OPTIMAL BUFFERING POLICIES FOR A SMALL TRADING COUNTRY

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I. Introduction

Trading situations subject to price or quantity fluctuations have always stimulated private and social attempts at buffering. Economic analysis of such behavior has been unsatisfactory because of its neglect of the nature of the utility function, implicitly or explicitly placing highly special restrictions on it.[1] Price variation induces both income and substitution effects in consumption (hence in utility), while buffering involves inter-temporal choice (again involving the utility function). This paper will analyze buffering where the utility function must satisfy only additivity over states of nature and time. It will show that non-zero buffering depends in a complex way on the degree of risk aversion, the rate of time preference, the structure of comparative advantage, and the income responsiveness of demand.

Most buffering studies have analyzed situations where intervention affects the variation of prices, indeed

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[1] One variant involves using expected social surplus as an objective function (for example, see Bieri and Schmitz (1973)) while another uses a quadratic in producers' income permitting E-V analysis (see Massell (1970)). Still another variant is Heckerman (1973), which allows for relative price effects and is formally a special case of our commodity storage model. In addition, while inter-temporal transfers lead to opportunity costs of buffering, this fact is obscured or not treated by all three of the above articles.

The studies above are also unsatisfactory in that they deal in partial equilibrium. The present paper undertakes a general equilibrium approach.

occasionally accepting this as the goal of buffering. The use of buffering is, however, to improve welfare and reduction of price variation is ancillary (may be pernicious) to this goal. Therefore, as a first step in a general analysis of buffering, we assume the country has all relevant prices given,[2] and faces either random shocks to prices or to domestic supply. Finally, since in a closed system buffering absorbs capital (and hence is costly), we make the simplifying assumption that the country is unable to borrow from abroad for purposes of buffering. It would be easy to modify this to allow borrowing at increasing cost (fixed cost could lead to infinite purchases where capital gains are expected).

Part II presents the case of exchange reserve accumulation. Part III takes up the cases of costless goods storage and futures market activity. Our results are compared with the work of others in Part IV.

II. Foreign Exchange Reserves

The subject of study throughout is a "one-person" economy to which all goods are costlessly tradable at given prices, $\bar{P}$. Domestic production is subject to a strictly concave efficient transformation function, conveniently represented by a vector of general equilibrium supply functions dependent on goods prices

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[2] The analysis could, of course with suitable modifications, be applied to any price taking entity--e.g., to a firm with a more complex objective function than expected profits.

only, $X(P)$.[3] Present period prices are known and next period's prices follow a known probability distribution, $F(P)$. Present period output is $X(P)$, and next period's output is $X(P)+\Theta$, where Θ is a random vector with distribution $F(\Theta)$.[4] The objective function to be maximized is a sum of present period utility plus discounted expected utility next period. The two period form is chosen to simplify the general stochastic control problem to manageable proportions as an initial step.

The crudest form of buffering occurs when only reserve accumulation is possible. Given P, the efficient production point $X(P)$ is selected, resulting in income $P \cdot X(P)$. Trade occurs at fixed prices subject to the balance of payments constraint. If t is the vector of excess demands and R the reserve accumulation, the balance of payments constraint in the present period is $P^0 \cdot t^0 + R \leq 0$, and in the future period $P \cdot t - R \leq 0$. Spending in the present is $P^0 \cdot X(P^0) + \Theta^0) - R$ and in the future $P \cdot [X(P)+\Theta]+R$. The optimal consumption vector is then $C(P^0 \cdot X(P^0)+\Theta^0)-R,\ P^0)$ in the present and $C(P \cdot [X(P)+\Theta]+R,\ P)$ in the future. We substitute consumption into the utility function, obtaining the indirect utility function. The optimal reserve choice problem is then:

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[3] We allow next period's production to be freely variable for convenience. Anderson-Riley (1973) analyzed the case where the production point must be picked *ex ante*; in the present context this is an inessential complication.

[4] This is of course a highly special type of supply disturbance akin to crop failures. In general, random shocks would "twist" the supply functions $X(P)$, and we would write $X(P,\Theta)$.

$$\underset{R}{\text{Max}}\ V(-R,\ P^0) + \frac{1}{1+r} \underset{P,\Theta}{E} [V(R,\ P\cdot\Theta,\ P)]$$

$$= V(P^0\cdot X(P^0) + \Theta^0) - R,\ P^0) + \frac{1}{1+r}E[V(P\cdot(X(P)+\Theta)+R,\ P)]$$

$$\text{s.t. } R \geq 0$$

where V is the indirect utility function, r is the discount rate and E is the expectations operator taken over P and Θ. The constraint is the requirement that there be no borrowing.

