Productivity and the Welfare of Nations[*]

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Abstract

We show that the welfare of a country's infinitely-lived representative consumer is summarized, to a first order, by total factor productivity (TFP), appropriately defined, and by the capital stock per capita. The result holds for both closed and open economies, regardless of the type of production technology and the degree of product market competition. Welfare-relevant TFP needs to be constructed with prices and quantities as perceived by consumers, not firms. Thus, factor shares need to be calculated using after-tax wages and rental rates. We use these results to calculate welfare gaps and growth rates in a sample of advanced countries with high-quality data on output, hours worked, and capital. We also present evidence for a broader sample that includes both advanced and developing countries.

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1 Introduction

Standard models in many fields of economics posit the existence of a representative household in either a static or a dynamic setting, and then seek to relate the household's welfare to observable aggregate data. A separate large literature examines the productivity residual defined by Solow (1957), and interprets it as a measure of technical change or policy effectiveness. Yet a third literature, often termed "development accounting," studies productivity differences across countries, and interprets them as measures of technology gaps or institutional quality. We show that these three literatures are intimately related. Starting from the standard framework of a representative household that maximizes intertemporal welfare over an infinite horizon, we derive methods for comparing economic well-being over time and across countries. Our methods minimize or avoid altogether the need to make parametric assumptions about preferences, technology or market structure. We show that under a wide range of assumptions, welfare can be measured using just two variables, productivity and capital accumulation. We then take our theory to the data, and measure welfare differences over time and across countries.

More precisely, we show that to a first-order approximation the welfare change of a representative household can be fully characterized by three objects: the expected present discounted value of total factor productivity (TFP) growth, the change in expectations of the level of TFP, and the growth in the stock of capital per person, all of which can be calculated using standard national accounting data. Welfare-relevant TFP needs to be constructed with prices and quantities as perceived by consumers, not firms, for example, using after-tax wages and rental rates. Our result applies to both closed and open economies, provided TFP is calculated using domestic absorption instead of GDP as the measure of output.

The result sounds similar to one that is proven in the context of a competitive optimal growth model with technology as the only source of stochastic shocks, which might lead one to ask what assumptions on technology and product market competition are required. The answer is, None, because in our setting TFP growth need not measure technical change in an aggregate production function, as it does in a neoclassical growth model. Our result holds for all types of technology and market structure, as long as consumers take prices as given and are not constrained in the amount they can buy or sell at those prices. Thus, for example, the same result holds whether the TFP growth is generated by exogenous technological change, as in the Ramsey model, or is the result of spillovers or profit-maximizing investment in R&D as in endogenous growth models.

Our basic result suggests a way to answer a pressing and long-standing question, the measurement of relative welfare across countries using National Income Accounts data and a method firmly grounded in economic theory. We show that data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, suffice to measure such differences.

The logic of our argument is based on a dynamic application of the envelope theorem, which implies that the welfare of a representative agent depends to a first order on the expected time paths of the variables that the agent takes as exogenous. In a dynamic growth context, these variables are the prices for factors the household supplies (labor and capital), the prices for the goods it purchases (consumption and investment), and beginning-of-period household assets,

which are predetermined state variables. Thus, the TFP that is directly relevant for household welfare is the price-based *dual* Solow residual, which we transform into the familiar primal Solow residual using the national income identity.

Our cross-country welfare result comes from using the link between welfare and exogenous prices implied by economic theory to ask how much a household's welfare would differ if it faced the sequence of prices, not of its own country, but of some other country. More precisely, we perform the thought experiment of having a US household optimizing while facing the expected time paths of all goods and factor prices in, say, France, and owning the initial stock of French assets rather than US assets. The difference between the resulting level of welfare and the welfare of remaining in the US measures the gain or loss to a US household of being moved to France.

Note that our thought experiment is a counterfactual one—the US household in France will choose different time paths for consumption, leisure and saving than the French household, because the two generally have different preferences. Yet we show that the welfare comparisons can be based on the productivity residuals constructed using just the observed, equilibrium data of both countries, without the need to construct any counterfactual quantities. By contrast, the method of comparing welfare by inserting the consumption and leisure of different countries into a fixed utility function requires one to assume that preferences are the same across countries. Otherwise, the observed French levels of consumption and leisure will not be those chosen by a US consumer facing French prices and initial conditions. Furthermore, our thought experiment takes dynamics into account, and allows the consumer to face time-varying paths of prices and assets.

Most importantly, our revealed-preference approach allows one to perform cross-country welfare comparisons without the need to know individuals' preference parameters (other than the discount rate) and without having to assume them equal across countries. Note that our welfare comparisons are from a definite point of view—in the above example, that of a US household. In principle, the result could be different if the USA-France comparison is made by a French household, with different preferences over consumption and leisure. Luckily, our empirical results are not greatly affected by the choice of the "reference country" used for these welfare comparisons.

Feenstra, Inklaar and Timmer (2015a) discuss the new data that have been introduced into the current Penn World Tables (PWT) to allow users to compute the welfare measures that we derive in this paper. In particular, they use our paper to motivate the addition of a measure of real domestic absorption (termed CDA in PWT8), as well as a TFP measure using absorption as the definition of output (CWTFP in PWT8). They note that a strength of our welfare measure is that it requires no assumptions about technology or market structure.[1]

We illustrate our methods using data for several industrialized countries for which high-quality data are available (Canada, France, Italy, Japan, Spain, the United Kingdom and the United States) and show how their relative welfare levels evolve over time. We also provide welfare comparisons for a larger set of countries that includes both advanced and developing

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[1] See Feenstra, Inklaar and Timmer (2015b) for a more extensive discussion and a comparison of our methods with other approaches to cross-country welfare measurement.

countries. However, we continue to analyze the smaller sample because it has better data: for example, information on hours worked is not available for the larger sample. In our benchmark case of optimal government spending and distortionary taxation, the US is the welfare leader among industrialized countries throughout our sample period (in our larger data set, only Luxembourg enjoys higher welfare). In our smaller sample with high-quality data, the US pulls away from all other advanced countries in terms of welfare. The only exception is the UK, which converges steadily to US levels of welfare over time. In both data sets, the welfare differences among countries are driven to a much greater extent by TFP gaps than by differences in capital intensity. This finding echoes the conclusion of the "development accounting" literature, but for welfare differences rather than GDP gaps.

The paper is structured as follows. The next section presents our analytical framework, and derives the results on the measurement of welfare within single economies and on welfare comparisons across countries. Section 3 extends the basic framework to allow for distortionary taxes and government expenditure, and summarizes our results in their more general form. It also contains an extension with human capital accumulation, using the framework of Lucas (1988). We take our framework to the data in Section 4. Section 5 relates our work to several distinct literatures. Finally, we summarize our findings and suggest avenues for future research.

