THE ROLE OF POLLUTEE TAXES
IN SEQUENTIALLY ADJUSTING
TO EXTERNALITIES PROBLEMS

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Discussion Paper #42

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In dealing with externalities problems, a "pure Pigouvian tax (or subsidy)" can be defined as one that is levied at socially optimal marginal damage (benefit) rates--rates that would be required to keep the parties involved in a globally optimum configuration of locations, output levels, and technologies once that optimum configuration has been reached. Information requirements severely limit the possibility of imposing pure Pigouvian taxes. In all but the simplest situations, approximating them with any degree of accuracy would be very costly if not impossible.

In part with these informational requirements in mind, several writers--Baumol (1972), Coase (1960), and Mohring-Boyd (1971) as examples--have analyzed the allocative effects of tax schemes in which externality imposing agents are taxed at rates reflecting current rather than socially optimal damage levels. Under these schemes, tax rates are altered recursively to reflect the changes in current damage levels that adjustments to prior taxes have induced. Provided, of course, that they converge to a social optimum, informational considerations make such adjustment processes considerably more attractive[1] than pure Pigouvian taxes.

In the context of a dynamic model, Baumol has shown that a sequential adjustment process does converge to a social optimum if the dynamic system is stable <u>and</u> there is a unique local optimum. Coase and Mohring-Boyd have found that, in models with multiple local optima (about which neither was explicit), convergence to a global optimum requires coupling a sequential damage tax with appropriate taxes on those affected by externalities. In this article, we attempt to show that, in the presence of multiple local optima, there do exist circumstances in which sequential imposition of bilateral taxes would effect optimal conditions, whereas imposition of unilateral taxes would not. However, the converse is also true. Moreover, situations exist in which neither system effects optimality. Our unhappy conclusion, then, is that, in externalities problems involving multiple local optima,[2] there is no inexpensive way to achieve global optimality.

I. The Need for a Double Tax System?

We start by reexamining the argument put forth by Coase, who drew on the following example (p. 41):

> Assume that a factory which emits smoke is set up in a district previously free from smoke pollution, causing damage valued at $100 per annum. Assume that the taxation solution is adopted and that the factory owner is taxed $100 per annum as long as the factory emits the smoke. Assume further that a smoke-preventing device costing $90 per annum to run is available. In these circumstances, the smoke preventing device would be installed . . . . Yet the position achieved may not be optimal. Suppose that those who suffer the damage could avoid it by moving to other locations or by taking various precautions which would cost them, or be equivalent to a loss in income of, $40 per annum. Then there would be a gain in the value of production of $50 if the factory continued to emit its smoke and those now in the district moved elsewhere or made other adjustments to avoid the damage. If the factory owner is to be made to pay a tax equal to the damage caused, it would clearly be desirable to institute a double tax system and to make residents of the district pay an amount equal to the additional cost incurred by the factory owner . . . in order to avoid the damage. In these conditions, people would not stay in the district or would take other measures to prevent the damage from occurring, when the costs of doing so were less than the costs that would be incurred by the producer to reduce the damage . . .

> A tax system which was confined to a tax on the producer for damage caused would tend to lead to unduly high costs being incurred for the prevention of damage. Of course this could be avoided if it were possible to base the tax, not on the damage caused, but on the fall in the value of production (in its widest sense) resulting from the emission of smoke.

Coase failed to elaborate on features of his model that are essential to his position, thereby making generalization difficult. In seeking to clarify matters, Baumol identified two distinctive features of the Coase model. These are the existence of multiple local optima and the dynamic adjustment process with which Coase was concerned. As for the latter, Coase seems to have had in mind that the per unit tax on the factory's output be equated to its current marginal social damage and that it be recomputed with each change in household location (imposition of the tax affects factory output affects household locations affects current marginal damage affects the tax). This process is to be distinguished from the pure Pigouvian tax which is equal to marginal social damage at the socially optimal rather than current level and which does not change with changes in household location.

