The Efficiency of the Global Markets for Final Goods and Productive Capabilities

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Abstract

Slow mean reversion of real exchange rates is commonly considered a result of border frictions that remain despite integration of financial and goods markets. This paper shows that even if border frictions decline, a contemporaneous decline in output shock variance can in fact slow down mean reversion. It proposes a new method of estimating border cost from time-series data only, without relying on within-country variation. Applying this method to the real exchange rate of final goods and a novel measure of the real exchange rate for productive capabilities, such as technology and know-how, gives very differential border cost estimates. During the years 1974–2008, a relocation reduces productive capability by 22% for the average country pair, whereas final goods by only 15%. The real exchange rate for final goods takes more than two years to revert to purchasing power parity, more than twice as long as productive capabilities.

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1 Introduction

During the past three decades not only final goods, but also capital goods, have become increasingly mobile. Despite globalization of financial and goods markets borders constitute a considerable friction for flows of goods. These are often measured by half-times of mean-reversion of the real exchange rate of final goods in a first-order autoregressive model, and appear excessively long by perfect market standards. For the time of a shock to decay by one-half, estimates of two years and more are no exception (Rogoff, 1996).

Several key question emerge: First, why does a decay of a real exchange rate shock still take this long despite fairly advanced globalization? Second, how can border frictions and times to revert to purchasing power parity (PPP) accurately be estimated from time-series data? And third, are the border costs and PPP-reversion times of final goods representative of all global markets, or do others, such as e.g. the market for capital goods, behave differently?

In this paper I show that the popular half-time measure cannot be interpreted as a measure of market integration in an economy in which country-idiosyncratic shock variances change over time. When arbitrage is costly, country-idiosyncratic shocks are needed to fully reestablish PPP. A decline of country-idiosyncratic shock variance (e.g. by a shift to a larger exposure to global shocks) thus counteracts a decline in border cost.

I propose a model-based method of estimating border costs, requiring only one price series per country. My method exploits the time series variation of prices, in particular nonlinearity in drift and diffusion, and does, unlike e.g. Engel and Rogers (1996), not require within-country variation. The proposed method is ideally suited to aggregate real exchange rates, but applies also to disaggregated price indices, which are only available at the national level. It provides estimates of the time the real exchange rate needs to completely revert to PPP, taking both border cost and shock variance into account.

I apply this method to two types of goods: final goods and productive capabilities. Final goods are similarly useful in all countries and do not require an excessive amount of localization. Productive capabilities include, but are not limited to, capital goods. They comprise, for example, machinery, technology, and human capital embedded in the productive sector. In contrast to final goods, they require considerable adaption effort to the local situation and operator training. Consider, for example, a firm which produces measurement instruments. Productive capabilities in this firm include operational know-how – both on how to build and on how to operate the instruments –, the patents, and the precision tools needed for assembly.

The nature of border costs between the two categories differs considerably: For final goods, on the one hand, transportation costs may explain most friction. For productive capabilities, on the other hand, a relocation can be very complex, even if hardly any physical goods move. Border cost in this case includes the cost of adaption to local use or of operator training. Differences in standards, culture, skill, and local technology levels bring about frictions that can exceed the plain transportation costs of final goods by a lot.

My border cost estimates in this paper confirm this conjecture. A relocation reduces productive capability by 22% for the average country pair, but final goods by only 15%. A subsequent comparison of the border cost estimates across country and goods category suggests transport cost for final goods and entrepreneurial barriers for productive capabilities as primary border costs. Accordingly, countries with low border cost for final goods do not necessarily have low costs for productive capabilities, and vice versa. The heterogeneity in relative market efficiency indicates which market in each country could benefit most from additional effort to reduce these frictions.

Previous studies of the market frictions embodied in borders focus on final goods. Estimates of market frictions typically rely on trade flows (e.g. Anderson and van Wincoop, 2003; McCallum, 1995), a comparison of within-country with between-country variation of consumer goods prices (e.g. Engel and Rogers, 1996; Gorodnichenko and Tesar, 2009), or a direct comparison of prices (e.g. in Broda and Weinstein, 2008). The latter approach rests on identifying identical products in different countries. Unfortunately, such product pairs are hard to identify. Even for countries as close as Canada and the USA, barcodes of products coincide for less than eight percent of products (Broda and Weinstein, 2008). The price-variation based studies compare the within-country variation of prices with their cross-country variation, which requires a detailed dataset of prices at many different locations (e.g. cities) within each country. Such detailed data is available only for few countries and/or narrowly defined sets of goods. In particular, this approach is infeasible for goods for which only one price per country is published.

Productive capabilities are an example. Their price is typically determined on a nationally centralized stock market, and is thus unique to each country. For this reason no estimates exist of this border cost. The market for productive capabilities, however, deserves particular attention as international flows of final goods are largely liberalized. The cost of adjusting productive capability by means of investing, i.e. the cost of converting final goods into productive capabilities and vice versa, is considerable, and unbundling and rebundling of productive capabilities for the sole purpose of relocating them is often economically unfeasible.

Thus the cost of *directly* relocating productive capabilities (Burstein and Monge-Naranjo, 2009; Keller and Yeaple, 2009) becomes decisive for the efficiency of their global allocation. This allocation is a key determinant of relative output and ultimately of relative welfare (Hsieh and Klenow, 2007; Lucas, 1990).

To estimate the border cost I follow an indirect approach. Already Heckscher (1916) observes a positive correlation between relocation costs and persistent deviations from purchasing power parity. Dumas (1992), as well as Sercu, Uppal, and van Hulle (1995), formally derive the link between relocation cost, allocations of goods, and the real exchange rate. Their fundamental insight allows me to analyze the real exchange rate in lieu of the allocation of goods across countries.

An important innovation of my approach is the estimation of relocation cost directly from the variation of the real exchange rate over time. Unlike previous studies, I neither rely exclusively on differences of price index *levels* between countries, nor soley on bounds on sustainable international price differences. Instead, I base the identification of relocation cost on the model's prediction of the evolution of drift and diffusion of the real exchange rate over time.

My continuous-time model, an extension of Dumas (1992), predicts that both the conditional drift and diffusion follow a nonlinear process, the feature that I exploit in the estimation. Nonlinearity in real exchange rates is an empirically well-documented phenomenon.[1] Prakash and Taylor (1997) and Obstfeld and Taylor (1997), for example, estimate a model with a hard cut-off between regimes.[2]

A parsimonious approximation of this nonlinearity is the exponential smooth transition autoregressive (ESTAR) model. I utilize ESTAR in this paper as a natural auxiliary model in an indirect inference framework, which allows me to translate nonlinearity into relocation costs and other structural parameters. Previous studies have used ESTAR as a stand-alone time-series model, which forgoes a lot of the economic interpretation that an indirect inference framework provides. Michael, Nobay, and Peel (1997), for example, apply an ESTAR model to monthly wholesale price indices (WPI) of four countries in the 1920s and annual

[1] The importance of a nonlinear specification is highlighted by the weak support for mean-reversion achievable with linear models. Froot and Rogoff (1995) and Sarno (2005) provide useful surveys. These studies imply extremely slow overall speeds of mean reversion (e.g. Murray and Papell, 2005; Rogoff, 1996). At the sectoral level, Imbs, Mumtaz, Ravn, and Rey (2003) find nonlinearity for two thirds of the sectors in their sample. Such nonlinear models provide a natural explanation for both observations. The real exchange rate mean-reverts whenever it has wandered far away from parity, but follows a random walk when it is close to parity.

[2] This threshold autoregressive model was introduced by Tong (1990) and Balke and Fomby (1997).

data for two countries over 200 years. Also based on an ESTAR model, Kilian and Taylor (2003) find predictability of the nominal exchange rate at horizons of two years or more.[3] All these studies report strong support for a nonlinear specification, with a random walk dominating over short horizons, but do not make further use of this nonlinearity to quantify border cost.

Caselli and Feyrer (2007) show the importance of price and availability of productive capabilities ("complementary factors"), such as technology (Eaton and Kortum, 1999) or human capital (Lucas, 1990), for differences in the marginal product of capital across countries. Despite their importance, any standard definition of a real exchange rate is based on a price index for final goods, usually the consumer price index (CPI) or the WPI. Its behavior over time reflects therefore the frictions for goods with small localization requirements, but contains no information on productive capabilities. Limiting the discussion about real exchange rate puzzles to final goods only might miss a lot – it ignores trade and arbitrage relationships in all other markets. To see if ignoring these markets misses anything, or if it suffices to analyze final goods markets in lieu of all other, I construct a real exchange rate for this very different market based on financial market data. In this sense, my work on productive capabilities is related to studies of mean reversion of international stock markets. Such studies, e.g. Richards (1995) and Balvers, Wu, and Gilliland (2000), find evidence of transitory country-specific effects in long-run relative stock returns. They focus on relative nominal international stock prices, however, not on real exchange rates.

The remainder of this paper is organized as follows: The following Section 2 outlines a model of real exchange rate behavior. Section 3 introduces the estimation and inference procedure, which exploits the nonlinearity predictions of the model. I introduce the real exchange rate for productive capabilities in Section 4. Section 5 presents and compares the empirical results for both goods categories. I conclude with a short summary in Section 6.

2 A Model of Real Exchange Rates

In this section I discuss a model of endogenous capital accumulation with depreciation shocks and relocation cost, which captures key features of real exchange rate data. It describes the link between real imbalances and the real exchange rate in a two-country economy with border friction. There is only one good in the economy, which I interpret as *either* a final

[3] Taylor, Peel, and Sarno (2001) work with monthly CPI data for five countries over the period 1973–1996. See also Taylor (2005) for a survey.

good *or* a productive capability. Trade in this model is limited to the relocation of goods between countries for arbitrage reasons, which by construction is directly reflected in the real exchange rate.[4]

I assume complete financial markets, i.e. all necessary securities are available and international financial flows are unconstrained. The counterpart of financial markets in the real economy, the market for goods, is subject to frictions. Relocating a generic good from one country to another entails an "iceberg" cost, $1 - r \in (0, 1)$, that is, of every unit relocated only $100 \cdot r$ percent arrive.[5]

Because any transfer of goods across the border between the two countries entails a loss, the good's location matters. Accordingly, I mark parameters and quantities of the foreign country with an asterisk (*). The stock of goods, K, can be either consumed, c, or invested in a constant returns to scale production with productivity α. The stock of goods in each country is subject to zero-mean country-idiosyncratic depreciation shock, $\sigma_{11} dz$ and $\sigma_{22} dz^*$, where dz and dz^* are increments of a standard Brownian motion process.

The marginal rates of substitution differ between countries at almost every instance because of the cost of crossing the border. Because the relocation cost is the only friction hindering the movement of goods, this cost bounds the possible valuation differences between countries. No relative valuation outside of the interval $[r; 1/r]$ can persist, because this would trigger immediate, risk-free, and profitable transfers of goods, dX, from a low-price to a high-price country, until the relative valuation has returned back into this interval.

Owing to the assumption of complete financial markets, the decentralized two-country problem is equivalent to the planner's problem[6]

$$V(K, K^*) = \max_{\substack{c(t), c^*(t), \\ \Xi(r)}} E_0 \int_0^\infty e^{-\rho u} \left(\frac{q}{\gamma} c(u)^\gamma + \frac{2 - q}{\gamma} c^*(u)^\gamma \right) du \qquad (1)$$

[4]This rules out trade due to specialization or love-of-variety. See B.1 for a larger model that also embeds trade due to specialization. A more general one-good economy is described in A.1. A provides also detailed calculations and discussion of many of the results in this section.

[5]This "iceberg" loss reflects shipping costs for final goods, and costs for transferring technology as in Keller and Yeaple (2009) for productive capabilities. This model does not intend to explain standard trade flows. See B.1.

[6]Basak and Croitoru (2007) show that a decentralized economy with country-specific bonds and a claim on the dividend flow of one country can equivalently be solved by (1).

s.t.

$$\begin{aligned} dK(t) &= [\alpha K(t) - c(t)]\, dt + K(t)\sigma_{11} dz(t) - dX(t) + r dX^*(t) \\ dK^*(t) &= [\alpha^* K^*(t) - c^*(t)]\, dt + K^*(t)\sigma_{22} dz^*(t) + r dX(t) - dX^*(t) \end{aligned}$$

where $K(0)$ and $K^*(0)$ are given, $c(t), c^*(t), K(t), K^*(t), X(t), X^*(t) \geq 0 \quad \forall t$, and where $\rho > 0$ denotes the discount rate, $1 - \gamma > 0$ the risk aversion and q the welfare weight of the home country. The relocation of goods is captured by $X(t)$, which is an adapted, nonnegative, right-continuous, nondecreasing stochastic process. The choice variable Ξ denotes the open region in the (K, K^*) space in which no goods are transferred, i.e. where $dX = 0$ and $dX^* = 0$. Optimal choice of $\Xi(r)$ determines the endogenous shipments dX and dX^*. I define the *imbalance* of goods as $\omega = K/K^*$.[7] Due to the homogeneity of the value function, the boundary of Ξ is fully characterized by the minimal and maximal imbalance levels, $\underline{\omega}$ and $\overline{\omega}$.

The symmetric version of this model has been developed by Dumas (1992). I extend this model by country heterogeneity in the ability to produce new goods (Hsieh and Klenow, 2007), which can result in persistent price differences between countries. Making the model asymmetric complicates it considerably, but helps explaining key features of the real exchange rate, such as reversion to a real exchange rate level different from PPP.

The homogeneity of the value function allows reposing the problem in terms of the imbalance alone by substituting $V(K, K^*) = K^{*\gamma} I(\omega)$. The resulting second-order ordinary differential equation governs the imbalance process in periods of no relocations.

$$\begin{aligned} 0 &= \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} \left[\gamma I(\omega) - \omega I'(\omega)\right]^{\frac{\gamma}{\gamma-1}} \\ &+ \left[\alpha^*\gamma - \rho + \frac{1}{2}\sigma_{22}^2 \gamma(\gamma - 1)\right] I(\omega) \\ &+ \left[\alpha - \alpha^* - (\gamma - 1)\sigma_{22}^2\right] \omega I'(\omega) \\ &+ \frac{1}{2}\left(\sigma_{11}^2 + \sigma_{22}^2\right) \omega^2 I''(\omega) \end{aligned} \tag{2}$$

By optimal choice of the boundary of Ξ, the unknown function $I(\omega)$ must satisfy value matching and smooth pasting conditions at both boundaries at all times. The value matching condition requires equalization of the value of the marginal good at the moment of relocation,

[7] Global shocks to both countries would have no effect on ω. Its effect cancels out and for this reason I do not include it in the model. Because the imbalance ω is the only state variable, $\sigma_{11} dz$ and $\sigma_{22} dz^*$ reflect the country-idiosyncratic component of total shocks only.

e.g. for the upper imbalance boundary, $\overline{\omega}$,

$$V_K(K, K^*) = rV_{K^*}(K, K^*).$$

The smooth pasting conditions require

$$V_{KK}(K, K^*) = rV_{KK^*}(K, K^*),$$

and

$$V_{K^*K}(K, K^*) = rV_{K^*K^*}(K, K^*).$$

The conditions for the lower imbalance boundary are analogous. Substituting for the value function I can express these conditions in terms of the unknown functional $I(\omega)$.

$$\frac{I'(\overline{\omega})}{\gamma I(\overline{\omega})} = \frac{r}{1 + r\overline{\omega}} \tag{3}$$

$$\frac{I''(\overline{\omega})}{\gamma I(\overline{\omega})} = \frac{r^2(\gamma - 1)}{(1 + r\overline{\omega})^2} \tag{4}$$

The differential equation (2) with boundary conditions (3) and (4) and the analogous conditions for the lower imbalance boundary can now be solved numerically for the function $I(\omega)$. I determine the optimal boundaries by guessing values for $\underline{\omega}$ and $\overline{\omega}$ and iterating both forward toward some intermediate imbalance level $\omega_0 \in (\underline{\omega}, \overline{\omega})$ using the Bulirsch-Stoer method (Stoer and Bulirsch, 2005) augmented by a Richardson extrapolation.[8] Richardson's deferred approach to the limit corrects for the estimated error stemming from finite accuracy. A solution has been found if the values of $(I'(\omega_0), I''(\omega_0))$ obtained by iterating from the upper boundary are equal to the values obtained by iterating from the lower boundary, i.e. if $\overline{I'(\omega_0)} = \underline{I'(\omega_0)}$ and $\overline{I''(\omega_0)} = \underline{I''(\omega_0)}$. Otherwise I retry with a new guess for the pair of boundaries. In a second step I use the results from the Bulirsch-Stoer method as starting values in a Newton algorithm for two-point boundary value problems.

[Table 1 about here.]

The upper panel of table 1 reports the maximum sustainable imbalance as a function of risk and risk aversion for modest relocation cost ($r = 0.9$) in a symmetric world. The smaller the risk aversion and the higher the risk, the larger are the imbalances. The lower panel

[8] See Press, Teukolsky, Vetterling, and Flannery (2001, p.376ff, p.718ff) for a detailed description of similar techniques.

of the same table shows that a larger relocation cost ($r = 0.75$) implies larger sustainable imbalances for any level of risk aversion and risk.

