Efficiency in the Market for Airline Services

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I. Introduction

Douglas and Miller\(^1\) and DeVany\(^2\) have recently developed a useful model of the market for airline passenger services and used it to draw social efficiency conclusions. Unfortunately, there are some serious problems with the conclusions even in the strict context of the model, and the regulator's game plan is neither fully nor correctly specified. This paper will develop a full and transparent treatment of social efficiency and regulation in the DeVany model, show that even purely competitive behavior produces a second best, uncover errors in DeVany's treatment of efficiency, and show that relaxing the assumption that all passengers have equal marginal value of time produces more complex though still manageable criteria for efficiency. An important restriction noted by Spence\(^3\) is that whatever the industry structure, it is assumed to exhibit naive Cournot-type behavior with respect to the regulator. While the focus here is on DeVany's airline model, slight modifications make the analysis applicable to any transportation mode.

Section II develops a full social optimum treatment for the "one consumer" case. A notable result is that provision of flights is a quasi-public good, so that ticket prices should reflect marginal

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passenger cost only, with flights being subsidized out of general revenues, like highway construction. Interestingly, the marginal efficiency condition for flights (marginal value = marginal cost) is the same in the full optimum and in the second-best optimum of Section III, where fares must cover average cost and the industry is competitive. With the industry oligopolistic the marginal condition differs, including the cooperative limit of oligopoly: monopoly. Unregulated (fare-and flight-maximizing) monopoly once again has the same marginal efficiency conditions on flights as the full social optimum. This is covered in Section IV. Section V relaxes the assumption that all passengers have the same marginal value of time and develops modified efficiency conditions. Section VI uses the conclusions drawn to critique DeVany's findings on efficiency based on his empirical work.

II. The Full Social Optimum

In this section we shall set out a full social optimum analysis in a model consistent with DeVany and Douglas-Miller. Several important assumptions are that trips create utility directly\(^1\), and that all sorts of time are identically treated\(^2\). The notation throughout follows DeVany's where possible. With DeVany and Douglas-Miller,

\(^{1}\) In business markets the correct treatment would be indirect, through creating income. For our purposes this is an inessential detail.

\(^{2}\) Breaking this assumption, as for example with a "white-knuckle" flier, is quite destructive, since it obviates any simple welfare analysis in practice.
assume that the full time involved in a trip involves not only flight time, but the difference between departure time and desired departure time. The more flights are scheduled, the lower the difference and, hence, total trip time. For simplicity, suppress the fact that the variable "flights" takes on integer values only and define a continuous differentiable function \( t(F) \) relating trip time to flights \( F \). By assumption, \( \frac{dt}{dF} < 0 \). Also define a continuously differentiable cost function \( C(Q,F) \), where \( Q \) is the number of trips.

Social choice involves selecting the output of trips and flights, together with expenditure on all other goods and leisure, subject to a budget constraint. Travel necessitates time, which detracts from time available for work or leisure. The social welfare function is \( U(X,Q,L) \), quasi-concave and differentiable; where \( X \) is the composite commodity "all other goods", \( Q \) is trips, and \( L \) is leisure. \( U \) is to be maximized subject to the budget constraint

\[
r X + C(Q,F) \leq I + w(H-L-t(F)Q),
\]

where \( r = \) price of the composite, \( w = \) wage rate, \( I = \) non-wage income, \( H = \) time available. The first order conditions for an interior maximum are (where \( \lambda \) is the Lagrange multiplier on the budget constraint)\(^1\):

\[
\frac{1}{\lambda} \frac{\partial U}{\partial X} = r \quad \quad \frac{\partial C}{\partial F} + Q \frac{dt}{dF} w = 0
\]

\[
\frac{1}{\lambda} \frac{\partial U}{\partial Q} = \frac{\partial C}{\partial Q} + wt \quad \quad \text{and the budget constraint}
\]

\[
\frac{1}{\lambda} \frac{\partial U}{\partial L} = w
\]

\(^1\) Convexity of \( C \) and \( t(F) \) ensures the second order conditions are met. This model where consumption of goods involves time is Becker's (G. Becker "A Theory of the Allocation of Time", Economic Journal, Sept., 1965).
Can a decentralized economy achieve the full social optimum? Evidently in competitive equilibrium with zero profits, average cost of a trip to the firm must equal the fare: \( \frac{C}{Q} = P \). For social optimum, the charge to the consumer must equal \( \frac{\partial C}{\partial Q} \). With competition of the atomistic sort, \( C \) is homogeneous of degree one, and thus \( \frac{\partial C}{\partial Q} Q + \frac{\partial C}{\partial F} F = C \). Therefore, decentralization requires \( \frac{\partial C}{\partial F} F = 0 \) —— flights are either a free good or worthless and hence set at zero.