The first order condition yields:

$$V_0(-R,\ P^0,\ \Theta^0) \geq E[V_0(R,\ P,\ \Theta)]\frac{1}{1+r}$$

where V_0 is the marginal utility of income. With equality holding, $R>0$, reserve accumulation occurs. The task of this section is to discover conditions under which this happens. If we assume risk aversion ($V_{00}<0$, decreasing marginal utility of income), reserve buffering occurs if:

$$(1)\quad V_0(P^0\cdot[X(P^0) + \Theta^0],\ P^0) < E[V_0(P\cdot[X(P) + \Theta],\ P)]\frac{1}{1+r}$$

This is of course intuitive--income whould be shifted forward if the expected discounted marginal utility of income next period exceeds the present marginal utility of income when the transfer is zero. If P^0 and Θ^0 were at the mean level, satisfaction of (1) depends on convexity properties of V_0 in P and Θ, by Jensen's Inequality.[5] The departure of P^0 and Θ^0 from their mean level in

[5]This is a straightforward application of the general results of Rothschild and Stiglitz (1971).

the present adds an additional term to be considered. The remainder of this section develops the analysis of (1) considering the convexity term and then the additional term involving initial conditions. Matters are simplified if we consider alternately $P\equiv\bar{P}$, supply disturbances only, and $\Theta\equiv 0$, price fluctuations only.

First consider the supply fluctuations problem, since it is much simpler. For simplicity, set $\Theta^0=0$. Setting aside the role of the discount rate, $\underset{\Theta}{E}[V_0(\bar{P}\cdot[X(\bar{P})+\Theta], \bar{P})] - V_0(\bar{P}\cdot X(\bar{P}), \bar{P}) > 0$ is the necessary condition for reserve accumulation. We can rewrite it as:

$$[\underset{\Theta}{E}[V_0(\bar{P}\cdot\Theta)] - V_0(\bar{P}\cdot\bar{\Theta})] + [V_0(\bar{P}\cdot\bar{\Theta}) - V_0(0)] > 0$$

The first term depends upon convexity of V_0 in Θ, by Jensen's Inequality. The second term may be evaluated by a Taylor's series expansion (though in the scalar Θ case its sign depends only on the first derivative where it is one-signed in the interval). It will dominate as higher derivatives are small or as disturbances in Θ are small. The former possibility at least makes reserve accumulation devolve upon $\frac{\partial V_0}{\partial\Theta}\cdot[\bar{\Theta}-0] = [V_{00}\bar{P}]\cdot\bar{\Theta} = V_{00}P\cdot\bar{\Theta}$. Since $V_{00}<0$ for risk aversion and $\bar{P}\cdot\bar{\Theta} = E(\bar{P}\cdot\Theta)$, the rule is to accumulate only if the expected income change is negative. If we allow at least non-negligible second derivatives, the convexity term matters. $\frac{\partial^2 V_0}{\partial\Theta^2} = V_{000}\bar{P}\bar{P}'$ and $\bar{P}\bar{P}'$ is positive semi-definite. $V_{000}>0$ is a necessary condition for decreasing absolute risk aversion,[6] so

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[6]Let $\sigma^* = -V_{00}/V_0$, the coefficient of absolute risk aversion; and let m = income. Then $d\sigma^*/dm = -V_{000}/V_0 + (\sigma^*)^2$, and $V_{000}>0$ is a necessary condition for decreasing risk aversion. This is generally accepted as a reasonable restriction.

V_0 is ordinarily convex in Θ. Noting that the second term of the expansion of $V_0 = \frac{1}{2}V_{000}\bar{\Theta}'\bar{P}\bar{P}'\bar{\Theta} \geq 0$, with negligible third and higher order derivatives, or small enough disturbances in Θ, we evaluate: $E[V_0(\Theta)] - V_0(0)$ is positive if the expected income change is negative, but may well still be positive even for positive expected income changes. The chances for this are enhanced the greater the rate of decrease of absolute risk aversion. Intuitively, the supply disturbance at fixed prices simply creates variation in lump sum income, and convexity of the marginal utility of income with respect to income suffices to pin down all major influences. Reserve accumulation should occur with a low discount rate if the expected income change next period is ≤ 0, and may occur with it > 0. Raising the discount rate of course lowers R; from the first order conditions (1): $-V_{00}dR = -\frac{1}{(1+r)^2}E(V_0)dr + E(V_{00})dR$, so

$$\frac{dR}{dr} = \frac{\frac{1}{1+r}E(V_0)}{V_{00} + E(V_{00})} < 0.$$

In contrast, the case of price fluctuations (with no supply disturbances) is more complex, causing not only variations in income as above, but twisting effects. For r=0 and risk aversion, reserve accumulation will occur if the sign of $E[V_0(P \cdot X(P), P)] - V_0(P^0 \cdot X(P^0), P^0) = [E(V_0) - V_0(\bar{P})] + [V_0(\bar{P}) - V_0(P^0)]$ is positive. $E(V_0) - V_0(\bar{P})$ depends on convexity of V_0 in P, while $V_0(P) - V_0(P^0)$ depends upon the