The tables reveal that, given some degree of risk aversion, the maximum sustainable imbalance approaches an asymptote as risk grows to infinity. For risk aversion larger than unity there exists a maximum risk level, beyond which the differential equation has no solution. If, contrary to what is shown in the table, two countries differ in their productivity, then the sustainable abundance of goods in the more productive country is higher, and correspondingly lower in the less productive country.

I am now able to define the *real exchange rate*, p, as the ratio of the marginal values of the good in two countries, i.e.

$$p(\omega) = \frac{V_K(K, K^*)}{V_{K^*}(K, K^*)} = \frac{I'(\omega)}{\gamma I(\omega) - \omega I'(\omega)}. \tag{5}$$

Note that the real exchange rate depends only on the imbalance, ω, but not on the stocks, K and K^*, themselves. Therefore, the real exchange rate tracks the *relative* scarcity of goods – a scarcity due to the frictions in the goods market, which emphasizes the strong impact of relative productivity on the real exchange rate.

Whereas the behavior of the real exchange rate is governed by the same parameters as the sustainable imbalance, its mean reversion time is extremely sensitive to them due to the nonlinear relationship (5). Table 2 shows the expected time until complete reversion of the real exchange rate to PPP, starting at the maximum sustainable imbalance $\overline{\omega}$. For all parameter values, except very small risk and high relocation cost, mean reversion speeds up with higher risk aversion. Higher relocation cost and lower risk both slow down mean reversion. If risk becomes small, mean reversion can take decades or centuries. Clearly, a decline in country-idiosyncratic risk can easily offset the effect of lower relocation costs.

[Table 2 about here.]

Using Ito's lemma, drift and diffusion of the natural logarithm of the real exchange rate process

$$dln(p) = \mu_p(p)dt + \sigma_p(p)dz$$

can be written as a function of ω, $I(\omega)$, $I'(\omega)$, $I''(\omega)$. The drift of $ln(p)$ at the boundaries with $\sigma_{11} = \sigma_{22} = \sigma$ is given by the following proposition.

Proposition 1 ***(Drift of Real Exchange Rate at the Boundary)*** *The drift of the* *real exchange rate at the upper imbalance boundary is*

$$\mu_p(\omega = \overline{\omega}) = \alpha^* - \alpha + \frac{\overline{\omega} - 1/r}{\overline{\omega} + 1/r}(1-\gamma)\sigma^2,$$

and at the lower imbalance boundary

$$\mu_p(\omega = \underline{\omega}) = \alpha^* - \alpha + \frac{\underline{\omega} - r}{\underline{\omega} + r}(1-\gamma)\sigma^2.$$

Proof: See A.3.

For realistic parameter values the mean reversion at the boundary gains in strength with shock diffusion, σ^2, and with risk aversion, $1-\gamma$.[9] The drift towards the center therefore shrinks as the economy is hit by smaller shocks. It further approaches $\alpha^* - \alpha$ as the economies get perfectly integrated ($r \to 1$). Therefore, because financial integration coincided with a period of "great moderation" (Kim and Nelson, 1999; Stock and Watson, 2002), it is not surprising that exchange rates do not appear more mean reverting today than in the past.

[Figure 1 about here.]

A key feature of my model is that drift and diffusion of the real exchange rate vary systematically with its level. The upper panel of Figure 1 shows the drift of two real exchange rate processes with different relocation costs. The process represented by the dashed line results from high relocation cost ($r = 0.75$), whereas the process represented by the solid line reflects low relocation cost ($r = 0.9$). Both processes share the property that a deviation from parity entails a drift of opposite sign. The diffusion, shown in the lower panel, decreases as real exchange rate deviations from parity become large. The real exchange rate process is therefore mean reverting at the boundary of Ξ, but indistinguishable from a random walk close to parity. Increases in relocation cost not only widen the range of sustainable deviations from PPP, but also lower the drift in absolute terms at all real exchange rate levels. Likewise, they increase diffusion at all real exchange rate levels.

[Figure 2 about here.]

The conditional drift is deterministic and non-zero except for one real exchange rate level, which I henceforth refer to as zero-drift point. The change of the real exchange rate

[9] Assuming $\alpha = \alpha*$, a sufficient condition for this to hold is $\underline{\omega} < r$ and $\overline{\omega} > 1/r$. Table 1 shows that this is satisfied except for very small σ in combination with $\gamma \leq 0$.

is therefore predictable. The predictability increases in the deviation of the real exchange rate from the zero-drift point for two reasons. First, the drift grows in absolute value, and second, the diffusion shrinks. As Figure 2 shows, only in a symmetric economy the zero-drift point coincides with PPP ($ln(p) = 0$). In all other situations, the real exchange rate change is predictable even at the PPP level. The agents in this model economy, however, cannot gain from this predictability. The expected appreciation of the abundant good, say, compensates for the lack of diversification and its frequent costly shipping.

Figure 2 illustrates the asymmetric, high relocation cost scenario. It compares the drift of the real exchange rate in a world in which productivities are equal ($\alpha = \alpha^*$, dashed line), with the drift in a world in which productivities differ ($\alpha < \alpha^*$, solid line). The solid line shows that small differences in growth rates can shift the reversion target level away from PPP ($ln(p) = 0$) to a considerably different level ($ln(p) \approx -0.25$). If the productivity gap between two countries is extremely wide as in Figure 2, then the drift can have the same sign at almost all real exchange rate levels. In this case the real exchange rate process is divergent for half of its support, although still bounded by Ξ.

3 Methodology

In this section I describe an estimation method, which captures the nonlinearity features of the model just discussed, and which achieves computational feasibility despite a tight link to the model.

3.1 Indirect Inference

The described model has no closed form solution. Drift and diffusion are time-varying and functions of the unobserved imbalance, ω. In principle the coefficients on $I'(\omega)^{\frac{\gamma}{\gamma-1}}$, $(\gamma I(\omega) - \omega I'(\omega))^{\frac{\gamma}{\gamma-1}}$, $I(\omega)$, $\omega I'(\omega)$, and $\omega^2 I''(\omega)$ of differential equation (2) can be estimated by maximum likelihood (Brown and Hewitt, 1975) after solving the nonlinear filtering problem of $\omega(t)$ based on the observations $p(t)$ (Bensoussan, 1992). However, because the differential equation for p is not available in closed form, this calculation of ω_0 from $p(\omega_0)$ is computationally very costly and practically infeasible. Further, even after filtering and obtaining the series of ω, the closed form of the density of the conditional likelihood is not available because of the discreteness of the data. In contrast, going in the opposite direction, i.e. calculating $p(\omega_0)$ from ω_0 by (5), is a simple task, because the functional's value at ω_0, $I(\omega_0)$, is a by-product of calculating ω_0 via (2). More productive is therefore a method that does not require calculating the conditional distribution of $p(t)$ given $p(t-1)$ directly from the original model.

This estimation problem ideally suits the indirect inference procedure, introduced by Smith (1993), Gouriéroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996). Indirect inference replaces the hard-to-evaluate likelihood function of the original model with the likelihood function of an auxiliary model which is easier to estimate. Importantly, the auxiliary model must pick up the key features of interest of the data, in particular nonlinearity in drift and diffusion. One can then generate independent simulated data sets from the structural model for various parameters, estimate the auxiliary model with these simulated data, and repeat this procedure until parameters are found for which the estimates of the auxiliary model based on the simulated data are close by some metric to the estimates based on the actual data. Further, indirect inference does not require the hard calculation of the unobservable ω from the observed p. It allows solving and simulating the model for ω, and then calculating the implied real exchange rate, $p(\omega)$.

3.2 Auxiliary Model

The crucial decision in indirect inference is choosing an appropriate auxiliary model. For the problem at hand, a natural auxiliary model is the ESTAR model of Haggan and Ozaki (1981) and Teräsvirta (1994). Whereas the process of the real exchange rate implied by my structural model is complicated and not available in closed form, its key feature, the smooth transition from a divergent to a mean reverting regime, can parsimoniously be modeled by the ESTAR specification. The ESTAR model has the following standard form:

$$\begin{aligned} p_t - p_{t-1} &= (1 - \Phi(\theta; p_{t-d} - \mu)) \left(\beta_0 + (\beta_1 - 1) p_{t-1} + \sum_{j=1}^{m} \beta_j p_{t-j} \right) + \\ &+ (\Phi(\theta; p_{t-d} - \mu)) \left(\beta_0^* + (\beta_1^* - 1) p_{t-1} + \sum_{j=1}^{m} \beta_j^* p_{t-j} \right) + \epsilon_t \qquad (6) \\ \epsilon_t &\sim N(0, \sigma_t^2) \end{aligned}$$

The transition function $\Phi(\theta; p_{t-d} - \mu)$, parametrized by the transition lag d and the transition parameter $\theta \geq 0$, governs the smooth transition between the inner autoregressive process with parameters β_i and the outer autoregressive process with parameters β_i^*, and is specified as

$$\Phi(\theta; p_{t-d} - \mu) = 1 - exp\left(-\theta (p_{t-d} - \mu)^2\right).$$

I generalize the standard ESTAR to allow for conditional variance dynamics which the structural model (1) predicts. The second transition function $\tilde{\Phi}\left(\tilde{\theta}; p_{t-d} - \mu\right)$ moves smoothly between an inner regime variance σ_1^2 and an outer regime variance σ_2^2.[10] In addition to time-varying conditional variance my specification restricts the outer regime mean dynamics.

$$
\begin{aligned}
p_t - p_{t-1} &= (1 - \Phi(\theta; p_{t-d} - \mu)) \left((\beta_1 - 1) p_{t-1} + \sum_{j=1}^{m} \beta_j p_{t-j} \right) + \\
&- \Phi(\theta; p_{t-d} - \mu) p_{t-1} + \epsilon_t \\
\sigma_t^2 &= \left(1 - \tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu)\right) \sigma_1^2 + \left(\tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu)\right) \sigma_2^2 \\
\tilde{\Phi}\left(\tilde{\theta}; p_{t-d} - \mu\right) &= 1 - exp\left(-\tilde{\theta}\left(p_{t-d} - \mu\right)^2\right) \\
\epsilon_t &\sim N(0, \sigma_t^2)
\end{aligned}
$$

I follow Teräsvirta (1994) in specifying the transition lag, d, and number of autoregressive terms, m, based on the nonlinearity test of Granger and Teräsvirta (1993), where I restrict $d \leq 12$ and $2 \leq m \leq 4$.

Michael et al. (1997) argue that real exchange rates based on CPI can be modeled by an ESTAR process. The same applies to real exchange rates based on productive capabilities. Table 3 shows that both real exchange rates have a nonstationary inner regime for most country pairs and a stationary outer regime for all country pairs.[11]

[Table 3 about here.]

3.3 Discretization, Simulation, and Optimization

Although the ESTAR estimates reveal nonlinearity in the data, they do not provide a natural interpretation of the autoregressive coefficients. Furthermore, they lack a natural benchmark for the transition parameter, θ. Only by estimating the structural model one can judge whether θ is "big" or "small". In the indirect inference framework here ESTAR assumes the role of the auxiliary model.

Inspection of the estimation equations (2) together with (3) and (4) reveals that in a country-by-country estimation not all parameters can be identified. Importantly, however,

[10] Studies that allow for time-varying conditional variance in an ESTAR setup are scarce. A notable exception are Lundbergh and Teräsvirta (2006), who augment an ESTAR-type model with a GARCH variance process.

[11] Individual coefficients of the mean equation are often insignificant. Conversely, coefficient estimates of the variance equation are typically significant, often with a higher variance in the outer regime. If an outer variance regime cannot be identified, I impose a fixed variance regime $\sigma_t^2 = \sigma_1^2$ as in the standard ESTAR setup.

the real exchange rate process may be asymmetric, reflecting differences in productivity between countries. For example, the real exchange rate process may be close to one boundary most of the time, and hardly ever reach the other. To allow for this possibility I keep the productivity differential $\Delta\alpha = \alpha - \alpha_{USA}$ as a parameter to be estimated. I calibrate $\alpha_{USA} = 0.33$ to the average aggregate productivity of the U.S. capital stock.[12] I assume equal variance $\sigma_{11} = \sigma_{22}$ and fix the discount rate at $\rho = 0.07$. The country weight q is calibrated to the average PPP-weighted GDP relative to the USA, based on the World Development Indicators provided by the Worldbank for the years 1980-2007. Finally, I fix γ at -0.5, the average of the estimates of a first stage estimation, which ensures a consistent U.S. risk aversion across country pairs.

Estimation requires the efficient simulation of many discrete trajectories of p for a given parameter set. I first simulate the $\tilde{\omega} = ln(\omega)$ process, which can be done with higher precision because the diffusion of this process is constant. Wagner and Platen (1978) and Platen (1981) introduce an Itô-Taylor scheme which strongly converges at rate 1.5.[13] Using the fact that the noise in my model is additive and the diffusion of $\tilde{\omega}$ is constant, this scheme can be written as

$$\begin{aligned}\tilde{\omega}_{t+1} &= \tilde{\omega}_t + \mu_{\tilde{\omega}}(\tilde{\omega}_t)\Delta + \sigma_{\tilde{\omega}} u_1 \sqrt{\Delta} + \frac{1}{2}\mu'_{\tilde{\omega}}(\tilde{\omega}_t)\sigma_{\tilde{\omega}}\left(u_1 + \frac{1}{\sqrt{3}}u_2\right)\Delta^{3/2} \\ &+ \frac{1}{2}\left(\mu_{\tilde{\omega}}(\tilde{\omega}_t)\mu'_{\tilde{\omega}}(\tilde{\omega}_t) + \frac{1}{2}\sigma^2_{\tilde{\omega}}\mu''_{\tilde{\omega}}(\tilde{\omega}_t)\right)\Delta^2,\end{aligned}$$

where $u_1 \sim N(0,1)$ and $u_2 \sim N(0,1)$ are independent.

Next, I calculate the process of the real exchange rate from the process of imbalances, using the interim results $(I(\omega), I'(\omega))$ obtained in the calculation of the imbalance process.

In summary, I proceed with inference by the following steps:

1. Estimate the ESTAR specification based on actual data by quasi maximum likelihood. Denote the set of parameter estimates by $A_0 = \{\theta, \tilde{\theta}, \beta_i, \mu, \sigma_1, \sigma_2\}$.

2. Draw sequences of iid random number pairs (u_1, u_2).

[12] I use annual 1980–2000 data from the World Development Indicators provided by the Worldbank to calculate α^* by the average of

$$\alpha^*_t = \frac{GDP_t}{\sum_{i=0}^{t}(GDP_i - C_i)\,0.95^{t-i}}.$$

Data from the Bureau of Economic Analysis for the USA 1995-2009 gives a similar estimate.

[13] This convergence excels the rate of 1.0 achieved by the well-known Milstein (1974) scheme. See also Kloeden and Platen (1999, p.351).

3. Pick starting values for the parameters of the structural model, $B = \{r, \Delta\alpha, \sigma\}$.

4. Solve the differential equation for optimal boundaries, $\overline{\omega}$ and $\underline{\omega}$, using the Bulirsch–Stoer method, augmented by subsequent Newton optimization.

5. Simulate $S = 500$ paths of the imbalance process, ω_t, by the Itô-Taylor scheme for 649 periods based on the structural model. The first 240 periods, or 20 years, are used as burn-in period. They are discarded to ensure that the results are independent from the starting values of the process. This leaves me with $T = 409$ simulated values.

6. Calculate the price process, $p(\omega_t)$, from the imbalance process.

7. Compute the indirect inference estimate of B by minimizing the distance between the data-based and the simulation-based estimate, measured by the score criterion

$$
\begin{aligned}
\hat{B} &= \underset{B}{\operatorname{argmin}} \left[\sum_{s=1}^{S} \sum_{t=1}^{T} \frac{\partial \ln f^{ESTAR}}{\partial A} \left(p_t^s(B) | p_{t-1}^s, A_0\right) \right]' \\
&\times \quad \Omega \times \left[\sum_{s=1}^{S} \sum_{t=1}^{T} \frac{\partial \ln f^{ESTAR}}{\partial A} \left(p_t^s(B) | p_{t-1}^s, A_0\right) \right],
\end{aligned}
$$

where Ω is a nonnegative, symmetric weighting matrix.

Applied jointly, this model and estimation approach provide multiple advantages over other approaches. First, one time series per country suffices to estimate the relocation cost. Second, it exploits the information in the time series at each point in time. Third – in contrast to an extremum estimator or a hard cut-off threshold model – drift and diffusion reveal information about the location of the threshold – and thus border cost – even when the process is far away from the threshold. Overall, the structural features of the economy, such as relocation cost, which bind real allocations only in the *long* run, are extracted at high frequency from the drift and diffusion of exchange rates of an arbitrarily *short* time series.

4 Productive Capabilities

The one-good model introduced in the previous section allows for two interpretations of the generic good: as final good or as productive capability.[14] Under the final good interpretation

[14] As shown in B.2, the one-good model of arbitrage-induced trade (1) can be seen as a reduced form of a two-good model featuring both final goods and productive capabilities. The two-good model reveals that

K is a consumption good, such as "grain", which reproduces itself at rate α and can be consumed. Under the productive capability interpretation the good is "knowledge", which also reproduces itself, and is, via an (not modelled) production process, converted into a consumable, e.g. "movies".

