International Trade and Macroeconomic Dynamics with Heterogeneous Firms

Technical Appendix

Fabio Ghironi[*]
Boston College
and EABCN

Marc J. Melitz[†]
Harvard University
CEPR and NBER

May 23, 2004

A The Steady State

We denote constant, steady-state levels of variables by dropping the time subscript and assume $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. Under these assumption, the steady state of the model is symmetric: $\tilde{Q} = Q = TOL = 1$ and the levels of all other endogenous variables are equal across countries.

Solving for $\tilde{z}_X$

Given the solution for the average export productivity $\tilde{z}_X$, we can obtain the cutoff level z_X from $\tilde{z}_X = \nu z_X$, where $\nu \equiv \{k/[k-(\theta-1)]\}^{1/(\theta-1)}$. We can solve for $\tilde{z}_X$ as follows. The Euler equation for share holdings yields:

$$\tilde{v} = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\left(\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X\right).$$

Combining this equation with the free entry condition $\tilde{v} = f_E w$ implies:

$$\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X = \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E w. \tag{A.1}$$

[*]Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3859, U.S.A. or Fabio.Ghironi@bc.edu. URL: http://fmwww.bc.edu/ec/Ghironi.php.

[†]Department of Economics, Harvard University, Littauer Center 120, Cambridge, MA 02138, U.S.A.

The steady-state zero profit export cutoff equation is:

$$\tilde{d}_X = w f_X \frac{\theta - 1}{k - (\theta - 1)}. \tag{A.2}$$

Also, steady-state profits from selling at home and abroad are $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, respectively. These two equations imply:

$$\tilde{d}_D = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{\theta-1} \left(\tilde{d}_X + w f_X\right). \tag{A.3}$$

Optimal pricing yields $\tilde{\rho}_D = [\theta/(\theta-1)]\, \tilde{z}_D^{-1} w$ and $\tilde{\rho}_X = [\theta/(\theta-1)]\, \tau \tilde{z}_X^{-1} w$. Hence, $\tilde{\rho}_X/\tilde{\rho}_D = \tau \tilde{z}_D/\tilde{z}_X$, and substituting this into (A.3), we have:

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1} \left(\tilde{d}_X + w f_X\right),$$

or, taking (A.2) into account,

$$\tilde{d}_D = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1} \left(w f_X \frac{\theta - 1}{k - (\theta - 1)} + w f_X\right). \tag{A.4}$$

The steady-state share of exporting firms in the total number of domestic firms is:

$$\frac{N_X}{N_D} = (z_{\min})^k (\tilde{z}_X)^{-k} \left[\frac{k}{k - (\theta - 1)}\right]^{\frac{k}{\theta-1}}. \tag{A.5}$$

Substituting equations (A.2), (A.4), and (A.5) into (A.1), using $\tilde{z}_D = \{k/[k - (\theta - 1)]\}^{\frac{1}{\theta-1}} z_{\min}$, and rearranging yields:

$$(\tilde{z}_X)^{1-\theta} (\tau z_{\min})^{\theta-1} \left[\frac{k}{k - (\theta - 1)}\right]^2 + (\tilde{z}_X)^{-k} (z_{\min})^k \left[\frac{k}{k - (\theta - 1)}\right]^{\frac{k}{\theta-1}} \frac{\theta - 1}{k - (\theta - 1)} = \frac{[1 - (1 - \delta)\beta]}{(1 - \delta)\beta} \frac{f_E}{f_X}.$$

This equation can be rewritten as:

$$\xi_1 (\tilde{z}_X)^{1-\theta} + \xi_2 (\tilde{z}_X)^{-k} = \xi_3, \tag{A.6}$$

where

$$\xi_1 \equiv (\tau z_{\min})^{\theta-1} \left[\frac{k}{k-(\theta-1)}\right]^2 > 0,$$
$$\xi_2 \equiv (z_{\min})^k \left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}} \frac{\theta-1}{k-(\theta-1)} > 0,$$
$$\xi_3 \equiv \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} \frac{f_E}{f_X} > 0.$$

The left-hand side of equation (A.6) is a hyperbola. This guarantees existence and uniqueness of $\tilde{z}_X > 0$, the exact value of which we obtain numerically.

