7041	0.8860	1.0419	1.1691	1.2446	1.3605	1.5130	1.7142	2.1422	2.6669	3.1703
AR(2)	0.1929	0.3648	0.4753	0.6117	0.5353	1.0171	1.1764	1.3453	1.5078	1.6587	1.7476	1.8680	2.1266	2.4250	2.6567
	0.1180	0.2108	0.3037	0.4808	0.6671	0.8064	0.8918	0.9989	1.1887	1.4044	1.5955	1.7616	2.0212	2.3054	2.4984
RW	0.1937	0.3637	0.4748	0.6160	0.8333	1.0145	1.1722	1.3392	1.4995	1.6487	1.7340	1.8524	2.1079	2.4065	2.6402
	0.1186	0.2110	0.3026	0.4801	0.6652	0.8042	0.8884	0.9909	1.1756	1.3889	1.5776	1.7413	2.0009	2.2857	2.4790

Column headings: k-Step Ahead Horizon (Number of Point Forecasts)

Notes: The test set consists of the last 347 observations for each series. The first entry of each cell is the root mean squared error (RMSE), while the second is the mean absolute deviation (MAD). AR(k) stands for an autoregression model of order k. RW stands for random walk (with drift). The long memory model consists of the fractional differencing parameter d and the order of the AR polynomial. The coefficient estimates and associated test statistics for the various AR models are available upon request. Those RMSEs and MADs obtained from the long memory models which are lower than those obtained from the RW model are underlined. The forecasting performance of the long memory model corresponding to d = 0.180 is not reported as it is essentially identical to the one for d = 0.181.

Table 3: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 3-Month Euromark Rate

Forecasting Model	k-Step Ahead Horizon (Number of Point Forecasts)														
	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
d = 0.149, AR(6)	0.0784	0.1659	0.2110	0.3305	0.4831	0.6360	0.8309	1.0574	1.3104	1.5316	1.7941	2.0119	2.2364	2.4591	2.6998
	0.0585	0.1262	0.1663	0.2657	0.4026	0.5626	0.7580	0.9895	1.2729	1.4713	1.7416	1.9858	2.2044	2.4149	2.6681
d = 0.181, AR(5)	0.0786	0.1664	0.2120	0.3304	0.4733	0.6084	0.7910	1.0046	1.2469	1.4624	1.7177	1.9246	2.1495	2.3696	2.6094
	0.0586	0.1263	0.1670	0.2665	0.3906	0.5298	0.7091	0.9263	1.2045	1.3946	1.6567	1.8900	2.1084	2.3130	2.5673
d = 0.172, AR(5)	0.0786	0.1660	0.2115	0.3299	0.4748	0.6140	0.7994	1.0159	1.2606	1.4770	1.7336	1.9427	2.1666	2.3867	2.6253
	0.0585	0.1262	0.1667	0.2659	0.3929	0.5366	0.7201	0.9408	1.2198	1.4115	1.6747	1.9108	2.1284	2.3340	0.5866
AR(1)	0.0781	0.1667	0.2195	0.3721	0.6120	0.8674	1.1336	1.4260	1.7284	1.9876	2.2885	2.5555	2.7886	3.0330	3.2888
	0.0584	0.1285	0.1717	0.2935	0.5488	0.8303	1.0966	1.3997	1.7106	1.9609	2.2689	2.5491	2.7778	3.0183	3.2804
RW	0.0778	0.1673	0.2194	0.3713	0.6105	0.8653	1.1307	1.4225	1.7244	1.9832	2.2836	2.5502	2.7830	3.0268	3.2835
	0.0583	0.1286	0.1719	0.2928	0.5470	0.8279	1.0935	1.3959	1.7065	1.9565	2.2637	2.5435	2.7720	3.0119	3.2749

See notes in Table 2 for explanation.

