Alternative Monetary Rules for a Small Open Economy:

The Case of Canada[*]

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First draft: November 21, 1998.
This draft: October 30, 2000

Abstract

I compare the performance of alternative monetary rules for Canada using an open economy model under incomplete markets. Different rules generate different paths for the markup and the terms of trade. A comparison of welfare levels suggests that flexible inflation targeting, the Bank of Canada's current policy, dominates strict targeting rules—among which a fixed exchange rate with the U.S.—and the Taylor rule. In contrast to other studies, strict targeting rules generate a more stable real economy by stabilizing markup dynamics. Flexible inflation targeting dominates because it yields a positive covariance between consumption and the labor effort, which provides agents with a source of risk diversification.

Keywords: Fixed exchange rates; Inflation targeting; Markup; Monetary rules; Taylor rule; Welfare
JEL Classification: C52, E52, F41

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1. Introduction

This paper contributes to a growing debate on the performance of alternative monetary rules for open economies. Several countries have started using inflation targeting in the recent years. An explicit quantitative target, either an interval or a point target, dictates what inflation should be in the economy. Until very recently, analyses of the advantages of following such a rule were based on closed economy models. However, given their economic size, the countries in question are generally better classified as small open economies. Ball (1998) and Svensson (1998) made a start at comparing the performance of inflation targeting with that of other rules in open economy models. They relied on traditional, non-microfounded, small open economy models. Gali and Monacelli (2000) and Gertler, Gilchrist, and Natalucci (2000) compare the performance of alternative policy rules for generic small open economies using microfounded, intertemporal models. Neither of these papers focuses on inflation targeting. I offer an empirical analysis of alternative monetary rules for Canada that relies on a rigorous, microfounded, intertemporal model. This allows me to highlight phenomena that are not featured in Ball and Svensson's studies, and that may be relevant also for other countries. As an important byproduct, this paper provides an attempt to face a model in line with the recent developments in the so-called "new open economy macroeconomics" with empirical evidence.

Because Canada is among the countries that are using inflation targeting, I treat this as the benchmark policy in my analysis. The Bank of Canada targets CPI inflation.[1] I obtain the benchmark rule by estimating a reaction function for the Canadian interest rate over the inflation targeting period for which I have data (1991-1997). The reaction function is specified similarly to Svensson's (1998) flexible inflation targeting regime. I compare the performance of this rule to that of several others, including strict inflation targeting, a fixed exchange rate with the U.S., a rule that stabilizes markup dynamics, and various versions of the Taylor rule.[2]

Although Canada has consistently floated its currency, the degree of trade and financial markets integration with the U.S. is substantial. To the extent that the North American Free Trade Agreement contributes to strengthen this integration, the U.S. and Canada may increasingly resemble an optimal currency area. This provides a rationale for considering the fixed-rate rule.[3] Under some conditions, markup stabilization maximizes agents' welfare.[4] The conditions are not satisfied in my exercise, but markup fluctuations are an important source of real volatility in my model. Hence, stabilizing the markup may yield a favorable welfare outcome. (And it turns out that markup stabilization is equivalent to a form of PPI inflation targeting in my model.) The Taylor rule has received and continues to receive much attention in policy debates.[5]

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[1] Information on the target and the procedures can be easily found in the Bank of Canada's website.

[2] I do not solve the problem of finding *the* optimal monetary rule for Canada. Rather, I compare the performance of rules that are commonly discussed in the literature and the policy debate. Gali and Monacelli (2000) compare the Taylor rule and a fixed exchange rate regime to the optimal monetary rule for a small open economy in a purely theoretical exercise.

[3] That this rationale is of relevance is exemplified by the recent policy debate in Canada, see for example Buiter (1999), Courchene (1998), and Laidler (1999).

[4] See, for example, Gali and Monacelli (2000).

[5] According to the Taylor rule, the central bank should set the interest rate to react to movements of inflation and output. I consider rules in which CPI inflation is the relevant measure of inflation. Answering a question asked during the conference "The ECB and Its Watchers," Frankfurt, June 17-18, 1999, John Taylor stated explicitly that the Taylor rule was born as a *normative* concept.

In my exercise, Canada is subject to exogenous volatility originating from three sources: shocks to domestic technology, disturbances to U.S. GDP, and shocks to U.S. monetary policy. Shocks to U.S. GDP also trigger a reaction by the Federal Reserve and movements in U.S. inflation. A simple VAR provides the empirical model for the U.S. variables of interest. Changes in these variables affect the Canadian economy.

I solve the model numerically, using values for the structural parameters obtained in a two-step estimation exercise that relies on single-equation regressions and system-wide, full information maximum likelihood regressions. Parameter estimates are generally characterized by small standard errors and are in line with the results of other studies. When I solve the model using these parameter values and the estimated reaction function for the Canadian interest rate, the model matches several key moments in the data for the inflation targeting period fairly well.

Different monetary rules generate different dynamics following exogenous shocks. Because firms react to CPI dynamics in their price-setting decisions, different rules generate different paths for producer prices, the terms of trade, and the markup. This also affects capital accumulation and asset prices. This channel of transmission of shocks is not featured in Ball (1998) and Svensson's (1998) analyses. Because Canada takes the world real interest rate as given, different rules cause differences in the real interest rate only in the very short run, when deviations from uncovered interest parity cause *ex post* real rates to differ internationally.

Ball (1998) argues that pure inflation targeting is dangerous in an open economy, because it generates large fluctuations in exchange rates and output. He argues that the Taylor rule must be adjusted to include a reaction to the exchange rate in order to stabilize the economy. Svensson (1998) finds that the real economy is more stable under flexible inflation targeting or the Taylor rule than under strict CPI inflation targeting. My results for the Canadian economy are different.

Comparisons of welfare levels under different rules suggest that abandoning flexible inflation targeting to adopt any of the other rules would not be optimal for Canada. The estimated flexible inflation targeting reaction functions I consider generate a multiplicity of stable solutions of the model. However, the solution obtained by using the smallest stable roots, which corresponds to McCallum's (1999) minimal state variable solution, yields the highest welfare level among the rules I consider. The reason differs from Ball and Svensson's arguments, though. Contrary to their results, the strict targeting rules—strict inflation targeting, a fixed exchange rate, markup stabilization—do yield a more stable economy by generating a less volatile path for the markup. However, the covariance between consumption and the labor effort is positive under flexible inflation targeting, negative under the strict targeting rules. Under flexible inflation targeting, agents enjoy more leisure in periods in which consumption is low. This source of risk diversification more than offsets the welfare effects of larger volatility. If only rules that generate a unique stable solution are considered, a Taylor rule in which the central bank reacts to anticipated inflation and current GDP dominates the version that adjusts for exchange rate depreciation and the strict targeting rules.

The structure of the paper is as follows. Section 2 presents the formal model of the small open economy under analysis. Section 3 illustrates the empirical model of the U.S. used to represent the dynamics of the rest-of-the-world economy. Section 4 describes the choice of parameter values for the exercise, the benchmark policy rule, and the evaluation of the empirical performance of the model. Section 5 presents the alternative monetary rules I consider. Section 6 discusses the welfare implications of different rules. Section 7 concludes.

2. The Model

The model builds on the analysis of macroeconomic interdependence under incomplete markets in Ghironi (2000*a*) by introducing nominal rigidity, endogenous markup dynamics, and accumulation of physical capital. The closest analogs are perhaps Gali and Monacelli (2000), Gertler, Gilchrist, and Natalucci (2000), and Smets and Wouters (2000). The main difference with the former is that markets are complete in Gali and Monacelli's model. The current account and foreign asset accumulation play no role in business cycle fluctuations. Investment and capital accumulation are not featured. Bonds denominated in domestic and foreign currency are the only internationally traded assets in my model. Net foreign asset accumulation matters for the transmission of shocks. Determinacy of the steady state and model stationarity are obtained by adopting a demographic structure that departs from the standard, representative agent framework.[6]

2.1. The Setup

The world is assumed to consist of two countries, home and foreign. Home is identified with Canada and foreign with the U.S. Variables referring to the foreign economy are denoted by an asterisk. World variables are denoted with a superscript *W*. In each period *t*, the world economy is populated by a continuum of distinct infinitely lived households between 0 and N_t^W. Each of these households consumes; supplies labor; and holds money balances, bonds, and shares in firms. Following Weil (1989), I assume that households come into being on different dates and are born owning no financial assets or cash balances. N_t —the number of households in the home economy—grows over time at the exogenous rate n, so that $N_{t+1} = (1+n)N_t$. I normalize the size of a household—or dynasty—to 1, so that the number of dynasties alive at each point in time is also the economy's population. Foreign population grows at the same rate as home. I assume that the ratio N_t/N_t^* is sufficiently small that home's population is small relative to the rest-of-the-world's. The world economy has existed since the infinite past. It is convenient to normalize world population at time 0 to the continuum between 0 and 1, so that $N_0^W = 1$.

At time 0, the number of households in the world economy is equal to the number of goods that are supplied. A continuum of goods $z \in [0, 1]$ is produced in the world by monopolistically competitive infinitely lived firms, each producing a single differentiated good. Over time, the number of households grows, but the commodity space remains unchanged. Thus, as time goes, the ownership of the firms spreads over a larger number of households.[7] I assume that the domestic economy produces goods in the interval [0, *a*]—which is also the size of the home population at time 0—whereas the foreign economy produces goods in the range (*a*, 1]. Because the ratio N_t/N_t^* is constant, it is always equal to $a/(1-a)$. Hence, the assumption that N_t/N_t^* is small is sufficient to ensure that home produces a small share of the goods available for consumption in each period.

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[6] Smets and Wouters (2000) adopt a somewhat similar approach. Gertler, Gilchrist, and Natalucci (2000) augment the Obstfeld-Rogoff (1995) framework by introducing a financial accelerator model of investment, whereas I rely on a more traditional *q* model. See also Bergin (1997) and Kollmann (1999) for (non-stationary) models including investment.

[7] Firms' profits are distributed to consumers via dividends. The structure of the market for each good is taken as given.

Consumers have identical preferences over a consumption index, leisure, and real money balances. At time t_0, the representative home consumer j born in period $\upsilon \in \left[-\infty,\ t_0\right]$ maximizes the intertemporal utility function:

$$U_{t_0}^{\upsilon j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{\left[\left(C_t^{\upsilon j}\right)^{\rho} \left(LE_t^{\upsilon j}\right)^{1-\rho}\right]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \chi \frac{\left(M_t^{\upsilon j}/P_t\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right\}, \tag{1}$$

where $0 < \rho < 1$, and χ and σ are strictly positive.[8] C is a real consumption index, LE denotes leisure, M is nominal money, and P is the price deflator.

The consumption index for the representative domestic consumer born in period υ is:

$$C_t^{\upsilon j} = \left[\int_0^a \left(c_t^{\upsilon j}(z)\right)^{\frac{\theta-1}{\theta}} dz + \int_a^1 \left(c_{t*}^{\upsilon j}(z)\right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},$$

with $\theta > 1$.[9] $c_{t*}^{\upsilon j}(z)$ is time t consumption by the representative home resident born in period υ of good z produced in the foreign country. Since a is small, the relative share of domestic goods in consumption is small.

The assumptions that the domestic population is small relative to the rest-of-the-world's, that the number of goods produced in the home economy is small, and that the relative weight of foreign goods in the consumption basket is large—combined with that of free international bond trading made below—are equivalent to the assumption that home is a small open economy, whose actions have a negligible impact on the rest of the world.

Consumers in the foreign economy are assumed to have identical preferences for consumption goods as those in the domestic country and with the same elasticity of substitution.

I assume that workers supply labor in competitive labor markets. The total amount of time available in each period is normalized to 1, so that:

$$LE_t^{\upsilon j} = 1 - L_t^{\upsilon j}, \tag{2}$$

and a similar constraint holds in the foreign economy.

The price deflator for nominal money balances is the consumption-based money price index. Letting $p_t(z)$ ($p_t^*(z)$) be the home (foreign) currency price of good z, the money price levels in home and foreign are, respectively:

$$P_t = \left[\int_0^1 \left(p_t(z)\right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_t^* = \left[\int_0^1 \left(p_t^*(z)\right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

I assume that there are no impediments to trade. If firms have no incentive to price discriminate across markets, as I assume below, the law of one price holds for each individual good. Letting ε

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[8] I restrict the intertemporal elasticity of substitution in utility from money holdings to equal the elasticity of substitution in utility from consumption and leisure as this makes it possible to aggregate the money demand equation across generations easily. The restriction has no significant consequence on my results.

[9] I assume that the degree of substitutability is the same across all goods, domestic and foreign. Tille (2000) discusses the consequences of monetary shocks in the presence of differences in the degree of substitutability between goods inside each country and between the baskets of goods that each country produces.

denote the domestic currency price of one unit of the foreign currency, this implies

$p_t(z) = \varepsilon_t p_t^*(z)$.[10]

Using the law of one price and recalling that the home economy produces goods in the range between 0 and a, while the foreign country produces goods between a and 1, makes it possible to show that $P_t = \varepsilon_t P_t^*$. Consumption-based PPP holds because preferences are identical across countries and there are no departures from the law of one price.

I assume that the only internationally traded assets are bonds issued by the two countries. Each country issues bonds denominated in units of the country's currency. These bonds are regarded as perfect substitutes and an arbitrage condition—uncovered interest parity (UIP)—holds in equilibrium:[11]

$$1 + i_{t+1} = \left(1 + i_{t+1}^*\right) \frac{\varepsilon_{t+1}}{\varepsilon_t} \ .$$

i_{t+1} is the date t nominal interest rate on bonds denominated in home currency. Letting r_t denote home's consumption-based real interest rate between t-1 and t, the familiar Fisher parity condition ensures that it is:

$$1 + i_{t+1} = \frac{P_{t+1}}{P_t}\left(1 + r_{t+1}\right), \ 1 + i_{t+1}^* = \frac{P_{t+1}^*}{P_t^*}\left(1 + r_{t+1}^*\right).$$

Perfect capital mobility and consumption-based PPP imply *ex ante* real interest rate equalization, so that $r_{t+1} = r_{t+1}^*$. Because home is small compared to the rest of the world, home takes the foreign nominal interest rate and the world real interest rate r_{t+1} as exogenous.

2.2. Consumers' Behavior

Agents in each country hold only units of the domestic currency. They are subject to lump-sum taxation T^{LS}, payable in units of the composite consumption good. V_t^i denotes the date t price of a claim to the representative domestic firm i's entire future profits (starting on date $t + 1$). V_t^i is denominated in units of home currency. I let $x_{t+1}^{\upsilon\ ji}$ be the share of the representative domestic firm i owned by the representative domestic consumer j born in period υ at the end of period t and d_t^i be the nominal dividends the firm issues on date t.

The consumer enters a period holding nominal bonds issued in the two countries, nominal money balances, and shares purchased the period before. He or she receives interest and dividends on the assets, may earn capital gains or losses on shares, earns labor income, is taxed, and consumes. Savings are divided between increases in assets and in the value of shares and money balances to be carried into the next period. Letting $A_{t+1}^{\upsilon\ j}$ ($A_{t+1*}^{\upsilon\ j}$) denote the representative home

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[10] Formally, the price index for each country solves the problem of minimizing total private spending evaluated in units of the country's currency subject to the constraint that the real consumption index be equal to 1. The assumption that consumers born at different points in time all share the same characteristics ensures that firms have no incentives to price discriminate across consumers of different ages.