Baumol demonstrated the indispensability of these two features to any rationale for bilateral taxes. He first used the Coase model to show that, even in the presence of multiple local optima, the pure Pigouvian tax suffices to effect an optimal allocation of resources. He then showed in the context of a dynamic model based on the sequential tax scheme that, ignoring the wastes and irreversibilities involved in such a learning process, the sequential adjustment process would converge to the social optimum <u>without</u> the imposition of a pollutee tax if the dynamic system were stable <u>and</u> there were a unique local optimum. He concluded the section (p. 315):

> The reason this process of simultaneous learning and adjustment does not work in Coase's example is that it involves (at least) two local maxima, as we have already noted. And in such a case, obviously, the adjustment mechanism may well take us to the wrong maximum.

But the Baumol article does not deal with the question, "Is there a mechanism by which an optimal allocation of resources might be attained with a sequential pollution tax in the presence of multiple local optima?" We are therefore left with neither sanction nor discredit for the rationale provided by Coase for a double tax system.

In another recent article, Mohring and Boyd also argued for bilateral taxes. They were more explicit than Coase not only as to the circumstances under which pollutee taxes were being advocated, but also as to the nature of the appropriate taxes. The specific example from which they generalized was that of the optimum location of a bathing beach along a river bank. Consider an initial situation in which only factories that sell their products and buy their inputs in competitive markets are located along it. Each factory benefits from using the river to dispose of its wastes. None requires pure water in its production process. Pollution of the ocean into which the river flows is of no concern. Under these circumstances, efficiency dictates that the river be used as a sewer. Each factory should increase its waste disposal to the point where its marginal physical product is zero. No expenditures that would improve the quality of river water would be justified.

Now suppose that a private entrepreneur or public agency has decided to undertake a new river oriented activity--a bathing beach, say--and is choosing between two sites, one upstream, the other downstream from all the factories. The bathing beach will generate no wastes. However, the value to it of the marginal product of water quality will be high. Suppose that the public authority responsible for the river basin responds to the bathing beach entrepreneur's location decision by putting into effect whatever system of pollution taxes and river basin capital improvements would be optimal given that decision.

Choice of the upstream location would require no changes in the behavior of factories. Choice of the downstream site would require the imposition of pollution taxes on factories to induce them to reduce waste disposal levels. Doing so would require either reduction in their output levels or the diversion of inputs from the production of final outputs to the elimination of wastes. Either way, the contribution of factories to "social welfare"--the difference between the aggregate market values of their outputs and the aggregate wages paid their inputs--would decline.

In brief, choice of the upstream site would not change the contributions of the factories to social welfare; choice of the downstream site would reduce it. Maximizing welfare, Mohring and Boyd argued, would require this fact to be reflected in the bathing beach entrepreneur's location decision.[3] The technique they suggested for accomplishing this end was making it known that location of the bathing beach downstream would result in the imposition of a franchise or site use tax equal to the difference between the contribution of the factories to social welfare with and without the bathing beach. To repeat, their "contribution to social welfare" in this context equals the difference between their aggregate revenues and the aggregate wages they pay their hired inputs.

Mohring and Boyd generalized this conclusion in the following terms (p. 355):

> The establishment or expansion of any activity in which some aspect of environmental quality has a positive marginal product may have opportunity costs not normally born directly by those undertaking it. The expansion of a quality-enjoying activity would reduce the socially optimal levels of activities that degrade the quality attribute in question. Efficiency would require this fact to be taken into account. One way of doing so would be to impose a site or franchise tax on each quality-enjoying activity equal to the increase in the net social product of polluting activities that would result if it went out of existence.

They went on to argue that the fault is not with the "tax only" solution in dealing with air and water pollution problems as such. "Tax only" would be fine if optimal (i.e., pure Pigouvian) taxes were imposed once and for all. Rather, they claimed, the fault is with "tax only" coupled with a sequential adjustment process in which taxes are recomputed so as, in effect, to make the services provided pollutees at one stage of the process depend on their location, scale, or other decisions during the preceding stage. Inefficiency is, they felt, inevitable if locational and other decisions are made without taking into account all of the costs (including those to polluters) contingent upon them.

Baumol's work makes it clear that Mohring and Boyd drew an erroneous generalization from their example. The sequential adjustment of pollution taxes does not in itself provide a valid rationale for pollutee taxes. Also necessary is a second ingredient that was present in their model but not appreciated by them, multiple local optima. But the question still remains, "Does a valid rationale exist for pollutee taxes?"

