The Macroeconomic Effects of Interest on Reserves

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Abstract

This paper uses a New Keynesian model with banks and deposits, calibrated to match the US economy, to study the macroeconomic effects of policies that pay interest on reserves. While their effects on output and inflation are small, these policies require important adjustments in the way that the monetary authority manages the supply of reserves, as liquidity effects vanish and households' portfolio shifts increase banks' demand for reserves when short-term interest rates rise. Money and monetary policy remain linked in the long run, however, since policy actions that change the price level must change the supply of reserves proportionately.

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1 Introduction

Slowly but surely over the three decades that have passed since the Federal Reserve's "monetarist experiment" of 1979 through 1982, the role of the monetary aggregates in both the making and analysis of monetary policy has eroded. Bernanke's (2006) historical account explains how and why Federal Reserve officials gradually deemphasized measures of the money supply as targets and indicators for monetary policy over these years. Taylor's (1993) highly influential work shows that, instead, Federal Reserve policy beginning in the mid-1980s is described quite well by a strikingly parsimonious rule for adjusting the short-term interest rate in response to movements in output and inflation. Taylor's insight has since been embedded fully into theoretical analyses of monetary policy and its effects on the macroeconomy, which now depict central bank policy as a rule for managing the short-term interest rate. Indeed, textbook New Keynesian models such as Woodford's (2003) and Gali's (2008) typically make no reference at all to any measure of the money supply, yet succeed nonetheless in providing a complete and coherent description of the dynamics of output, inflation, and interest rates.

Still, as discussed by Ireland (2008) with reference to both practice and theory, the central bank's ability to manage short-term interest rates has rested, ultimately, on its ability to control, mainly through open market purchases and sales of government bonds, the quantity of reserves supplied to the banking system. Recently, however, Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008) have all suggested that to some extent, even this last remaining role for a measure of money in the monetary policymaking process can vanish when the central bank pays interest on reserves. In the United States, interest on reserves moved quickly from being a theoretical possibility to becoming an aspect of reality when, first, the Financial Services Regulatory Relief Act of 2006 promised to grant the Federal Reserve the power to pay interest on reserves starting on October 1, 2011, second, the Emergency Economic Stabilization Act of 2008 brought that starting date forward to October 1, 2008, and third, the Federal Reserve announced on October 6, 2008 that it would, in fact, begin paying interest on reserves.

Figure 1 illustrates how the mechanics of the Federal Reserve's federal funds rate targeting procedures change with the introduction of interest payments on reserves. In each panel, the quantity of reserves gets measured along the horizontal axis and the federal funds rate along the vertical axis. Panel (a) depicts the traditional case, in which no interest is paid on reserves. The demand curve for reserves slopes downward, since as the federal funds rate falls those banks that typically borrow reserves find that the cost of doing so has declined and those banks that typically lend reserves find that the benefit of doing so has declined: all banks, therefore, wish to hold more reserves. The notation, $DR(FFR; RR = 0, P_0)$, makes clear that while the demand curve describes a relationship between banks' desired holdings of reserves and the federal funds rate FFR, this relationship also depends on the fact that, by assumption, the interest rate RR paid on reserves equals zero. Moreover, because reserves are denominated in units of dollars, this relationship also depends on the aggregate price level P_0. In other words, a change in the federal funds rate leads to a movement along the downward-sloping demand curve, whereas a change in either the interest rate paid on reserves or the aggregate price level results in a shift in the demand curve.

Panel (a) therefore shows that with $RR = 0$ and the price level P_0 taken as fixed in the short run, the Federal Reserve hits its target FFR_0 by conducting open market operations that leave QR_0 dollars of reserves to circulate among banks in the system. Panel (b) then elaborates: if the Fed wants to lower its federal funds rate target from FFR_0 to FFR_1, it must conduct an additional open market purchase of US Treasury securities that increases the supply of reserves from QR_0 to QR_1. In this way, the Federal Reserve's ability to manage the short-term interest rate depends on its ability to control the supply of reserves as well.

Panel (c) of figure 1 then shows how the payment of interest on reserves places a floor under the federal funds rate. For if the federal funds rate does fall below the rate RR_0 at which the Fed pays interest on reserves, any individual bank can earn profits by borrowing reserves from another bank and depositing them at the Fed; this excess demand for reserves then pushes the funds rate back to RR_0. If there is a satiation point beyond which banks will carry no more reserves, then the demand curve in panel (c) terminates when the funds rate falls to RR_0; if, instead, banks become willing to hold arbitrarily large stocks of reserves when the opportunity cost of doing so falls to zero, then the demand curve flattens out and follows the horizontal dotted line when the funds rate reaches RR_0. Of course, these observations simply generalize those that could have been made when describing panels (a) and (b) for the case without interest on reserves: there, the lower bound for the federal funds rate equals zero, since no bank will lend reserves at a negative interest rate when those funds can be held without opportunity cost either as vault cash or as deposits at the Fed.

When, as in panel (c), the Federal Reserve's funds rate target FFR_0 lies above the interest rate RR_0 paid on reserves, the Fed must still conduct open market operations to make the quantity of reserves supplied, QR_2, equal to the quantity demanded. But, with interest on reserves, the level of reserves QR_2 required to support the funds rate target FFR_0 in panel (c) differs from the level of reserves QR_0 required to support the same funds rate target shown in panel (a) for the case without interest on reserves. This is one of the points emphasized by Goodfried (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008): the authority to pay interest on reserves gives the Federal Reserve an additional tool of monetary policy that provides another degree of freedom in the policymaking process since, by adjusting the interest rate paid on reserves, the Fed can achieve different combinations of settings for both the federal funds rate and the quantity of reserves.

Panel (d) of figure 1 highlights another manifestation of this same basic phenomenon. If the Fed holds the interest rate it pays on reserves fixed at RR_0, it must still conduct an open market operation, changing the supply of reserves, to support a change in its federal funds rate target; this remains exactly as before, in panel (b), where the interest rate on reserves is also held fixed, more specifically, at zero. But suppose instead, as shown in panel (d), that the Fed lowers both its funds rate target and the interest rate it pays on reserves, so as to maintain a constant spread between the two. In this case, the lower rate RR_1 on reserves shifts the demand curve for reserves downward and to the left. A new short-run equilibrium is established at the new funds rate target FFR_1 without any change in the quantity of reserves. To use Keister, Martin, and McAndrews' (2008) apt words, paying interest on reserves works to "divorce money" (meaning the quantity of reserves) from "monetary policy" (meaning the federal funds rate).

Importantly, however, panels (a)-(d) hold other determinants of the demand for reserves fixed. And, in particular, while the Keynesian assumption of a fixed aggregate price level may be perfectly justified when looking at the effects of monetary policy actions over short horizons, measured in days or weeks, the question remains as to what will happen over longer intervals, as weeks blend into months and then quarter years and prices begin to change. Going back to panels (a) and (b) for the case without interest on reserves, one possibility is that the Fed simply reverses its policy action, using open market sales of US Treasury securities it purchased previously to drain reserves from the banking system and restore the initial equilibrium in which the funds rate rises again to FFR_0 and the quantity of reserves falls back to QR_0. Another possibility, though, arises when the Fed leaves the supply of reserves at the new, higher level QR_1. The fractional reserve banking system will then use these additional reserves to make new loans and create additional deposits. Broader measures of the money supply will rise and, in the long run, will be matched by a rise in prices that shifts the demand curve for reserves to the right as shown in panel (e). To avoid dynamic instability, the Fed will have to allow the funds rate to return to its initial, higher level FFR_0. In this case, the monetary expansion has all of its classic effects: it decreases interest rates and increases the money supply and output in the short run, but leaves interest rates and output unchanged while increasing money and prices in the long run.

Going back to panels (c) and (d) for the case with interest on reserves, again one possibility is that the Federal Reserve reverses its initial actions, raising both its federal funds rate target and the interest rate it pays on reserves so that the initial equilibrium gets restored without a change in prices. Suppose, however, that the Fed holds interest rates low enough, long enough, so that prices begin to rise. In panel (f), the rising price level shifts the demand curve for reserves back to the right. To maintain an equilibrium and avoid dynamic instability, it appears that the Fed must now do two things: raise its target for the funds rate back to FFR_0 and use open market operations to accommodate the increased demand for reserves brought about by the rising price level. And if, in addition, the Fed wants to maintain a constant spread between the funds rate and the interest rate it pays on reserves, it will of course have to return the interest rate on reserves back to its original setting RR_0 as well. This last example reveals that, even with interest on reserves, monetary policy actions that have macroeconomic effects, changing prices in the long run, still require open market operations that change the quantity of reserves and the broader monetary aggregates. In this example, money and monetary policy get divorced in the short run, but appear happily reunited by the story's close.

Above all, however, the series of examples considered in figure 1 illustrates how tricky it can be to think about the dynamic effects of monetary policy using diagrams that hold many endogenous variables fixed. Although, in several cases, the interest rate on reserves and even the aggregate price level are allowed to vary together with the federal funds rate and the quantity of reserves, all of the graphs ignore the effects that changes in output, brought about by changes in monetary policy, have on the demand for reserves. Likewise, to the extent that changes in the interest rate paid on reserves get passed along to consumers through changes in retail deposit rates, and to the extent that changes in deposit rates then set off portfolio rebalancing among households, additional effects that feed back into banks' demand for reserves get ignored as well. One cannot tell from these graphs whether changes in the federal funds rate, holding the interest rate on reserves fixed either at zero or some positive rate, have different effects on output and inflation than changes in the federal funds rate that occur when the interest rate on reserves is moved in lockstep to maintain a constant spread between the two; if that spread between the federal funds rate and the interest rate on reserves acts as a tax on banking activity, those differences may be important too. Finally, it is of course impossible to say much about the dynamic stability or instability of equilibria under different monetary policymaking strategies with these two-dimensional diagrams. Assessing the full, dynamic macroeconomic effects of monetary policies that involve the payment of interest on reserves requires a fully dynamic and stochastic macroeconomic model. The purpose of this paper is to build and analyze such a model, so as to explore the macroeconomic effects of interest on reserves in more detail.

In previous work, Sargent and Wallace (1985) and Smith (1991) use overlapping generations models of money to see whether the payment of interest on reserves gives rise to problems of equilibrium indeterminacy; here, these same issues are revisited, but with the help of a New Keynesian model that resembles more closely the newer, textbook models of Woodford (2003) and Gali (2008). Berentsen and Monnet (2008) also use a dynamic, general equilibrium model to investigate the workings of monetary policy systems that pay interest on reserves. In particular, Berentsen and Monnet employ a search-theoretic framework that highlights, in great detail, how schemes involving the payment of interest on reserves can make systems of payment operate more efficiently and thereby improve resource allocations supported by decentralized markets in which money serves as a medium of exchange. Here, as in Belongia and Ireland (2010) but in contrast to most other New Keynesian models, the medium of exchange role played by currency and bank deposits receives some attention. But, by generating a demand for money through a more stylized shopping-time specification as opposed to an explicit description of decentralized trade, the model used here can go beyond Berentsen and Monnet's in other ways, allowing for a more detailed analysis of the dynamics of macroeconomic variables including output, inflation, and interest rates that compares to similar analyses conducted with more conventional New Keynesian models.

Finally and most recently, Kashyap and Stein (2010) develop a detailed model of the financial sector, in which the spread between the federal funds rate and the interest rate paid on reserves acts as a time-varying tax, and show how a central bank can use this time-varying tax to optimally stabilize a fractional reserve banking system. Here, the spread between the federal funds rate and the interest rate paid on reserves also gets modeled like a tax on banks. Once again, however, the description of the banking system provided here remains more stylized so that, while some attention is paid below to shocks that destabilize the financial sector, issues relating to the optimal design, structure, and regulation of the financial system cannot receive the very detailed consideration that they get in Kashyap and Stein (2010) and the three other studies mentioned previously: Goodfriend (2002), Ennis and Weinberg (2007), and Keister, Martin, and McAndrews (2008). Here, however, banks' activities get modeled together with those of all other households and firms in the economy, so that the broader focus can be on the macroeconomic effects of interest on reserves.

2 The Model

2.1 Overview

Belongia and Ireland (2010) extend the standard New Keynesian framework, exposited by Woodford (2003) and Gali (2008) and used by many others, to incorporate roles for currency and bank deposits in providing monetary services to households. There, the objective is to revisit issues first raised by Barnett (1980) concerning the ability of simple-sum versus Divisia monetary aggregates to track movements in the true quantity of monetary services provided by liquid assets supplied by both the government and the private banking system. Here, the same model gets extended still further to consider the macroeconomic effects of monetary policies that manage both a short-term market rate of interest, like the federal funds rate in the United States, and the rate of interest on reserves. This extended model allows the host of issues, raised above with the help of figure 1, to be addressed head on, directly and fully, with a dynamic, stochastic general equilibrium model, but requires a somewhat more elaborate description of how banks optimally manage their holdings of reserves; the previous model in Belongia and Ireland (2010) simply posits an exogenously-varying reserve ratio that affects other aspects of bank behavior but it not itself an explicit choice variable as it is here.

The model economy consists of a representative consumer, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0,1]$, a representative bank, and a monetary authority. During each period $t = 0, 1, 2, ...$, each intermediate goods-producing firm produces a distinct, perishable intermediate good. Hence, the intermediate goods may also be indexed by $i \in [0,1]$, where firm i produces good i. The model features enough symmetry, however, to allow the analysis to focus on the behavior of a representative intermediate goods-producing firm that produces the generic intermediate good i. The activities of each of these agents will now be described in turn.

2.2 The Representative Household

The representative household enters each period $t = 0, 1, 2, ...$ with M_{t-1} units of currency, B_{t-1} bonds, and $s_{t-1}(i)$ shares in each intermediate goods-producing firm $i \in [0,1]$. At the beginning of the period, the household receives T_t additional units of currency in the form of a lump-sum transfer from the monetary authority. Next, the households bonds mature, providing B_{t-1} more units of currency. The household uses some of this currency to purchase B_t new bonds at the price of $1/r_t$ dollars per bond, where r_t denotes the gross nominal interest rate between t and $t+1$, and $s_t(i)$ new shares in each intermediate goods-producing firm $i \in [0,1]$ at the price of $Q_t(i)$ dollars per share.

