

Expanding Demand through Price Advertisement*

Hideo Konishi[†]

Michael T. Sandfort[‡]

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Abstract

Retail stores frequently advertise prices. When consumer search is costly, advertising low prices expands the number of consumers who visit and buy. We show that this demand enhancement may offset a retailer's loss in margins from the low price, making advertisement desirable. We further show that a multi-product monopolist may choose to advertise low prices only for select products, even when advertised and non-advertised products are substitutes. For duopolist retailers at a common location, an equilibrium exists in which (for some range of advertising costs) one store advertises and the other free-rides on its competitor.

JEL Codes: D4, L0, M3.

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[†]Corresponding author. Department of Economics, Boston College, Chestnut Hill, MA 02467-3806, (phone): 617-552-1209, (fax): 617-552-2308, (e-mail): hideo.konishi@bc.edu

[‡]Antitrust Division, US Department of Justice, 600E St. NW, Suite 10000, Washington DC 20530, (phone): 202-307-6376, (fax): 202-514-8862, (e-mail): michael.sandfort@usdoj.gov

1 Introduction

Price advertisement by retail stores is pervasive. Computer and audio stores advertise the prices of specific models or configurations. Car dealers advertise discounts for specific secondhand cars. These advertisements reach consumers via television, newspaper and direct mail, the latter in the form of sales circulars or coupons. Often, these advertised prices are substantially lower than commonly observed prices. This paper explores an incentive for retail stores to advertise (occasionally very low) prices for the commodities they sell.

The idea of the paper can be explained by describing our basic model, presented formally in Section 2. There is one retail store which sells one type of commodity. This commodity is a so-called search good: a commodity for which each consumer discovers her willingness-to-pay after search (a visit to the retail location), but before purchase (Nelson, 1970). Unless advertised by the retailer, the commodity's price is unknown to consumers before they visit the store. Visits to the store are costly and all such search costs, once incurred, are sunk. Search costs vary across consumers, and each consumer knows her own search cost.

If the retailer does not advertise its price, each consumer must infer the retailer's price in computing her expected net surplus from search. Since consumers know that search costs are sunk, they know that a visit to the retailer's location exposes them to opportunistic behavior (monopoly prices) by the retail monopolist. Thus, only consumers who have very low search costs choose to visit the store location, so the store has few customers. If, on the other hand, the retailer advertises its price, then laws against false advertisement commit the retailer to the advertised price. Although a low advertised price reduces the retailer's profit margin, the increased number of consumers choosing to visit the store may increase the retailer's total profit. Without advertisement, the retailer cannot commit to a low price, even if it knows that the low price, if honored, would result in higher profits.

Using the basic idea outlined above, we can answer more interesting questions. For example, it is commonly observed that computer stores, music instrument stores, and audio stores advertise the prices of a selected number of items among the ones they carry. These items are usually heavily

discounted, while others are not. Unlike the commodities sold by grocery stores, however, the commodities sold by the above retailers are substitutes for one another and consumers normally buy at most one unit. Why, then, do stores discount only a few items heavily? It seems likely that, given a choice between the discounted and undiscounted commodities, consumers would purchase only the discounted one.

In order to answer these questions, we generalize the basic model in Section 3, considering a monopolist retailer who sells multiple commodities (two commodities, for simplicity). Each consumer buys at most one of these two commodities, so the commodities are substitutes. Moreover, because there is no quality difference between the two commodities apart from consumer tastes, the commodities are *ex ante* identical. Finally, we assume that each consumer's willingnesses-to-pay for the commodities are uncorrelated. The retailer has three options: (a) do not advertise any commodity price, and thus pay no advertising costs; (b) advertise exactly one commodity price, paying the associated advertising cost; (c) advertise both commodity prices, paying the cost of advertising two commodities. Clearly, the parameters of the advertising cost function will affect the retailer's advertising decision. Nonetheless, our model shows that, for a wide range of advertising cost parameters, the retailer chooses to advertise only one commodity price.

The intuition underlying the above result is as follows. Consumers may not know exactly how much they prefer each of two different computers sold by a given retailer, for example, until they have an opportunity to visit the retail location and examine them. Some consumers may visit the retailer attracted by the low price of the advertised computer, but realize upon arriving at the store that they actually like the non-advertised computer much better than the advertised one. Even if the price of the non-advertised computer is higher, they will purchase it if their valuation is sufficiently high. Thus, advertising one commodity may pay, if (i) price advertisement can substantially increase the number of customers who visit the retail location, (ii) advertising one commodity is not excessively costly, while the marginal cost of advertising the second commodity is nonnegligible, and (iii) each consumer's willingnesses-to-pay for the two commodities are not too positively correlated.

Finally, Section 4 treats the case of two competing retail stores at the same site (e.g., in the

same shopping mall). Each retailer sells one commodity and can choose whether or not to advertise its price. The resulting advertising decision game may have asymmetric equilibria. When one retailer advertises, the other may not follow suit. A non-advertising retailer does not pay the advertising cost, yet benefits from the increase in consumer visits brought about by its competitor's advertisements, suggesting free-riding. Indeed, for some values of the advertising cost parameter, the retailer who does not advertise obtains a higher profit than the retailer who does.

Another interesting feature of the duopoly case is the diminished incentive for price advertisement in the presence of collocation. In the monopoly case of Section 2, the monopolist chooses to advertise mainly because rational consumers expect a non-advertised price to be the monopoly price. The case where the retailer faces a competitor at its location is very different, since consumers instead expect lower, duopoly, prices at the retail location. More consumers may therefore choose to visit the retail location, even without price advertisement.¹ Thus, the gain from advertising a low price in a duopoly setting is less than it is in the monopoly case. Consequently, both price advertisement and collocation with other stores act as commitment devices.² In the remainder of this section, we review the literature on price advertisement. Concluding remarks may be found in Section 5.

1.1 A Summary of the Literature

There is a huge literature on various kinds of advertising (see Schmalensee, 1986; Tirole, 1988; and Carlton and Perloff, 1994) and many authors have examined price advertisement in particular. In his pioneering work, Butters (1977) assumes that advertising provides consumers with information about the existence of retail firms selling homogeneous products and their prices.³ Each firm sends (multiple) adverts, which consumers receive randomly. A consumer who receives adverts from different firms buys from the firm advertising the lowest price. Market structure is monopolistically

¹This idea was first expressed in Dudey (1990) as an explanation for the geographical concentration of retail stores. Dudey's model is described more fully in the literature review below.

²Wernerfelt (1994) mentions this point as well. In a much different model, he shows that bargaining and collocation together act as a substitute for price advertisement, yet price competition with collocation does not perform well.

³There are papers that focus on persuasive advertisement (i.e., advertisement affects consumers' utilities directly). Dixit and Norman (1978) argue that there is too little advertisement relative to the socially optimal level in this model. For a recent contribution, see Bloch and Manceau (1999).

competitive. Surprisingly, Butters finds that (i) the advertisement level is socially optimal and that (ii) price dispersion occurs, although result (i) is sensitive to the setup of the model (see Grossman and Shapiro, 1984; Stegeman, 1991; and Stahl, 1994). In contrast to Butters, we assume that all consumers receive all advertised commodity prices. Moreover, while the above papers assume homogeneous goods, we assume that commodities are search goods.⁴ This assumption is particularly appealing for computer stores and audio shops.

It has been known since Diamond (1971) that firms' inability to commit to prices can inhibit consumer search. In Diamond's oligopolistic market for a homogeneous good with positive consumer search costs, consumers do not know the retail price set by each store before they visit the store. Equilibrium is characterized by monopoly prices and no consumers search. That is, consumers expect monopoly prices and so do not search, and consumers' failure to search provides firms with no price-cutting incentive.⁵

Using different models, Lal and Matutes (1994), Wernerfelt (1994), and Chen and Rosenthal (1996a,b) have considered the commitment role of price advertisement. Lal and Matutes (1994) and Wernerfelt (1994) assume that, before they visit a store, consumers know their willingnesses-to-pay (so the commodities in their model are not search goods) and consumers do not know the price of any commodity which has not been advertised. Wernerfelt assumes further that consumers have heterogeneous willingnesses-to-pay which are private information, and that each consumer pays an identical cost c to visit a monopoly store.⁶ Assuming that consumers expect the selling price to be p^e , the store knows that any consumer who visits the store has willingness-to-pay no smaller than $p^e + c$. Consequently, the store charges a price not less than $p^e + c$. As a result, Wernerfelt argues, there is no equilibrium price for which consumers visit the store (see Stiglitz, 1979, for a seminal

⁴An exception is Grossman and Shapiro (1984), who consider a model with horizontally differentiated commodities, but in which consumers know the attributes of the relevant commodities before they visit stores. Grossman and Shapiro employ the Butters (1977) advertising technology. Consumers are certain about their willingnesses-to-pay before they visit a retailer, so commodities are not search goods.

⁵Recent work by Anderson and Renault (1999a) has nicely reconciled this "Diamond paradox" with that of Bertrand by introducing product heterogeneity (search good). Stahl (1996) has also reconciled these two paradoxes, but by introducing heterogeneous consumer search costs into a homogeneous commodity model. The symmetric mixed strategy Nash equilibrium of his model has neither monopoly nor marginal cost pricing.