Taylor's series expansion of V_0 about $\bar{P}$. We shall consider alternately the cases where $P^0=\bar{P}$ and where V_0 is (nearly) linear in P to focus on the role of the two terms separately.

Taking up the role of the first term (setting $P^0=\bar{P}$) first, we note that general convexity depends upon a second order matrix that is too messy for useful results. In the case of variation in a single price (a diagonal element of the general matrix), however, we obtain a useful characterization in terms of comparative advantage, risk aversion, and the nature of demand conditions. Some manipulation establishes:[7]

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[7]First noting $V_0 = V_0(P\cdot X(P),\ P)$ and $P\cdot\frac{\partial X}{\partial P_i} = 0$ by the production efficiency assumption, and using the properties of the Slutsky equations, we have

$$(2)\quad \frac{\partial V_0}{\partial P_i} = \left.\frac{\partial V_0}{\partial P_i}\right|_{comp} - t_i V_{00} = -V_0\,\frac{\partial C_i}{\partial m} - t_i V_{00}, \text{ where } m = \text{income.}$$

Then: $$\frac{\partial^2 V_0}{\partial P_i^2} = -\frac{\partial C_i}{\partial m}\,\frac{\partial V_0}{\partial P_i} - V_0\,\frac{\partial^2 C_i}{\partial m\partial P_i} - V_{00}\,\frac{\partial t_i}{\partial P_i} - t_i\,\frac{\partial V_{C0}}{\partial P_i}\ .$$

To evaluate this, we need some intermediate steps. First, note that:

$$\frac{\partial V_{00}}{\partial P_i} = -t_i V_{000} + \left.\frac{\partial V_{00}}{\partial P_i}\right|_{comp}$$

We have already noted $V_{C00}>0$ is a necessary condition for decreasing absolute risk aversion. We evaluate the second term by noting:

$$\left.\frac{\partial V_{00}}{\partial P_i}\right|_{comp} = \frac{\partial\left(\left.\frac{\partial V_0}{\partial P_i}\right|_{comp}\right)}{\partial m} = \frac{\partial\left(-V_0\,\frac{\partial C_i}{\partial m}\right)}{\partial m} = -V_{00}\,\frac{\partial C_i}{\partial m} - V_0\,\frac{\partial^2 C_i}{\partial m^2}$$

by properties of the Slutsky matrix and rules of differentiation. Also, we note that:

(Cont.)

$$(3) \quad \frac{1}{V_0}\frac{\partial^2 V_0}{\partial P_i^2} = (\frac{\partial C_i}{\partial m})^2 + t_i^2((\sigma^*)^2 - \frac{\partial \sigma^*}{\partial m}) + {}^*[\frac{\partial C_i}{\partial P_i}\bigg|_{comp} - 3t_i\frac{\partial C_i}{\partial m} - \frac{\partial X_i}{\partial P_i}]$$

$$+ [2t_i \frac{\partial^2 C_i}{\partial m^2} + \frac{\partial^2 C_i}{\partial m \partial P_i}\bigg|_{comp}]$$

The last term in brackets is one about which economic theory tells us nothing a priori, and it appears to have no useful "uncertainty economics" interpretation. Setting aside its influence, the first two terms are positive for normal goods in the class of decreasing absolute risk aversion functions. The third term is negative for imports of good i ($t_i \geq 0$) but may be positive for exports of good i, provided demand is normal (inferiority will switch the role of trade). The <u>ceteris paribus</u> influence of trade in the first three terms may be toward concavity when the good is imported but is definitely toward convexity when the good is exported. Convexity is most likely when the range of price variation is confined to the heavy export range, but is possible in the heavy import range.

$$-V_0 \frac{\partial^2 C_i}{\partial P_i \partial m} = -V_0[-t_i \frac{\partial^2 C_i}{\partial m^2} + \frac{\partial^2 C_i}{\partial m \partial P_i}\bigg|_{comp}].$$