2　The Productivity Residual and Welfare

Both intuition and formal empirical work link TFP growth to increases in the standard of living, at least as measured by GDP per capita.[2] The usual justification for studying the Solow productivity residual is that, under perfect competition and constant returns to scale, it measures technological change, which contributes to GDP growth, one major determinant of welfare. Indeed, in the basic version of the neoclassical growth model with fixed or variable labor supply, the time path of technology and the initial capital stock are sufficient statistics for consumer welfare. But the intuition based on this result suggests that we should not care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions where the residual as defined by Solow (1957) no longer measures technology correctly. It also suggests that the Solow residual and the capital stock would not be sufficient statistics for welfare if there are shocks to the model other than technology.

We show that the link between Solow's residual and welfare is immediate and robust, even when the residual does not measure technical change. Here we build on the intuition of Basu and Fernald (2002) and derive rigorously the relationship between a modified version of the productivity residual and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, utility reflects the present discounted value of productivity residuals (plus the initial stock of assets), regardless of the production technology and the degree of product market competition. Our result also establishes that these two variables comprise a sufficient statistic for consumer welfare (to a first order) even if technology is not the only shock. For example, our result holds even with shocks to tax

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[2]For a review of the literature linking TFP to GDP per worker, in both levels and growth rates, see Weil (2008).

rates, government expenditure, tariff rates, changes in price-cost markups, and changes in the flexibility of prices, to name only a few of the cases we cover.

Our results are complementary to those in Solow's classic (1957) paper. Solow established that if there was an aggregate production function with constant returns to scale and if all markets were competitive, then his index measured its rate of change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the key component of an intertemporal welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm market structure. We do not need to assume anything about the firm side (which includes technology and market structure) in order to give a welfare interpretation, but we do need to assume the existence of a representative consumer.[3] Moreover, as we shall show, our approach does not require knowledge of the parameters of the utility function (besides the discount rate). Essentially, the key parameters of consumer preferences are revealed by the expenditure and distributional shares used to construct the productivity residual.

An obvious alternative for calculating consumer welfare would be to use calibrated or estimated dynamic stochastic general equilibrium (DSGE) models. They routinely assume a representative consumer as well. The advantage of our approach is that imposes less structure on the problem, and therefore it is much less dependent on parametric assumptions. Furthermore, understanding the actual welfare changes in a given historical period requires the modeler to incorporate all of the possible shocks that were relevant: changes in technology, trade policy, market structure, and so on. Our measure allows for a great many shocks that we do not even have to specify—basically, shocks to anything other than the utility function itself.[4] However, only a fully-specified DSGE model allows going beyond historical comparisons to conduct counterfactual policy experiments. Thus, there are trade-offs between the two approaches. They are complementary rather than competing, and the choice of which to use depends upon the purpose of the analysis.

2.1 Measuring welfare changes over time

We begin by assuming the familiar objective function for a representative household that maximizes intertemporal utility. In a growth context one often neglects the dependence of welfare on leisure, but Nordhaus and Tobin (1972) suggest that this omission is not innocuous (see the discussion in Section 5). Thus, we assume the household derives utility from both consumption and leisure:

$$W_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \frac{1}{1-\sigma} C_{t+s}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{1}$$

where W_t denotes the total welfare of the household, C_t is the per-capita consumption, L_t are per-capita hours of work and $\overline{L}$ is the per-capita time endowment. N_t is population and H is the number of households, assumed to be fixed and normalized to one from now on. Initially,

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[3]At a technical level, both results assume the existence of a potential function (Hulten, 1973), and show that TFP growth is the rate of change of that function.

[4]In any environment, DSGE or otherwise, welfare comparisons with time-varying preferences raise problematic philosophical issues.

we assume there is no government. Population grows at a constant rate n. To ensure the existence of a well-defined steady-state in which hours of work are constant while consumption and the real wage share a common trend, we assume that the utility function has the King, Plosser and Rebelo (1988) form with $\sigma > 0$ and $\nu(.) > 0$.[5] In a setting with no government, the budget constraint facing the representative consumer and the capital accumulation equation are respectively:

$$P_t^I K_t N_t + B_t N_t = (1 - \delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} N_{t-1} + \Pi_t N_t - P_t^C C_t N_t \tag{2}$$

$$K_t N_t = (1 - \delta) K_{t-1} N_{t-1} + I_t N_t \tag{3}$$

K_t, B_t and I_t denote per-capita capital, net asset holdings and investment; P_t^K, P_t^L, P_t^C and P_t^I denote, respectively, the user cost of capital, the hourly wage, the price of consumption goods and of new capital goods; i_t^B is the nominal rate of returns on assets and Π_t denotes per-capita profits, which are paid lump-sum from firms to consumers. So far we have implicitly assumed, for ease of notation, that there is only one type of consumption and one type of investment good. However, we show in the (2012) working paper version that we can allow for multiple consumption and investment goods.[6] In this case, C_t and I_t should be interpreted as homothetic aggregators of individual consumption and investment goods. P_t^C and P_t^I are the corresponding price indexes.

In a closed economy B_t would denote domestic inside debt. In equilibrium, it would equal zero. It does not matter whether B_t represents a single asset or a full menu of state-contingent assets. In turn, in an open-economy setting, the set of available assets and whether there is full consumption risk-sharing matters for many questions. However, our results hold, even in the open economy, regardless of the structure of asset markets and the degree of risk sharing, as one can verify by inspecting the proofs we present later.[7] For ease of notation, we continue to treat B_t and i_t^B as scalars, but it is important to keep in mind that they may be vectors, and the set of assets in B_t may or may not span the full state space.

Define "equivalent consumption" per person, denoted by C_t^*, as the level of consumption per-capita at time t that, if growing at the steady-state rate g from t onward, with leisure set at its steady-state level, delivers the same per-capita intertemporal utility as the actual stream of consumption and leisure. More precisely, C_t^* satisfies:

$$\begin{aligned} \frac{W_t}{N_t} &= V_t = \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \frac{(C_t^*(1+g)^s)^{1-\sigma}}{(1-\sigma)} \nu(\overline{L} - L)) \tag{4} \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L} - L) \end{aligned}$$

where V_t denotes per-capita utility and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ is the discount rate. We will measure welfare changes over time in terms of equivalent consumption per-capita and relate them to

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[5] If $\sigma = 1$, then the utility function must be $U(C, \overline{L} - L) = \log(C) + \nu(\overline{L} - L)$. See King, Plosser and Rebelo (1988).