The problem is that no market exists for flights —— they are a pure public good in this model. The solution is for the public authority to subsidize flights fully (up to the social optimum point) and allow the firms to competitively determine fares at marginal trip cost. The recognition that flights are a public good (outside the model, a quasi-public good, since not everyone shares in the benefits) seems at first peculiar. All strangeness vanishes when the analogy is drawn with highways. The mathematics of the model above could equally well represent stylized social planning for a city road system, with \( F \) denoting highway capacity. Marginal use fares with public subsidy is a familiar principle here.

It is rather strange that the public good aspect of flights does not appear to have been recognized in the debates over CAB fare structure. I conjecture that "public good" has tended to be identified in the transportation context with something producible only in large segments (lumpy) like rail or highway systems. There is no necessary connection.

The optimal provision of flights is attained when, with all other variables at the optimum, \( \frac{\partial C}{\partial F} = -Q_{w} \frac{dt}{dF} \), marginal cost of flights
equals marginal benefit. Is this simple condition still relevant in the second-best situation where no subsidies are paid and fares must cover costs? Interestingly, if the industry is competitive, this is still the relevant condition. Sections III and IV deal with second-best regulation fare policies when the industry is and is not competitive.

III. Optimal Fares for a Competitive Industry

Assume the regulatory body sets fares for a competitive industry. Each firm in the industry sets flights to maximize profits for given fares and costs and in competitive equilibrium, profits are zero. Changing the fare alters the flight offer of each firm and, hence, the industry. The regulator faces an implicit function relating flights to fares and must set fares in a "second-best" optimal way.

The consumer in our one-person economy will maximize utility subject to a budget constraint \( rX + PQ \leq I + w(H - L - tQ) \). His demand function for trips is \( Q(P + wt, r, w, I, H) \). The regulator, by altering \( P \), the fare, alters flights and thus total trip time \( t \). Maximizing welfare formally involves substituting all the demand functions back into the utility function and maximizing the indirect utility function over \( P \). This is equivalent to minimizing \( P + wt \), the "full" price (Douglas-Miller's criterion) or to maximizing \( Q \) (DeVany's criterion) both with respect to \( P \). All versions produce
the marginal condition \( 1 + w \frac{dt}{dF} \frac{dF}{dp} = 0 \), or \( \frac{dF}{dp} = -\frac{1}{w \frac{dt}{dF}} \). The task is then to identify \( \frac{dF}{dp} \), the regulators implicit marginal trade-off of dollars for flights\(^1\).

DeVany has identified \( \frac{dF}{dp} \) for competitive and monopoly industries under the assumption of a linear cost function, so that what follows here and in Section IV generalizes his treatment of cost and extends the analysis to oligopolistic cases.

Define the profit function for the \( i \)th firm, \( \Pi_i \), as \( Pq_i - C(q_i, f_i) \) where \( q_i = \text{trips sold by firm } i \) and \( f_i = \text{flights offered by firm } i \).

Assume that every firm has a load factor equal to the industry average: \( q_i = \frac{Q}{F} f_i \). \( Q \equiv \Sigma q_i \) and \( F \equiv \Sigma f_i \). For a purely competitive firm, \( \frac{\partial q_i}{\partial f_i} = \frac{Q}{F} \), the firm can sell seats on flights at the industry load factor\(^2\). The firm maximizes \( \Pi_i \) with respect to \( f_i \) to determine the optimal number of flights and at an interior maximum:

\[ 1 \quad \text{Second order conditions are assumed in all versions. Even in the competitive case, it is difficult to find weak conditions under which they hold.} \]

\[ 2 \quad \text{In the next section this is verified as the limit of an oligopolistic industry response as } f_i / F \text{ goes to zero.} \]
\[
\frac{\partial \Pi_j}{\partial f_j} = \Pi_f = p \frac{Q}{F} - \frac{3C}{\partial q_i} \frac{Q}{F} - \frac{3C}{\partial f_i} = 0
\]

