Unraveling Results from *Comparable* Demand and Supply: An Experimental Investigation*

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Abstract

Markets sometimes unravel, with offers becoming inefficiently early. Often this is attributed to competition arising from an imbalance of demand and supply, typically excess demand for workers. However this presents a puzzle, since unraveling can only occur when firms are willing to make early offers and workers are willing to accept them. We present a model and experiment in which workers' quality becomes known only in the late part of the market. However, in equilibrium, matching can occur (inefficiently) early only when there is comparable demand and supply: a surplus of applicants, but a shortage of high quality applicants.

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1 Introduction

In many professional labor markets most entry-level hires begin work at around the same time, e.g. soon after graduating from university, or graduate or professional school. Despite a common start time, offers can be made and contracts can be signed at any time prior to the start of employment. Such markets sometimes *unravel*, with offers becoming earlier and more dispersed in time, not infrequently well over a year before employment will begin.[1]

Unraveling is sometimes a cause of market failure, particularly when contracts come to be determined before critical information is available.[2] Attempts to prevent or reverse unraveling are often a source of new market design in the form of new rules or market institutions.[3]

It is commonly suggested, both by economists and lay participants in these markets, that unraveling results from competition related to an imbalance of demand and supply. For example, Roth (1984), writing about the market for new physicians around 1900, writes

> "The number of positions offered for interns was, from the beginning, greater than the number of graduating medical students applying for such positions, and there was considerable competition among hospitals for interns. One form in which this competition manifested itself was that hospitals attempted to set the date at which they would finalize binding agreements with interns a little earlier than their principal competitors."

Describing the growth of early admissions to colleges, Avery, Fairbanks, and Zeckhauser (2003, p32) quote a 1990 U.S. News and World Report story[4]

> "Many colleges, experiencing a drop in freshman applications as the population of 18-year-olds declines, are heavily promoting early-acceptance plans in recruiting visits to high schools and in campus tours in hopes of corralling top students sooner."

[1] See Roth and Xing (1994) for many examples, including markets other than labor markets in which contracts are fulfilled at around the same time but can be finalized substantially earlier, such as the market for college admissions, or for post-season college football bowls.

[2] See e.g. Niederle and Roth (2003) on how the market for gastroenterology fellows fragmented when it unraveled, or Fréchette, Roth, and Ünver (2007) on how college football bowls lost television viewership in years when teams and bowls were matched too early.

[3] See e.g. Niederle, Proctor, and Roth (2006, 2008) and Niederle and Roth (2004, 2005, 2009) on the market for gastroenterologists, Roth and Peranson (1999) on the market for new American medical graduates, Roth (1991), Ünver (2001, 2005) on markets for new British doctors, and Fréchette, Roth, and Ünver (2007) on college football bowl selections.

[4] Titled "A Cure for Application Fever: Schools Hook More Students with Early Acceptance Offers." (April 23).

There is also no shortage of commentary by market participants linking excess demand for workers to unraveling.[5] And indeed, a number of unraveling events seem to have been initiated by a shock to supply or demand. See e.g. Niederle and Roth (2003) and McKinney, Niederle, and Roth (2005) for a description of just such a development in the market for gastroenterology fellows, in which a shock that produced excess demand for fellows (i.e. a shock that led, for the first time, to fewer fellowship applicants than fellowship positions) led to unraveling that lasted for a decade.

But, when looking at a labor market, it is not uncommon for participants on both sides of the market to be nervous about their prospects, and it can be difficult to be sure which is the short side of the market. Even in a market with more applicants than positions there may be a shortage of the most highly qualified applicants.

For example, the market for law clerks has experienced serious unraveling, with positions for new graduates in some recent periods being filled two years before graduation (see Avery, Jolls, Posner, and Roth 2001, 2007, and Haruvy, Roth, and Ünver 2006). Wald (1990) writes of the judges' perception of that market as follows:

> "But why the fervent competition for a handful of young men and women when our law schools spawn hundreds of fine young lawyers every year? Very simply, many judges are not looking just for qualified clerks; they yearn for neophytes who can write like Learned Hand, hold their own in a discussion with great scholars, possess a preternatural maturity in judgment and instinct, are ferrets in research, will consistently outperform their peers in other chambers and who all the while will maintain a respectful, stoic, and cheerful demeanor.
>
> ... Thus, in any year, out of the 400 clerk applications a judge may receive, a few dozen will become the focus of the competition; these few will be aggressively courted by judges from coast to coast. Early identification of these "precious few" is sought and received from old-time friends in the law schools – usually before the interview season even begins."

In just the same way, there are many federal appellate court judges, but only a few dozen are considered to have highly prestigious clerkships.

A similar story can be told about college admissions. For example, Menand (2003) says

> "There are more than two thousand four-year colleges in the United States. Only about two hundred reject more students than they accept. The vast majority of American

[5] See for example Santos and Sexson (2002) on the market for new psychiatrists, or Gorelick (1999) on gastroenterologists.

colleges accept eighty per cent or more of those who apply. But among the top fifty there is a constant Darwinian struggle to improve selectivity."

So both law clerks and college admissions are examples of markets in which participants on each side of the market can think of themselves as being on the long side, i.e. as facing a shortage of appropriate opportunities, if they focus on an elite subset of participants on the other side of the market. In such markets it can be a little hard to assess the relative balance of demand and supply in field data.

We are going to argue that such a situation, which we will call comparable demand and supply, is in fact characteristic of markets in which we should expect to see costly, inefficient unraveling. The intuition is that unraveling requires both that firms want to make early offers and that applicants want to accept them. If information about match quality evolves over time, and if one side of the market knows that it will be in short enough supply to attract a good match when the good matches become clear, then there will not be unraveling, as one side of the market will want to await the resolution of the uncertainty.[6]

Unraveling will cause changes in who is matched with whom, but whether this will cause big efficiency losses will depend on whether the demand and supply imbalances are such that some "mismatches" are inevitable (in which case unraveling may just cause redistribution of mismatches), or whether they can largely be avoided by awaiting information on match quality (in which case too-early matches are likely to be inefficient).

This paper reports an experiment on these issues. The control available in the laboratory allows us to precisely vary the conditions of demand and supply.[7] Section 2 outlines the elements of a

[6]In any model in which offers are made over time, unraveling will occur at equilibrium only if early offers are made and accepted. The theoretical model closest in spirit to the one explored here is that of Li and Rosen (1998) in which firms and workers unravel to each insure the other against an outcome that leaves their side of the market in excess supply (in an assignment model in which agents on the long side of the market earn zero). Damiano, Li, and Suen (2005) look at a model with a continuum of agents in which unraveling is driven at equilibrium by the fact that it makes later contracting less desirable because of the difficulty of finding a match when everyone else contracts early. Unraveling as insurance is further explored in Li and Suen (2000, 2004) and Suen (2000). Other theoretical models have unraveling (under conditions of fixed supply and demand) determined by the competition for workers as determined by how correlated firms' preferences are (Halaburda 2008), or by how well connected firms are to early information about workers' qualities (Fainmesser 2009), or by the establishment of certain kinds of centralized matching mechanisms (Roth and Xing 1994, Sönmez 1999, and Ünver 2001). In prior experimental studies, Kagel and Roth (2000), Ünver (2005) and Haruvy, Roth, and Ünver (2006) look at unraveling as a function of what kinds of centralized market clearing mechanism are available at the time when matching may be done efficiently. Niederle and Roth (2009) look at unraveling as a function of the rules governing exploding offers in a decentralized market.

[7]The usual difficulties of measuring supply and demand in the field are compounded when the supply of workers of the highest quality must be evaluated.

theoretical model, sufficiently to apply it to the experimental design presented in Section 3. Although we will concentrate here on the predictions about unraveling and inefficiency as a function of demand and supply, we will also note that the theoretical framework we develop enriches the standard model of stable matching in ways that allow for some novel predictions about the effect on one side of the market of adding participants to the other side.

2 The Model

Consider a market with n_F firms and n_A applicants. Denote a typical firm by f, and a typical applicant by a. Every firm wants to hire up to one applicant, and every applicant can work for up to one firm. Firms and applicants can be of one of two possible qualities, high (h) or low (ℓ). A **matching** is a mapping that (i) maps each agent to a partner or leaves her unmatched, and (ii) for any firm f and applicant a, maps firm f to applicant a if and only if it maps applicant a to firm f. The interpretation is that a firm f and an applicant a are matched if firm f hired applicant a and applicant a works for firm f.

Unmatched firms and applicants receive a payoff of 0. If firm f and applicant a are matched with each other, they receive payoffs that depend upon their own quality, the quality of the matched partner, and a random variable. We assume every agent prefers a high quality partner to a low quality partner, and being unmatched is the least preferred alternative for each agent. For each firm f of quality i, the payoff for being matched with an applicant a of quality j is $V_f(a) = u_{ij} + \varepsilon_a$, where ε_a is a random variable with mean 0, and $u_{ij} > 0$. For each applicant a of quality i, the payoff for being matched with a firm f of quality j is $V_a(f) = u_{ij} + \varepsilon_f$, where ε_f is a random variable with mean 0. ε_f and ε_a are identically distributed with a marginal distribution G in the interval $[-\alpha, \alpha]$ where $\alpha > 0$. The joint distribution of $\{\varepsilon_a\}$ is such that there is no tie in ε_a, that is, there are no two applicants a_1 and a_2 such that $\varepsilon_{a_1} = \varepsilon_{a_2}$ occurs with positive probability. Similarly, the joint distribution of $\{\varepsilon_f\}$ is such that there is no tie in ε_f.

A matching is **ex-post stable** if there is no firm-applicant pair such that, after all uncertainty is resolved, each prefers one another to her match. A matching is **efficient** if the sum of the payoffs of the agents is highest among all matchings after all uncertainty is resolved. A matching is **qualitywise-efficient** if the sum of the payoffs of agents excluding ε's is highest among all matchings. Since ex-post there are no ties in ε_f and ε_a with probability one, firms and applicants can be strictly rank-ordered with respect to preferences. A matching is **assortative** if for all $k \leq \min\{n_F, n_A\}$, the k^{th} best applicant is matched with the k^{th} best firm. The market has a unique assortative matching with probability one. A matching is **qualitywise-assortative** if there is no firm f of quality i and applicant a of quality j such that f is matched with an applicant of lower quality than j or is unmatched, and a is matched with a firm of lower quality than i or is unmatched.[8]

We assume that

$$u_{hh} - u_{h\ell} > u_{\ell h} - u_{\ell\ell} > 0.^{9}$$

We choose α (the bound of the ε's) small enough such that with the above assumptions, an efficient matching is qualitywise-assortative; i.e. no agent ever prefers a low quality partner to any high quality partner. This implies that a matching is qualitywise-assortative if and only if it is qualitywise-efficient. Moreover, there is a unique ex-post stable matching (the unique assortative matching, due to the random variables ε), which is efficient.