If, on the one hand, the model is in terms of final goods, then the relocation cost, r, reflects first and foremost the cost of shipping. If, on the other hand, the model is in terms of productive capabilities, then r contains a second component: the cost of relocating organizational structures and knowledge necessary for operating the physical good. The overall relocation cost for productive capabilities can therefore be higher than for final goods, even if the shipping cost for productive capabilities is small. In order to empirically verify this hypothesis, I derive in this section a measure of the real exchange rate for productive capabilities and compare its properties with a standard real exchange rate for final goods.

4.1 Measuring Real Exchange Rates

The model described in section 3 allows estimating the relocation cost directly from real exchange rate time-series data. Typically the real exchange rate is calculated for final goods, and based on the CPI, WPI, or deflators of the gross domestic product (GDP). Each of these excludes a large share of productive capabilities, in particular immaterial goods. To compare the border effect of final goods with that of productive capabilities, I need one real exchange rate series for each.

For final goods I use the standard real exchange rate based on the WPI, which captures the bulkiness and business-to-business nature of these goods in international transactions.

Unfortunately, for productive capabilities no appropriate real exchange rate is readily available. Productive capabilities are factors in operation in the productive sector of the economy. They are typically owned by a firm, need to be combined with other capital goods in order to be fully useful, and are often intangible (e.g. in the form of patents and know-how, as in Burstein and Monge-Naranjo (2009)). A major share of productive capabilities, except machinery, can be considered complementary factors. In the case of information technology investment, for example, only about one third of expenditures are invested in hardware and prepackaged software, whereas the rest is spent for complementary capital such as training, support, and custom software, which is a necessary requirement for the hardware to be useful within the productive sector (Basu, Fernald, Oulton, and Srinivasan, 2004; Kiley, 1999).

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the dynamics of the exchange rate for each good separately can be described by the one-good model.

Clearly, none of the aforementioned real exchange rates focuses on productive capabilities, and for the most part they do not contain any productive capabilities at all. I therefore construct a real exchange rate series tailored to capture the valuation differences of productive capabilities between countries.

The market value of all productive capabilities, or invested capital goods, in a given country can be written as

$$M(t) = V_K\left(K(t), K^*(t)\right) K(t) e(t),$$

where $e(t)$ is the nominal exchange rate to a numeraire currency. Likewise, for completely equity-funded firms, book values of all capital goods in a given country are

$$B(t) = V_G(t) K(t) e(t) \varphi,$$

where $V_G(t) \equiv V_{G^*}(t)$ denotes the exogenous world market price of an uninvested capital good, and φ a time-invariant, country-specific accounting constant. Therefore $\frac{M(t)}{B(t)} = \frac{V_K(K(t),K^*(t))}{V_G(t)\varphi}$. Solving for V_K and plugging into (5) I get

$$p(\omega) = \frac{M(t)/B(t)}{M^*(t)/B^*(t)} \frac{\varphi}{\varphi^*}. \tag{7}$$

The real exchange rate for productive capabilities can therefore be written in terms of (inflation-adjusted) market-to-book ratios, M/B. Stock indices measure the total market *value* of productive capabilities of firms included in this stock index. The underlying *quantity* is captured by the aggregate book values, after adjusting it for the effect of inflation. Normalizing the stock indices by adjusted book values removes the effect of nominal exchange rates and of quantity changes via e.g. retained earnings or international relocation of productive capabilities, because the book values share the currency unit of the market value M, and return a measure of the value of one unit of productive capability. For countries with identical accounting standards the real exchange rate of productive capabilities is thus simply the ratio of average market-to-book values.[15] In the end, the crucial assumption for (7) to be a meaningful measure of the real exchange rate of productive capabilities is that after removing the global stock market cycle and both countries' inflation processes the remaining variation in this ratio is not meaningless financial market noise, but rather a noisy-looking

[15]If countries differ in accounting standards or leverage levels, $p(\omega)$ can be corrected for the constant factor $\frac{\varphi}{\varphi^*}$ by setting midrange of $log(p(\omega))$ to zero.

process *regulated* by the arbitrage bounds imposed by model (1).

My use of market-to-book ratios differs from its interpretation in standard q-theory (Hayashi, 1982). Tobin's q in my model is always unity, because there are no adjustment costs of investment *within* a given country. The ratio of two countries' market-to-book ratios, however, measures the *relative* price of productive capabilities in these two countries and thus captures the "adjustment cost" of investment *across* a national border. Nevertheless, the stylized fact that market-to-book ratios are inversely related to future equity returns (Fama and French, 1992, 1998; Pontiff and Schall, 1998) applies in my model: High relative market-to-book ratios between two countries lower future relative returns.

This concept of the real exchange rate based on productive capabilities has two advantages. Firstly, it allows studying the properties of the market for productive capabilities in isolation from markets for other goods. In combination with my estimation approach, it enables me to estimate relocation costs of productive capabilities from macroeconomic data. With microeconomic data this has not been possible so far, because data on the cost of relocating productive capabilities across borders on a per-project basis is usually not publicly available.[16]

Secondly, because one component of market-to-book ratios is determined in financial markets, it responds quickly to new information. This mitigates problems with CPI and WPI data, which are subject to aggregation bias (Imbs, Mumtaz, Ravn, and Rey, 2005) and non-synchronous sampling (Taylor, 2001). The valuation component of this real exchange rate, the market values, are synchronously sampled worldwide in centralized markets in a standardized and automated manner. It is collected in real time, and not subject to revisions. Further, aggregation to a country stock index is transparent and largely internationally comparable.

4.2 Data

I collect data from various editions of Capital International Perspective (1975-2008). This dataset from Morgan Stanley Capital International contains monthly nominal exchange rates, stock price indices and consistently calculated market-to-book ratios for these indices of 18 developed countries for the period December 1974 to December 2008.

[Figure 3 about here.]

[16] Available data, such as data on FDI, which might be used to identify frictions between countries, measures only financial flows, but not the underlying flow of productive capabilities.

The market-to-book ratios vary substantially over time. The solid line in figure 3 shows that the equal-weighted average market-to-book ratio trended upward over the last 30 years, with large transitory upward bursts, but returned close to unity during the 2008 financial crisis. The variation across countries does not show any trend during the same period. In periods of a high market-to-book ratio average, however, the variation increases temporarily. This indicates that except in the recent 2005–2007 boom not all countries participated in these transitory upward bursts. After a few years variation returns back to the long-term base level. This foreshadows mean reversion of relative market-to-book values between countries, and thus of the real exchange rate for productive capabilities.

I correct book values for the effect of inflation, using WPI and CPI data provided by the International Monetary Fund's (IMF) International Financial Statistics database.[17] This correction adjusts, for example, the original bookvalues for Germany upwards during the high inflation periods of the 1970s.

I calculate the real exchange rate for final goods based on the monthly WPI from the IMF International Financial Statistics database for the same countries and time period. Subsequently, I demean the data and remove any seasonality effects by regression on a set of monthly dummies to account for cyclicality in economic activity or reporting.

[Figure 4 about here.]

Figure 4 compares my two measures of the real exchange rate.[18] The real exchange rate based on productive capabilities, represented by the solid line, moves less steadily than the real exchange rate for final goods, represented by the dashed line. Quite striking is the difference in evolution of the real exchange rate of productive capabilities during and after the communication technology boom. Canadian productive capabilities, shown in the top panel, reached their 30-year low in value relative to the USA shortly before the peak of the communication technology boom, reflecting the delayed growth of this sector in Canada. In contrast, German productive capabilities in the middle panel reached their all-time low in the after-technology-boom recession, which suggests that the 2002 recession had freed up more productive capabilities in Germany than in the USA.

[17] Inflation adjustment is based on a hypothetical firm investment cycle. I assume a degressive depreciation schedule at a depreciation rate δ of 10%, i.e. $B_t = \sum_{i=0}^{\infty} \delta^i I_{t-i}$, which lets me calculate the approximate path of investment over time, $I_t = B_t - \delta B_{t-1}$. Adjusted bookvalues are therefore $\tilde{B}_t = \sum_{i=0}^{\infty} \delta^i I_{t-i} \Pi_{t-i}^t$, where Π_{t-i}^t denotes the WPI price deflator between period t and $t-i$. C provides graphs of the effect of this book value correction.

[18] F presents graphs for additional country pairs.

My model predicts a nonlinear relationship between returns and current levels of the real exchange rate. This kind of relationship is known to exist for final goods (Michael et al., 1997),[19] and I find a similar relationship for productive capabilities. I test the real exchange rate series of each of the 153 possible country pairs for ESTAR-type nonlinearity, using a Granger and Teräsvirta (1993)-type test. This test is based on a second-order Taylor approximation of the ESTAR function around $\theta = 0$.[20] As expected, real exchange rates based on productive capabilities of most country pairs follow a nonlinear process as well. 52% of country pairs reject linearity at the 5% level, and 76% at the 10% level, in favor of ESTAR-type nonlinearity. As illustration, figure 5 plots two-year changes in the log real exchange rate of productive capabilities against the initial log levels. Both country pairs feature random walk behavior close to the parity level and strong mean reversion away from parity. Visual inspection thus already indicates nonlinearity with two regimes.[21]

[Figure 5 about here.]

My approach differs in important ways from the study of Engel and Rogers (1996) on the "width of the border", addressing concerns pointed out by Gorodnichenko and Tesar (2009). For example, I do not rely on the unconditional variance of the real exchange rate as dependent variable. My estimates are identified by time variation of country aggregates, not by within-country cross-section variation, and are therefore unaffected by differences in within-country price dispersion. Further, the real exchange rate for productive capabilities is – by virtue of the speed of financial markets – less persistent than a real exchange rate based on the CPI, and by its definition immune to pure nominal exchange rate changes.

5 Empirical Results

I apply my estimation procedure to the real exchange rates of both final goods and productive capabilities with the USA as base country. The estimated relocation cost for final goods given

[19] Broda and Weinstein (2008) find nonlinearity at the product level. At such a disaggregated level, however, nonlinearity may primarily be driven by strategic pricing: Temporary sale events by manufacturers or correlated sale events of distributors in limited marketing regions are very common. It is the very essence of a sale event that sale prices revert quickly back to their long-run levels. This type of mean reversion at the product level, however, does then not reflect international arbitrage or reversion back to an international PPP.

[20] D describes this test in more detail.

[21] Further, the variance appears higher in the center than in the outer regime, in line with the predictions of my model. This may, however, be an effect of the relatively small number of observations in the outer regime.

in the first column of table 4 ranges from 1% for Canada–USA up to 26% for France–USA, with a median of about 14%.

The picture changes dramatically when I shift my focus to productive capabilities. The right half of table 4 ranks the same countries again, but now by their cost of relocating productive capabilities to and from the USA. This heavily reshuffles the ordering of countries, which means that for these countries the two markets are very different from each other. The distant large economy Japan, for example, has a high relocation cost for final goods (22%) to the USA, but only 5% for productive capabilities. Similarly, borders to Australia and the UK seem to impose less frictions on productive capabilities than on final goods. Hongkong and Norway, on the contrary, are economies which are quite open for trade in final goods, but largely closed for transfers of productive capabilities. The variation of relocation costs of productive capabilities across countries is considerably higher as well, with costs ranging from 5% to 56%. This implies that despite the similar median (17%) of productive capabilities, the average relocation cost is with 22% much higher than the one for final goods (15%). The tradeability of productive capabilities appears much more location-specific than the one of final goods.

But there are also similarities: The direct neighbor of the USA, Canada, has a very low relocation cost to and from the USA for both classes of goods, 1% for final goods and 6% for productive capabilities. For countries like Belgium, Denmark, Germany, Italy, and Switzerland both goods are equally costly to relocate, implying that both markets are equally easy to access.

Figure 6 compares the relocation cost of final goods with the corresponding cost of productive capabilities directly. The bulk of relocation costs is in the range of 10% to 20%. Relocation cost between Canada and the USA are minimal - no country is closer geographically, culturally, and on top of this member of the same trade block. In contrast, France appears to be very distant from the US along any of these dimensions, causing a very high relocation cost for final goods and productive capabilities.

The close cultural ties between the Anglosaxon countries, Australia and UK, with the USA are reflected in a relocation cost of productive capabilities that is even lower than the cost of moving final goods. Norway and Hongkong display the very pronounced cost pattern already mentioned. Whereas its trade costs are relatively low, their relocation costs for productive capabilities are much larger. In the case of Norway, this is most likely due to an industry specific to local natural resources. In the case of Hongkong, this points to differences in openness between the two types of goods. Surprisingly, Japan shows the opposite pattern.

Whereas its goods market is hard to access from the USA (and vice versa), its productive capabilities flow relatively freely to and from the USA.

Overall, relocation costs are only weakly correlated between the two types of goods, because Hongkong, Norway, Spain, and Sweden appear to be somewhat disconnected from the market for productive capabilities.

[Figure 6 about here.]

[Table 4 about here.]

Returning to the left half of table 4, which ranks countries by their cost of transferring final goods to and from the USA, reveals that by far the lowest border cost obtains between the USA and Canada. Only 1% of the quantity of final goods crossing this border is lost in the form of iceberg costs. On the other end, a transfer of final goods from Japan to the USA incurs a cost of more than 20%. Besides the distant economy Japan, relatively small or protected markets such as Austria, Belgium, France, or Sweden are among the countries with the highest trade cost to and from the USA.

I compare my estimates with the ratio of cost-insurance-freight (*cif*) and free-on-board (*fob*) prices provided by the IMF's International Financial Statistics database for a few countries. For countries in my sample both *cif* and *fob* values of imports are available for Australia, France, Germany, UK, and USA. The average *civ*/*fob* ratio for these countries fell from approximately 6% in 1975 to approximately 4.5% today. All relocation cost estimates except for Canada exceed this level. This emphasizes that transfer cost consists of more than just insurance and freight. Other expenses, such as e.g. administration, customs, distribution, and market access costs devour up to another 20% of the transferred quantity in the case of France and Japan.[22]

In order to put my estimates for productive capabilities into perspective, I compare them with estimates of adjustment costs. The adjustment cost of capital goods within a country, i.e. the difference between the productive capability's value as part of a firm and the proceeds from selling the dismantled capital good, provide an upper bound on how large relocation costs of productive capabilities within a country can be. In a setup similar to Abel and Eberly (1994) and Abel and Eberly (1996), where the only form of adjustment

[22]These border cost estimates are somewhat smaller than the average estimate in the literature. The transfer cost equivalent of the estimate in the survey of Anderson and van Wincoop (2004) is more than 20% for a representative rich country. Gopinath, Gourinchas, Hsieh, and Li (2009) find for a grocery chain price differences of 20% for even the US-Canadian border.

cost is a gap between buying and selling prices of capital goods, Cooper and Haltiwanger (2006) find for the USA an adjustment loss of about 20%. Many of these intra-US adjustment losses are larger than my estimates for relocation costs. For most country pairs in my sample a direct relocation of productive capabilities is therefore cheaper than a disinvestment and subsequent transfer as final good, whereas for Hongkong, Norway, and Spain the opposite is likely the case.

The relocation cost parameter, r, captures most of the nonlinearity of the real exchange rate processes – the key feature of the auxiliary model– and is thus estimated with a small standard error. The relocation discounts differ significantly from zero and unity for all country pairs. That is, for both real exchange rates I reject both of the extremes random walk ($r = 0$) and constant ($r = 1$).[23]

My parameter estimates allow me to graphically compare the drift and diffusion processes. As an example, I discuss here the real exchange rates of the pair Germany–USA. Figure 7 emphasizes the very different behavior of the exchange rates for final goods and productive capabilities despite similar border cost. The real exchange rate process for final goods, shown by the dashed line, follows a much narrower band with lower diffusion than the process for productive capabilities. The mean reversion properties of final goods resemble those of productive capabilities, but cut off at tighter boundaries, which limits the maximum mean reversion of the real exchange rate of final goods.

[Figure 7 about here.]

The upper panel of this figure also shows that the drift of the real exchange rates of productive capabilities at parity is negative, which is an effect of the higher productivity in productive capabilities in Germany relative to the USA and the, as a result, relative abundance in Germany. Because the USA in this particular example reproduces productive capabilities a lower rate, only a large abundance in Germany, i.e. a small p, triggers a relocation of productive capabilities from Germany to the USA.

Figures 8 and 9 compare the real exchange rate process for productive capabilities of Germany–USA, shown as dashed line, with other country pairs.

[23] Whereas the other parameters primarily serve to fit the model, their averages deserve some attention on their own: The median estimated productivity differential relative to the USA, $\Delta\alpha$, is negative for both types of goods. This indicates that during the sample period the USA was better than other countries in the sample at producing final goods as well as productive capabilities, e.g. ideas. The mean volatility estimate, σ, for final goods of 0.44 is considerably smaller than the estimate for productive capabilities of 0.74. Clearly, the creation process of productive capabilities involves more risk (Schumpeter (1942)'s "creative destruction"). E provides detailed results for all 17 country pairs.

[Figure 8 about here.]

The process for Japan–USA in Figure 8 moves with a larger variance within a much wider band. Its mean reversion is smaller than for Germany–USA. The large diffusion obtains despite the lower shock estimate (σ) for Japan. The high relocation cost for Japan and the thus infrequent relocations amplify the effect of shocks on $p(\omega)$.