[6] See Section 3.3.

[7] The intuition is that the observed time paths of consumption and leisure summarize all of the relevant information about the degree of risk sharing allowed by asset markets.

observable economic variables.

First we define a few of the key variables used in our analysis. Consider a modified definition of the Solow productivity residual:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} \tag{5}$$

where $\Delta \log Y_t = s_C \Delta \log C_t + s_I \Delta \log I_t$. $\Delta \log Y_t$ is a Divisia index of domestic absorption growth (in real per-capita terms), where demand components are aggregated using constant steady-state shares. s_C and s_I denote the steady-state values of $s_{C,t} = \frac{P_t^C C_t}{P_t^Y Y_t}$ and $s_{I,t} = \frac{P_t^I I_t}{P_t^Y Y_t}$ respectively, and $P_t^Y Y_t$ represents per-capita absorption in current prices. Distributional shares are also defined as the steady-state values, s_L and s_K, of $s_{L,t} \equiv \frac{P_t^L L_t}{P_t^Y Y_t}$ and $s_{K,t} \equiv \frac{P_t^K K_{t-1} N_{t-1}}{P_t^Y Y_t N_t}$ (note that the household receives remuneration on the capital stock held at the end of the last period). We use the word "modified" in describing the productivity residual for several reasons. First, we use real absorption rather than GDP as the measure of output. Second, all shares are calculated at their steady-state values, and hence are not time varying, which is sometimes assumed when calculating the residual.[8] Third, the residual is stated in terms of per-capita rather than aggregate variables, although it should be noted that Solow himself defined the residual on a per-capita basis (1957, equation 2a). Correspondingly, define the log level productivity residual as:

$$\log PR_{t+s} = s_C \log C_{t+s} + s_I \log I_{t+s} - s_L \log L_{t+s} - s_K \log K_{t+s-1} \tag{6}$$

The prices in the budget constraint, equation (2), are defined in nominal terms. It will often be easier to work with relative prices. Taking the purchase price of new capital goods, P_t^I, as numeraire, define the following relative prices: $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I}$ and $p_t^C = \frac{P_t^C}{P_t^I}$. Real per-capita profits are defined as $\pi_t = \frac{\Pi_t}{P_t^I}$. Our approximations are taken around a steady-state path where the first three relative prices are constant and the wage p^L grows at rate g, as in standard one-sector models of economic growth. We also assume that all per-capita quantity variables other than labor hours (for example Y_t, C_t, I_t, etc.) grow at a common rate g in the steady-state. Note these assumptions imply that all of the shares we have defined above are constant in the steady-state and so is the capital-output ratio, whose nominal steady-state value will be denoted by $\frac{P^I K}{P^Y Y}$.[9]

Under these assumptions we can show that welfare changes, as measured by equivalent consumption, C_t^*, are, to a first-order approximation, a linear function of the expectation of present and future total factor productivity growth (and its revision), and of the initial capital stock. This first key result is summarized in:

Proposition 1 *Assume that the representative household in a closed or open economy with no government maximizes (1) subject to (2), taking prices, profits and interest rates as exogenously given. Assume also that population grows at a constant rate n, and the wage and*

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[8]Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant.

[9]We conjecture that all our results could be proved in the household environment corresponding to a two-sector growth model as laid out, for example, in Whelan (2003)—assuming that the steady-state shares are also constant, as in Whelan's setup.

all per-capita quantities other than hours worked grow at rate g in the steady-state. To a first-order approximation, the growth rate of equivalent consumption can be written as:

$$\Delta \log C_t^* = \frac{(1-\beta)}{s_c} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} \right]$$

(7)

Proof. Proofs of all propositions and extensions are in Appendix A (Sections A.1 through A.3).
■

Proposition 1 implies that the expected present value of current and future Solow productivity residuals (or their revision), together with the change in the initial stock of capital per-capita, is a sufficient statistic for the welfare of a representative consumer (where we measure welfare as the log change in equivalent consumption). Note that the term $\Delta E_t \log PR_{t+s} = E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}$ represents the revision in expectations of the log level of the productivity residual, based on the new information received between $t-1$ and t and it will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time t.[10]

Proposition 1 is a statement about the value function. A similar result has been proven in the context of the Ramsey growth model, where productivity measures technology. In that setting, one can show that the maximized value depends on current and expected future technology and current capital. Under a Markov assumption about the evolution of technology, all the expected technology terms can be summarized by technology today. Thus, in this simple context our result nests one that is already known. But our result is much more general. Since we have not made any assumptions about production technology and market structure, the productivity terms may or may not measure technical change. For example, Solow's residual does not measure technical change in economies where firms have market power, or produce with increasing returns to scale, or where there are Marshallian externalities. Even in these cases, Proposition 1 shows that productivity and the capital stock are jointly a sufficient statistic for welfare.

While the proof of the proposition requires somewhat complex notation and algebra, in the remainder of this sub-section we shall try to convey the economic reasoning for the result by considering the simpler case of a closed economy with a zero steady-state growth rate ($g = 0$). (Of course, the formal proof of Proposition 1 allows for $g > 0$ and for the economy to be either open or closed.) We begin by taking a first-order approximation to the level of utility of the household (normalized by population) around the steady state.[11] We then use the household's first-order conditions for optimality and the trasversality condition to obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_L \widehat{p_{t+s}^L} + s_K \widehat{p_{t+s}^K} + s_\pi \widehat{\pi}_t - s_C \widehat{p_{t+s}^C} \right] + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \widehat{K}_{t-1}$$

(8)

Hatted variables denote log deviations from the steady-state ($\widehat{x}_t = \log x_t - \log x$). Variables without time subscripts denote steady-state values. Since $g = 0$, $\beta = \frac{1+n}{1+\rho}$. λ is the Lagrange

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[10] If we assume that the modified log level productivity residual follows a univariate time-series process, then only the innovation of such a process matters for the expectation revision, and the first summation is simply a function of current and past values of productivity.

[11] We approximate the level of V rather than its log because $V < 0$ if $\sigma > 1$.

multiplier associated with the budget constraint expressed, like utility, in per-capita terms. Equation (8) follows almost directly from the Envelope Theorem. An atomistic household maximizes taking as given the sequences of current and expected future prices, lump-sum transfers (Π_t), and predetermined variables (in our environment, just K_{t-1}). Thus, only fluctuations in these objects affect welfare to a first order. The Envelope Theorem plus a bit of algebra shows that each change in exogenous prices or profits needs to be multiplied by its corresponding share to derive its effect on welfare—for example, the larger is s_C the more the consumer suffers from a rise in the relative price of consumption goods. (It may appear that the investment price is missing, but since we normalized the relative price of investment goods to 1 it never changes.) The terms within the summation can be thought of as the dual version of the productivity residual, as we will show shortly.