⁶The model described here is a simplified version of Wernerfelt's (1994) model, presented to clarify the role of price advertisement. In the full model, Wernerfelt attempts to give a comprehensive treatment of a variety of marketing strategies, including return rights, collocation of stores and bargaining. Lal and Matutes (1994) use the same mechanism as well.

contribution in a model with identical consumers). Hence, in these two models, price advertisement as a commitment device is necessary for stores to obtain positive profits.⁷ Chen and Rosenthal (1996a,b), on the other hand, consider an impatient seller who has only one unit of an indivisible good to sell. They assume that goods are search goods. Consumers arrive according to a Poisson process and each must decide if she will inspect the commodity, paying an inspection cost (identical across consumers). After the commodity has been inspected, the consumer’s willingness-to-pay is realized (so the good is a search good). Chen and Rosenthal then show that the optimal mechanism (under some surplus sharing rules) involves a ceiling price which works as a commitment device.⁸

Alternatively, retail stores can use collocation (geographical concentration) as a demand-enhancing form of price commitment. Dudey (1990) presented a homogeneous commodity model in which retail stores can choose their geographical locations. The location decision is followed by Cournot competition among all collocated stores. Dudey shows that retail stores have an incentive to concentrate as long as consumers are not informed of retail prices. That is, given that consumers need to infer retail prices before deciding where to shop, consumers tend to visit a concentration of stores, since a large number of stores at a single site signals lower prices. As a result, stores would rather collocate than stand alone. In contrast with the Dudey model, we assume that the stores choose prices instead of quantities and that commodities are search goods.⁹ Price setting behavior by stores is essential in our model since we analyze price advertisement.

The most related work to this paper is Anderson and Renault (1999b), although the focus of their paper is different from ours. We focus on the positive externality, on the sales of one of two commodities sold at a common location, when the second commodity is advertised at a low price.

⁷This implies that, in Wernerfelt (1994), stores *always* choose to advertise. A desirable feature of our model is that a store may choose not to advertise a price depending on the cost of advertisement. This is because we assume that willingness-to-pay is uncertain before search. See Anderson and Renault (1999b) for related discussions.

⁸Chen and Rosenthal’s (1996a,b) “ceiling prices” can be negotiated down after the inspection cost is incurred (e.g., an automobile advertisement of “\$5,000 or best offer”), while our advertised prices (as is common in sales circulars) will be non-negotiable. Consequently, our work is complementary in treating different types of markets. The first of the two Chen and Rosenthal papers assumes homogeneous inspection costs across consumers, while the latter allows for heterogeneous costs of inspection.

⁹If commodities are search goods, firms have an additional incentive to collocate. Because consumers’ valuations are unknown before search, a consumer visiting a concentration of stores has increased likelihood of finding a commodity for which her valuation is high. Consequently, collocated stores face enhanced demand curves (see Stahl, 1982, and Wolinsky, 1983). Fischer and Harrington (1996) and Konishi (1999) consider price competition models with both the Dudey and Stahl-Wolinsky effects. Our model is based on Konishi (1999), but allowing for price advertisement.

The two commodities can be sold by the same store (the multiproduct monopoly case), or by two different stores (the duopoly case). By contrast, Anderson and Renault (1999b) assume that each firm sells only one commodity, employing the sequential search model of Wolinsky (1986): i.e., a consumer incurs a search cost in every (sequential) search. Thus, collocated stores cannot be analyzed in their framework. Although our paper focuses only on price advertising, Anderson and Renault (1999b) assume that firms can choose price advertising, match advertising, or both, if they wish. The main finding of their paper is that firms may choose no advertising, price advertising, or price and match advertising depending on the level of consumers' search costs.¹⁰

Finally, our monopoly result with two commodities complements Lal and Matutes's (1994) analysis of "loss-leader pricing."¹¹ Lal and Matutes assume that there are two independent commodities (neither substitutes nor complements) and every consumer has identical willingness-to-pay for each. There are two stores located at the edges of an interval, consumers are distributed over the interval and commuting costs are proportional to distance. Stores can advertise prices of two goods, one good, or none. Consumers do not know prices unless advertised and can choose any shopping strategy. Lal and Matutes show that there is a loss-leader pricing equilibrium of this model, in which each store advertises only one commodity price, however both stores advertise the same commodity. Consumers buy both commodities at the same store in equilibrium in order to economize the transportation costs. Consumer search is encouraged by the surplus received from purchasing the loss-leader. While stores lose money on the loss-leader, they make money on the non-advertised commodity. Thus, Lal and Matutes's (1994) paper successfully explains loss-leader pricing by grocery stores, but it is difficult to extend their rationale for advertised discounts to the case of computer and audio stores. In computer and audio stores, consumers usually buy at most one commodity, so there is no incentive to economize transportation costs. Our model, on the other hand, explains why a multi-product firm selling substitutes might choose to significantly discount one of the products it sells. Note, however, that our discounted prices will be higher than marginal costs, so our discounts do not constitute loss-leader pricing.

¹⁰We further differ from Anderson and Renault (1999b) in that we assume heterogeneous search costs, while they assume search costs are identical across buyers.

¹¹Loss-leader pricing means that a retail store sells some items under marginal costs in order to attract customers.

2 Monopoly with One Commodity

In this section, we start with the most basic model to get insight into the benefits of price advertisement.

2.1 The Basic Monopoly Model

There is a retail store in the market that sells a single type of indivisible commodity. The marginal cost of production is zero. The price of the commodity is denoted by $p \geq 0$. There is a continuum of consumers, all of whom have *ex ante* identical tastes, but differ in their search costs. Each consumer's willingness-to-pay for the commodity, v , is independently distributed uniformly over the closed interval $[0, 1]$. That is, consumers are not informed about how much they like the commodity that is sold at the store, and their willingnesses-to-pay are stochastically independent. To find out her willingness-to-pay for the commodity, each consumer must visit the store, incurring a search cost. One natural interpretation is that this search cost reflects differences in consumers' locations relative to the location of the store. We assume that consumers' search costs are distributed uniformly over the closed interval $[0, 1]$, and the retail store is located at 0.¹² In summary, each consumer knows her own search cost before search, but must pay the search cost to learn both her willingness-to-pay, v , and (absent advertising) the price of the commodity, p .

The store chooses the price p , and may choose to advertise the price or not. In the former case, the store incurs an extra advertising cost $c > 0$. It is illegal to advertise a false price. That is, the store commits to its price by advertising. The store's advertising decision problem can be described by the following extensive form game.

1. The monopoly store chooses if it advertises or not.

2. There are two cases:

- (a) The store does not advertise p : In this case, p is not a part of public information.

¹²Those uniform distribution assumptions are used only for calculation purposes. Nevertheless, they play essential roles in our analysis. Since a store's choice variable is not continuous, we need to calculate the resulting profit of each case in order to see what the store's decision on advertising would be.

- (b) The store advertises p : In this case, p becomes a part of public information.
- 3. Consumers decide if they search or not by utilizing public information (independently).
- 4. For each consumer who decided to search, nature plays and her willingness-to-pay is realized.
- 5. Each consumer decides whether or not to buy the commodity.

In the following subsections, we analyze the optimal strategy for the retail stores by investigating two cases in order.

2.2 No Advertisement

Consider first the case in which the store chooses not to advertise the retail price p . Since search costs are sunk costs, each consumer must decide whether or not to search (travel to the store's location) in advance of learning her willingness-to-pay and the store's retail price. Thus, when the store sets its retail price, it cannot affect consumers' sunk search decisions through subsequent price cutting.¹³ As a result, the store's optimization problem is very simple. We write the profit as $\Pi^m(p) = R^m(p)\mu(p)$, where $R^m(p)$ represents the **profit per unit market size** (the profit when the number of consumers who incurred search costs is unity), and $\mu(p)$ represents the **market size** (the number of consumers who incur search costs). This multiplicative separability of the profit function follows from the assumption of constant (zero) marginal costs of production. The profit per unit market size is given by

$$R^m(p) = p \int_p^1 dv = p(1 - p).$$

Note that willingness-to-pay is uniformly distributed in the interval $[0, 1]$. Since the store knows that it cannot affect the market size by changing its price, it chooses to maximize $R^m(p)$ for any market size. Thus, the monopoly price is $p = \frac{1}{2}$ and $R^m(\frac{1}{2}) = \frac{1}{4}$ (a standard result for monopoly price

¹³Even if the store decides to cut the price, consumers have no way to know in advance what has been done to the price, and the number of consumers who incur search costs is not affected. That is, the market size that the store faces is fixed irrespective of the price charged, and consumers expect that the store decides its price for a fixed market size.

under linear demand). To find a Nash equilibrium in this subgame, we need to find the market size which results from consumers' search decisions when the price is $p = \frac{1}{2}$. When $p = \frac{1}{2}$, the expected utility (gross of search costs) for each consumer is $E(\max\{0, v - \frac{1}{2}\}) = \int_{\frac{1}{2}}^1 (v - \frac{1}{2}) dv = \frac{1}{8}$. Therefore, a consumer who decides to visit the store's location necessarily has a search cost no larger than $\frac{1}{8}$. Hence, $\mu(\frac{1}{2}) = \frac{1}{8}$, and the store's profit without advertising its price is $\Pi^m(\frac{1}{2}) = R^m(\frac{1}{2})\mu(\frac{1}{2}) = \frac{1}{32}$.