Substituting these relations into the above expression for $\frac{\partial^2 V_0}{\partial P_i^2}$, using $\sigma^* = -V_{00}/V_0$ and expanding $\partial t_i/\partial P_i$, we obtain:

$$(3) \quad \frac{1}{V_0}\frac{\partial^2 V_0}{\partial P_i^2} = (\frac{\partial C_i}{\partial m})^2 + t_i^2((\sigma^*)^2 - \frac{\partial \sigma^*}{\partial m}) + \sigma^*[\frac{\partial C_i}{\partial P_i}\bigg|_{comp} - 3t_i\frac{\partial C_i}{\partial m} - \frac{\partial X_i}{\partial P_i}]$$

$$+ [2t_i \frac{\partial^2 C_i}{\partial m^2} + \frac{\partial^2 C_i}{\partial m \partial P_i}\bigg|_{comp}].$$

The Appendix sharpens these results by considering the Cobb-Douglas utility function, $U = \frac{1}{1-\sigma}[C_1^{\alpha}C_2^{1-\alpha}]^{1-\sigma}$, and its indirect utility function, $V = \frac{1}{1-\sigma}[(P\cdot X)P_1^{-\alpha}P_2^{-(1-\alpha)}]^{1-\sigma}$. The coefficient $\sigma = -(P\cdot X)V_{00}/V_0$, the coefficient of relative risk aversion. For $\sigma \leq 1$, V_0 is everywhere convex in P_i if the elasticity of supply is negligible. For $\sigma > 1$, V_0 is concave in P_i in a range running from no production of good i up to a positive production point which may lie on either side of the no trade point, after which it is convex. The influence of the supply elasticity, ceteris paribus, is toward concavity. We expect in general a concavity range of heavy imports, a convexity range of heavy exports, with no definite result in the neighborhood of no trade.

Turning again to the issue of evaluating

$$0 < \{E[V_0(P\cdot X(P),\ P)] - V_0(\bar{P}\cdot X(\bar{P}),\ \bar{P})\} + V_0(\bar{P}\cdot X(\bar{P}),\ \bar{P})$$
$$- V_0(P^0\cdot X(P^0),\ P^0)$$

we have seen that for $P_i^0 = \bar{P}_i$, reserve accumulation under our assumptions was most likely if the range of price variation were such that the good was a heavy export, but could occur elsewhere. We now integrate consideration of $P_i^0 \neq \bar{P}_i$, first assuming higher derivatives are negligible or disturbances to P_i are small so $E(V_0) = V_0(\bar{P})$. In this case a Taylor's series expansion of V_0 about $\bar{P}$ yields:

$$V_0(\bar{P}) - V_0(P^0) = \nabla_P V_0(\bar{P})[\bar{P} - P^0]$$

We have already noted $\nabla_p V_0 = -V_0 \frac{\partial C}{\partial m} - tV_{00}$. The ith element of $\nabla_p V_0$ is negative for the risk averse, normal, exports case but can become positive as trade goes to heavy imports (or risk aversion grows). Above (below) average prices for exports act toward (against) reserve accumulation, and this is intuitive. The surprise is that above (below) average prices for imports may act against (toward) reserve accumulation but <u>need not</u> if risk aversion is low. The second term of $\partial V_0/\partial P_i$ incorporates income effects and is intuitive: $\frac{\partial m}{\partial P_i} = -t_i$ and the effect on V_0 is $-t_i V_{00}$. The first term, $\left.\frac{\partial V_0}{\partial P_i}\right|_{comp} = -V_0 \frac{\partial C_i}{\partial m}$, reinforces for the export case and offsets for the import case. This is what may produce the "surprising" result for the latter. We can summarize the results in a table:

Influences on Reserve Accumulation
(normal, risk averse case)

	$P_i^0 = \bar{P}_i$	$V_0(P)$ nearly linear, $P_i^0 > \bar{P}_i$
prices in import range	- (?)	- (?)
prices in export range	+ (?)	+
risk aversion	+ (?)	+

We note finally that the desirability of buffering has no necessary relation to whether price fluctuations <u>per se</u> are beneficial. This depends, for single price fluctuations, on the convexity of $V(P \cdot X(P), P)$ in P_i. A little manipulation (see Anderson-Riley) yields:

$$\frac{\partial^2 V}{\partial P_i^2} = -V_0 \frac{\partial t_i}{\partial P_i} - t_i[-V_0 \frac{\partial C_i}{\partial m} - t_i V_{00}]$$

$$= -V_0 \frac{\partial t_i}{\partial P_i} + t_i V_0 \frac{\partial C_i}{\partial m} + t_i^2 V_{00}$$

As a simple example, if we assume $\partial t_i/\partial P_i < 0$, V is convex in P_i at the no-trade point ($t_i=0$). Even so, for $P_i^0=\bar{P}_i$ and r=0 we still cannot ascertain whether buffering is desirable without the information in (3). This suggests that the preoccupation of some authors with price fluctuations *per se* is a mistake, and that in a more general analysis where intervention affects the probability distribution of prices, a tradeoff is possible between benefits of buffering and costs of price stability.