After this initial securities-trading session, the household is left with

$$M_{t-1} + T_t + B_{t-1} + \int_0^1 Q_t(i)s_{t-1}(i)\,\mathrm{d}i - B_t/r_t - \int_0^1 Q_t(i)s_t(i)\,\mathrm{d}i$$

units of currency. It keeps N_t units of this currency to purchase goods and deposits the rest in the representative bank. At the same time, the household also borrows L_t dollars from the bank, bringing the total nominal value of its deposits to

$$D_t = M_{t-1} + T_t + B_{t-1} + \int_0^1 Q_t(i)s_{t-1}(i)\,\mathrm{d}i - B_t/r_t - \int_0^1 Q_t(i)s_t(i)\,\mathrm{d}i - N_t + L_t. \qquad (1)$$

During period t, the household supplies $h_t^g(i)$ units of labor to each intermediate goods-producing firm $i \in [0,1]$, for a total of

$$h_t^g = \int_0^1 h_t^g(i)\,\mathrm{d}i.$$

The household also supplies h_t^b units of labor to the representative bank. The household therefore receives $W_t h_t$ in labor income, where W_t denotes the nominal wage rate and $h_t = h_t^g + h_t^b$ denotes total hours worked in goods production and banking.

Also during period t, the household purchase C_t units of the finished good at the nominal price P_t from the representative finished goods-producing firm. Making this transaction requires

$$h_t^s = \frac{1}{\chi}\left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi} \tag{2}$$

units of shopping time, where M_t^a is an aggregate of monetary services provided from currency N_t and deposits D_t according to

$$M_t^a = [(v^n)^{1/\omega} N_t^{(\omega-1)/\omega} + (1-v^n)^{1/\omega} D_t^{(\omega-1)/\omega}]^{\omega/(\omega-1)}. \tag{3}$$

In the shopping-time specification (2), the parameter $\chi > 1$ governs the rate at which the effort required to purchase goods and services increases as the household economizes on its holdings of monetary assets. The shock v_t^a impacts on the total demand for monetary services; it follows the autoregressive process

$$\ln(v_t^a) = (1-\rho_v^a)\ln(v^a) + \rho_v^a \ln(v_{t-1}^a) + \varepsilon_{vt}^a, \tag{4}$$

where $v_a > 0$ helps determine the steady-state level of real monetary services demanded relative to consumption, the persistence parameter satisfies $0 \leq \rho_v^a < 1$, and the serially uncorrelated innovation ε_{vt}^a has mean zero and standard deviation σ_v^a. In the monetary aggregation specification (3), the parameter $\omega > 0$ measures the elasticity of substitution between currency and deposits in creating liquidity services and the parameter v^n, satisfying

$0 < v^n < 1$, helps determine the steady-state share of currency versus deposits in creating the monetary aggregate.

At the end of period t, the household owes the bank $r_t^l L_t$ dollars, where r_t^l is the gross nominal interest on loans. At the same time, however, the bank owes the household $r_t^d D_t$ dollars, where r_t^d is the gross nominal interest rate on deposits. The household also receives a nominal dividend payment $F_t(i)$ for each share that it owns in each intermediate goods-producing firm $i \in [0, 1]$. After all these payments get sent and received, the household carries M_t units of currency into period $t+1$, where

$$M_t = N_t + W_t h_t + \int_0^1 F_t(i) s_t(i)\,\mathrm{d}i + r_t^d D_t - P_t C_t - r_t^l L_t. \tag{5}$$

The household, therefore, chooses sequences for B_t, $s_t(i)$ for all $i \in [0, 1]$, N_t, D_t, L_t, h_t, C_t, h_t^s, M_t^a, and M_t for all $t = 0, 1, 2, ...$ to maximize the expected utility function

$$E \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \eta(h_t + h_t^s)], \tag{6}$$

where the discount factor satisfies $0 < \beta < 1$ and $\eta > 0$ measures the weight on leisure versus consumption. The preference shock a_t in (6) follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \tag{7}$$

where the persistence parameter satisfies $0 \leq \rho_a < 1$ and the serially uncorrelated innovation ε_{at} has mean zero and standard deviation σ_a. The household makes its optimal choices subject to the constraints (1)-(3) and (5), each of which must hold for all $t = 0, 1, 2, ...$, taking as given the behavior of the exogenous shocks described by (4) and (7) for all $t = 0, 1, 2, ...$

A convenient way to characterize the solution to the household's problem is to substitute the shopping-time specification (2) into the utility function (7) and to express the remaining constraints (1), (3), and (5) in real terms by dividing through by the nominal price level P_t

to obtain

$$\frac{M_{t-1} + T_t + B_{t-1} - B_t/r_t - N_t + L_t}{P_t} + \int_0^1 \left[\frac{Q_t(i)}{P_t}\right] [s_{t-1}(i) - s_t(i)]\,\mathrm{d}i \geq \frac{D_t}{P_t}, \tag{8}$$

$$\left[(v^n)^{1/\omega}\left(\frac{N_t}{P_t}\right)^{(\omega-1)/\omega} + (1-v^n)^{1/\omega}\left(\frac{D_t}{P_t}\right)^{(\omega-1)/\omega}\right]^{\omega/(\omega-1)} \geq \frac{M_t^a}{P_t}, \tag{9}$$

and

$$\frac{N_t + W_t h_t + r_t^d D_t}{P_t} + \int_0^1 \left[\frac{F_t(i)}{P_t}\right] s_t(i)\,\mathrm{d}i \geq C_t + \frac{r_t^l L_t + M_t}{P_t}, \tag{10}$$

after allowing for free disposal. Letting Λ_t^1, Λ_t^2, and Λ_t^3 denote the nonnegative Lagrange multipliers on these three constraints, the first-order conditions for the household's problem can be written as

$$\frac{\Lambda_t^1}{r_t} = \beta E_t\left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}}\right), \tag{11}$$

$$\Lambda_t^1\left[\frac{Q_t(i)}{P_t}\right] = \Lambda_t^3\left[\frac{F_t(i)}{P_t}\right] + \beta E_t\left\{\Lambda_{t+1}^1\left[\frac{Q_{t+1}(i)}{P_{t+1}}\right]\right\} \tag{12}$$

for all $i \in [0,1]$,

$$\frac{N_t}{P_t} = v^n\left(\frac{M_t^a}{P_t}\right)\left(\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3}\right)^{\omega}, \tag{13}$$

$$\frac{D_t}{P_t} = (1-v^n)\left(\frac{M_t^a}{P_t}\right)\left(\frac{\Lambda_t^2}{\Lambda_t^1 - r_t^d\Lambda_t^3}\right)^{\omega}, \tag{14}$$

$$\Lambda_t^1 = r_t^l \Lambda_t^3, \tag{15}$$

$$\eta a_t = \Lambda_t^3\left(\frac{W_t}{P_t}\right), \tag{16}$$

$$\frac{a_t}{C_t}\left[1 - \eta\left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi}\right] = \Lambda_t^3, \tag{17}$$

$$\eta a_t\left(\frac{v_t^a P_t C_t}{M_t^a}\right)^{\chi} = \Lambda_t^2\left(\frac{M_t^a}{P_t}\right), \tag{18}$$

and

$$\Lambda_t^3 = \beta E_t\left(\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}}\right), \tag{19}$$

together with (2) and (8)-(10) with equality for all $t = 0, 1, 2, ...$ The implications of these optimality conditions for issues relating to monetary aggregation and the demand for monetary assets are discussed below.

2.3 The Representative Finished Goods-Producing Firm

During each period $t = 0, 1, 2, ...$, the representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta-1)\theta}\,\mathrm{d}i\right]^{\theta/(\theta-1)} \geq Y_t,$$

where $\theta > 1$ measures the elasticity of substitution between the various intermediate goods in producing the final good. Thus, the finished goods-producing firm chooses $Y_t(i)$ for all $i \in [0, 1]$ to maximize its profits, given by

$$P_t\left[\int_0^1 Y_t(i)^{(\theta-1)\theta}\,\mathrm{d}i\right]^{\theta/(\theta-1)} - \int_0^1 P_t(i)Y_t(i)\,\mathrm{d}i,$$

for all $t = 0, 1, 2, ...$ The first-order conditions for this problem are

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \tag{20}$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, ...$

Competition drives the finished goods-producing firm's profits to zero in equilibrium. This zero profit condition implies that

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta}\,\mathrm{d}i\right]^{1/(1-\theta)}$$

for all $t = 0, 1, 2, ...$

2.4 The Representative Intermediate Goods-Producing Firm

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm hires $h_t^g(i)$ units of labor from the representative household to manufacture $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Z_t h_t^g(i) \geq Y_t(i). \tag{21}$$

The aggregate technology shock follows a random walk with positive drift:

$$\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{zt}, \tag{22}$$

where $z > 1$ and the serially uncorrelated innovation ε_{zt} has mean zero and standard deviation σ_z.

Since the intermediate goods substitute imperfectly for one another in producing the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market. Hence, during each period $t = 0, 1, 2, ...$, the intermediate goods-producing firm sets the nominal price $P_t(i)$ for its output, subject to the requirement that it satisfy the representative finished goods-producing firm's demand, described by (20). In addition, following Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in units of the finished good and given by

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t,$$

where the parameter $\phi_p > 0$ governs the magnitude of the price adjustment cost and where $\pi > 1$ denotes the gross, steady-state inflation rate.

The cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: the firm chooses a sequence for $P_t(i)$ for all $t = 0, 1, 2, ...$ to maximize its total, real market value, which from the equity-pricing relation (12) implied by the household's optimizing behavior is proportional to

$$E\sum_{t=0}^{\infty}\beta^t\Lambda_t^3\left[\frac{F_t(i)}{P_t}\right]$$

where

$$\frac{F_t(i)}{P_t}=\left[\frac{P_t(i)}{P_t}\right]^{1-\theta}Y_t-\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{W_tY_t}{P_tZ_t}\right)-\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 Y_t \tag{23}$$

for all $t=0,1,2,....$ The first-order conditions for this problem are

$$\begin{aligned}0 &= (1-\theta)\Lambda_t^3\left[\frac{P_t(i)}{P_t}\right]^{-\theta}Y_t+\theta\Lambda_t^3\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\left(\frac{W_tY_t}{P_tZ_t}\right)\\ &-\phi_p\Lambda_t^3\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]\left[\frac{Y_tP_t}{\pi P_{t-1}(i)}\right]\\ &+\beta\phi_pE_t\left\{\Lambda_{t+1}^3\left[\frac{P_{t+1}(i)}{\pi P_t(i)}-1\right]\left[\frac{Y_{t+1}P_{t+1}(i)P_t}{\pi P_t(i)^2}\right]\right\}\end{aligned} \tag{24}$$

for all $t=0,1,2,....$

2.5 The Representative Bank

During each period $t=0,1,2,...$, the representative bank accepts deposits worth D_t dollars from the representative household. Creating these deposits requires N_t^v dollars in reserves and h_t^d units of labor, where

$$x_t^a\left[(x^n)^{1/\nu}\left(\frac{N_t^v}{P_t}\right)^{(\nu-1)/\nu}+(1-x^n)^{1/\nu}(Z_th_t^d)^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)}\geq\frac{D_t}{P_t}. \tag{25}$$

In (25), the parameter $\nu>0$ measures the elasticity of substitution between reserves and labor in the deposit creation process and the parameter x^n, satisfying $0<x^n<1$, helps determine the share of reserves relative to labor in producing deposits. The shock x_t^a to productivity in the banking sector follows the autoregressive process

$$\ln(x_t^a)=(1-\rho_x^a)\ln(x^a)+\rho_x^a\ln(x_{t-1}^a)+\varepsilon_{xt}^a, \tag{26}$$

where $x^a > 0$, $0 \leq \rho_x^a < 1$, and the serially uncorrelated innovation ε_{xt}^a has mean zero and standard deviation σ_x^a. In addition, to manage its stock of reserves worth N_t^v/P_t in real terms, the bank must hire h_t^v additional units of labor, where

$$Z_t h_t^v \geq \phi_v \left(\frac{N_t^v}{P_t} \right). \tag{27}$$

When the parameter $\phi_v > 0$ is very small but still strictly positive, the additional labor requirement embedded into the specification through (27) has little effect on real resource allocations, but ensures that banks' holdings of reserves remain finite and uniquely-determined even when the central bank pays interest on those reserves at a rate that equals the market rate r_t on bonds. This model feature thereby eliminates one potential source of equilibrium indeterminacy by invoking the plausible assumption that looking after a larger stock of reserves always requires at least a small amount of additional time and effort from a bank's funds manager; the same assumption would make the demand curves for reserves in the various panels of figure 1 terminate at the point where the federal funds rate falls to the rate of interest paid on reserves, instead of extending infinitely far out, parallel to the horizontal axis. Also, the assumption, reflected in both (25) and (27), that labor productivity in banking activities grows at the same stochastic rate as it does in the production of intermediate goods as described by (21) and (22), ensures that the model remains consistent with balanced growth.