[Figure 9 about here.]

The solid line in Figure 9 displays the pair UK–USA, which has a mean reversion as strong as, but somewhat slower than, Germany–USA. Its real exchange rate process is, unlike Germany–USA, almost symmetric.

The finding that the cost of relocating productive capabilities from one country to another is larger than the corresponding cost for final goods raises the question: What are the underlying causes for these costs? To answer this, I regress the border cost measures on four explanatory factors commonly used in international trade. The independent variables are distance, the absolute 2003 GDP difference, a dummy for common language, and the 2008 indicator of barriers to entrepreneurship taken from the 2008 OECD Product Market Regulation Database.

[Table 5 about here.]

Table 5 reveals that geographic distance and language barriers affect the relocation cost for final goods, but not for productive capabilities. For every 1000 kilometers of geographic distance, the relocation cost for final goods increases by more than one percent of the transferred quantity. Language barriers increase this cost by about seven percent.

For productive capabilities, neither of these matters significantly. Instead, barriers to entrepreneurship, such as regulatory opacity, administrative burdens on startups, and barriers to competition, as well as a small country size significantly increase the relocation cost of productive capabilities.

[Table 6 about here.]

These results admittedly have to be taken with a grain of salt given the small sample. Nevertheless, they show that there is more to a border than just geographic distance, a point made by e.g. McCallum (1995) and Engel and Rogers (1996). For final goods this “border effect” are primarily obstacles to trade, partly reflected in the “common language” dummy in the regression. But the "border effect" is not limited to physical goods. Relocation of productive capabilities, by their typically intangible nature, is not impaired by distance or language at all. Instead, what hinders the flow of productive capabilities across borders appear to be barriers to entrepreneurship in one of the countries. Furthermore, the economic size of a country determines the range of productive capabilities in use. Small countries (relative to the USA) might specialize in a few sectors. The lack of exchange of productive capabilities in the other sectors might explain the high overall border cost in the aggregate for smaller countries.

The difference of my two measures of relocation cost, r_{WPI} and r_{CAP}, becomes even more vivid when they serve as explanatory variable to crossborder mergers. Table 6 reveals that relocation cost of final goods does not explain merger and acquisition (M&A) activity, simply because M&A does not involve the relocation of physical goods. In contrast, columns (3) and (4) show that the cost reflected in prices of productive capabilities have additional explanatory power of the M&A activity with the U.S., even after controlling for distance, language, and GDP. This is not surprising, because M&A – despite not moving any physical goods – usually requires a transfer of knowledge.

Having established mean reversion as a property of the real exchange rate, it is natural to ask how long complete reversion to the mean or to PPP takes. For this purpose I calculate the time that it takes the real exchange rate process on average to hit a boundary when started at a balanced level, and the time it takes to revert to this level if started at either boundary. Because the exchange rate processes are typically asymmetric, these so-called "hittimes" depend on the direction of the initial PPP violation. Hittimes are obtained by integrating over the appropriately scaled speed density (Karlin and Taylor, 1981).

[Table 7 about here.]

A good example are the hittimes for Japan-USA in Table 7. The table makes an important distinction between the time to revert to a quantity-wise balanced world ($\omega = 0$), shown in the upper panel, and the time to revert to PPP ($ln(p) = 0$), shown in its lower panel. PPP, in general, does not imply balanced quantities across countries. In fact, some countries never reach complete balance relative to the USA. Therefore, whereas the reversion time to PPP is defined for all country pairs, because it is a value-based measure, the reversion time to a balanced level is not.

No matter whether one cares more about reversion to quantity-balance, or reversion to PPP – Japan's mean reversion time for final goods qualifies for the label "glacial" (Rogoff,

1996) – reversion to PPP takes five years and more. But thanks to a much smaller relocation cost and higher variance the hittime for productive capabilities is much shorter, in the range of two to three months.

The hittimes discussed so far differ from the frequently used "half-times" calculated from first-order autoregressive models in that they measure the time until the desired level is actually reached, not just the time until the process is halfway there. For comparison with common "half-time" measures, the lower panel of Table 7 also reports the time it takes the real exchange rate process to progress from one of its extremes half-way to PPP. Because of the divergent behavior of the underlying real imbalances, the halftimes are less than one half of the time of complete reversion to PPP. That is, for real exchange rates that follow the model described in Section 2, the halftime measure understates the persistence of real exchange rates.

[Table 8 about here.]

This is a result that also holds for the average of all country pairs: Table 8 shows implied times until a complete reversion to PPP from a maximum imbalance for all country pairs, again with the USA on one side. The PPP level, to which each exchange rate reverts, is an estimate itself and differs among countries and type of good. For the median country reversion to PPP for productive capabilities takes about one year. This is much faster than the reversion of the real exchange rate of final goods, which is more than two years, despite the lower border cost estimates for final goods.

A more detailed look at the table reveals for about half of the countries the PPP-reversion from either imbalance takes about the same time. But this is not a necessity. For the other countries a non-zero productivity differential combined with a PPP-level at a non-zero imbalance causes the PPP-reversion times to differ widely between the two starting imbalances. For final goods, PPP-reversion is quickest for Norway with less than six months, and slowest for Sweden with several decades. PPP-reversion for productive capabilities is generally much quicker – for Canada and Japan is takes only about three months. For slow reversion countries, like Hongkong, Norway and Spain, this can take nevertheless up to almost four decades.

Table 8 also illustrates that a narrow range of the real exchange rate is *not* equivalent with fast mean reversion. In fact, a narrow range for p does not necessarily imply a narrow range for ω. The deterministic drift weakens as the real exchange rate approaches the balanced level, and it is completely up to random shocks to push the exchange rate through its zero-drift level. Canada's small border cost for final goods, for example, estimated at one percent, does not lead to a faster PPP-reversion than of Norway with a border cost of six percent.

The hittime distribution is typically very skewed. Figure 10 shows that a large number of shocks to PPP decay quickly within, say, a year. This is offset by a smaller number of extremely slowly decaying deviations, with hittimes of three years and more. Taken together this may explain the inconclusiveness of the data about halftimes of mean reversion documented by Kilian and Zha (2002).

[Figure 10 about here.]

In summary, borders affect productive capabilities more than final goods. Whereas the classic trade frictions, geographic distance and language barriers, impair trade in final goods, they have no significant effect on productive capabilities. Instead, relative economic size and barriers to entrepreneurship hinder the relocation of productive capabilities. Despite different causes, in effect, for both goods returning from a maximum imbalance to a balanced position can take several years, but is typically much shorter for productive capabilities because of the larger volatility of the underlying shocks. In effect, country-idiosyncratic shocks are as important for mean reversion speeds as are border costs.

6 Conclusion

This paper provides three important insights: First, as further integration of the world economy continues to shrink the barriers to trade, the range of values assumed by the real exchange rate will shrink as well. But if country-idiosyncratic volatility shrinks at the same time, mean reversion might in fact *not* accelerate, despite shrinking barriers. Thus no matter how easy it eventually will become to relocate goods, the random walk behavior in the inner exchange rate regime will ensure that reversion to PPP will remain slow.

Second, even if relative prices never reach the arbitrage points, these unobserved points can nevertheless be estimated from their effect on the nonlinear dynamics of the real exchange rate process far away from the arbitrage points. Thus border cost can be estimated from time series only, without observing any arbitrage activity. Embedding the time series dynamics in a structural model of real imbalances allows calculating the time complete reversion to PPP takes. This measure is more meaningful than the popular halftime which rest on the counterfactual assumption that PPP will be reestablished only in the infinite future. The results also reveal that there is long-run predictability of real exchange rates in line with

Kilian and Taylor (2003) despite short-run random walk behavior, albeit unfortunately not useful for risk-free arbitrage. Nevertheless, the key parameters border cost and country-idiosyncratic shock variance allow to calculate the expected real exchange rate return and variance for each observable exchange rate level.

Third, the global markets for final goods and productive capabilities behave markedly different. Borders hinder the relocation of productive capabilities considerably more than the one of final goods, and the former are subject to larger shocks. Thus whereas the final goods market is without doubt a very important one, limiting the analysis of real exchange rates to final goods misses a lot. In particular, my work emphasizes that for a meaningful forecast the *definition* of the real exchange rate must be tailored to the market of interest. After all, the real exchange rate for final goods might have little to say about foreign direct investment.

Table 1: Maximum Sustainable Imbalance ($\overline{\omega}$)

(a) for Small Relocation Cost ($r = 0.9$)

Risk av. $(1-\gamma)$	Risk ($\sigma_{11} = \sigma_{22}$) 0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	1.23	1.64	2.63	3.25	3.29	3.30
1	1.11	1.41	2.11	2.56	2.59	2.59
1.5	1.07	1.34	1.94	2.29	n.a.	n.a.
2	1.05	1.31	1.86	2.13	n.a.	n.a.
3	1.04	1.28	1.75	n.a.	n.a.	n.a.

(b) for Large Relocation Cost ($r = 0.75$)

Risk av. $(1-\gamma)$	Risk ($\sigma_{11} = \sigma_{22}$) 0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	1.78	2.32	3.95	5.88	6.06	6.13
1	1.33	1.69	2.76	4.12	4.26	4.31
1.5	1.21	1.54	2.46	3.53	n.a.	n.a.
2	1.15	1.47	2.31	3.21	n.a.	n.a.
3	1.10	1.41	2.15	n.a.	n.a.	n.a.

The other parameter values are $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$.

Table 2: Expected Time until Reversion to PPP, in years

(a) for Small Relocation Cost ($r = 0.9$)

risk av. $1-\gamma$	risk σ 0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	∞	3317	81.07	2.96	0.74	0
1	∞	2675	52.06	1.83	0.46	0
1.5	∞	2226	39.79	1.40	n.a.	n.a.
2	∞	1911	32.86	1.16	n.a.	n.a.
3	∞	1507	25.12	n.a.	n.a.	n.a.

(b) for Large Relocation Cost ($r = 0.75$)

risk av. $1-\gamma$	risk σ 0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	∞	6653	195.41	7.09	1.76	0
1	∞	7433	131.77	4.33	1.07	0
1.5	∞	8564	104.21	3.31	n.a.	n.a.
2	∞	10012	87.93	2.75	n.a.	n.a.
3	∞	13609	69.01	n.a.	n.a.	n.a.

The other parameter values are $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$. PPP is given by $ln(p) = 0$. The process is started at the maximum imbalance $\omega_0 = \overline{\omega}$ given by Table 1.

Table 3: Stationarity of ESTAR Regimes

	inner regime (AR)		outer regime (AR $\times\Phi$)	
	avg. root	non-stationary	avg. root	non-stationary
final goods	1.009	29%	1.068	0
productive capabilities	0.999	53%	1.080	0

Table 4: Relocation Cost vs. USA

Final Goods: r_{WPI}		Productive Capabilities: r_{CAP}	
Canada	0.99 (0.00)	**Japan**	0.95 (0.00)
Norway	0.94 (0.01)	**Canada**	0.94 (0.00)
Hongkong	0.92 (0.01)	**UK**	0.90 (0.00)
Netherlands	0.89 (0.02)	Australia	0.88 (0.01)
Italy	0.89 (0.02)	Italy	0.88 (0.01)
Germany	0.88 (0.02)	**Germany**	0.87 (0.01)
Denmark	0.86 (0.02)	Denmark	0.86 (0.01)
Singapore	0.86 (0.02)	Switzerland	0.84 (0.01)
UK	0.86 (0.02)	Austria	0.83 (0.01)
Australia	0.84 (0.01)	Belgium	0.82 (0.02)
Switzerland	0.84 (0.02)	Singapore	0.81 (0.01)
Spain	0.82 (0.01)	Netherlands	0.80 (0.00)
Belgium	0.81 (0.01)	France	0.77 (0.01)
Austria	0.79 (0.01)	Sweden	0.63 (0.03)
Japan	0.78 (0.01)	Spain	0.52 (0.03)
Sweden	0.78 (0.00)	Norway	0.50 (0.02)
France	0.74 (0.07)	Hongkong	0.44 (0.04)

Inflation-adjusted bookvalues, demeaned and deseasonalized, 1974:12–2008:12.
ESTAR as auxiliary model with $2 \le p \le 4$ and $d \le 12$ chosen by nonlinearity test.
Standard errors in parentheses.

Table 5: Components of Relocation Cost

	r_{WPI} (1)	r_{WPI} (2)	r_{CAP} (1)	r_{CAP} (2)
Distance	-0.09***	-0.11***	-0.06	0.01
	(0.03)	*(0.03)*	*(0.10)*	*(0.08)*
Common language	-0.08**	-0.07**	-0.06	-0.09
	(0.03)	*(0.03)*	*(0.08)*	*(0.07)*
Δ GDP	0.01	0.01	-0.07**	-0.07**
	(0.01)	*(0.01)*	*(0.02)*	*(0.02)*
Barriers to Entrepreneurship		0.03		-0.19***
		(0.02)		*(0.04)*
Constant	0.79***	0.79***	1.55***	1.68***
	(0.12)	*(0.13)*	*(0.25)*	*(0.23)*
obs.	17	16	17	16
Adj. R^2	0.24	0.20	0.04	0.27

Dependent variable: relocation cost r, independent variables: distance (in 10000 km), common language (0 if common language, 1 otherwise), absolute GDP difference (million USD at PPP, 2003), barriers to entrepreneurship (0-6, where 6 is most restrictive). Robust standard errors in parentheses. Asterisks indicate the level of significance, (*) for 10% level, (**) for 5% level, and (***) for significance at the 1% level.

Table 6: Relocation Cost as Determinant of Crossborder Mergers

	Total Mergers/ Total GDP			
	(1)	(2)	(3)	(4)
r_{WPI}		-275		-186
		(168)		*(151)*
r_{CAP}			136***	122***
			(32)	*(35)*
Distance	-187***	-211***	-176***	-194***
	(30)	*(33)*	*(30)*	*(32)*
Common language	-130***	-152***	-124***	-140***
	(27)	*(29)*	*(24)*	*(29)*
Constant	181***	430**	68**	249
	(25)	*(160)*	*(26)*	*(152)*
obs.	17	17	17	17
Adj. R^2	0.75	0.77	0.82	0.83

Dependent variable: Total crossborder mergers between any pair of countries divided by the sum of GDP (in USD millions) of these two countries; independent variables: relocation cost of final goods r_{WPI}, relocation cost of productive capabilities r_{CAP}, distance (in 10000 km), common language (0 if common language, 1 otherwise). Robust standard errors in parentheses. Asterisks indicate the level of significance, (*) for 10% level, (**) for 5% level, and (***) for significance at the 1% level.