Table 4: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 6-Month Euromark Rate

	k-Step Ahead Horizon (Number of Point Forecasts)														
Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
d = 0.109, AR(2)	0.0810	0.1582	0.2020	0.2966	0.4445	0.6086	0.7845	0.9899	1.2022	1.3968	1.6253	1.8441	2.0825	2.2994	2.5294
	0.0596	0.1226	0.1602	0.2387	0.3757	0.5502	0.7327	0.9464	1.1749	1.3554	1.5908	1.8208	2.0582	2.2699	2.5134
d = 0.148, AR(2)	0.0814	0.1600	0.2033	0.2942	0.4229	0.5593	0.7131	0.8995	1.0944	1.2764	1.4874	1.6866	1.9173	2.1237	2.3423
	0.0600	0.1240	0.1618	0.2397	0.3533	0.4919	0.6473	0.8402	1.0576	1.2195	1.4382	1.6522	1.8825	2.0804	2.3162
d = 0.130, AR(2)	0.0812	0.1591	0.2025	0.2947	0.4316	0.5807	0.7447	0.9400	1.1432	1.3310	1.5503	1.7588	1.9933	2.2047	2.4286
	0.0598	0.1233	0.1609	0.2388	0.3626	0.5179	0.6853	0.8882	1.1109	1.2819	1.5086	1.7302	1.9639	2.1685	2.4079
AR(1)	0.0806	0.1593	0.2136	0.3464	0.5850	0.8431	1.0944	1.3638	1.6327	1.8769	2.1613	2.4382	2.7071	2.9633	3.2349
	0.0590	0.1230	0.1665	0.2801	0.5293	0.8126	1.0712	1.3457	1.6211	1.8590	2.1482	2.4301	2.6973	2.9527	3.2305
RW	0.0815	0.1601	0.2137	0.3453	0.5827	0.8395	1.0896	1.3580	1.6261	1.8695	2.1526	2.4287	2.6966	2.9518	3.2235
	0.0592	0.1235	0.1670	0.2789	0.5270	0.8083	1.0659	1.3394	1.6140	1.8512	2.1391	2.4202	2.6867	2.9409	3.2188

See notes in Table 2 for explanation.

Table 5: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 3-Month Euroswiss Franc Rate

	k-Step Ahead Horizon (Number of Point Forecasts)														
Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
$d = 0.215$, AR(4)	0.1169	0.1781	0.2181	0.3195	0.3987	0.4436	0.4914	0.4876	0.5364	0.6280	0.6080	0.6567	0.7001	0.9051	1.1476
	0.0837	0.1292	0.1619	0.2492	0.3049	0.3138	0.3637	0.3469	0.4126	0.4702	0.4379	0.4741	0.5190	0.6897	0.9210
$d = 0.152$, AR(4)	0.1164	0.1747	0.2127	0.3081	0.3968	0.4685	0.5291	0.5549	0.6211	0.7135	0.7516	0.8345	0.9501	1.1778	1.4410
	0.0831	0.1263	0.1582	0.2402	0.3024	0.3403	0.3824	0.4104	0.4837	0.5469	0.5888	0.6751	0.8610	1.0653	1.3333
AR(1)	0.1156	0.1809	0.2222	0.3446	0.5395	0.7320	0.8911	1.0399	1.1981	1.3639	1.5476	1.7362	1.9685	2.2524	2.5640
	0.0815	0.1295	0.1680	0.2656	0.4375	0.6228	0.7908	0.9540	1.1224	1.2906	1.4879	1.6924	1.9508	2.2298	2.5452
RW	0.1142	0.1796	0.2213	0.3445	0.5405	0.7340	0.8938	1.0435	1.2027	1.3692	1.5538	1.7433	1.9766	2.2612	2.5734
	0.0800	0.1287	0.1674	0.2656	0.4388	0.6253	0.7943	0.9583	1.1276	1.2965	1.4947	1.7000	1.9592	2.2390	2.5550

Notes: The forecasting performance of the long memory model corresponding to $d = 0.210$ is not reported as it is essentially identical to the one for $d = 0.215$. See notes in Table 2 for additional explanation of the table.

Table 6: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 6-Month Euroswiss Franc Rate