The response to a change in fares is:

\[
\frac{df_j}{dp} = -\frac{\Pi f_i p}{\Pi f_i f_i}
\]

\[
= - \frac{\frac{Q}{F} + (p - \frac{3C}{\partial q_i}) \frac{3Q}{F} \frac{1}{F}}{(p - \frac{3C}{\partial q_i})(-\frac{Q}{F^2}) + (p - \frac{3C}{\partial q_i}) \frac{3Q}{F} \frac{1}{F} - \left(\frac{\partial C}{\partial q_i}\right)^2 \frac{\partial^2 C}{\partial q_i \partial f_i} \frac{Q}{F} - \frac{\partial^2 C}{\partial f_i^2}
\]

using the first order condition and the homogeneity of degree one of \(C(q_i, f_i)\), hence, of degree zero of its first derivatives implied by atomistic competition^1.

With a competitive industry this is also the response of the industry, so we write:

\[
\frac{df}{dp} = \frac{Q - (\frac{3C}{\partial Q} - p) \frac{3Q}{F}}{\frac{3Q}{F} (\frac{3C}{\partial Q} - p) + \frac{3C}{\partial F}}
\]

^1 The zero profit condition together with the first order condition implies this.
The second-best optimum is attained when \( \frac{dF}{dP} = - \frac{1}{w} \frac{dt}{dF} \) and with the demand function for \( Q \) as written, \( w \frac{dt}{dF} = \frac{\partial Q}{\partial F} / \frac{\partial Q}{\partial P} \).

The social optimum requires

\[
\frac{\partial Q}{\partial P} = Q - \frac{\partial C}{\partial Q} - P \frac{\partial Q}{\partial P} + \frac{\partial C}{\partial F}
\]

which simplifies to:

\[
\frac{1}{Q} \frac{\partial C}{\partial F} = - \frac{\partial Q}{\partial F} / \frac{\partial Q}{\partial P} = - w \frac{dt}{dF}
\]

This is the marginal condition for the full social optimum as well, though naturally the level of flights implied differs. Douglas and Miller report this condition, though their derivation is not as clear as it might be. DeVany erroneously claims the term on the left should be average cost of flights (p. 334). His justification is confusing, which is not surprising, since it is attempting to validate an error\(^1\).

The reason why the full social optimum condition survives into the second-best is that "full price" is additive in money price and

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1 On pp. 335-6 DeVany correctly derives \( \frac{dF}{dP} \) for a competitive industry and correctly states that the optimal policy would maximize output. He fails, however, to see the inconsistency between this statement and his earlier mistake.
and time price. Price can be at the wrong level and thus time, but
the rate of substitution between dollars and time in consumption should
still equal the rate of substitution in the social budget constraint,
\[
\frac{\partial C}{\partial F} \left[ \frac{dt}{df} \right]^{-1}
\].

IV. Second-Best Fares with Imperfect Competition

Departures from perfect competition affect the regulator's
problem only in altering the function relating flights to fares. As
in the previous section, firms maximize profits over selection of
flights given the fare, and a change in fare alters the optimal flight
offer. Adding up the changes gives the industry response to a fare
change. A problem which is evaded here has been uncovered by Spence.
As the number of firms shrink toward one, the "naive" response by
firms outlined above, which assumes the regulator's fare response
cannot be affected by the firms' flight decisions, becomes increas-
ingly unrealistic. Whether the airline industry is sufficiently
competitive so that Cournot behavior with respect to the regulator
is the rule is an important empirical issue\(^1\). In any case, Cournot
behavior is assumed here.

\[\text{Spence has sketched out a duopoly game for the regulator and a monopoly. It would be interesting to extend this to several firms and a regulator, but this is a complex topic for future research.}\]
For the ith firm the profit function is written as before, and the decision rule on flights is derived from:

\[
\frac{\partial \Pi_i}{\partial f_i} = \Pi_i f_i = P \frac{\partial q_i}{\partial f_i} - \frac{\partial C}{\partial q_i} \frac{\partial q_i}{\partial f_i} - \frac{\partial C}{\partial f_i} = 0
\]