[11] This condition can be derived from the first-order conditions governing the consumer's optimal choices of bond holdings once indifference on the margin between domestic and foreign bonds is imposed. Uncovered interest parity will be violated *ex post* in periods in which unexpected shocks happen.

consumer's holdings of domestic (foreign) bonds entering time $t + 1$, the period budget constraint expressed in units of the domestic currency is:

$$A_{t+1}^{\upsilon\ j} - A_t^{\upsilon j} + \varepsilon_t\left(A_{t+1*}^{\upsilon\ j} - A_{t\ *}^{\upsilon j}\right) + \int_0^a\left(V_t^i x_{t+1}^{\upsilon\ ji} - V_{t-1}^i x_t^{\upsilon ji}\right)di + M_t^{\upsilon j} - M_{t-1}^{\upsilon\ j} =$$

$$= i_t\, A_t^{\upsilon j} + \varepsilon_t i_t^* A_{t\ *}^{\upsilon j} + \int_0^a d_t^i x_t^{\upsilon ji} di + \int_0^a\left(V_t^i - V_{t-1}^i\right)x_t^{\upsilon ji}\, di + W_t\, L_t^{\upsilon j} - P_t\, C_t^{\upsilon j} - P_t\, T_t^{LS\,\upsilon}. \qquad (3)$$

W_t is the nominal wage paid for one unit of labor, taken as given by workers. $M_{t-1}^{\upsilon\ j}$ is the agent's holdings of nominal money balances entering period t. I assume that $A_\upsilon^{\upsilon j} = A_{\upsilon\ *}^{\upsilon j} = x_\upsilon^{\upsilon\ ji} = M_{\upsilon-1}^{\upsilon} = 0$. Newly born individuals are not linked by altruism to individuals born in previous periods. Hence, individuals are born owning no financial wealth or cash balances.[12] This assumption is crucial to ensure that the model has an endogenously determined steady state, to which the economy returns following temporary shocks.

The representative domestic consumer maximizes the intertemporal utility function (1) subject to the constraints (2) and (3). Dropping the j superscript, because symmetric agents make identical equilibrium choices, optimal supply of labor is determined by the labor-leisure tradeoff equation:

$$L_t^{\upsilon} = 1 - LE_t^{\upsilon} = 1 - (1-\rho)C_t^{\upsilon}/(\rho W_t/P_t).$$

When agents are optimizing, the marginal cost of supplying more labor equals the marginal utility of the additional consumption that the increase in labor income allows to afford.

Making use of the previous equation, the first-order conditions for the optimal holdings of domestic and foreign bonds reduce to the Euler equation:

$$C_t^{\upsilon} = \frac{1}{\beta^{\sigma}(1+r_{t+1})^{\sigma}} C_{t+1}^{\upsilon}\left(\frac{W_t/P_t}{W_{t+1}/P_{t+1}}\right)^{(1-\rho)(1-\sigma)}, \quad \upsilon \le t.$$

Unless $\sigma = 1$, in which case period utility is additively separable in consumption and leisure, the rate of consumption growth depends on the rate of growth of the net real wage. Depending on whether σ is smaller or larger than 1, net real wages that grow over time introduce an upward or downward tilt in the path of consumption, respectively.

Combining the first-order condition for the optimal choice of $x_{t+1}^{\upsilon\ ji}$ with the Euler equation shows that, at an optimum, consumers are indifferent between bonds and shares provided the gross rate of return on shares equals the gross real interest rate:

$$1 + r_{t+1} = \frac{V_{t+1}^i + d_{t+1}^i}{V_t^i}\frac{P_t}{P_{t+1}}. \qquad (4)$$

Demand for real balances is given by:

$$\frac{M_t^{\upsilon}}{P_t} = \left(\frac{\chi}{\rho}\right)^{\sigma} C_t^{\upsilon}\left(\frac{1+i_{t+1}}{i_{t+1}}\right)^{\sigma}\left(\frac{1-\rho}{\rho W_t/P_t}\right)^{(1-\rho)(1-\sigma)}.$$

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[12] Although they are born owning the present discounted value of their net labor income.

Real balances increase with consumption and decrease with the opportunity cost of holding money. The impact of a higher real wage depends on σ. If $\sigma < 1$, a higher real wage causes demand for real balances to decrease for any given level of consumption.[13]

2.3. Firms' Behavior

2.3.a. Output Supply

Production requires labor and physical capital. Capital is a composite good, whose composition is the same as the consumption bundle, with the same elasticity of substitution. Output supplied at time t by the representative domestic firm i is:

$$Y_t^{Si} = Z_t \left(K_t^i \right)^{\gamma} \left(E_t \, L_t^i \right)^{1-\gamma}. \tag{5}$$

Because all firms in the world economy are born in period $-\infty$, after which no new goods appear, it is not necessary to index output production and factor demands by the firms' date of birth. K_t^i is the firm's capital stock, and L_t^i is labor employed by the firm. Z_t measures economy-wide, exogenous shocks to productivity. E_t is exogenous, worldwide, labor-augmenting technological progress. I assume that $E_t = (1 + g) E_{t-1}$, where g turns out to be the steady-state rate of growth of aggregate per capita output. I also assume that $1 + r > (1 + n)(1 + g)$, where r is the steady-state world real interest rate.[14]

Rotemberg and Woodford (1993) argue that, when competition is not perfect, it is important to explicitly consider materials as an input distinct from capital. In their model, material inputs are a basket of all goods in the economy, with the same composition as the consumption bundle. I choose not to consider materials as a distinct input for two reasons. First is my desire to keep the model relatively simple. Second, I assume below that purchases of goods are necessary to install new capital and make it operational and for marketing reasons. This will provide a channel through which materials affect firms' costs, and thus production, even if they do not enter the production function directly. Another difference between my model and Rotemberg and Woodford's is that I do not allow for the possibility of increasing returns and stick to a constant returns Cobb-Douglas technology. Again, reasons of simplicity motivate my choice. In the context of my model, increasing returns would pose problems of aggregation that would complicate the analysis.

2.3.b. Output Demand and Price Stickiness

Demand for the firms' output comes from several sources. The demands of goods produced in the two countries by the representative home consumer born in period υ are:[15]

$$c_t^{\upsilon}(h) = \left(\frac{p_t(h)}{P_t} \right)^{-\theta} C_t^{\upsilon}, \ \ c_{t\,*}^{\upsilon}(f) = \left(\frac{p_t(f)}{P_t} \right)^{-\theta} C_t^{\upsilon}.$$

Given identity of preferences, expressions for the foreign consumers' demands are analogous.

At time t, total demand for home good z coming from domestic consumers is:[16]

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[13] As usual, first-order conditions and the period budget constraint need to be combined with an appropriate transversality condition to ensure optimality.

[14] This condition is necessary for the model's stability.

[15] Demand functions are obtained by maximizing C subject to a spending constraint.

$$c_t(i) = a \left[\cdots \frac{n}{(1+n)^{t+1}} c_t^{-t}(i) + \cdots + \frac{n}{(1+n)^2} c_t^{-1}(i) + \frac{n}{1+n} c_t^0(i) + \right.$$
$$\left. + nc_t^1(i) + n(1+n)c_t^2(i) + \cdots + n(1+n)^{t-1} c_t^t(i) \right] =$$

$$= \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \left\{ a \left[\cdots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \cdots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 + \right. \right. \\ \left. \left. + nC_t^1 + n(1+n)C_t^2 + \cdots + n(1+n)^{t-1} C_t^t \right] \right\} = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \left[a(1+n)^t C_t \right]$$

where C_t is aggregate private home consumption per capita, defined by:

$$C_t \equiv \frac{a \left[\cdots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \cdots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 + \right. }{a(1+n)^t} \\ \left. + nC_t^1 + n(1+n)C_t^2 + \cdots + n(1+n)^{t-1} C_t^t \right]}{a(1+n)^t}.$$

Similarly, total demand for the same good by foreign consumers is:

$$c_t^*(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \left[(1-a)(1+n)^t C_t^* \right]$$

where $C_t^* \equiv \dfrac{(1-a) \left[\cdots \frac{n}{(1+n)^{t+1}} C_t^{-t^*} + \cdots + \frac{n}{(1+n)^2} C_t^{-1^*} + \frac{n}{1+n} C_t^{0^*} + \right. }{(1-a)(1+n)^t} \\ \left. + nC_t^{1^*} + n(1+n)C_t^{2^*} + \cdots + n(1+n)^{t-1} C_t^{t^*} \right]}{(1-a)(1+n)^t}$.

Investment is modeled as in the familiar Tobin's q framework. Capital accumulation by firm i obeys:

$$K_{t+1}^i - K_t^i = I_t^i - \delta \ K_t^i, \tag{6}$$

where I is investment and δ is the rate of depreciation. Investment is a composite index of all the goods produced in the world economy, defined as the private consumption index, C.

Adjusting the capital stock is costly. In order to install new capital and make it operational, the firm needs to purchase materials in the amount $CAC_t^i = \dfrac{\eta}{2} \dfrac{(I_t^i)^2}{K_t^i}$. This quantity—measured in units of the composite consumption good—represents the real cost of adjusting the firm's capital stock. The cost is convex in the amount of investment. Faster changes in the capital stock are accompanied by more than proportional increases in installation costs. A larger amount of capital in place reduces adjustment costs because larger firms can absorb a given amount of new capital at a lower cost. Note the different interpretation of the adjustment cost relative to

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[16] Vintage $\upsilon = 0$ of home consumers, born at time 0, has size a. Home population in the next period is $a(1+n)$, of which an individuals are new-born. In the following period, population contains $N_2 - N_1 = an(1+n)$ individuals born in that period. Continuing with this reasoning shows that generation t consists of $an(1+n)^{t-1}$ households. Going back in time from $t = 0$, population at time -1 is $a/(1+n)$. Hence, generation 0 consists of $an/(1+n)$ households. And so on. Vintage $-t$ consists of $an/(1+n)^{t+1}$ households.

more traditional versions of the *q* model. There, the adjustment cost is measured as a reduction in the firm's output due to the investment activity. Here, investment causes costs due to the need of purchasing a set of goods that are required to make the installation possible and new capital operational.

Changing the price of its output is another source of costs for the firm. I introduce nominal rigidities in the model, consistent with the strong evidence in favor of sluggish adjustment of prices. Specifically, I assume that the real cost of output-price inflation volatility around a

steady-state level denoted by $\bar{\pi}$ is: $PAC_t^i = \frac{\phi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1 - \bar{\pi}\right)^2 K_t^i$. This cost is measured in

units of the composite good. When the firm changes the price of its output, a set of material goods—new catalogs, price tags, etc.—need to be purchased. PAC_t^i can be thought of as the amount of marketing materials that the firm needs to purchase when a change in price is implemented. Because the amount of these materials is likely to increase with the size of the firm, the cost of adjusting the price increases with the firm's capital stock, which is taken as a proxy for size. The cost is convex in inflation. Faster price movements are more costly to the firm. More marketing activity is likely to be required to preserve demand from falling too much as a consequence of a large price increase. Symmetrically, a large price cut gives the firm incentives to do more marketing as a way of letting a larger fraction of the public know about the lower price. The quadratic specification for the cost of adjusting prices yields dynamics for the aggregate economy that are similar to those resulting from staggered price setting a' la Calvo (1983). Thus, the specification can also be thought of as an approximation for a mechanism of price staggering.[17]

I assume that domestic firms face an identical degree of nominal rigidity domestically and abroad. Combined with the assumption of identical elasticity of substitution across goods in the two economies, this assumption implies that, even if firms were allowed to price-discriminate across markets and set prices in the consumers' currency, no pricing to market would actually result in equilibrium. The law of one price would emerge as an equilibrium outcome (see Benigno, 2000). For this reason, I do not consider pricing to market as a source of deviations from the law of one price and PPP in my model.

Total demand for good *i* produced in the home country is obtained by adding the demands for that good originating in the two countries. Making use of the results above, it is:

$$Y_t^{Di} = \left(p_t(i)/P_t\right)^{-\theta} \hat{Y}_t^{DW}, \tag{7}$$

where a hat on a variable denotes the aggregate level of the variable and $\hat{Y}_t^{DW}$ is aggregate world demand of the composite good, defined by $\hat{Y}_t^{DW} \equiv \hat{C}_t^W + \hat{I}_t^W + C\hat{A}C_t^W + P\hat{A}C_t^W$. $\hat{C}^W$ is world consumption, $\hat{C}_t^W \equiv (1+n)^t\left[aC_t + (1-a)C_t^*\right]$.[18] Because the number of firms is constant, world aggregate investment is given by $\hat{I}_t^W \equiv aI_t^i + (1-a)I_t^{*i}$. $C\hat{A}C_t^W$, and $P\hat{A}C_t^W$ are defined similarly.

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[17] The consequences of quadratic costs of adjusting prices have been first explored by Rotemberg (1982). See also Ireland (1997, 1999).

[18] In deriving (7), I have used the facts that consumption-based PPP and the law of one price imply

$\dfrac{p(z)}{P} = \dfrac{p^*(z)}{P^*}$ for each good *z* and that symmetric agents make identical choices in equilibrium.

In the expression for $\hat{I}_t^W$, I have used the fact that symmetric firms make identical equilibrium choices. However, I have maintained the i superscript for individual firms' investment, and I will do the same below, when the first-order condition for the representative firm's problem are presented. The reason is my desire of economizing on notation. Firms are not indexed by their date of birth. Hence, I maintain the i superscript for individual firms' variables because this allows me to denote aggregate per capita variables referring to firms simply by dropping the i superscript.

2.3.c. Optimality Conditions

Equation (4) implies that, on date t_0,

$$\frac{V_{t_0}^i}{P_{t_0}} = \frac{d_{t_0+1}^i/P_{t_0+1}}{1+r_{t_0+1}} + \frac{V_{t_0+1}^i/P_{t_0+1}}{1+r_{t_0+1}}.$$

Assuming that a no speculative bubbles condition holds, forward iteration yields:

$$\frac{V_{t_0}^i}{P_{t_0}} = \sum_{t=t_0+1}^{\infty} R_{t_0,t}\, \frac{d_t^i}{P_t}, \tag{8}$$

where $R_{t_0,t} \equiv 1 \Big/ \prod_{u=t_0+1}^{t}(1+r_u)$. Equation (8) states the familiar result that a firm's market value on

date t_0 is the present discounted value of the dividends the firm will pay shareholders over the future, starting on date $t_0 + 1$.

The real dividends paid by the representative domestic firm in period t are equal to

revenues—$\dfrac{p_t(i)}{P_t}Y_t^i$—minus expenditures—$\dfrac{W_t}{P_t}L_t^i + I_t^i + \dfrac{\eta}{2}\dfrac{(I_t^i)^2}{K_t^i} + \dfrac{\phi}{2}\left(\dfrac{p_t(i)}{p_{t-1}(i)}-1-\bar{\pi}\right)^2 K_t^i$. Hence,

the present discounted value of current and future real dividends at time t_0 is:

$$\frac{d_{t_0}^i + V_{t_0}^i}{P_{t_0}} = \sum_{t=t_0}^{\infty} R_{t_0,t}\left\{\frac{p_t(i)}{P_t}Y_t^i - \left[\frac{W_t}{P_t}L_t^i + I_t^i + \frac{\eta}{2}\frac{(I_t^i)^2}{K_t^i} + \frac{\phi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1-\bar{\pi}\right)^2 K_t^i\right]\right\},$$

where R_{t_0,t_0} is interpreted as 1. The representative firm chooses the price of its product, labor, investment, and capital in order to maximize this expression subject to the constraints (5), (6), (7), and the market clearing condition $Y_t^i = Y_t^{Si} = Y_t^{Di}$. The firm takes the wage, the aggregate price index, Z, E, world aggregates, and the rate of taxation of its revenues as given.

The first-order condition with respect to $p_t(i)$ returns the pricing equation:

$$p_t(i) = \Psi_t^i P_t\, \lambda_t^i. \tag{9}$$

In each period, firms charge a price which is equal to the product of the (nominal) shadow value of one extra unit of output—the (nominal) marginal cost, $P_t \lambda_t^i$—times a markup. The latter depends on output demand as well as on the impact of today's pricing decision on today's and tomorrow's cost of adjusting the output price:

$$\Psi_t^i \equiv \theta \ Y_t^i \left\{ (\theta-1)Y_t^i + \phi \ \frac{P_t}{p_t(i)} \left[\begin{matrix} K_t^i \ \frac{p_t(i)}{p_{t-1}(i)} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 - \bar{\pi} \right) + \\ - \frac{K_{t+1}^i}{(1+r_{t+1})} \ \frac{p_{t+1}(i)}{p_t(i)} \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 - \bar{\pi} \right) \end{matrix} \right] \right\}^{-1} . \qquad (10)$$

If $\phi = 0$, *i.e.*, if prices are fully flexible, Ψ_t^i reduces to $\theta/(\theta-1)$, which is the familiar constant-elasticity markup.