Two comments are in order. First, the left hand side of the equation has an interesting interpretation: it is the money value of the deviation of per-person utility from its steady-state level, expressed as a fraction of steady-state absorption per person. To understand this interpretation, consider the units. The numerator is in "utils," which we divide by λ, which has units of utils per investment good (since investment goods are our numeraire). The division gives us the deviation of utility from its steady-state value measured in units of investment goods in the numerator. We then scale the result by the real value of per-capita absorption, also stated in terms of investment goods (recall that p^Y is a relative price).

Second, note that we can express welfare change without knowing the parameters of the utility function, other than the discount factor β. The right-hand side contains only expectations of the Solow residual (and its revision) and the initial capital stock per capita. Essentially, the parameters of the utility function are embedded in observed choices for consumption, labor and capital, in the expenditure shares, in the distributional shares and in the capital-output ratio. Our basic idea is to use a revealed-preference approach, akin to the logic behind the economic approach to index-number theory. This approach uses observed choices to infer welfare parameters (as, for example, expenditure shares reveal the relative importance of different prices to the household).

However, we find it more convenient and intuitive to express the left hand side of (8) in terms of equivalent consumption. Using the definition in (4) and taking a first order approximation of $V_t - V$ in terms of $\log C_t^*$ we obtain:

$$\frac{(V_t - V)}{\lambda p^Y Y} = \frac{s_C}{(1 - \beta)}(\log C_t^* - \log C) \tag{9}$$

where we have used the fact that in the steady state $C^* = C$ and $U_C = \lambda p^c$.

The right hand side of (8) is written as a function of the log deviation from the steady-state of prices, profits and the initial capital stock. Our results can also be presented using the familiar primal productivity residual rather than stating them in terms of prices and transfers, as in (8). However, if one uses a consistent data set, there is literally no difference between the two. Using the per-capita version of the household budget constraint (2), the capital accumulation equation (3), and the fact that in a closed economy $B_t = 0$ in equilibrium, one can show that

the following relationship must hold at all points in time:

$$s_L \widehat{p_{t+s}^L} + s_K \widehat{p_{t+s}^K} + s_\pi \widehat{\pi}_t - s_C \widehat{p_{t+s}^C} = s_C \widehat{C}_{t+s} + s_I \widehat{I}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{K_{t+s-1}}. \tag{10}$$

Equation (10) says that in any data set where national income accounting conventions are enforced, the primal productivity residual identically equals the dual productivity residual.[12] Thus we can express our results in either form, but using the dual result would require us to provide an empirical measure of lump-sum transfers, which is not needed for results based on the primal residual. For this reason, we work with the primal.

Using (9) and (10) in (8), we can write:

$$(\log C_t^* - \log C) = \frac{(1-\beta)}{s_C} \left[E_t \sum_{s=0}^{\infty} \beta^s \widehat{PR}_{t+s} + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \widehat{K}_{t-1} \right] \tag{11}$$

where now the log deviation of consumption is expressed as a function of the log deviation from steady-state of the productivity residual, defined in equation (6). Taking differences of equation (11) and using the definition of the Solow residual in (5) gives the key equation of Proposition 1, whose proof we have just sketched for the case of $g = 0$.

So far we have used absorption as a measure of output. How would we have to modify our proposition, if one were to use, instead, a standard measure of output, real GDP, defined as consumption, plus investment, plus net exports? Then, we show in Appendix A that the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, the initial conditions should include the lagged value of the change of the stock of net foreign assets.

More precisely, now the absorption based productivity residual, $\Delta \log PR_{t+s}$, must be replaced by $\Delta \log PRTT_{t+s}$ defined as:

$$\begin{aligned}
\Delta \log PRTT_{t+s} &= \Delta \log GDP_{t+s} - s_L' \Delta \log L_{t+s} - s_K' \Delta \log K_{t+s-1} \\
&\quad + s_X' \Delta \log p_{t+s}^{EX} - s_M' \Delta \log p_{t+s}^{IM} + \left(\frac{Br}{P^Y Y} \right) \Delta \log r_{t+s}
\end{aligned} \tag{12}$$

where $\Delta \log GDP_{t+s}$ is the applicable Divisia index of per-capita GDP growth, defined as $\Delta \log GDP_t = s_C' \Delta \log C_t + s_I' \Delta \log I_t + s_X' \Delta \log EX_t - s_M' \Delta \log IM_t$, where s_C', s_I', s_X' and s_M' are respectively the steady-state shares of consumption, investment, exports and imports out of GDP. s_L' and s_K' are also factor shares out of GDP, r is the real rate of return on foreign assets, p_{t+s}^{EX} and p_{t+s}^{IM} are the relative prices of exports and imports, and $\left(\frac{Br}{P^Y Y} \right)$ is the steady-state ratio of foreign asset income to GDP.[13] Finally, the initial conditions now include not only the (domestic) capital stock, but also a term reflecting the lagged change in the stock of foreign assets, $\frac{1}{\beta} \left(\frac{P^I}{P^Y Y} \right) \Delta \frac{B_{t-1}}{P_{t-1}^I}$.

Conceptually, it makes very good sense that all these extra terms come into play when

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[12] See, for example, Barro and Sala-i-Martin (2004, section 10.2).

[13] Recall that our results hold with multiple consumption and investment goods, in which case p_t^{EX} and p_t^{IM} are conventionally defined price indexes of exports and imports (relative to the price index of the investment goods).

taking the GDP route to the measurement of welfare in the open economy. The effects of an improvement in the terms of trade, as captured by $s'_X \Delta \log p_t^{EX} - s'_M \Delta \log p_t^{IM}$ in (12), are analogous to those of an increase in TFP: both give the consumer higher consumption for the same input of capital and labor (and therefore higher welfare).[14] In turn, the term in $\Delta \log r_{t+s}$ captures present and expected future changes in the rate of return on foreign assets, including capital gains and losses on net foreign assets due either to exchange rate movements or to changes in the foreign currency prices of the assets.

However, measuring these extra terms empirically poses major challenges. In contrast, all these problems disappear if the measurement of welfare is based on real absorption rather than GDP as the measure of output. This is why we have put the absorption-based productivity residual at the center of our theoretical analysis and empirical application.