2.3 Price Advertisement

Next, suppose that the store commits to its price, before consumers search, by advertising its price. By advertising a low price, it can increase the market size, since consumers will make their search decisions accordingly. That is, the store can now affect the market size. The profit is given by (c is the advertisement cost):

$$\begin{aligned}\Pi^m(p) - c &= R^m(p)\mu(p) - c \\ &= p \left(\int_p^1 dv \right) \left(\int_p^1 (v - p) dv \right) - c \\ &= \frac{1}{2}p(1 - p)^3 - c.\end{aligned}$$

By taking the first order condition, we obtain:

$$(1 - p)^3 - 3p(1 - p)^2 = 0.$$

Thus, it is easy to see (by checking the second order condition) that the profit maximizing price is $\frac{1}{4} = 0.25$ and the profit is $\Pi^m(\frac{1}{4}) = \frac{27}{512} - c \approx 0.0527 - c$. The resulting market size is $\mu(\frac{1}{4}) = \frac{9}{32} \approx 0.2813$. Note that the advertised price is a half of the monopoly price without advertisement.

2.4 The Advertisement Decision

Summarizing the previous two subsections' results, we obtain the following table.

	Price	Market Size	Profit
Advertise	0.25	0.2813	$0.0527 - c$
Don't Advertise	0.5	0.125	0.0313

From the table, it is easy to see that the store's advertisement decision depends on the value of c . The critical c is obtained by taking the difference between these two profits: $\Pi^m(\frac{1}{4}) - c - \Pi^m(\frac{1}{2}) = \frac{11}{512} - c (\approx 0.0215 - c)$. That is, if $c < \frac{11}{512}$, then the monopolist advertises its price ($p = \frac{1}{4}$). If $c = \frac{11}{512}$, then it is indifferent between advertising and not. If $c > \frac{11}{512}$, then it does not advertise, and sets its price at $\frac{1}{2}$. The critical value $\frac{11}{512}$ is fairly large relative to the profit that the store earns: 68% of the profit without advertisement. Thus, we conclude from the above table that the store will advertise a low price unless the advertising cost is too high. As an explanation for this phenomenon, recall that advertising a low price has two effects: first on the profit per unit market size and second on the market size itself. Since the store must honor its advertised price, advertising a low price reduces the profit per unit market size ($R^m(p)$). That is, the store can no longer opportunistically charge the monopoly price to any consumer who visits. Second, by advertising a low price, the store can convince consumers with higher search costs to visit the store, thus expanding its market size ($\mu(p)$). Since profit (gross of advertising costs) is the product of $R^m(p)$ and $\mu(p)$, the total impact of advertising a low price will be positive as long as the market size effect is sufficiently strong. Finally, note that price advertisement by the retailer is not wasteful, since any advertisement which takes place necessarily improves retail profits and also (at least in the *ex ante* sense) enhances consumers' surplus.¹⁴

¹⁴See also Anderson and Renault (1999b) for a welfare analysis of price advertising in a different model.

3 Monopoly with Two Commodities

It is often observed that a store advertises very low prices for a small number of commodities, yet also carries other non-discounted commodities. Our analysis may be easily extended to explain such behavior by stores. Toward this end, we depart from the basic model by allowing the monopolist retailer to sell two types of commodities, $i = 1, 2$. The marginal cost of producing either commodity is zero and the price of commodity i is denoted by p_i . Each consumer's willingness-to-pay for commodity i , v_i , is an i.i.d. (over consumers, and across commodities) random variable, distributed uniformly over the closed interval $[0, 1]$. That is, a consumer who visits the store (paying a one-time search cost which is sunk) makes two stochastically independent draws from the uniform distribution, each of which determines her willingness-to-pay for one of the commodities $i = 1, 2$. As above, before visiting the retail location, the consumer knows neither her willingness-to-pay for either commodity nor the price of any commodity which has not been advertised. Consumers' search costs are again distributed uniformly over the closed interval $[0, 1]$, the monopolist retailer is located at 0, and each consumer knows her own search cost. The store can advertise zero, one or two commodity prices. If the store chooses to advertise one commodity price, it pays c . If it chooses to advertise two commodity prices, it pays $2c$.¹⁵ In subsections 3.2–3.4 below, we consider the price/search subgame between consumers and the store, returning in the final subsection to consider the monopolist's advertising decision.

3.1 The Description of the Monopolist's Decision Problem

The monopolist's advertising decision problem can be described as a game with consumers who cannot observe the monopolist's price decision unless the monopolist chooses to advertise. The timing of moves in the game is as follows:

1. The monopoly store chooses the number of commodities to advertise (0, 1 or 2).
2. There are three cases:

¹⁵This is a benchmark case. The same assumption is adopted by Lal and Matutes (1994). We discuss other cases briefly in subsection 3.5.

- (a) No commodity price is advertised: In this case, p_1 and p_2 are chosen simultaneously, and these prices are not a part of public information.
 - (b) Only one commodity price (w.l.o.g., say p_1) is advertised: In this case, p_1 is first chosen and p_2 is chosen accordingly, but only p_1 is a part of public information (p_2 is not).¹⁶
 - (c) Both commodity prices are advertised: In this case, p_1 and p_2 are chosen simultaneously, and these prices are both a part of public information.
3. Consumers decide if they search or not by utilizing public information (independently).
 4. For consumers who decided to search, nature plays and willingnesses-to-pay are realized.
 5. Consumers decide which commodity to buy or to buy neither.

3.2 Case 1: No Advertisement

First, we consider the case where the store advertises no commodity price. In this case, by the same logic as in the previous section, there is no way for the store to affect the market size by choosing a lower price than the monopoly price. Thus, the store simply maximizes its profit per unit market size $R^m(p_1, p_2)$ such that (w.l.o.g. $p_1 \leq p_2$)

$$R^m(p_1, p_2) = p_1 \left(\int_{p_1}^{1-p_2+p_1} (v_1 - p_1 + p_2) dv_1 + \int_{1-p_2+p_1}^1 dv_1 \right) + p_2 \int_{p_2}^1 (v_2 - p_2 + p_1) dv_2. \quad (1)$$

Figure 1 may be helpful in the interpretation of the above equation. Areas A and B of Figure 1 represent the consumers who purchase commodity 1 ($v_1 \geq p_1$ and $v_1 - p_1 \geq v_2 - p_2$). Given that willingnesses-to-pay for commodities $i = 1, 2$ are independent uniform random variables, areas A and B can be regarded as the measure of consumers who purchase commodity 1. As a result, the first term of (1) is the profit made by selling commodity 1 to a unit size market. The second term can be interpreted in the same way, but for commodity 2. It is easy to show that the store's revenue

¹⁶This means that there are two sequential decision nodes for the monopolist. Since p_1 is observable by consumers, a subgame starting from that node forms a proper subgame, and we can refine the set of Nash equilibria by utilizing the standard notion of subgame perfection in order to exclude unreasonable equilibria. If we assume that the store chooses p_1 and p_2 simultaneously at the same node, then even perfect Bayesian Nash equilibrium is not enough. We need further refinement of equilibrium (see Appendix A.6).

per unit market size achieves the unique maximum at $p^* = p_1 = p_2 = \frac{\sqrt{3}}{3} \approx 0.5774$. Note that this price is higher than the non-advertising monopoly price ($\frac{1}{2}$) in the one-commodity model of Section 2. The monopolist's behavior can be explained by noting that, by providing two commodities, the likelihood increases that the consumer will find at least one commodity for which her willingness-to-pay exceeds the price (given our assumption of the stochastic independence of willingnesses-to-pay). Consequently, a monopolist retailer of two commodities could charge somewhat higher prices and make no fewer sales than a monopolist retailer of a single commodity.

Market size is determined by consumers' expected utility from searching for commodities at the retail store's location. Therefore, for any price pair (p_i, p_j) , market size $\mu(p_i, p_j)$ can be found by calculating consumers' expected utility from searching at the store. To compute the expected utility, we use the fact that consumers' valuations for the two commodities are i.i.d. and that, as before, $F(v)$ represents the probability that the willingness-to-pay turns out to be less than v . The expected utility (gross of search costs) at prices (p_i, p_j) is then given by

$$\begin{aligned} E(\max\{0, v_i - p_i, v_j - p_j\}) &= 0 \times F(p_i) \times F(p_j) \\ &\quad + \int_{p_i}^1 (v_i - p_i) F(\min\{v_i - p_i + p_j, 1\}) dv_i \\ &\quad + \int_{p_j}^1 (v_j - p_j) F(\min\{v_j - p_j + p_i, 1\}) dv_j. \end{aligned}$$

The first term of the RHS describes the case in which a consumer cannot find a buyable commodity, while the second and third terms describe the cases in which she buys goods i and j , respectively. Note that the consumer buys good i only when $v_i \geq p_i$ and $v_i - p_i \geq v_j - p_j$. The second condition is satisfied with probability $F(v_i - p_i + p_j)$ if $v_i - p_i + p_j \leq 1$, and with probability one, otherwise. Without loss of generality, we assume $p_i \leq p_j$, which makes the equation as follows (again, see Figure 1 for intuition):

$$\begin{aligned} E(\max\{0, v_i - p_i, v_j - p_j\}) &= \int_{p_i}^{1-p_j+p_i} (v_i - p_i)(v_i - p_i + p_j) dv_i + \int_{1-p_j+p_i}^1 (v_i - p_i) dv_i \\ &\quad + \int_{p_j}^1 (v_j - p_j)(v_j - p_j + p_i) dv_j. \end{aligned}$$