Introducing price rigidity generates endogenous fluctuations of the markup even in the presence of a constant elasticity specification of demand that is consistent with the assumptions of Rotemberg and Woodford's (1993) basic model, where the markup is constant. Because Ψ_t^i depends on $p_{t+1}(i), \ p_t(i)$, and $p_{t-1}(i)$—as well as on Y_t^i—if $\phi \neq 0$, equation (9) defines $p_t(i)$ implicitly as solution to a second-order non-linear difference equation. It is possible to verify that, if it is optimal to have relatively higher (weighted) price inflation today than tomorrow, the firm will find it optimal to react to an increase in demand today by raising its markup. If instead today's optimal inflation is lower than tomorrow's, an increase in demand will be accompanied by a decrease in the markup. The introduction of nominal price rigidity in a constant-elasticity framework that would otherwise be characterized by a constant markup makes some of the predictions of my model resemble those of the implicit collusion model put forth by Rotemberg and Woodford (1990).

If θ approaches infinity, firms have no monopoly power, and the markup reduces to the competitive level—1—regardless of the (finite) value of ϕ. [19] Under perfect competition, the presence of a cost of adjusting the price level is *de facto* irrelevant for the firm's decisions. Some degree of monopoly power is necessary for the nominal rigidity to matter. When the elasticity of substitution across goods is finite, $\Psi_t^i > 1$ as long as the real net revenue from output sale is larger that the real marginal cost of a price change, a condition that must be satisfied for the firm to be optimizing.

The first-order condition for the optimal choice of L_t^i yields:

$$\lambda_t^i (1-\gamma) (Y_t^i / L_t^i) = W_t / P_t .$$

At an optimum, the real wage index must equal the shadow value of the extra output produced by an additional unit of labor.

In a model in which firms have no market power and they take the price as given, labor demand is determined by the familiar equality between the real wage index—W_t/P_t—and the marginal product of labor measured in units of the composite good—$(p_t(i)/P_t)(1-\gamma)(Y_t^i/L_t^i)$. Here, the combination of pricing and labor demand decisions yields:

$$\frac{W_t}{P_t} = \frac{p_t(i)}{P_t \Psi_t^i} (1-\gamma) \frac{Y_t^i}{L_t^i} , \qquad (11)$$

which reduces to the familiar condition that determines labor demand in a competitive framework when θ approaches infinity. The presence of monopoly power introduces a wedge between the

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[19] I am implicitly assuming that all variables in the definition of Ψ_t^i have finite limit values as θ approaches infinity.

real wage index and the marginal product of labor. Because $\Psi_t^i > 1$ when θ is finite, monopoly power causes firms to raise the marginal product of labor above the real wage, *i.e.*, to demand less labor than they would under perfect competition, as in Rotemberg and Woodford (1993). The wedge between the real wage index and the marginal product of labor reflects also the presence of costs of adjusting the price level. A finite value of θ causes price stickiness to have a direct effect on labor demand, which would disappear if firms had no monopoly power.

The cyclical behavior of the markup in my model, and its impact on labor demand, are consistent with Rotemberg and Woodford's (1990) result that markup variations play an important role in business cycle fluctuations. When the markup can move with the business cycle, the real wage is not tied to the marginal product of labor. As a consequence, the real wage can in principle adjust procyclically to shocks, letting the markup move in a countercyclical fashion.

The first-order condition for the optimal choice of I_t^i implies that firm i's investment is positive if and only if the shadow value of one extra unit of capital in place at the end of period t—q_t^i—is larger than 1:

$$I_t^i = \frac{K_t^i}{\eta}\left(q_t^i - 1\right). \tag{12}$$

q_t^i obeys the difference equation:

$$q_t^i = \left(\frac{1}{1+r_{t+1}}\right)\left[q_{t+1}^i(1-\delta) + \frac{W_{t+1}}{P_{t+1}}\frac{\gamma}{1-\gamma}\frac{L_{t+1}^i}{K_{t+1}^i} + \frac{\eta}{2}\left(\frac{I_{t+1}^i}{K_{t+1}^i}\right)^2 - \frac{\phi}{2}\left(\frac{p_{t+1}(i)}{p_t(i)} - 1 - \bar{\pi}\right)^2\right].$$

The shadow price of one unit of capital in place at the end of period t is the discounted sum of the shadow price of capital at time $t + 1$ net of depreciation, of the shadow value of the incremental output generated by capital at $t + 1$, and of the marginal contribution of capital in place at the end of period t to the costs of installing capital and changing the price of the firm's output at time $t + 1$. Solving this equation under the assumption of no speculative bubbles yields:

$$q_t^i = \sum_{s=t+1}^{\infty} R_{t,s}(1-\delta)^{s-(t+1)}\left[\frac{W_s}{P_s}\frac{\gamma}{1-\gamma}\frac{L_s^i}{K_s^i} + \frac{\eta}{2}\left(\frac{I_s^i}{K_s^i}\right)^2 - \frac{\phi}{2}\left(\frac{p_s(i)}{p_{s-1}(i)} - 1 - \bar{\pi}\right)^2\right].$$

The shadow value of an additional unit of capital installed during period t is equal to the present discounted value of its marginal contributions to production and the firms' costs.

It is possible to show that an alternative expression for q is given by:

$$q_t^i = \left[\frac{V_t^i}{P_t} + \sum_{s=t+1}^{\infty} R_{t,s}\left(\frac{1}{\Psi_s^i} - 1\right)\frac{p_s(i)}{P_s}Y_s^{Si}\right]\Big/ K_{t+1}^i. \tag{13}$$

This is the result first obtained by Hayashi (1982). The ratio of the firms' real equity to the capital stock—$\left(V_t^i/P_t\right)\Big/K_{t+1}^i$—is the so-called average q—$q^{AVG i}$. Under perfect competition (when θ is infinite), the markup reduces to 1, and marginal and average q coincide. When firms have monopoly power, Ψ is higher than 1, and marginal q is smaller than average q. The shadow value of an additional unit of capital installed at the end of period t is smaller under monopolistic competition because a larger capital stock causes production to increase and the output price to decrease. This conflicts with a monopolist's incentive to keep the price higher and supply less output than it would be optimal in the absence of monopoly power. Markup fluctuations affect

firms' investment decisions by generating fluctuations in the discrepancy between average and marginal q.

2.4. Monetary Policy, Markup Dynamics, and Welfare: A Preview

Monetary policy is conducted by setting the nominal interest rate i_{t+1} according to interest setting rules discussed below.[20] Different monetary rules generate different degrees of CPI inflation volatility. Because firms react to CPI dynamics in their price setting (equation (9)), different CPI inflation volatility translates into different degrees of volatility in producer prices and, in turn, in markup dynamics (equation (10)). But the more volatile the markup, the more volatile labor demand and investment (equations (11), (12), and (13)). Labor market equilibrium and the labor-leisure tradeoff tie labor demand to consumption dynamics. Hence, volatile CPI inflation ends up causing consumption to be more volatile via its impact on the markup. Put differently, alternative policy rules can produce different dynamics for the real economy by causing differences in the behavior of the relative price of the representative domestic good, $p(i)/P$, which can be taken as a measure of the economy's terms of trade.[21] In a world in which *ex ante* real interest rates are equalized across countries, this is the main channel through which different monetary regimes have different implications for welfare.

2.5. Steady-State Determinacy and Stationarity: Intuition

It is possible to obtain the equations that describe the behavior of (detrended) aggregates per capita from those for individual consumers and firms using the linear aggregation procedure across generations described in Section 2.3.*b*. Once these equations are obtained, it is possible to verify that—given steady-state levels for exogenous foreign variables and the domestic nominal interest rate—domestic variables have unique, endogenously determined, constant steady-state levels. The breakup of Ricardian equivalence implicit in the assumption that newborn agents have no financial assets provides a realistic solution to the indeterminacy of the steady state that affects several open economy models under incomplete markets, and to the non-stationarity that follows. Under sensible parameter restrictions, endogenous variables return to their steady-state levels following temporary shocks that cause them to move away from their long-run equilibrium levels. The intuition for this result is simple. In most models of macroeconomic interdependence under incomplete markets, the steady state is indeterminate, because consumption growth does not depend on the country's net foreign asset holdings. Hence, setting consumption to be constant does not pin down a steady-state distribution of net foreign assets. Because countries cannot fully insure themselves against the consequences of unexpected shocks, all shocks have permanent real effects via redistribution of assets across countries. This is true regardless of the nature of the shock, whether temporary or monetary. In my model, aggregate per capita consumption growth depends on asset holdings because of the discrepancy between asset holdings of agents already

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[20] The government budget constraint simply requires the government to rebate seignorage revenue to consumers via lump-sum transfers.

[21] The terms of trade are actually given by $p(i)/\varepsilon\ p^*(f)$, where $p^*(f)$ is the foreign PPI. Under my assumptions, the fraction of domestic goods in the world consumption bundle is negligible. Hence, $p^*(f)$ is only marginally different from P^*. Because of purchasing power parity, $p(i)/\varepsilon\ P^* = p(i)/P$.

alive at each point in time and of newborn agents—who have no assets. Hence, setting consumption to be constant endogenously pins down a stationary steady state.[22]

3. The U.S. Economy

I consider three sources of exogenous volatility for the Canadian economy: shocks to U.S. GDP, exogenous changes in U.S. monetary policy, and domestic technology shocks. Because the U.S. can be taken as a rough approximation of the rest-of-the-world economy from the perspective of Canada, shocks to U.S. variables are interpreted as shocks to world aggregates.

Shocks to U.S. variables cannot be taken in isolation. I have not modeled the structure of the U.S. economy as explicitly as Canada's but—at a minimum—one must recognize that four variables that feature in the equations for Canadian variables will be affected by shocks to the U.S. GDP or interest rate: besides these, the U.S. CPI inflation rate and the real interest rate will change. One cannot analyze the consequences of a shock to U.S. output or the interest rate without explicitly accounting for the comovements in all relevant variables that are triggered by the initial disturbance.

In the exercise of this paper, I impose a minimal amount of structure on the U.S. economy. I take the Federal Funds Rate to be the relevant nominal interest rate. The Federal Reserve is assumed to set this rate as its policy instrument. Following Rotemberg and Woodford (1997), I assume that the Fed sets the nominal interest rate based on a reaction function that depends on past levels of the rate and on the current and past levels of CPI inflation and GDP. In terms of deviations from the steady state:[23]

$$\tilde{i}_{t+1}^* = \sum_{k=1}^{n_{\tilde{i}^*}} \varphi_{1k} \tilde{i}_{t+1-k}^* + \sum_{k=0}^{n_{\tilde{\pi}^*}} \varphi_{2k} \tilde{\pi}_{t-k}^{CPI*} + \sum_{k=0}^{n_{y^W}} \varphi_{3k} y_{t-k}^W . \tag{14}$$

Shocks to this equation are exogenous shocks to monetary policy. I assume that the Fed reacts to changes in U.S. inflation and GDP that happen in the same period. McCallum (1997) argues that this is an unrealistic assumption, based on evidence collected by Ingenito and Trehan (1996). I follow Hall (1996, p. 67), who observes:

"I agree that the within-month effects of money and interest on output and prices are probably fairly small, but that does not make them literally zero. *And I have an even stronger suspicion that the Fed gets some inkling about output and prices within the month and responds to that inkling.*" (Emphasis added.)

Because Canada is small relative to the U.S., the Fed's reaction function does not incorporate any Canadian variable. The negligible impact of Canadian GDP on world aggregates allows me to identify U.S. GDP with y^W in the model.

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[22] Because this paper focuses on a policy application of the model, I do not dwell on the details of aggregation, solution for the steady state, and analysis of stationarity. A detailed analysis of these topics for a simplified, real version of the model in which the two countries have comparable size can be found in Ghironi (2000*a*), along with a discussion of related literature and alternative approaches to the non-stationarity issue. For the sake of brevity, here I limit myself to observing that several alternative solutions to the non-stationarity problem under incomplete markets yield only limited tractability gains and appear more *ad hoc* than assuming that a small number of households with no assets enter the economy in each period.

[23] Remember that i_{t+1}^* is the time *t* nominal interest rate. Steady-state levels of U.S. variables are calculated as averages over the 1980:1-1997:4 sample. Arial variables denote percent deviations from the steady state. A tilde denotes the percent deviation of a *gross* rate from its steady state level.

I model the U.S. economy as a recursive structural VAR that includes equation (14) and equations for GDP and inflation. The state vector is $\left[\tilde{\pi}_t^{CPI*}, y_t^W, \tilde{i}_{t+1}^* \right]'$, and the causal ordering of variables is the order in which they are listed. I follow Rotemberg and Woodford (1997)—and not Hall (1996)—in assuming that the interest rate affects output and inflation only with a lag, but I do not include future inflation and GDP in the time-t state vector.

I estimate the VAR with three lags using full information maximum likelihood. I use data between 1980:1 and 1997:4 from the IMF International Financial Statistics. The estimated coefficients for the three equations and the standard errors are in the columns of Table 1. Seasonal dummies were not significant, as well as further lags. The estimated coefficients for the Fed's reaction function suggest behavior in line with a generalized Taylor rule, consistent with the findings of Rotemberg and Woodford (1997).

To illustrate an example, Figure 1 shows the responses of GDP, inflation, and the Federal Funds Rate to a 1 percent increase in U.S. GDP. [24] Periods are interpreted as quarters.[25] The deviation of GDP from the steady state increases in the first two quarters. Inflation reacts with a lag, and subsequently rises. The Fed reacts immediately by raising the Federal Funds Rate to cool the economy.[26] Over time, all variables go back to the steady state.

The paths of U.S. variables generated by the shocks constitute the paths of the world-economy variables following the initial impulses in my model of the Canadian economy. The estimated VAR equations are included in the system of equations that describes the world economy, along with the equations that govern the dynamics of Canadian endogenous variables and the monetary rule followed by the Bank of Canada.

4. Model Solution and Evaluation

4.1. Parameter Values and Solution

In the remainder of the paper, I use the model to analyze the performance of alternative monetary rules for Canada by means of numerical methods. I estimate the structural parameters of the model using quarterly data from Canada and the United States for the period 1980:1-1997:4.[27] The estimates are obtained in two steps. First, I run single-equation regressions based on log-linear equations for the behavior of detrended aggregates per capita.[28] I use U.S. variables and lagged Canadian variables where instruments are necessary. In the second step, I take the estimates from the single-equation procedure as initial values for full information maximum likelihood regressions based on the systems of the first-order conditions for the behavior of

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[24] Because markets clear in the model, an exogenous increase in U.S. GDP can be interpreted both as a generalized increase in world demand for goods and as a favorable supply shock. I interpret the shock as an exogenous expansion in demand. This is consistent with the fact that U.S. inflation increases following the disturbance.

[25] In the impulse responses, the level of the interest rate at each point in time is the value chosen by the monetary authority at that date.

[26] A measure of the U.S. real interest rate can be obtained by using the response of the inflation rate to deflate that of the Federal Funds Rate. The real interest rate reacts with a lag. It rises above the steady state in the first quarter after the shock and remains higher than its long-run level until it eventually returns to it.

[27] Sources: IMF and CANSIM. See Appendix A.

[28] See Appendix B.

consumers and firms. The purpose of the second step is to verify whether estimates are significantly affected by a multi-equation approach.[29]

4.1.a. The Supply Side

The elasticity of output to capital, γ: Regressing Canadian detrended per capita GDP on the ratio of the industrial price index to the CPI, the capital stock, hours, and a set of seasonal dummies, yields an estimate for $1-\gamma$ of approximately .9, with standard error (henceforth, s.e.) = .067.[30] This is a high value for the elasticity of output to hours. However, it seems reasonable that GDP be much more sensitive to hours than capital on a quarterly basis.