The function relating seats sold to flights is:

\[
q_i = \frac{\int (P, F)}{F} f_i
\]

and, hence, the marginal seat response is:

\[
\frac{\partial q_i}{\partial f_i} = \frac{\partial Q}{\partial F} \frac{f_i}{F} \left(1 + \sum_{j \neq i} \frac{\partial f_j}{\partial f_i} \right) + \frac{F - f_i (1 + \sum_{j \neq i} \frac{\partial f_j}{\partial f_i})}{F^2} \int \left[ \frac{\partial f_j}{\partial f_i} \right]
\]

The terms \( \frac{\partial f_j}{\partial f_i} \) represent conjectural variations: the assumed response of other firms to one's own changes in flights.

Note that \( \lim_{f_i \rightarrow 0} \frac{\partial q_i}{\partial f_i} = \frac{Q}{F} \), the competitive response used in the previous section. As before, the response to a fare change is given by:
\[
\frac{df_i}{dP} = -\frac{\prod_{j \neq i} f_j}{f_i} = - \frac{\frac{\partial q_i}{\partial f_i} + (P - \frac{\partial C}{\partial q_i}) \frac{\partial^2 q_i}{\partial f_i^2}}{(P - \frac{\partial C}{\partial q_i}) \frac{\partial^2 q_i}{\partial f_i^2} - 2 \frac{\partial q_i}{\partial f_i} \frac{\partial^2 C}{\partial q_i \partial f_i} - (\frac{\partial q_i}{\partial f_i})^2 \frac{\partial^2 C}{\partial f_i^2} - \frac{\partial^2 C}{\partial f_i \partial q_i}}
\]

The quadratic term in the second order derivatives of the cost function no longer in general vanishes, and the second order derivatives of the passenger function are now more complex. Two special cases are of particular interest: Cournot behavior \((\frac{\partial f_j}{\partial f_i} = 0)\) and market share preserving behavior \((\frac{\partial f_j}{\partial f_i} = \frac{f_j}{F})\). In addition, we impose homogeneity of degree one on the cost function (firm size is then determined by a combination of barriers to entry, and historic market position).

For the Cournot case, \(^1\)

\(^1\) For the Cournot assumption, \(\frac{\partial q_i}{\partial f_i} = \frac{q_i}{f_i} [1 + (\frac{\partial Q}{\partial F} \frac{F}{Q} - 1) \frac{f_i}{F}]\), and

\[
\frac{\partial^2 q_i}{\partial f_i \partial P} = \frac{\partial Q}{\partial F} \frac{f_i}{F}^2 + \frac{\partial Q}{\partial P} (1 - \frac{f_i}{F}) \frac{1}{F}, \text{ and } \frac{\partial^2 q_i}{\partial f_i^2} = \frac{\partial Q}{\partial F^2} \frac{f_i}{F} + 2 \frac{\partial Q}{\partial F} (1 - \frac{f_i}{F}) \frac{1}{F} - 2 \frac{\partial Q}{\partial F^2} (1 - \frac{f_i}{F})^2.
\]

Homogeneity of degree one of \(C\) implies homogeneity of degree zero of its first derivatives, and \(\frac{\partial^2 C}{\partial q_i} q_i + \frac{\partial^2 C}{\partial q_i \partial f_i} f_i = 0 = \frac{\partial^2 C}{\partial f_i^2} f_i + \frac{\partial^2 C}{\partial q_i \partial f_i} q_i\).

This permits substitutions with second derivatives of the cost function. Also, \(q_i \frac{f_i}{F} = Q\), the equal load factors assumption, is used in simplifying.
\[
\frac{df_i}{dP} = -\left(\frac{Q}{F} (1 + (e-1) \frac{f_i}{F}) + (P - \frac{3C}{3q_i}) \left[ \frac{2Q}{3F^2} \frac{f_i}{F} + \frac{1}{F} \frac{3Q}{3P} (1 - \frac{f_i}{F}) \right] \right.
\]
\[
- \frac{3Q}{3P} \frac{f_i}{F} (l - e) \frac{3^2C}{3q_1 \delta f_i} / \{(P - \frac{3C}{3q_i}) \left[ \frac{3^2Q}{3P^2} \frac{f_i}{F} \right]
\]
\[
+ 2 \frac{Q}{F^2} (e - 1) (1 - \frac{f_i}{F}) \} + \frac{Q}{F} (1 - e) \frac{f_i}{F} \frac{3^2C}{3q_1 \delta f_i}
\]

where \( e = \frac{3Q}{3F} \frac{F}{Q} \).