We will be mostly interested in qualitywise efficiency or qualitywise inefficiency rather than efficiency or inefficiency, since the values of random $\{\varepsilon\}$ will be rather small and this will cause small welfare losses when a qualitywise-efficient matching is not efficient.[10] However, qualitywise-inefficient matchings will lead to comparatively large welfare losses.

While we denote qualities as high and low, it would be misleading to think of those as e.g. Ivy League universities and community colleges. Rather, the analogy would be between the very best colleges and some only slightly less prestigious ones. Note that in the model the firms literally hire from the same pool of applicants. Similarly, each applicant has a chance to be among the very best, or not quite make it (hence they are far from applicants who would have no chance to be accepted to a top university, no matter the final information available about them).

[8] The assortative matching is qualitywise-assortative, although the converse doesn't hold. Note that the ε's define an absolute standard of efficiency, but we look at quality-wise efficiency and sorting. This is because the ε's play three roles in our treatment: 1. as a technical assumption to give us uniqueness, 2. to reflect the fact that, in future field work, we expect the data might be able to distinguish quality in the large, but not preferences among applicants with similar observables (so that quality would be observable only up to the ε's); and 3. to make clear in our model that we are not claiming that just because a market isn't unraveled it is efficient, rather we are only claiming that avoiding unraveling avoids a large source of inefficiency, quality-wise inefficiency.

[9] That is, high quality firms (and applicants) have a strictly higher marginal expected payoff from increasing their partner's quality than low quality firms (and applicants). I.e., if the production function regarding a firm and worker is the sum of the pay-offs of the firm and the worker, then it is supermodular.

[10] For example, a qualitywise efficient matching may be inefficient when the applicant with the largest ε value remains unmatched while an applicant of the same quality is matched. Qualitywise efficiency or its absence is also what can generally be assessed from evaluating field data such as that obtained by using revealed preferences over choices in marriage or dating markets for estimating preferences over observables (i.e. such estimates can determine how important a potential mate's education is in forming preferences, but cannot observe which among identically educated potential mates will have the best personal chemistry (see e.g. Banerjee et al. 2009; Hitsch et al. 2009; Lee 2009).

2.1 The Game

Firms and applicants can match in a market that is characterized by early and late hiring stages. Each stage consists of several hiring periods. The firms' qualities, high or low, are revealed and become common knowledge to all market participants at the beginning of the first period in the early hiring stage. Let $n_F^h \leq n_F$ be the number of high quality firms in the market and $n_F - n_F^h$ be the number of low quality firms. The applicants' qualities, however, are uncertain and revealed only at the first period of the late hiring stage when they become common knowledge. Furthermore, each market is characterized by a common knowledge deterministic final proportion of high and low quality applicants. That is, from the beginning, it is known that a proportion $\frac{n_A^h}{n_A}$ of applicants for some integer $n_A^h < n_A$ will be of high quality, and the remaining applicants, $n_A - n_A^h$ will be of low quality. (For example, there might be a special distinction attached to hiring a "top ten" applicant.) To simplify the exposition, and the later experiments, we assume that all applicants have the same ex-ante probability of being of high quality, namely $\frac{n_A^h}{n_A}$.[11]

At the beginning of the late hiring stage, the qualities of the applicants are revealed, and become common knowledge. Furthermore, we assume that the values of the random variables $\{\varepsilon\}$ are realized at the beginning of the late hiring stage, and become common knowledge. The distribution of $\{\varepsilon\}$ is common knowledge at the beginning of the early hiring stage.

The market is one in which firms make offers to applicants, and applicants have to decide whether to accept or reject those offers right away (they cannot hold on to offers). Once an applicant has accepted an offer from a firm, the firm and the applicant are matched with each other: the applicant cannot receive any other offers, and the firm cannot make any other offers.

In every period, firms that are not yet matched may decide to make up to one offer to any available applicant. Once all the firms have decided what if any offers to make, the offers become public information. Then, applicants can decide how to respond to offers they have received; whether to accept at most one offer, or reject all offers. After all applicants have decided, their actions become public information.

Therefore, the market is a multi-period game in which firms (and applicants) make simultaneous decisions with public actions, and a move by nature in the middle of the game which determines the qualities of the applicants and the $\{\varepsilon\}$ perturbations of preferences over different firms and applicants.

There are T^E periods in the early hiring stage, and T^L periods in the late hiring stage.

[11] For risk-neutral market participants we can equivalently assume that each applicant has a probability $\frac{n_A^h}{n_A}$ to be of high quality. We instead choose a fixed fraction to reduce variance in the experiment that follows.

2.2 Analysis of Unraveling

This game has many (Nash) equilibria, including ones in which every match is concluded in period 1 of the early hiring stage. For example, a strategy profile in which for each applicant a there is only one firm f_a whose offer a accepts in period 1, and in which firm f_a only makes an offer in period 1 to applicant a, and no other offers are made or accepted, is an equilibrium. However, such equilibria will in general not be subgame-perfect. We restrict our theoretical analysis to subgame perfect equilibria (SPEs).

We partition the space of possible demand and supply relations into 3 different scenarios representing excess supply, comparable demand and supply, and excess demand for labor, respectively. The important variables will be n_F, the total number of firms, n_F^h, the number of high quality firms, n_A, the total number of applicants, and n_A^h, the total number of high quality applicants:

Case 1. $n_A^h \geq n_F$: Every firm can be matched with a high quality applicant, some high quality applicants remain unmatched (if $n_A^h > n_F$). There is *excess supply.*

Case 2. $n_A > n_F > n_A^h$: Excess applicants, but shortage of high quality applicants. There is *comparable demand and supply.*

Case 3. $n_F \geq n_A$: Each applicant can be matched with a firm. There is *excess demand.* We analyze excess demand in two subcases:

a. $n_F \geq n_A > n_F^h$: Excess firms, but shortage of high quality firms.

b. $n_F^h \geq n_A$: Every applicant can be matched with a high quality firm, some high quality firms remain unmatched (if $n_F^h > n_A$).

We will show that the SPEs of this game support the following intuition. In case 1, no firm will wish to make an early offer, because by waiting until applicants' qualities are known, every firm can hire a high quality applicant. Thus, there is *excess supply* of labor.

In Case 2, in the early stage, a given applicant does not know if she will be on the long or short side of the market, since her quality is still unknown (and since there will be too few high quality applicants, and too many low quality applicants). This is why we say that there is *comparable* demand and supply. An applicant may therefore find it attractive to take an early offer from (even) a low quality firm, and making such an offer is profitable for a low quality firm, since this is the only way to possibly hire a high quality applicant. So Case 2 may be subject to unraveling.

Case 3a looks superficially symmetric to Case 2 but with the critical difference that the role of firms and applicants are reversed. Since the qualities of firms are already known in the early stage, no high quality firm is in any doubt that it will be on the short side of the market, and no applicant is in any doubt that she can find a position in the late period. So no applicant will take an early offer from a low quality firm (since such offers will remain available even if the applicant turns out to be low quality), and no high quality firm will make an early offer.[12] Case 3b looks like Case 1 with firms and workers reversed, but in this case there is much less difference between early or late matching, since even early matchings cannot be qualitywise-inefficient, as every applicant, regardless of quality, will match to a high quality firm. Thus, we refer to Cases 3a and 3b as *excess demand* conditions.

We are concerned about the inefficiency (or efficiency) of SPE outcomes. We will mostly focus on "large scale" inefficiency, namely *qualitywise inefficiency*. Our first result characterizes the demand and supply condition in which there are qualitywise-inefficient outcomes under SPEs:

Theorem 1 *A qualitywise-inefficient early matching is an outcome of a subgame perfect equilibrium only if the market is one of* comparable *demand and supply (Case 2), i.e. if* $n_A > n_F > n_A^h$.

Note that not all early matching equilibria are qualitywise-inefficient. For example, when $n_F^h \geq n_A$, high firms can hire applicants early under SPE as described by Lemma 3, but the outcome will not be qualitywise-inefficient. That is the reason we concentrate on qualitywise-inefficient early hiring (as opposed to all early hiring) in Theorem 1, since only qualitywise-inefficient hiring creates large welfare problems.[13]

We will prove Theorem 1 using several lemmas. The proofs of all results are given in Appendix A.

Lemma 1 *Any subgame perfect equilibrium produces assortative matching among the firms and applicants still unmatched at the beginning of the late hiring stage.*

[12] There is another asymmetry between cases 2 and 3a, namely that the firms make offers, and applicants accept or reject them. However, this will not be important to determine the SPEs. It may however be important empirically, in the experiment and in the field.

[13] In their pioneering theoretical investigation of unraveling, Li and Rosen (1998) study an assignment market with continuous payoffs in which the supply and demand are assumed to always fall in Case 2. In the early period of their model each worker has a probability of being a productive worker in period 2 (and in period 2 all workers are either productive or unproductive, and only firms matched with a productive worker have positive output). In this context, their assumption that supply and demand fall in Case 2 is that there are more workers than firms, but a positive probability that there will be fewer productive workers than firms. They find, among other things, that inefficient unraveling is more likely "the smaller the total applicant pool relative to the number of positions." Our framework allows us to see how this conclusion depends on supply and demand remaining in Case 2. When the total applicant pool declines sufficiently, the market enters Case 3a (when the number of workers falls below the number of firms), and inefficient unraveling is no longer predicted. Moreover, we find a characterization of supply and demand conditions necessary for sequentially rational unraveling.

The next two lemmas are about SPE outcomes for Cases 1, 3a, and 3b.

Lemma 2 *When $n_A^h \geq n_F$ (Case 1), and $n_F \geq n_A > n_F^h$ (Case 3a), the unique subgame perfect equilibrium outcome is late, assortative matching.*

Lemma 3 *When $n_F^h \geq n_A$ (Case 3b), the outcome of any subgame perfect equilibrium is qualitywise-efficient.*

In the next result, we state some necessary and sufficient conditions of comparable *demand and supply* that lead to unraveling under SPEs. That is, Theorem 1 establishes that qualitywise-inefficient SPEs can only exist in Case 2, but whether one will in fact exist depends on whether, in stage 1, an applicant will find it attractive to accept an offer from a low quality firm. This decision will of course depend both on the number of high and low quality applicants and firms, and on the expected utility of having a high or low quality position.