The market size when prices are set at p_i and p_j , $\mu(p_i, p_j)$, is determined by consumers whose search costs are less than $E(\max\{0, v_i - p_i, v_j - p_j\})$. Since consumers' search costs are distributed by unit density, we have $\mu(p_i, p_j) = E(\max\{0, v_i - p_i, v_j - p_j\})$: i.e.,

$$\begin{aligned} \mu(p_i, p_j) = & \int_{p_i}^{1-p_j+p_i} (v_i - p_i + p_j)(v_i - p_i) dv_i + \int_{1-p_j+p_i}^1 (v_i - p_i) dv_i \\ & + \int_{p_j}^1 (v_j - p_j + p_i)(v_j - p_j) dv_j. \end{aligned} \quad (2)$$

By symmetry, the market size equation for $p_i \geq p_j$ can be found easily by interchanging p_i and p_j . Consumers correctly expect the monopolist retailer to charge p^* for both commodities, so the market size becomes

$$\mu(p^*, p^*) = \frac{1}{3} \left((p^*)^3 - 3p^* + 2 \right) \approx 0.1535.$$

Note that the market size in the case of a single-commodity monopoly without advertisement was $\frac{1}{8} = 0.125$. Thus, neither consumers' expected utility nor market size increases significantly when the monopolist carries two commodities rather than one. While consumers are more likely to receive at least one "good" realization of their willingness-to-pay when they have two independent draws, the increased likelihood of high gross surplus is largely offset by the monopolist's price increase. The store's total profits are given by $\Pi^m(p^*, p^*) = R^m(p^*, p^*)\mu(p^*, p^*) \approx 0.0591$.

3.3 Case 2: Advertising One Commodity Price

We assume (without loss of generality) that the store advertises p_1 but does not advertise p_2 . Since p_2 has not been advertised, we apply same logic as that used in the basic model to conclude that the store takes market size as given when choosing p_2 . Since p_1 has already been chosen at the time p_2 is chosen, the store simply tries to maximize its profit per unit market size $R^m(p_1, p_2)$. For each p_1 , define $\beta(p_1) = \arg \max_{p_2 \in [0,1]} R^m(p_1, p_2)$ as the store's best response conditional on p_1 . Consumers, knowing both the advertised value p_1 and the response function $\beta(\cdot)$, correctly infer the monopolist's choice $p_2 = \beta(p_1)$ and make their search decisions accordingly in every subgame.¹⁷ Knowing its

¹⁷Inference must be correct in each subgame. Each subgame starts with the store's decision p_1 , and consumers make their search decisions without knowing p_2 . The store also does not know consumers' decisions when it sets p_2 .

choice of advertised price p_1 will therefore affect the market size, the monopolist retailer chooses its advertised price p_1 to maximize $\tilde{\Pi}^m(p_1) \equiv \Pi^m(p_1, \beta(p_1)) - c = R^m(p_1, \beta(p_1))\mu(p_1, \beta(p_1)) - c$. Thus, the store's decision making process has a Stackelberg structure, although it is a monopoly decision problem.

One can show that the best response function $\beta : [0, 1] \rightarrow [0, 1]$ has the following form (see Appendix A.1 for the derivation):

$$\beta(p_1) = \begin{cases} \frac{1}{3} \left(3p_1 + 2 - \sqrt{9p_1^2 + 1} \right) & \text{if } p_1 \leq \frac{\sqrt{3}}{3}, \\ \frac{1}{4} \left(-3p_1^2 + 4p_1 + 1 \right) & \text{if } p_1 \geq \frac{\sqrt{3}}{3}. \end{cases}$$

Since consumers know this relationship, market size is given by $\mu(p_1, \beta(p_1))$ for an advertised price $p_1 \in [0, 1]$. Thus, the store chooses p_1 to maximize

$$\tilde{\Pi}^m(p_1) = R^m(p_1, \beta(p_1))\mu(p_1, \beta(p_1)) - c.$$

A closed form for the derivative of the profit function is presented in Appendix A.1. Because finding roots of the derivative is analytically impossible, we employ a numerical approach to find the optimum, finding that the payoff function has a unique maximizer at $p_1^* \approx 0.2537$. We conclude that prices $(p_1^*, \beta(p_1^*)) \approx (0.2537, 0.5015)$ are optimal for the store, yielding a market size $\mu(p_1^*, \beta(p_1^*)) \approx 0.3306$ and total profit $\tilde{\Pi}^m(p_1^*) \approx 0.0938 - c$.

3.4 Case 3: Advertising Both Commodities

Finally, we consider the case where the store advertises both commodity prices. Since consumers will know both prices in this case, the store can take into account the influence of both prices on market size. Thus, the store's payoff function is given by $\Pi^m(p_1, p_2) - 2c = R^m(p_1, p_2)\mu(p_1, p_2) - 2c$. This function is not well behaved due to the fact that $\mu(p_1, p_2)$ is a convex function. Each component

Thus, once p_1 is chosen, the store and consumers essentially play a simultaneous move game. A Nash equilibrium in this simultaneous game must require consistent (correct) inferences to the other party's strategies. Since subgame-perfection requires that we assign a Nash equilibrium to each subgame, and inference needs to be correct in each subgame with p_1 .

of $\Pi^m(p_1, p_2)$ (that is, on each of the regions $p_1 \leq p_2$ and $p_1 \geq p_2$) is a fifth order polynomial in p_1 and p_2 . As a consequence, equilibria cannot be computed in closed form, so we characterize optima numerically (see Appendix A.2). We identify a local maximum at the symmetric prices $p^{**} = p_1 = p_2 \approx 0.3143$. The market size is $\mu(p^{**}, p^{**}) \approx 0.3627$, and the total profit is $\tilde{\Pi}^m \equiv \Pi^m(p^{**}, p^{**}) - 2c \approx 0.1027 - 2c$.

3.5 The Advertisement Decision

Based on results from the analysis of the above three cases, the store's payoffs may be summarized by the following table.

	Prices	Market Size	Profit
Advertising Both Prices	$p_1 = 0.3143$ $p_2 = 0.3143$	$\mu = 0.3627$	$0.1027 - 2c$
Advertising One Price	$p_1 = 0.2537$ $p_2 = 0.5015$	$\mu = 0.3306$	$0.0938 - c$
Non-Advertising	$p_1 = 0.5774$ $p_2 = 0.5774$	$\mu = 0.1535$	0.0591

The store's advertisement decision is affected by the value of c . Three distinct cases can be identified (with the exception of indifference cases):

1. When $c < 0.0089$, the store advertises both prices.
2. When $0.0089 < c < 0.0347$, the store advertises one commodity price.
3. When $0.0347 < c$, the store advertises neither commodity's price.

Note that $c = 0.0089$ and $c = 0.0347$ are about 16.8% and 58.7% of the profit when neither commodity price is advertised (respectively). Thus, in a fairly wide range of c , we observe the store advertising only one commodity price. Note also that we have assumed an advertising cost function which is linearly homogeneous in the number of commodities. If on the other hand, the

incremental advertising cost for the second commodity were sufficiently small (e.g., zero), then clearly the store would never choose to advertise only the first commodity's price. Nonetheless, if $c_1 = 0.0347$ represents the cost of advertising the first commodity, then any advertising cost for the second commodity larger than $c_2 = 0.0089$ (or 25% of c_1) will induce the firm to advertise only one commodity.

Another interesting observation is that the advertised price is significantly smaller than the non-advertised price when only one commodity is advertised. Consumers are drawn to the retail store by the low advertised price. Nonetheless, some consumers, upon arriving at the store and discovering that their willingness-to-pay for the non-advertised commodity is quite high, may purchase the non-advertised commodity instead. In particular, any consumer who arrives at the store and discovers that $v_1 \leq p_1$ (p_1 advertised) and $v_2 \geq p_2$ would (weakly) prefer the non-advertised commodity. Like Lal and Matutes (1994) treatment of loss-leader pricing, consumers in our model are drawn to the firm's location by low advertised prices. Unlike Lal and Matutes, however, each consumer in our model buys at most one commodity and the monopolist sets price above marginal cost on both commodities.

4 Duopoly

In this section, we expand our basic model by introducing a second store, focusing our analysis on the strategic interaction between the two stores. The next subsection describes our duopoly model, and three subsequent subsections analyze subgame perfect Nash equilibria of the pricing and advertising game played by the firms.

4.1 The Duopoly Model

The model is essentially the same as the one in the previous section, except that there are two stores, each of which sells one commodity. There are two retail stores, $i = 1, 2$, and they are located at the same site. Each store sells a single type of an indivisible commodity. The marginal costs of production are zero. The price set by store i is denoted by p_i . Each consumer's willingness-to-pay

for commodity i , v_i , is an i.i.d. (over consumers, and across commodities for a given consumer) random variable, distributed uniformly over the closed interval $[0, 1]$. As above, this assumption implies that any consumer who visits the retail stores' location (paying a one-time sunk search cost) discovers her willingnesses-to-pay for commodities $i = 1, 2$ as two independent draws from the uniform distribution. Before visiting the retail stores' location, she knows neither the price of any non-advertised commodity nor her willingness-to-pay for that commodity. Upon visiting the retail stores' location, she learns both of these things. Consumers' search costs are again distributed uniformly over the closed interval $[0, 1]$, and the retail stores are located at 0.