	k-Step Ahead Horizon (Number of Point Forecasts)														
Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
d = 0.285, AR(6)	0.1212	0.1777	0.2263	0.3170	0.3979	0.4740	0.5338	0.5398	0.5677	0.6374	0.5911	0.6308	0.6124	0.7706	0.9305
	0.0810	0.1316	0.1691	0.2477	0.3032	0.3406	0.4102	0.3889	0.4438	0.4950	0.4331	0.4903	0.4532	0.5935	0.7160
d = 0.308, AR(6)	0.1214	0.1789	0.2289	0.3239	0.4079	0.4844	0.5484	0.5604	0.6007	0.6842	0.6457	0.6882	0.6465	0.7860	0.9251
	0.0812	0.1323	0.1708	0.2518	0.3083	0.3509	0.4236	0.4043	0.4711	0.5374	0.4846	0.5449	0.4884	0.6173	0.7214
d = 0.211, AR(5)	0.1210	0.1765	0.2211	0.3028	0.3854	0.4734	0.5361	0.5457	0.5566	0.6050	0.5872	0.6550	0.7512	0.9613	1.1785
	0.0806	0.1306	0.1658	0.2394	0.2970	0.3351	0.4060	0.4088	0.4335	0.4505	0.4334	0.5072	0.5973	0.7969	1.0372
AR(2)	0.1203	0.1796	0.2290	0.3437	0.5552	0.7689	0.9434	1.0838	1.2058	1.3538	1.5381	1.7530	2.0228	2.3250	2.6317
	0.0789	0.1305	0.1715	0.2726	0.4663	0.6479	0.8234	0.9730	1.1217	1.2831	1.4785	1.7079	2.0041	2.3049	2.6213
RW	0.1241	0.1835	0.2327	0.3455	0.5570	0.7707	0.9461	1.0874	1.2101	1.3592	1.5439	1.7591	2.0299	2.3331	2.6419
	0.0799	0.1327	0.1730	0.2730	0.4673	0.6502	0.8263	0.9769	1.1264	1.2877	1.4835	1.7136	2.0108	2.3126	2.6309

See notes in Table 2 for explanation.

Table 7: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 3-Month Euroyen Rate

	k-Step Ahead Horizon (Number of Point Forecasts)														
Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
Long Memory															
$d = 0.250$, AR(5)	0.0484	0.0846	0.1161	0.1920	0.2675	0.3371	0.4096	0.4293	0.3682	0.3464	0.3663	0.3889	0.5098	0.5625	0.5102
	0.0345	0.0606	0.0856	0.1477	0.2156	0.2827	0.3313	0.3387	0.2807	0.2865	0.3101	0.3158	0.4387	0.5331	0.4890
$d = 0.190$, AR(4)	0.0483	0.0841	0.1149	0.1884	0.2639	0.3344	0.4060	0.4274	0.3776	0.3683	0.4000	0.4487	0.5730	0.6532	0.6248
	0.0343	0.0601	0.0840	0.1446	0.2111	0.2806	0.3323	0.3431	0.2873	0.2964	0.3332	0.3875	0.4983	0.6345	0.6112
AR(1)	0.0482	0.0844	0.1181	0.2022	0.3196	0.4361	0.5476	0.6177	0.6577	0.7293	0.8341	0.9758	1.1426	1.2814	1.3397
	0.0331	0.0576	0.0828	0.1479	0.2437	0.3634	0.4771	0.5539	0.6172	0.6956	0.8033	0.9579	1.1253	1.2768	1.3349
RW	0.0491	0.0847	0.1184	0.2023	0.3194	0.4358	0.5473	0.6175	0.6575	0.7289	0.8336	0.9752	1.1419	1.2808	1.3390
	0.0334	0.0578	0.0832	0.1481	0.2437	0.3632	0.4767	0.5536	0.6167	0.6951	0.8027	0.9572	1.1245	1.2761	1.3341

Notes: The forecasting performance of the long memory model corresponding to $d = 0.244$ is not reported as it is essentially identical to the one for $d = 0.250$. See notes in Table 2 for additional explanation of the table.

Table 8: Out-of-Sample Forecasting Performance of Alternative Modeling Strategies: 6-Month Euroyen Rate

Forecasting Model	1 (347)	5 (343)	10 (338)	24 (324)	48 (300)	72 (276)	96 (252)	120 (228)	144 (204)	168 (180)	192 (156)	216 (132)	240 (108)	264 (84)	288 (60)
	k-Step Ahead Horizon (Number of Point Forecasts)														
Long Memory															
$d = 0.298$, AR(6)	0.0458	0.0806	0.1129	0.1885	0.2725	0.3534	0.4447	0.4929	0.4805	0.4663	0.4105	0.3935	0.5175	0.5843	0.5375
	0.0325	0.0592	0.0836	0.1456	0.2259	0.2841	0.3585	0.3989	0.3693	0.3680	0.3416	0.3115	0.4209	0.5293	0.5154
$d = 0.237$, AR(4)	0.0455	0.0841	0.1149	0.1884	0.2639	0.3344	0.4060	0.4274	0.3776	0.3683	0.4000	0.4487	0.5730	0.6532	0.6248
	0.0323	0.0601	0.0840	0.1446	0.2111	0.2806	0.3323	0.3431	0.2873	0.2964	0.3332	0.3875	0.4983	0.6345	0.6112
AR(1)	0.0456	0.0808	0.1158	0.2024	0.3320	0.4583	0.5740	0.6518	0.7056	0.7755	0.8583	0.9830	1.1522	1.2996	1.3725
	0.0314	0.0579	0.0866	0.1631	0.2632	0.3829	0.4786	0.5492	0.6231	0.7132	0.8193	0.9582	1.1290	1.2898	1.3669
RW	0.0465	0.0809	0.1160	0.2020	0.3308	0.4563	0.5714	0.6486	0.7016	0.7706	0.8524	0.9762	1.1445	1.2912	1.3634
	0.0315	0.0583	0.0869	0.1627	0.2622	0.3810	0.4762	0.5455	0.6184	0.7078	0.8130	0.9512	1.1210	1.2812	1.3577