Theorem 2 *In the case of* comparable *demand and supply, i.e. $n_A > n_F > n_A^h$ (Case 2), all high quality firms hire in the late stage under any subgame perfect equilibrium. At every subgame perfect equilibrium, at least one low quality firm hires early if and only if*

$$0 > \begin{cases} n_A^h u_{hh} - n_A^h u_{h\ell} + (n_F^h - n_A^h) u_{\ell h} - \left((n_A - n_A^h) - (n_F - n_F^h) \right) u_{\ell\ell} & \text{if } n_F^h \geq n_A^h \\ n_F^h u_{hh} - n_F^h u_{h\ell} - (n_A - n_F) u_{\ell\ell} & \text{otherwise} \end{cases}$$

and

$$\frac{n_A^h}{n_A} > \frac{n_A^h - n_F^h}{n_F - n_F^h}.$$

Note that when $n_F^h \geq n_A^h$, then whenever a low quality firm succeeds in hiring early, there is a positive probability that a qualitywise-inefficient matching will result, since the applicant hired by the low quality firm may turn out to be of high quality.

The last result of this section finds a sufficient condition under which all low quality firms hire early in the case of *comparable* demand and supply (Case 2). We will use this proposition in our experimental design:

Proposition 1 *In the case of* comparable *demand and supply with $n_A > n_F > n_A^h$ (Case 2), if*

$$0 > n_F^h u_{hh} + \left(n_F - n_F^h - n_A^h\right) u_{h\ell} - \left(n_A - n_A^h\right) u_{\ell\ell} \text{ and } n_F^h \geq n_A^h,$$

then all low quality firms hire in the early hiring stage under any subgame perfect equilibrium, leading to a qualitywise-inefficient matching outcome with positive probability.

2.3 Effects of Changing Labor Supply

A robust result in the literature of static matching models is about the welfare of agents on one side of the market when the number of agents on the other side of the market increases or decreases. Using the firm-optimal stable matching as the solution concept for centralized models of matching markets, Gale and Sotomayor (1985) showed that increasing the number of applicants in the market while keeping the firm preferences unchanged within the old applicant set cannot make any firm worse off (Proposition 2, p. 264).[14] We can state a similar result on our domain, if we confine our attention to the late hiring stage.

Proposition 2 *For a given set of firms that remain unmatched until the late hiring stage, if the number of available applicants increases and the number of available high quality applicants does not decrease, no available firm will be ex-ante worse off under any subgame perfect equilibrium.*

However, if we consider our full two-stage matching game, this conclusion no longer holds. An increase in the number of applicants can move the market conditions from Case 3a, when all equilibria yield late matching, to Case 2, and an unravelled market, and this can harm some firms. (A change in the number of applicants could also simply increase the amount of unraveling within Case 2. Note that Proposition 2 implies that firms can become worse off from an increase in the number of applicants only when this leads to a decrease in the number of high quality applicants available in the late hiring stage, through unraveling.)

Proposition 3 *It is possible that, for a given set of firms, if the number of applicants increases and the number of high quality applicants does not decrease, a firm can be ex-ante worse off at a subgame perfect equilibrium outcome.*

Note also that a decrease in the number of high quality applicants can hurt firms in two ways. First, as in static models (and as in Proposition 2) a reduction in the number of potential high quality partners hurts firms in the usual way. Second, if the decrease in the number of applicants moves the market from Case 1 to Case 2, it may also change the SPE outcome from late matching to one with some unraveling, and this may further hurt the high quality firms by decreasing the amount of assortative matching. (Something like this happened in the market for gastroenterologists in the late 1990's.[15])

[14] The firm-optimal stable matching is a stable matching that makes every firm best off among all stable matchings. It always exists in the general Gale and Shapley (1962) matching model, and in many of its generalizations (see Roth and Sotomayor, 1990). In our model, there is a unique stable matching, which is therefore the firm-optimal stable matching.

[15] See Niederle and Roth (2009) for a survey.

	$n_A = 6$ $n_A^h = 2$	$n_A = 12$ $n_A^h = 4$
$n_F = 4$ $n_F^h = 2$	*THIN COMPARABLE MARKET* *Case 2* (baseline design) SPE Prediction: Low quality firms hire early qualitywise-inefficient outcome	*MARKET WITH EXCESS SUPPLY* *Case 1* SPE Prediction: Late and assortative matching efficient outcome
$n_F = 8$ $n_F^h = 4$	*MARKET WITH EXCESS DEMAND* *Case 3* SPE Prediction: Late and assortative matching efficient outcome	*THICK COMPARABLE MARKET* *Case 2* SPE Prediction: Low quality firms hire early qualitywise-inefficient outcome

Table 1: Treatments

3 Experimental Design

The theoretical analysis of this simple market shows that there are a multitude of Nash-equilibria. However, restricting attention to SPEs provides clear predictions. Early, qualitywise-inefficient matching can occur only when demand and supply are *comparable* in the sense that there is initial uncertainty whether a given applicant will be on the long or short side of the market.

However, behavior doesn't always conform to the predictions of SPE. Consequently we also test the theoretical predictions in laboratory experiments.

Apart from testing the SPE predictions, the experiments also allow us to distinguish these from alternative hypotheses about unraveling. As noted in the introduction, in many markets we have studied, the intuition of market participants is that unraveling results simply from a shortage of applicants.

The experimental markets have two hiring stages each of which consists of 4 periods, so that there is time to make multiple offers at each stage (the market is uncongested). The qualities of applicants and ε values are determined at the beginning of period 5, i.e. the first period of the late hiring stage.

We consider a design with four treatments in which there are 4 or 8 firms, and 6 or 12 applicants (in all possible combinations). In each case, half the firms are of high quality, and a third of the applicants are eventually of high quality. It will be helpful to consider the treatment with the fewest firms and applicants as the baseline treatment, to which others will be compared. We will refer to the baseline treatment as the ***thin comparable*** market treatment, since it satisfies the Case 2 demand

and supply condition, comparable demand and supply. We then change the market conditions by increasing the number of firms and workers. Doubling the number of low and high quality applicants yields the treatment referred to as the market with ***excess supply***, satisfying the Case 1 demand and supply condition. Doubling the number of high and low quality firms, keeping the number of applicants at the baseline level, yields the market with ***excess demand*** (Case 3, in particular Case3a). The largest treatment is obtained by doubling both the number of high and low quality firms and applicants. When the number of firms and workers are both at their highest levels, we are again in the case of *comparable* demand and supply (Case 2), so we refer to this treatment as the ***thick comparable*** market (see Table 1).

In each market an agent earns points when she gets matched to a partner. We set $u_{hh} = 36$ points, $u_{h\ell} = u_{\ell h} = 26$ points, and $u_{\ell\ell} = 20$ points for the preferences, which satisfy that $u_{hh} - u_{hl} > u_{lh} - u_{ll} > 0$. We choose the distribution of ε as follows: When there are 2 agents in a quality type (high quality applicants, high quality firms or low quality firms in various treatments), we set $\{\varepsilon_1, \varepsilon_2\} = \{-1.2, 1.2\}$, and each permutation, i.e. $(\varepsilon_1, \varepsilon_2) = (-1.2, 1.2)$ and $(\varepsilon_1, \varepsilon_2) = (1.2, -1.2)$, is chosen with equal probability. When there are 4 agents in a quality type (high quality applicants, low quality applicants, high quality firms or low quality firms in various treatments), we have $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\} = \{-1.2, -0.4, 0.4, 1.2\}$, and each permutation is chosen with equal probability. When there are 8 agents in a quality type (such as low applicants in the market with *excess supply*), we set $\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8\} = \{-1.4, -1, -0.6, -0.2, 0.2, 0.6, 1, 1.4\}$ and each permutation is chosen with equal probability.

At the beginning of each session, participants were randomly assigned their role as applicant and firm, which they kept throughout the session. Participants who were firms also kept their quality types as high quality or low quality throughout each session. In each session, the market game was played 20 consecutive times by generating new ID numbers for applicants in each market. Subjects received a \$10 participation fee plus their total earnings in the session except in the treatment with *excess supply*, where we also gave an additional \$5 participation fee to the subjects in the roles of applicants at the end of the session. The experimental points were converted to dollars at the exchange rate \$1=20 points.

The experimental sessions were run at the Computer Laboratory for Experimental Research (CLER) of the Harvard Business School using a subject pool consisting of college students (primarily from Harvard, Boston, Tufts, and Northeastern Universities) and at the Pittsburgh Experimental Economics Laboratory (PEEL) of the University of Pittsburgh using a subject pool consisting of college students (primarily from Pittsburgh, Carnegie Mellon, and Duquesne Universities). Each participant could only participate in one session. The experiment was conducted using z-Tree (Fischbacher 2007). We ran 4 sessions of the treatments requiring the most subjects, namely the *excess* *supply* treatment and the *thick comparable* market treatment. For the other treatments we collected 7 sessions. The instructions for the *excess supply* market treatment are given in Online Appendix C. Instructions for the other treatments are similar.

The theoretical properties of the general model apply to the experimental design. Hence, the *thin* and *thick comparable* market treatments each fall under Case 2, the treatment with *excess supply* falls under Case 1, and the treatment with *excess demand* falls under Case 3a of the demand-supply conditions described in the previous section. It is trivial to check that our *comparable* market treatments satisfy the conditions of Proposition 1. Therefore, the SPE predictions for each treatment are given in Table 1 via Theorem 1 and Proposition 1. The SPE predictions are thus that, we should see qualitywise-inefficient early matching in the baseline - *thin comparable* market treatment. As we add either only applicants or only firms (the treatments with *excess supply* and *excess demand*) efficiency should be restored. Restoring the relative balance (in the *thick comparable* market treatment), that is adding both firms and applicants, is predicted to bring back qualitywise-inefficient early matching.

Note that this design allows us to distinguish the SPE predictions from the alternative hypothesis that excess demand for workers (i.e. a shortage of workers) causes unraveling, since in that case there should be the most unraveling in the *excess demand* treatment. The *excess supply* treatment could well be similar according to this hypothesis, despite the asymmetric roles of firms and workers and how their qualities are revealed. The design also allows us to distinguish the SPE predictions from the *congestion hypothesis* that increasing the number of firms or of applicants will by itself facilitate unraveling, since the equilibrium predictions are that, moving from cell to cell in the experiment, an increase in the size of one side of the market will in some cases increase and in other cases decrease unraveling.[16]

4 Experimental Results

4.1 The Analysis of Matches and Unraveling

We first analyze when firms and workers match, and whether the market experiences unraveling. In every market, in all cohorts, the maximal number of matches were formed, that is, it was never the case that both a firm and a worker were unmatched. Hence, we can focus the analysis in this section on how many matches were formed in the early hiring stage, the amount of unraveling.