Each store can choose to advertise or not to advertise its price. If a store chooses to advertise, it must decide its retail price, which becomes a part of public knowledge. Consumers then know the price before they decide whether or not to search. If a store chooses not to advertise its price, then consumers would not know the price before their search decision. The game and the timing of the moves is as follows:

1. Stores $i = 1, 2$ choose if they advertise or not simultaneously.
2. There are three cases:
 - (a) Neither store advertises. In this case, each store i chooses p_i simultaneously, and these prices are not a part of public information.
 - (b) One store (say i) advertises and the other ($j \neq i$) does not. In this case, p_i is first chosen and p_j is chosen accordingly, but p_i is a part of public information while p_j is not.
 - (c) Both stores advertise. In this case, each store i chooses p_i simultaneously, and both prices become a part of public information.
3. Consumers decide if they search or not by utilizing public information (independently).
4. For consumers who decided to search, nature plays and willingnesses-to-pay realize.
5. Consumers decide which commodity to buy or to buy neither.

In the following subsections, we analyze each of the subgames 2(a), 2(b) and 2(c), finding subgame perfect Nash equilibria.

4.2 Case 1: Neither Store Advertises

In the first case, neither store commits to its price through advertisement. As above, each store's profit can be written as $\Pi_i(p_i, p_j) = R_i(p_i, p_j)\mu(p_i, p_j)$ where $R_i(p_i, p_j)$ and $\mu(p_i, p_j)$ denote, respectively, profit per unit market size and the market size when prices are p_i and p_j .¹⁸ As in the monopoly case discussed above, if a store does not commit to its price through advertisement, then the store cannot affect consumers' search decisions through its choice of prices. Consequently, each store in this case takes the market size $\mu(p_i, p_j)$ as given, maximizing only the profit per unit market size $R_i(p_i, p_j)$. For store $i = 1, 2$ ($j \neq i$), the profit per unit market size is given by (again, see Figure 1)

$$R_i(p_i, p_j) = \begin{cases} R_i^+(p_i, p_j) \equiv p_i \left(\int_{p_i}^{1-p_j+p_i} (v_i - p_i + p_j) dv_i + \int_{1-p_j+p_i}^1 dv_i \right) & \text{if } p_i \leq p_j, \\ R_i^-(p_i, p_j) \equiv p_i \int_{p_i}^{1-p_j+p_i} (v_i - p_i + p_j) dv_i & \text{if } p_i \geq p_j. \end{cases} \quad (4)$$

These are the payoff functions of each store in this case. Of course, consumers rationally expect Nash equilibrium prices of the above game and use these prices (p_1^*, p_2^*) to decide whether to visit the stores' location. That is, the market size is $\mu(p_1^*, p_2^*)$, where μ is as given in (2). There is a unique symmetric Nash equilibrium price $p^* = p_1^* = p_2^*$ in this subgame (see Appendix A.3 for the proof).

Lemma 1:

The symmetric Nash equilibrium price p^ of the subgame in which neither store advertises is unique and satisfies $p^* = \sqrt{2} - 1$.¹⁹*

¹⁸Since stores are symmetric, $R_i(p_i, p_j) = R_j(p_{j'}, p_{i'})$ where $p_{j'} = p_i$ and $p_{i'} = p_j$, and $\mu(p_i, p_j) = \mu(p_j, p_i)$ follow.

¹⁹Actually, we can show that there is no asymmetric equilibrium, so the symmetric Nash equilibrium is the unique Nash equilibrium. We can show that this particular subgame is a supermodular game (see Milgrom and Roberts, 1990, and Vives, 1990). Since the set of Nash equilibria in a supermodular game has a lattice structure, the unique symmetric Nash equilibrium must also be the unique Nash equilibrium (Milgrom and Roberts, 1990, and Vives, 1990).

Since consumers correctly expect that $p^* = \sqrt{2} - 1$ will be the equilibrium price for each store, the market size is $\mu(p^*, p^*) = 2 \int_{p^*}^1 v(v - p^*) dv \approx 0.2761$. Since $R_i(p^*, p^*) \approx 0.1716$, we have $\Pi_i(p^*, p^*) = R_i(p^*, p^*)\mu(p^*, p^*) \approx 0.0474$.

4.3 Case 2: One Advertises, One Does Not

Next, we consider the case where one store commits to its price through advertisement, while the other does not. Without loss of generality, suppose that store 1 advertises, while store 2 does not. The sequence of moves in this subgame therefore begins with an advertised price p_1 set by store 1. Knowing that store 1 must honor its advertised price p_1 , store 2 decides its price p_2 . Store 2, however, knows that there is no way for it to influence the market size by changing its price p_2 , since without advertisement it has no ability to commit to p_2 . Consequently, it simply maximizes $R_2(p_1, p_2)$ by taking the market size μ as given. Consumers calculate p_2 , knowing that it will be store 2's best response to store 1's choice of p_1 . As a result, this case has a Stackelberg structure in which store 1 moves first, leading to a sub-subgame in which store 2 makes its pricing decision. A subgame perfect equilibrium in Case 2 is therefore a pair (p_1^*, β_2) , where $p_1^* \in [0, 1]$ and $\beta_2 : [0, 1] \rightarrow [0, 1]$, such that

$$\begin{aligned} R_2(p_1, \beta_2(p_1)) &\geq R_2(p_1, p_2) && \text{for any } p_2 \in [0, 1], \\ \Pi_1(p_1^*, \beta_2(p_1^*)) - c &\geq \Pi_1(p_1, \beta_2(p_1)) - c && \text{for any } p_1 \in [0, 1]. \end{aligned}$$

We have the following result (for which the proof may be found in Appendix A.4):

Lemma 2:

The subgame perfect Nash equilibrium of the subgame in which only store 1 advertises is unique and described by a pair (p_1^, β_2) such that*

$$p_1^* \approx 0.2913$$

and

$$\beta_2(p_1) = \begin{cases} \frac{1}{3} \left(2 + 2p_1 - \sqrt{1 + 2p_1 + 4p_1^2} \right) & \text{if } p_1 \leq \sqrt{2} - 1, \\ \frac{1}{4} (1 + 2p_1 - p_1^2) & \text{if } p_1 \geq \sqrt{2} - 1. \end{cases}$$

In this equilibrium, the profit of store 1 is $\Pi_1(p_1^*, \beta_2(p_1^*)) - c \approx 0.0523 - c$. Store 2's price on the equilibrium path is $\beta_2(p_1^*) \approx 0.3987$. Market size is therefore $\mu(p_1^*, \beta_2(p_1^*)) \approx 0.3400$, so store 2's profit is $\Pi_2(p_1^*, \beta_2(p_1^*)) = R_2(p_1^*, \beta_2(p_1^*))\mu(p_1^*, \beta_2(p_1^*)) \approx 0.0483$.

4.4 Case 3: Both Stores Advertise

Finally, consider the case when both stores advertise. In this case, both p_1 and p_2 are part of public information. Thus, the market size is $\mu(p_1, p_2)$ for a given price pair (p_1, p_2) . Stores take this into account when they choose prices. Thus, the payoff functions are described by $\Pi_i(p_i, p_j) - c = R_i(p_i, p_j)\mu(p_i, p_j) - c$. We have the following result (the proof is given in Appendix A.5):

Lemma 3:

*The symmetric Nash equilibrium price p^{**} of the subgame in which both stores advertise is unique and satisfies*

$$p^{**} = \frac{1}{15} \left(1441 + 30\sqrt{2055} \right)^{\frac{1}{3}} + \frac{61}{15} \left(1441 + 30\sqrt{2055} \right)^{-\frac{1}{3}} - \frac{14}{15} \approx 0.2949.$$

Given that each store sets price $p^{**} \approx 0.2949$, the equilibrium market size is $\mu(p^{**}, p^{**}) \approx 0.3803$, and the equilibrium payoff to each store is then given by $\Pi_i(p^{**}, p^{**}) - c \approx 0.0512 - c$.

4.5 The Advertisement Decision

Based on the Nash equilibria in the above subgames, the advertising game played in the first stage may be represented in the following strategic form (for convenience, we also list equilibrium prices and market size):

Store 2 Store 1	Advertise	Don't Advertise
Advertise	$0.0512 - c$ $0.0512 - c$ $p_1 = 0.2949$ $p_2 = 0.2949$ $\mu = 0.3803$	$0.0523 - c$ 0.0483 $p_1 = 0.2913$ $p_2 = 0.3987$ $\mu = 0.3400$
Don't Advertise	0.0483 $0.0523 - c$ $p_1 = 0.3987$ $p_2 = 0.2913$ $\mu = 0.3400$	0.0474 0.0474 $p_1 = 0.4142$ $p_2 = 0.4142$ $\mu = 0.2761$

From the table, we can see that, dependent on the values of c , the equilibrium strategy configuration changes. There are three cases (except for indifference cases):

1. When $c < 0.0029$, advertising becomes a dominant strategy for both firms, and the unique Nash equilibrium is (Advertise, Advertise).
2. When $0.0029 < c < 0.0049$, the game becomes one of “battle of sexes.” Thus, there are two pure strategy equilibria: (Advertise, Don't Advertise) and (Don't Advertise, Advertise).²⁰ If $0.0029 < c < 0.0040$, the advertising store obtains a higher profit in the equilibrium. If $0.0040 < c < 0.0049$, the non-advertising store obtains a higher profit.
3. When $0.0049 < c$, non-advertising becomes a dominant strategy for both firms, i.e., the unique Nash equilibrium is (Don't Advertise, Don't Advertise).