III. Commodity Storage and Futures Contracts

Reserve accumulation is a common form of buffering against variable income or prices due to low storage and transactions costs. But where commodity storage and commodity futures transaction costs are low, these activities become beneficial. With costs equal (here assumed zero), they may be (but are not necessarily) preferred to reserve accumulation, since they afford the chance of making the income transfer in a commodity where a capital gain is expected. In Part II the transfer must be made in terms of the numéraire commodity (foreign exchange) with an expected price change of zero. Commodity buffering is also undertaken to affect price fluctuations, but we have ruled this out by making the small country assumption.[8]

We shall assume alternately that all goods are storable at no cost, and that the futures market produces given prices unrelated to the distribution of spot prices next period. Price fluctuations alone are studied, since Section II shows they are richer.

The commodity storage case formally is:

$$\max_{Z} V(P^0 \cdot X(P^0) - P^0 \cdot Z,\ P^0) + \frac{1}{1+r} E[V(P \cdot [X(P) + Z],\ P)]$$

$$\text{s.t.} \quad Z \geq 0$$

Z is the vector of stored commodities and the other variables are as previously defined. The first order conditions for maximization yield:

$$\frac{1}{1+r} \underset{P}{E}[V_0(P)P] \leq V_0(P^0)P^0$$

A sufficient condition for storage to occur in at least one good (equality to hold at least once) is derived by examining $\frac{1}{1+r} E(V_0 P_j) - V_0(P^0)P_j^0$ when no storage occurs. Expanding the expression, we have:

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[8] In Anderson-Riley (1973) and above, it is shown that the welfare effects of reducing price fluctuations *per se* may be of either sign. The other benefits of buffering are those analyzed here: reduced variability of income, and expected capital gain.

$$0 \geq \frac{1}{1+r} [E(V_0 P_j) - E(V_0)E(P_j)] + \frac{1}{1+r} E(V_0)E(P_j) - V_0(P^0)P_j^0$$

$$0 \geq \frac{1}{1+r} C(V_0, P_j) + \frac{\bar{P}_j}{1+r} [E(V_0) - V_0(\bar{P})] + \frac{\bar{P}_j}{1+r} [V_0(\bar{P}) - V_0(P^0)]$$

$$+ V_0(P^0)[\frac{\bar{P}_j}{1+r} - P_j^0]$$

where $\bar{P}_j \equiv E(P_j)$, $j=1,\ldots,n$

The second and third terms were studied in Section II, while the fourth has the sign of $\frac{\bar{P}_j}{1+r} - P_j^0$. This large expression is not too useful, but we can say a little.[9] Since even these partial results depend upon variation in a single price, and hence require some fixed price, it is worthwhile to take a different tack and establish <u>necessary</u> conditions for storage to occur in a variable priced good. If the necessary conditions are all violated we can return to Section II to examine sufficient conditions for storage in a fixed price good. We thus focus on the essence of storage--the opportunity to make the transfer in a good where a capital gain is expected.

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[9]The first term, if P_j is the only variable price, has the sign of $\partial V_0/\partial P_j$ (provided it is one-signed in the interval of variation) and we have seen that $\partial V_0/\partial P_j = -V_0 \frac{\partial C_j}{\partial m} - t_j V_{00}$, which is negative for the risk averse, normal, exports case and may be positive for imports. For exports, the first term is negative, the second (probably) positive, the third has the sign of $-[\bar{P}_j - P_j^0]$, and the fourth has the sign of $[\frac{\bar{P}_j}{1+r} - P_j^0]$. The results are thus indefinite. For the imports case of $\partial V_0/\partial P_i > 0$, if $\frac{\bar{P}_j}{1+r} > P_j^0$ all terms are positive if V_0 remains convex in P_j; and thus storage will not occur. Concavity of V_0 could turn even this around. With concavity and $\frac{\bar{P}_j}{1+r} < P_j^0$ we can insure that storage <u>will</u> occur in the highly risk averse heavy imports case.