[16]While this market experiment has been designed with multiple periods so that congestion should not be a big issue, there are markets in which there is insufficient time for the market to clear after the information needed for efficient matching has been realized, i.e. because there are too many offers that need to be made and considered. Congestion can be an incentive for unraveling (cf. Roth and Xing 1997 for an account in the field, and Kagel and Roth 2000 for an experiment focused on this cause of unraveling).

Actual (SPE) **% Firms Hiring Early** In the last five markets - Medians	*THIN COMPARABLE*	*EXCESS SUPPLY*
Low Firms	**100%** (100%)	**25%** (0%)
High Firms	**0%** (0%)	**0%** (0%)
	EXCESS DEMAND	*THICK COMPARABLE*
Low Firms	**0%** (0%)	**87.5%** (100%)
High Firms	**0%** (0%)	**0%** (0%)

Table 2: Median percentage of firms hiring early in the last five markets, with subgame perfect equilibrium predictions in parentheses.

Figure 1 shows over time the number of firms that hire in the early stage, as they gain experience. In Table 2, we report the median of the median percentage of firms that hire early in the last five markets for each treatment.[17,18] Especially in the final five markets, the observed behavior in the experiments are similar to the SPE predictions for risk-neutral subjects outlined in Table 1, although we do not observe that hiring behavior is fully consistent with SPE.

Figure 1 shows that the hiring behavior moves in the direction of the SPE predictions, suggesting that there is learning over time. In the two *comparable* demand and supply treatments, more and more low quality firms hire early, while more and more high quality firms hire late, approaching the SPE predictions in the last five markets.

[17]For the graphs and tables, for each session, for each of the markets, we compute the median of the variable in question, in this case the number of firms that hire early. We then compute the median of each market block, for markets 1-5, 6-10, 11-15, 16-20 for each session, and then the median for each market block taking these session medians as data points, to report the final variable.

For example, in Session 7 of the *thin comparable* market treatment, in markets 1 to 5, 100%, 0%, 50%, 0%, and 0% of the high type firms are hiring early, respectively, which results in a 0% median hiring rate. In Sessions 1 to 6, medians are similarly calculated for markets 1-5 as 0%, 50%, 50%, 0%, 50%, and 50%, respectively. The median of these six sessions and Session 7 is 50%, which is marked in the top graph of Figure 1 for markets 1-5.

[18]In Appendix B, we report alternative analyses using *means* instead of *medians*. The figures and statistical test results are similar. In general, the *mean* statistics are noisier than *medians* due to the fact that the mean takes extreme outcomes into consideration for these samples with relatively small sizes.

In the main text, we use *medians* instead of *means* for two reasons:

First, our statistical test is an ordinal non-parametric median comparison test (i.e. Wilcoxon rank sum test) and not a cardinal parametric mean comparison test. We chose an ordinal test based on the small sample sizes, 7 or 4 for each treatment.

Second, many of the empirical distributions are truncated. I.e., even if the efficiency measure is centered around 100%, there will be no observations above 100% while depending on the variance, we will observe lower efficiency levels. Since we use percentages to compare treatments of different size, the appropriate measure of the center of a distribution seems to be a *median* rather than the *mean* due to the inevitable skewedness of the empirical distributions.

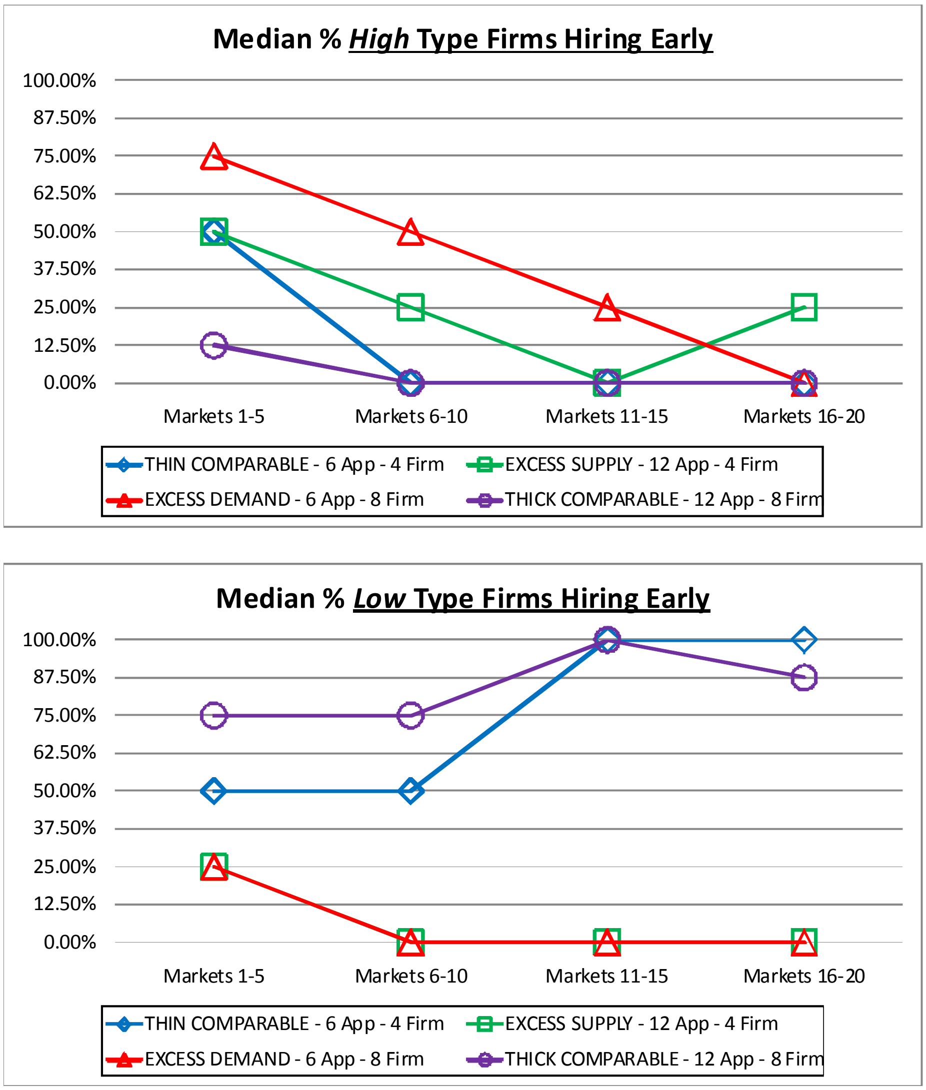

Figure 1: Median percentage of high and low type firms hiring early across treatments.

H_0 (For median % high/low firms hiring early)	sample sizes	p-value: High	p-value: Low
Thin Comparable = Thick Comparable	7,4	1	0.79
Excess Supply = Excess Demand	4,7	0.67	0.76
Thin Comparable = Excess Supply	7,4	0.56	<0.01**
Thin Comparable = Excess Demand	7,7	0.44	<0.01**
Thick Comparable = Excess Supply	4,4	0.43	0.03*
Thick Comparable = Excess Demand	4,7	0.42	<0.01**

Table 3: Testing equivalence of median high firm and low firm early hiring percentages in the last five markets. We denote significance regarding the rejection of the null hypotheses at 95% level with * and significance at 99% level with ** after the reported p-values.

We increase excess demand when moving from the *comparable* to the *excess demand* treatment. The amount of early hiring in the *excess demand* condition refutes the hypothesis that excess demand, i.e. a shortage of applicants generates unraveling. Furthermore, unraveling in these markets is not driven primarily by congestion, since neither the *excess demand* or *excess supply* conditions experience as much early hiring as the *thin comparable* condition, despite having more participants.

In the *excess demand* condition, both high and low quality firms approach the SPE prediction of no early hiring. Using the non-parametric two-sample and two-sided Wilcoxon rank sum test, we test whether the median early hiring levels of the *excess demand* and *comparable* treatment are the same (in the last five markets), taking each session median for the last five markets as an independent observation. The p-values reported in Table 3 confirm that there is significantly more early hiring by low quality firms in the *comparable* compared to the *excess demand* treatments.

Similarly, the *excess supply* treatment also approaches SPE predictions of late hiring by both high and low quality firms (see Figure 1 and Table 3).

As predicted by SPE, we cannot reject the null hypothesis that median early hiring by high quality firms in the last five markets is the same across all treatments under pairwise comparison. On the other hand, the *comparable* treatments have significantly higher proportions of low quality firms hiring early than the *excess demand* and *excess supply* treatments. The *comparable* treatments do not significantly differ from each other. Neither do the *excess demand* and *supply* treatments differ from each other.

4.2 The Analysis of Early Offer and Acceptance Rates

In what follows, we examine not only the outcomes, but also the observed offer making and acceptance behavior. The top two graphs of Figure 2 show the percentage of high and low quality firms making