First, note that the threshold advertising costs 0.0029, 0.0040 and 0.0049 are 6.1%, 8.4%, and 10.3% of the profit when neither store advertises. Thus, in the range between 6.1% and 10.3%, an asymmetric equilibrium prevails. Especially important is the range between 8.4% and 10.3%, in which the non-advertising store obtains greater equilibrium profit than does the advertising store. That is, the non-advertising store free-rides at the advertising store's expense. The former does not need to pay the advertising cost, yet can enjoy the expanded market size without significantly reducing its market price (from 0.4142 to 0.3987).

²⁰We focus exclusively on pure strategy Nash equilibria.

Second, note that advertisement does not (in the *ex ante* sense) make anyone in the economy worse off. Neither store receives less profit than it does in the case where no store advertises.²¹ Consumers are clearly better off as well, since μ can be regarded as an index for consumer surplus. Price advertisement simply improves imperfect information in the economy. Third, in the asymmetric equilibrium, the advertising store's price is lower than in any other equilibrium. As in the monopoly case, the advertising firm must charge a low price so that loss of profit per unit market size is offset by market expansion (an increase in consumer visits). However, it must also take into account that the non-advertising store will not reduce its price much, since the non-advertising store cannot alter market size through its price.

Finally, note that the incentive to advertise prices here is much weaker than in the monopoly case. In the monopoly case (with two goods), if the advertising cost is less than 58.7% of its profit without advertisement, then the store chooses to advertise at least one commodity. Here, unless the advertising cost is less than 10.3% of its profit without advertisement, a store does not advertise its price. Of course, one explanation is that the other store's profit increase is not internalized in the duopoly case. However, the main reason is that the market size is already fairly large without advertisement in the duopoly case, due to price competition between the two stores. Without advertisement, monopoly prices are 0.5774, yet duopoly prices are 0.4142. Thus, in the duopoly case, an additional profit increase by price advertisement is much more limited than in the monopoly case. This implies that collocation of stores can signal low prices to consumers. Thus, collocation and price advertisement can be regarded as substitutable marketing strategies (see also Wernerfelt, 1994).

5 Concluding Remarks

In this paper, we discussed the incentives for retail stores to advertise their prices. We demonstrated that price advertisement works as a commitment device in a simple one commodity monopoly model with consumer search and provided two extensions of this basic model. We first extended the model

²¹This result is specific to the assumption of stochastically independent willingnesses-to-pay. See the concluding remarks for more on this issue.

to a monopoly selling two commodities, showing that a monopoly retail store may advertise only one (deeply discounted) commodity price, even if both commodities are substitutes. Second, we extended the model to a duopoly setting. In this case, we showed that the advertising game may exhibit a free-riding incentive in some situations. Moreover, we observed that both collocation of stores and price advertisement work as commitment devices.

Our results in Sections 3 and 4 may be sensitive to the assumption of stochastic independence of consumers' willingnesses-to-pay for the two commodities. If, instead, consumers' willingnesses-to-pay are highly positively correlated with one another, a monopoly store might not have incentive, when it advertises only one price, to charge a highly discounted price for the advertised commodity. Since most consumers who bought any commodity at all would buy the advertised (discounted) one, the retailer would sell fewer units of the non-advertised (higher-priced) commodity. In the duopoly case, if a consumer's willingnesses-to-pay are highly correlated, then the game may no longer be "battle of the sexes," but rather "prisoners' dilemma." Each store has advertisement as its dominant strategy, but the outcome will be profit-reducing (as in a homogeneous-commodity Bertrand game).

In Section 4, we analyzed a duopoly model in which both stores sell from the *same* location. The collocation assumption was made in order to analyze a free-rider problem in price-advertising. Collocation implies that, once a consumer has visited one store and paid the search cost, she need not pay a search cost to visit the other store. Consequently, a store may have an incentive not to advertise its price when the other store chooses to advertise, since consumers who visit one store would visit the other as well. If there is an additional search cost in visiting the second store, then some consumers who visited the advertising store may not go on to visit the non-advertising store, so the non-advertising store may lose a considerable number of customers. This gives the non-advertising store a greater incentive to advertise its price. See Anderson and Renault (1999b) for an analysis on such an oligopolistic market.

In this paper, we analyzed a model with at most two commodities that are *ex ante* homogeneous. It may be interesting to increase the number of commodities, but the analysis would become much more complicated. It would also be interesting to assume heterogenous commodities, where one

commodity gives higher willingness-to-pay on average but is more risky than the other commodity. In this case, we would be interested to know (if a monopoly) which commodity the monopolist would choose to advertise or (if a duopoly) which store would choose to advertise its price. These questions will be analyzed in future research.

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Appendix

A.1 Monopoly with Two Commodities, One Advertised

In this section, we provide the derivation of the best response function β , and a closed form for the derivative of the profit function of a monopolist selling two commodities and advertising one. First, we derive $\beta(\cdot)$. By taking derivatives of (1), we obtain

$$\frac{\partial R^m}{\partial p_2}(p_1, p_2) = \begin{cases} \frac{3}{2}p_2^2 - 3p_1p_2 + 2p_1 - 2p_2 + \frac{1}{2} & \text{if } p_2 \geq p_1, \\ -\frac{3}{2}p_1^2 + 2p_1 - 2p_2 + \frac{1}{2} & \text{if } p_2 \leq p_1. \end{cases}$$

By taking second order derivatives, it is easily shown that $R^m(p_1, \cdot)$ is strictly concave for $p_2 \in [0, p_1 + \frac{2}{3})$. Moreover, $\frac{\partial^2 R^m}{\partial (p_2)^2}(p_1, p_2)$ is always positive for $p_2 \in (p_1 + \frac{2}{3}, 1]$, and we have $\frac{\partial R^m}{\partial p_2}(p_1, p_1 + \frac{2}{3}) < 0$ and $\frac{\partial R^m}{\partial p_2}(p_1, 1) < 0$. This implies that any profit maximizing p_2 must lie in the interval $[0, p_1 + \frac{2}{3})$. Thus, the best response $\beta(p_1)$ is unique for any p_1 , and is found by setting $\frac{\partial R^m}{\partial p_2}(p_1, p_2) = 0$ if there is such $p_2 \in [0, p_1 + \frac{2}{3})$ (note, however, that $\frac{\partial R^m}{\partial p_2}(p_1, 0) > 0$, $\frac{\partial R^m}{\partial p_2}(p_1, p_1 + \frac{2}{3}) < 0$, and $\frac{\partial R^m}{\partial p_2}$ is strictly concave in p_2). Thus, the best response function β takes the following form:

$$\beta(p_1) = \begin{cases} \frac{1}{3} \left(3p_1 + 2 - \sqrt{9p_1^2 + 1} \right) & \text{if } p_1 \leq \frac{\sqrt{3}}{3}, \\ \frac{1}{4} (-3p_1^2 + 4p_1 + 1) & \text{if } p_1 \geq \frac{\sqrt{3}}{3}. \end{cases}$$

This best response function β crosses the line $p_1 = p_2$ at the solution of $\frac{\partial R^m}{\partial p_2}(p, p) = 0$, which is $p = \frac{\sqrt{3}}{3}$. Now, we move to the derivatives of the profit function. On the interval $[\frac{\sqrt{3}}{3}, 1]$, the derivative of the profit function is given by

$$\frac{d\tilde{\Pi}^m(p_1)}{dp_1} = -\frac{27}{64}p^7 + \frac{63}{32}p^6 - \frac{47}{64}p^5 - \frac{175}{32}p^4 + \frac{1177}{192}p^3 + \frac{13}{32}p^2 - \frac{153}{64}p + \frac{49}{96}. \quad (5)$$

It is easily shown that the maximum cannot occur on $(\frac{\sqrt{3}}{3}, 1]$ (where $p_1 \geq \beta(p_1)$). It can be

numerically verified that (5) has seven roots, five of which are real, and only one of which falls in $[\frac{\sqrt{3}}{3}, 1]$ (at $p = 1$). The unit root cannot be a maximizer, since $\tilde{\Pi}^{m'}(\frac{\sqrt{3}}{3}) < 0$ and the derivative does not change sign on $[\frac{\sqrt{3}}{3}, 1)$.

On $[0, \frac{\sqrt{3}}{3}]$ (where $p_1 \leq \beta(p_1)$), the derivative of the profit function is given by

$$\begin{aligned} \frac{d\tilde{\Pi}^m(p_1)}{dp_1} = \frac{1}{162\phi(p)} & \left(1944p^6 + 2025p^5 - 2484p^4 - 648p^5\phi(p) - 675p^4\phi(p) + 540p^3\phi(p) \right. \\ & \left. + 480p^2\phi(p) - 484p\phi(p) + 79\phi(p) + 900p^3 - 258p^2 + 83p - 2 \right), \end{aligned} \quad (6)$$

where $\phi(p) = \sqrt{9p^2 + 1}$ and the superscript on p_1 has been suppressed for clarity. Unfortunately, it is difficult to ensure that all roots of a high-order non-polynomial expression have been found. Nevertheless, it is easy to show that (6) can be written as $\psi_1(p)\phi(p) + \psi_2(p) = 0$ where ψ_1 and ψ_2 are polynomials in p , from which it follows (after simplification) that all roots of (6) will be roots of

$$\begin{aligned} 139968p^{10} - 174960p^9 - 245133p^8 + 498780p^7 - 212592p^6 \\ - 54060p^5 + 80222p^4 - 37820p^3 + 13272p^2 - 2820p + 231, \end{aligned}$$

with the possible introduction of spurious roots due to squaring both sides of an expression. It can be numerically verified that the above expression has three pairs of complex conjugate roots and four real roots, two of which ($p \approx 0.2537$ and $p \approx 0.1903$) are in $[0, \frac{\sqrt{3}}{3}]$. It is also easily verified that the root $p \approx 0.2537$ solves (6), while $p \approx 0.1903$ does not. Since $\tilde{\Pi}^{m'}(0) > 0$ and $\tilde{\Pi}^{m'}(\frac{\sqrt{3}}{3}) < 0$, it follows that $p_1^* \approx 0.2537$ is the unique profit maximizing price.