In principle, all these terms are identifiable. \( \frac{dF}{dP} = \sum_i \frac{df_i}{dP} \).

For the market share preserving case, it is straightforward to show, as DeVany does, that it is equivalent to joint profit maximization, or monopoly. In this case,

\[
\frac{dF}{dP} = -\frac{\frac{3Q}{3F} + (P - \frac{3C}{3Q}) \frac{3^2Q}{3F^2} \frac{3^2C}{3F^2} - \frac{3^2C}{3F} \frac{3Q}{3P} - \frac{3^2C}{3P} \frac{3Q}{3P}}{(P - \frac{3C}{3Q}) \frac{3^2Q}{3F^2} - 2 \frac{3Q}{3F} \frac{3^2C}{3Q^2} - (\frac{3Q}{3F})^2 \frac{3^2C}{3Q^2} - \frac{3^2C}{3F^2}}
\]

1 The quadratic form in the cost function in the denominator simplifies slightly if homogeneity of degree one is assumed.
None of the imperfectly competitive models reduce, with substitution in \( \frac{dF}{dp} = - \frac{1}{w} \frac{dt}{df} = - \frac{\partial Q/\partial P}{\partial Q/\partial F} \), to a formula whereby \( \frac{\partial C}{\partial F} \)

may be related to \( - \frac{\partial Q/\partial P}{\partial Q/\partial F} \). But it is evident in general that the solution departs from any simple equation of marginal benefit with marginal cost.

It is of interest to note, on the other hand, that the unregulated monopoly will attain the marginal efficiency condition on flights which holds in the optimum competitive solution and in the full optimum. Writing \( \Pi = PQ - C \) and optimizing over both \( P \) and \( F \), we have

\[
\Pi_p = 0 = Q + \frac{\partial Q}{\partial F} p - \frac{\partial Q}{\partial F} \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial F} \\
\Pi_F = 0 = p \frac{\partial Q}{\partial F} - \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial F} - \frac{\partial C}{\partial F}
\]

which implies \( \frac{1}{Q} \frac{\partial C}{\partial F} = - \frac{\partial Q/\partial F}{\partial Q/\partial P} \), the full social optimum condition.

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1 For example, with monopoly and \( \frac{d^2 t}{dF^2} = 0 \), and using homogeneity of degree one in the cost function, \( \frac{dF}{dp} = - \frac{\partial Q/\partial F}{\partial Q/\partial P} \) reduces to

\[
- \frac{\partial Q/\partial F}{\partial Q/\partial P} = \frac{F}{Q} \frac{\partial^2 C}{\partial F^2} \left( \frac{1-e}{e} \right).
\]
The analysis of Sections III and IV may be summarized in 2 diagrams.

Figure 1

\[ C(Q^*, F) \]
\[ Q^*(P, F) \]
\[ P \]
\[ F \]

Figure 1 shows the competitive second-best solution at point A. The average cost function \( C/Q \) is decreasing in \( Q \), since

\[ \frac{\partial C}{\partial Q} = \frac{1}{Q} \left( \frac{\partial C}{\partial Q} - \frac{C}{Q} \right) < 0 \] with homogeneity of degree one. For given \( Q \), it increases in \( F \) at an increasing rate and for convex \( C \), we assume \( C > 0 \) if \( F = 0 \). The demand function is \( Q(P + wt(F)) \) and \( \frac{dt}{dF} < 0 \) and we assume \( \frac{d^2t}{dF^2} > 0 \). The demand isoquants are then vertically parallel, concave, and indicate higher \( Q \) as we move to the right. The regulator for a competitive industry faces an implicit function \( F(P) \) (not drawn) which is the locus of intersection points of the average cost function with the demand function at equal \( Q \) in the 3 dimensional \( Q - P - F \).
space projected into \( P - F \) space. \( P + \text{wt}(F) \) is minimized where