The rate of depreciation of capital, δ: A non-linear least squares regression based on the law of motion for detrended aggregate per capita capital (equation (B.10) in the appendix) yields an estimate of δ of .031 (s.e. = .01, R^2 = .89). Controlling for seasonal effects raises the estimate of the rate of depreciation to approximately .04.

The cost of adjusting capital, η: The equilibrium value of the markup can be written as $\Psi_t = (1-\gamma)y_t/(w_t L_t)$, where y denotes detrended aggregate per capita GDP in units of consumption, w is the detrended real wage, and L is aggregate per capita labor. Given the estimate of the elasticity of GDP with respect to hours, a series of the markup can be calculated using this expression. Figure 2 shows the growth rates of markup and employment hours. Consistent with the model, the markup is strongly countercyclical.

  Using the series of the markup makes it possible to construct a proxy for the discrepancy between average and marginal q in equation (13)—the "monopoly adjustment factor" in investment decisions. I use an OLS regression in first differences of the investment-capital ratio on capital, average q, the monopoly adjustment factor, and a set of seasonal dummies to obtain a baseline value for η.[31] The coefficient on average q is approximately equal to $(\bar{k}\bar{q}^{AVG})/(\eta\,\overline{inv})$. Given the estimated coefficient on average q in the regression, it is possible to obtain an approximate estimate for η. The procedure suggests that values of η as high as 2 are consistent with the estimated coefficient on average q in equations that include lagged capital as a regressor.[32]

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[29] Details on the estimation procedure and the construction of proxies for unobservable variables are in Ghironi (2000*b*).

[30] Given the production function (5) for the representative Canadian good, real GDP *in units of consumption* is obtained by aggregating individual firms' output multiplied by its price in terms of the consumption basket. (See equation (B.17) in the appendix.) I construct the trend series $E_t = (1+g)E_{t-1}$ by assuming that g is the average rate of growth of Canadian real per capita GDP during the sample period and letting $E_{1980:2} = 1$. The rate of growth of population, n, is set to the average over the sample period. Steady-state levels of variables in the data are calculated as the unconditional means of the corresponding series over the sample. Variables in the regressions are defined as percentage deviations from the steady state, to match the concepts in the model.

[31] Regressions in levels did not yield reliable results.

[32] Bergin (1997) argues that a value of η as high as 20 would be required in a calibration of his model to generate results that are consistent with the empirical evidence on adjustment costs for Japanese firms reported by Hayashi and Inoue (1991). If compared to such value, my choice of 2 for the exercise below appears conservative.

The size of nominal rigidity, ϕ: Estimates of ϕ from various specifications of PPI inflation dynamics are large and significantly different from zero, but also characterized by large standard errors. Hence, I calibrate this parameter with the help of the data, remaining in the estimated range. For values of θ between 3.68 and 12.08 (see below), ϕ as high as 200 is required to generate a pattern of deviations of PPI inflation from the steady state that matches the behavior of the observed series. Would $\phi = 200$ be absurdly large? The cost of adjusting prices is $(\phi/2)(\pi^{PPI} - \bar{\pi})^2 K$. If steady-state quarterly inflation is about 1 percent, increasing inflation by 10 percent—to 1.1 percent—would require the representative firm to purchase materials in an amount equal to .01 percent of its capital stock. Although the value of ϕ is large, the actual cost borne by the firm for a substantial acceleration in its output price inflation does not seem unrealistic.

To verify the reliability of the estimates obtained above, I ran full information maximum likelihood regressions of the system of equations that govern the production side of the economy. I took the following as starting values for the procedure: $\delta = .035$, $\phi = 200$, $\gamma = .1$, $\eta = 2$, $\theta = 3.68$ (see below). The results supported my choice of these values as empirically reasonable for Canada.

4.1.b. Consumption and Leisure

The elasticity of substitution across goods, θ: Letting overbars denote steady-state levels of variables, the steady-state markup is equal to $\theta/[(\theta-1)(1-\bar{\tau}^F)]$, where $\bar{\tau}^F$ is the unconditional mean of a series of the tax rate on firms' revenues (not included in the model). Equating this expression to the unconditional mean of the markup series yields an estimate of θ around 12.08. Schmitt-Grohé (1998) calibrates the steady-state markup to 1.4 in her analysis of the transmission of U.S. business cycles to the Canadian economy. This yields an estimate for θ of 3.68. To allow an easier comparison of my results with Schmitt-Grohé's, I set θ to 3.68 in my exercise.

The intertemporal elasticity of substitution in utility from consumption and leisure, σ: Non-linear least squares regressions based on an approximation of the log-linear Euler equation for detrended aggregate per capita consumption yield *negative* and significant estimates for σ, with the likelihood function increasing for higher values of ρ.[33] To understand what motivates this result, I ran an unrestricted OLS regression of the type:

$$c_t = A_1 c_{t-1} + A_2 \tilde{r}_t + A_3 w_t + A_4 (w_t - w_{t-1}) + \sum_{i=5}^{8} A_i D_i + error_t,$$

where the D_is are seasonal dummies. The current level of the real wage proxies for consumption by newborn agents at time t. The estimates are: $A_1 = 1.04$ (s.e. = .038); $A_2 = -.18$ (.047); $A_3 = .28$ (.098); $A_4 = -.099$ (.17); $R^2 = .96$, DW = 2.17. Contrary to what the theory suggests, the real interest rate (measured by a series of the T-bills rate deflated by the CPI) has a negative and significant impact on consumption. This result resembles the findings of a previous study of the

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[33] I set $\beta = .99$ and do a grid search over plausible values of ρ. The log-linear Euler equation for aggregate per capita consumption includes an adjustment for consumption by newborn agents due to the economy's demographic structure. The result is robust to alternative estimation techniques (GMM, IV) and to the use of U.S. variables rather than lagged Canadian ones as instruments.

Canadian economy by Altonji and Ham (1990). The coefficient on real wage growth is insignificantly different from zero. This is consistent with a high value of ρ in the log-linear labor-leisure tradeoff (equation (B.8) in the appendix). The current real wage has a positive and significant effect on consumption.[34] The real interest rate has a negative and significant impact on consumption, which explains the negative estimate of σ in the initial non-linear least squares regressions. The coefficient on the real interest rate in the Euler equation is equal to σ times the coefficient on lagged consumption. Because the latter is positive, the negative effect of the real interest rate on consumption translates into a negative estimate of σ.

Following Altonji and Ham, I separate the impact of the nominal interest rate and inflation. The impact of both variables on consumption turns out only marginally significant, though the effect of inflation is larger.[35] I thus drop the nominal interest rate from the regressions used to estimate σ and focus on the effect of inflation. Dropping insignificant terms from the Euler equation, one can run the non-linear least squares regression:

$$c_t = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left(c_{t-1} - \sigma\tilde{\pi}_t^{CPI}\right) + \left[1 - \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\right]w_t + error_t.$$

β and ρ are set to .99, r equals the average of the real interest rate series described above. The estimated intertemporal elasticity of substitution is .23, with a standard error of .1 (R^2 = .95, DW = 1.64).

Alternative regressions using U.S. variables as instruments as in Altonji and Ham (1990) and doing a grid search over a range of values for ρ yield estimates of σ between .14 and .25. Altonji and Ham concluded in favor of a range between .1 and .2. My results are fairly consistent with their findings.

The relative importance of consumption and leisure in utility, ρ: Regressions based on the log-linear labor-leisure tradeoff (equation (B.8) in the appendix) suggest a range of values for ρ between .55 and .8. This range is confirmed by regressions based on the Euler equation for labor supply, obtained by combining the labor-leisure tradeoff with the consumption Euler equation. .55 seems too low a weight for consumption in utility. The consumption regressions actually suggest that values of ρ as high as .99 cannot be dismissed.

As for the production side, I used values in the ranges above for ρ and σ as starting points for FIML regressions of the system of the consumers' first-order conditions to verify whether multi-equation regressions would yield different results. The results of the FIML estimation confirmed those of the single-equation regressions. I use ρ = .75 and σ = .16 in the exercise below, along with β = .99 and χ = .0001 (to ensure that real balances have a second-order direct effect on welfare).

4.1.c. Solution

I solve the log-linear model in Appendix B, combined with the VAR equations for the U.S. and the relevant monetary rule, using routines described in Uhlig (1997). Canada is subject to shocks

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[34] This result—which conflicts with the basic random walk hypothesis—is consistent with the dynamics of population in the model.
[35] Details can be found in Ghironi (2000*b*).

to domestic technology, U.S. GDP, and the Federal Funds Rate in all periods. Given steady-state levels of U.S. variables and the domestic nominal interest rate (set to the average over the 1980:1-19974:4 sample), steady-state levels of endogenous Canadian variables can be calculated from the non-linear equations for aggregates per capita (detrended when necessary) using the parameter values obtained above and values for n, g, and r.[36]

For the overall model to generate sensible dynamics around the steady state following any of the three shocks, I am forced to raise n substantially above the average quarterly rate of growth of Canadian population over the sample (.003). I set $n = .03$, reinterpreting it as a measure of the size of the departure from Ricardian equivalence in the Canadian economy.[37] Raising n implies that the steady-state real interest rate must rise too, as the condition $1 + r > (1 + n)(1 + g)$ must be satisfied ($g = .0013$). PPP and UIP imply that real interest rates are equalized across Canada and the U.S. in the model. The quarterly U.S. real rate obtained by deflating a quarterly series of the Federal Funds Rate by CPI inflation would be too low to ensure $1 + r > (1 + n)(1 + g)$ with $n = .03$. Using the interest rate on Canadian T-bills and the series of Canadian CPI inflation shows that there is a persistent, sizable differential between Canadian and U.S. real interest rates. For consistency with real interest rate equalization in the model, I simply use the annual U.S. real interest rate as the relevant value of r in the solution of the model ($r = .066$).

Depending on the interest setting rule used by the Bank of Canada, the model can have either a unique stable solution or a multiplicity of stable solutions for the dynamics of Canadian variables. In principle, the economy is subject to the risk of sunspot equilibria in the latter situation, with random shifts taking place between alternative stable solutions. Following McCallum (2000), when multiple stable solutions exist, I focus on the one that corresponds to the smallest stable roots. In most circumstances, this corresponds to the minimal state variable solution of McCallum (1999). This is the only solution that is consistent with expectational stability and learnability as defined in Bullard and Mitra (2000) and Evans and Honkapohja (1995).

4.2. Evaluation

There are several criteria on which to base an evaluation of the empirical performance of the model. In the previous sub-section, I pointed out that the estimates of the structural parameters used in the exercise are characterized by small standard errors in most cases and are in line with the findings of other studies of the Canadian economy. Thus, the choice of parameter values captures relevant empirical features of Canada.

On a different level, one can ask whether, given those parameter values, the model yields sensible impulse responses and is capable of replicating empirical regularities observed in Canadian data. To evaluate that, I perform the following exercise. I estimate an interest rate reaction function for Canadian policymaking over the period 1991:1-1997:4—the inflation targeting period for which I have data. This rule, which I will refer to as flexible inflation targeting (*FIT*), is taken as the benchmark for comparison in the analysis of the next sections. Given the estimated policy rule, I use the frequency domain technique in Uhlig (1997) to obtain estimates of

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[36] I assume $\bar{Z} = 1$.

[37] n is indeed the rate of growth of the number of households in the economy. As the size of households tends to decrease over time in industrial economies, growth in the number of independent households may well be faster than population growth.

the unconditional second moments of a number of variables generated by the model. The disturbances in the log-linear equations are assumed to be normally *i.i.d.* with zero averages and standard deviations σ_Z, σ_{y^W}, and $\sigma_{\tilde{i}^*}$, respectively. Estimates of the variances of the shocks to the U.S. economy are obtained as the ratios of the sum of squared residuals of the VAR regression equations for y^W and $\tilde{i}^*$ divided by the number of observations in the estimation procedure. The estimated variance of the productivity shock comes from the regression used to estimate the elasticity of output to capital, γ. The estimated standard deviations are $\sigma_Z = 1.5866$, $\sigma_{y^W} = .5954$, and $\sigma_{\tilde{i}^*} = .6323$.

4.2.a. The Benchmark Rule

I consider two alternatives for the *FIT* rule. In Svensson (1998), flexible CPI inflation targeting is consistent with a reaction function in which the interest rate is a function of its own lag; current CPI inflation, GDP, natural output, and risk premium; expected CPI inflation for the next period; the past level of the real exchange rate; and the current levels of a set of foreign variables including the interest rate, GDP, and CPI inflation. [38] In the spirit of Svensson's paper, I estimate a reaction function of the type:

$$\tilde{i}_{t+1} = \alpha_1 \tilde{i}_t + \alpha_2 \tilde{\pi}_t^{CPI} + \alpha_3 \tilde{\pi}_{t+1}^{CPI} + \alpha_4 e_t + \alpha_5 y_t + \alpha_6 \tilde{i}_t^* + \alpha_7 \tilde{\pi}_t^{CPI*} + \alpha_8 y_t^W . \qquad (15)$$

I proxy expected inflation for the next quarter with its actual level. Estimated coefficients (and standard errors) are: $\alpha_1 = .67$ (.14), $\alpha_2 = -.78$ (.39), $\alpha_3 = .23$ (.34), $\alpha_4 = .23$ (.09), $\alpha_5 = -.38$ (.17), $\alpha_6 = .74$ (.3), $\alpha_7 = .31$ (.66), $\alpha_8 = .44$ (.31), $R^2 = .92$, DW = 1.18. The reactions to expected inflation and to foreign output and inflation are not significant. The central bank lowers the interest rate in reaction to higher CPI inflation. Surprising as this may appear, it is not necessarily inconsistent with flexible inflation targeting. In Svensson's model, the central bank reacts to higher expected inflation by lowering the interest rate substantially under flexible inflation targeting. From now on, I will refer to rule (15) as *SFIT* regime, for Svensson-*FIT*.

In alternative to *SFIT*, I estimate a reaction function in which I keep only the variables that are found to be (at least marginally) statistically significant. I allow the domestic central bank to react to lagged variables (one lag). The resulting *FIT* rule has the form:

$$\tilde{i}_{t+1} = \alpha'_1 \tilde{i}_t + \alpha'_2 \tilde{\pi}_t^{CPI} + \alpha'_3 e_t + \alpha'_4 y_t + \alpha'_5 \tilde{i}_t^* + \alpha'_6 \tilde{i}_{t-1}^* , \qquad (16)$$

with estimated coefficients: $\alpha'_1 = .83$ (.12), $\alpha'_2 = -.59$ (.27), $\alpha'_3 = .32$ (.06), $\alpha'_4 = -.22$ (.14), $\alpha'_5 = 1.49$ (.29), $\alpha'_6 = -.88$ (.27), $R^2 = .94$, DW = 1.88. When this procedure is followed, the Canadian central bank is found to react more aggressively to the Federal Reserve's policy than under the *SFIT* rule.

Because of its better statistical performance, I take rule *FIT* as the benchmark for evaluation of the model, using *SFIT* as a "control" alternative and a candidate alternative reaction function on welfare grounds. When the model is solved using either one of rules *FIT* and *SFIT* and the parameter values above, multiple stable solutions exist. I focus on the solution associated to the smallest stable roots for the reasons mentioned above.

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[38] Svensson (1998) analyzes both CPI and PPI inflation targeting. I limit my attention to CPI inflation targeting for two reasons. First, as mentioned in the introduction, this is what Canada is actually doing. Second, as we shall see, a rule that stabilizes markup dynamics is equivalent to a rule that targets PPI inflation in my model.