2.2 Implications for Cross-Country Analysis

Proposition 1 pertains to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. However, in this sub-section we show that similar methods can be used to do a rigorous welfare comparison across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

A comparison of welfare across two countries, i and j, requires either assuming that their respective representative agents possess the same utility function, or making the comparison from the perspective of the representative agent in a reference country, j, that optimizes facing the prices, per-capita profits and initial assets of country i. The former assumption is clearly unappealing, but the latter approach seems difficult since it appears that we need to construct counterfactual choices of consumption and leisure for the household from the reference country, moving to country i. However, we show that our approach does not require the construction of such counterfactual quantities, and can be implemented using only observed cross-country differences in productivity.

We can then study the cross-country difference in the utility of a representative consumer. As in the within-country case, we conduct the comparison using the concept of equivalent consumption. In this context for the representative agent of the reference country j living in country i, equivalent per-capita consumption, $\widetilde{C}_t^{*,i}$ satisfies:

$$\widetilde{V}_t^i = \frac{1}{\left(1 - \sigma^j\right)\left(1 - \beta^j\right)} \left(\widetilde{C}_t^{*,i}\right)^{1-\sigma^j} \nu(\overline{L} - L^j) \tag{13}$$

where $\widetilde{V}_t^i$ denotes per-capita utility of the individual from country j, facing country i's relative prices, per-capita profits and per-capita initial capital stock (we use $\sim$ to denote these counterfactual quantities). Note that $\widetilde{C}_t^{*,i}$ is defined for a constant level of leisure fixed at country j's steady-state level. We will use V_t^j and $C_t^{*,j}$ to denote per-capita utility and equivalent consumption of the individual of country j living in country j. We take first-order approximations of $\widetilde{V}_t^i$, V_t^j, and the budget constraints around the steady state of country j. This gives us:

---

[14]See Kohli (2004) for a static version of this result.

Proposition 2 *Assume that in a reference country, country j, the representative household maximizes (1) subject to (2), under the assumptions of Proposition 1. Assume now that the household of country j is confronted with the sequence of prices, per-capita profits and initial capital stock of country i. In a closed or open economy with no government, to a first order approximation, the difference in equivalent consumption between living in country i versus country j is:*

$$
\log \widetilde{C}_t^{*,i} - \log C_t^{*,j} = \frac{(1-\beta^j)}{s_c^j} E_t \sum_{s=0}^{\infty} (\beta^j)^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^j \right)
$$
$$
+ \frac{(1-\beta^j)}{\beta^j s_c^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K_{t-1}^j \right) \tag{14}
$$

where the productivity terms are constructed in the following fashion:

$$
\log \overline{PR}_{t+s}^i = \left(s_C^j \log C_{t+s}^i + s_I^j \log I_{t+s}^i \right) - s_L^j \log L_{t+s}^i - s_K^j \log K_{t+s-1}^i \tag{15}
$$

$$
\log PR_{t+s}^j = \left(s_C^j \log C_{t+s}^j + s_I^j \log I_{t+s}^j \right) - s_L^j \log L_{t+s}^j - s_K^j \log K_{t+s-1}^j \tag{16}
$$

Welfare differences across countries are therefore summarized by two components. The first component is related to the well-known log difference between TFP levels, which accounts empirically for most of the difference in per-capita income across countries (Hall and Jones (1999)), although here it is the present value of the gap that matters for welfare. In the development accounting literature, this gap is interpreted as a measure of technological or institutional differences between countries. This interpretation, however, is valid only under restrictive assumptions on market structure and technology (perfect competition, constant returns to scale, no externalities, etc.). We provide a welfare interpretation of cross-country differences in TFP that applies even when these assumptions do not hold. The second component of the welfare difference reflects the difference in capital intensity between the two countries; *ceteris paribus*, a country with more capital per person can afford more consumption or higher leisure. The development accounting literature also uses capital intensity as the second variable explaining cross-country differences in per-capita income.

Our result holds for any kind of technology and market structure, as long as a representative consumer exists, takes prices as given and is not constrained in the amount he can buy and sell at those prices. (We discuss quantity rationing and violations of the law of one price in our (2012) working paper). Notice however, that our measure of per-capita TFP is modified with respect to the traditional growth accounting measure in three ways. First, measuring welfare differences requires comparing not only current log differences in TFP but the present discounted value of future ones as well. Second, the distributional and expenditure shares used to compute the log differences in TFP between countries need to be calculated at their steady-state values in the reference country.[15] Third, we use real absorption rather than GDP as the measure of output.

As in the case of Proposition 1, we shall try to convey the economic reasoning for the result by considering the simple case of a closed economy with a zero steady-state growth rate ($g = 0$).

---

[15] It is standard in the development accounting literature to assume that all countries have the same capital and labor shares in income (often one-third and two-thirds), but to use country-specific shares in expenditure.

Assume we confront the household from country j with the prices, profits and the initial per-capita capital stock of country i. If we expand the utility of a representative member of the household, denoted by $\widetilde{V}_t^i$, around the steady state of his own country, we obtain:

$$
\begin{aligned}
\frac{(\widetilde{V}_t^i - V^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p^{L,j}) + s_K^j (\log \ p_{t+s}^{K,i} - \log \ p^{K,j}) \\
&\quad + s_\Pi^j (\log \pi_{t+s}^i - \log \pi^j) - s_C^j (\log p_{t+s}^{C,i} - \log p^{C,j})] \\
&\quad + \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K^j \right)
\end{aligned}
\tag{17}
$$

Now expand per-capita utility for country j's household around its own steady-state and subtract from (17). This yields:

$$
\begin{aligned}
\frac{(\widetilde{V}_t^i - V_t^j)}{\lambda^j p^{Y,j} Y^j} &= E_t \sum_{s=0}^{\infty} (\beta^j)^s [s_L^j (\log p_{t+s}^{L,i} - \log p_{t+s}^{L,j}) + s_K^j (\log \ p_{t+s}^{K,i} - \log \ p_{t+s}^{K,j}) \\
&\quad + s_\Pi^j (\log \pi_{t+s}^i - \log \pi_{t+s}^j) - s_C^j (\log p_{t+s}^{C,i} - \log p_{t+s}^{C,j})] \\
&\quad + \frac{1}{\beta^j} \left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j} \right) \left(\log K_{t-1}^i - \log K_{t-1}^j \right)
\end{aligned}
\tag{18}
$$

Differences in welfare across countries are, therefore, due to differences in their relative prices, per-capita profits and capital intensities. Use (13) to express differences in utility across countries in term of log differences in equivalent consumption on the left hand side of (18). Now linearize two budget constraints around country j's steady state: first, the budget constraint for the household from country j if moved to country i and, second, its budget constraint when living in its own country. Subtracting one from the other allows us to write the right hand side of (18) in terms of productivity differences and differences in the initial capital stock. This yields equations (14), (15), and (16) in Proposition 2.[16]

Notice that in stating Proposition 2, we have not needed to assume that either the population growth rate n or the per-capita growth rate g is common across countries.[17] Most importantly, we do not need to know the parameters of the utility function (other than β) to perform cross country welfare comparisons, nor do we need to assume that any of those parameters (including β) is common across countries. This is because we are making the comparisons in terms of differences in equivalent consumption and from the point of view of the representative individual in the reference country, who is faced with the exogenous (to the household) prices, lump-sum transfers and initial conditions of country i.