Table 7: Selected Hittimes, Japan–USA, in years

(a) Real Imbalance Levels

ω-process first hit of	ω_0	final goods	productive capabilities
any boundary, $\overline{\omega}$ or $\underline{\omega}$	0	3.6	0.2
upper boundary, $\overline{\omega}$	0	8.3	0.7
lower boundary, $\underline{\omega}$	0	21.6	0.8
balanced level, $\omega = 0$	$\overline{\omega}$	3.2	0.2
balanced level, $\omega = 0$	$\underline{\omega}$	8.6	0.3

(b) PPP-based Levels

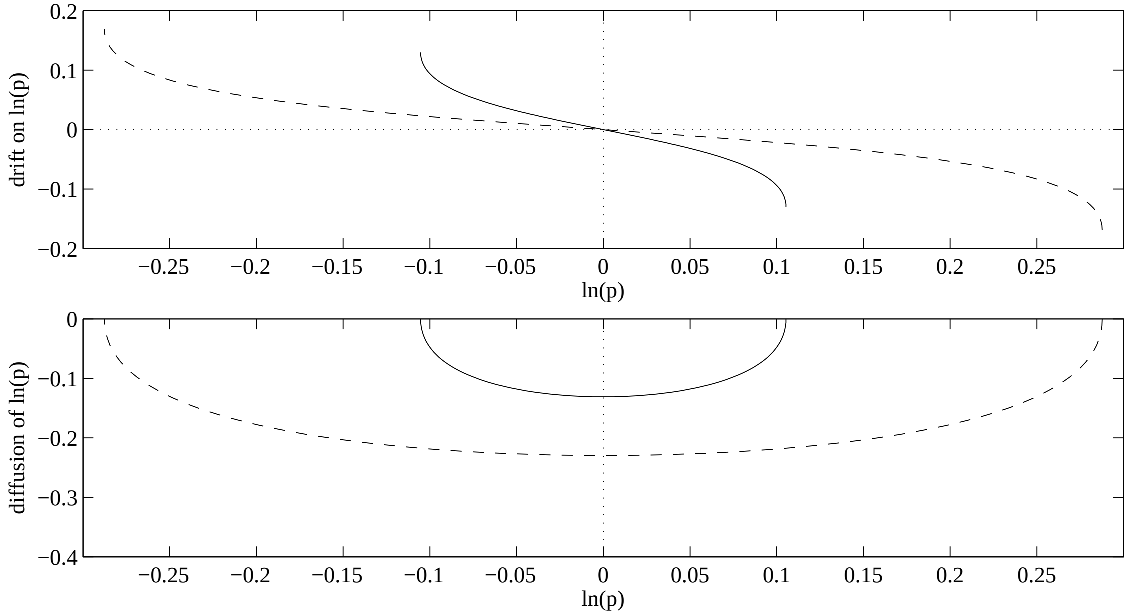
p-process first hit of	ω_0	final goods	productive capabilities
any boundary, $p(\overline{\omega})$ or $p(\underline{\omega})$	ω_{PPP}	4.2	0.2
upper boundary, $p(\overline{\omega})$	ω_{PPP}	11.9	0.7
lower boundary, $p(\underline{\omega})$	ω_{PPP}	17.5	0.7
PPP, $p = 1$	$\overline{\omega}$	7.2	0.2
PPP, $p = 1$	$\underline{\omega}$	5.0	0.2
PPP "halftime", $\sqrt{p(\overline{\omega})}$	$\overline{\omega}$	3.0	0.1
PPP "halftime", $\sqrt{p(\underline{\omega})}$	$\underline{\omega}$	2.1	0.1
ω_{PPP}		-0.37	-0.04

Table 8: Years Until Full Reversion to PPP

	Final Goods		Productive Capabilities	
	$\overline{\omega}$	$\underline{\omega}$	$\overline{\omega}$	$\underline{\omega}$
Australia	3.3	1.4	0.6	0.6
Austria	8.1	2.1	0.8	0.8
Belgium	5.7	2.2	1.0	1.4
Canada	0.7	25.7	0.3	0.3
Denmark	2.6	2.3	0.7	0.7
France	12	8.6	3.2	1.0
Germany	2.1	1.6	1.0	0.6
Hongkong	3.7	2.6	7.3	10.7
Italy	1.8	1.5	0.4	0.4
Japan	7.2	5.0	0.2	0.2
Netherlands	1.8	1.5	38.0	3.9
Norway	0.4	0.4	7.6	14.7
Singapore	9.1	6.8	1.0	0.9
Spain	5.3	1.2	5.0	18.9
Sweden	243	8.6	3.3	7.8
Switzerland	2.6	2.2	1.2	1.9
UK	2.6	2.4	0.9	0.5
Median	3.3	2.2	1.0	0.9

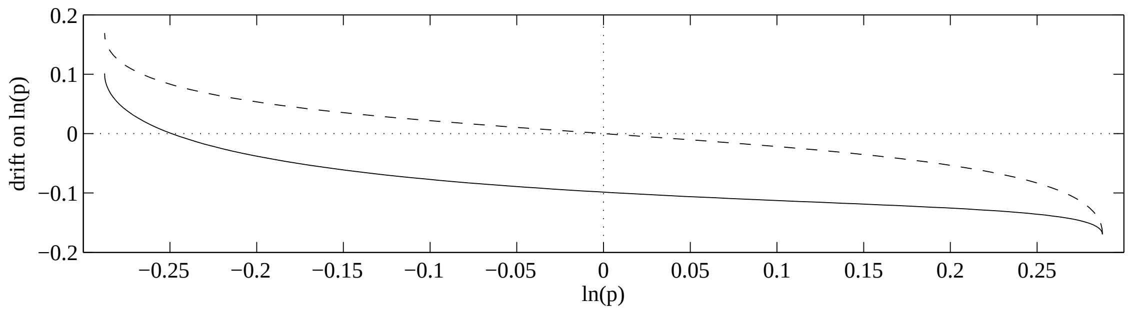
Time until full reversion of real exchange rate vs. USA to PPP in years, starting from a situation of maximum imbalance ($\overline{\omega}$ or $\underline{\omega}$).

Figure 1: Drift and Diffusion of Price Process, Effect of Relocation Cost



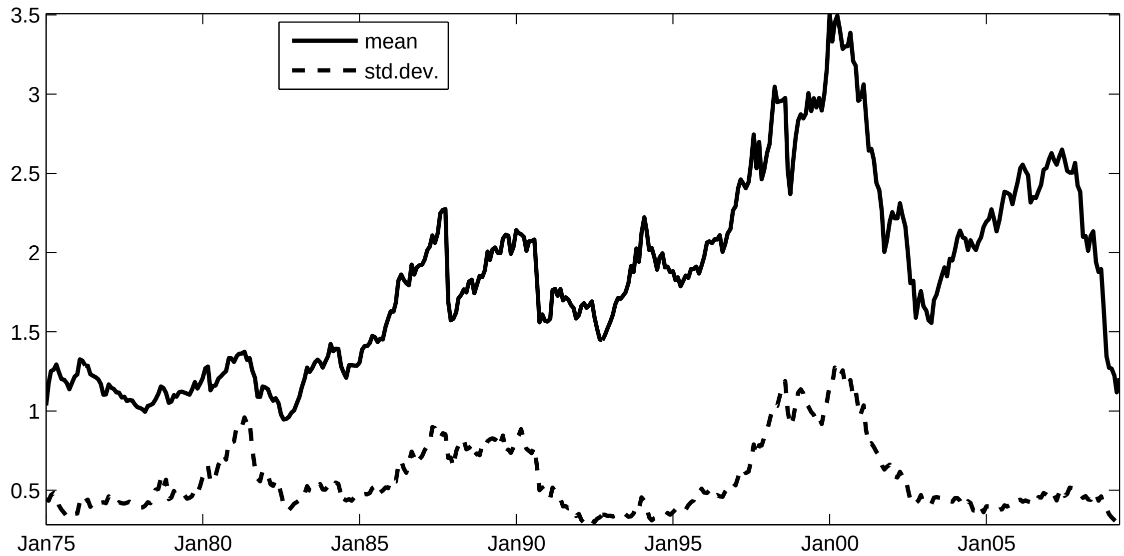
The upper panel shows the drift of the natural logarithm of the real exchange rate process as a function of the natural logarithm of the current real exchange rate, $ln(p)$. The lower panel shows the corresponding (signed) diffusion. It compares the process for a low relocation cost ($r = 0.9$, solid line) with the process for a high relocation cost ($r = 0.75$, dashed line). The other parameter values used for this graph are $\gamma = -0.5$, $\rho = 0.07$, $q = 1$, $\alpha = \alpha^* = 0.1$, and $\sigma_{11} = \sigma_{22} = 0.5$.

Figure 2: Drift of Price Process, Effect of Productivity Differences



The graph shows the drift of the natural logarithm of the real exchange rate process as a function of the natural logarithm of the current real exchange rate, $ln(p)$. The dashed line shows the process for the case when both countries are identical ($\alpha = \alpha^* = 0.1$). The solid line obtains if country 1 has a higher productivity than country 2 ($\alpha = 0.3 > \alpha^* = 0.1$). The other parameter values used for this graph are $\gamma = -0.5$, $\rho = 0.07$, $q = 1$, $r = 0.75$, $\sigma_{11} = \sigma_{22} = 0.5$.

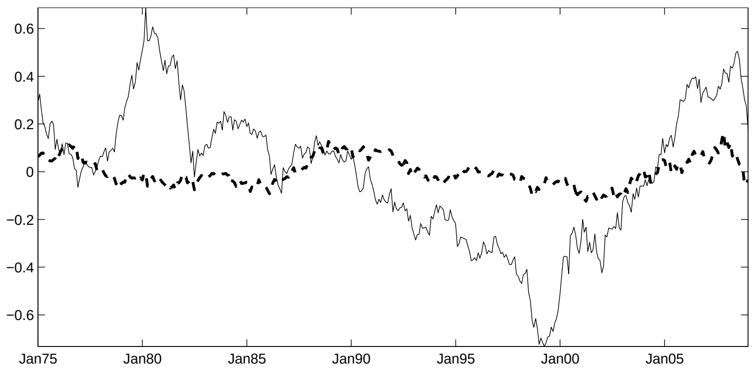
Figure 3: Average and Standard Deviation of Market-to-Book Ratios Across Countries



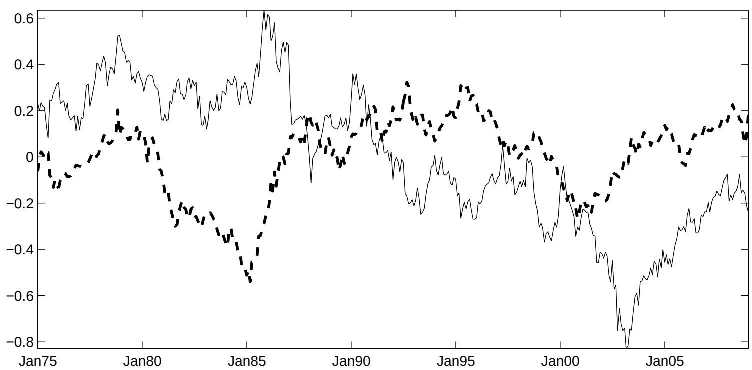
The graph shows the cross-section equal-weight mean and standard deviation of the market-to-book value for all 153 country pairs for the period 1974:12-2008:12.

Figure 4: Real Exchange Rates, 1974–2008

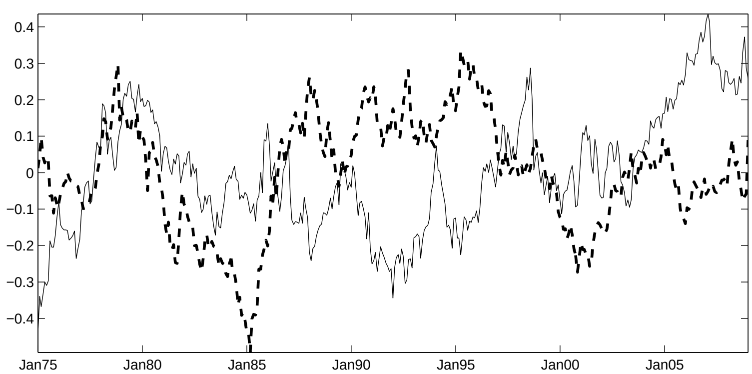
(a) Canada–USA



(b) Germany–USA



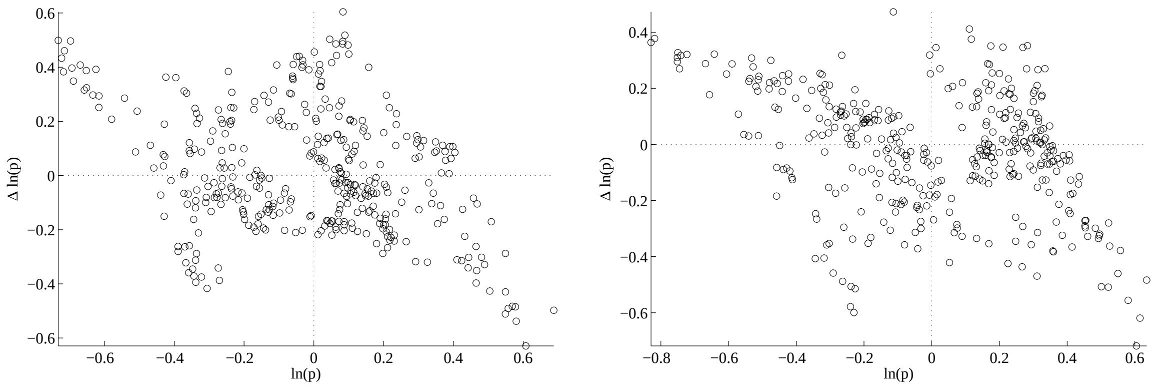
(c) UK–USA



The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the time period 1974:12–2008:12.

Figure 5: 24-month Changes in Real Exchange Rate of Productive Capabilities vs. Initial Levels

(a) Canada–USA (b) Germany–USA



The graphs show the changes in the log real exchange rate of productive capabilities in the two years following a given level for the country pairs Canada-USA and Germany-USA during the period 1974:12-2008:12.

Figure 6: Relocation Costs of Final Goods vs. Productive Capabilities

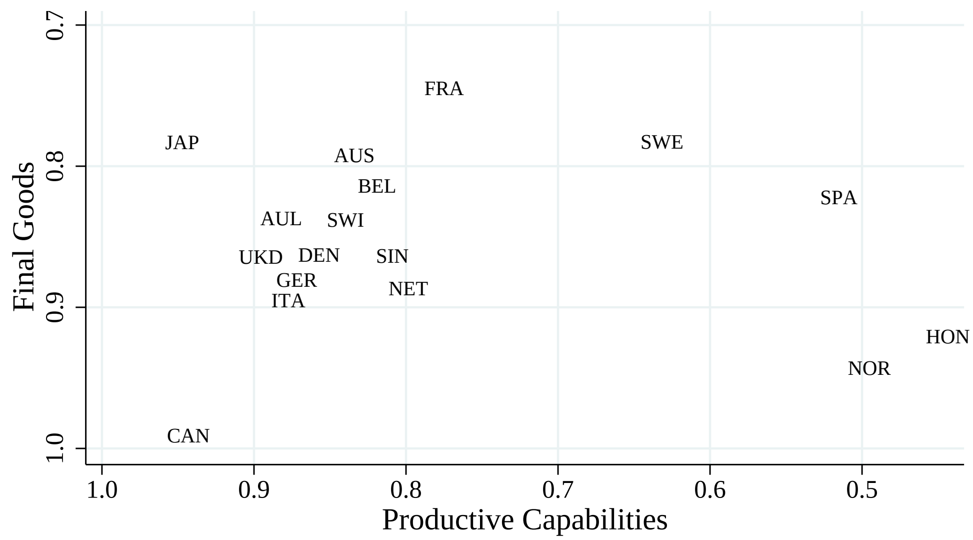
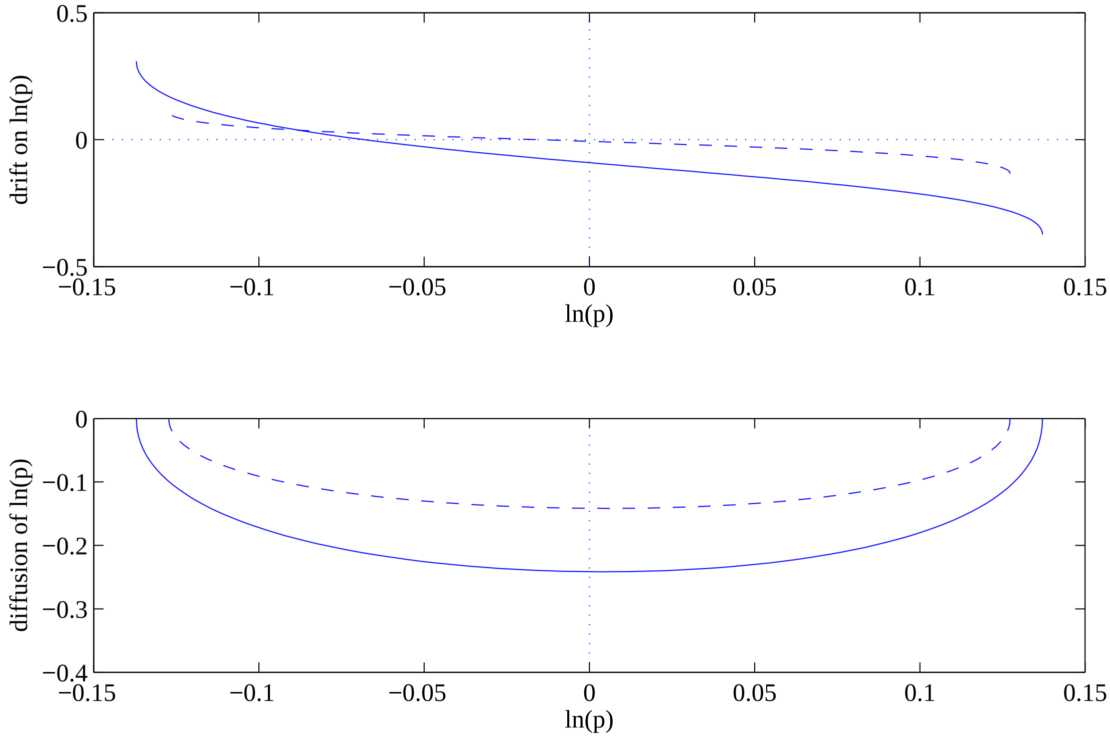
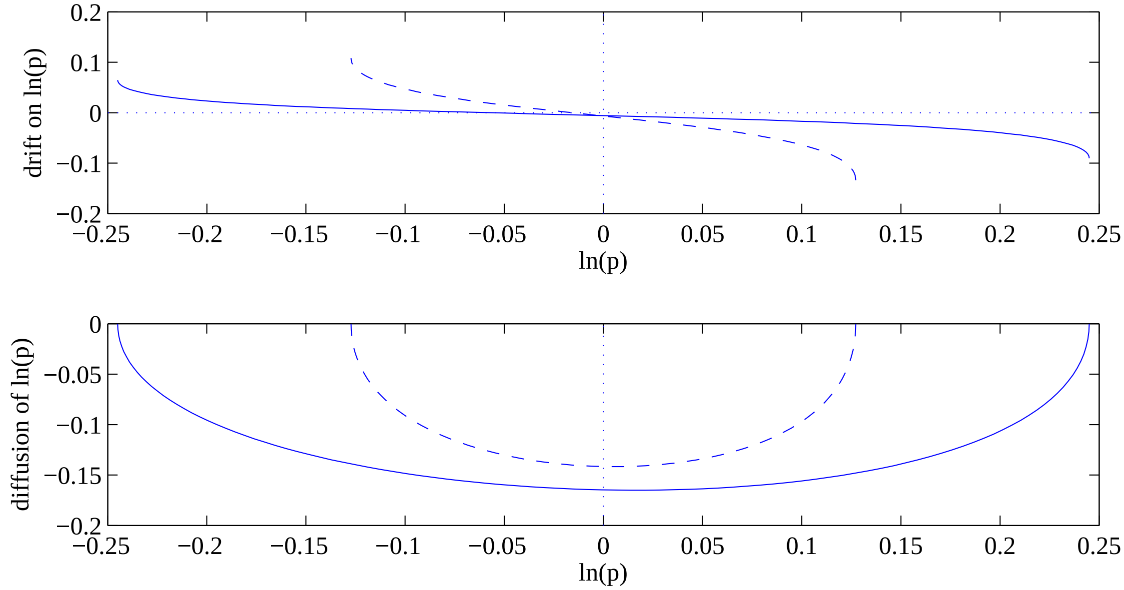