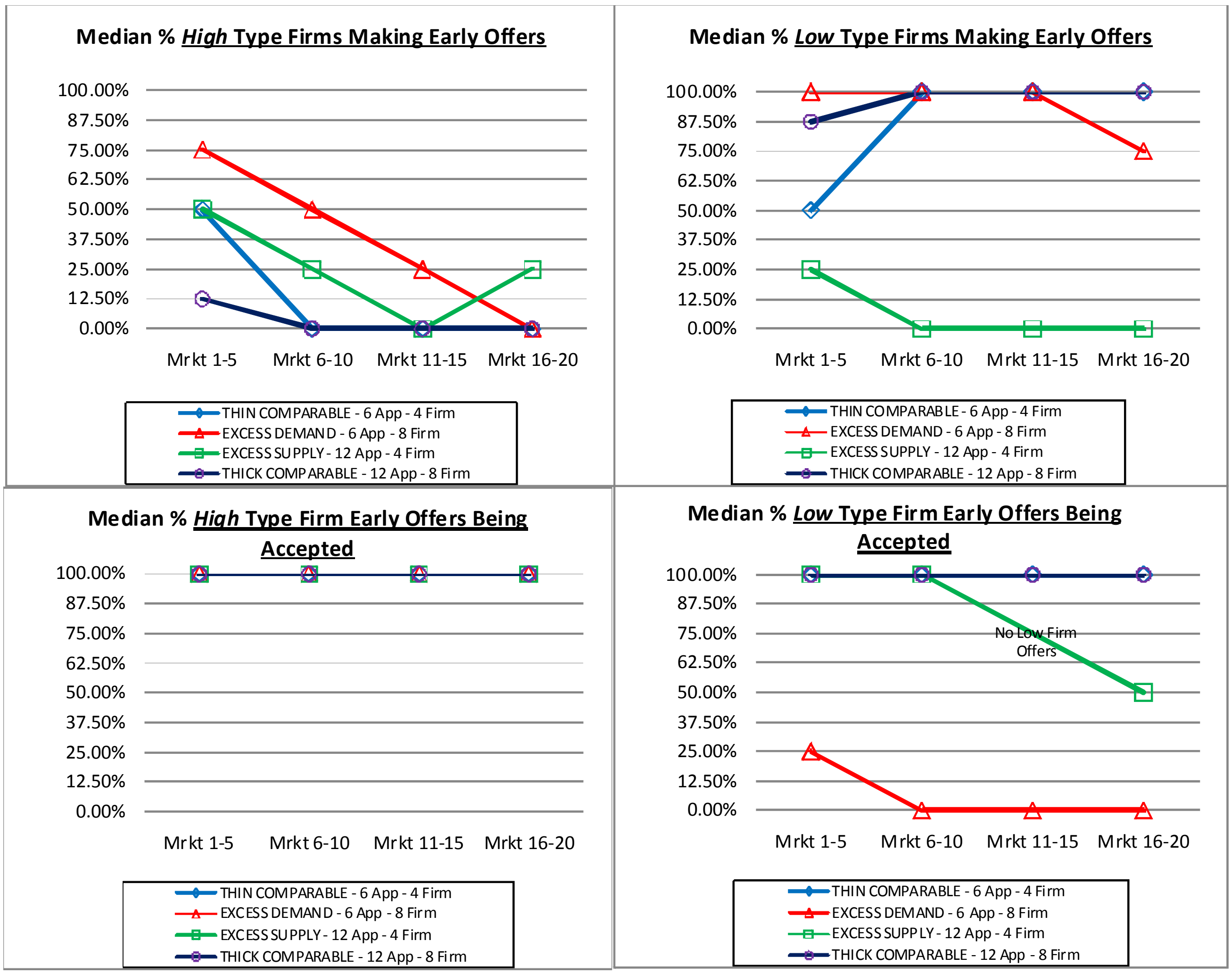

Figure 2: Median early offer rates and acceptance rates across treatments.

offers in the early stage across treatments. As before, we represent markets in groups of five and report the medians of the median rates across these five market groups for each treatment. The first median mentioned is obtained across cohorts of data for each treatment. If a firm makes one or more offers in the early stage, it is counted as a firm that is making an early offer. The bottom two graphs of Figure 2 show the acceptance rates by applicants of the early offers. That is, we determine for each early offer whether it was accepted or rejected. We use the same reporting convention as before with one important difference. When there are no early offers by any firm in a market, the acceptance rate is not defined. In our median calculations, we only consider markets in which offers were actually made. This technique results in well-defined acceptance rates, even though the median offer rate is 0% in many instances. (The only exception is markets 11-15 in the *excess supply* treatment: no low type firms make any offers in any of these markets in any of the cohorts of data, hence we omit that acceptance rate, see the bottom right graph of Figure 2.)

We observe that the number of early offers from high type firms is decreasing and mostly converging to zero in all treatments, and that acceptance rates of high firm offers are 100%. Both of these findings are consistent with the SPE predictions both on and off the equilibrium paths. In the *comparable* demand and supply treatments, these percentages converge to 0% very fast. However, in the *excess supply* and *excess demand* treatments, convergence takes more time, and in the *excess supply* treatment we do not observe full convergence.

For early offers from low type firms and the applicants' acceptance behavior we observe different behavior across different treatments. On and off the SPE path, predictions are such that in the *comparable* demand and supply treatments all low quality firms make early offers and all applicants always accept them, whenever such offers are made. Behavior in the experiment is consistent with the SPE predictions.

In the *excess demand* treatment, applicants should not accept early offers from low quality firms on the SPE paths. Thus, low quality firms are indifferent between making early offers and not making them at all. However, when applicants make mistakes and accept such offers, we expect low quality firms to make early offers (i.e., on a trembling hand-perfect equilibrium path). Indeed in markets 1-5, 25% of applicants accept an early offer from a low quality firm. This reinforcement seems to cause 100% of low quality firms to make early offers throughout the early markets. However, towards the final markets, early offers from low quality firms are never accepted anymore. This seems to reduce the amount of early offers from low quality firms, only 75% of them make early offers.

In the *excess supply* treatment, we expect low quality firms not to make early offers under SPE. Yet, off the SPE paths, when they make early offers, we expect the applicants to accept these offers. Initially, this is exactly what happens. Low quality firms almost never make early offers. However, when they make mistakes and make early offers (which is a more frequent mistake in markets 1-10, never happens in market 11-15, and only rarely in markets 16-20), applicants tend to accept those offers (as seen in the bottom right graph of Figure 2). Only in the final markets, markets 16-20, when early offers are very rare, are they accepted only 50% of the time (though this is a very small number phenomenon).

In summary, in all treatments the behavior of the firms and applicants are mostly consistent not only with SPE predictions, but also with trembling-hand perfection predictions.

4.3 The Analysis of Efficiency

We conclude our analysis of the experimental results with qualitywise efficiency (i.e. ignoring ε's). A **full matching** is an assignment under which the maximum number of firms and applicants are matched. Since for both firms and workers, matching to a high quality partner is worth more than to a low quality partner, we use the welfare of firms to provide a measure of total welfare.[19] We aim to measure the proportion of gains achieved compared to the payoffs achieved by a random match that matches as many agents as possible, that is the payoffs achieved by a randomly generated full matching, where each possible full matching is equally likely. We call the sum of these expected firm payoffs the random match payoffs. **Normalized (qualitywise) efficiency** is defined as the sum of firm payoffs (disregarding ε values) minus the random match payoffs divided by the maximum sum of possible expected firm payoffs minus the random match payoffs (see Table 4). A value of 100% indicates that the matching in the lab achieved all possible gains compared to the average random full matching and is hence the matching that maximizes firm payoffs.[20] The random match payoffs and maximum sums of possible expected firm payoffs of a full matching in each treatment are given in Table 4 along with the median of the sum of actual firm payoffs (disregarding ε values) and the normalized efficiency of the last five markets. We also report in the same table the predictions of SPE outcomes in terms of efficiency.

First note that both *comparable* demand and supply treatments lack full qualitywise efficiency. The median SPE outcomes predict 57% and 79% efficiency for the thin and thick *comparable* treatment. The outcomes are exactly aligned with the theoretical predictions.[21] This reflects the earlier

[19]We could also have chosen the welfare of applicants or the sum of applicant and firm payoffs in our efficiency measure. Note that all of these measures will give the same efficiency level, since a firm and a worker who are matched receive exactly the same payoff.

[20]In each treatment, the best full matching is the qualitywise assortative matching, that is, as many high quality firms as possible are matched with high quality applicants, and as many high quality applicants as possible are matched before matching low quality applicants.

[21]The difference in the median of the *comparable* treatments is due to the small market size of the *thin comparable* treatment. The mean of both SPE predictions generates 71.38% efficiency. Under a typical matching (i.e. a median SPE matching), in the *thick comparable* treatment, one low quality firm hires a high quality applicant and the other

In the last five markets - Medians	*THIN COMPARABLE*	*EXCESS SUPPLY*
Actual Firm Welfare [Random, Max.]	**108** [102.67, 112]	**124** [102.67, 124]
Actual (SPE) **N. Efficiency**=$\frac{\textbf{W. - Random W.}}{\textbf{Max. W.- Random W.}}$	**57%** (57%)	**100%** (100%)
	EXCESS DEMAND	*THICK COMPARABLE*
Actual Firm Welfare [Random, Max.]	**164** [154, 164]	**220** [205.34, 224]
Actual (SPE) **N. Efficiency**=$\frac{\textbf{W. - Random W.}}{\textbf{Max. W. - Random W.}}$	**100%** (100%)	**79%** (79%)

Table 4: Median of the sum of firm payoffs in the last five markets, with random match payoffs and maximum firm payoffs under full matchings in parentheses, and normalized efficiency with the theoretical SPE prediction in parentheses.

$\mathbf{H_0}$ **(For median n. efficiency)**	**sample sizes**	**p-value**
Thin Comparable = Thick Comparable	7,4	0.12
Excess Supply=Excess Demand	4,7	0.73
Thin Comparable=Excess Supply	7,4	0.33
Thin Comparable=Excess Demand	7,7	0.021*
Thick Comparable=Excess Supply	4,4	0.49
Thick Comparable=Excess Demand	4,7	0.048*

Table 5: Testing equality of median normalized efficiency in the last five markets.

results that both early offers and acceptances, and hence early matches follow SPE predictions (low quality firms hire early, while high quality firms hire late).

When we inspected the data, we found that there is less overall early hiring in the *excess demand* treatment compared to the *comparable* demand and supply treatments. Furthermore, there is also a significantly higher proportion of efficiency realized, compared to both the thin and thick *comparable* treatments (the two of which are not significantly different from one another). See Table 5 for p-values of the Wilcoxon rank sum tests of median efficiency.[22]

In the *excess supply* treatment, we found that, in the last five markets, a median of 25% of high

three low quality firms hire low quality applicants through unraveling, where as in the *thin comparable* treatment, one low quality firm hires a high quality applicant and the other low quality firm hires a low quality applicant through unraveling.

[22] In our analysis with the means for the last market block reported in Appendix B, we observe a mean of 20-21% of high type firms making early offers which are accepted in the *excess demand* treatment. SPE predicts that no high quality firms hire early. However, in the *excess demand* condition, there are only 2 high quality applicants for 4 high quality firms. So, even if 50% of high quality firms were to hire early, efficiency would not be affected given that the other agents follow SPE strategies. And indeed, all seven of our *excess demand* sessions have close to average 100% efficiency despite of relatively high average of high type firms hiring early.

type firms hire early, while low type firms do not hire early. SPE predicts that no high quality firms hire early. In the *excess supply* treatment, there are four high quality applicants for four firms, including the two high quality firms. A high quality firm that hires early, will however match with a high type applicant only one third of the time. That is two thirds of the time there will be an efficiency loss of 10 points (when the high type firm only receives 26 points from hiring a low type applicant compared to the 36 from hiring a high type applicant). Since only once in every other market does a high type firm hire early, we expect about one third of all sessions to yield median efficiency below 100%. And indeed, the median efficiencies are 100, 100, 100, and 53.11% (which results from one high type firm hiring a low type worker, and all other firms hiring high type workers). Hence while the median efficiency of the excess supply treatment is 100%, there is some variance, such that the efficiency is not significantly higher in the *excess supply* treatment compared to the *comparable demand and supply* treatments (see Table 5).