After deciding on its optimal holding of reserves N_t^v, the bank lends out its remainin funds L_t; its balance sheet constraint requires that

$$\frac{D_t}{P_t} \geq \frac{N_t^v + L_t}{P_t} \tag{28}$$

As noted above, deposits pay interest at the gross rate r_t^d and loans earn interest at the gross rate r_t^l; in addition, reserves earn interest at the gross rate r_t^v. Hence, during each period t, the bank chooses D_t, N_t^v, L_t, h_t^d, and h_t^v to maximize its profits, given in nominal terms by

$$(r_t^l - 1)L_t + (r_t^v - 1)N_t^v - (r_t^d - 1)D_t - W_t(h_t^d + h_t^v), \tag{29}$$

subject to the constraints (25), (27) and (28) and taking as given the behavior of the exogenous shocks as described by (22) and (26) for all $t = 0, 1, 2, ...$

A convenient way to characterize the solution to the bank's problem is to substitute the constraints (25), (27), and (28) into the expression (29) for profits, which can be rewritten in real terms as

$$\begin{aligned}&(r_t^l - r_t^d)x_t^a \left[(x^n)^{1/\nu} \left(\frac{N_t^v}{P_t} \right)^{(\nu-1)/\nu} + (1 - x^n)^{1/\nu} (Z_t h_t^d)^{(\nu-1)/\nu} \right]^{\nu/(\nu-1)} \\ &- \left[r_t^l + \phi_v \left(\frac{W_t}{P_t Z_t} \right) - r_t^v \right] \left(\frac{N_t^v}{P_t} \right) - \left(\frac{W_t}{P_t} \right) h_t^d.\end{aligned} \tag{30}$$

Equation (30) serves to highlight that the bank earns revenue from charging a higher interest rate on its loans than it must pay on its deposits, but also incurs both an opportunity and real resource cost of holding reserves and must compensate its workers for their efforts in creating deposits. The first-order conditions for the bank's problem can be written as

$$\frac{N_t^v}{P_t} = (r_t^l - r_t^d)^\nu (x_t^a)^{\nu-1} x^n \left(\frac{D_t}{P_t} \right) \left[r_t^l + \phi_v \left(\frac{W_t}{P_t Z_t} \right) - r_t^v \right]^{-\nu}, \tag{31}$$

$$h_t^d = (r_t^l - r_t^d)^\nu (x_t^a)^{\nu-1} (1 - x^n) \left(\frac{D_t}{P_t} \right) Z_t^{\nu-1} \left(\frac{W_t}{P_t} \right)^{-\nu}, \tag{32}$$

and (25), (27), and (28) with equality for all $t = 0, 1, 2, ...$ Note that (11), (15), and (19) imply that $r_t = r_t^l$ for all $t = 0, 1, 2, ...$: since households can obtain additional funds either by selling bonds or borrowing from banks, the interest rate on bonds must equal the interest rate on loans. Using this no-arbitrage relationship, (25), (31), and (32) can be combined to obtain

$$r_t^d = r_t - \frac{1}{x_t^a}\left\{x^n\left[r_t + \phi_v\left(\frac{W_t}{P_tZ_t}\right) - r_t^v\right]^{1-\nu} + (1-x^n)\left(\frac{W_t}{P_tZ_t}\right)^{1-\nu}\right\}^{1/(1-\nu)}, \tag{33}$$

which shows how the cost of deposit creation, which in turn depends on the opportunity cost of holding reserves and the cost of labor, causes the competitively-determined interest rate on deposits to fall short of the interest rate on bonds and loans.

2.6 The Monetary Authority

As usual in New Keynesian models like this one, the central bank will be assumed to conduct monetary policy by adjusting the short-term market rate of interest r_t in response to movements in inflation $\pi_t = P_t/P_{t-1}$ and a stationary measure of real economic activity, in this case the rate of output growth

$$g_t = Y_t/Y_{t-1}, \tag{34}$$

since the level of output inherits a random walk from the nonstationary technology shock (22). The modified Taylor (1993) rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_{t-1}/\pi) + \rho_g \ln(g_{t-1}/g) + \varepsilon_{rt} \tag{35}$$

allows, in addition, for interest rate smoothing through the lagged interest rate term on the right-hand side. In (35), the constants r, π, and g denote the steady-state values of the short-term nominal interest rate, the inflation rate, and the output growth rate, the Taylor rule coefficients $\rho_r \geq 0$, $\rho_\pi \geq 0$, and $\rho_g \geq 0$ are chosen by the central bank, and the serially uncorrelated innovation ε_{rt} has mean zero and standard deviation σ_r.

In addition, here, the central bank must also choose a rule for determining the interest rate r_t^v it pays on reserves. By specifying a general rule of the form $r_t^v = \tau_t r_t^\alpha$ or, in logs,

$$\ln(r_t^v) = \ln(\tau_t) + \alpha \ln(r_t), \tag{36}$$

where $\alpha \geq 0$ is a parameter, the new variable τ_t follows the autoregressive process

$$\ln(\tau_t) = (1 - \rho_\tau)\ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t} \tag{37}$$

with $\tau \geq 1$, $0 \leq \rho_\tau < 1$, and the serially uncorrelated innovation $\varepsilon_{\tau t}$ has mean zero and standard deviation σ_τ, the general model allows flexibly for a number of special cases, including: (i) the standard case $\alpha = 0$, $\tau = 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$ in which no interest is paid on reserves, (ii) the case $\alpha = 0$, $\tau > 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$ in which interest is paid on reserves at the constant, gross rate τ, (iii) the case $\alpha = 1$, $0 < \tau < 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$ depicted in panels (c), (d), and (f) of figure 1 in which there is a constant, $100(1 - \tau)$ percentage-point spread between the market rate and the interest rate on reserves, (iv) the case $\alpha = 1$, $\tau = 1$, $\rho_\tau = 0$, and $\sigma_\tau = 0$ in which interest is paid on reserves at the market rate, and (v) a variety of cases with $0 \leq \rho_\tau < 1$ and $\sigma_\tau > 0$ in which there is independent, stochastic variation in the rate of interest on reserves, giving rise to a time-varying spread between the market rate and the rate of interest on reserves.

2.7 Monetary Aggregation

In this model with currency and deposits, the variable M_t^a represents the true aggregate of monetary services demanded by the representative household during each period $t = 0, 1, 2, ...$. Note that (11) and (19), describing the representative household's optimizing behavior, imply that

$$\Lambda_t^1 = r_t \Lambda_t^3 \tag{38}$$

for all $t = 0, 1, 2, ...$ Substituting (38), together with (13) and (14), into (9) then yields

$$\frac{\Lambda_t^2}{\Lambda_t^3} = [v^n(r_t - 1)^{1-\omega} + (1 - v^n)(r_t - r_t^d)^{(1-\omega)}]^{1/(1-\omega)}. \tag{39}$$

Define the own rate of return r_t^a on the monetary aggregate M_t^a with reference to the right-hand side of (39):

$$r_t - r_t^a = [v^n(r_t - 1)^{1-\omega} + (1 - v^n)(r_t - r_t^d)^{(1-\omega)}]^{1/(1-\omega)}. \tag{40}$$

Then one can verify that if the household's choices of currency and deposits are not of independent interest, the household's problem can be stated more simply as one of choosing sequences for B_t, $s_t(i)$ for all $i \in [0, 1]$, L_t, h_t, C_t, M_t^a, and M_t for all $t = 0, 1, 2, ...$ to maximize the expected utility function obtained by substituting (2) into (6) subject to the constraints

$$\frac{M_{t-1} + T_t + B_{t-1} - B_t/r_t + L_t}{P_t} + \int_0^1 \left[\frac{Q_t(i)}{P_t}\right] [s_{t-1}(i) - s_t(i)] \,\mathrm{d}i \geq \frac{M_t^a}{P_t},$$

and

$$\frac{W_t h_t + r_t^a M_t^a}{P_t} + \int_0^1 \left[\frac{F_t(i)}{P_t}\right] s_t(i) \,\mathrm{d}i \geq C_t + \frac{r_t^l L_t + M_t}{P_t}$$

for all $t = 0, 1, 2, ...$, confirming that M_t^a represents a true economic aggregate of monetary services.

Equations (38)-(40) also allow (16) and (18) to be combined to obtain

$$\ln\left(\frac{M_t^a}{P_t}\right) = \frac{\chi}{1+\chi}\ln(C_t) + \frac{1}{1+\chi}\ln\left(\frac{W_t}{P_t}\right) - \frac{1}{1+\chi}\ln(r_t - r_t^a) + \frac{\chi}{1+\chi}\ln(v_t^a), \tag{41}$$

and (13) and (14) to be rewritten as

$$\frac{N_t}{P_t} = v^n\left(\frac{u_t^a}{u_t^n}\right)^{\omega}\left(\frac{M_t^a}{P_t}\right) \tag{42}$$

and

$$\frac{D_t}{P_t} = (1 - v^n) \left(\frac{u_t^a}{u_t^d}\right)^{\omega} \left(\frac{M_t^a}{P_t}\right), \tag{43}$$

where

$$u_t^a = \frac{r_t - r_t^a}{r_t}, \tag{44}$$

$$u_t^n = \frac{r_t - 1}{r_t}, \tag{45}$$

and

$$u_t^d = \frac{r_t - r_t^d}{r_t}, \tag{46}$$

use Barnett's (1978) formula to define the user costs u_t^a, u_t^n, and u_t^d of the monetary aggregate M_t^a, currency N_t, and deposits D_t. Equation (41) takes the form of a demand curve for monetary aggregate M_t^a, which having been derived from a shopping-time specification has the real wage as well as consumption as its scale variables, a result that echos Karni's (1973), and has $r_t - r_t^a$ as its opportunity cost term. Meanwhile, (42) and (43) show how the household's optimal choices of currency and deposits in creating the monetary aggregate depend on the share parameter v^n as well as the user cost of each individual monetary asset relative to the aggregate as a whole. A larger value of the parameter ω, implying more substitutability between currency and deposits in creating the aggregate, naturally makes the demand for each individual asset more responsive to changes in user costs.

While the true monetary aggregate M_t^a and its user cost u_t^a are well-defined and observable within the model, their magnitudes depend on not only on the quantities of currency and deposits but also on functional forms and parameters that may not be known to outside agents, including analysts at the monetary authority and applied econometricians more generally. Belongia and Ireland (2010) show, however, that in a model like this one, but without interest on reserves, movements in both the true aggregate and its user cost are approximated very closely by movements in Divisia quantity and price indices for monetary services like those proposed by Barnett (1980); and the advantage of these Divisia aggregates is that they can be constructed without reference to unknown functional forms and parameters. Until 2006, the Federal Reserve Bank of St. Louis compiled and released data on these monetary services indices, building closely on Barnett's work as described by Anderson, Jones, and Nesmith (1997*a*, 1997*b*); below, therefore, data on the St. Louis Fed's monetary services indices will be used as a proxy for data on M_t^a. The more familiar, simple-sum aggregate

$$M_t^s = N_t + D_t \tag{47}$$

is, of course, constructed quite easily both in the model and the data: it, too, does not depend on unknown parameters. As emphasized by Barnett (1980), however, simple-sum aggregates do not represent true, economic aggregates except under the extreme and counterfactual assumption that currency and deposits are perfect substitutes in providing liquidity services and therefore have the identical user costs; and as shown both in Belongia and Ireland (2010) and here, below, the simple-sum aggregate M_t^s often behaves quite differently from the true monetary aggregate M_t^a and, by extension, to the preferred, Divisia approximations to the true aggregate.

Two other monetary variables considered below are the reserve ratio and the monetary base. The former can be measured in the usual way, dividing bank reserves by deposits:

$$rr_t = N_t^v / D_t. \tag{48}$$

The latter is measured most easily by observing that since, for simplicity, households in this model do not carry deposits across periods and banks do not carry reserves across periods either, the variable M_t that keeps track of the amount currency possessed by the representative household at the end of each period $t = 0, 1, 2, ...$ also equals the monetary base. And since, within each period, the monetary base gets increased both through the lump-sum transfer made by the monetary authority to households and the interest payments on reserves made by the monetary authority to banks, it evolves according to

$$M_t = M_{t-1} + T_t + (r_t^v - 1)N_t^v \tag{49}$$

for all $t = 0, 1, 2, ...$

2.8 Symmetric Equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $Y_t(i) = Y_t$, $h_t^g(i) = h_t^g$, $P_t(i) = P_t$, $F_t(i) = F_t$, and $Q_t(i) = Q_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, ...$ In addition, the market-clearing conditions $B_t = 0$, $s_t(i) = 1$ for all $i \in [0, 1]$,

$$h_t = h_t^g + h_t^g \tag{50}$$

and

$$h_t^b = h_t^d + h_t^v \tag{51}$$

must hold for all $t = 0, 1, 2, ...$ After imposing these conditions, (2), (4), (7)-(19), (21)-(28), (31), (32), (34)-(37), (40), and (44)-(51) can be collected together to form a system of 38 equations determining the equilibrium behavior of the 38 variables C_t, Y_t, g_t, h_t^s, h_t, h_t^g, h_t^b, h_t^d, h_t^v, F_t, Λ_t^1, Λ_t^2, Λ_t^3, M_t, T_t, N_t, D_t, L_t, M_t^a, N_t^v, M_t^s, P_t, W_t, Q_t, r_t^l, r_t^d, r_t, r_t^v, r_t^a, u_t^a, u_t^n, u_t^d, rr_t, v_t^a, a_t, Z_t, x_t^a, and τ_t.

This system implies that many of these variables will be nonstationary, with real variables inheriting a unit root from the nonstationary process (22) for the technology shock and nominal variables inheriting a unit root from the conduct of monetary policy as described by the Taylor rule (35). However, the real variables become stationary when scaled by the lagged technology shock Z_{t-1} and the nominal variables become stationary when expressed in growth rates. When the 38-equation system is rewritten in terms of these appropriately-transformed variables, it implies that the economy has a balanced growth path, along which all of the stationary variables remain constant in the absence of stocks. The transformed system can therefore by log-linearized around its steady state to form a set of linear expectational difference equations that can be solved using methods outlined by Blanchard and Kahn (1980) and Klein (2000).

3 Results

3.1 Calibration

Numerical implementation of the solution procedure just described requires that specific values be assigned to each of the model's 29 parameters: χ, β, η, θ, ω, ν, ϕ_p, ϕ_v, π, ρ_r, ρ_π, ρ_g, α, τ, v^a, v^n, z, x^a, x^n, ρ_v^a, ρ_a, ρ_x^a, ρ_τ, σ_v^a, σ_a, σ_z, σ_x^a, σ_τ, and σ_r. Hence, the exercise continues by calibrating a version of the model without interest on reserves to match various statistics from the United States economy, mainly during the period extending from the fourth quarter of 1987 through the third quarter of 2008. This sample period starts with the appointment of Alan Greenspan as Federal Reserve Chairman and continues through Ben Bernanke's term until the onset of the financial crisis; throughout this period, likewise, the Federal Reserve did not pay interest on reserves.