Figure 7: Drift and Diffusion of Real Exchange Rate, Final Goods vs. Productive Capabilities, Germany–USA



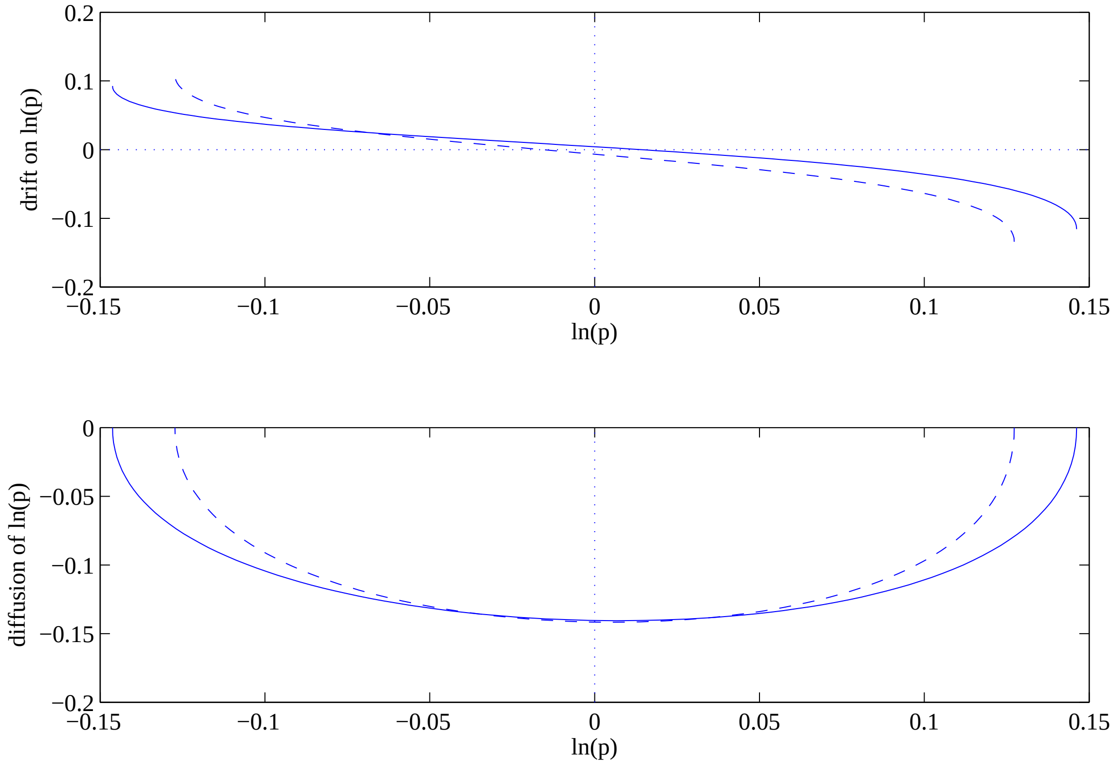
The graph plots the estimated drift and diffusion of the real exchange rate process, based on final goods (dashed line) and based on productive capabilities (solid line), for the pair Germany–USA during the period 1974:12–2008:12. Parameter values are taken from tables 9 and 10.

Figure 8: Drift and Diffusion of Real Exchange Rate based on Final Goods, Japan–USA



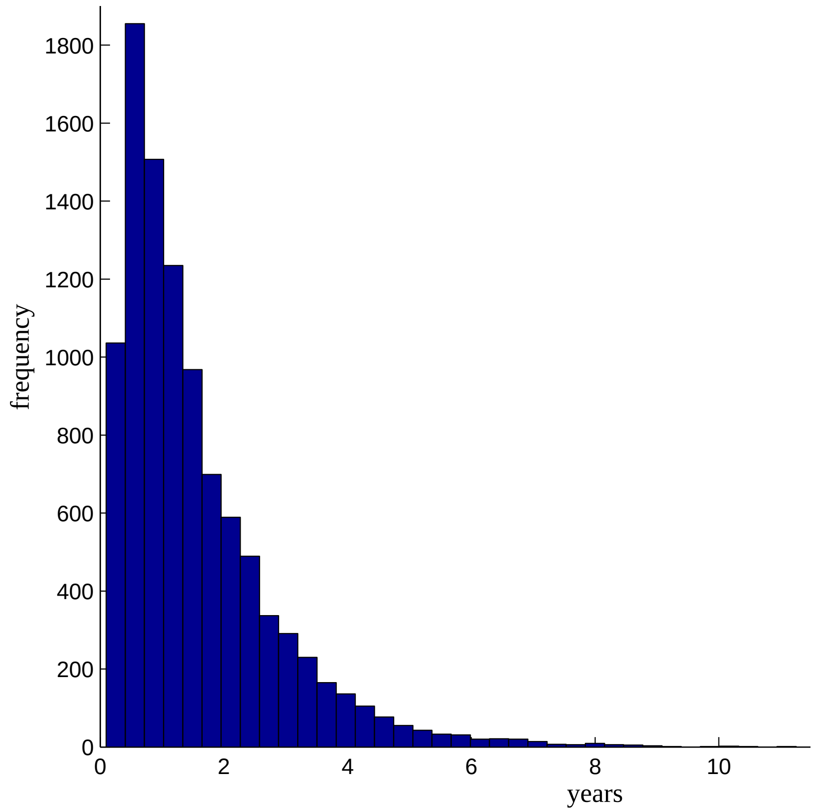
The solid line is the estimated drift and diffusion of the real exchange rate of final goods for the country pair Japan–USA during the period 1974:12–2008:12. The dashed line plots the same for Germany–USA. Parameter values are taken from tables 9 and 10.

Figure 9: Drift and Diffusion of Real Exchange Rate based on Final Goods, UK–USA



The solid line is the estimated drift and diffusion of the real exchange rate for final goods for the country pair UK–USA during the period 1974:12–2008:12. The dashed line plots the same for Germany–USA. Parameter values are taken from tables 9 and 10.

Figure 10: Sample Hittime Distribution



The histogram shows the distribution of the time span until the real exchange rate process reaches the PPP level when starting at an extreme. The histogram is based on 10000 simulated sample paths, and has an average hittime of about 1.4 years. The parameter values used for this graph are $r = 0.9$, $\gamma = -0.5$, $\sigma_{11} = \sigma_{22} = 0.5$, $\rho = 0.07$, $\alpha = \alpha^{*} = 0.1$, and $q = 1$.

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A One-Good Model

In this appendix I discuss and solve an extension of Dumas (1992). In particular, I introduce nonzero covariance between the country-specific shocks and country-specific productivity.

A.1 General One-Good Model

Suppose z and z^* are two standard Brownian motion processes[24] and Ω is a positive definite and symmetric matrix.

$$\begin{pmatrix} d\tilde{z} \\ d\tilde{z}^* \end{pmatrix} = \Omega \begin{pmatrix} dz \\ dz^* \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \begin{pmatrix} dz \\ dz^* \end{pmatrix} = \begin{pmatrix} \sigma_{11} dz + \sigma_{12} dz^* \\ \sigma_{12} dz + \sigma_{22} dz^* \end{pmatrix}$$

Plugging this process of productivity shocks and the first order conditions for c into the Hamilton-Jacobi partial differential equation implies that capital stocks K and K^* follow the differential equation

$$\begin{aligned} 0 = & \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} V_K^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} V_{K^*}^{\frac{\gamma}{\gamma-1}} - \rho V \\ & + V_K \alpha K + V_{K^*} \alpha^* K^* + \frac{1}{2} V_{KK} \sigma_{11}^2 K^2 + V_{KK^*} \sigma_{12}^2 K K^* + \frac{1}{2} V_{K^* K^*} \sigma_{22}^2 K^{*2}. \end{aligned}$$

Using $V(K, K^*) = K^{*\gamma} I(\omega)$, the capital imbalance $\omega = K/K^*$ then follows the differential equation

$$\begin{aligned} 0 \quad = \quad & \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} \left(\gamma I(\omega) - \omega I'(\omega) \right)^{\frac{\gamma}{\gamma-1}} \\ + \quad & \left[\alpha^* \gamma - \rho + \frac{1}{2} \left(\sigma_{12}^2 + \sigma_{22}^2 \right) \gamma (\gamma - 1) \right] I(\omega) \\ + \quad & \left[\alpha - \alpha^* + (\gamma - 1)(-\sigma_{22}^2 - \sigma_{12}^2 + \sigma_{12}(\sigma_{11} + \sigma_{22})) \right] \omega I'(\omega) \\ + \quad & \left[\frac{1}{2} \left(\sigma_{11}^2 + \sigma_{12}^2 \right) + \frac{1}{2} \left(\sigma_{22}^2 + \sigma_{12}^2 \right) - \sigma_{12}(\sigma_{11} + \sigma_{22}) \right] \omega^2 I''(\omega). \end{aligned} \tag{8}$$

For estimation I use $\sigma_{12} = 0$ and $\sigma_{11} = \sigma_{22} = \sigma$, which leads to equation (2) in the main

[24] dz and dz^* may be called white noise with $dz \sim N(0,1)$, $dz^* \sim N(0,1)$

text. In the symmetric case $\alpha = \alpha^*$, $\sigma_{11} = \sigma_{22} = \sigma$, $q = 1$, and equation (8) reduces to

$$\begin{aligned} 0 &= \frac{1-\gamma}{\gamma} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} \left[\gamma I(\omega) - \omega I'(\omega)\right]^{\frac{\gamma}{\gamma-1}} \\ &+ (\alpha\gamma - \rho) I(\omega) + \frac{1}{2}\gamma(\gamma-1)(\sigma^2 + \sigma_{12}^2) I(\omega) \\ &- (\gamma-1)(\sigma - \sigma_{12})^2 \omega I'(\omega) + (\sigma - \sigma_{12})^2 \omega^2 I''(\omega). \end{aligned} \tag{9}$$

The last three terms capture the effect of a nonzero covariance.

A.2 Model Solution

Optimal choice of the two boundaries of $\Xi(r)$ requires that the good's valuation, as well as the marginal valuation before and after a relocation must be equal. This imposes three boundary conditions on each side of the differential equation. For the upper boundary these conditions are

$$V_K(K, K^*) = r V_{K^*}(K, K^*), \tag{10}$$

$$V_{KK}(K, K^*) = r V_{KK^*}(K, K^*), \tag{11}$$

$$V_{K^*K}(K, K^*) = r V_{K^*K^*}(K, K^*). \tag{12}$$

By homogeneity of the value function, the latter two conditions are identical. These conditions can be rewritten in terms of $I(\omega)$ as

$$\begin{aligned} I(\overline{\omega}) &= \frac{1}{\gamma} (1 + r\overline{\omega})^\gamma \left[(1-\gamma) q^{\frac{1}{1-\gamma}} r^{\frac{\gamma}{\gamma-1}} + (1-\gamma)(2-q)^{\frac{1}{1-\gamma}}\right]^{1-\gamma} \\ &\times \left\{\rho - \alpha^*\gamma - \frac{1}{2}(\sigma_{12}^2 + \sigma_{22}^2)(\gamma-1)\gamma \right. \\ &- \frac{r\overline{\omega}\gamma}{1 + r\overline{\omega}} \left[\alpha - \alpha^* + (\gamma-1)(-\sigma_{22}^2 - \sigma_{12}^2 - \sigma_{12}(\sigma_{11} + \sigma_{22}))\right] \\ &- \left. \frac{r^2\overline{\omega}^2\gamma(\gamma-1)}{(1 + r\overline{\omega})^2} \left[\frac{1}{2}\left(\sigma_{11}^2 - \sigma_{12}^2\right) + \frac{1}{2}\left(\sigma_{22}^2 - \sigma_{12}^2\right) - \sigma_{12}(\sigma_{11} + \sigma_{22})\right]\right\}^{\gamma-1}, \end{aligned} \tag{13}$$

$$\frac{I'(\overline{\omega})}{\gamma I(\overline{\omega})} = \frac{r}{1 + r\overline{\omega}},$$

and

$$\frac{I''(\overline{\omega})}{\gamma I(\overline{\omega})} = \frac{r^2(\gamma-1)}{(1 + r\overline{\omega})^2}.$$

For the lower boundary I have, analogously,

$$
\begin{aligned}
I(\underline{\omega}) &= \frac{1}{\gamma}(r+\underline{\omega})^{\gamma}\left[(1-\gamma)\,q^{\frac{1}{1-\gamma}}+(1-\gamma)\,(2-q)^{\frac{1}{1-\gamma}}\,r^{\frac{\gamma}{\gamma-1}}\right]^{1-\gamma} \\
&\times \left\{\rho-\alpha^*\gamma-\frac{1}{2}(\sigma_{12}^2+\sigma_{22}^2)(\gamma-1)\gamma\right. \\
&- \frac{\underline{\omega}\gamma}{r+\underline{\omega}}\left[\alpha-\alpha^*+(\gamma-1)(-\sigma_{22}^2-\sigma_{12}^2-\sigma_{12}(\sigma_{11}+\sigma_{22}))\right] \\
&- \left.\frac{\underline{\omega}^2\gamma(\gamma-1)}{(r+\underline{\omega})^2}\left[\frac{1}{2}\left(\sigma_{11}^2-\sigma_{12}^2\right)+\frac{1}{2}\left(\sigma_{22}^2-\sigma_{12}^2\right)-\sigma_{12}(\sigma_{11}+\sigma_{22})\right]\right\}^{\gamma-1},
\end{aligned}
$$

$$
\frac{I'(\underline{\omega})}{\gamma I(\underline{\omega})}=\frac{1}{1+r\underline{\omega}},
$$

and

$$
\frac{I''(\underline{\omega})}{\gamma I(\underline{\omega})}=\frac{\gamma-1}{(r+\underline{\omega})^2}.
$$

In the symmetric case $\alpha=\alpha^*$, $\sigma_{11}=\sigma_{22}=\sigma$, and $q=1$, with $\sigma_{12}=0$, I have $\overline{\omega}=1/\underline{\omega}=\lambda$, $I(\overline{\omega})=\overline{\omega}^{\gamma}I(\underline{\omega})$, and (13) reduces to

$$
I(\lambda)=\frac{(1+r\lambda)^{\gamma}}{\gamma}\left(1+r^{\frac{\gamma}{\gamma-1}}\right)^{1-\gamma}\left(\frac{\rho-\alpha\gamma}{1-\gamma}+\frac{\sigma^2\gamma}{2}\cdot\frac{1+r^2\lambda^2}{(1+r\lambda)^2}\right)^{\gamma-1}.
$$

Applying Ito's formula for multiple standard processes I obtain the process of the capital imbalance ω inside of Ξ

$$
\begin{aligned}
d\omega &= \left[-\frac{c(t)}{K(t)}+\frac{c^*(t)}{K^*(t)}-\sigma_{12}(\sigma_{11}+\sigma_{22})+\sigma_{12}^2+\sigma_{22}^2+\alpha-\alpha^*\right]\omega dt \\
&+ (\sigma_{11}-\sigma_{12})\omega dz-(\sigma_{22}-\sigma_{12})\,\omega dz^*,
\end{aligned} \tag{14}
$$

which in terms of $I(\omega)$ can be written as

$$
\begin{aligned}
d\omega &= \left(-\frac{1}{\omega}\left(\frac{I'(\omega)}{q}\right)^{\frac{1}{\gamma-1}}+\left(\frac{\gamma I(\omega)-\omega I'(\omega)}{2-q}\right)^{\frac{1}{\gamma-1}}-\sigma_{12}(\sigma_{11}+\sigma_{22})+\sigma_{12}^2+\sigma_{22}^2+\alpha-\alpha^*\right)\omega dt \\
&+ \sqrt{(\sigma_{11}-\sigma_{12})^2+(\sigma_{22}-\sigma_{12})^2}\omega dz',
\end{aligned} \tag{15}
$$

where z' is again standard Brownian motion.