If storage occurs in the jth good we have:

$$\frac{1}{1+r} \underset{P}{E}[V_0(P)P_j] = V_0(P^0)P_j^0$$

$$\frac{1}{1+r} \underset{P}{E}[V_0(P)P_j]\frac{1}{P_j^0} = V_0(P^0)$$

Making the nth good the numéraire and setting its price $\equiv 1$, we have $V_0(P^0) \geq \frac{1}{1+r} \underset{P}{E}[V_0(P)]$ and thus if good j is stored (necessary condition):

$$\frac{1}{1+r} E[V_0(P)P_j]\frac{1}{P_j^0} \geq \frac{1}{1+r} E(V_0)$$

$$E(V_0P_j) - E(V_0)P_j^0 \geq 0$$

$$C(V_0, P_j) + E(V_0)[E(P_j) - P_j^0] \geq 0$$

This is a very simple necessary condition if P_j is the only variable price. In this case $C(V_0(P_j), P_j)$ takes the sign of $\frac{\partial V_0}{\partial P_j} = V_{00}Z_j - V_0 \frac{\partial C_j}{\partial m} - t_jV_{00}$. All influences are negative for the risk averse, normal, exports case, implying that $E(P_j) > P_j^0$ is necessary for storage of good j. If good j is an import it is possible for storage to occur with above normal prices provided risk aversion and trade are sufficiently high. Intuitively, with heavy dependence on trade in the good it may pay to store even at above average prices because of the possibility of still worse prices next period.

For P_j not the only variable price, $C(V_0, P_j)$ is evaluated by a Taylor's series and if we have approximate linearity $C(V_0, P_j) = \frac{\partial V_0}{\partial P_j} \mathrm{Var}(P_j) + \sum_{i \neq j} \frac{\partial V_0}{\partial P_i} C(P_j, P_i)$. The derivatives are evaluated as above.

The possibility of futures markets in every good at least adds to the storage example the potential for the price of "stored" goods to be different from today's price of current consumption. Let q be the vector of prices for future delivery, b(s) the vector of futures purchases (sales), and b-s=f. For simplicity we assume 100% margin requirements. The formal problem is:

$$\max_f \; V(P^0 \cdot X(P^0) - q \cdot f,\ P^0) + \frac{1}{1+r} E[V(P \cdot X(P) + P \cdot f,\ P)]$$

$$\text{s.t.} \quad q \cdot f \geq 0$$

If we required in addition that no sales contracts be allowed (s=0) the correspondence would be very great with the previous problem. Instead we restrict futures market activity to no borrowing, which is not too unrealistic if we think of our small country working mostly with official hedging under the burden of a bad credit rating. Its debtor position might then be carefully watched. Requiring s=0 would be the extreme assumption, owing to a record of default, perhaps. 100% margin requirements is assumed for simplicity only; any constant fraction would work the same. Variable margin requirements would introduce elements of increasing transaction cost, but not add anything essential.

Letting α = the Lagrange multiplier on the borrowing constraint, the first order conditions with respect to b and s are:

$$V_0(P^0)q \geq \alpha q + \frac{1}{1+r} E[V_0(P)P]$$

$$V_0(P^0)q \leq \alpha q + \frac{1}{1+r} E[V_0(P)P]$$

Evidently equality holds; there is always futures activity in some direction. Note $\alpha=0$ if there is a net transfer of income forward, but $\alpha>0$ is consistent with futures hedging. The present problem thus differs from the previous ones in removing the necessity of a transfer.

To study conditions under which forward purchases ($b_i>0$) or sales ($s_i>0$) will occur, we can develop sufficient conditions. Assume for simplicity that the nth good has a certain spot price of unity and that the futures price for good n = $\frac{1}{1+r}$.[10] Then

$$V_0(P^0) = \alpha + E(V_0)$$

$$V_0(P^0)q_j = \alpha q_j + \frac{1}{1+r} E[V_0(P)P] \qquad j=1,\ldots,n-1$$

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[10]This is almost strictly a convenience assumption. V_0 is homogeneous of degree minus one in prices. Therefore if we define $\pi = \frac{1}{P_n}P$, we have $E[V_0(\pi)] = E[V_0(P)P_n]$ and $E[V_0(\pi)\pi_1] = E[V_0(P)P_1]$. We can solve the nth equation for $V_0(P^0) = \alpha + E[V_0(\pi)]\frac{1}{1+r}\cdot\frac{1}{q_n}$, substitute in the remaining equations and obtain:

$$E[V_0(\pi)][\frac{q_i}{q_n} - \frac{1}{1+r}E(\pi_i)] = \frac{1}{1+r}C(V_0(\pi),\ \pi_i) \qquad i=1,\ldots,n-1$$

(cont.)