Equation (41) reveals that $1/(1+\chi)$ measures the elasticity of demand for monetary services with respect to the opportunity cost variable $r_t - r_t^a$; the same money demand relationship constrains the coefficient on the opportunity cost variable to be equal in absolute value to the coefficient on the real wage, which in term equals one minus the coefficient on consumption. Using data on the M2 monetary services quantity index and the associated price index compiled by the Federal Reserve Bank of St. Louis to measure M_t^a and $r_t - r_t^a$, as well as data on real personal consumption expenditures and the associated chain-type price index to measure C_t and P_t and the index of compensation per hour in the nonfarm business sector assembled by the Bureau of Labor Statistics for its report on Productivity and Costs to measure W_t, an ordinary least squares regression of this form, with all of the constraints imposed, yields

$$\ln(M_t^a/P_t) = -4.5 + 0.84\ln(C_t) + 0.16\ln(W_t/P_t) - 0.16\ln(r_t - r_t^a),$$

suggesting a calibrated value of $\chi = 5$. Since the St. Louis Fed discontinued its monetary services series in 2005:4, the data used to estimate this equation run from 1987:4 through 2005:4. Likewise, (42) indicates that the parameter ω measuring the elasticity of substitution between currency and deposits in creating the true monetary aggregate M_t^a can be calibrated based on a regression of the ratio of currency to the M2 monetary services index on the ratio of the user cost index associated with M2 monetary services to the user cost of currency. Again, these data are available from the St. Louis Fed from 1987:4 through 2005:4 and yield the estimated equation

$$\ln(N_t/M_t^a) = -4.3 + 0.53\ln(u_t^a/u_t^n),$$

suggesting the calibrated value $\omega = 0.50$. Note that this setting for ω makes the elasticity of substitution between currency and deposits in creating the true monetary aggregate smaller than that implied by the Cobb-Douglas specification that represents the special case of (3) with $\omega = 1$. In principle, a setting for ν, measuring the elasticity of substitution between reserves and labor in deposit creation, might be pinned down from a detailed study of bank productivity, but a search of the literature yielded no estimate covering the 1987-2008 period. Introspection suggests that there is likely to be very little substitutability between these two quite different inputs, and the calibrated value $\nu = 0.25$ reflects this idea.

Data on the federal funds rate and the growth rates of real GDP and its deflator, 1987:4-2008:3, yield ordinary least squares estimates of the coefficients of the Taylor rule (35):

$$\ln(r_t) = -0.0018 + 0.95\ln(r_{t-1}) + 0.21\ln(\pi_{t-1}) + 0.13\ln(g_t),$$

suggesting the settings $\rho_r = 0.95$, $\rho_\pi = 0.20$, and $\rho_g = 0.15$. Of course, under the benchmark regime where interest is not paid on reserves, (36) and (37) get specialized by setting $\alpha = 0$

and $\tau = 1$. The analysis below considers two alternative policy regimes under which interest does get paid on reserves. In the first alternative regime, $\alpha = 1$ and $\tau = 1 - 0.000625$; under this policy, the monetary authority maintains an average spread of 25 basis points between the market rate of interest r_t and the interest rate on reserves r_t^v, when both rates are expressed in annualized terms. In the second alternative regime, $\alpha = 1$ and $\tau = 1$, so that interest gets paid on reserves at the market rate.

Interpreting each period in the model as a quarter year in real time, the settings $z = 1.005$ and $\pi = 1.005$ imply an annualized, steady-state growth rate for real, per-capita variables of 2 percent and an annualized, steady-state inflation rate of 2 percent as well. Given these choices, the setting $\beta = 0.995$ then implies a steady-state market rate of interest of about 6 percent per year. The settings $\theta = 6$ and $\phi_p = 50$, drawn from previous work by Ireland (2000, 2004*a*, 2004*b*), make the steady-state markup of price over marginal cost equal to 20 percent and, as explained in Ireland (2004*a*), imply a speed of price adjustment in this model with quadratic price adjustment costs that is the same as the speed of price adjustment in a model with staggered price setting following Calvo's (1983) specification in which individual goods' prices are adjusted, on average, every 3.75 quarters, that is, just slightly more frequently that once per year.

Values for the the next six parameters, η, v^a, v^n, x^a, x^n, and ϕ_v, get selected to match six facts. First, the steady-state value of hours worked equals 0.33, meaning that the representative household allocates 1/3 of its time to labor. Second, the steady-state ratio of the simple-sum monetary aggregate M_t^s to nominal consumption P_tC_t equals 3, matching the fact that during the 1987:4-2008:3 period, the average ratio of simple-sum M2 to quarterly nominal personal consumption expenditure equals 3.04; since the St. Louis Fed's monetary services indices are just that, namely index numbers for monetary services, they track growth rates not levels and therefore cannot be used to match the ratio of M_t^a to P_tC_t in the model. Third, the steady-state ratio of currency to deposits equals 0.10, approximately matching the fact that the average ratio of currency to simple-sum deposits in M2 equals 0.1076.

Fourth, the steady-state ratio of reserves to deposits equals 0.02, matching the fact that the average ratio of Federal Reserve Bank of St. Louis adjusted reserves to simple-sum deposits in M2 equals almost exactly 0.02. Fifth, the steady-state ratio of employment in banking to total employment equals 0.007, or seven-tenths of one percent. In data from the Bureau of Labor Statistics' Current Employment Survey, 1.4 percent of all workers on total nonfarm payrolls were employed in depository credit intermediation on average over the period from 1990 through 2010. Of course, those employees engaged in a range of banking activities that extends beyond deposit creation; hence, the smaller, 0.7 percent figure is taken as the one to be matched by the model. Sixth, in data covering 2008:4 through 2010:3, the ratio of Federal Reserve Bank of St. Louis adjusted reserves to simple-sum deposits in M2 averaged 13 percent and peaked at a level just below 17 percent. During most of that period, the Federal Reserve paid interest on reserves at a rate intended to match the federal funds rate. Based on these observations, ϕ_v is chosen so that in the steady state of the model in which $\alpha = 1$ and $\tau = 1$, so that interest is paid on reserves at the market rate, the ratio of reserves to deposits equals 15 percent. Thus, a 15 percent reserve ratio is interpreted as the one that is optimally chosen by banks when the opportunity cost of holding reserves falls to zero.

Searching over various parameter combinations with these targets in mind leads to the settings $\eta = 2.5$, $\upsilon^a = 0.90$, $\upsilon^n = 0.20$, $x^a = 65$, $x^n = 0.75$, and $\phi_v = 0.000005$. Interestingly, these parameter values also imply, through the relationship shown in (33), a spread between the market rate of interest r_t and the deposit rate r_t^d equal in annualized terms to 0.97, just below one percentage point, in the model's steady state without interest on reserves. In US data, 1987:4-2008:3, the average spread between the three-month Treasury bill rate and the own rate of return on the deposit component of simple-sum M2 equals 1.23 percent. Hence, the model and data are not too far out of line along this added dimension; indeed, that the spread between market and deposit rates in the data exceeds the same spread in the model provides reassurance that the real resource cost of deposit creation in the model is based on a conservative estimate.

Since the technology shock in (22) follows a random walk, it is highly persistent by assumption. The settings $\rho_v^a = 0.95$ and $\rho_a = 0.95$ make the shocks to the demand for monetary services and to preferences highly persistent as well. The additional settings $\rho_x^a = 0.50$ and $\rho_\tau = 0.50$ imply a more modest amount of persistence in the shocks to productivity in the banking system and to the spread between the market interest rate and the interest rate paid on reserves. Since most of the analysis that follows focuses on impulse responses from the log-linearized model, the settings $\sigma_v^a = 0.01$, $\sigma_a = 0.01$, and $\sigma_z = 0.01$ are really just normalizations that make one-standard-deviation money demand, preference, and technology shocks, equivalently, into one-percentage-point shocks. The setting $\sigma_r = 0.000625$, however, means that the monetary policy shock leads to a 25-basis-point change in the annualized, short-term interest rate. The setting $\sigma_\tau = 0.0003125$ is half that size, so that the rate of interest paid on reserves remains below the market rate of interest after a one-standard-deviation shock to the interest rate spread, even under the alternative policy considered below in which $\alpha = 1$ and $\tau = 1 - 0.000625$, so that the monetary authority maintains an average 25-basis-point spread between the annualized market rate of interest and the annualized interest rate on reserves. Finally, the setting $\sigma_x^a = \ln(10)$ is used below to capture some of the effects of a financial crisis, in which an adverse shock reduces the productivity of reserves and labor in producing bank deposits by an entire order of magnitude.

3.2 Equilibrium Determinacy

The larger size of this model with currency, deposits, and banks precludes the derivation of analytic results like those obtained by Woodford (2003) and Bullard and Mitra (2005), identifying conditions on the coefficients of Taylor rules like (35) that ensure the determinacy of rational expectations equilibria in smaller-scale New Keynesian models. Numerical analysis indicates, however, that for this model, familiar conditions for determinacy apply, both with and without interest on reserves. Specifically, a grid search over 2001 evenly-spaced values for ρ_r between 0 and 2, 2001 evenly-spaced values for ρ_π between 0 and 2, and 11 evenly-spaced values for ρ_g between 0 and 1, making a total of more than 44 million cases in all, reveals that for all three policy regimes described above, without interest on reserves ($\alpha = 0$ and $\tau = 1$), with interest paid on reserves at an annualized rate that is, on average, 25 basis points below the market rate ($\alpha = 0$ and $\tau = 1 - 0.000625$), and with interest paid on reserves at the market rate ($\alpha = 1$ and $\tau = 1$), the exact same condition

$$\rho_r + \rho_\pi > 1 \tag{52}$$

is both necessary and sufficient on the grid for the system to have a unique dynamically stable rational expectations equilibrium according to the criteria of Blanchard and Kahn (1980). Condition (52), of course, requires the monetary authority to satisfy what Woodford (2003) calls the "Taylor Principle," increasing the short-term market rate of interest more than proportionately in response to any change in inflation.

Evidently, the payment of interest on reserves, at a rate below or at the market rate, does not give rise to special problems of equilibrium indeterminacy in this New Keynesian model as it does in the overlapping generations models studied by Sargent and Wallace (1985) and Smith (1991). Along these lines it should be noted, however, that for the case where interest on reserves is paid at the market rate, the small but positive labor cost (27) of managing larger stocks of reserves plays a key role. Without this additional cost, a well-defined equilibrium would fail to exist in the first place, as the representative bank facing the technology described by (25) alone would want to hold an unboundedly large stock of reserves in order to drive the labor requirement of creating deposits down to zero. And if, instead, the technology described by (25) were modified to make banks indifferent between holding any level of reserves beyond some finite satiation point when the opportunity cost of doing so equals zero, as they are along the horizontal, dashed segments of the demand curves shown in panels (c), (d), and (f) of figure 1, then the model would feature of continuum of steady states when interest is paid on reserves at the market rate. This last source of indeterminacy, however, could be eliminated if the monetary authority lowers the rate of interest it pays on reserves ever so slightly below the market rate; the arbitrarily small but still positive interest rate spread would then play the same role as the arbitrarily small but positive labor requirement measured by the parameter ϕ_v in the model as it stands now.

3.3 The Steady-State Effects of Paying Interest on Reserves

Table 1 compares the steady-state values of a range of variables under the benchmark policy that does not pay interest on reserves to the steady-state values of the same variables under the alternative policies of paying interest on reserves, either at a rate that, in annualized terms, lies 25 basis points below the market rate or that coincides with the market rate. In the model, the steady-state rate of output growth gets pinned down by the steady-state rate of technological change, as measured by the parameter z in (22), describing the process for the technology shock. The steady-state rate of inflation π gets chosen by the monetary authority at the same time it fixes the coefficients of the Taylor rule (35). The steady-state market rate of interest then gets determined by the Fisher relationship: in gross terms, it equals the product of the inflation rate π and the real interest rate z/β. Hence, the first three rows of table 1 confirm that none of those steady-state values depends on whether or not interest is paid on reserves.

Instead, a decision by the monetary authority to pay interest on reserves has its steady-state effects on banks' demand for reserves and, through the pricing relationship shown in (33), the interest rate that banks pay on deposits. Changes in the deposit rate then set off portfolio adjustments by households, which also have implications for the levels of output and hours worked. Not surprisingly, table 1 reveals that the biggest effects in percentage terms are on banks' holdings of reserves, which more than double moving from the steady state without interest on reserves to the steady state in which interest is paid at a 25-basis-point spread; and, as noted above, the calibrated parameters are chosen partly so that, as shown in table 1, reserves rise by a factor of seven when interest on reserves is paid at the market rate. As reflected in table 1, the levels of most real variables are determined in steady state relative to the level of lagged productivity Z_{t-1}, as those levels grow steadily over time at the constant, gross rate z along the model's balanced growth path, which again is invariant to changes in policies relating to the payment of interest on reserves. The technological specification (27) implies that the amount of labor that banks use to manage reserves rises proportionately with the size of the stock of reserves; hence, table 1 shows that large percentage changes in h_t^v also appear across steady states. In all cases, however, the very small value for the parameter ϕ_v selected above implies that the management of reserves imposes nonzero but extremely small resource costs.

Competitive pressures in the banking system imply, through (33), that reductions in the opportunity cost that banks incur when holding reserves and interest is paid on those reserves get passed along to households in the form of higher deposit rates. Table 1 shows that, in particular, the annualized interest rate on deposits rises by 15 or 16 basis points, depending on whether the interest rate on reserves is held 25 basis points below or set equal to the market rate of interest. These changes seem modest, but imply sizable reductions in the user cost of deposits, which according to Barnett's (1978) formula shown in (46) depends not on the level of the deposit rate but rather on the spread between the market and deposit rates. Hence, according to the demand relationships (41)-(43), households shift out of currency and into deposits when interest gets paid on reserves, and their overall demand for monetary services as reflected in the level of the true monetary aggregate increases as well. Shopping time, while always small relative to the household's other time commitments, falls by 8 or 9 percent across steady states when interest gets paid on reserves.