Solving for $\tilde{\rho}_X$

The law of motion for the total number of domestic firms implies:

$$N_E = \frac{\delta}{1-\delta} N_D. \tag{A.7}$$

Steady-state aggregate accounting yields $C = wL + N_D\tilde{d}_D + N_X\tilde{d}_X - N_E w f_E$. Using (A.1) and (A.7), this can be rewritten as:

$$\frac{C}{w} = L + N_D f_E \frac{1-\beta}{(1-\delta)\beta}. \tag{A.8}$$

Equation (A.2) and the expression for average export profits, $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - wf_X$, imply:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta k}{k-(\theta-1)} f_X. \tag{A.9}$$

The price index equation $N_D\tilde{\rho}_D^{1-\theta} + N_X\tilde{\rho}_X^{1-\theta} = 1$ yields:

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{\theta-1} + \frac{N_X}{N_D},$$

or, using $\tilde{\rho}_X/\tilde{\rho}_D = \tau\tilde{z}_D/\tilde{z}_X$ and equation (A.5),

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tau\tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1} + \left(\frac{z_{\min}}{\tilde{z}_X}\right)^k \left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}. \tag{A.10}$$

Together, equations (A.8), (A.9), and (A.10) yield the following equation for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta k}{k-(\theta-1)} f_X - K^{-1} f_E \frac{1-\beta}{(1-\delta)\beta}\right] L^{-1}.$$

where K is the right-hand side of equation (A.10).

Special Case: All Firms Export

In this case, equation (A.9) no longer holds since the zero cutoff profit condition (A.2) no longer applies. Using $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - wf_X$, equation (A.1) can be written as:

$$\tilde{\rho}_X^{1-\theta} \frac{C}{\theta} \left(\tau^{\theta-1} + 1\right) - wf_X = \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E w,$$

which implies:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta}{\tau^{\theta-1}+1} \left\{ f_X + \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E \right\}. \tag{A.11}$$

Equation (A.11) now replaces equation (A.9) when solving for $\tilde{\rho}_X$. This yields the following expression for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta}{\tau^{\theta-1}+1} \left\{ f_X + \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E \right\} - K^{-1} f_E \frac{1-\beta}{(1-\delta)\beta} \right] L^{-1}.$$

Solving for the Remaining Variables

The solutions for other endogenous variables are straightforward

- $N_D = K^{-1} \tilde{\rho}_X^{\theta-1}$;
- $\tilde{\rho}_D = \frac{\tilde{z}_X}{\tau \tilde{z}_D} \tilde{\rho}_X$ using $\tilde{\rho}_X / \tilde{\rho}_D = \tau \tilde{z}_D / \tilde{z}_X$;
- $w = \tilde{\rho}_X \frac{\theta-1}{\theta\tau} \tilde{z}_X$ using $\tilde{\rho}_X = [\theta/(\theta-1)] \tau \tilde{z}_X^{-1} w$;
- $C = w \left[L + N_D f_E \frac{1-\beta}{(1-\delta)\beta} \right]$ using (A.8);
- $N_E = \frac{\delta}{1-\delta} N_D$;
- $N_X = N_D (z_{\min})^k (\tilde{z}_X)^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$ using (A.5);
- $\tilde{d}_D = \frac{1}{\theta} (\tilde{\rho}_D)^{1-\theta} C$;
- $\tilde{d}_X = \frac{1}{\theta} (\tilde{\rho}_X)^{1-\theta} C - wf_X$;
- $\tilde{v} = wf_E$ (using the free entry condition);
- $1 + r = 1/\beta$ (using the Euler equation for bond holdings).

Symmetry of the steady state ensures $\tilde{z}_X^* = \tilde{z}_X$, $\tilde{\rho}_X^* = \tilde{\rho}_X$, $N_D^* = N_D$, $N_E^* = N_E$, $N_X^* = N_X$, $\tilde{d}_D^* = \tilde{d}_D$, $\tilde{d}_X^* = \tilde{d}_X$, $\tilde{v}^* = \tilde{v}$, in addition to $C^* = C$, $w^* = w$, and $r^* = r$.