It is also important to emphasize that this thought experiment does not simply assign to the household from the reference country the consumption and leisure choices made by the household from country i. Rather, our approach allows the reference-country household to

---

[16]We can show that, in the special case in which all countries are in the steady state, share a common growth rate, and consumers do not derive utility from leisure, cross-country welfare comparisons reduce to comparing Net National Product. This result is in the spirit of Weitzman (1976, 2003). We thank Chad Jones for this observation.

[17]Both introspection and the results of Kremer, Onatski and Stock (2001) suggest that it is implausible to assume that countries will diverge perpetually in per-capita terms. Thus, even though we do not need to assume a common g, we would not view it as a restrictive assumption.

re-optimize when facing the conditions of country i. Even if faced with the same exogenous variables, the choices of the two households will generally differ, unless their preferences are identical. However, to a first order approximation, the algebraic sum of the terms in prices, profits, and initial conditions for the household from country j facing country i's prices, profits, and endowments equals the algebraic sum of the terms in consumption, labor supply and capital chosen by the individual from country i. We exploit this fact to obviate the need to know counterfactual quantities in calculating the welfare change for a household of country j moving to country i.

If we based our cross-country comparison on consumption and leisure, rather than productivity and the capital stock, we would be left with the insurmountable problem of providing an empirical counterpart for the (unobservable) quantities of the former variables chosen by the household of country j moving to country i. Only if preferences were assumed identical across countries – a rather restrictive assumption – could the issue be resolved by using the actual quantities of country i. Our derivation shows that we can, instead, base the comparison on the actual observed productivity levels and capital stocks of the two countries, even if preferences differ across countries.[18]

3   Extensions

We now show that our method of using TFP to measure welfare can be extended to allow for the presence of taxes and government expenditure. We also extend the problem of the household to allow for human capital accumulation. These extensions require modifications in the formulas for welfare comparisons over time and across countries, and we state the changes to the basic framework that are needed in each case[19]. These results prove that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but they also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household. For brevity, we only discuss the generalization of our measure of welfare changes over time (Proposition 1). Similar results apply to the measure of cross-country welfare differences (Proposition 2).

3.1   Taxes

Consider first an environment with distortionary and/or lump-sum taxes. Since the prices in the budget constraint (2) are those faced by the consumer, in the presence of taxes all prices should be interpreted as after-tax prices. At the same time, the variable that we have been calling "profits," Π_t, can be viewed as comprising any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates. Finally one should think of B_t as including both government and private bonds (assumed to be perfect substitutes, for ease of notation).

More precisely, in order to modify (2) to allow for taxes, let τ_t^K be the tax rate on capital income, τ_t^R be the tax rate on revenues from bonds, τ_t^L be the tax rate on labor income, τ_t^C

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[18]See the equations leading up to (A.35) in Appendix A, Section A.3.

[19]Detailed derivations for the case of government expenditure are given in Appendix A (section A.3).

be the *ad valorem* tax on consumption goods, and τ_t^I be the corresponding tax on investment goods.[20] Also, let $P_t^{C'}$ and $P_t^{I'}$ respectively denote the pre-tax prices of consumption and capital goods, so that the tax-inclusive prices faced by the consumer are $P_t^{C'}\left(1+\tau_t^C\right)$ and $P_t^{I'}\left(1+\tau_t^I\right)$. We assume for the time being that the revenue so raised is distributed back to individuals using lump-sum transfers; we consider government expenditures in the next subsection. The representative household's budget constraint now is:

$$
\begin{aligned}
0 = {} & -P_t^{I'}\left(1+\tau_t^I\right)K_tN_t - B_tN_t + (1-\delta)P_t^{I'}\left(1+\tau_t^I\right)K_{t-1}N_{t-1} + \left(1+i_t^B\left(1-\tau_t^R\right)\right)B_{t-1}N_{t-1} \\
& +P_t^L\left(1-\tau_t^L\right)L_tN_t + P_t^K\left(1-\tau_t^K\right)K_{t-1}N_{t-1} + \Pi_tN_t - P_t^{C'}\left(1+\tau_t^C\right)C_tN_t \qquad (19)
\end{aligned}
$$

Thus, extending our benchmark case, the exogenous variables in the household's maximization are not only the prices and the initial stocks of capital and bonds, but also the tax rates on labor and capital income, consumption and investment. However, it can be easily shown that the basic results (7) and (14) continue to hold. The only modification is that in defining the Solow productivity residual we need to take account of the fact that the national accounts measure factor payments as perceived by firms – that is, before income taxes – while nominal expenditure is measured using prices as perceived from the demand side, thus inclusive of indirect taxes (subsidies) on consumption and investment. Hence, letting $P_t^C = P_t^{C'}\left(1+\tau_t^C\right)$ and $P_t^I = P_t^{I'}\left(1+\tau_t^I\right)$ denote the tax-inclusive prices of consumption and investment goods, the expenditure shares s_C and s_I defined earlier are fully consistent with those obtained from national accounts data, but the factor shares s_L and s_K defined above use the gross income of labor and capital rather than their after-tax income. Thus, to be data consistent, with taxes the welfare residual needs to be redefined in terms of the shares of after-tax returns on labor and capital. Specifically, equation (5) should be re-written as:

$$
\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - \left(1-\tau^L\right)s_L\Delta \log L_{t+s} - \left(1-\tau^K\right)s_K\Delta \log K_{t+s-1} \qquad (20)
$$

and an analogous modification applies to (15) and (16). τ^L and τ^K are the steady-state values of τ_t^L and τ_t^K With these modifications, our results generalize to a setting with distortionary, time-varying taxes on consumption and investment goods and on the household's income coming from labor, capital or financial assets.