5 Discussion

It has been known at least since Roth and Xing (1994) that many markets unravel, so that offers become progressively earlier as participants seek to make strategic use of the timing of transactions. It is clear that unraveling can have many causes, because markets are highly multidimensional and time is only one dimensional (and so transactions can only move in two directions in time, earlier or later). So there can be many different reasons that make it advantageous to make transactions earlier.[23]

Thus the study of factors that promote unraveling is a large one, and a number of distinct causes have been identified in different markets or in theory, including instability of late outcomes (which gives blocking pairs an incentive to identify each other early), congestion of late markets (which makes it difficult to make transactions if they are left until too late), and the desire to mutually insure against late-resolving uncertainty. There has also been some study of market practices that may facilitate or impede the making of early offers, such as the rules and customs surrounding "exploding" offers, which expire if not accepted immediately.

In this paper we take a somewhat different tack, and consider conditions related to supply and demand that will tend to work against unraveling, or to facilitate it. There seems to be a widespread perception, in markets that have experienced it, that unraveling is sparked by a shortage of workers.

But for inefficient unraveling to occur, firms have to be willing to make early offers *and* workers have to be willing to accept them. Our experiment supports the hypothesis that a shortage of

[23]There can also be strategic reasons to *delay* transactions; see e.g. Roth and Ockenfels (2002) on late bidding in internet auctions.

workers is not itself conducive to unraveling, since workers who know that they are in short supply need not hurry to accept offers by lower quality firms. Instead, in the model and in the experiment, it is *comparable* supply and demand that leads to unraveling, in which attention must be paid not only to the overall demand and supply, but to the supply and demand of workers and firms of the highest quality. An important feature of our model and experiments is that when there is inefficient unraveling, this is due to low type firms, but not high types, hiring early.

We emphasize again that the qualities of workers and firms in our model should not be taken literally as "low" and "high" while mapping our predictions to real markets. Being hired in our model can refer to being hired by one of the elite firms in a real market, some better than the others, and being unmatched can refer to being hired by one of the ordinary firms in a secondary market. For example, clerkships for federal appellate judges are elite positions for law school graduates. Yet, there is unraveling in this market, and it seems to start in the 9th Circuit, whose judges are a little disadvantaged compared to the East Coast circuits.

Our results seem to reflect what we see in many unraveled markets, in which competition for the elite firms and workers is fierce, but the quality of workers may not be reliably revealed until after a good deal of hiring has already been completed.

A Appendix: Proofs

Proof of Lemma 1: First recall that there is a unique strict rank-ordering of firms and applicants with respect to preference with probability one, when all uncertainty is resolved at the beginning of the late hiring stage.

We prove the lemma by backward induction.

- We first show that in the last period T^L of the late hiring stage, every SPE involves assortative matching among the remaining firms and applicants:
  - In every subgame starting with applicants' information sets in period T^L of the late hiring stage, it is a dominant strategy for the applicant to accept the best incoming offer, since otherwise she will either remain unmatched or be matched with a worse firm.
  - The best remaining unmatched firm, by making an offer to the best applicant, will be accepted by the applicant, and receive the highest possible payoff. The second best remaining firm, will be rejected if she makes an offer to the best applicant, since the applicant will get an offer from the best firm. The highest applicant who will accept the second best firms' offer is the second best applicant. Similarly, the k^{th} best remaining unmatched firm maximizes her payoff from making an offer to the k^{th} highest remaining unmatched applicant.

  We showed that the outcome of any SPE will involve assortative matching of agents who are available in any last period subgame of the late hiring stage.

- Let us assume that we showed for a period $t + 1 < T^L$ that SPE strategies for any subgame starting in period $t+1$ involve assortative matching among the remaining unmatched firms and applicants. We now show that this implies that for any subgame starting in period t the SPE involve assortative matching among the unmatched applicants in period t. Let us relabel the remaining firms and applicants, such that the remaining firms are $f^1, f^2, ..., f^m$ and available applicants $a^1, a^2, ..., a^n$ such that f^k is better than f^{k+1} for any $k < m$ and a^k is better than a^{k+1} for any $k < n$.
  - We show that at any SPE in period t involves applicant a^1 not to accept an offer from firm f^j for any $j > 1$ if firm 1 did not make an offer to any applicant. By rejecting firm f^j, applicant a^1 will be the highest quality remaining applicant in period $t + 1$, and f^1 will be the highest quality unmatched firm. That is, by the inductive assumption, a^1 can expect to be matched to firm f^1 in the SPE.

- We show that at any SPE in period t, firm f^1, either makes no offer, or an offer to applicant a^1 in period t. This guarantees that f^1 is either accepted by applicant a^1 in period t, or else both $f^1$and applicant a^1 are unmatched in period t, in which case firm f^1 will be matched to applicant a^1 before the end of the game by the inductive assumption.
- We show that at any SPE in period t applicant a^2 does not accept an offer from f^j, $j > 2$ if firms f^1 and f^2 both did not make an offer to an applicant a^l for any $l > 2$. As in the case of applicant a^1; by rejecting firm f^j, a^2 can expect to be matched to either f^1 or f^2 in period $t + 1$, since at least one of the two firms will be unmatched.
- We now show that any SPE in period t involves firm f^2, not to make an offer to an applicant a^j, $j > 2$.

We can follow this line of iterative argument to show that any SPE strategies in any subgame starting in period t involve no matches that are not assortative in period t. Therefore, by the inductive assumption, we have that any SPE starting in period t involves assortative matching. ■

Proof of Lemma 2: We prove the lemma for Case 1 and Case 3a separately.

- $n_A^h \geq n_F$ (Case 1): We already established by Lemma 1 that once participants are in the late hiring stage, the unique SPE outcome is assortative matching among the remaining firms and applicants. When $n_A^h \geq n_F$, by not hiring any applicant in the early hiring stage each firm guarantees to hire a high quality applicant in the late hiring stage under a SPE. For a firm f of quality $i \in \{h, \ell\}$, her expected payoff of hiring an applicant in the early stage is given by
$$\frac{n_A^h}{n_A} u_{ih} + \frac{n_A - n_A^h}{n_A} u_{i\ell}$$
which is strictly smaller than her expected payoff of hiring a high quality applicant in the late stage, u_{ih}. Therefore, under any SPE no firm will make any early hiring, and thus, by Lemma 1 the outcome will be assortative.

- $n_F \geq n_A > n_F^h$ (Case 3a)**:** By Lemma 1, once participants are in the late hiring stage, the unique SPE outcome is assortative matching among the remaining firms and applicants. Since $n_F \geq n_A$, under a SPE, every applicant will at least be matched with a low quality firm by waiting for the late hiring stage. We will show that no matches will occur in the early hiring stage under a SPE by backward iteration.

  First consider the last period (period T^E) of the early hiring stage. We will show that no high quality firm is matched under a SPE in this period as long as more applicants than high quality firms are available. We prove this with two claims. Consider an information set $\mathcal{I}$ of applicants located in this period.

Claim 1 Under a SPE, no available applicant will accept a low quality firm's offer in $\mathcal{I}$ if there is a high quality firm who did not make an offer in period T^E.

Proof of Claim 1 Consider a SPE profile and an applicant a available in $\mathcal{I}$. Suppose that there is at least one high quality firm that did not make an offer in T^E. Then the applicant has a chance to be of high quality and be matched to a high quality firm (of which at least one is available in the late hiring stage), if she is of low quality, she will receive a low quality firm by Lemma 1. Hence her expected payoff from waiting is strictly larger than her expected payoff from accepting a low quality firm offer in period T^E. □

Consider a subgame Γ starting in the last period (period T^E) of the early stage.

Claim 2 Under a SPE, no high quality firm makes an offer to an applicant, unless the number of remaining applicants is equal to the number of remaining high quality firms (in which case we do not determine the strategies fully).

Proof of Claim 2 Suppose there are l high quality firms left, and $k > l$ applicants. If a high quality firm f^h makes an offer to an applicant, it will hire her and this will be an average quality applicant. If firm f^h makes no offer this period, then we have seen that no applicant will accept an offer from a low quality firm. Suppose r_H high quality firms are left after the end of early stage, including the high quality firm f^h. This implies that there are $r_A = k - (l - r_H) > r_H$ applicants unmatched at the end of period T^E. Since firm f^h has equal chance to be ranked in any place amongst the remaining r_H high quality firms, and since by Lemma 1, any SPE matching in the late hiring stage is assortative, firm f^h, by not matching early, will match to one of the best r_H applicants in the remaining r_A, and has an equal chance to match with any of them. The average applicant's quality amongst the r_H $(< r_A)$ best applicants of all r_A is strictly better than the unconditional average applicant quality. Therefore, f^h, by not making an offer, is matched with an applicant with an expected quality higher than the average, receiving higher expected earnings in an SPE than by making an early offer. □

We showed that in the last period of the early hiring stage no matches will occur, if the number of high quality firms available is smaller than the number of applicants available under a SPE profile. By iteration, we can similarly prove that in period $T^E - 1$ of the early hiring stage no matches will occur, if the number of high quality firms available is smaller than the number of applicants available under a SPE profile. By backward iteration, we conclude the proof of no matches will occur in the early hiring stage under a SPE. Therefore, all hirings occur in the late hiring stage and these hirings are assortative under any SPE by Lemma 1. ■

Proof of Lemma 3: Let $n_F^h \geq n_A$. By Lemma 1, every applicant guarantees to be matched with a high quality applicant by waiting for the late hiring stage under a SPE. Therefore, no applicant will accept an offer from a low quality firm in the path of a SPE. Therefore, all applicants will be matched with high quality firms in every SPE. Every such matching is qualitywise-efficient. ■

Proof of Theorem 2: Let the market be of *comparable* demand and supply with $n_A > n_F > n_A^h$. Let σ be a SPE profile. We will prove the theorem using three claims:

Claim 1 Under σ restricted to any subgame, no high quality firm is matched in the early hiring stage.