In this shopping-time model as in Cooley and Hansen's (1989) cash-in-advance model and Belongia and Ireland's (2006) real business cycle model with currency and deposits, inflation acts like a tax on market activity, since households must use monetary assets that pay interest at below-market rates to purchase consumption but do not receive nominal wage payments in exchange for their labor until the end of each period. Here, however, the focus lies not on changes in the overall levels of inflation and interest rates, as it does in those previous studies, but on the incremental changes in the inflation tax effects brought about by the payment of interest on reserves. And since, under the benchmark policy of no interest on reserves, reserves are small when compared to both the monetary base and the level of deposits, these incremental effects of the inflation tax on output and employment are relatively small as well: as shown in table 1, goods output rises by about one-tenth of a percentage point when interest gets paid on reserves.

3.4 The Dynamic Effects of Macroeconomic Shocks

Figure 2 plots the the impulse responses of output Y_t, inflation π_t, and the market rate of interest r_t to the preference shock a_t, the technology shock Z_t, and the monetary policy shock ε_{rt}, both under the benchmark policy that does not pay interest on reserves (solid lines) and the alternative policy under which interest is paid on reserves at a rate that lies 25 basis points below the market rate (dashed lines). Since the impulse responses for the case where the central bank pays interest on reserves at the market rate resemble so closely those for the case with a 25-basis-point spread, results for this third case are not shown. The panels express output in log levels and inflation and the interest rate in annualized terms. These are the variables and shocks that hold center stage in most New Keynesian analyses, and here they display their usual behavior.

The preference shock acts as an exogenous, non-monetary, demand-side disturbance, increasing both output and inflation and, under the Taylor rule (35), calling forth a tightening of monetary policy in the form of higher short-term interest rates. The technology shock increases output and decreases inflation. The random walk specification (22) implies that the technology shock exerts a permanent effect on the level of output, and the increases in output dominates the decrease in inflation so that, under the Taylor rule (35), the monetary authority responds with a modest, 8-basis-point increase in the market rate of interest. Finally, the monetary policy shock generates a 25-basis-point increase in the market interest rate that, in this purely forward-looking model, reduces output and inflation immediately. These movements in output growth and inflation then imply that the interest rate returns quite quickly to its steady-state level, despite the large calibrated value $\rho_r = 0.95$ assigned to the interest rate smoothing parameter in the Taylor rule.

The new results shown in figure 2 can be summarized by observing that the solid and dashed lines in each panel overlap to the extent that they become virtually indistinguishable. While, in fact, the changes in the market rate of interest shown in the figure's bottom row give rise to changes in banks' opportunity cost of holding reserves under the benchmark policy of no interest on reserves but not under the alternative policy in which the positive interest rate on reserves tracks those changes in the market rate to maintain the 25-basis-point spread, and while, in principle, these differences in the cost of holding reserves might translate into variable inflation tax effects that then impact differently on output and inflation as well, these effects turn out, quantitatively, to be quite small. These results for the model's dynamics echo those for the steady states described earlier in table 1.

Again as in table 1, however, measures of money, and particularly bank reserves, behave very differently across policy regimes. Thus, figure 3 extends the analysis from figure 2 by plotting the growth rates of various measures of money, in annualized terms, in response to the same three macroeconomic shocks. Although, in figure 3, differences appear in the aftermath of preference and technology shocks as well, they become most striking in the case of a monetary policy shock. In the traditional case where interest is not paid on reserves, the monetary authority must drain reserves from the banking system to bring about the outcome in which the market rate of interest rises by 25 basis points. The middle, right-hand panel of figure 2 shows that when the monetary authority manages the market rate according to the Taylor rule (35), inflation returns to its steady-state level after this contractionary monetary policy shock, but the price level remains permanently lower. Hence, the top, right-hand panel of figure 3 reveals that while the monetary authority subsequently reserves part of the decrease in reserves that is required to engineer the initial monetary tightening, this reversal is incomplete. In the long-run, the level of reserves declines in proportion to the price level.

These effects when interest is not paid on reserves are consistent with the intuition built up with the help of the diagrams in figure 1. But figure 3, showing results from the full, dynamic model, reveals that by inappropriately holding other variables constant, the diagrams in figure 1 tell only part of the story for the case with interest on reserves. When the monetary authority increases its target for the market rate by 25 basis points, the user cost of currency, measured as shown in (45), rises as well. Hence, in figure 3, households' demand for currency falls sharply after a monetary policy shock, regardless of whether or not interest is paid on reserves. On the other hand, the next figure 4 reveals that when the monetary authority also pays interest on reserves, and manages that interest rate to maintain a 25-basis-point spread with the market rate, the user cost of deposits actually falls after a contractionary monetary policy shock.

Equation (33) explains this surprising result. In the model, banks create deposits with a combination of reserves and labor. Hence, the wedge between the market interest rate and the competitively-determined interest rate on deposits depends on both the opportunity cost of holding reserves, $r_t - r_t^v$, and the real wage relative to productivity, $(W_t/P_t)/Z_t$. Without interest on reserves, the rise in the market rate increases the opportunity cost term, more than offsetting the decline in the real wage brought about by the contractionary macroeconomic effects of the monetary policy shock. When the monetary authority increases r_t^v in lockstep with r_t, however, the opportunity cost of holding reserves gets held fixed and the only effect that remains works through wages: although the deposit rate still goes up when the market rate of interest rises, it does so by a smaller amount, so that the spread $r_t - r_t^d$ declines, as does the user cost of deposits as given by (46). Figure 3 then shows that as households substitute more strongly into deposits after a monetary policy shock, the monetary authority must actually increase the supply of reserves to prevent the market rate from rising still further. Not only does the liquidity effect vanish when the central bank pays interest on reserves, but in fact reserves and the short-term interest rate have to move in the same direction following a monetary policy shock.

Still, both with and without interest on reserves, the Taylor rule (35) associates a contractionary monetary policy shock with a transitory fall in inflation but a permanent decline in the price level. Hence, the top right-hand panel of figure 3 also shows that even when the central bank pays interest on reserves, it must in the long run contract the supply of reserves after a monetary policy shock. Although the dynamic behavior of reserves differs quite dramatically depending on whether or not interest is paid on reserves, the long-run effects coincide: a monetary policy action that decreases the price level always requires a proportionate reduction in the supply of bank reserves.

Finally, in figure 3, the simple-sum monetary aggregate M_t^s typically fails to accurately track movements in the true monetary aggregate M_t^a following each macroeconomic shock. Belongia and Ireland (2010) discuss these results in more detail, showing also that by contrast, movements in the Divisia monetary aggregates proposed by Barnett (1980) mirror movements in the true aggregate almost exactly, even though like simple-sum aggregates, they can be constructed without knowledge of either the functional forms or the parameter values observable within the model through equations such as (3), but potentially unobservable to agents operating outside the model.

3.5 The Effects of Financial Shocks

Figures 5-7 repeat the impulse response analysis from figures 2-4, but for the remaining three shocks to money demand v_t^a, bank productivity x_t^a, and the spread τ_t between the market rate and the interest rate paid on reserves. The left-hand columns of each figure confirm that Poole's (1970) classic result, showing that by holding the market rate of interest fixed in the face of shocks to money demand, the monetary authority automatically accommodates the shifts in demand with appropriate shifts in the supply of liquid assets and thereby insulates the macroeconomy from the effects of those disturbances, carries over to this setting just as it does to the simpler New Keynesian model studied by Ireland (2000). In particular, figure

6 shows how under the Taylor rule (35), the stock of monetary assets of all kinds expands to meet the additional demand generated by an increase in v_t^a; in figures 5 and 7, therefore, output, inflation, interest rates, and the user costs of these same monetary assets remain virtually unchanged.

The calibrated value $\sigma_x^a = \ln(10)$ is selected above to make the banking productivity shock very large and thereby simulate the effects of a financial crisis that makes it much more difficult for private financial institutions to supply households with highly liquid assets like bank deposits. Hence, the middle columns of figure 5-7 trace out the effect of an adverse shock of this kind. Under the Taylor rule (35), the monetary authority floods the economy with reserves and currency to help offset the negative effects this shock has on the quantity and user cost of deposits. As in figure 3, the monetary growth rates shown in figure 6 are annualized; hence, the 275 percentage-point decline shown for deposits and the 100 percentage-point decline shown for the true monetary aggregate imply that despite the monetary authority's dramatic action, the volume of deposits created by banks gets reduced by almost 70 percent and the flow of liquidity services provided to households declines by 25 percent in the initial quarter when the shock hits. In figure 5, aggregate output falls by 1 percent, even as the monetary authority lowers the market rate of interest by more than 60 basis points to provide further macroeconomic stabilization. These effects remain much the same, regardless of whether the monetary authority also pays interest on reserves. Evidently, what matters most in shaping the effects of this financial-sector shock is how the monetary authority expands the supply of reserves and currency to help banks and households cope with the increased cost of creating liquid deposits.

Finally, the right-hand columns of figures 4-7 show the effects of a 12.5-basis-point increase in the interest rate that the monetary authority pays on reserves. The solid lines in figures 6 and 7 show that starting from the benchmark case in which no interest gets paid on reserves, this small and temporary increase in the interest rate on reserves has only modest effects on the quantity and user cost of deposits and can therefore be supported by a small increase in the monetary authority's supply of reserves. Starting from the alternative case in which there is a 25-basis-point spread between the market rate of interest and the interest rate of reserves, this same shock cuts banks' opportunity cost of holding reserves in half, and therefore sets off much large responses in all of the monetary variables. Figure 5 confirms that once again, however, changes in the interest rate paid on reserves have very small effects on macroeconomic variables like output and inflation.

3.6 Robustness

Running through all of the results displayed in table 1 and figures 2-7 is the basic finding that while the monetary authority's decision to pay interest on reserves can have very important effects on both the average levels and dynamic behavior of reserves and other monetary assets, the effects on output and inflation are by contrast quite small. Behind these results lie some very basic features of the contemporary United States economy, which are reflected in the model's calibration. In the US prior to 2008, when the Federal Reserve paid no interest on reserves, the stock of reserves was small, relative both to the monetary base and the level of deposits. Moreover, inflation and market rates of interest remained low. Putting these two sets of facts together: with a small tax base as measured by the stock of reserves and a small tax rate as measured by the market rate of interest relative to the zero rate of interest paid on reserves, the distortionary effects on the macroeconomy stemming from banks' demand for reserves were modest and, by extension, the effects of changes in the opportunity cost of holding reserves appear more modest still.

To show that these basic results reflect, not the inevitable workings of the model itself, but rather the way in which the model gets calibrated to match the most relevant aspects of the US economy, figure 8 displays impulse responses generated after two sets of changes are made to the model's parameter values. First, new values $v^a = 3.75$, $v^n = 0.225$, $x^a = 11$, and $x^n = 0.98$ increase the steady-state ratio of the simple-sum monetary aggregate M_t^s to nominal consumption P_tC_t from 3 to 10 and the steady-state ratio of reserves N_t^v to deposits

D_t from 0.02 to 0.10, greatly increasing the importance of reserves in creating deposits and deposits in providing transaction services, while holding the steady-state ratio of currency to deposits constant at 0.10 and the steady-state ratio of employment in banking to total employment at 0.007, the values used in the calibration exercise before. This first set of changes, therefore, has the effect of dramatically enlarging the tax base that gets hit when the monetary authority does not pay interest on reserves. Second, new values $\rho_r = 0.50$, $\rho_\pi = 0.75$, and $\rho_g = 0$ for the coefficients of the Taylor rule (35) make monetary policy shocks more persistent. Since, when the monetary authority does not pay interest on reserves, movements in the market rate translate directly into movements in the opportunity cost of holding reserves, this second set of changes makes swings in the distortionary tax rate on reserves more persistent as well.

In figure 8, therefore, output responds quite differently to shocks, depending on whether or not the monetary authority pays interest on reserves. The additional inflation tax effects, for instance, make the decline in output that follows the monetary policy shock depicted in the figure's third column significantly larger when interest is not paid on reserves. Conversely, the rise in interest rates that follows a preference shock turns the initial output expansion into a subsequent contraction when interest is not paid on reserves. These results suggest that, in other economies with different basic features, the monetary authority's decision to pay or not to pay interest on reserves might have larger macroeconomic consequences. And, of course, with this alternative calibration just as before, the dynamic behavior of reserves themselves shifts most dramatically when the monetary authority decides to pay interest on reserves.

4 Conclusion

The analysis performed here, with the help of a dynamic, stochastic, general equilibrium New Keynesian model, shows that the Federal Reserve's recent decision to begin paying interest on reserves is unlikely to have large effects on the behavior of macroeconomic variables such as aggregate output and inflation, once normal times return. These results serve to confirm the basic thrust of arguments advanced, using a variety of quite different models, by Goodfriend (2002), Ennis and Weinberg (2007), Keister, Martin, and McAndrews (2008), and Kashyap and Stein (2010), all suggesting that the ability to manipulate the spread between the federal funds rate and the interest rate it pays on reserves provides the Fed with an additional degree of freedom that it can use to expand its policymaking strategies and objectives. Specifically, the results obtained here show how the Fed can continue to adjust its target for the federal funds rate to achieve its goals for macroeconomic stabilization, while independently varying the interest on reserves, as necessary, to help enhance the efficiency of and reinforce the stability of private financial institutions and the financial sector as a whole.

On the other hand, the results obtained here also show that the decision to pay interest on reserves requires rather dramatic changes in the way in which the Fed, or any other central bank, must manage the supply of reserves in support of its broader policy objectives. In particular, with interest on reserves, the traditional liquidity effect, associating a monetary policy tightening with higher interest rates engineered through a reduction in the supply of reserves, vanishes. Instead, as discussed above, portfolio reallocations by households may actually require the monetary authority to initially increase the supply of reserves when raising its target for the short-term interest rate.