From (14), using $p(\omega)=\frac{V_K}{V_{K^*}}=\frac{I'(\omega)}{\gamma I(\omega)-\omega I'(\omega)}$ the process dp can be written as a function of

ω, $I(\omega)$, $I'(\omega)$, $I''(\omega)$, and $I'''(\omega)$. The process for $ln(p)$ is

$$\begin{aligned}
& dln(p(\omega)) \\
= \; & dt\left\{\left[-\frac{c}{K}+\frac{c^*}{K^*}+\alpha-\alpha^*+\sigma_{12}^2+\sigma_{22}^2-\sigma_{12}(\sigma_{11}+\sigma_{22})\right]\left[\frac{I''}{I'}-\frac{(\gamma-1)I'-\omega I''}{\gamma I-\omega I'}\right]\omega\right. \\
+ \; & \frac{(\sigma_{11}-\sigma_{12})^2+(\sigma_{12}-\sigma_{22})^2}{2} \\
\times \; & \left.\left[\frac{I'''I'-I''^2}{I'^2}-\frac{((\gamma-2)I''-\omega I''')(\gamma I-\omega I')-((\gamma-1)I'-\omega I'')^2}{(\gamma I-\omega I')^2}\right]\omega^2\right\} \\
+ \; & dz'\sqrt{(\sigma_{11}-\sigma_{12})^2+(\sigma_{12}-\sigma_{22})^2}\left[\frac{I''}{I'}-\frac{(\gamma-1)I'-\omega I''}{\gamma I-\omega I'}\right]\omega. \qquad (16)
\end{aligned}$$

The optimality conditions (10), (11), (12), and their counterparts for the lower boundary directly imply that the price level at both boundaries is identical, even if $\overline{\omega} \neq 1/\underline{\omega}$. At the upper boundary $\overline{\omega}$, for example, I calculate

$$p(\overline{\omega})=\frac{I'(\overline{\omega})}{\gamma I(\overline{\omega})-\overline{\omega}I'(\overline{\omega})}=\frac{\frac{r\gamma}{1+r\overline{\omega}}}{\gamma-\overline{\omega}\frac{r\gamma}{1+r\overline{\omega}}}=r.$$

In the symmetric case ($\alpha=\alpha^*$, $\sigma_{11}=\sigma_{22}=\sigma$, and $q=1$) equation (15) reduces to

$$\begin{aligned}
d\omega \; = \; & \left(-\frac{1}{\omega}I'(\omega)^{\frac{1}{\gamma-1}}+(\gamma I(\omega)-\omega I'(\omega))^{\frac{1}{\gamma-1}}+(\sigma-\sigma_{12})^2\right)\omega dt \\
+ \; & \sqrt{2}(\sigma-\sigma_{12})\omega dz'.
\end{aligned}$$

Utilizing the useful property of the symmetric case that $I'(1)=\frac{\gamma}{2}I(1)$, I find that at $\omega=1$ the differential equation (9) implies

$$\begin{aligned}
0 \; = \; & (1-\gamma)I'(1)^{\frac{1}{\gamma-1}} \\
+ \; & \alpha\gamma-\rho+\gamma(\gamma-1)\sigma\sigma_{12}+(\sigma-\sigma_{12})^2\frac{I''(1)}{I(1)}.
\end{aligned}$$

Simplifying further by setting $\sigma_{12} = 0$, the price process (16) reduces to

$$\begin{aligned}
&dln(p(\omega)) \\
&= dt\left\{\left[-\frac{c}{K}+\frac{c^*}{K^*}+\sigma^2\right]\left[\frac{I''}{I'}-\frac{(\gamma-1)I'-\omega I''}{\gamma I-\omega I'}\right]\omega\right. \\
&+\sigma^2\left[\frac{I'''I'-I''^2}{I'^2}-\frac{((\gamma-2)I''-\omega I''')(\gamma I-\omega I')-((\gamma-1)I'-\omega I'')^2}{(\gamma I-\omega I')^2}\right]\left.\omega^2\right\} \\
&+dz'\sigma\sqrt{2}\left[\frac{I''}{I'}-\frac{(\gamma-1)I'-\omega I''}{\gamma I-\omega I'}\right]\omega. \qquad (17)
\end{aligned}$$

This special case allows calculating the exact value of the price level at $\omega = 1$ by

$$p(1) = \frac{\frac{\gamma}{2}}{\gamma - \frac{\gamma}{2}} = 1.$$

A.3 Proof of Proposition 1

Proposition (Drift of Real Exchange Rate at the Boundary) The drift of the real exchange rate at the upper boundary is

$$\mu_p(\omega = \overline{\omega}) = \alpha^* - \alpha + \frac{\overline{\omega} - 1/r}{\overline{\omega} + 1/r}(1-\gamma)\sigma^2,$$

and at the lower boundary

$$\mu_p(\omega = \underline{\omega}) = \alpha^* - \alpha + \frac{\underline{\omega} - r}{\underline{\omega} + r}(1-\gamma)\sigma^2.$$

Proof: I calculate the drift of the real exchange rate, $ln(p)$, at the boundary. Let $\overline{\omega} > 1$ denote the upper imbalance level. One can show that the drift of $dln(p) = \mu_p(p)dt + \sigma_p(p)dz$ at $\omega = \overline{\omega}$ with $\sigma_{11} = \sigma_{22} = \sigma$ and $\sigma_{12} = 0$ is

$$\mu_p(r) = \sigma^2\overline{\omega}^2\left[\frac{I'''(\overline{\omega})}{I'(\overline{\omega})}(1+r\overline{\omega}) - (\gamma-1)(\gamma-2)\frac{r^2}{1+r\overline{\omega}}\right].$$

Further,

$$\frac{I'''(\overline{\omega})}{I'(\overline{\omega})} = \frac{\gamma-1}{\overline{\omega}^2(1+r\overline{\omega})^2}\left[r^2\overline{\omega}^2(\gamma-2) - r\overline{\omega} + 1 - \frac{1+r\overline{\omega}}{\gamma-1}\frac{\alpha-\alpha^*}{\sigma^2}\right].$$

Therefore the drift of $ln(p)$ at $\omega = \overline{\omega}$ simplifies to

$$\mu_p(r) = \sigma^2(\gamma - 1)\frac{1 - r\overline{\omega}}{1 + r\overline{\omega}} - \alpha + \alpha^*.$$

q.e.d.

Note that the drift at the boundary depends on ρ only indirectly via $\overline{\omega}$, but directly on the productivity differential $\Delta\alpha = \alpha - \alpha^*$. At $\omega = \overline{\omega} > 1$ I have $V_K < V_{K^*}$ and therefore $p(\lambda) = \frac{V_K}{V_{K^*}} < 1$. For mean reversion to hold I need therefore a positive drift of p, which requires at $\alpha = \alpha^*$ that $1 - r\overline{\omega} < 0$ and therefore $\overline{\omega} > \frac{1}{r}$.[25]

B Extended Models

B.1 A Model with both Trade due to Specialization and Trade due to Arbitrage

This appendix presents a two-country model in which goods are traded for two different reasons. The global undifferentiated good K is produced in both countries, and is traded for arbitrage reasons, whereas the local goods A and B are produced only in country A and B respectively, and are traded due to complete specialization combined with love-of-variety in both countries. Goods located in country B are marked with an asterisk.

$$V(K, K^*, A, A^*, B, B^*) = \max_{\substack{c(t), c^*(t), \\ \Xi(r)}} E_0 \int_0^\infty e^{-\rho u}\left(\frac{q}{\gamma} c_K(u)^\gamma c_A(u)^\gamma c_B(u)^\gamma + \frac{2-q}{\gamma} c_K^*(u)^\gamma c_A^*(u)^\gamma c_B^*(u)^\gamma\right) du$$

s.t.

$$\begin{aligned}
dK(t) &= [\alpha_K K(t) - c_K(t)]\, dt + K(t)\sigma dz(t) - dX_K(t) + r dX_K^*(t) \\
dA(t) &= [\alpha_A A(t) - c_A(t)]\, dt - dX_A(t) + r dX_A^*(t) \\
dB(t) &= -c_B(t) dt - dX_B(t) + r dX_B^*(t)
\end{aligned}$$

[25]The values for $\overline{\omega}$ in table 1 show that $\forall \gamma < 0\ \exists \sigma_0(\gamma)$ s.t. $\forall \sigma < \sigma_0(\gamma)$ the process of $ln(p)$ is in fact divergent.

$$
\begin{aligned}
dK^*(t) &= \left[\alpha_K^* K^*(t) - c_K^*(t)\right] dt + K^*(t)\sigma dz^*(t) + r dX_K(t) - dX_K^*(t) \\
dA^*(t) &= -c_A^*(t)dt + r dX_A(t) - dX_A^*(t) \\
dB^*(t) &= \left[\alpha_B B^*(t) - c_B^*(t)\right] dt + r dX_B(t) - dX_B^*(t)
\end{aligned}
$$

Storing good B in country A yields no return, it is merely consumed there. Thus absent any fixed cost of shipping and $\alpha_B > 0$ the optimal allocation implies no shipment of good B from country A to B ($dX_B = 0$), hence $rdX_B^* = c_B dt$ and $dB = 0$. Analogously $dX_A^* = 0$, $rdX_A = c_A^* dt$, $dA^* = 0$, and $dA = \left(\alpha_A A - c_A - \frac{1}{r} c_A^*\right) dt$. Thus under an optimal consumption plan, goods A and B are shipped at every instant to satisfy the consumption need in the other country. The real exchange rate for these two goods is fixed, for good A $\frac{V_A}{V_{A^*}} = \frac{V_A}{rV_A} = \frac{1}{r}$ and for good B $\frac{V_B}{V_{B^*}} = r$ hold always. Thus for goods which are traded for specialization reasons, there is no time variation in the real exchange rate. As part of an aggregate real exchange rate composed of many goods, these goods affect only the average exchange rate, e.g. the long-run PPP level.

To the contrary, good K is shipped only if due to random shocks this good becomes very differently valued in the two countries. This trade for arbitrage reasons comes with a systematic time variation in real exchange rates, as shown by equation (17) in appendix A. Because goods A and B contribute nothing to the time-variation of exchange rates, on which my identification approach rests, I drop these goods from my model, and focus my model on goods with arbitrage trade only.

B.2 Two-Good Model

The model described in the paper can be seen as a reduced form of a two-good model. Consider an economy identical to the one described in the paper, but now I explicitly model the difference between a final good, G, and a productive capability, K. Investment, I, the conversion of a final good into a productive capability in the same country carries no cost, but the reverse entails a loss of setup and training costs. These costs are proportional to the amount disinvested, D, so that of one unit of K only $q \in (0,1]$ units of G remain. Relocation of final goods, X_G, between the two countries entails – as before – a proportional relocation cost of $1 - r_G$, and direct relocation of productive capabilities, X_K, a relocation cost of $1 - r_K$. Only the final good can be consumed in the respective country, and only the productive capability can be used in production.[26] The following four equations describe

[26] Otherwise the notation is the same as in equation (1). In the empirical part, home variables with asterisk represent the USA.

the dynamics of the stock of final goods and productive capabilities in the home and foreign country, respectively:

$$\begin{aligned} dG(t) &= -c(t)dt + G(t)\sigma dz(t) - dX_G(t) + r_G dX_G^*(t) - dI(t) + qdD(t) = 0 \\ dK(t) &= \alpha K(t)dt + K(t)\sigma dz(t) - dX_K(t) + r_K dX_K^*(t) + dI(t) - dD(t) \end{aligned}$$

$$\begin{aligned} dG^*(t) &= -c^*(t)dt + G^*(t)\sigma dz^*(t) - dX_G^*(t) + r_G dX_G^*(t) - dI^*(t) + q^* dD^*(t) = 0 \\ dK^*(t) &= \alpha^* K^*(t)dt + K^*(t)\sigma dz^*(t) - dX_K^*(t) + r_K dX_K(t) + dI^*(t) - dD^*(t) \end{aligned}$$

Under the same assumptions as in the model in the paper, in particular $\alpha > 0$, $\alpha^* > 0$, $G(0) = 0$, $G^*(0) = 0$, and non-increasing disinvestment cost coefficients, q and q^*, the stock of final goods in both countries is always zero. This obtains because only the productive capabilities generate output and are no more risky than final goods. Therefore, under continuous time and absent any fixed costs, it is never optimal to disinvest more than what is needed for instantaneous consumption. Any addition to (G, G^*) is either immediately consumed, relocated, or invested. The state vector of this model thus collapses to (K, K^*).

Despite the zero stock of final goods I can of course calculate the real exchange rate for these goods at all times. The valuation of the final goods is linked to the valuation of productive capabilities via relocation and (dis-)investment by $V_G = min\left[\frac{V_K}{q}, \frac{V_{K^*}}{q^* r_G}\right]$ and $V_{G^*} = min\left[\frac{V_{K^*}}{q^*}, \frac{V_K}{q r_G}\right]$.[27] These two conditions imply that the real exchange rates for final goods, $p_G(t) \equiv \frac{V_G(t)}{V_{G^*}(t)}$, and for productive capabilities, $p_K(t) \equiv \frac{V_K(t)}{V_{K^*}(t)}$, are linked by

Proposition 2

$$p_G(t) = min\left[max\left(p_K(t)\frac{q^*}{q}, r_G\right), \frac{1}{r_G}\right].$$

Proof: From

$$p_G(t) \equiv \frac{V_G(t)}{V_{G^*}(t)} = \frac{min\left[\frac{V_K}{q}, \frac{V_{K^*}}{q^* r_G}\right]}{min\left[\frac{V_{K^*}}{q^*}, \frac{V_K}{q r_G}\right]}$$

[27] This holds with equality because $G(t) = G^*(t) = 0 \ \forall t$.

I get

$$p_G(t) = \begin{cases} r_G & \text{if } \frac{V_K}{q} < min\left[\frac{r_G V_{K^*}}{q^*}, \frac{V_{K^*}}{r_G q^*}\right] \\ \frac{V_K}{V_{K^*}}\frac{q^*}{q} & \text{if } \frac{r_G V_{K^*}}{q^*} < \frac{V_K}{q} < \frac{V_{K^*}}{r_G q^*} \\ \frac{V_{K^*}}{V_K}\frac{q}{q^*} & \text{if } \frac{r_G V_{K^*}}{q^*} > \frac{V_K}{q} > \frac{V_{K^*}}{r_G q^*} \\ \frac{1}{r_G} & \text{if } \frac{V_K}{q} > max\left[\frac{r_G V_{K^*}}{q^*}, \frac{V_{K^*}}{r_G q^*}\right] \end{cases},$$

which simplifies to

$$p_G(t) = \begin{cases} r_G & \text{if } \frac{V_K}{V_{K^*}}\frac{q^*}{q} < r_G \\ \frac{V_K}{V_{K^*}}\frac{q^*}{q} & \text{if } r_G \leq \frac{V_K}{V_{K^*}}\frac{q^*}{q} \leq \frac{1}{r_G} \\ \frac{1}{r_G} & \text{if } \frac{V_K}{V_{K^*}}\frac{q^*}{q} > \frac{1}{r_G} \end{cases}.$$

This directly gives the equation stated in the proposition. *q.e.d.*

The proposition shows that the differential behavior of the two real exchange rates stems from a scale factor, $\frac{q^*}{q}$, due to differential disinvestment cost, and from different cutoff points, $(\frac{1}{r_G}, r_G)$ vs. $(\frac{1}{r_K}, r_K)$, due to relocation costs. If $r_G > r_K$ then the link between the two real exchange rates is not only affected by the scale factor, but also by the frequently binding relocation cutoff.

C Inflation Adjustment of Book Values

Book values record the value of capital goods and productive capabilities at the time of acquisition or production by the firm. In order to properly measure the current value of these goods, that is, in order to match the overall inflation reflected in the market values, I correct the book values for inflation. Figure 11 shows the effect of this inflation correction for Germany, Japan and the USA.

My correction procedure adjusts the original bookvalues (dashed line) for Germany upwards only in the high inflation periods of the 1970s. The new series is shown by the solid line. Persistent inflation periods lead to a substantial upward adjustment in bookvalues, as in the case of the USA in the late 1970s in the lower panel of figure 11. A deflation, as in Japan in the 1990s, has the opposite effect. The corrected book values for Japan in the middle panel of figure 11 are smaller than the original ones.

[Figure 11 about here.]

D Nonlinearity Test

I test the null hypothesis of linearity against the alternative of ESTAR-type nonlinearity using a heteroskedasticity-consistent Lagrange-multiplier-type test (Franses and van Dijk, 2000; Granger and Teräsvirta, 1993; Teräsvirta, 1994). For a given transition variable p_{t-d} the test is based on a second-order Taylor approximation of the ESTAR model (6)

$$p_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 q_{t-1} p_{t-d} + \beta_3 q_{t-1} p_{t-d}^2 + \varepsilon_t, \tag{18}$$

where $q_{t-1} = (p_{t-1}, p_{t-2}, \ldots, p_{t-m})$ denotes the vector of independent AR(m) variables and β_0, β_1, β_2 and β_3 are functions of the parameters of the ESTAR model. The null hypothesis of linearity is then $H_0 : \beta_2 = \beta_3 = 0$. A heteroskedasticity-robust test statistic is

$$F = \frac{T - 3m - 1}{2mT} \mathbf{1}' D R (R'R)^{-1} R' D' \mathbf{1},$$

where $\mathbf{1} = (1, \ldots, 1)'$, D is a $T \times T$ matrix with the residuals from the linear model on the diagonal, and the matrix R contains the residuals from the regression of the $2m$ interaction terms in (18) on $(1 \; q_{t-1})$. Under the null hypothesis this test statistic is approximately $F(2m, T - 3m - 1)$ distributed.