II. A Simple Pollution Model

In an attempt to resolve this problem, we have developed a simple example which replicates the results of Baumol's more general models, but, at the same time, demonstrates the virtues and deficiencies of site use taxes. Briefly, the scenario is as follows: Firm 2, a laundry, is situated downwind of a city in which it sells its services in a competitive market. The unit cost of transporting clothing between the laundry and the market, borne by the laundry, is proportional to the distance between the two. Firm 1, a competitive Scotch distillery located in the city, emits smoke in fixed proportion to its output. This smoke increases the unit costs of laundering by an amount that is inversely proportional to the distance between the two firms. Specifically, the profit function of the laundry is:

$$\Pi_2 = Y_2 - A_2Y_2^{\,2}/2 - K_1Y_2D - K_2Y_1^{\,2}Y_2/D \qquad (1b)$$

where Y_i is Firm i's output, with a price of $1 per unit, D is the distance between Firm 2 and the market, and A_2, K_1, and K_2 are constants.

Initially, no constraints are imposed on the distillery's Scotch/smoke output. Its profit function under these circumstances is

$$\Pi_1 = Y_1 - A_1 Y_1^{\,2}/2 \tag{1a}$$

The distillery therefore establishes that output/emission rate which yields zero marginal (private) profits. These profits are $\Pi_1^{\,o}$ per week. The laundry establishes that output and location which maximize its profits given the distillery's behavior. Its optimum with respect to location involves balancing the increase in transportation costs with greater distance against the decrease in laundering costs. Its profits in this equilibrium are $\Pi_2^{\,o}$.

After this equilibrium has been established, a tax is imposed on the distillery reflecting the damage puffs of smoke impose on the laundry given the latter's initial output and location. This tax induces the distillery to reduce its Scotch/smoke output. This change makes a new location and output level optimal for the laundry. The distillery tax is adjusted to reflect this new location and output level and the process repeats itself. In this process, neither the distillery nor the laundry becomes aware of the effects its actions have on the pollution tax and whatever tax on the laundry may accompany it.

Answers to the following questions are of interest:

1. What tax system would be required to effect, once and for all, an optimal location for the laundry and an optimal allocation of resources to the two firms?

2. Would the sequential adjustment process described in the preceding paragraph converge to the optimum identified in the answer to question 1?

3. Suppose that (a) during this adjustment process or (b) after the new equilibrium to which it gives rise has been reached, an unpolluted site for the laundry becomes available upwind of the distillery. Would sequentially adjusted smoke emission and site use taxes effect an optimal resource allocation?

Since most of the implications of the formal model used in seeking the answers to these questions can be described with the aid of the simple geometry developed by Turvey (1963), presentation of the formal model itself is relegated to an appendix.

The distillery's marginal benefit per puff of smoke is shown by ABC in Figure 1, while the laundry's marginal damage function is given by DBE. In the absence of controls on the distillery and of negotiations between it and the laundry--again, it seems reasonable to ignore the possibility of negotiation in most real world pollution problems of any consequence--the distillery would emit C puffs of smoke a week. This is the level at which its marginal benefit from smoke emission is zero. Efficiency would dictate that, if