The full-blown dynamic model studied here, however, also serves to highlight that, while a monetary authority's decision to pay interest on reserves does open up new dimensions for policymaking and does yield important shifts in the dynamic behavior of reserves and other monetary variables, it is simply not the case that the payment of interest on reserves completely divorces reserves from the monetary policymaking process. Importantly, it continues to be true that a monetary policy action intended to raise or lower the aggregate price level requires a proportionate change in the volume of reserves supplied to the banking system. In this way, a central bank's most important task, of controlling nominal variables in the long run, remains dependent on its ability and willingness to manage the supply of reserves appropriately.

Finally, the analysis shows how these results, particularly those having to do with the small macroeconomic effects of paying interest on reserves, depend on some specific features of the United States economy. Before 2008, in the years leading up to the Fed's decision to begin paying interest on reserves, banks operated successfully with fairly small stocks of reserves. Moreover, inflation, interest rates, and by extension banks' opportunity cost of holding reserves, remained low as well. With a relatively small base taxed at a relatively low rate to begin with, incremental changes in the opportunity cost of holding reserves, brought about through independent variation in the policy rate paid on reserves, have only modest effects outside the banking system. Building on this same intuition, however, the model also demonstrates how, in other economies where banks operate less efficiently and are more dependent on reserves in their own efforts to create additional liquidity and where inflation and market interest rates are higher on average, paying interest on reserves may have more profound macroeconomic consequences.

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Table 1. Steady-State Effects of Paying Interest on Reserves

	No Interest Paid On Reserves $\alpha = 0$ $\tau = 1$	Interest Paid At 25 Basis Points Below Market Rate $\alpha = 1$ $\tau = 1 - 0.000625$		Interest Paid At Market Rate $\alpha = 1$ $\tau = 1$	
Variable	Steady-State Value	Steady-State Value	Percentage Change	Steady-State Value	Percentage Change
Output Growth Y_t/Y_{t-1}	1.0050	1.0050	0.00	1.0050	0.00
Inflation P_t/P_{t-1}	1.0050	1.0050	0.00*	1.0050	0.00*
Market Interest Rate r_t	1.0151	1.0151	0.00*	1.0151	0.00*
Output Y_t/Z_{t-1}	0.3314	0.3317	0.09	0.3317	0.10
Shopping Time h_t^s	0.0009	0.0008	−7.97	0.0008	−8.78
Hours Worked h_t	0.3320	0.3324	0.09	0.3324	0.10
Hours in Goods Production h_t^g	0.3297	0.3300	0.09	0.3300	0.10
Hours in Banking h_t^b	0.0023	0.0023	0.88	0.0023	0.97
Hours in Deposit Creation h_t^d	0.0023	0.0023	0.87	0.0023	1.00
Hours in Reserves Management h_t^v	0.0000	0.0000	122.46	0.0000	683.50
Real Reserves $(N_t^v/P_t)/Z_{t-1}$	0.0191	0.0425	122.46	0.1496	683.50
Real Monetary Base $(M_t/P_t)/Z_{t-1}$	0.1113	0.1323	18.87	0.2408	116.27
Real Currency $(N_t/P_t)/Z_{t-1}$	0.0922	0.0893	−3.23	0.0890	−3.56
Real Deposits $(D_t/P_t)/Z_{t-1}$	0.9202	0.9676	5.15	0.9729	5.72
Real True Monetary Aggregate $(M_t^a/P_t)/Z_{t-1}$	0.8856	0.9013	1.76	0.9029	1.95
Real Simple-Sum Monetary Aggregate $(M_t^s/P_t)/Z_{t-1}$	1.0125	1.0568	4.38	1.0618	4.88
Real Wage $(W_t/P_t)/Z_{t-1}$	0.8375	0.8375	0.00	0.8375	0.00
Interest Rate on Reserves r_t^v	1.0000	1.0145	5.79*	1.0151	6.04*
Interest Rate on Deposits r_t^d	1.0127	1.0130	0.15*	1.0131	0.16*
Own Rate on True Monetary Aggregate r_t^a	1.0110	1.0114	0.16*	1.0114	0.17*
User Cost of Currency u_t^n	0.0149	0.0149	0.00	0.0149	0.00
User Cost of Deposits u_t^d	0.0024	0.0020	−15.29	0.0020	−16.79
User Cost of True Monetary Aggregate u_t^a	0.0040	0.0036	−9.57	0.0036	−10.53
Reserve Ratio rr_t	0.0207	0.0439	111.57	0.1537	641.09

Notes: Each row shows the steady-state value of the variable indicated under the benchmark policy of no interest on reserves and the alternative policies of paying interest on reserves either at an annualized rate that is 25 basis points below the annualized market rate or at the market rate itself. "Percentage Change" refers to the percentage change in the steady-state value of each variable under each alternative policy with interest on reserves compared to the value of the same variable under the benchmark policy of no interest on reserves, except starred (*) entries that show percentage-point changes in annualized inflation and interest rates.

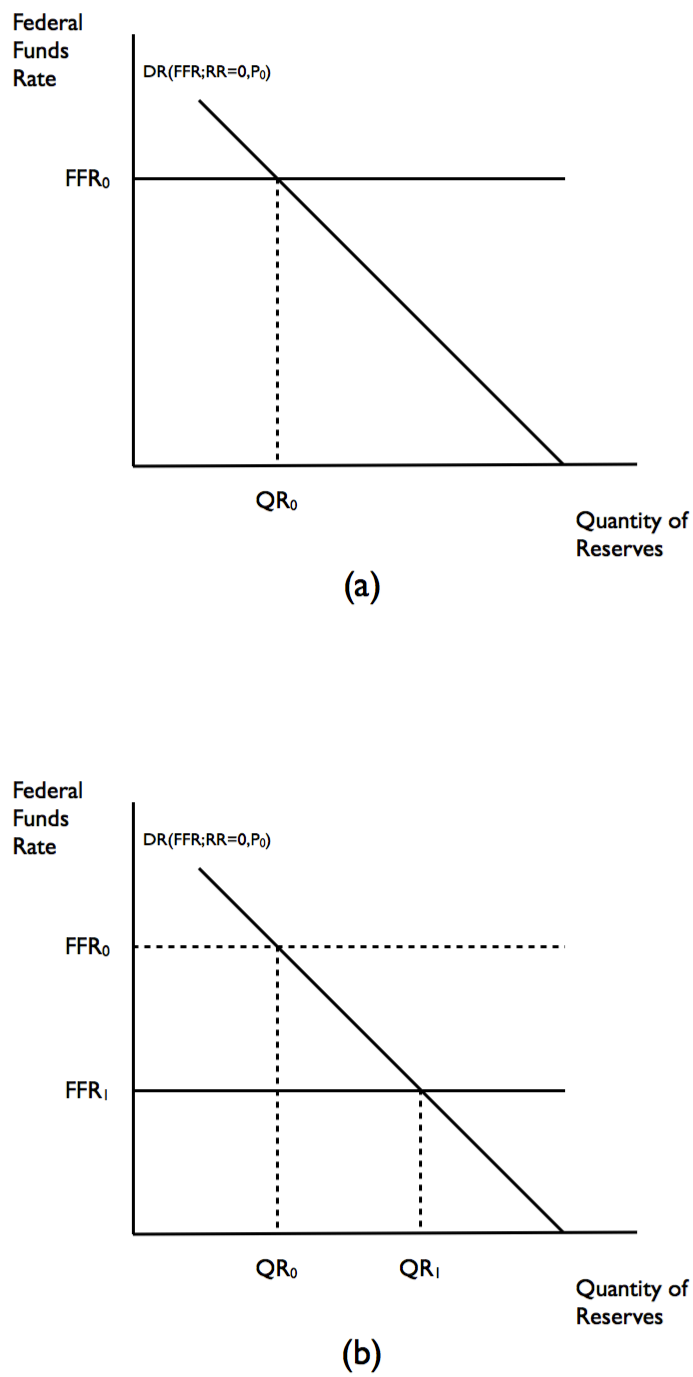

Figure 1. Panels (a) and (b)

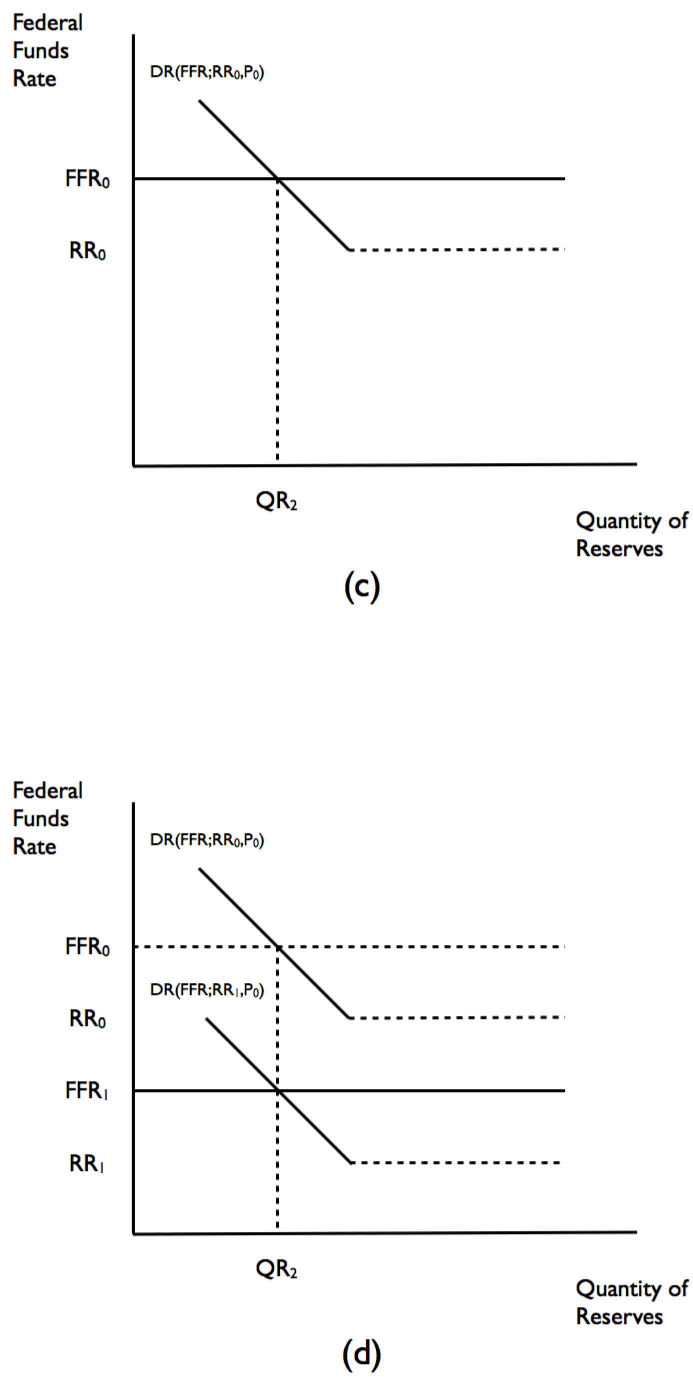

Figure 1. Panels (c) and (d)

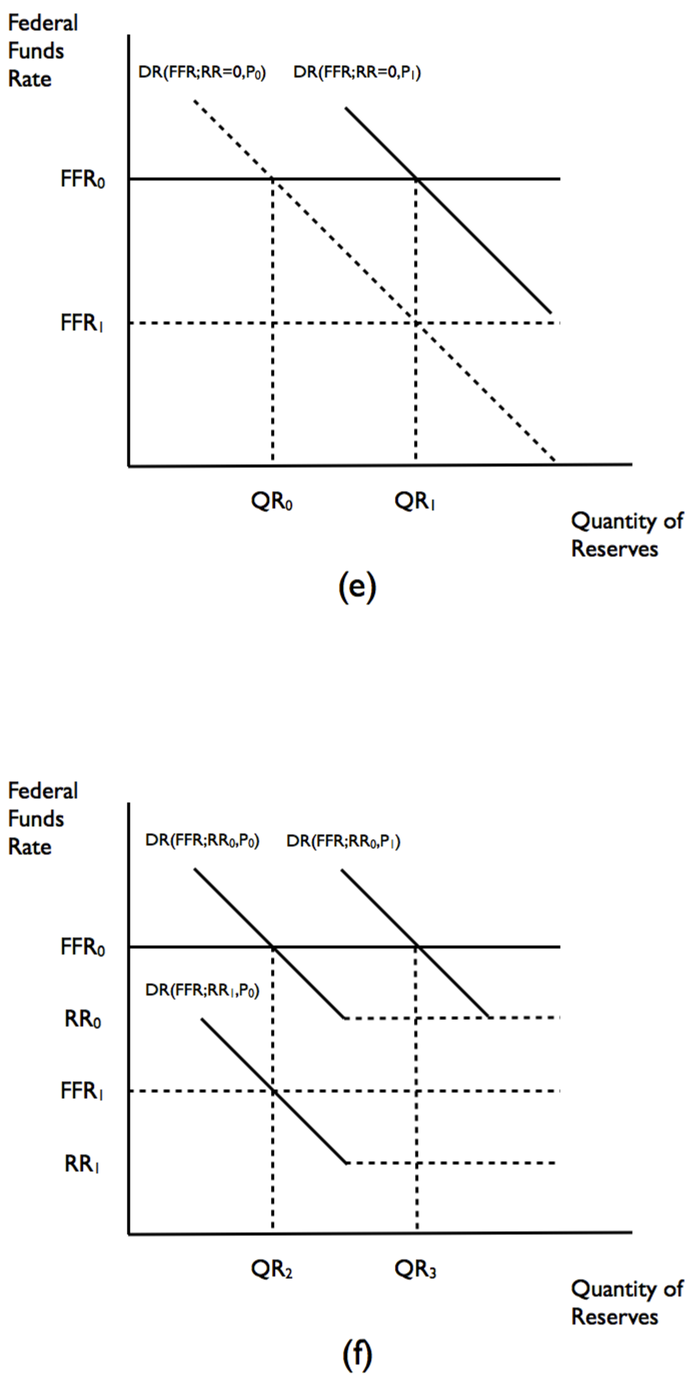

Figure 1. Panels (e) and (f)