3.2   Government Expenditure

With some minor modification, our framework can be likewise extended to allow for the public provision of goods and services (see Appendix A for details). We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite instantaneous utility as:

---

[20]For simplicity, we are assuming no capital-gains taxes and no expensing for depreciation. These could obviously be added at the cost of extra notation.

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1-\sigma}\Omega(C_{t+s}; C_{G,t+s})^{1-\sigma}\nu(\overline{L} - L_{t+s}) \tag{21}$$

where C_G denotes per-capita public consumption and $\Omega(.)$ is homogenous of degree one in its arguments. Total absorption now includes public consumption: that is, $P_t^Y Y_t = P_t^C C_t + P_t^G C_{G_t} + P_t^I I_t$, where P^G is the public consumption deflator.

In this setting, our earlier results need to be modified to take account that public consumption may not be set by the government at the level that consumers would choose. Intuitively, in such circumstances the value that consumers attach to public consumption may not coincide with its observed value as included in domestic absorption, and therefore in the productivity residual as conventionally defined. Formally, let $s_{c_{Gt}} = \frac{P_t^G C_{G_t}}{P_t^Y Y_t}$ denote the share of public consumption out of domestic absorption, and let $s_{c_{G_t}}^*$ be the share that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household, rather than using its deflator P_t^G.[21] The welfare-relevant modified Solow residual (5) now is:

$$\Delta \log PR_{t+s} = \Delta \log Y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log K_{t+s-1} + \left(s_{c_G}^* - s_{c_G}\right) \Delta \log C_{G,t+s} \tag{22}$$

and an analogous modification applies to (15) and (16). Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption is under- or over-provided (i.e., $s_{c_G}^* > s_{c_G}$ or $s_{c_G}^* < s_{c_G}$ respectively). If the government sets public consumption exactly at the level the utility-maximizing household would have chosen if confronted with the price P_t^G, then $s_{c_G}^* = s_{c_G}$ and no correction is necessary. For want of a better term, we shall refer to this case as 'optimal government consumption.' In turn, in the case in which public consumption is pure waste $s_{c_G}^* = 0$, the welfare residual should be computed on the basis of private absorption, consumption and investment. With the residual redefined in this way, the growth rate of equivalent consumption now is:[22]

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s_{C_G}^*)}\left[E_t \sum_{s=0}^{\infty}\beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty}\beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right)\Delta \log K_{t-1}\right] \tag{23}$$

3.3 Summing up

We can now go back to the two propositions stated earlier. They were formulated for the special case of an economy with no government. In light of the discussion in this section, we can now restate them in a generalized form for an economy with a government sector, which is more appropriate for empirical implementation.

Proposition 1' *Assume a closed or open economy, with public consumption, taxes on labor*

---

[21] It is easy to verify that $s_{c_{G_t}}^* = \frac{U_{C_{Gt}} P_t^C C_{G_t}}{U_{C_t} P_t^Y Y_t}$.

[22] Government purchases might also yield productive services to private agents. For example, the government could provide education or health services, or public infrastructure, which – aside from being directly valued by consumers – may raise private-sector productivity. In such case, the results in the text remain valid, but it is important to note that the contribution of public expenditure to welfare would not be fully captured by the last term in the modified Solow residual as written in the text. To this term we would need to add a measure of the productivity of public services, which is implicitly included in the other terms in the expression.

and capital income at rates τ_{t+s}^L and τ_{t+s}^K, and taxes on consumption and investment expenditure at rates τ_{t+s}^C and τ_{t+s}^I. Assume also that the representative household maximizes intertemporal utility, taking prices, profits, interest rates, tax rates and public consumption as exogenously given. Lastly, assume that population grows at a constant rate n, and the wage and all per-capita quantities other than hours worked grow at rate g in the steady state. To a first-order approximation, the growth rate of equivalent consumption can be written as:

$$\Delta \log\left(C_t\right)^* = \frac{(1-\beta)}{(s_C + s_{C_G}^*)}\left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} + \frac{1}{\beta}\left(\frac{P^I K}{P^Y Y}\right)\Delta \log K_{t-1}\right]$$

(24)

where productivity growth is defined as:

$$\begin{aligned}
\Delta \log PR_{t+s} &= s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s_{C_G}^* \Delta \log C_{G,t+s} \\
&\quad - \left(1 - \tau^L\right) s_L \Delta \log L_{t+s} - \left(1 - \tau^K\right) s_K \Delta \log K_{t+s-1}
\end{aligned}$$

(25)

Shares and tax rates are evaluated at their steady-state values, and $s_{C_G}^*$ denotes the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household.

As explained earlier, the value of $s_{C_G}^*$ depends on the assumptions made about government consumption. This implies that the proposition can encompass a variety of cases with respect to taxation and government spending: 1) wasteful government spending with lump sum taxes (in which case distortionary taxes are set to zero in the productivity equation); 2) optimal government spending with lump sum taxes; 3) wasteful government spending with distortionary taxes; 4) optimal government spending with distortionary taxes.

Our main result regarding welfare differences across countries can be restated in a similar way:

Proposition 2' *Assume that in a reference country j the representative household maximizes intertemporal utility under the assumptions of Proposition 1'. Assume now that the household of country j is confronted with the sequence of prices, tax rates, per-capita profits, other lump sum transfers, public consumption, and endowment of country i. In a closed or open economy with distortionary taxation and government spending, the difference in equivalent consumption between living in country i versus country j can be written, to a first order approximation, as:*

$$\begin{aligned}
\ln \widetilde{C}_t^{*,i} - \ln C_t^{*,j} &= \frac{(1-\beta^j)}{(s_C^j + s_{C_G}^{*j})} E_t \sum_{s=0}^{\infty} (\beta^j)^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^j\right) \\
&\quad + \frac{(1-\beta^j)}{\beta^j (s_C^j + s_{C_G}^{*j})}\left(\frac{P^{I,j} K^j}{P^{Y,j} Y^j}\right)\left(\log K_{t-1}^i - \log K_{t-1}^j\right)
\end{aligned}$$

(26)

where $s_{C_G}^{*j}$ the steady-state share of public consumption in total absorption that would obtain if public consumption were valued according to its marginal contribution to the utility of the representative household. The two productivity terms are constructed with all shares and tax

rates evaluated at the reference country's steady-state values:

$$\log PR_t^j = s_C^j \log C_t^j + s_I^j \log I_t^j + s_{CG}^{*j} \log C_{G,t}^j - \left(1 - \tau^{L,j}\right) s_L^j \log L_t^j - \left(1 - \tau^{K,j}\right) s_K^j \log K_{t-1}^j \quad (27)$$

$$\log \overline{PR}_t^i = s_C^j \log C_t^i + s_I^j \log I_t^i + s_{CG}^{*j} \log C_{G,t}^i - \left(1 - \tau^{L,j}\right) s_L^j \log L_t^i - \left(1 - \tau^{K,j}\right) s_K^j \log K_{t-1}^i \quad (28)$$

Proposition 2' will be the basis for our cross-country welfare comparisons. The derivation shows a result that would be hard to intuit ex ante, which is that to a first-order approximation only the tax rates of the reference country enter the welfare comparison.[23] This asymmetry implies that welfare rankings may depend on the choice of reference country. In our empirical application in Section 4.3 below we take the US as our reference country, but check the robustness of the results by using France instead.[24]

So far we have assumed that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. In our (2012) working paper, we show that under some conditions our approach extends to environments where the household does not behave as a price taker, faces a distribution of prices rather than a single price, or faces quantity constraints. Our working paper also shows that our results hold when the model is extended to allow for multiple types of consumption and investment goods.