4.2.b. Impulse Responses

To illustrate the functioning of the model, Figure 3 shows the impulse responses of the main variables to a 1 percent increase in U.S. GDP under the *FIT* rule.[39] The exchange rate depreciates initially[40], which translates into faster CPI inflation. Employment rises on impact in reaction to increased output demand. More labor demand causes the real wage to be higher. GDP rises, but not as much as the real wage bill. The markup falls, countercyclical to labor demand. Even if the Federal Reserve is raising the interest rate to cool the U.S. economy (Figure 1), the Canadian central bank reacts to higher CPI inflation and GDP by lowering its interest rate on impact. PPI inflation is faster, but its dynamics are smoothed by the falling markup. The initial fall of the latter causes the relative price of domestic output to fall below the steady state. Dividends drop substantially. Lower profitability and the increased wage bill cause firms to invest substantially less than in steady state, so that the capital stock shrinks[41], along with the equity value of the home economy (*v*). Higher labor income causes consumption to rise. Higher GDP and significantly smaller investment ensure that Canada accumulates net foreign assets against the U.S. However, consumer assets (the aggregate of net foreign bond holdings and the equity value of domestic firms) fall, as a consequence of the lower value of domestic equity and larger consumption. After the period of the shock, in which UIP can be violated *ex post*, the domestic real interest rate is tied to the U.S. real interest rate regardless of the policy rule. As the Fed tightens policy and the U.S. real rate rises, U.S. GDP peaks and starts moving back to the steady state (Figure 1). Domestic consumption falls and remains below its long-run level throughout the transition dynamics. Employment falls, too, but it returns above the steady state one year after the shock. It remains higher than the long-run level during the rest of the transition, mirroring the behavior of markup and real wage. GDP tracks employment closely. PPI inflation is above the steady state through the entire transition. CPI inflation falls below for two quarters after the initial shock, as the exchange rate appreciates in response to the initial interest differential. Net foreign bond holdings and consumer assets return to the steady state gradually.

The response to a 1 percent increase in domestic productivity (Figure 4) displays substantially less persistence, as the shock has no impact on U.S. variables. Labor demand falls on impact, along with the real wage. This causes consumption to fall. The markup rises, and so does the real price of domestic output. Higher profitability spurs investment and capital accumulation, but GDP falls. Net foreign assets fall, along with consumer assets. PPI and CPI inflation fall on impact. They then oscillate around the steady state. All variables return to their long-run levels within four years.[42]

Overall, impulse responses appear consistent with economic intuition. They display reasonable shape, range of fluctuation, and persistence in most cases. Schmitt-Grohé (1998) uses a VAR approach to study how shocks to U.S. GDP are transmitted to the Canadian economy. She compares the predictions of the estimated VAR to those of alternative flexible price models. She finds that the models can explain the observed responses of output, hours, and capital only if

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[39] Some responses are shown in more than one panel to facilitate interpretation.

[40] I use the word depreciation (appreciation) to refer to an acceleration (deceleration) of the rate of depreciation above (below) the steady state.

[41] In the figure, capital at time *t* is capital at the *end* of that period. The deviation of investment from the steady state can be made smaller by raising the value of η. However, this would imply an absurdly large capital adjustment cost as a fraction of GDP.

[42] The responses to a U.S. monetary shock are omitted. They are available upon request.

larger-than-realistic movements in relative prices are allowed. She raises the question of whether relaxing the assumption of price flexibility might help remove the puzzle. My exercise provides a positive answer to the question.

4.2.c. Moments

Table 2 displays the contemporaneous correlations of several key variables calculated from the data for the inflation targeting period (1991:1-1997:4) and those generated by the model. Correlations are grouped to facilitate a comparison between the data and the building blocs of the model. The latter matches the sign of several key correlations in the data quite well under the *FIT* rule.[43] In many cases, also the size of the correlation is matched to a reasonable extent.[44] Although the assumption of PPP is certainly a weakness of the model—as suggested by the absence of any match in the correlation between depreciation and CPI inflation—the model matches the sign of several correlations involving currency depreciation. The framework matches the correlations between consumption and GDP and consumption and employment to a reasonable degree. There is a very good match between data and model for the correlation between consumption and the value of domestic equity. The same is true of the correlations involving employment, GDP , the domestic interest rate, and U.S. interest rate and GDP. The consumption-interest rate correlation is matched only in terms of its sign. Generally, there is a poor match between data and model as far as the correlations between consumption and prices are concerned. This notwithstanding several correlations involving the markup and the relative price are matched quite well, including the negative correlation between employment and the markup. The performance of the investment side of the model is not good. This is not surprising, given the generally poor performance of the *q* model. Whereas GDP and investment are positively correlated in the data, the model yields a negative correlation. The reason is that (in the data and in the model) firms tend to invest in periods in which the markup and the relative price are high. These are the periods in which increased profitability spurs investment.[45] However, in the model, these periods are also those in which firms substitute capital for labor. Given the high elasticity of GDP to labor in the exercise, declining employment yields lower GDP in the periods in which investment is higher.

Table 3 presents the standard deviations of key macro variables in the 1991:1-1997:4 sample and under the *FIT* rule. The model generates substantially larger volatility for the relatively more volatile variables than it is observed in the data. However, if standard deviations are ranked in decreasing order, the framework captures some important qualitative features. Investment, the markup, and the value of equity are among the more volatile variables both in the data and in the exercise. U.S. variables are relatively stable in the data and in the model. So are the interest rate, depreciation, and inflation. Instead, the model does poorly in terms of the ranking of the standard deviation of the real wage (far too high), capital (substantially above GDP), and consumption (more volatile than GDP, the domestic interest rate, the Federal Funds Rate, and PPI inflation).

Finally, Table 4 presents serial autocorrelations of key macro variables with GDP up to 3 leads and lags calculated from the data and the model under the *FIT* rule. Again, results are

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[43] Good matches are highlighted by using boldface fonts.

[44] Particularly poor size matches are highlighted by using italic fonts. The SFIT rule performs very similarly in terms of matching the sign of correlations. It does somewhat worse in terms of size, which lends support to the choice of *FIT* as benchmark.

[45] See also the correlations between markup and equity value and the latter and investment.

mixed. The model matches several correlations and misses others.[46] Serial correlations implied by the model are often smaller than the data would suggest. As in Table 2, investment and capital accumulation do not yield good matches. However, the model matches the sign of the serial correlation of Canadian GDP with the U.S. interest rate and GDP quite well.

4.2.d. Discussion

All in all, the moment-based evaluation exercise yields reasonably good results, although there are certainly directions in which the framework could be improved.

It is a known fact that PPP does not hold in the short run between the U.S. and Canada.[47] This may help explain some problems in matching sample moments. Nonetheless, introducing PPP deviations would require modeling them in a satisfactory way. Local currency pricing is not convincing if one believes that the U.S. and Canada are structurally similar in terms of nominal rigidity and characteristics of demand. Simply assuming that there are exogenous deviations from the law of one price, such that $p_t(i) = dev_t \varepsilon_t p_t^*(i)$, would not do the job. Given identical preferences for consumption across countries, it would be $P_t = dev_t \varepsilon_t P_t^*$. Hence, the deviations from the law of one price would not cause the real price of good i to differ in the domestic and foreign market. It would still be $\dfrac{p_t(i)}{P_t} = \dfrac{p_t^*(i)}{P_t^*}$. The firm's maximization problem would be unaffected. Assuming existence of a sector producing non-traded goods would appear preferable. This notwithstanding, it could still be argued that the current version of the model may be a proper characterization of the structure of the tradable, manufacturing sector, thus capturing substantial features of the Canadian economy.

The q model of investment is another weak point of the framework. A financial accelerator model along the lines of Gertler, Gilchrist, and Natalucci (2000) may improve the performance of the overall model.

It is not clear a priori what direction would be more successful to match more correlations, whether to introduce non-tradables or to adopt a different model of investment. Indeed, it may well be the case that the current version of the overall framework would yield a better match with the data if more sources of shocks were included. For example, absence of a domestic demand shock (say, of fiscal nature) from the model may well explain the poor match of correlations between some quantities and prices to a substantial degree. Allowing the productivity process, domestic government consumption, and the exogenous component of U.S. GDP to display realistic persistence could also improve the match between sample moments and model estimates.

In summary, the exercise of this paper uses parameter values that reflect information on the Canadian economy. The model does match several key moments in the data even with a very simple structure of exogenous processes. For these reasons, although the findings of the following sections are not meant to be in any way conclusive as far as the debate over Canadian monetary policy is concerned, they may provide insights into relevant features of the Canadian economy, with the advantage of being based on a rigorous, yet relatively tractable model.

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[46] Sign matches are highlighted by boldface fonts.
[47] See Engel and Rogers (1996).

5. Alternative Monetary Rules for Canada

I do not solve the problem of finding the rule that maximizes agents' welfare in the model. This is complicated by the distortions generated by market incompleteness. I focus on two types of alternative interest rate rules: (1) Rules that achieve simple, clearly specified policy targets, such as stability of a specific variable. (2) Taylor-type rules. In particular, I consider the following alternatives to rules *FIT* and *SFIT*: strict inflation targeting, a fixed exchange rate with the U.S., a rule that stabilizes markup dynamics, and various versions of the Taylor rule.

5.1. Strict Inflation Targeting

Under strict inflation targeting (*SIT*), the Bank of Canada sets the Canadian nominal interest rate to keep CPI inflation at its steady-state level in *all* periods, including when an unexpected shock happens: $\tilde{\pi}_t^{CPI} = 0 \ \forall t \geq t_0$.[48]

   If uncovered interest parity held *ex post* in all periods, the Bank of Canada could achieve this goal by setting its instrument equal to the U.S. real interest rate in all periods:[49]

$$\tilde{i}_{t+1} = \tilde{i}_{t+1}^* - \tilde{\pi}_{t+1}^{CPI*}.\tag{17}$$

Because i_{t+1} is determined at time t, the Bank of Canada should target the next period's real interest rate. However, unexpected shocks can cause uncovered interest parity to be violated *ex post* when a disturbance happens. Hence, the operational rule (17) is not sufficient to guarantee that the inflation rate will remain at its steady state in *all* periods—a deviation will be observed at the time of the shock. A rule that solves this problem and is consistent with strict inflation targeting as defined above can be obtained from the money demand equation. Inflation is constant at its steady-state level in all periods if the Bank of Canada sets its interest rate as:

$$\tilde{i}_{t+1} = \tilde{i}_t - \frac{\bar{i}}{\sigma}\tilde{g}_t^M + \frac{\bar{i}}{\sigma}(c_t - c_{t-1}) - \bar{i}(1-\rho)\left(\frac{1-\sigma}{\sigma}\right)(w_t - w_{t-1}),\tag{18}$$

where $\tilde{g}^M$ is nominal money growth. Given rule (18), the log-linear money demand equation (B.9) in the appendix implies $\tilde{\pi}_t^{CPI} = 0 \ \forall t \geq t_0$. Strict inflation targeting requires the Bank of Canada to raise its interest rate if it did so in the previous period and if consumption is growing, and to lower it if money growth and/or the real wage are rising.[50] From PPP, it follows that, in this case, the Canadian dollar depreciates according to: $e_t = -\tilde{\pi}_t^{CPI*} \ \forall t$.

   When the model is solved under rule (18), a unique stable equilibrium is found. Hence, for the parameter values I work with, strict inflation targeting gets rid of the multiplicity of equilibria found under the *FIT* and *SFIT* rules.[51]

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[48] An important dimension of inflation targeting in the real world is the choice of the steady-state inflation rate, *i.e.*, the actual target, given by $\bar{\pi}$ in my model. When evaluating the consequences of inflation targeting, one would want to discuss the implications of different levels of the target. My model is not appropriate to perform such analysis. Because firms face costs of output price inflation volatility around $\bar{\pi}$, the choice of the steady-state level of inflation has no consequence for welfare in the economy, because it does not affect the steady-state level of the markup or the dynamics of relative prices. Things would be different if firms' costs of price adjustment were defined around a different inflation rate.

[49] This follows from UIP and PPP.

[50] The latter as long as $\sigma < 1$.

[51] Impulse responses under this and other rules are available upon request.

5.2. A Fixed Exchange Rate with the U.S.

The possibility of *ex post* violations of uncovered interest parity poses similar problems for a fixed exchange rate regime. Under fixed exchange rates (*FR*), the Bank of Canada sets the interest rate to keep the depreciation rate constant at its steady-state level in *all* periods, including when unexpected shocks happen: $e_t = 0 \;\; \forall t \geq t_0$.[52] If the Bank of Canada sets its interest rate equal to the Federal Funds Rate in all periods— $\tilde{i}_t = \tilde{i}_t^*$, the operational rule that is often thought consistent with fixed exchange rates—UIP guarantees that it will be $e_t = 0$ *ex ante* in all periods. But, because UIP needs not hold *ex post* at the time of a disturbance, $\tilde{i}_t = \tilde{i}_t^*$ is not sufficient to ensure that the exchange rate remains fixed at the time of a shock. A short-run deviation of the exchange rate from the target will be observed, resulting in exchange-rate indeterminacy. A rule that ensures exchange-rate stability in all periods can again be obtained from the money demand equation:

$$\tilde{i}_{t+1} = \tilde{i}_t + \frac{\bar{i}}{\sigma}\left(\tilde{\pi}_t^{CPI*} - \tilde{g}_t^M\right) + \frac{\bar{i}}{\sigma}(c_t - c_{t-1}) - \bar{i}(1-\rho)\left(\frac{1-\sigma}{\sigma}\right)(w_t - w_{t-1}). \tag{19}$$

In addition to interest-rate changes that are consistent with strict inflation targeting, this rule requires the Bank of Canada to raise its interest rate if U.S. inflation is higher. U.S. and Canadian CPI inflation are equalized by purchasing power parity, $\tilde{\pi}_t^{CPI} = \tilde{\pi}_t^{CPI*}$, and the rule yields $\tilde{i}_t = \tilde{i}_t^*$ *endogenously* in all periods.[53] As the *SIT* regime, *FR* delivers a unique stable solution for the model.

5.3. Markup Targeting

Several papers have made the point that, if the steady-state markup is removed through an appropriate system of distortionary subsidies to firms, a rule that holds the markup constant at its steady-state level in all periods maximizes agents' welfare by removing the effect of the monopolistic distortion and yielding an allocation that corresponds to the flexible price, competitive equilibrium.[54] It is not clear that stabilizing the markup would maximize agents' welfare in my model, due to the additional distortion generated by international and intergenerational market incompleteness. However, markup volatility does play an important role in business cycle fluctuations in my model economy. Thus, I consider an alternative interest setting function, which I label *MKUP*, which stabilizes the markup at its steady-state level in all periods. I do not assume that this level is adjusted to the competitive equilibrium through subsidies, as these belong in the realm of fiscal policy, and the central bank has no control over them. I am interested in the potentially stabilizing (or destabilizing) effects on the economy of a smooth path of the markup. Equation (B.14) in the appendix implies that the markup is at its steady state in all periods if and only if PPI inflation is such that:

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[52] If $\bar{i}$ is set equal to $\bar{i}^*$, the regime is a truly fixed exchange rate arrangement. If the steady-state levels of the nominal interest rates differ, the regime can be identified with a crawling peg. Because $\bar{i}$ belongs to the Bank of Canada's choice set, I will refer to this regime as fixed exchange rates.

[53] See Benigno, Benigno, and Ghironi (2000) for more details on interest rate rules that implement a fixed exchange rate regime.