Proof of Claim 1 If a firm matches early, she receives average quality, i.e. the expected quality she receives is $\frac{n_A^h}{n_A}h + \frac{n_A - n_A^h}{n_A}l$. If the firm does not match early, and if there are r_F firms left, then there will be $r_A > r_F$ applicants left (since $n_A > n_F$), and the firm, instead of receiving the average quality of r_A receives the average quality of r_F, which is strictly better. □

Claim 2 Let $\mathcal{I}$ be an information set in period t of the early stage for an applicant a such that if applicant a does not accept any offers in $\mathcal{I}$, she does not receive any more offers in the remainder of the early hiring stage under σ. Suppose no other applicants accepted any offers until $\mathcal{I}$ under σ. Then applicant a accepts the best offer she receives in all such $\mathcal{I}$ under all SPE if and only if

$$0 > \begin{cases} n_A^h u_{hh} - n_A^h u_{h\ell} + (n_F^h - n_A^h) u_{\ell h} - \left((n_A - n_A^h) - (n_F - n_F^h) \right) u_{\ell\ell} & \text{if } n_F^h \geq n_A^h \\ n_F^h u_{hh} - n_F^h u_{h\ell} - (n_A - n_F)\, u_{\ell\ell} & \text{otherwise} \end{cases} \quad (1)$$

Proof of Claim 2 An applicant a strictly prefers to accept an offer from a low quality firm in a period in the early stage compared to only matching in the late stage, in case no other applicant has accepted and will accept an offer from a low quality firm in such $\mathcal{I}$ under all SPE if and only if

$$\begin{array}{lr} \frac{n_A^h}{n_A} u_{hl} + \frac{n_A - n_A^h}{n_A} u_{ll} > \frac{n_A^h}{n_A} u_{hh} + \frac{n_A - n_A^h}{n_A} \frac{n_F^h - n_A^h}{n_A - n_A^h} u_{lh} + \frac{n_A - n_A^h}{n_A} \frac{n_F - n_F^h}{n_A - n_A^h} u_{ll} & \text{if } n_F^h \geq n_A^h \text{, and} \\ \frac{n_A^h}{n_A} u_{hl} + \frac{n_A - n_A^h}{n_A} u_{ll} > \frac{n_A^h}{n_A} \frac{n_F^h}{n_A^h} u_{hh} + \frac{n_A^h}{n_A} \frac{n_A^h - n_F^h}{n_A^h} u_{hl} + \frac{n_A - n_A^h}{n_A} \frac{n_F - n_A^h}{n_A - n_A^h} u_{ll} & \text{if } n_F^h < n_A^h \end{array} . \quad (2)$$

In both cases, when $n_F^h \geq n_A^h$ or $n_F^h < n_A^h$, the expected payoff on the left hand side is the expected payoff of accepting a low quality firm offer. In each case, the expected payoff on the right hand side of Inequality 2 can be interpreted as follows: When $n_F^h \geq n_A^h$, by Lemma 1, the applicant will be matched with a high quality firm, if she is of high quality, or one of the best $n_F^h - n_A^h$ low quality applicants, and she will be matched with a low quality firm, if she is one the next $n_F - n_F^h$ best low quality applicants. When $n_F^h < n_A^h$, the applicant will be matched with a high quality firm, if she is one the best n_F^h high quality applicants, and she will be matched with a low quality applicant, if she is one of worst $n_A^h - n_F^h$ high quality applicants or one of the best $n_F - n_A^h$ low quality applicants. Inequality 2 can be rewritten as

$$\begin{array}{ll} \frac{n_A^h}{n_A}u_{hl} + \frac{n_A - n_A^h}{n_A}u_{ll} > \frac{n_A^h}{n_A}u_{hh} + \frac{n_F^h - n_A^h}{n_A}u_{lh} + \frac{n_F - n_F^h}{n_A}u_{ll} & \text{if } n_F^h \geq n_A^h \text{, and} \\ \frac{n_A^h}{n_A}u_{hl} + \frac{n_A - n_A^h}{n_A}u_{ll} > \frac{n_F^h}{n_A}u_{hh} + \frac{n_A^h - n_F^h}{n_A}u_{hl} + \frac{n_F - n_A^h}{n_A}u_{ll} & \text{if } n_F^h < n_A^h \end{array},$$

which is equivalent to Condition 1. □

Claim 3[24] All low quality firms make *early* offers to applicants under σ given that their last early offer will be accepted if and only if

$$\frac{n_A^h}{n_A} > \frac{n_A^h - n_F^h}{n_F - n_F^h}.$$

Proof of Claim 3 Suppose $k \geqslant 0$ low quality firms made early offers that were accepted under σ. Then a low quality firm strictly prefers to make an early offer (in case it is accepted) if and only if the expected probability of hiring a high quality applicant early is higher than the expected probability of hiring a high quality applicant in the late hiring stage, which is equal to the expected number of high quality applicants that remain after the high quality firms hired high quality applicants, divided by the number of remaining low quality firms, that is, if and only if

$$\frac{n_A^h}{n_A} > \underbrace{\frac{n_A^h - n_F^h - \frac{n_A^h}{n_A}k}{n_F - n_F^h - k}}_{=p(k)}$$

Therefore, when $k = 0$, one low quality firm will always make an offer (if that will be accepted in the early stage) under SPE σ in the early stage if and only if

$$\frac{n_A^h}{n_A} > \frac{n_A^h - n_F^h}{n_F - n_F^h}. \tag{3}$$

Moreover $p(k)$ is decreasing in k, implying that all low quality firms will make offers in the early stage if they will be accepted under SPE σ if and only if Condition 3 holds. □

Claims 1-3 complete the proof of the theorem. ■

Proof of Proposition 1: Let the market be of *comparable* demand and supply with $n_A > n_F > n_A^h$. Let σ be a SPE profile. By Claim 1 of Theorem 2, no high quality firm will hire early under σ. Since

[24] This claim is more strong than what we need for the proof of this Theorem. However, we will make use of this claim in the proof of Proposition 1.

$n_F^h \geq n_A^h$, no low quality firm will go late under σ as long as their offers are accepted early by Claim 3 of Theorem 2.

We will show that any applicant is ready to accept offers from low quality firms in the early hiring stage under σ if $0 > n_F^h u_{hh} + \left(n_F - n_F^h - n_A^h\right) u_{h\ell} - \left(n_A - n_A^h\right) u_{\ell\ell}$.

Claim 1 Let

$$0 > n_F^h u_{hh} + \left(n_F - n_F^h - n_A^h\right) u_{h\ell} - \left(n_A - n_A^h\right) u_{\ell\ell}.$$

Also let $\mathcal{I}$ be an information set in period t of the early stage for an applicant a such that if applicant a does not accept any offers in $\mathcal{I}$, she does not receive any more offers in the remainder of the early hiring stage under SPE σ. Then applicant a accepts the best offer she receives in $\mathcal{I}$ under σ.

Proof of Claim 1 Applicant a will be better off by accepting the best offer if it is from a high quality firm, because she may remain unmatched with positive probability when she remains available in the late hiring stage given that $n_A > n_F$ by Lemma 1. Suppose that the best offer is from a low quality firm in period t and suppose that she rejects all offers in $\mathcal{I}$ under σ. Let Γ be the subgame such that information set $\mathcal{I}$ is located at the beginning of Γ. Let r_F^h be the number of high quality firms available, r_F^ℓ be the number of low quality firms available, and r_A be the number of applicants available in the late hiring stage under σ restricted to Γ. By Lemma 1, applicant a will be matched with a high quality firm if she turns out to be one of the best r_F^h applicants, she will be matched with a low quality firm if she turns out to be one of the $r_F^\ell + r_F^h$ applicants who are not among the best r_F^h applicants. Under σ restricted to Γ, applicant a's expected payoff is

$$v^\sigma = \frac{r_F^h}{r_A} w_h + \frac{r_F^\ell}{r_A} w_\ell$$

where w_h is the expected payoff a gets by being matched with a high quality firm and w_ℓ is the expected payoff she gets by being matched with a low quality firm. The lowest upper-bound for w_h is u_{hh}, and the lowest upper-bound for w_ℓ is $u_{h\ell}$. Thus, v^σ is bounded above by

$$\bar{v}^\sigma = \frac{r_F^h}{r_A} u_{hh} + \frac{r_F^\ell}{r_A} u_{h\ell}$$

Since the number of applicants available, r_A, satisfies $r_A = n_A - \left(n_F - r_F^h - r_F^\ell\right)$, we have

$$\bar{v}^\sigma = \frac{r_F^h u_{hh} + r_F^\ell u_{h\ell}}{n_A - \left(n_F - r_F^h - r_F^\ell\right)}$$

Upper-bound $\bar{v}^\sigma$ is an increasing function of both r_F^h and r_F^ℓ. Therefore, $\bar{v}^\sigma$ achieves its highest value when $r_F^h = n_F^h$, and $r_F^\ell = n - n_F^h$ and this value is given by

$$\tilde{v}^\sigma = \frac{n_F^h u_{hh} + \left(n_F - n_F^h\right) u_{h\ell}}{n_A}$$

Consider a deviation in applicant a's strategy such that she accepts the best offer in $\mathcal{I}$ and her actions in other information sets coincide with those under σ. The expected payoff of applicant a under this deviation restricted to subgame Γ is given by

$$v = \frac{n_A^h u_{h\ell} + \left(n_A - n_A^h\right) u_{\ell\ell}}{n_A}$$

Since $0 > n_F^h u_{hh} + \left(n_F - n_F^h - n_A^h\right) u_{h\ell} - \left(n_A - n_A^h\right) u_{\ell\ell}$, rearranging the terms and dividing them by n_A, we obtain that

$$v = \frac{n_A^h u_{h\ell} + \left(n_A - n_A^h\right) u_{\ell\ell}}{n_A} > \frac{n_F^h u_{hh} + \left(n_F - n_F^h\right) u_{h\ell}}{n_A} = \tilde{v}^\sigma \geq \bar{v}^\sigma \geq v^\sigma,$$

contradicting σ being a SPE profile. Therefore, applicant a accepts one of the offers from low quality firms in information set $\mathcal{I}$ under SPE σ. □

This concludes the proof of Proposition 1. ■

Proof of Proposition 2: By Lemma 1, there is a unique SPE outcome that is assortative among available firms and applicants in the late hiring stage. When there are more available applicants and not less high quality applicants, each firm's partner quality weakly increases. Hence, each firm's expected payoff weakly increases. ■

Proof of Proposition 3: We prove the proposition with an example. Let $n_F = 4$, $n_F^h = 2$, $n_A = 4$, $n_A^h = 2$, $u_{hh} = 36$, $u_{h\ell} = u_{\ell h} = 26$, $u_{\ell\ell} = 20$. Since $n_F = n_A > n_F^h$, this market satisfies demand and supply condition Case 3a, and therefore by Theorem 1, all SPE outcomes are assortative. So each high quality firm's expected payoff is u_{hh}. Suppose that the number of applicants increases to $n'_A = 6$ and number of high quality applicants does not change. The new market is the *thin comparable* market treatment in our experiment, i.e. a market of *comparable* demand and supply (Case 2). By Proposition 2 since $n_F^h = n_A^h$, and $n_F^h u_{hh} - n_F^h u_{h\ell} - (n_A - n_F) u_{\ell\ell} = -20 < 0$, all low quality firms hire early under a SPE, causing that a high quality firm gets matched with a low quality applicant with positive probability. This expected payoff of a high quality firm is lower than u_{hh} in the new market under any SPE. ■

B Appendix: Alternative Analyses with the *Means*

In this appendix, we present our alternative analyses of the experimental data using the *mean* statistic instead of the ones using the *median* statistic which were presented in Figure 1, Table 2, Table 3, Figure 2, Table 4, and Table 5 of the main text, respectively. These results are consistent with the analyses using medians.