Notes: The forecasting performance of the long memory model corresponding to $d = 0.290$ is not reported as it is essentially identical to the one for $d = 0.298$. See notes in Table 2 for additional explanation of the table.

Table 9: Relative Forecasting Performance of the Long Memory and the Random Walk Models

	k-Step Ahead Horizon														
Forecasting Model	1	5	10	24	48	72	96	120	144	168	192	216	240	264	288
CD (3-month), $d = 0.213$	1.0015	1.0302	1.0647	1.1245	1.1060	1.1771	1.2055	1.2191	1.2115	1.2167	1.2129	1.2072	1.2008	1.2530	1.3584
	0.9992	1.0318	1.0238	1.0102	1.0585	1.1017	1.1728	1.1798	1.0587	0.9796	0.9591	0.9844	1.0706	1.1668	1.2789
GM (3-month), $d = 0.181$	1.0103	0.9946	0.9663	0.8898	0.7753	0.7031	0.6996	0.7062	0.7247	0.7374	0.7522	0.7547	0.7723	0.7829	0.7947
	1.0051	0.9821	0.9715	0.9102	0.7141	0.6399	0.6485	0.6636	0.7058	0.7128	0.7319	0.7431	0.8289	0.7680	0.7839
GM (6-month), $d = 0.148$	0.9988	0.9994	0.9513	0.8520	0.7258	0.6662	0.6545	0.6624	0.6730	0.6827	0.6910	0.6944	0.7110	0.7195	0.7266
	1.0135	1.0040	0.9689	0.8594	0.6704	0.6086	0.6073	0.6273	0.6553	0.6588	0.6723	0.6827	0.7007	0.7074	0.7196
SF (3-month), $d = 0.215$	1.0236	0.9916	0.9855	0.9274	0.7377	0.6044	0.5498	0.4673	0.4460	0.4587	0.3913	0.3767	0.3542	0.4003	0.4459
	1.0463	1.0039	0.9671	0.9383	0.6948	0.5018	0.4579	0.3620	0.3659	0.3627	0.2930	0.2789	0.2649	0.3080	0.3605
SF (6-month), $d = 0.308$	0.9782	0.9749	0.9837	0.9375	0.7323	0.6285	0.5120	0.5154	0.4964	0.5034	0.4182	0.3912	0.3185	0.3369	0.3502
	1.0163	0.9970	0.9873	0.9223	0.6597	0.5397	0.4247	0.4139	0.4182	0.4173	0.3267	0.3180	0.2429	0.2669	0.2742
JY (3-month), $d = 0.250$	0.9857	0.9988	0.9806	0.9491	0.8375	0.7735	0.7484	0.6952	0.5600	0.4752	0.4394	0.3988	0.4464	0.4392	0.3810
	1.0329	1.0484	1.0288	0.9973	0.8847	0.7784	0.6950	0.6118	0.4552	0.4122	0.3863	0.3299	0.3901	0.4178	0.3665
JY (6-month), $d = 0.298$	0.9849	0.9963	0.9733	0.9332	0.8238	0.7745	0.7783	0.7599	0.6849	0.6051	0.4816	0.4031	0.4522	0.4525	0.3942
	1.0317	1.0154	0.9620	0.8949	0.8616	0.7457	0.7528	0.7313	0.5972	0.5199	0.4202	0.3275	0.3755	0.4131	0.3796

Notes: The long memory model for each series is the one corresponding to the highest d estimate. Similar results are obtained for the other long memory models reported in previous tables. The first (second) entry in each cell is the ratio of the RMSE (MAD) value achieved by the long memory model to that of the random-walk-with-drift model. See Table 2 for additional explanation of the table.