B Labor Market Clearing

Recall that a firm with productivity z produces $Z_t z$ units of output per unit of labor employed. Consider separately the labor used to produce goods for the domestic and export markets: let $l_{D,t}(z)$ and $l_{X,t}(z)$ represent the amount of labor hired to produce goods for each market. These only represent labor used in production; in addition, each new entrant hires $f_{E,t}/Z_t$ units of labor to cover the entry cost, and each exporter hires $f_{X,t}/Z_t$ units of labor to cover the fixed export cost in every period. The profits earned from domestic sales for a firm with productivity z are then given by:

$$d_{D,t}(z) = \rho_{D,t}(z) Z_t z l_{D,t}(z) - w_t l_{D,t}(z) = \frac{1}{\theta - 1} w_t l_{D,t}(z),$$

using $\rho_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ from optimal pricing. This relationship holds for a firm with average productivity $\tilde{z}_D$, and also for averages across all domestic firms. This implies that the average amount of production labor hired to cover domestic sales is $(\theta - 1)\,\tilde{d}_{D,t}/w_t$. The total amount of such labor hired at home is thus $N_{D,t}\,(\theta - 1)\,\tilde{d}_{D,t}/w_t$.

The profits earned from export sales for an exporting firm with productivity z are given by:

$$d_{X,t}(z) = Q_t \rho_{X,t}(z) \frac{Z_t z l_{X,t}(z)}{\tau_t} - w_t \left[l_{X,t}(z) + \frac{f_{X,t}}{Z_t} \right] = \frac{1}{\theta - 1} w_t l_{X,t}(z) - w_t \frac{f_{X,t}}{Z_t},$$

using $\rho_{X,t}(z) = Q_t^{-1} \tau_t \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ from optimal pricing. Note that only $Z_t z l_{X,t}(z)/\tau_t$ export units are sold, although $Z_t z l_{X,t}(z)$ are produced (the remaining fraction having "melted" away in an iceberg fashion while crossing the border). Again, this relationship holds for a firm with average export productivity $\tilde{z}_{X,t}$, and also for averages across all exporters. The average amount of production labor hired to cover export sales is thus $(\theta - 1)\,\tilde{d}_{X,t}/w_t + (\theta - 1)\,f_{X,t}/Z_t$. Multiplying by $N_{X,t}$ yields the total amount of such labor for the home economy.

The total amount of production labor hired in the home economy is then

$$\frac{\theta - 1}{w_t} N_{D,t} d_{D,t}(z) + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta - 1}{Z_t} N_{X,t} f_{X,t}.$$

Adding the total amount of labor hired by new entrants, $N_{E,t} f_{E,t}/Z_t$, and that hired by exporters to cover the fixed costs, $N_{X,t}f_{X,t}/Z_t$, yields the aggregate labor demand for the home economy:

$$L_t = \frac{\theta-1}{w_t} N_{D,t}\tilde{d}_{D,t} + \frac{\theta-1}{w_t} N_{X,t}\tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$

Equating L_t to labor supply (L) yields the equilibrium condition for home's labor market. The derivation for foreign is analogous.

Balanced Trade Implies Labor Market Clearing

We now demonstrate that balanced trade under financial autarky implies labor market clearing.

Using the home price index equation $1 = N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta} + N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta}$, the balanced trade condition $Q_t N_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^{*} = N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta} C_t$ can be written:

$$Q_t N_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^{*} = \left[1 - N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta}\right] C_t.$$

This condition can be re-written as

$$C_t = \theta N_{X,t}\left(\tilde{d}_{X,t} + w_t \frac{f_{X,t}}{Z_t}\right) + \theta N_{D,t}\tilde{d}_{D,t},$$

since $\tilde{d}_{D,t} = \left(\tilde{\rho}_{D,t}\right)^{1-\theta} C_t/\theta$ and $\tilde{d}_{X,t} = Q_t\left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^{*}/\theta - w_t f_{X,t}/Z_t$. Combining this with aggregate accounting ($C_t = w_t L + N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t} - N_{E,t} w_t f_{E,t}/Z_t$) yields the labor market clearing condition for the home economy:

$$L = \frac{\theta-1}{w_t} N_{D,t}\tilde{d}_{D,t} + \frac{\theta-1}{w_t} N_{X,t}\tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$

The proof for the foreign economy follows the same steps.