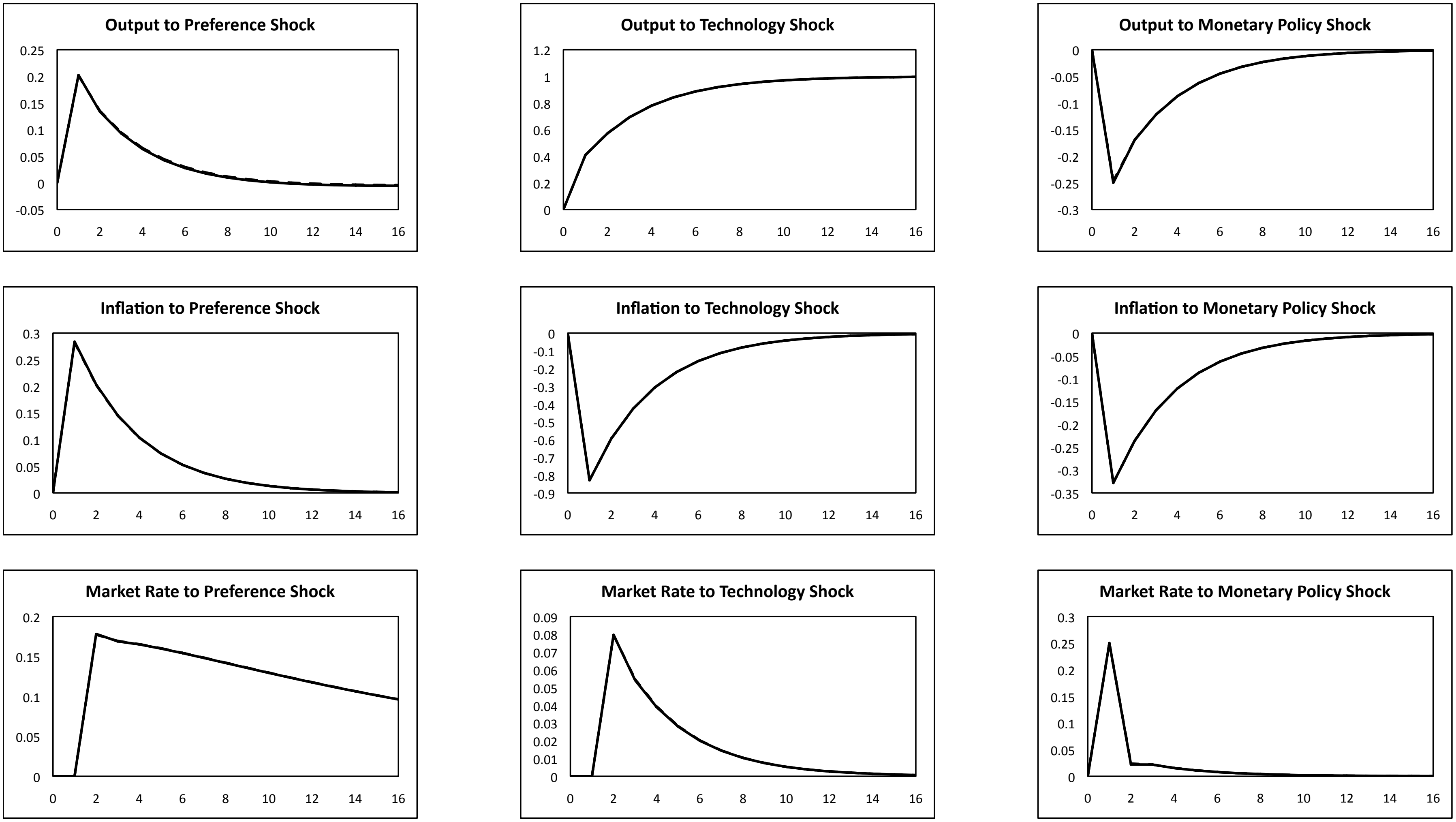

Figure 2. Responses of Macroeconomic Variables to Macroeconomic Shocks. Each panel shows the percentage-point response of the indicated variable to the indicated shock. Output is in log-levels; the inflation and interest rates are in annualized terms. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

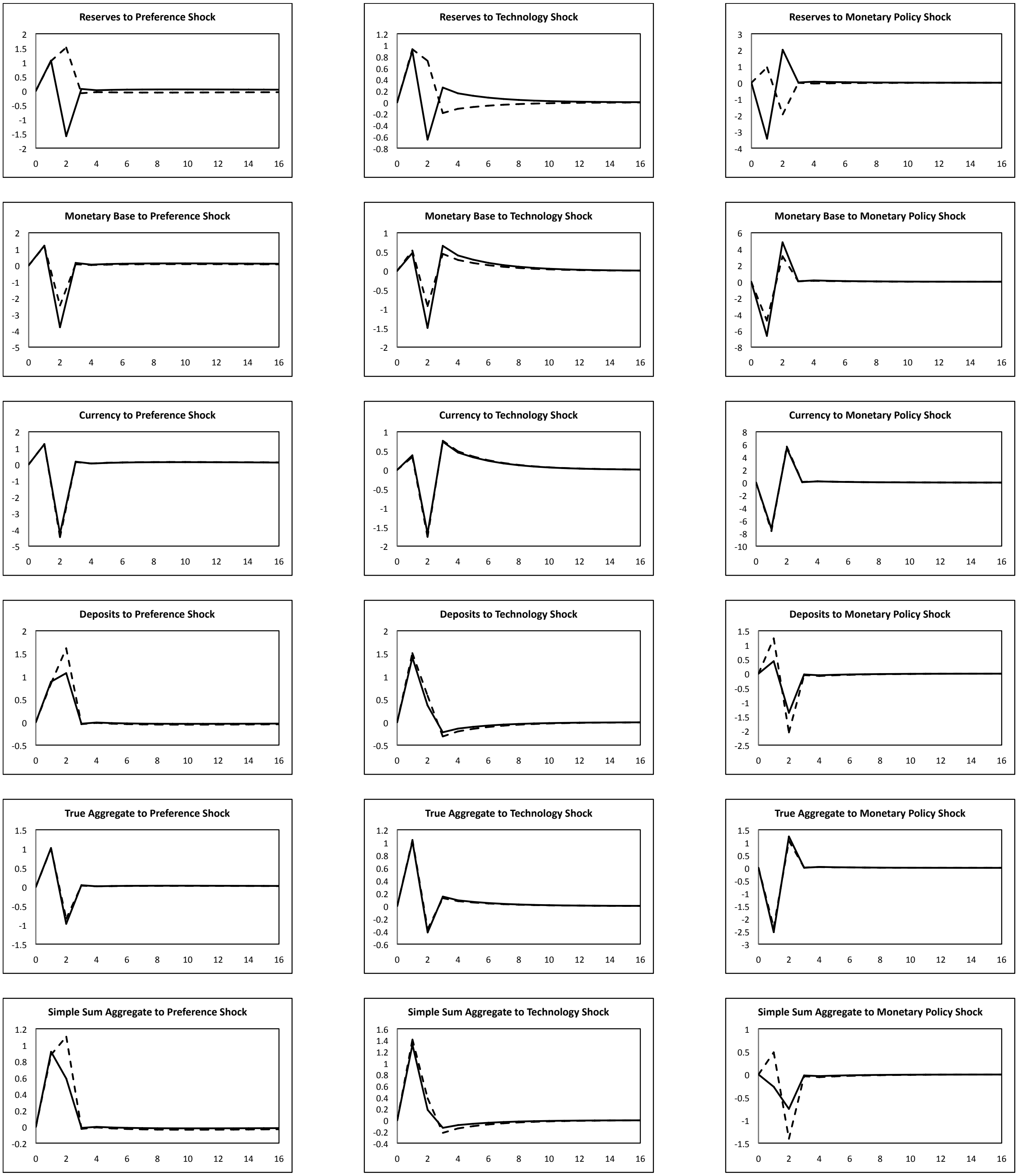

Figure 3. Responses of Money Growth to Macroeconomic Shocks. Each panel shows the percentage-point response of the annualized growth rate of the indicated monetary asset or aggregate to the indicated shock. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

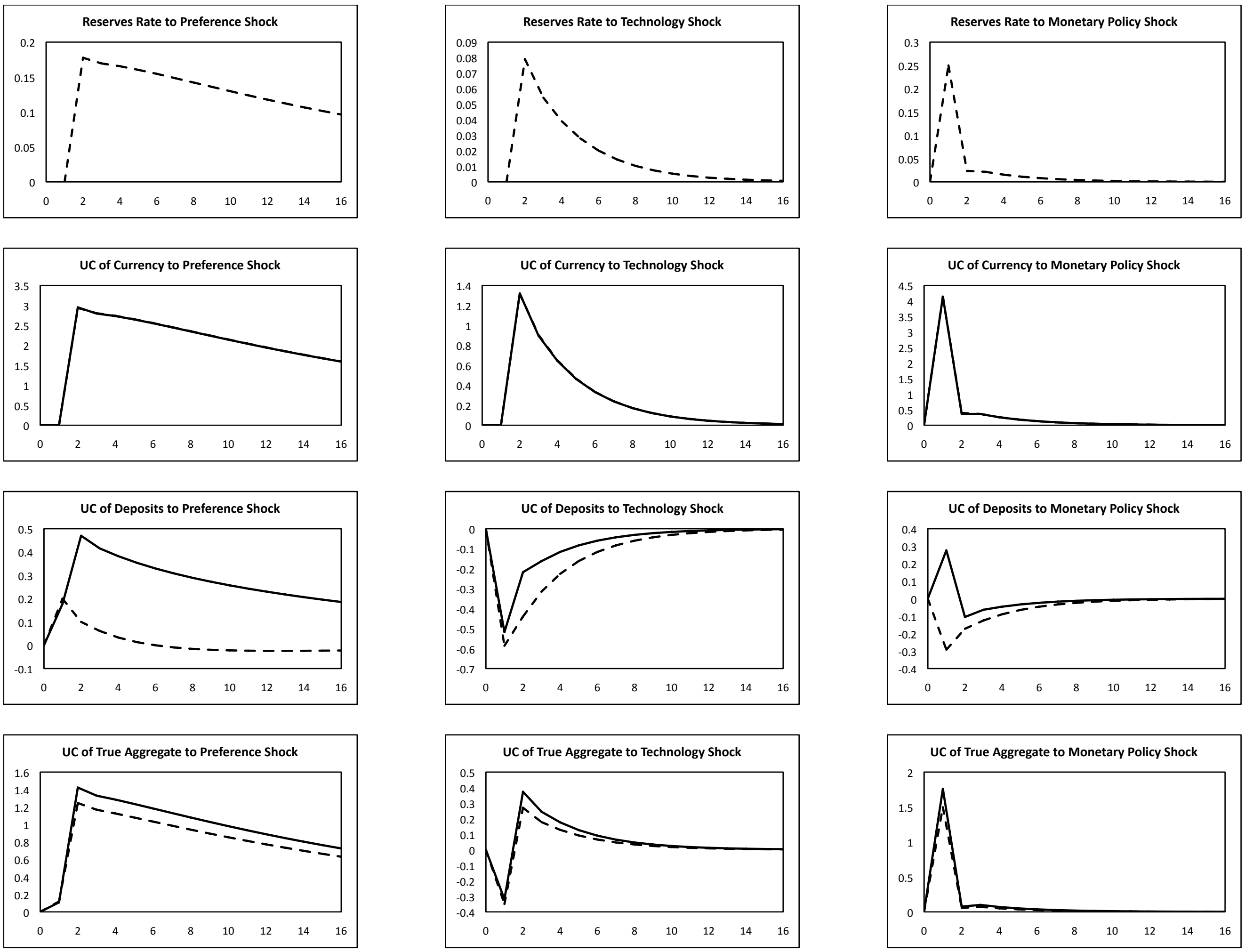

Figure 4. Responses of Financial Variables to Macroeconomic Shocks. Each panel shows the percentage-point response of the indicated variable to the indicated shock. The reserves rate is annualized; the other variables are not. Solid lines track responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

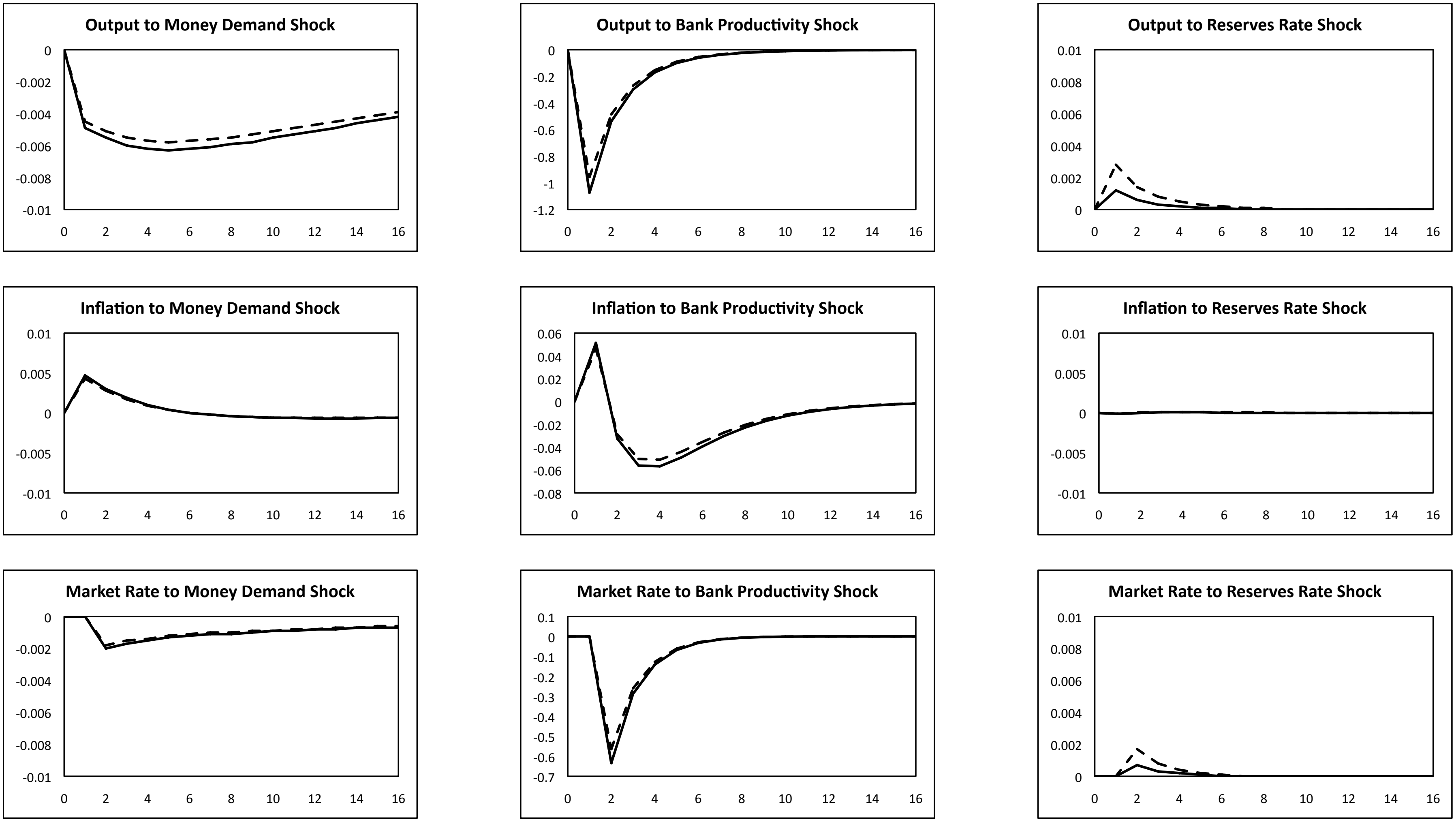

Figure 5. Responses of Macroeconomic Variables to Financial Shocks. Each panel shows the percentage-point response of the indicated variable to the indicated shock. Output is in log-levels; the inflation and interest rates are in annualized terms. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