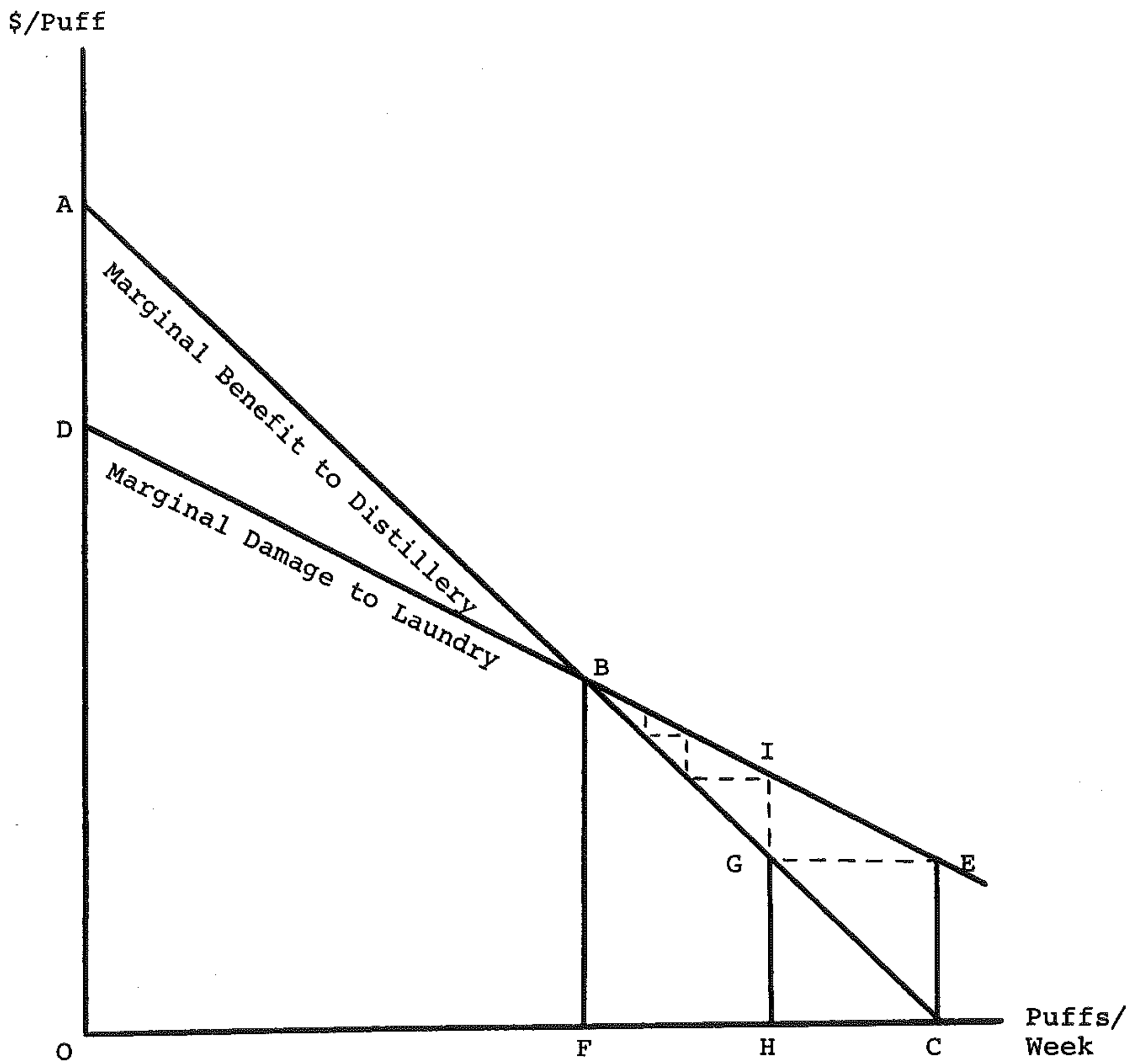
$/Puff
A
D
Marginal Benefit to Distillery
Marginal Damage to Laundry
B
I
G
E
O
F
H
C
Puffs/
Week

Figure 1

the laundry is to remain downwind, smoke emission be reduced to F puffs per week. Again ignoring negotiation, this could be accomplished by imposing on the distillery a tax which, at the margin, yields BF per puff of smoke.[4] Imposition of this tax would yield a net social gain of BCE--the difference between the damage to the laundry and the gain to the distillery from emitting the last FC puffs of smoke per week. In the absence of a tax on the laundry, it would receive this entire gain plus the loss, BCF, incurred by the distillery in reducing its emissions. That is, the following relationship holds:

$$\Pi_2^{\,*} = \Pi_2^{\,o} + BFCE = \Pi_2^{\,o} + BCE + (\Pi_1^{\,o} - \Pi_1^{\,*})$$

where $\Pi_1^{\,*}$ and $\Pi_2^{\,*}$ denote the weekly profits of the two firms in this new equilibrium.

Suppose, alternatively, that the effects on the laundry of the distillery's smoke emissions had not been incorporated into the pricing system by an initial tax of BF but rather through a sequential adjustment process of the sort described by Baumol. Suppose, specifically, that the initial marginal tax levied on the distillery was not BF, the marginal damage it would do to the laundry under optimum conditions, but rather EC, marginal damage at the original C puff per week equilibrium. Imposition of such a tax would induce the distillery to reduce its smoke output to H puffs per week. The laundry's profit maximizing response would be to adjust its output and location so as to increase the marginal damage caused by H puffs per week to IH. Increasing the marginal tax on the distillery to this amount would lead it to reduce its smoke output further resulting in another adjustment by the laundry and so forth. Clearly, this sequence of adjustments--the dotted lines shown in Figure 1--would converge to the F puff per week optimum and would do so, as Baumol demonstrated, in the absence of site taxes.