Substituting

$$[\alpha + E(V_0)]q_j = \alpha q_j + \frac{1}{1+r} E[V_0(P)P_j]$$

$$E(V_0)[q_j - \frac{1}{1+r} E(P_j)] = \frac{1}{1+r} C(V_0, P_j) \qquad j=1,\ldots,n-1$$

For uncertainty in a single price, whether large <u>or</u> small, we know the sign of $C(V_0, P_j)$ depends only on the sign of $\partial V_0/\partial P_j$ so long as it is one-signed in the interval of variation. $\partial V_0/\partial P_j$ in this case is $-V_0 \frac{\partial C_j}{\partial m} - t_j V_{00} + (b_j - s_j)V_{00}$. For the risk averse, normal, exports case the first two terms are negative. $q_j > \frac{1}{1+r} \bar{P}_j$ therefore necessitates forward <u>sales</u>, which is intuitive. $q_j < \frac{1}{1+r} \bar{P}_j$ will initially necessitate sales as well, but at some point, as the futures price falls further below the discounted expected spot price, purchases will result. If the jth good is a normal import, and risk aversion is sufficiently low, the pattern is the same. If risk aversion is high, however, so the net of the first two terms is positive, forward sales may result if q_j is sufficiently greater than $\frac{1}{1+r} \bar{P}_j$ but there will always be a region of $q_j > \frac{1}{1+r} \bar{P}_j$ where purchases are made. $q_j < \frac{1}{1+r} \bar{P}_j$ necessitates purchases.

For the multi-price uncertainty case, we can evaluate $C(V_0, P)$ by a Taylor's series expansion about the mean and for approximately linear V_0 (or small price disturbances), we obtain:

Since $q\cdot(b-s)=0$ if $\alpha>0$, we can solve for the transaction in the numéraire good using the n-1 equations above. If $\alpha=0$, we can use $V_0(\pi^0) = E \frac{[V_0(\pi)]P_n/q_n}{1+r}$ to evaluate. Locally, this will depend chiefly on $\partial V_0/\partial \pi$.

$$V_0(\bar{P})[q - \frac{1}{1+r}\bar{P}] = \frac{1}{1+r}[-V_0(\bar{P})\frac{\partial C}{\partial m} - tV_{00} + (b - s)V_{00}]'\Omega_{PP}$$

where Ω_{PP} is the variance-covariance matrix of the prices.

This is usefully rewritten as:

$$(3) \quad q - \frac{1}{1+r}\bar{P} = \frac{1}{1+r}[-\frac{\partial C}{\partial m} + \sigma^*(t - (b - s))]'\Omega_{PP}$$

where σ^* is the coefficient of absolute risk aversion and σ^*, $\partial C/\partial m$, and t are evaluated at the mean.

This equation can be solved for the optimal vector b-s. More general results depend on higher derivatives of V_0 and higher moments of P.

In comparison with the commodity storage case (and *a fortiori* with reserve accumulation), futures market activity is welfare-improving, scarcely a surprising result. In practice, countries use a mixture of all three stabilization methods, and the reasons are to be sought in various increasing cost elements here assumed away for simplicity. In addition, relaxing the small country assumption will make combinations of methods desirable, but appropriate treatment of the large country case is too complex to be taken up here.

IV. Other Approaches

Previous studies of buffering have usually analyzed partial equilibrium cases where prices may be influenced by buffering activity (an exception is Heckerman (1973)); indeed frequently contrasting price fixity by the stabilization fund with the non-intervention fluctuation (Bieri-Schmitz). The goal of buffering activity always is to increase real income (utility), however, with decreased variability of prices being a (not necessarily good) by-product. Therefore, making the small country assumption is a useful first step in moving away from the restrictive assumptions of previous authors.

It is seen above that under the assumptions made, optimal buffering hangs almost exclusively on the behavior of the marginal utility of income. Assuming constancy, as in the expected social surplus analysis of Bieri-Schmitz, is clearly inappropriate; but the quadratic form of E-V analysis is also unduly restrictive (Massell). Counsels of perfection probably deserve to be ignored where no alternative exists; we hope to have demonstrated that this is no longer the case. It remains to be seen whether useful results can be extracted in the non-price-taking case, but about the crucial role of the marginal utility of income in this attempt, there can be no doubt.