\[
\frac{\text{d}F}{\text{d}P} = - \frac{1}{w} \frac{\text{d}t}{\text{d}F} \quad \text{which implies} \quad \frac{1}{Q} \frac{\partial C}{\partial F} = - \frac{\partial Q/\partial F}{\partial Q/\partial P} \quad \text{at point A.} \quad F(P) \quad \text{has slope equal to the inverse of the slope of} \quad C/Q \quad \text{at} \quad A. \quad \text{The full social optimum would set} \quad P = \frac{\partial C}{\partial Q}, \quad \text{and retain} \quad \frac{1}{Q} \frac{\partial C}{\partial F} = - \frac{\partial Q/\partial F}{\partial Q/\partial P}. \quad \text{If} \quad \frac{1}{Q} \frac{\partial C}{\partial F} \quad \text{were independent of} \quad Q, \quad \text{an unreasonable stipulation for airlines, this would involve moving due south from} \quad A \quad \text{on the demand surface with flights remaining at the same value.}

The various imperfectly competitive solutions may be represented as in Figure 2. \( F(P) \) incorporates \( Q = Q(P + \text{wt}(F)) \)

and the industry's flight offer response to a change in \( P \). \( F(P) \) is no longer traced out on the average cost surface (though it is of course on the demand surface), and it is difficult to determine its shape. It is drawn as an increasing concave function, which it must be in
the neighborhood of a local minimum in full price, which locally maximizes welfare.

V. Optimal Fares with Many Consumers

An important fact for even a simplified approach to optimal fare policies is that different travelers have different marginal values of time. Relaxing the assumption that all consumers have the same value of time (and hence flights) fortunately still permits a feasibly simple approach. The value of time (or flights) to be used is simply a weighted average of individual values of time (or flights), where the weights are trip shares. The marginal value of flights will not in general, however, equal the ratio of derivatives of the aggregate trip demand function $\frac{\partial Q}{\partial F}/\frac{\partial Q}{\partial P}$ used in previous sections.

Consider a full social optimum policy. Planners maximize a Bergson utility function of individual utilities. Each individual $k$ has a budget constraint $rX_k + w_k \left[ t(F)Q_k + L_k \right] \leq I_k + w_kH$. The aggregate budget constraint is $\sum_k I_k + C(\sum_k Q_k, F) \leq I + \sum_k w_k(H - L_k)$.

The first order condition with respect to $F$ is $\sum_k \lambda_k w_k Q_k \frac{dt}{dF} + \beta \frac{\partial C}{\partial F} = 0$ where $\beta$ is the marginal aggregate utility of income and the $\lambda_k$'s are the individual marginal utilities of income. With optimal income distribution, $\lambda_k = \beta, \forall k$. Then the condition becomes

$$\frac{1}{Q} \frac{\partial C}{\partial F} = - \sum_k w_k \frac{Q_k dt}{dF}.$$  
Marginal flight cost per trip should equal a

The same condition is derived by H. Mohring in Transportation Economics, Ballinger, 1975.
a weighted sum of individual marginal values of flights, with weights equal to individual trip shares.

The same marginal benefit implication is derivable ignoring distributional problems if we use social surplus analysis. Define surplus as:

\[ S = \int_{\mathbb{P}} \sum_{k} Q_k (v + w_k t(F(P))) \, dv \]

\( F(P) \) is the implicit regulator's trade off function, dependent on industry structure and aggregate travel demand. Surplus is maximized when (necessary condition):

\[ \frac{dS}{dP} = 0 = - \sum_{k} Q_k + \int_{\mathbb{P}} \left[ \sum_{k} \frac{\partial Q_k}{\partial v} w_k \frac{dt}{dF} \frac{dF}{dP} \right] \, dv \]

\[ = - Q + \frac{dt}{dF} \frac{dF}{dP} \int_{\mathbb{P}} \sum_{k} \frac{\partial Q_k}{\partial v} w_k \, dv \]

\[ = - Q + \frac{dt}{dF} \frac{dF}{dP} \sum_{k} w_k \int_{\mathbb{P}} \frac{\partial Q_k}{\partial v} \, dv \]

\[ = - Q - \frac{dt}{dF} \frac{dF}{dP} \sum_{k} w_k Q_k \]
The regulator's rule is now:

\[
\frac{dF}{dP} = \left[- \sum_k \frac{w_k dF}{Q_k} \right]^{-1} = \left[ - \sum_k \left( \frac{\partial Q_k/\partial F}{\partial Q_k/\partial P} \right) \frac{Q_k}{Q} \right]^{-1} \neq \left[ - \frac{\partial Q/\partial F}{\partial Q/\partial P} \right]^{-1}
\]