A.2 Monopoly with Two Commodities, Two Advertised

In this section, we consider the decision problem of a monopolist seller of two commodities, both of which are advertised. It is evident that (1) and (2) are symmetric across $p_1 = p_2$, so we limit

our attention to $p_1 \leq p_2$. The derivatives of the profit function are given by

$$\begin{aligned}\frac{\partial \Pi^m}{\partial p_1} = & -\frac{11}{6}p_1 + \frac{5}{6}p_2 - 2p_1^3 + \frac{1}{3} - \frac{7}{4}p_2^2 - \frac{3}{2}p_2p_1 + 6p_2p_1^2 \\ & + \frac{9}{4}p_1^2 - 4p_2^2p_1 + \frac{8}{3}p_2^3 + \frac{35}{6}p_2^3p_1 - \frac{3}{2}p_2^4p_1 \\ & - \frac{15}{4}p_2^2p_1^2 - \frac{25}{12}p_2^4 + \frac{1}{2}p_2^5,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Pi^m}{\partial p_2} = & \frac{5}{6}p_1 - \frac{11}{6}p_2 + 2p_1^3 + \frac{1}{3} + \frac{13}{4}p_2^2 - \frac{7}{2}p_2p_1 - 4p_2p_1^2 - \frac{3}{4}p_1^2 \\ & + 8p_2^2p_1 - \frac{10}{3}p_2^3 - 3p_2^3p_1^2 - \frac{5}{2}p_2p_1^3 - \frac{25}{3}p_2^3p_1 \\ & + \frac{5}{2}p_2^4p_1 + \frac{35}{4}p_2^2p_1^2 + \frac{25}{12}p_2^4 - \frac{1}{2}p_2^5.\end{aligned}$$

Solving this system (using computer algebra) gives a collection of solutions which lie outside $[0, 1]^2$, a collection of complex solutions, the solution $(1, 1)$ and a collection of solutions consisting of pairs of roots of the expression $p^3 + 2p^2 + \frac{1}{3}p - \frac{1}{3}$ ($p \in \mathbb{R}$). Since the only one of the three real roots of this expression which lies in $[0, 1]$ is the root $p^{**} \approx 0.3143$ and (p^{**}, p^{**}) satisfies second order sufficient conditions, we conclude that this point is a local maximum. Moreover, $\Pi^m(p^{**}, p^{**}) > \Pi^m(1, 1)$, so the only other critical point identified cannot be a global maximizer. Unfortunately, it is difficult to confirm that the above list of solutions is exhaustive, given the limitations of the software. Note, however, that Π^m is polynomial on the compact set $[0, 1]^2$, from which it immediately follows that Π^m is Lipschitz continuous (i.e., there exists a real number M such that $|\Pi^m(p) - \Pi^m(p')| \leq M\|p - p'\|$ for any $p, p' \in [0, 1]^2$). Consequently, the optimizer of Π^m may be verified, to arbitrary precision, by a grid search.

A.3 Duopoly, No Advertising

In this section, we provide the proof of Lemma 1. A Nash equilibrium in prices is an ordered pair $(p_1^*, p_2^*) \in [0, 1]^2$ such that

$$\begin{aligned} R_1(p_1^*, p_2^*) &\geq R_1(p_1, p_2^*) \quad \text{for any } p_1 \in [0, 1], \\ R_2(p_1^*, p_2^*) &\geq R_2(p_1^*, p_2) \quad \text{for any } p_2 \in [0, 1]. \end{aligned}$$

To find a symmetric Nash equilibrium, note that for any $i = 1, 2$ and any $p \in [0, 1]$, we have

$$\left. \frac{\partial R_i^+}{\partial p_i} \right|_{p_i=p_j=p} = \left. \frac{\partial R_i^-}{\partial p_i} \right|_{p_i=p_j=p} = -\frac{1}{2}p_2 - p + \frac{1}{2}. \quad (7)$$

Any candidate for a symmetric equilibrium price p^* therefore satisfies $-\frac{1}{2}(p^*)^2 - p^* + \frac{1}{2} = 0$ and, *a fortiori*, $p^* = \sqrt{2} - 1$. Since $R_i(p_i, p_j)$ is strictly quasi-concave (actually strictly log-concave), we have $R_i(p^*, p^*) > R_i(p_i, p^*)$ for any $p_i \in [0, 1] \setminus \{p^*\}$, so p^* is the unique best response to p^* and thus the unique symmetric Nash equilibrium.²² ■

A.4 Duopoly, One Advertising

In this section of the Appendix, we provide a proof of Lemma 2. The best response function β_2 can be calculated easily by recalling that the store 2's payoff function, $R_2(p_1, p_2)$, is given by (4).

The derivatives of the payoff function is therefore

$$\begin{aligned} \frac{\partial \Pi_2^+}{\partial p_2} &= \frac{3}{2}p_2^2 - 2p_1p_2 + p_1 - 2p_2 + \frac{1}{2}, \\ \frac{\partial \Pi_2^-}{\partial p_2} &= -\frac{1}{2}p_1^2 + p_1 - 2p_2 + \frac{1}{2}. \end{aligned}$$

Applying our analysis from case 1 (namely, strict log-concavity of payoffs and the uniqueness of the best response) we can solve the above for a unique best response $p_2 = \beta_2(p_1) \in [0, 1]$. Note

²²Quasi-concavity can be checked by taking the second derivative. Actually, we can show that for any log-concave density (e.g., the uniform) on willingness-to-pay, $R_i(p_i, p_j)$ is log-concave and thus quasi-concave. The technique (which relies on the Prékopa-Borell Theorem) may be found in Caplin and Nalebuff (1991) and Dierker (1991) (also Anderson *et al.*, 1992). For this particular model, see Konishi (1999).

that when $p_1 = \sqrt{2} - 1$, $\beta_2(p_1) = \sqrt{2} - 1$ as well. Since $p^* = \sqrt{2} - 1$ is the only solution of (7), it is clear β_2 crosses the 45-degree line exactly once at $p_1 = p_2 = \sqrt{2} - 1$, leading to the best response function given.

Knowing this best response function, store 1 maximizes its profit. Store 1 is aware of how p_1 affects market size, both through its direct effect on consumer search choices and through its indirect effect on consumers' expectations of store 2's response. Thus, store 1's payoff function can be written as:

$$\hat{\Pi}_1(p_1) = \Pi_1(p_1, \beta_2(p_1)) - c = R_1(p_1, \beta_2(p_1))\mu(p_1, \beta_2(p_1)) - c,$$

where $\mu(\cdot, \cdot)$ is defined in (2). Since $\hat{\Pi}_1$ is continuous, it attains a maximum on $[0, 1]$. It remains to show that $p_1^* \approx 0.2913$ is the maximizer. Note that $\hat{\Pi}_1$ is first-order continuously differentiable except at $p_1 = \sqrt{2} - 1$. Thus, p_1 that maximizes $\hat{\Pi}_1$ must satisfy the first order condition unless $p_1 = \sqrt{2} - 1$. On the interval $(\sqrt{2} - 1, 1]$, the derivative of store 1's profit function is given by

$$\frac{d\tilde{\Pi}_1(p_1)}{dp_1} = -\frac{3}{16}p^7 + \frac{203}{384}p^6 + \frac{37}{64}p^5 - \frac{335}{128}p^4 + \frac{109}{96}p^3 + \frac{287}{128}p^2 - \frac{139}{64}p + \frac{55}{128}. \quad (8)$$

It can be numerically verified that (8) has seven roots, five of which are real, and none of which falls in $(\sqrt{2} - 1, 1]$. Moreover, $\frac{d\tilde{\Pi}_1}{dp_1}(1) < 0$, so $p_1 = 1$ cannot be a maximizer. On the interval $[0, \sqrt{2} - 1)$, the derivative of the profit function is given by

$$\begin{aligned} \frac{d\tilde{\Pi}_1^-(p)}{dp_1} = \frac{1}{1458\rho(p)} & \left(3840p^6 + 6520p^5 - 6898p^4 - 1920p^5\rho(p) - 2780p^4\rho(p) + 1408p^3\rho(p) \right. \\ & \left. + 4440p^2\rho(p) - 3776p\rho(p) + 676\rho(p) + 871p^3 - 509p^2 + 545p - 53 \right), \quad (9) \end{aligned}$$

where $\rho(p) = \sqrt{4p^2 + 2p + 1}$ (again, the superscript on p_1 is suppressed for notational simplicity).