Now suppose that, after the F puff per week equilibrium is established, an alternative upwind site becomes available to the laundry, and that its weekly profits at this new site--Π_2^{**}--would exceed those in the original C puff per week equilibrium by an amount somewhat greater than area BCE but somewhat less than area BFCE. Relocation of the laundry would obviate the need for restrictions on the distillery's output and hence would increase the total contribution of the two firms to social welfare--

$$\Pi_1^{o} + \Pi_2^{**} > \Pi_1^{*} + \Pi_2^{*}$$

But the laundry would have no incentive to make this welfare increasing move <u>if</u> the F puff per week equilibrium is maintained only by taxing the distillery. The laundry <u>would</u> relocate, however, if the distillery tax is accompanied by a site tax,

$$S^{*} = \Pi_1^{o} - \Pi_1^{*} = BCF$$

of the sort advocated by Mohring and Boyd. Furthermore, if Π_2^{**} exceeds Π_2^{o} by an amount somewhat less than area BCE, so that relocation of the laundry is not in society's interest, this same site tax would not induce an upwind move.

Now suppose that the potential upwind laundry site becomes available during the Baumol sequential adjustment process, say after the first round with the distillery emitting H puffs of smoke per week. The efficiency implications of imposing site taxes should again be considered under two alternative possibilities: that the upwind location (a) is and (b) is not socially optimal--i.e., that the laundry's relocation there (a) would and (b) would not increase the joint profits of the two firms. Suppose first that relocation would be efficient--i.e.,

$$\Pi_2^{**} > \Pi_2^{o} + BCE \qquad (2)$$

and that the appropriate site tax, GCH, is imposed. The laundry's after-tax downwind profits would be Π_2^{o} + IGCE, which, by (2), would be less than Π_2^{**}. This being the case, the laundry would relocate. Indeed, it might even relocate in the absence of a site tax. Assuming that

$$\Pi_2^{**} < \Pi_2^{o} + BFCE$$

does not rule out the possibility that

$$\Pi_2^{**} > \Pi_2^{o} + IHCE$$

In the absence of a site tax, efficient relocation might come about if the upwind site were discovered at an early enough stage of adjustment. But with a site tax, efficient relocation would occur at whatever stage of adjustment the upwind site availed itself.

Alternatively, if

$$\Pi_2^{**} < \Pi_2^{o} + BCE \tag{3}$$

efficiency would dictate establishing a downwind equilibrium with F puffs of smoke per week. With a site tax of GCH, the laundry would remain downwind only if

$$\Pi_2^{**} < \Pi_2^{o} + IGCE \tag{4}$$

But there is no guarantee that satisfaction of (3) implies satisfaction of (4). _If_ a site tax is incorporated into the sequential adjustment process, and _if_ relationship (4) does _not_ hold when, during this process, the upwind site becomes available, _then_ an inefficient relocation of the laundry would result. Indeed, having inequality (3) hold does not even guarantee that

$$\Pi_2^{**} < \Pi_2^{o} + IHCE$$

That is, there is no guarantee that discovery of the upwind location early in the sequential adjustment process would not lead to an inefficient relocation even if no site tax is imposed. If inefficient relocation would result without a site tax, it would certainly result with one; the converse, however, is not true.

III. Conclusions

Coase and Mohring-Boyd have found pollutee taxes to be desirable. We have found that these taxes possess both desirable and undesirable properties. What, then, is the relationship between our findings and those of Coase and Mohring-Boyd?

The answer can be seen by noting that, in answering question 3 of Section II, had we confined ourselves only to question 3b, we would have reached the same conclusions as did Coase and Mohring-Boyd. That is, had we, in the context of our dynamic model, confined our attention only to equilibrium positions, we would have found site taxes to have the desirable properties suggested by these writers.