[54] See, for example, Gali and Monacelli (2000) and Woodford (1999).

$$\tilde{\pi}_t^{PPI} = \frac{(1+n)(1+g)}{1+r}\tilde{\pi}_{t+1}^{PPI} \ \forall t . \tag{20}$$

Markup stabilization is equivalent to targeting PPI inflation, ensuring that today's inflation rate equals the discounted value of tomorrow's inflation, adjusted for growth in productivity and the number of households. Equation (B.15) in the appendix implies that, if PPI inflation obeys equation (20), the following equation must hold (and vice versa):

$$\tilde{\pi}_t^{CPI} - RP_{t-1} + w_t + \gamma \ L_t - \gamma \ k_t - Z_t = \frac{(1+n)(1+g)}{1+r}\left(\tilde{\pi}_{t+1}^{CPI} - RP_t + w_{t+1} + \gamma \ L_{t+1} - \gamma \ k_{t+1} - Z_{t+1}\right), \tag{21}$$

where k is the percentage deviation of detrended aggregate per capita capital from the steady state. Thus, the following interest setting rule implements markup stability:

$$\tilde{i}_{t+1} = \tilde{i}_t - \frac{\bar{i}}{\sigma}\tilde{g}_t^M + \frac{\bar{i}}{\sigma}(c_t - c_{t-1}) - \bar{i}(1-\rho)\left(\frac{1-\sigma}{\sigma}\right)(w_t - w_{t-1}) +$$

$$+ \frac{\bar{i}}{\sigma}\left[RP_{t-1} - w_t - \gamma \ L_t + \gamma \ k_t + Z_t + \frac{(1+n)(1+g)}{1+r}\left(\tilde{\pi}_{t+1}^{CPI} - RP_t + w_{t+1} + \gamma \ L_{t+1} - \gamma \ k_{t+1} - Z_{t+1}\right)\right]. \tag{22}$$

Combining rule (22) with the money demand equation (B.9) yields equation (21), which holds if and only if the markup is at the steady state in all periods. Note also that equation (20) has unique stable solution $\tilde{\pi}_t^{PPI} = 0 \ \forall t$, so that, in equilibrium, a regime that stabilizes the markup at its steady-state level is indeed equivalent to a strict PPI inflation targeting regime, which holds PPI inflation at $\bar{\pi}$ in all periods.

Rule (22) is substantially more complicated than rules (18) and (19), which already require the central bank to track money growth, consumption, the real wage, and foreign inflation—and to know the exact values of ρ and σ. In addition, rule (22) requires the monetary authority to follow the evolution of the relative price, employment, and the capital stock—and to know the exact value of γ. Because the focus of this paper is on the stabilizing (or destabilizing) properties of perfectly implemented rules, rather than on problems that may arise in their implementation, I keep the assumption that the central bank can deliver markup stability according to rule *MKUP*. Also this rule generates a unique stable solution for the model.

5.4. The Taylor Rule

I consider several versions of the Taylor rule. In its most widely known form, the Taylor rule says that the central bank should set the nominal interest rate to react to CPI inflation and GDP movements with coefficients 1.5 and .5:

$$\tilde{i}_{t+1} = 1.5\tilde{\pi}_t^{CPI} + .5y_t .$$

When the model is solved under this rule, the economy is not stable. Stability is obtained only with an absurdly large reaction to current inflation.

The purpose of a more than proportional reaction of the interest rate to inflation is to generate an adjustment in the real interest rate that will stabilize inflation in the following period. Because the real interest rate between t and $t + 1$ is given by $\tilde{r}_{t+1} = \tilde{i}_{t+1} - \tilde{\pi}_{t+1}^{CPI}$, and $\tilde{i}_{t+1}$ is determined at time t, for an increase in the time t nominal interest rate to be sure to generate a

higher real interest rate at $t + 1$, $\tilde{i}_{t+1}$ should react to $\tilde{\pi}^{CPI}_{t+1}$ rather than $\tilde{\pi}^{CPI}_t$ in my model. Rule *TR* amends the basic Taylor rule to take this observation into account:[55]

$$\tilde{i}_{t+1} = 1.5\tilde{\pi}^{CPI}_{t+1} + .5y_t. \tag{23}$$

Indeed, this rule yields stable dynamics and a unique equilibrium.

Because Canada is a small open economy, it may be argued that the interest rate should react to the behavior of the exchange rate in order to properly stabilize the economy. Ball (1998) argues in favor of adjusting the Taylor rule in open economies to include a reaction to exchange-rate movements. A simple alternative Taylor-type reaction function that the central bank may consider is given by rule *ATR* (for "adjusted Taylor rule"):

$$\tilde{i}_{t+1} = 1.5\tilde{\pi}^{CPI}_{t+1} + .5y_t + 1.5e_t. \tag{24}$$

Because of the large impact of exchange rate movements on inflation, the correction for exchange rate depreciation is as large as the interest rate change prompted by inflation. This rule too is found consistent with a unique stable equilibrium.

A different alternative consists in the adoption of a fully forward-looking version of the Taylor rule, in which the central bank reacts to anticipated changes in CPI inflation *and* GDP (*FLTR*):[56]

$$\tilde{i}_{t+1} = 15\tilde{\pi}^{CPI}_{t+1} + .5y_{t+1}. \tag{25}$$

Perhaps not surprisingly, rule *FLTR* generates a multiplicity of stable equilibria. For the reasons mentioned in Section 4.1.c, I will focus on the solution that corresponds to the smallest stable roots in what follows.

6. Welfare

I assume that the weight attached to real balances in consumers' utility—χ—is sufficiently small that the volatility of real money holdings has second-order direct effects on welfare. I use the following measure to evaluate the consequences of different rules on the economy's welfare:

$$\tilde{E}[u(c, \ LE)] = \tilde{E}\left[\frac{\left(c^\rho LE^{1-\rho}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right], \tag{26}$$

where $\tilde{E}$ is the unconditional expectation operator, c is detrended aggregate per capita consumption, *LE* is aggregate per capita leisure, and the time index has been omitted.[57] [58]

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[55] By allowing the central bank to react to expected inflation, the rule makes it possible for the monetary authority to counteract the consequences of today's interest rate choice on CPI inflation between today and tomorrow. In particular, the central bank is indirectly adjusting its action for the consequences of today's policy on future currency depreciation.

[56] Clarida, Gali, and Gertler (1997) focus their attention on forward-looking versions of Taylor-type rules. They argue that this is consistent with the empirical evidence.

[57] Calvo and Obstfeld (1988) show how to derive an intertemporal social welfare function in terms of aggregate consumption in a continuous time version of the model used here. A formal derivation of the discrete time counterpart can be found in Ghironi (2000*a*). The period welfare function in terms of aggregates per capita is as $u(c, LE)$ in equation (26). Because the choice of the monetary regime takes place from an *ex ante* perspective, the unconditional expectation of period social welfare is a proper choice criterion.

For sufficiently small deviations from the steady state, the welfare criterion (26) can be rewritten (omitting unimportant constants) as:

$$\tilde{E}\left\{ \frac{\exp\left[\rho\left(1-\frac{1}{\sigma}\right)c - (1-\rho)\left(1-\frac{1}{\sigma}\right)\frac{\overline{L}}{1-\overline{L}}L \right]}{1-\frac{1}{\sigma}} \right\}.$$

Hence, under assumptions of normality—and continuing to neglect unimportant terms—the welfare criterion becomes:

$$\frac{\left\{\left[\tilde{E}(c)\right]^{\rho}\left[\tilde{E}(L)\right]^{-(1-\rho)\frac{\overline{L}}{1-\overline{L}}}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp\left\{ \begin{array}{l} -\frac{1}{2}\rho\left(1-\frac{1}{\sigma}\right)\left[1-\rho\left(1-\frac{1}{\sigma}\right)\right]\sigma_c^2 + \\[2mm] +\frac{1}{2}(1-\rho)\left(1-\frac{1}{\sigma}\right)\frac{\overline{L}}{1-\overline{L}}\left[1+(1-\rho)\left(1-\frac{1}{\sigma}\right)\frac{\overline{L}}{1-\overline{L}}\right]\sigma_L^2 + \\[2mm] -\rho(1-\rho)\left(1-\frac{1}{\sigma}\right)^2\frac{\overline{L}}{1-\overline{L}}\sigma_{c,L} \end{array} \right\},$$

where σ_x^2 is the variance of variable x and $\sigma_{c,L}$ is the covariance between the deviations of consumption and labor from the respective steady-state levels. The first part of this expression is not affected by the monetary rule, which is neutral in the long run. Starting from the steady state, the unconditional expected values of consumption and the labor effort are given by the respective steady-state levels. The policy rule affects welfare by causing different values for the variances of the deviations of consumption and labor from the steady state and for their covariance, *i.e.*, by affecting the expression in curled brackets. Because σ is smaller than 1, welfare is higher the smaller this expression. A higher value of σ_c^2 causes it to be larger. Hence, it has a negative effect on welfare. The same is true of a negative covariance between c and L. A negative covariance between consumption and labor implies that consumption and leisure tend to move in the same direction. When agents are risk averse, their welfare is higher if movements of consumption and leisure in opposite directions provide a source of risk diversification. The effect of σ_L^2 on welfare is more ambiguous. More uncertainty in leisure tends to decrease welfare directly. However, it also causes $\exp[E(L)] = [E(L)]\exp\left(-\sigma_L^2/2\right)$ to be smaller, which has a positive effect on utility because agents enjoy more leisure in expected value. For the parameter values in the exercise[59], the direct effect dominates. The coefficient of σ_L^2 is equal to 3.32. The coefficient of σ_c^2 is 9.72, and the coefficient of $\sigma_{c,L}$ is –12.30. The covariance between consumption and the labor effort is the most important moment in determining welfare.

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[58] In principle, it is not appropriate to apply the welfare criterion of a stochastic setting to a perfect foresight model, whose linearized equations do not include variance and covariance terms, usually interpreted as risk premia. The inaccuracy is mitigated in Ghironi and Rebucci (2000), where risk premia are modeled empirically. Here, I appeal to the broadly satisfactory empirical performance of the model to justify the procedure. Construction of a rational expectation version of the model is complicated by the issue of applying a linear aggregation procedure across generations to non-linear first-order conditions in which the expectation operator appears.

[59] $\rho = .75$, $\sigma = .16$, $\overline{L} = .7$.

Table 5 shows the standard deviations of Canadian macro variables in the model under the rules I consider, the covariance between consumption and the labor effort, and the implied welfare ranking.[60] Rule *SFIT* ranks first, followed by rule *FIT*. Either one of these rules can be used as a characterization of what the Bank of Canada has being doing since 1991. The match between the model and the data is somewhat better under rule *FIT*, so this is taken as benchmark. According to the model, flexible inflation targeting dominates the alternatives for Canada. If anything, holding *FIT* as benchmark, the model suggests that moving to a rule that closely matches Svensson's (1998) characterization of flexible inflation targeting would be welfare improving.

It is interesting that rules *SFIT* and *FIT* do not deliver the smallest volatility of consumption, nor do they yield relatively small volatility of the labor effort. Rule *TR* yields the less volatile consumption path. Not surprisingly, rule *MKUP* yields the smallest employment volatility. Both strict inflation targeting and a fixed exchange rate regime with the U.S. do better than flexible inflation targeting on consumption and labor volatility grounds. The markup, inflation, the relative price, GDP, and a number of other variables are more stable under these rules than under flexible inflation targeting. This is intuitive: Firms' pricing in the model is such that CPI inflation volatility generally translates into more PPI inflation volatility and into a less stable markup. This tends to cause more volatility in the real economy. *SIT* removes CPI inflation variability altogether. *FR* ties domestic inflation to the very stable U.S. inflation. Thus, it is not surprising that these rules deliver smaller standard deviations than flexible inflation targeting. Rules *SFIT*, and *FIT* (and *FLTR*) dominate because they generate a positive covariance between consumption and the labor effort that provides a source of risk diversification for agents. Keeping the markup at its steady-state level in all periods has a stabilizing impact on several macro aggregates relative to rules *SFIT* and *FIT*. However, it generates a large, negative covariance between consumption and labor effort. The Taylor rules *TR* and *ATR* do better than *SIT* and *FR*. In the case of *TR*, a less volatile consumption path and smaller absolute value of the covariance between consumption and the labor effort more than offset a somewhat more volatile employment. The adjustment for exchange-rate depreciation in *ATR* does not improve the performance of the Taylor rule, contrary to what one would expect based on Ball (1998).

The rules ranking in the first three positions are also those that generate multiple stable solutions for the Canadian economy. As I mentioned, when multiple stable solutions exist, results are based on the solution that corresponds to the smallest stable roots of the model. In most cases, this coincides with McCallum's (1999) minimal state variable solution. Rule *TR* ranks first among the rules that generate a unique stable equilibrium for the model. A Taylor rule in which the policymaker reacts to expected inflation and current GDP would be the best among those I consider if rules *SFIT*, *FIT*, and *FLTR* were discarded on the grounds that they do not deliver a unique equilibrium.

Ball (1998) and Svensson (1998) conclude in favor of flexible inflation targeting rather than strict targeting alternatives because these tend to destabilize the real economy to a harmful extent in their models. Here, the conclusion in favor of flexible inflation targeting—or the Taylor rule—and away from strict targeting regimes hinges on agents' attitude towards risk. The strict targeting rules—*SIT*, *FR*, *MKUP*—do yield a more stable economy, different from what happens in Svensson and Ball's traditional, non-microfounded models. Strict targeting rules generate more stable markup dynamics, and markup fluctuations have a major role in the business cycle in my

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[60] THETA is the reciprocal of the consumption to wealth ratio, inc is the present discounted value of the detrended real wage stream. See Appendix B.

model. This channel of transmission of business cycle dynamics is not featured in Ball and Svensson's models. However, flexible inflation targeting yields higher welfare by generating positive covariance between consumption and the labor effort. Agents enjoy more leisure in periods in which consumption is low. This source of risk diversification ensures that, as far as my exercise is concerned, the current policy of the Bank of Canada dominates the alternatives in the ongoing debate.

7. Conclusions

I have applied a state of the art model of macroeconomic interdependence between a small open economy and the rest of the world to the issue of monetary policy in Canada. Prices are sticky in the model, and the markup reacts endogenously to the business cycle. Foreign asset accumulation plays a role in the transmission of shocks. Stationarity of the model is achieved by adopting an overlapping generations structure in which new households are born in each period, and newborn households have no assets. The rest-of-the-world economy is proxied by an estimated VAR model for the U.S. Canada is subject to three sources of exogenous volatility in my exercise: shocks to domestic technology, U.S. GDP, and the Federal Funds Rate. The model is solved numerically under alternative interest setting rules at the center of the current debate on monetary policy in open economies. Values of the structural parameters of the model are obtained through estimation and calibration. Given the parameter values, the model matches several key moments in the data for the flexible inflation targeting period fairly well.

Flexible CPI inflation targeting dominates the alternatives I consider. Strict targeting rules (strict CPI inflation targeting, a fixed exchange rate with the U.S., markup targeting—which is equivalent to strict PPI inflation targeting in the model) generate less volatility in the economy, because they yield a less volatile markup. So does the Taylor rule. However, flexible inflation targeting dominates because of agents' attitude towards risk. Consumption and the labor effort are more volatile than under alternative rules. But, differently from the alternatives, the covariance between consumption and the labor effort is positive under flexible inflation targeting. Consumption is low in periods in which leisure is high. Risk diversification ensures that flexible inflation targeting ranks first among the rules I consider.

Ball (1998) and Svensson (1998) argue in favor of flexible rules because strict targeting regimes destabilize the real economy in their traditional, non-microfounded models. Strict targeting rules have a stabilizing effect through the markup channel in my model—a channel that is not featured in more traditional analyses. Differently from Ball and Svensson, the conclusion in favor of the policy currently followed by the Bank of Canada hinges on welfare gains through risk diversification in a more volatile environment.