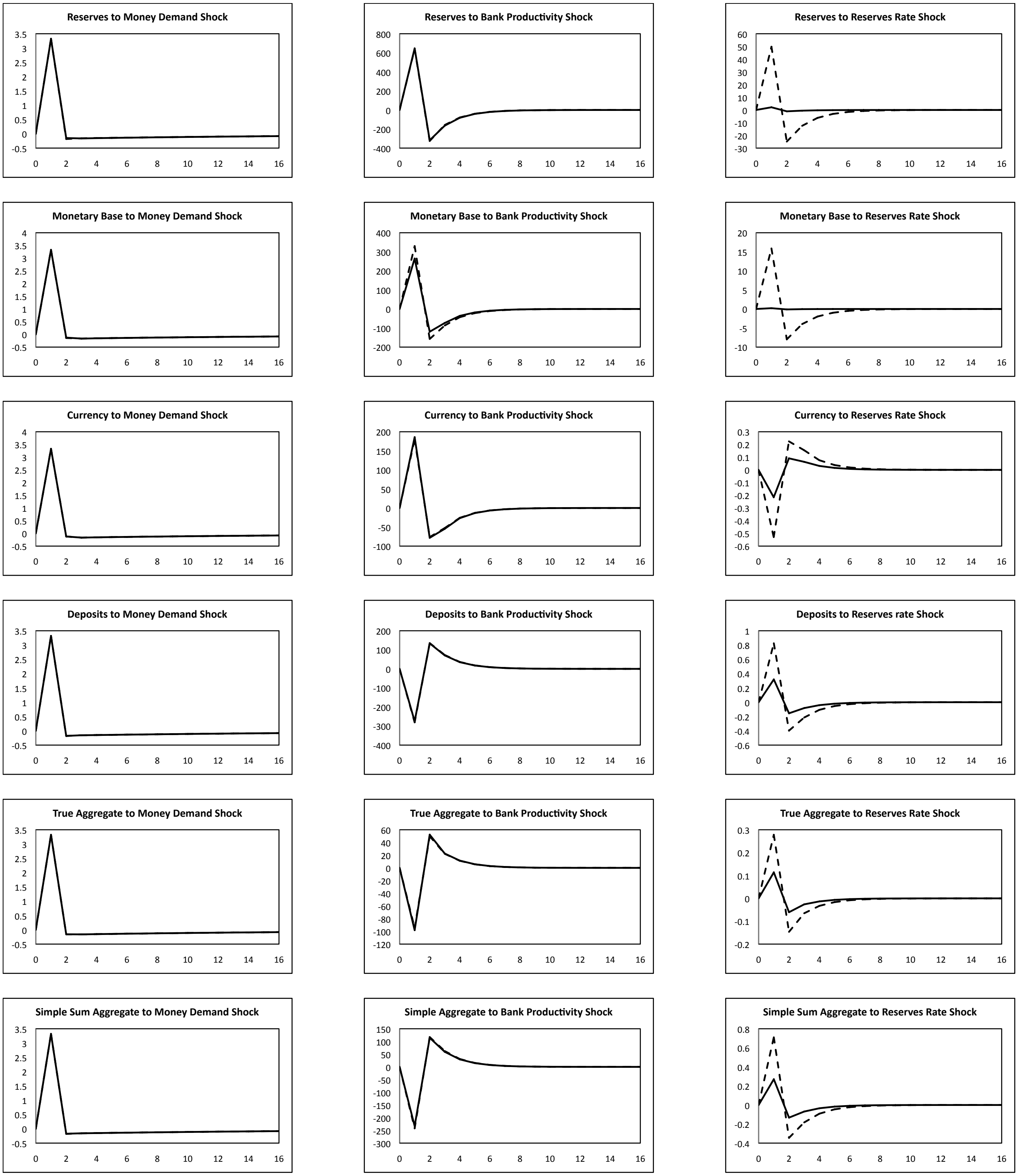

Figure 6. Responses of Money Growth to Financial Shocks. Each panel shows the percentage-point response of the annualized growth rate of the indicated monetary asset or aggregate to the indicated shock. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

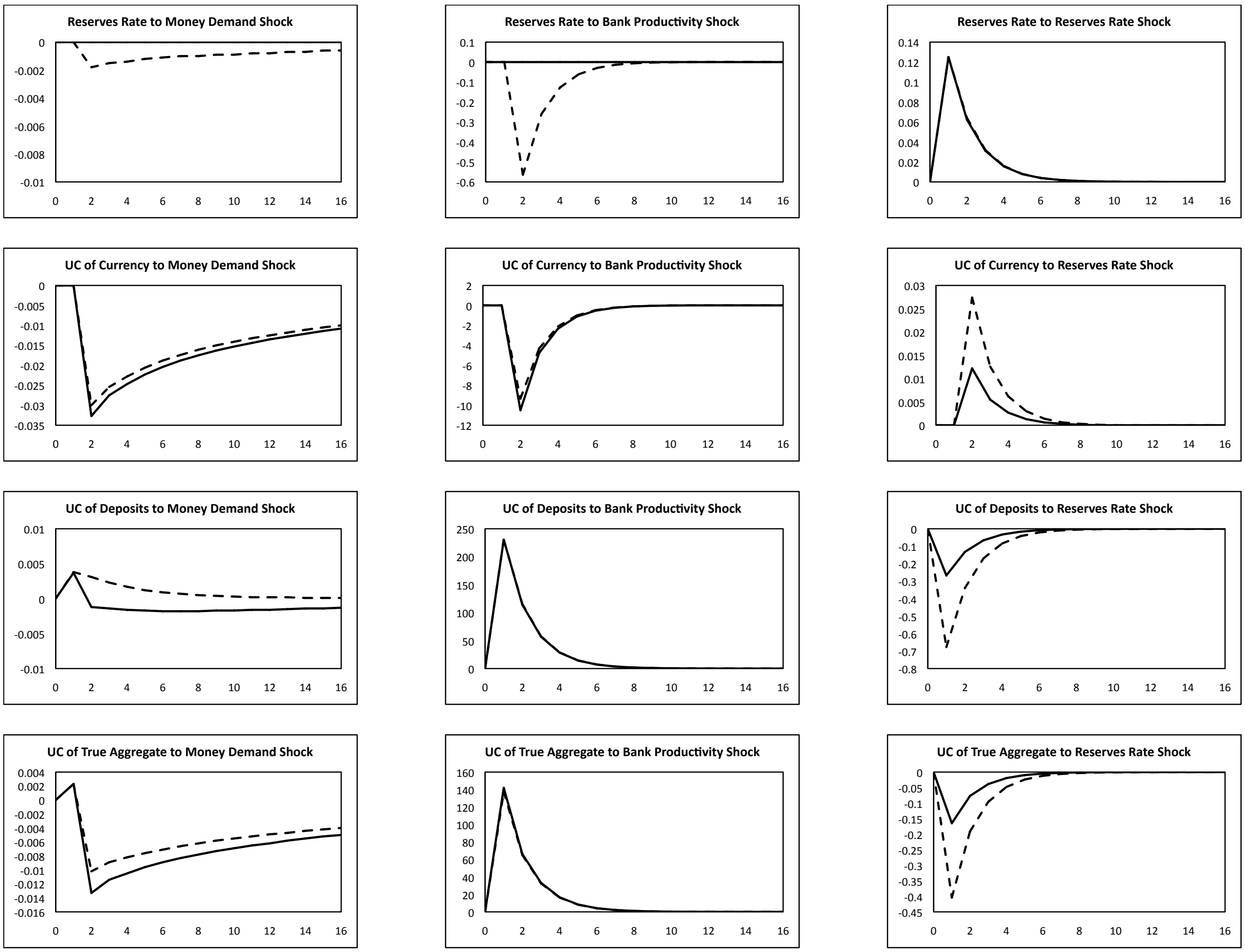

Figure 7. Responses of Financial Variables to Financial Shocks. Each panel shows the percentage-point response of the indicated variable to the indicated shock. The reserves rate is annualized; the other variables are not. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.

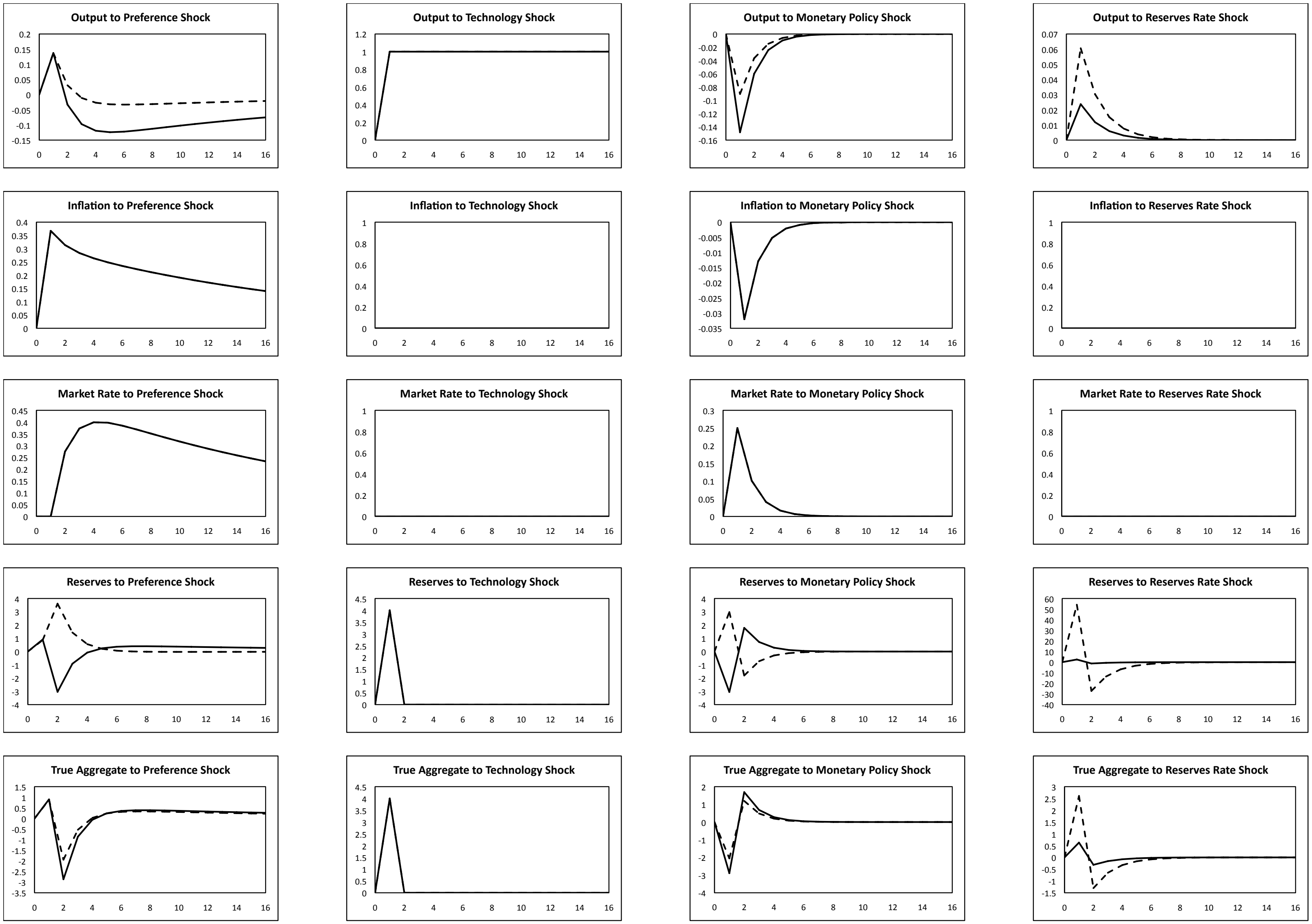

Figure 8. Responses of Macroeconomic and Money Growth Variables to Macroeconomic and Financial Shocks. Derived using the alternative calibration described in the text. Each panel shows the percentage-point response of the indicated variable to the indicated shock. Output is in log-levels; the inflation, interest, and money growth rates are in annualized terms. Solid lines track the responses under the benchmark policy without interest on reserves; dashed lines track the responses under the alternative policy when interest is paid on reserves at a rate that is 25 basis points below the market rate.