Although generalizing from simple models is often risky, it seems reasonable to draw the following conclusions from the models discussed in this and the Baumol papers:

1. In a world not troubled by the existence of multiple local optima, the imposition of suitable taxes on polluters only would suffice to achieve a social optimum. This conclusion holds regardless of whether taxes are imposed at the socially optimum rate from the start or at rates reflecting current damage levels that are sequentially adjusted.

2. In a world of multiple local optima, neither sequentially adjusted polluter taxes alone nor such taxes

coupled with sequentially adjusted site taxes on pollutees could be relied on to achieve a global optimum. A system of paired sequential taxes would assure relocation if (but unfortunately not only if) it is efficient. Sequential pollution taxes only might result in some inefficient relocation but also would likely result in failure to relocate when relocating would be efficient.

3. In brief, there appears to be no inexpensive mechanism for obtaining global optimality in externalities problems involving multiple local optima.

Appendix: The Algebra of the Pollution Model Described in Section II

To repeat, the specific profit functions of Firms 1 and 2 in the absence of taxes are respectively:

$$\Pi_1 = Y_1 - A_1 Y_1^{2}/2 \tag{1a}$$

$$\Pi_2 = Y_2 - A_2 Y_2^{2}/2 - K_1 Y_2 D - K_2 Y_1^{2} Y_2/D \tag{1b}$$

Since the algebra involved in answering the questions raised in Section II is straightforward if sometimes tedious, only the final results are given. Equating to zero the partial derivative of (1b) with respect to D yields

$$D = (K_2/K_1)^{1/2} Y_1 \tag{5}$$

as the profit maximizing downwind location for Firm 2. This relationship holds regardless of the tax environment in which the firms operate. That is, if conditions are such that it is profitable for Firm 2 to locate downwind, the location chosen will satisfy (5).

Using (5) to eliminate D in equation (1b) and setting the derivative with respect to Y_2 of the resulting expression equal to zero yields

$$Y_2 = [1 - 2(K_1 K_2)^{1/2} Y_1]/A_2 \tag{6}$$

as the output which would maximize Firm 2's profits given Firm 1's output. Substituting (5) and (6) into (1b) and differentiating with respect to Y_1 yields

$$\partial \Pi_2/\partial Y_1 = -\partial L_2/\partial Y_1 = -2(K_1K_2)^{1/2}Y_2 \tag{7}$$

where $\partial L_2/\partial Y_1$ denotes the marginal loss to Firm 2 from a change in Firm 1's output. Since Y_2 is a diminishing function of Y_1, equation (7) implies that Firm 2's marginal loss is a diminishing function of Firm 1's output.[5]

Equating to zero the partial derivatives of the sum of (1a) and (1b) with respect to Y_1, Y_2, and D yields equation (5) and

$$1 - A_1Y_1 - 2K_2Y_1Y_2/D = 0 \tag{8a}$$

$$1 - A_2Y_2 - K_1D - K_2Y_1{}^2/D = 0 \tag{8b}$$

Solving these equations for the joint profit maximizing output levels as functions of system parameters yields

$$Y_i{}^* = [A_j - 2(K_1K_2)^{1/2}]/[A_1A_2 - 4K_1K_2]; \quad i \neq j; \quad i, j = 1, 2 \tag{9}$$

For these output levels to be positive, the numerator on the right hand side of equation (9) must be positive for j = 1 and 2. If these expressions <u>are</u> positive, the solution given by equations (5) and (9) is a maximum. That is, the matrix of second derivatives of the sum of (1a) and (1b) is negative definite.

The third term on the left side of equation (8a) is the marginal effect of Firm 1's output (or, equivalently, of the smoke emitted in order to produce it) on Firm 2's profits. If this effect is evaluated under joint profit maximizing conditions, and a tax of

$$T^* = 2K_2Y_1^*Y_2^*/D^* = 2(K_1K_2)^{1/2}Y_2^* \tag{10}$$

is imposed on each unit of Firm 1's output, its profit function would change from equation (1a) to

$$\Pi_1 = (1 - T^*)Y_1 - A_1Y_1^2/2 \tag{11}$$

If Firm 1 chooses Y_1 so as to maximize this function, the socially optimum output of commodity 1 would result. In the absence of any special incentives, Firm 2 would act to satisfy equations (5) and (8b). With the socially optimum value of Y_1 substituted in them, satisfaction of these equations would lead to the socially optimum levels of Y_2 and D. Thus, the only tax required to generate socially optimal output levels is one levied on Firm 1's output at a rate equal to the marginal damage that the associated smoke imposes on Firm 2--"marginal damage" being evaluated, of course, under optimal conditions. No tax--site or otherwise--on Firm 2 would be required.