I determine the lag, d, of the transition variable as the lag which leads to the strongest rejection of linearity (Teräsvirta, 1994). When the significance of rejection is similar for multiple lags, I choose the lowest, and the one that is robust against inclusion of additional AR lags.

E Indirect Inference Results

This appendix provides the full estimation results for final goods (Table 9) and for productive capabilities (Table 10).

[Table 9 about here.]

[Table 10 about here.]

F Real Exchange Rate Data

Figures 12, 13, and 14 show the natural logarithm of the real exchange rate series in my dataset. The solid line represents the real exchange rate for for productive capabilities, and the dashed line the real exchange rate for final goods.

[Figure 12 about here.]

[Figure 13 about here.]

[Figure 14 about here.]

Table 9: Indirect Inference Estimates for Final Goods vs. USA

	r	$\Delta\alpha$	σ	q
Australia	**0.84**	0.02	0.52	0.05
	(0.01)	(0.23)	(0.11)	
Austria	**0.79**	0.02	0.45	0.05
	(0.01)	(0.05)	(0.05)	
Belgium	**0.81**	-0.01	0.44	0.06
	(0.01)	(0.04)	(0.04)	
Canada	**0.99**	-0.14	0.32	0.17
	(0.00)	(0.02)	(0.03)	
Denmark	**0.86**	-0.13	0.48	0.03
	(0.02)	(0.96)	(0.27)	
France	**0.74**	-0.03	0.31	0.29
	(0.07)	(0.66)	(0.43)	
Germany	**0.88**	-0.03	0.48	0.39
	(0.02)	(0.76)	(0.12)	
Hongkong	**0.92**	-0.09	0.36	0.03
	(0.01)	(0.24)	(0.07)	
Italy	**0.89**	-0.06	0.48	0.27
	(0.02)	(1.5)	(0.05)	
Japan	**0.78**	-0.01	0.36	0.53
	(0.01)	(0.02)	(0.02)	
Netherlands	**0.89**	-0.08	0.51	0.09
	(0.02)	(0.91)	(0.06)	
Norway	**0.94**	-0.07	0.80	0.04
	(0.01)	(5.7)	(0.14)	
Singapore	**0.86**	-0.10	0.33	0.02
	(0.01)	(0.02)	(0.03)	
Spain	**0.82**	0.15	0.54	0.17
	(0.01)	(0.19)	(0.12)	
Sweden	**0.78**	-0.01	0.23	0.05
	(0.00)	(0.01)	(0.00)	
Switzerland	**0.84**	-0.11	0.50	0.05
	(0.02)	(0.51)	(0.08)	
UK	**0.86**	-0.07	0.44	0.28
	(0.02)	(0.80)	(0.09)	

WPI, demeaned and deseasonalized, 1974:12–2008:12, $\gamma = -0.5$, $\rho = 0.07$. ESTAR as auxiliary model with $2 \le p \le 4$ and $d \le 12$ chosen by nonlinearity test. q is calibrated as described in the paper. Standard errors in parentheses.

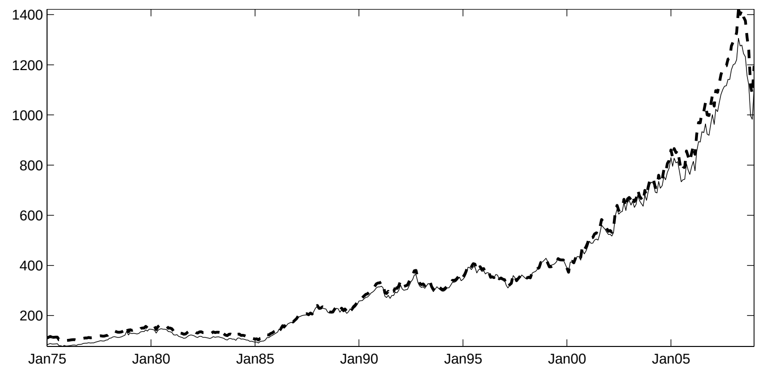
Table 10: Indirect Inference Estimates for Productive Capabilities vs. USA

	r	$\Delta\alpha$	σ	q
Australia	**0.88**	-0.13	0.85	0.05
	(0.01)	(2.3)	(0.22)	
Austria	**0.83**	-0.15	0.84	0.05
	(0.01)	(0.31)	(0.08)	
Belgium	**0.82**	-0.31	0.75	0.06
	(0.02)	(1.3)	(0.45)	
Canada	**0.94**	-0.02	0.93	0.17
	(0.00)	(2.0)	(0.07)	
Denmark	**0.86**	-0.11	0.81	0.03
	(0.01)	(2.4)	(0.17)	
France	**0.77**	0.28	0.71	0.29
	(0.01)	(0.09)	(0.05)	
Germany	**0.87**	0.27	0.80	0.39
	(0.01)	(0.54)	(0.19)	
Hongkong	**0.44**	-0.18	0.58	0.03
	(0.04)	(0.44)	(0.22)	
Italy	**0.88**	0.14	1.03	0.27
	(0.01)	(1.6)	(0.16)	
Japan	**0.95**	-0.02	0.93	0.53
	(0.00)	(2.4)	(0.06)	
Netherlands	**0.80**	0.00	0.30	0.09
	(0.00)	(0.01)	(0.01)	
Norway	**0.50**	-0.20	0.52	0.04
	(0.02)	(0.11)	(0.08)	
Singapore	**0.81**	-0.08	0.82	0.02
	(0.01)	(1.4)	(0.07)	
Spain	**0.52**	-0.24	0.54	0.17
	(0.03)	(0.50)	(0.35)	
Sweden	**0.63**	-0.30	0.61	0.05
	(0.03)	(0.38)	(0.24)	
Switzerland	**0.84**	-0.31	0.66	0.05
	(0.01)	(0.40)	(0.16)	
UK	**0.90**	0.31	0.80	0.28
	(0.00)	(0.31)	(0.12)	

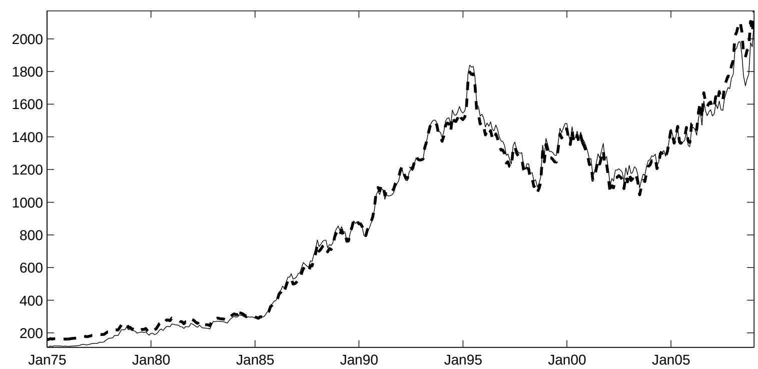
Inflation-adjusted bookvalues, demeaned and deseasonalized, 1974:12–2008:12, $\gamma = -0.5$, $\rho = 0.07$. ESTAR as auxiliary model with $2 \leq p \leq 4$ and $d \leq 12$ chosen by nonlinearity test. q is calibrated as described in the paper. Standard errors in parentheses.

Figure 11: Book Value Correction

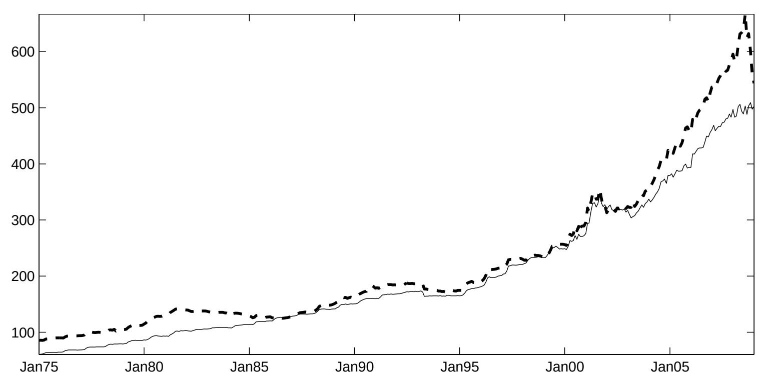
(a) Germany



(b) Japan



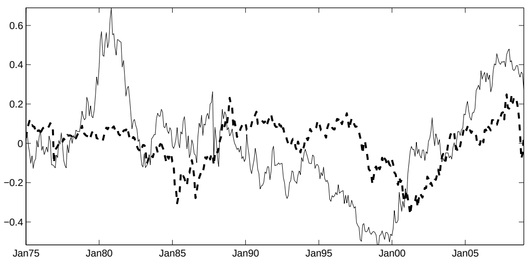
(c) USA



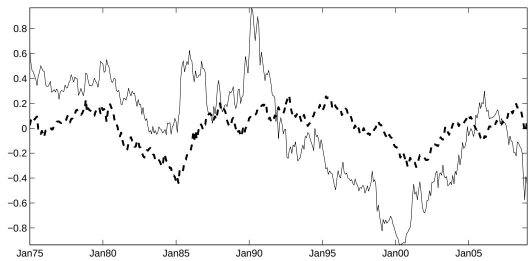
The solid line shows the reported book value for companies included in the respective MSCI country index during 1974:12–2008:12. The dashed line shows the same book value after correcting for inflation.

Figure 12: Real Exchange Rates, 1974-2008

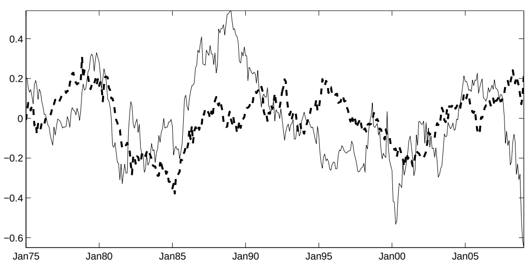
(a) Australia–USA



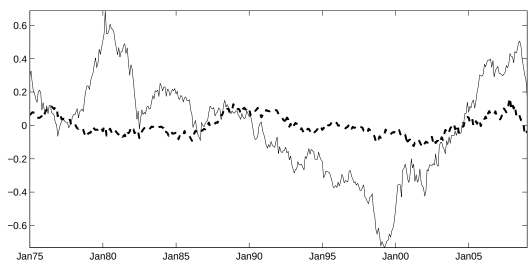
(b) Austria–USA



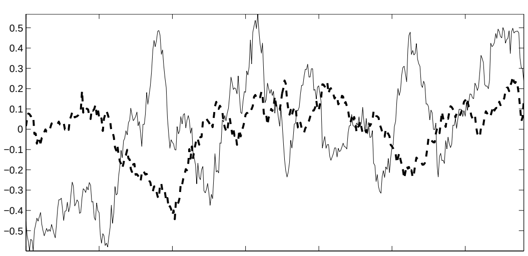
(c) Belgium–USA



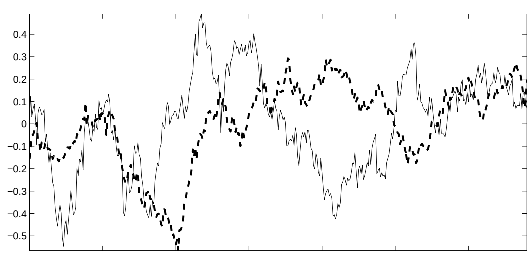
(d) Canada–USA



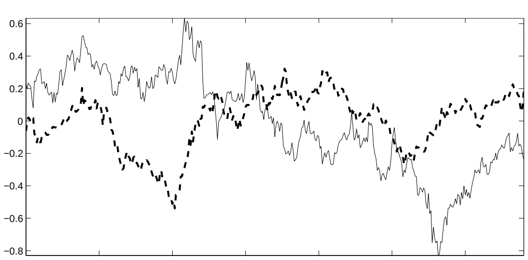
(e) Denmark–USA



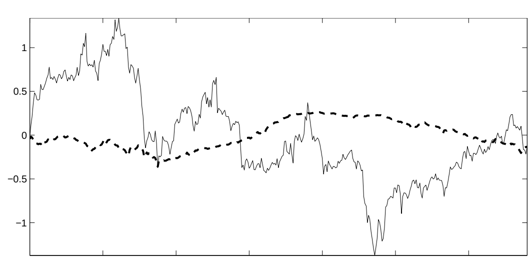
(f) France–USA



(g) Germany–USA



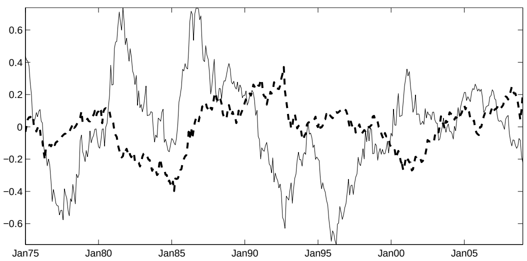
(h) Hongkong–USA



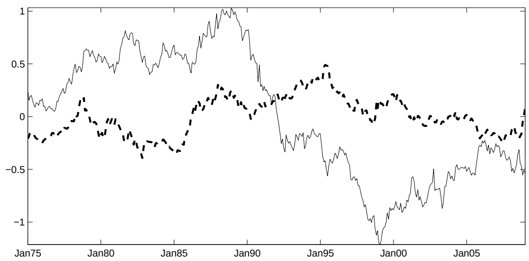
The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.

Figure 13: Real Exchange Rates, 1974-2008 (continued)

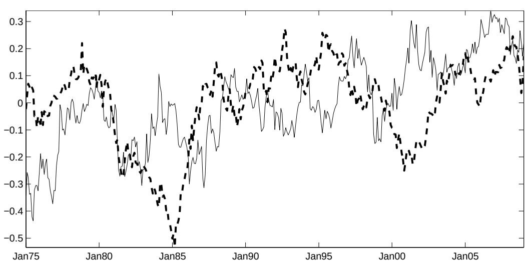
(a) Italy–USA



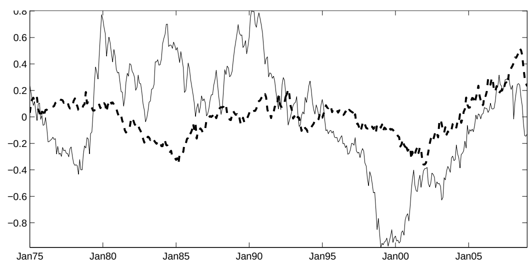
(b) Japan–USA



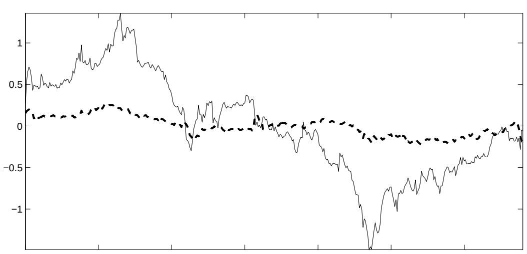
(c) Netherland–USA



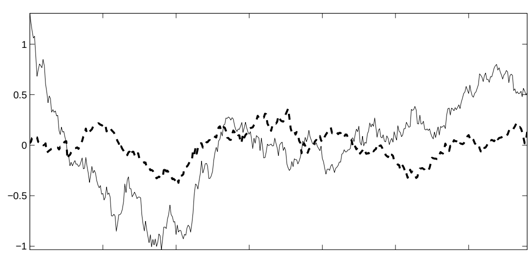
(d) Norway–USA



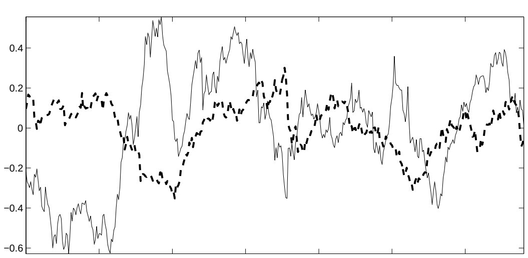
(e) Singapore–USA



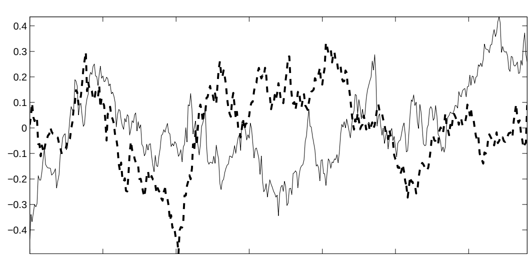
(f) Spain–USA



(g) Sweden–USA



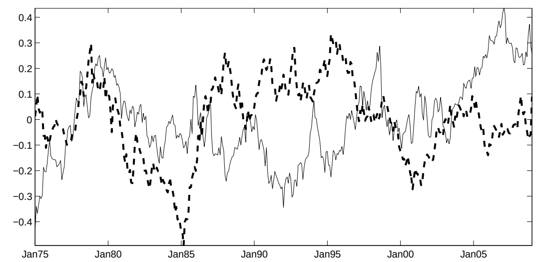
(h) Switzerland–USA



The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.

Figure 14: Real Exchange Rates, 1974-2008 (continued)

(a) United Kingdom–USA



The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.