Marginal benefit is calculated as in the full social optimum case, but demands individual level information.¹

An implication of the modified marginal benefit calculation is that even for the competitive industry of Section II, optimal policy no longer reduces to \(\frac{1}{Q} \frac{\partial C}{\partial F} = -\frac{\partial Q/\partial F}{\partial Q/\partial P}\). In the competitive case

\[
\frac{dF}{dP} = \frac{Q - (\frac{\partial C}{\partial Q} - P) \frac{\partial Q}{\partial F}}{\frac{\partial Q}{\partial F} (\frac{\partial C}{\partial Q} - P) + \frac{\partial C}{\partial F}} + \frac{\partial C}{\partial F}, \text{ to be equated with } \left[ - \sum_k \left( \frac{\partial Q_k/\partial F}{\partial Q_k/\partial P} \right) \frac{Q_k}{Q} \right]
\]

This reduces to:

\[
\frac{1}{Q} \frac{\partial C}{\partial F} = -\sum_k \frac{\partial Q_k/\partial F}{\partial Q_k/\partial P} \frac{Q_k}{Q} + (\frac{\partial C}{\partial Q} - P) \left[ \frac{\partial Q}{\partial P} \sum_k \frac{\partial Q_k/\partial F}{\partial Q_k/\partial P} \frac{Q_k}{Q} - \frac{\partial Q}{\partial F} \right]
\]

Douglas and Miller in their assessment of fare efficiency have used the criterion of Section II, correct in context, but inappropriate

¹ Note that Spence's problem of needing more than marginal individual benefit for proper evaluation is avoided in this context because the qualitative variable \(F\) enters so simply.
with the realistic case of many consumers, each with different marginal values of flights.

VI. A Critique of Devany's Findings

Devany's development of the airline model is very useful, and it may seem a bit unfair to underline his failure to adequately treat social efficiency for the regulator. Nevertheless, his finding that fares are near the socially efficient level is erroneous based on a correct use of his own data. Such an important false conclusion should not be allowed to stand.

Devany partitioned his sample into a competitive and a monopoly market. The marginal value of flights \( \frac{\partial Q}{\partial F} \) was $3.09 in monopoly markets and $.27 in competitive markets. The inverse of these numbers should equal \( \frac{dF}{dP} \) for the two market structures. Calculations reported in the appendix show that for monopoly markets \( \frac{dF}{dP} = -1.63 \times 10^{-7} \) and for competitive markets \( \frac{dF}{dP} = -.0143 \). Fares are not optimally set. In addition, for monopoly markets \( \frac{d^2 F}{dP^2} < 0 \) while in competitive markets \( \frac{d^2 F}{dP^2} > 0 \). Maximizing welfare is equivalent to minimizing full price \( Z = P + wt(F(P)) \). \( \frac{dZ}{dP} = 1 + w \frac{dt}{dF} \frac{dF}{dP} > 0 \) in both markets. \( \frac{d^2 Z}{dP^2} = w \frac{dt}{dF} \frac{d^2 F}{dP^2} \) assuming \( \frac{d^2 t}{dP^2} = 0 \), and for monopoly markets \( \frac{d^2 Z}{dP^2} > 0 \) while for competitive markets \( \frac{d^2 Z}{dP^2} < 0 \).
In either case, welfare would improve by lowering fares, since flights would rise and fares would be lower. The competitive case may not be in the region of a local minimum, since the second-order condition is violated. Some idea of the magnitude of the distortion is afforded by DeVany's figures in competitive markets for \(- Q \frac{\partial Q}{\partial P}\), the marginal value of flights and \(\frac{\partial C}{\partial P}\), the marginal cost of flights. For optimality, they should be equal. Instead, marginal value = $1260 and marginal cost = $2048. Lowering fares and increasing flights will increase marginal cost, but will increase Q by enough so that marginal value of flights comes closer to marginal cost. A simulation analysis could discover optimal fares and compare them with the 1968 fares on which the analysis is based. I suspect there would be a large difference.
Appendix: Calculation of Derivatives of $F(P)$ in DeVany's Model

DeVany's results are reported in Tables 2 and 3 (pp. 341-2) for the model $Q^\beta = p^\beta_1 F^\beta_2 W$, where $W$ need not concern us, and $C = \alpha_o^\beta_1$. We reproduce them here.

Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Monopoly</th>
<th>Competitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>-1.91</td>
<td>-1.63</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.20</td>
<td>0.98</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>2.18</td>
<td>1.58</td>
</tr>
</tbody>
</table>

Table 3 (Partial)

<table>
<thead>
<tr>
<th>Type of Market</th>
<th>Marginal Value of Flights</th>
<th>Average Value of Flights</th>
<th>Marginal Revenue of Flights</th>
<th>Marginal Cost of Flights</th>
<th>Average Cost Per Flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly</td>
<td>2573</td>
<td>4070</td>
<td>4884</td>
<td>6638</td>
<td>3501</td>
</tr>
<tr>
<td>Competitive</td>
<td>1260</td>
<td>2105</td>
<td>2062</td>
<td>2048</td>
<td>1296</td>
</tr>
</tbody>
</table>

For the monopoly case,

$$\frac{dF}{dP} = \frac{-\beta_2 + \beta_1 \beta_2}{\frac{PQ}{F} \beta_2 (\beta_2 - 1) - \alpha_1 (\alpha_1 - 1) \frac{C}{F} \frac{1}{Q}}$$

since

$$\frac{dF}{dP} = \frac{\beta Q + P \frac{\beta Q}{F} \frac{\beta Q}{F}}{P \frac{\beta Q}{F} \frac{\beta Q}{F} - \frac{\beta C}{F} \frac{\beta C}{F}}$$

with DeVany's assumptions.
Taking the ratio of marginal value of flights, 2573, and marginal value of flights per passenger, 3.09, we find \( Q = 832.686 \). Then:

\[
\frac{dF}{dP} = \frac{- (1.20)(- .91)}{4070(1.20)(.20) - (2.18)(1.18) 3501} \cdot \frac{1}{832.686} = \frac{1.092}{976.8 - 9005.97} \left( \frac{1}{832.686} \right) = -1.63 \times 10^{-7}
\]

Differentiating the above expression with respect to \( P \), we find (after simplification):

\[
\frac{d^2F}{dP^2} = - \left( \frac{dF}{dP} \right)^2 \left\{ \frac{1}{\beta_2(\beta_2 - 1)} \left( \frac{1}{F} - \frac{P}{F^2} \frac{dF}{dP} \right) \right\}
\]

\[
- \alpha_1(\alpha_1 - 1)(\alpha_1 - \beta_2 - 1) \frac{CQ}{(QF)^2} \frac{dF}{dP} = (+) \left[ \frac{1}{(-)} \left[ (+) (+) (+) \right] - (+) (+) (-) (+) (-) \right] < 0
\]

For the competitive case, with simplification,

\[
\frac{dF}{dP} = \frac{1 + \beta_1}{P \beta_2 + \frac{C}{Q} \alpha_1} = \frac{1 - 1.63}{P \ (98) + 24(1.58)}
\]

DeVany reports average cost of output on competitive routes as $24.
P/F is found by noting \( \frac{PQ}{F} = 2105 \) and \( Q = 1260/27 \). Therefore,

\( P/F = 6.1875 \). Substituting, \( \frac{dF}{dP} = -0.0143 \). Differentiating the above expression for \( \frac{dF}{dP} \) with respect to \( P \) we obtain

\[
\frac{d^2 F}{dP^2} = -\left( \frac{dF}{dP} \right)^2 \frac{1}{1+\beta_1} \frac{\beta_2}{F^2} - \frac{\beta_2}{F^2} \frac{dF}{dP} + \alpha_1 \frac{1}{Q} \frac{3C}{Q} - \alpha_1 \frac{C}{Q^2} \left( \frac{3Q}{dP} + \frac{3Q}{dF} \frac{dF}{dP} \right)
\]

\[
= (-)(-)[(+)-(+) + (+) - (-)] > 0.
\]