We conclude by noting that the work of Heckerman (1972, 1973) is a special and rather oddly specified case of our commodity storage model, though he is dealing with a simple 2 period consumption-savings model. He assumes (in our notation)

$Z > 0$, that assets are accumulated without enquiring why (or whether) this would always be true. The fixed nominal wealth selected outside his model must be spent on two assets today whose price tomorrow is random. The objective is to maximize expected utility of tomorrow's consumption assuming assets will be sold and the proceeds spent on two goods whose price tomorrow is random. In the 1973 paper, each asset has constant purchasing power in terms of one commodity, completing the identity with our commodity storage market model. By assuming homothetic utility Heckerman is able to define a real wealth variable which reflects terms of trade effects in consumption, and he is then able to proceed with maximizing the expected utility of real wealth. By further restricting the utility function (1972), he is able to extract useful results. It is a defect of his analysis that one is no longer quite sure what utility function in terms of goods is implied. It is homothetic in goods, and in real wealth (with prices normally distributed):

$$E[V(W_{R_K})] = -\exp\{-r_K[\bar{W}_{R_K} - r_K \frac{\sigma^2 W_{R_K}}{2}]\}$$

where $\bar{W}_{R_K}$ = real wealth of individual K (including terms of trade effects) at the mean

r_K = Pratt-Arrow measure of risk aversion, our σ^*, assumed constant

$\sigma^2 W_{R_K}$ = variance of Kth individual's real wealth

The analysis of Section III therefore simply generalizes his results without implicitly restricting the utility function.

We saw there that the question of at least one $Z_i > 0$ is quite complex, indicating the limitations of his analysis.

Appendix: Convexity of V_0 in P for the Cobb-Douglas Case

Consider the Cobb-Douglas utility function

$U = \frac{1}{1-\sigma}[C_1^{\alpha}C_2^{1-\alpha}]^{1-\sigma}$, which has as its indirect utility function

$V = \frac{1}{1-\sigma}[(P\cdot X)P_1^{-\alpha}P_2^{-(1-\alpha)}]^{1-\sigma}$. With some manipulation, we obtain:

$$\left(\frac{P_1}{V_0}\right)^2 \frac{\partial^2 V_0}{\partial P_1^2} = 2\Theta_1^2 - 2\Theta_1\alpha\,\frac{\sigma-1}{\sigma} - \alpha\,\frac{\sigma-1}{\sigma}[1 - \alpha\,\frac{\sigma-1}{\sigma}] - \Theta_1\varepsilon_{X1}$$

where $\Theta_1 = \dfrac{P_1X_1}{P\cdot X}$ and $\varepsilon_{X1} = \dfrac{P_1}{X_1}\,\dfrac{\partial X_1}{\partial P_1}$

Θ_1 represents the structure of comparative advantage. If $\varepsilon_{X1}\to 0$, the expression is always positive for $\sigma\leq 1$. Differentiating with respect to Θ_1, $\dfrac{d\left(\left(\frac{P_1}{V_0}\right)^2 \frac{\partial^2 V_0}{\partial P_1^2}\right)}{d\Theta_1} = 4\Theta_1 - 2\alpha\,\dfrac{\sigma-1}{\sigma}$.

For $\sigma\leq 1$, this derivative is always positive and the convexity derivative above is always positive if we neglect ε_{X1}. Therefore V_0 is convex in P_1 and is more convex the smaller imports or the larger exports (implied by increasing Θ_1). For $\sigma>1$, matters are more complex. V_0 is concave in a region ranging from $\Theta_1=0$ to some critical value, after which it is convex. The critical value may lie on either side of the no trade point (where $\Theta_1=\alpha$), depending on the values of α and σ. Increasing risk aversion (σ) makes the critical value of Θ_1 increase. Also of interest, the effect of increasing Θ_1 within the concavity side of its range of values initially <u>increases</u>

concavity, then decreases it, of course ultimately causing convexity. Neglecting the effect of ε_{X1}, we have discovered clean results of the general sort expected: V_0 is convex in an export range, concave in an import range, and no generalization can be made for small trade. Also, the effect of larger trade in the range of imports can either increase or decrease concavity.

Incorporating the influence of ε_{X1}, we note that it acts toward concavity, and the more so the greater the specialization in X_1. Intuitively, the reason it appears is because the second order effects on income of changing P_1 require it. $P \cdot X(P)$ is convex in P, so $E(P \cdot X(P))$ is raised by price fluctuation relative to $\bar{P} \cdot X(\bar{P})$, and the more so the greater the second order effects. The effect of this on V_0, ceteris paribus, is to lower it due to $V_{00} < 0$. Therefore the influence of ε_{X1} is toward concavity.

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