Suppose that Firm 1 initially produces $Y_1^{(o)}$ units of output. $Y_1^{(o)}$ could but need not necessarily be Firm 1's profit maximizing output in the absence of pollution taxes. Taking this value parametrically, Firm 2 selects it location and output so as to satisfy equations (5) and (8b), respectively. Its output is given by

$$Y_2^{(o)} = [1 - 2(K_1K_2)^{1/2}Y_1^{(o)}]/A_2 \tag{12}$$

Now suppose that a public authority imposes a per unit tax on Firm 1's output equal to the marginal rate at which its smoke currently reduces Firm 2's profits. This tax (see the discussion of equation (10)) is equal to $T^{(o)} = 2(K_1K_2)^{1/2}Y_2^{(o)}$. Firm 1 takes this tax parametrically and selects its new profit maximizing level of output. Accepting this new output, $Y_1^{(1)}$, as parametric, Firm 2 selects a new profit maximizing location and output level. The tax on Firm 1 is adjusted to reflect these new values for Firm 2, and this process repeats itself.

In the i-th iteration of this process, Firm 1's profit maximizing output level equals

$$Y_1^{(i)} = [1 - T^{(i-1)}]/A_1 = [1 - 2(K_1K_2)^{1/2}Y_2^{(i-1)}]/A_1.$$

Substituting this expression in the equation for Firm 2's profit maximizing output level in period i (an equation identical to (12) except for the replacement of $^{(o)}$ by $^{(i)}$) yields an expression which can be written

$$Y_2^{(i)} = B + CY_2^{(i-1)} \tag{13}$$

where $B = [A_1 - 2(K_1K_2)^{1/2}]/A_1A_2$, and $C = 4K_1K_2/A_1A_2$. If the joint profit maximizing output levels of the two firms are to be positive, the numerator on the right side of the equation (9) must, to repeat, be positive for $j = 1$ and 2. This being the case, $0 < C < 1$, and the first order difference equation (13) has a solution $Y_2^{(i)} = C^iY_2^{(o)} + B[(1 - C^i)/(1 - C)]$. Then the limit of Firm 2's output as i approaches infinity is $B/(1 - C) = [A_1 - 2(K_1K_2)^{1/2}]/[A_1A_2 - 4K_1K_2]$, which is precisely the value given by equation (9). Thus, a site tax would not be required to maximize welfare even if a sequential adjustment process is employed.

FOOTNOTES

[1] Ignoring the wastes and irreversibilities that they entail.

[2] An all too likely situation. See the article by Baumol and the references cited there.

[3] One might argue with Coase that negotiations between factories and the bathing beach entrepreneurs could be relied on to achieve this result. But entirely apart from other objections to negotiation (see Section III of Mohring-Boyd), in most real world pollution problems of any significance, the numbers involved are so great as to make negotiation totally unrealistic.

[4] Given the assumed immobility of the distillery, an excise tax on smoke output would suffice. An appropriate excise tax on Scotch output would also suffice, given our assumption of a fixed coefficient between Scotch/smoke output. The appendix development is in terms of an excise tax on Scotch output.

[5] On inserting (6) for Y_2^* in equation (7), the intercept of $\partial L_2/\partial Y_1$ with the \$/puff axis can be seen to equal $2(K_1K_2)^{1/2}/A_2$--a number which (see the discussion of equation (9) below) must be less than 1. On differentiating (1a) with respect to Y_1, $\partial \Pi_1/\partial Y_1$ can be seen to have a \$/Puff axis intercept of 1 and a slope of $-A_1$.

The slope of $\partial L_2/\partial Y_1$ is $-4K_1K_2/A_2$ which (again see the discussion of equation (9) below) must be smaller in magnitude than A_1. Hence, $\partial \Pi_1/\partial Y_1$ and $\partial L_2/\partial Y_1$ have the general shapes shown in Figure 1.

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