As in Appendix A.1, all roots of (9) will also be roots of

$$\begin{aligned} 184320p^{10} - 3840p^9 - 556716p^8 + 398308p^7 + 248585p^6 \\ - 312210p^5 + 87127p^4 - 44768p^3 + 47439p^2 - 17010p + 1869, \end{aligned}$$

with the possible introduction of spurious roots. It can be numerically verified that the above expression has three pairs of complex conjugate roots and four real roots, two of which ($p \approx 0.2405$ and $p \approx 0.2913$) are in $[0, \sqrt{2}-1)$. The root $p_1^* \approx 0.2913$ solves (9), while $p \approx 0.2405$ does not. Since $\frac{d\tilde{\Pi}_1^-}{dp_1}(0) > 0$, it is clear that $p_1 = 0$ cannot be a maximizer. Finally, note that $\tilde{\Pi}_1(\sqrt{2}-1) < \tilde{\Pi}_1(p_1^*)$, from which it follows that the best policy is uniquely given by $p_1^* \approx 0.2913$. ■

A.5 Duopoly, Both Advertising

In this section, we provide a proof of Lemma 3. From (4) and (2), we can explicitly calculate the derivatives of $\Pi_i^+(p_i, p_j) = R_i^+(p_i, p_j)\mu(p_i, p_j)$ and $\Pi_i^-(p_i, p_j) = R_i^-(p_i, p_j)\mu(p_i, p_j)$. Since we are interested in a symmetric Nash equilibrium price, we evaluate the first order conditions of these two at $p = p_i = p_j$, giving

$$\left. \frac{\partial \Pi_i^+}{\partial p_i} \right|_{p_i=p_j=p} = \left. \frac{\partial \Pi_i^-}{\partial p_i} \right|_{p_i=p_j=p} = -\frac{5}{12}p^5 - \frac{1}{3}p^4 + \frac{7}{6}p^3 + \frac{2}{3}p^2 - \frac{17}{12}p + \frac{1}{3}. \quad (10)$$

Setting (10) equal to zero yields five solutions: p^{**} , a double root at $p = 1$, and a pair of complex conjugate roots. Consequently, the only candidate for a symmetric Nash equilibrium is p^{**} (neither $p = 0$ nor 1 can be a Nash equilibrium since $\Pi_i(0, 0) = \Pi_i(1, 1) = 0$ and store i can do better than zero profit). Now, we show that (p^{**}, p^{**}) is a Nash equilibrium. We need to investigate the shapes of $\Pi_i^-(p_i, p^{**})$ in $p_i \in [0, p^{**})$ and $\Pi_i^+(p_i, p^{**})$ in $p_i \in (p^{**}, 1]$. First, note

$$\frac{\partial \Pi_i^-}{\partial p_i}(p_i, p^{**}) = A_3(p^{**})p^3 + A_2(p^{**})p^2 + A_1(p^{**})p + A_0(p^{**}) \quad (11)$$

where

$$\begin{aligned} A_3(p^{**}) &= -2 \\ A_2(p^{**}) &= -\frac{9}{4}(p^{**})^2 + \frac{9}{2}p^{**} + \frac{9}{4} \\ A_1(p^{**}) &= -(p^{**})^4 + \frac{7}{3}(p^{**})^3 - 2(p^{**})^2 - p^{**} - \frac{11}{6} \\ A_0(p^{**}) &= \frac{1}{12}(p^{**})^5 - \frac{5}{12}(p^{**})^4 + \frac{2}{3}(p^{**})^3 - \frac{7}{12}(p^{**})^2 + \frac{5}{12}p^{**} + \frac{1}{3}. \end{aligned}$$

We can show numerically that (11) has one real root at p^{**} and one pair of complex conjugate roots. Thus, the sign of (11) does not change in the interval $[0, p^{**})$. Since (11) is positive at $p_i = 0$, $\Pi_i(p^{**}, p^{**}) > \Pi_i(p_i, p^{**})$ for any $p_i \in [0, p^{**})$. Second, note

$$\frac{\partial \Pi_i^+}{\partial p_i}(p_i, p^{**}) = B_5(p^{**})p^5 + B_4(p^{**})p^4 + B_3(p^{**})p^3 + B_2(p^{**})p^2 + B_1(p^{**})p + B_0(p^{**}) \quad (12)$$

where

$$\begin{aligned} B_5(p^{**}) &= \frac{1}{2} \\ B_4(p^{**}) &= -\frac{25}{12}(p^{**}) - \frac{25}{12} \\ B_3(p^{**}) &= 2(p^{**})^2 + \frac{20}{3}p^{**} - \frac{10}{3} \\ B_2(p^{**}) &= -\frac{21}{4}(p^{**})^2 - 6p^{**} - \frac{13}{4} \\ B_1(p^{**}) &= (p^{**})^3 + 2(p^{**})^2 + \frac{7}{3}p^{**} - \frac{11}{6} \\ B_0(p^{**}) &= \frac{1}{2}(p^{**})^3 + \frac{1}{4}(p^{**})^2 + \frac{5}{12}p^{**} + \frac{1}{3}. \end{aligned}$$

We can show numerically that (12) has three real roots and one pair of complex conjugate roots. Of the real roots, one is p^{**} and the other two are greater than one. We again conclude that the sign of (12) cannot change on $(p^{**}, 1]$. Since (12) is negative at $p_i = 1$, $\Pi_i(p^{**}, p^{**}) > \Pi_i(p_i, p^{**})$ for any $p_i \in (p^{**}, 1]$. Hence, we conclude that p^{**} is unique symmetric Nash equilibrium. ■

A.6 Simultaneous Price Setting Case

In this appendix, we analyze a simultaneous choice version of the model in Section 3, in particular, case 2(b) of 3.1. We modify the description of a subgame (2(b)) in the following way:

Only one commodity price (w.l.o.g., say p_1) is advertised: In this case, p_1 and p_2 are chosen simultaneously, but only p_1 is a part of public information (p_2 is not).

Although this modification may seem innocuous, it weakens the predictive power of the standard equilibrium concept. We will show that even perfect Bayesian equilibrium can be powerless in

prediction: there can be a continuum of perfect Bayesian equilibria, although in our original game, subgame perfect equilibrium is enough to pin down a unique outcome. Let $\nu(p_2|p_1)$ be consumers' conditional beliefs over p_2 when p_1 is announced by the store. Consider the following class of assessments (combinations of beliefs and strategies): Let $\hat{p}_1 \in [0, 1]$,

Beliefs:

- (i) For $p_1 = \hat{p}_1$, $\nu(\beta(\hat{p}_1)|\hat{p}_1) = 1$ and $\nu(p_2|\hat{p}_1) = 0$ for any $p_2 \neq \beta(\hat{p}_1)$,
- (ii) For $p_1 \neq \hat{p}_1$, $\nu(1|p_1) = 1$ and $\nu(p_2|p_1) = 0$ for any $p_2 \neq 1$.

Strategies:

- 1. The store's strategy is $(\hat{p}_1, \beta(\hat{p}_1))$.
- 2. If $p_1 = \hat{p}_1$, a consumer visits the store if and only if her search cost is less than $\mu(\hat{p}_1, \beta(\hat{p}_1))$; and if $p_1 \neq \hat{p}_1$, she visits the store if and only if her search cost is less than $\mu(p_1, 1)$.

It is easy to see that this belief and strategy configuration forms a perfect Bayesian equilibrium for any \hat{p}_1 satisfying

$$R^m(\hat{p}_1, \beta(\hat{p}_1))\mu(\hat{p}_1, \beta(\hat{p}_1)) \geq \max_{p_1 \neq \hat{p}_1} R^m(p_1, \beta(p_1))\mu(p_1, 1).$$

For the store, it does not pay to deviate from $(\hat{p}_1, \beta(\hat{p}_1))$, since (i) if $p_1 = \hat{p}_1$, $p_2 = \beta(\hat{p}_1)$ maximizes $R^m(\hat{p}_1, p_2)$ and given the beliefs, the market size is $\mu(\hat{p}_1, \beta(\hat{p}_1))$; and (ii) if $p_1 \neq \hat{p}_1$, the store can only (at best) earn $R^m(p_1, \beta(p_1))\mu(p_1, 1)$ given consumers' beliefs. Moreover, given consumers' beliefs, consumers' decisions should be the ones that the strategy configuration describes. Thus, we have to conclude that there are a continuum of perfect Bayesian equilibria.

The problem with perfect Bayesian equilibrium as a refinement in our model is that it does not restrict beliefs off the equilibrium path. Consequently, for any $p_1 \neq \hat{p}_1$ the beliefs can be anything. On the other hand, in the original game, subgame perfection is strong enough to eliminate equilibrium paths with $\hat{p}_1 \neq p_1^*$, since off-equilibrium paths need to form Nash equilibria in

subgames (so the belief needs to be consistent with the store's actual action). Is there any way to strengthen perfect Bayesian equilibrium to attain the same outcome in the simultaneous move case? One way is to use the idea of trembling hand (consistency requirement in Kreps-Wilson's sequential equilibrium) so that with a positive probability every off-equilibrium path p_1 realizes, and consumers are forced to have a belief with $\nu(\beta(p_1)|p_1) = 1$. However, unfortunately, since we have a continuum of actions, we cannot assign positive probabilities to all actions. We can probably define an equilibrium refinement to obtain the same outcome as in the sequential move case. However, the equilibrium concept may be very specific to the game, and it may not be quite useful to do that. This is the reason why we used our specific game form and subgame perfect equilibrium as its solution concept.

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