Appendix A. Data Sources

U.S. GDP	IMF IFS
U.S. CPI	IMF IFS
U.S. Federal Funds Rate	IMF IFS
U.S. T-Bills Rate	IMF IFS
U.S. Population	IMF IFS
CAN CPI	IMF IFS

CAN PPI	IMF IFS
CAN Bank Rate	IMF IFS
CAN T-Bills Rate	IMF IFS
CAN Dollar/U.S. Dollar Exchange Rate	IMF IFS
CAN Money	IMF IFS
CAN GDP	IMF IFS
CAN Households' Consumption	IMF IFS
CAN Gross Fixed Capital Formation	IMF IFS
CAN Change in Inventories	IMF IFS
CAN Hourly Wages	IMF IFS
CAN Population	CANSIM D1
CAN Hours	CANSIM D980214
CAN Net Capital Assets	CANSIM D86146
CAN Equity	CANSIM D86162
CAN Sales of Goods and Services	CANSIM D86167
CAN Corporate Income Taxes	CANSIM D86184

Appendix B. The Log-Linear Economy

Detrended aggregate per capita real net foreign assets entering period $t + 1$:

$$as_{t+1} = \frac{1+r}{(1+n)(1+g)}\frac{\overline{as}}{|\overline{as}|}\left(\tilde{i}_t - \tilde{\pi}_t^{CPI}\right) + \frac{1+r}{(1+n)(1+g)}as_t + \frac{\overline{y}}{(1+n)(1+g)|\overline{as}|}y_t - \frac{\overline{c}}{(1+n)(1+g)|\overline{as}|}c_t +$$

$$- \frac{\overline{inv}}{(1+n)(1+g)|\overline{as}|}inv_t - \frac{\eta}{2}\frac{\overline{inv}^2}{(1+n)(1+g)|\overline{as}|\overline{k}}\left(2inv_t - k_t\right).$$

(B.1)

The deviation of as from the steady state is defined by $as_t \equiv \left(as_t - \overline{as}\right)/|\overline{as}|$ to allow for the possibility of negative steady-state net foreign asset holdings.[61]

Detrended aggregate per capita real equity value of the home economy entering period $t +$ 1:

$$v_t = -\tilde{i}_{t+1} + \tilde{\pi}_{t+1}^{CPI} + \frac{(1+n)(1+g)}{1+r}v_{t+1} + \frac{\overline{d}}{(1+r)\overline{v}}d_{t+1}.$$

(B.2)

Detrended aggregate per capita real dividends:

$$d_t = \frac{\overline{y}}{\overline{d}}y_t - \frac{\overline{w}\overline{L}}{\overline{d}}(w_t + L_t) - \left(\frac{\overline{inv}}{\overline{d}} + \eta\frac{\overline{inv}^2}{\overline{k}\overline{d}}\right)inv_t + \frac{\eta}{2}\frac{\overline{inv}^2}{\overline{k}\overline{d}}k_t.$$

(B.3)

Detrended aggregate per capita real consumer assets entering period t:

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[61] Equation (B.1) is obtained by aggregating a real version of the budget constraint (3) across generations (recalling that newborn households have no assets), dividing by population, detrending, and combining the resulting equation with the law of motion for the economy's real detrended aggregate per capita equity value and the government's real budget constraint in detrended, aggregate per capita terms (which simply states that real seignorage revenue is rebated to consumers via the lump-sum transfers).

$$asc_t = \frac{\left|\overline{as}\right|}{\left|\overline{asc}\right|} as_t + \frac{\overline{v}}{\left|\overline{asc}\right|} v_{t-1}.$$ (B.4)

The deviation of *asc* from the steady state is defined by $asc_t \equiv \left(asc_t - \overline{asc}\right)/\left|\overline{asc}\right|$ to allow for the possibility of negative steady-state consumer asset holdings.

Detrended aggregate per capita consumption:

$$c_t = -\hat{\Theta}_t + \frac{(1+r)\overline{asc}}{(1+r)\overline{asc} + \frac{1+r}{r-g}\overline{w}}\left(\tilde{i}_t - \tilde{\pi}_t^{CPI}\right) + \frac{(1+r)\left|\overline{asc}\right|}{(1+r)\overline{asc} + \frac{1+r}{r-g}\overline{w}} asc_t + \frac{\frac{1+r}{r-g}\overline{w}}{(1+r)\overline{asc} + \frac{1+r}{r-g}\overline{w}} inc_t.$$ (B.5)

Θ^{-1} is the consumption to wealth ratio:

$$\Theta_t \equiv \sum_{s=t}^{+\infty} \beta^{\sigma(s-t)} R_{t,s}^{1-\sigma} (1+g)^{(1-\rho)(1-\sigma)(s-t)} \left(\frac{w_s}{w_t}\right)^{(1-\rho)(1-\sigma)}.$$ $\hat{\Theta}$ is the percent deviation of Θ from the steady state:

$$\hat{\Theta}_t = \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{1+r}\left[\hat{\Theta}_{t+1} - (1-\sigma)\left(\tilde{i}_{t+1} - \pi_{t+1}^{CPI}\right) + (1-\rho)(1-\sigma)\left(w_{t+1} - w_t\right)\right].$$ (B.6)

inc is the present discounted value of the detrended real wage stream:

$$inc_t \equiv \sum_{s=t}^{+\infty} R_{t,s}(1+g)^{s-t} w_s.$$

$$inc_t = \frac{1+g}{1+r}\left(inc_{t+1} - \tilde{i}_{t+1} + \tilde{\pi}_{t+1}^{CPI}\right) + \frac{r-g}{1+r} w_t.$$ (B.7)

Aggregate per capita labor supply:

$$L_t = \frac{1-\rho}{\rho}\frac{\overline{c}}{\overline{wL}}(w_t - c_t).$$ (B.8)

Growth rate, detrended nominal money balances:

$$\tilde{g}_t^M = \tilde{\pi}_t^{CPI} + c_t - c_{t-1} - \frac{\sigma}{\overline{i}}\left(\tilde{i}_t - \tilde{i}_{t-1}\right) - (1-\rho)(1-\sigma)(w_t - w_{t-1}).$$ (B.9)

Detrended aggregate per capita capital:

$$k_{t+1} - k_t = \frac{(1+n)(1+g)-(1-\delta)}{(1+n)(1+g)}(inv_t - k_t).$$ (B.10)

Detrended aggregate per capita investment:

$$inv_t - k_t = \frac{1+\eta[(1+n)(1+g)-(1-\delta)]}{\eta[(1+n)(1+g)-(1-\delta)]} q_t.$$ (B.11)

Tobin's *q*:

$$q_t = -\left(\tilde{i}_{t+1} - \tilde{\pi}_{t+1}^{CPI}\right) + \frac{1-\delta}{1+r}q_{t+1} + \frac{\gamma}{1-\gamma}\frac{\overline{w}\overline{L}}{\overline{q}\overline{k}(1+r)}\left(w_{t+1} + L_{t+1} - k_{t+1}\right) + \frac{\eta\left[(1+n)(1+g)-(1-\delta)\right]^2}{\overline{q}(1+r)}\left(inv_{t+1} - k_{t+1}\right).$$

(B.12)

Aggregate per capita labor demand:

$$L_t = y_t - \psi_t - w_t.$$

(B.13)

ψ is the percent deviation of the markup from the steady state:

$$\psi_t = -\frac{\phi(1+\overline{\pi})^2 \overline{k}}{(\theta-1)\overline{y}}\left[\tilde{\pi}_t^{PPI} - \frac{(1+n)(1+g)}{1+r}\tilde{\pi}_{t+1}^{PPI}\right].$$

(B.14)

PPI inflation:

$$\tilde{\pi}_t^{PPI} = \tilde{\pi}_t^{CPI} + \psi_t - \psi_{t-1} + w_t - w_{t-1} + \gamma\left[L_t - L_{t-1} - (k_t - k_{t-1})\right] - (Z_t - Z_{t-1}).$$

(B.15)

The relative price:

$$RP_t = -\frac{\gamma}{\theta}k_t - \frac{1-\gamma}{\theta}L_t + \frac{1}{\theta}y_t^W - \frac{1}{\theta}Z_t.$$

(B.16)

Detrended aggregate per capita GDP (in units of consumption):

$$y_t = RP_t + \gamma\, k_t + (1-\gamma)L_t + Z_t.$$

(B.17)

PPP:

$$\tilde{\pi}_t^{CPI} = e_t + \tilde{\pi}_t^{CPI^*}.$$

(B.18)

UIP:

$$\tilde{i}_{t+1} - \tilde{i}_{t+1}^* = e_{t+1}.$$

(B.19)

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Figure 1. Impulse responses, 1% increase in U.S. GDP


Impulse responses to a shock in WGDP
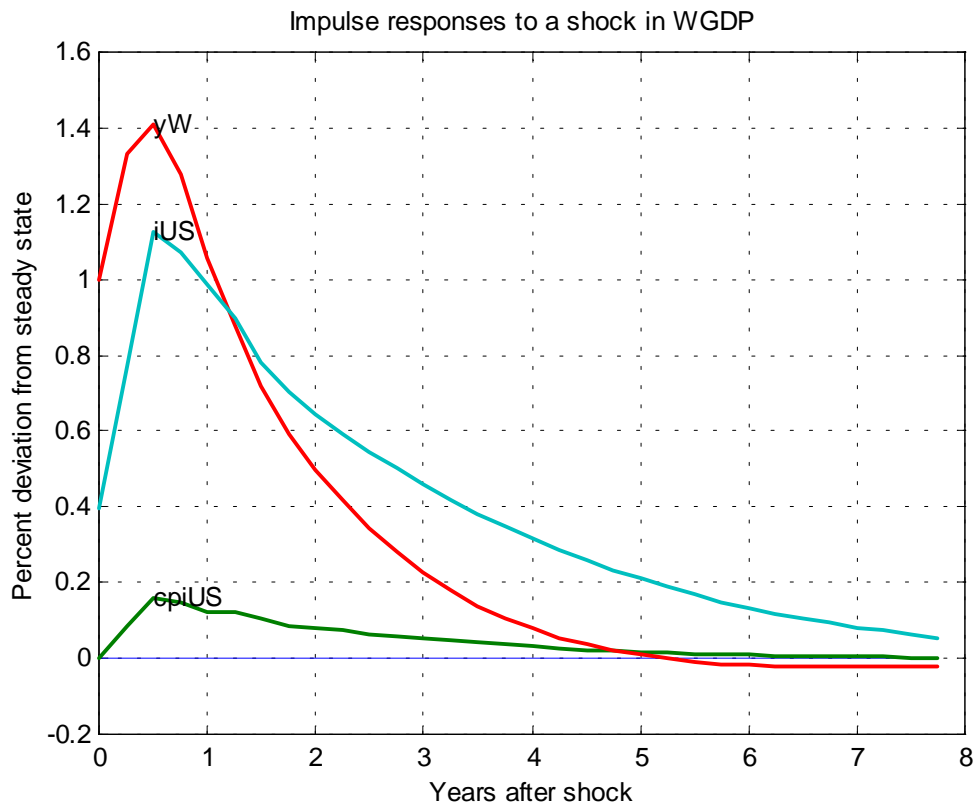
Figure 2. Markup and hours
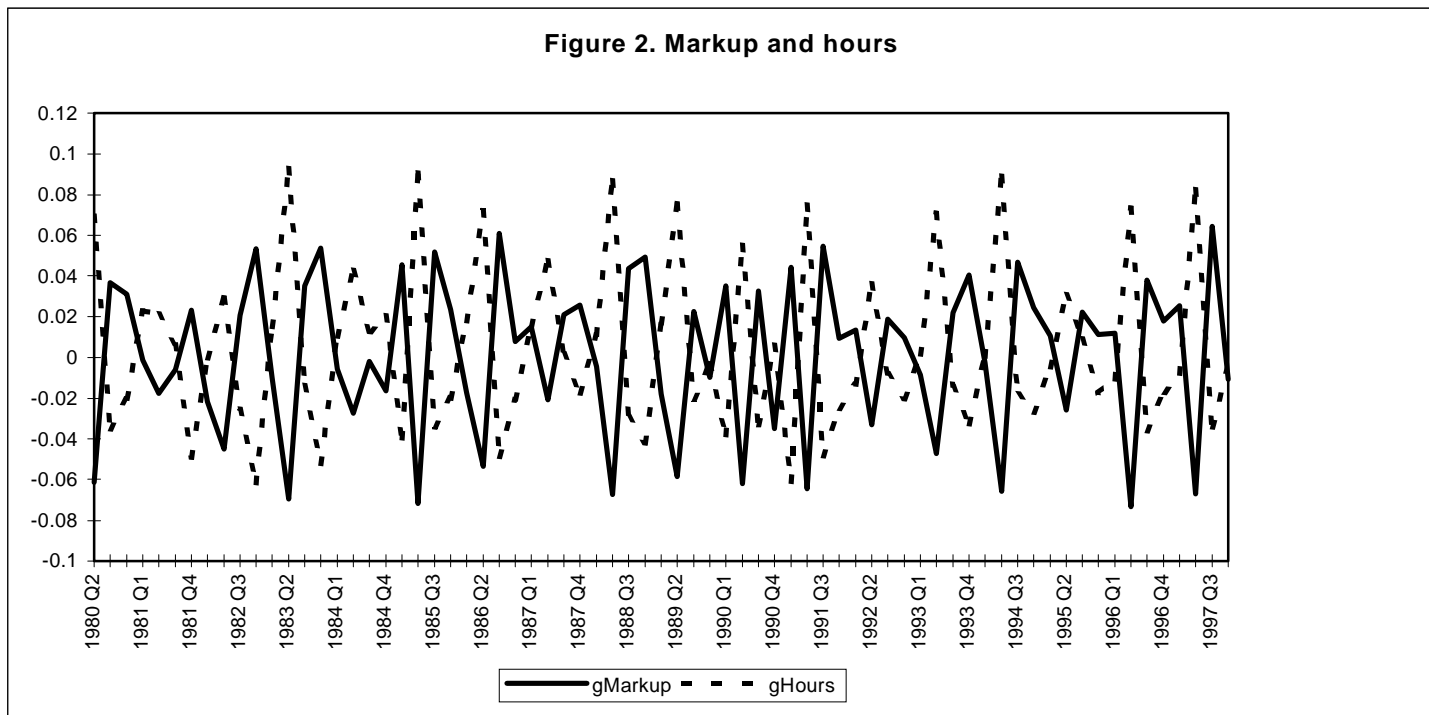
Figure 3. Impulse responses, 1% increase in U.S. GDP
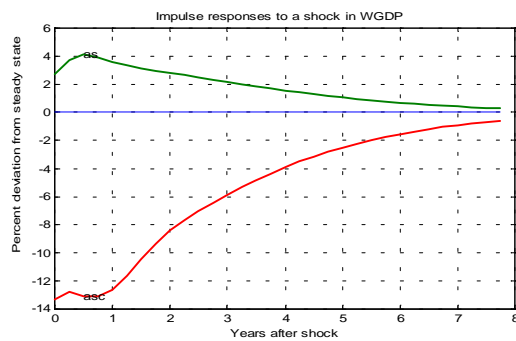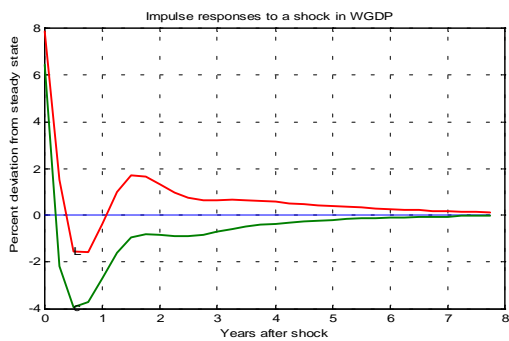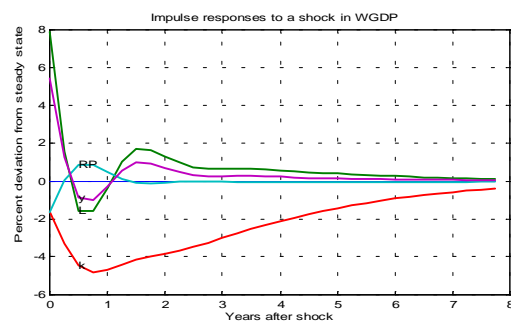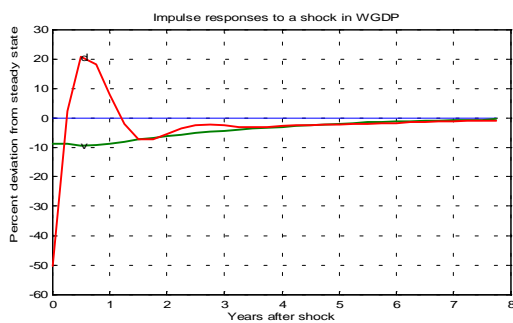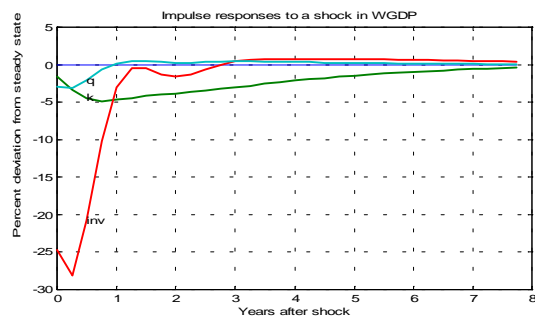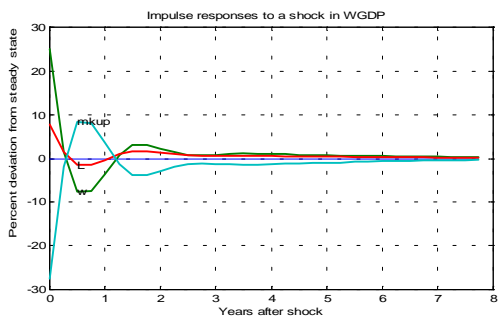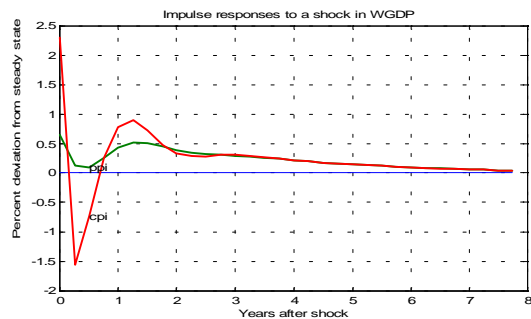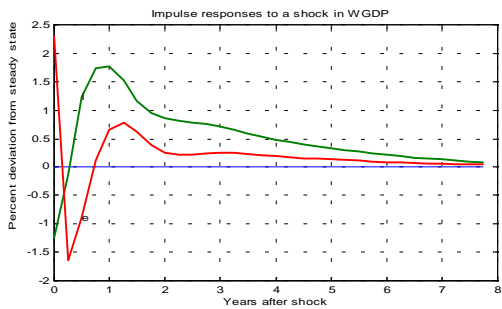
**Figure 4.** Impulse responses, 1% increase in domestic productivity
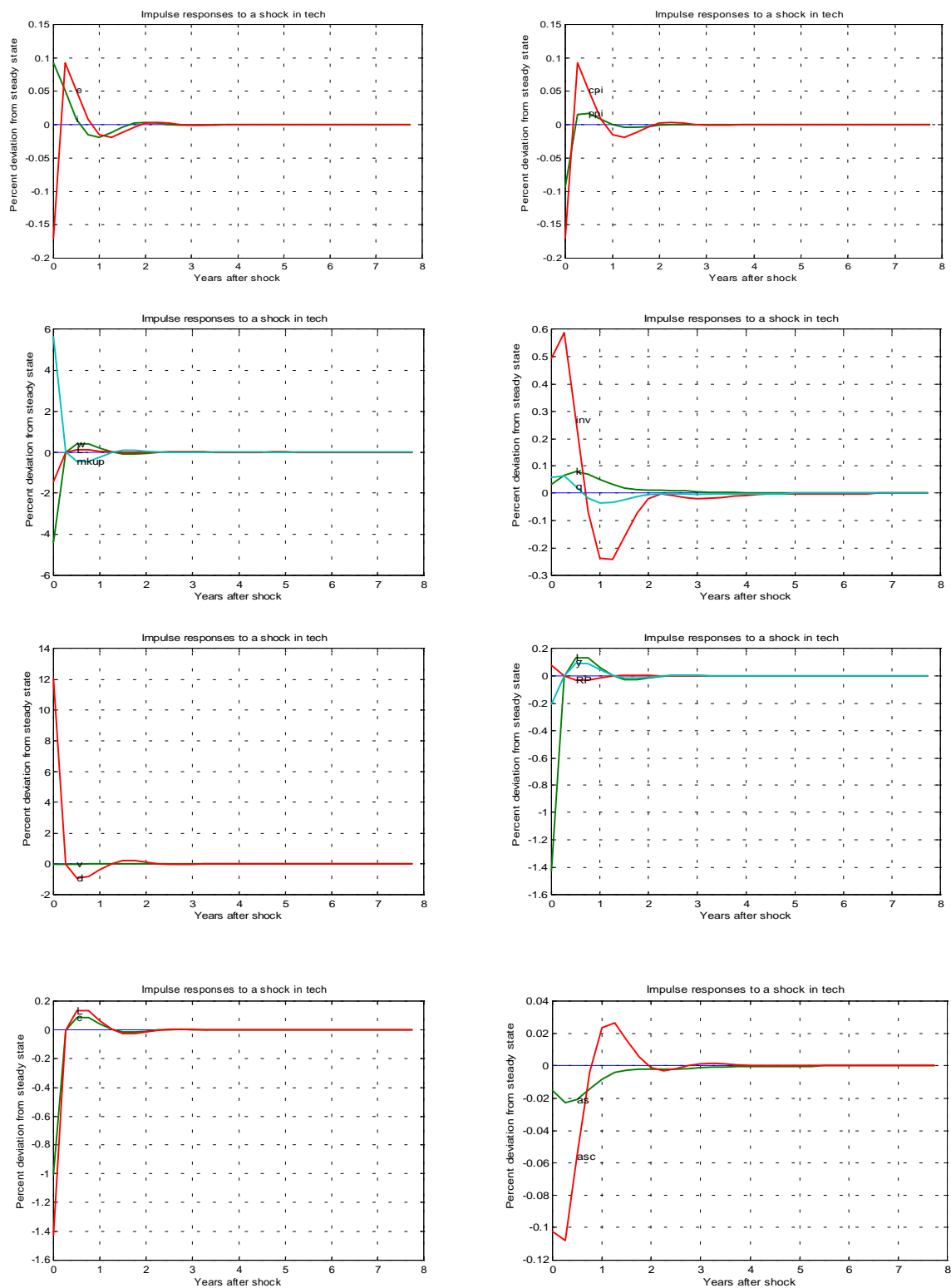
Table 1. The U.S. economy