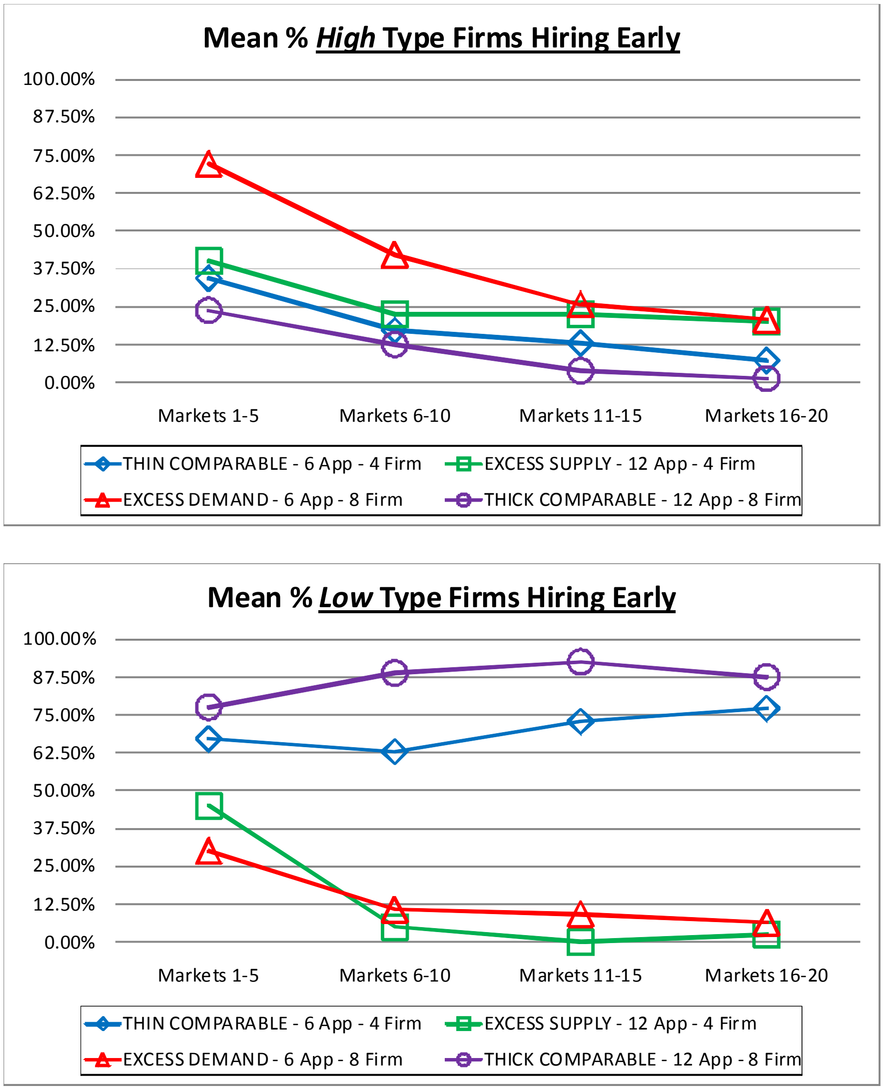

Figure 3: Mean percentage of high and low type firms hiring early across treatments.

Actual (SPE) **% Firms Hiring Early** In the last five markets - Means	*THIN COMPARABLE*	*EXCESS SUPPLY*
Low Firms	**77.14%** (100%)	**2.5%** (0%)
High Firms	**7.14%** (0%)	**20%** (0%)
	EXCESS DEMAND	*THICK COMPARABLE*
Low Firms	**6.43%** (0%)	**87.5%** (100%)
High Firms	**20.71%** (0%)	**1.25%** (0%)

Table 6: Mean percentage of firms hiring early in the last five markets, with subgame perfect equilibrium predictions in parentheses.

H_0 (For mean % high/low firms hiring early)	**sample sizes**	**p-value: High**	**p-value: Low**
Thin Comparable = Thick Comparable	7,4	1	0.33
Excess Supply = Excess Demand	4,7	0.86	0.64
Thin Comparable = Excess Supply	7,4	0.56	<0.01**
Thin Comparable = Excess Demand	7,7	0.085	<0.01**
Thick Comparable = Excess Supply	4,4	0.43	0.029*
Thick Comparable = Excess Demand	4,7	0.15	<0.01**

Table 7: Testing equivalence of mean high firm and low firm early hiring percentages in the last five markets.

In the last five markets - Means	*THIN COMPARABLE*	*EXCESS SUPPLY*
Actual Firm Welfare [Random, Max.]	**108** [102.67, 112]	**124** [102.67, 124]
Actual (SPE) **N. Efficiency** $=\frac{\text{W. - Random W.}}{\text{Max. W. - Random W.}}$	**71.21%** (71.38%)	**86.87%** (100%)
	EXCESS DEMAND	*THICK COMPARABLE*
Actual Firm Welfare [Random, Max.]	**164** [154, 164]	**220** [205.34, 224]
Actual (SPE) **N. Efficiency**$=\frac{\text{W. - Random W.}}{\text{Max. W. - Random W.}}$	**96.57%** (100%)	**77.49%** (71.38%)

Table 8: Mean of the sum of firm payoffs in the last five markets, with random match payoffs and maximum firm payoffs under full matchings in parentheses, and normalized efficiency with the theoretical SPE prediction in parentheses.

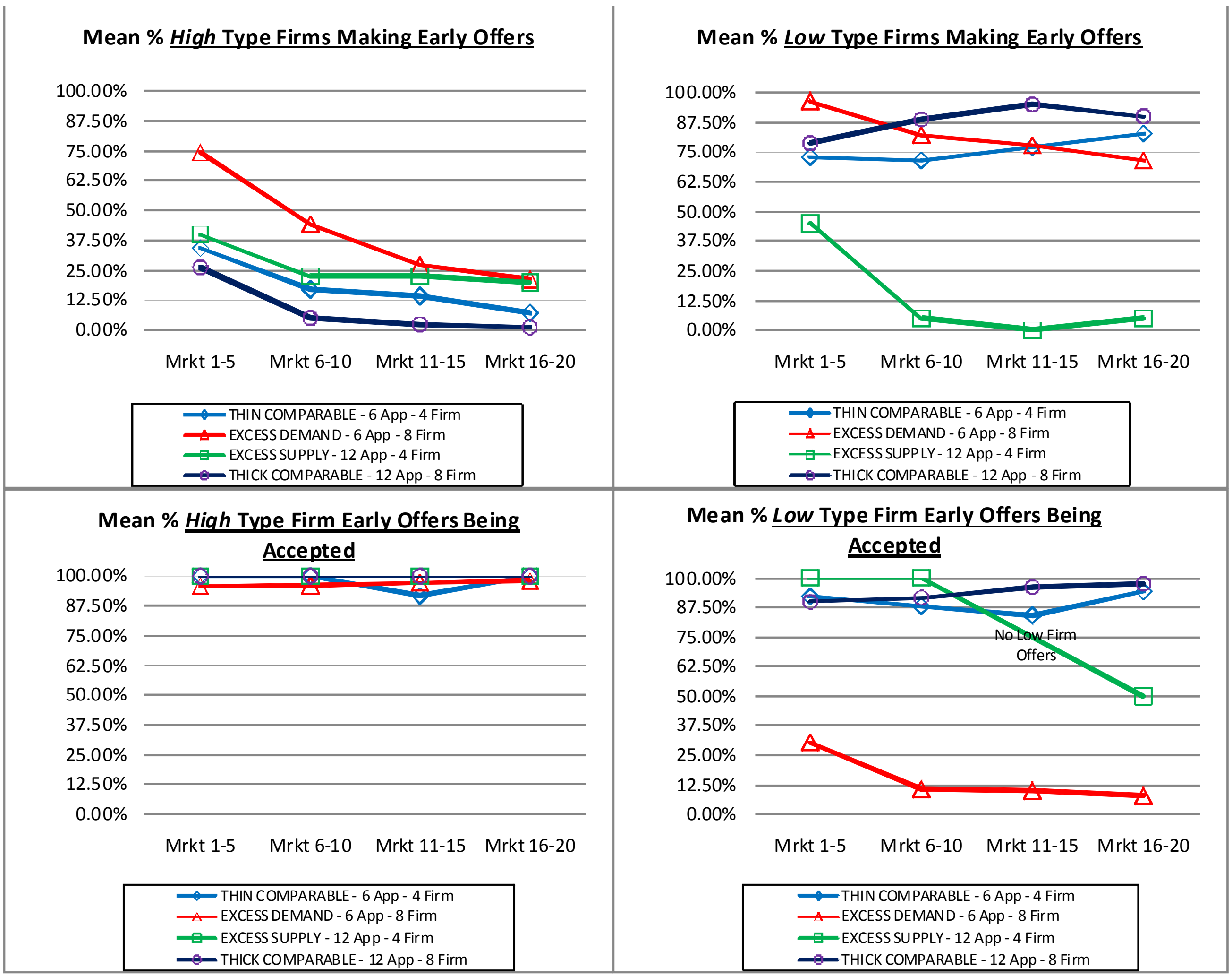

Figure 4: Mean early offer rates and acceptance rates across treatments.

H_0 (For mean n. efficiency)	sample sizes	p-value
Thin Comparable = Thick Comparable	7,4	0.65
Excess Supply=Excess Demand	4,7	0.94
Thin Comparable=Excess Supply	7,4	0.28
Thin Comparable=Excess Demand	7,7	0.014*
Thick Comparable=Excess Supply	4,4	0.31
Thick Comparable=Excess Demand	4,7	0.012*

Table 9: Testing equality of mean normalized efficiency in the last five markets.

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