	$\tilde{\pi}_t^{CPI*}$	y_t^W	$\tilde{i}_{t+1}^*$
y_t^W			.397 (.187)
$\tilde{\pi}_t^{CPI*}$		-.446 (.537)	1.090 (.592)
$\tilde{\pi}_{t-1}^{CPI*}$	.370 (.196)	.244 (.544)	.063 (.484)
y_{t-1}^W	.047 (.134)	1.386 (.240)	-.057 (.340)
$\tilde{i}_t^*$	.100 (.082)	-.037 (.283)	.511 (.313)
$\tilde{\pi}_{t-2}^{CPI*}$	-.022 (.206)	.186 (.362)	.971 (.473)
y_{t-2}^W	.030 (.211)	-.243 (.300)	.028 (.393)
$\tilde{i}_{t-1}^*$	-.106 (.114)	-.293 (.399)	.090 (.472)
$\tilde{\pi}_{t-3}^{CPI*}$	.373 (.176)	.111 (.325)	.266 (.563)
y_{t-3}^W	-.053 (.133)	-.180 (.232)	-.337 (.273)
$\tilde{i}_{t-2}^*$	.018 (.101)	.274 (.212)	.108 (.223)

Table 2. Correlations

	Data	Model, FIT		Data	Model, FIT
Consumption			**Investment**		
c, v	**0.270**	**0.318**	inv, v	**0.398**	**0.609**
c, y	**0.314**	**0.716**	inv, RP	**0.241**	**0.258**
c, L	**0.309**	**0.721**	mkup, v	**0.416**	**0.228**
c, w	-0.260	0.846	inv, mkup	**0.177**	**0.391**
c, mkup	-0.004	-0.830	inv, c	0.843	-0.069
c, RP	0.053	-0.901			
c, cpiinfl	-0.142	0.515	**Real wage**		
c, intrate	***-0.080***	***-0.693***	w, y	-0.435	0.952
c, iUS	0.263	-0.287	w, mkup	**-0.214**	***-0.999***
c, yUS	0.763	-0.366			
			Prices		
Employment			RP, mkup	**0.407**	**0.934**
L, y	**0.469**	**0.970**	ppi, mkup	0.128	-0.572
L, w	-0.412	0.979	cpi, mkup	0.024	-0.685
L, mkup	**-0.572**	**-0.984**	RP, ppiinfl	0.176	-0.390
L, RP	0.372	-0.891	RP, cpiinfl	**-0.065**	***-0.616***
L, intrate	**-0.103**	**-0.127**	RP, w	**-0.417**	**-0.948**
L, iUS	**0.442**	**0.413**	RP, v	**0.835**	***0.016***
L, yUS	**0.343**	**0.268**	RP, iUS	0.623	-0.075
			RP, yUS	**0.457**	**0.116**
GDP			RP, cpiinflUS	-0.218	0.202
y, v	0.967	-0.376	cpiinfl, cpiinflUS	**0.397**	**0.102**
y, inv	0.455	-0.540			
y, k	0.722	-0.210	**Depreciation**		
y, RP	0.865	-0.910	e, cpiinfl	-0.202	0.996
y, mkup	0.415	-0.940	e, c	**0.017**	***0.564***
y, intrate	**-0.434**	**-0.180**	e, L	-0.260	0.681
y, iUS	**0.732**	**0.300**	e,w	**0.260**	**0.688**
y, yUS	**0.651**	**0.390**	e, inv	-0.129	0.010
			e, mkup	**-0.035**	***-0.688***
			e, RP	**-0.278**	**-0.638**

Table 3. Standard deviations

Data, decreasing		Model, FIT	Model, FIT, decreasing	
v	5.391	16.762	inv	28.606
RP	4.743	1.712	mkup	26.140
inv	4.743	28.606	w	23.275
L	3.016	7.526	v	16.762
mkup	2.827	26.140	k	9.269
y	2.198	4.774	L	7.526
k	2.020	9.269	c	6.781
intrate	1.728	2.663	y	4.774
e	1.690	2.596	intrate	2.663
ppiinfl	1.064	1.017	cpiinfl	2.609
iUS	1.053	1.898	e	2.596
c	1.006	6.781	yUS	1.955
w	0.893	23.275	iUS	1.898
cpiinfl	0.585	2.609	RP	1.712
yUS	0.560	1.955	ppiinfl	1.017
cpiinflUS	0.176	0.234	cpiinflUS	0.234

Table 4. Autocorrelation with GDP

Data

	t-3	t-2	t-1	t	t+1	t+2	t+3
y	0.82	0.87	0.94	1.00	0.93	0.86	0.82
c	-0.14	0.00	0.22	0.31	0.02	0.13	0.21
w	-0.27	-0.37	-0.46	-0.43	-0.33	-0.45	-0.38
L	0.24	0.39	0.57	0.47	0.37	0.40	0.43
mkup	0.41	0.31	0.32	0.41	0.37	0.43	0.37
RP	0.92	0.92	0.90	0.86	0.81	0.74	0.66
ppiinfl	0.17	0.11	-0.16	-0.15	-0.18	-0.33	-0.46
cpiinfl	-0.09	0.07	0.00	-0.11	0.20	0.29	0.13
v	0.84	0.89	0.94	0.97	0.94	0.90	0.89
inv	0.13	0.31	0.43	0.45	0.20	0.23	0.30
k	0.29	0.55	0.74	0.72	0.77	0.79	0.85
intrate	-0.52	-0.46	-0.48	-0.43	-0.30	-0.34	-0.40
e	-0.28	-0.23	-0.47	-0.30	-0.31	-0.36	-0.31
cpiinflUS	-0.17	-0.14	-0.27	-0.39	-0.08	-0.06	-0.25
yUS	0.37	0.50	0.58	0.65	0.48	0.51	0.51
iUS	0.47	0.66	0.78	0.73	0.82	0.82	0.79

Model, FIT

	t-3	t-2	t-1	t	t+1	t+2	t+3
y	-0.21	-0.14	**0.29**	**1.00**	**0.29**	-0.14	-0.21
c	**-0.29**	-0.19	**0.19**	**0.72**	-0.08	-0.33	-0.35
w	**-0.24**	**-0.15**	0.27	0.95	0.18	**-0.19**	**-0.24**
L	-0.20	-0.13	**0.28**	**0.97**	0.27	-0.13	-0.18
mkup	**0.23**	**0.15**	-0.27	-0.94	-0.19	**0.18**	**0.22**
RP	**0.27**	**0.17**	-0.26	-0.91	-0.16	**0.27**	**0.32**
ppiinfl	**0.05**	**0.09**	0.30	0.63	0.16	0.07	0.17
cpiinfl	**-0.05**	**0.10**	**0.40**	**0.67**	-0.43	-0.25	**0.03**
v	-0.13	-0.08	-0.14	-0.38	-0.34	-0.33	-0.33
inv	**0.09**	**0.20**	**0.04**	-0.54	-0.63	-0.41	-0.14
k	-0.17	-0.12	-0.11	-0.21	-0.32	-0.38	-0.38
intrate	0.27	0.24	0.08	**-0.18**	**-0.00**	0.28	0.43
e	**-0.06**	0.09	0.40	0.66	**-0.47**	**-0.28**	0.01
cpiinflUS	0.17	0.09	0.06	0.02	0.42	0.34	0.27
yUS	**0.12**	**0.06**	**0.09**	**0.30**	**0.33**	**0.28**	**0.24**
iUS	**0.10**	**0.05**	**0.13**	**0.39**	**0.38**	**0.38**	**0.35**

Table 5. Standard deviations under alternative rules and welfare

	FIT	SFIT	IT	FR	MKUP	TR	ATR	FLTR
as	7.67	7.36	12.36	12.14	14.24	8.73	11.07	6.29
asc	23.72	25.79	26.70	27.63	27.82	31.11	29.16	27.47
v	16.76	17.69	13.78	14.44	13.76	17.24	15.60	18.35
c	6.78	8.43	5.90	5.75	6.13	5.26	5.54	5.77
w	23.28	33.73	5.12	4.89	1.59	9.74	5.82	18.54
L	7.53	11.09	2.59	2.72	2.16	4.30	3.12	6.29
mkup	26.14	37.76	6.50	6.48	0.00	11.84	7.70	21.06
intrate	2.66	5.11	1.69	1.90	1.82	2.72	2.28	2.85
k	9.27	6.97	11.37	10.43	11.39	6.66	8.99	6.17
gM	6.41	18.92	2.84	2.99	3.09	3.79	3.23	5.34
inv	28.61	27.83	25.68	23.82	25.90	16.14	21.14	15.45
RP	1.71	2.62	0.37	0.34	0.67	0.75	0.48	1.27
inc	9.77	8.91	11.70	11.21	11.79	9.13	10.41	8.30
THETA	11.64	12.45	10.04	10.02	9.80	10.27	9.98	11.47
e	2.60	5.06	0.23	0.00	0.82	1.15	0.46	2.12
y	4.77	7.17	1.43	1.46	1.89	2.47	1.60	3.94
d	50.76	72.27	14.36	14.29	5.71	24.17	16.75	41.85
ppiinfl	1.02	1.25	0.12	0.24	0.00	1.12	0.59	1.54
cpiinfl	2.61	5.06	0.00	0.23	0.79	1.32	0.65	2.22
q	3.28	3.27	2.79	2.60	2.82	1.78	2.32	1.72
cov(c,L)	36.80	77.88	-9.77	-10.73	-12.92	-7.86	-10.88	18.27

	Welfare, exponent
SFIT	141.12
FIT	182.13
FLTR	229.76
TR	427.20
ATR	464.25
FR	477.37
IT	480.31
MKUP	539.70