A potential concern with our main results, as stated in Proposition 1' and 2', is that they are proved using first-order approximations. This approach may seem especially problematic for cross-country comparisons, where gaps in living standards are often large. In Appendix B (and in our (2012) working paper), we consider a set of workhorse models that are standard in the macroeconomic literature, such as the neoclassical growth model with variable labor supply, augmented, in turn, with distortionary taxes, government expenditure, and production externalities. We solve these models either analytically or with third order approximation methods, and then compare the calculated welfare values to our approximated measures. Reassuringly, we find that our approximation is very accurate for assessing welfare differences, both over time and across countries.

Taking our measures to the data requires that outputs and inputs be measured accurately. It is well known that there are significant problems with the measurement of quality change and the impact of new goods on measures of real output and its components; similar issues apply to the measurement of capital and labor input. These problems can be particularly vexing when one tries to quantify gains from trade. But conceptually the issues are well understood. For example, it would be easy to extend our theory by introducing changing product variety into our consumption aggregator. The main problems are empirical: the measurement problems are often difficult, and as a result, such corrections have not been applied to all the relevant series.

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[23] Of course, the tax rates of the comparison country will generally change output and input levels in that country through general-equilibrium effects, which will influence the welfare gap between the two countries. However, the tax rates of the comparison country do not enter the formula directly.

[24] We conjecture that the asymmetry may be eliminated by moving to second-order approximations, where instead of using the tax-adjusted shares of the reference country only, one might take an arithmetic average of the shares of the reference and comparison countries.

3.4 An Extension with Human Capital

We will use the results summarized in Propositions 2' to compare welfare across countries. In so doing it may be useful to account for differences in human capital. It goes beyond the scope of this paper to provide an exhaustive analysis of the implications of human capital accumulation for welfare measurement. However, in this subsection we develop an extension of our model in the spirit of Lucas (1988). As in Lucas, we assume that non-leisure time can be used either to work or to accumulate human capital, and that the accumulation of human capital is linear in the stock of human capital.[25] Specifically, assume that the per-period utility function of the representative individual is now $U(C_t; \overline{L} - L_t - E_t)$, where E_t denotes the amount of time devoted to human capital accumulation. We assume that labor income per person can be written as $P_t^L H_{t-1} L_t$, where L_t continues to denote hours worked, H_{t-1} the initial per-capita level of human capital, and P_t^L the hourly price of one unit of human capital. The human capital accumulation equation is assumed to be:

$$(H_t - H_{t-1}) + \delta_H H_{t-1} = F(E_t) H_{t-1} \tag{29}$$

where $F'(E_t) > 0$ and $F(0) = 0$. It is possible to show that the change in equivalent consumption now also depends upon the change in the initial level of human capital. Moreover, labor input must be adjusted for human capital growth in the definition of productivity growth. Thus, equations (24) and (25) now become:

$$
\begin{aligned}
\Delta \log (C_t)^* &= \frac{(1-\beta)}{(s_C + s_{C_G}^*)} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s-1} \right. \\
&\quad \left. + \frac{1}{\beta} \left(\frac{P^I K}{P^A A} \right) \Delta \log K_{t-1} + \left(\frac{1}{1-\beta} \right) \left(1 - \tau^L \right) s_L \Delta \log H_{t-1} \right]
\end{aligned} \tag{30}
$$

and:

$$
\begin{aligned}
\Delta \log PR_{t+s} &= s_C \Delta \log C_{t+s} + s_I \Delta \log I_{t+s} + s_{C_G}^* \Delta \log C_{G,t+s} \\
&\quad - \left(1 - \tau^L \right) s_L \Delta \log(L_{t+s} H_{t+s-1}) - \left(1 - \tau^K \right) s_K \Delta \log K_{t+s-1}
\end{aligned} \tag{31}
$$

Summarizing, in the presence of human capital, the definition of productivity must account for the effect on total labor input of both hours and human capital changes. Note that human capital investment does not show up as part of domestic absorption because in the Lucas formulation it is only a subtraction from leisure and does not require any other physical input. Moreover, human capital must now be included among the initial conditions, alongside physical capital. In the empirical section, we will present results accounting for human capital in the measurement of welfare differences across countries.

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[25]See Appendix A, Section A.4, for details. Lucas showed that this formulation of the capital accumulation equation generates a positive steady-state growth rate, even if there is no exogenous technological progress. If the production function of goods is constant returns in physical and human capital, then all the relevant quantities grow at the same rate. These results extend to the case in which the leisure choice is endogenized, although in this case there are some parameter configurations for which multiple steady-state balanced growth paths exist (Ladron de Guevara, Ortigueira and Santos (1999)).

4 Empirical Results

4.1 Data and Measurement

We illustrate the potential of our methodology by computing welfare indexes over the period 1985-2005 for a set of large, developed countries for which high quality time series data are available: US, UK, Japan, Canada, France, Italy and Spain. We use two different data sets to compare welfare within a country and across countries. Given the interest in welfare comparisons for a more heterogeneous group of countries, we use a third data set for cross country comparisons for 63 developed and less developed countries. However, we do not use this sample for our baseline results because consistent data on hours of work (as opposed to employment) are not available for most countries, particularly outside the OECD.

To analyze welfare changes over time for our sample of advanced countries, we combine data coming from the OECD Statistical Database with the EU-KLEMS dataset.[26] We construct the log of absorption from the OECD dataset, using data on household final consumption, gross capital formation and government consumption (where appropriate) at constant national prices, and their respective nominal shares of absorption as weights. Since our theory requires steady-state shares, we use the averages of the observed shares across the twenty years in our sample.

The growth rate of our modified productivity residual is constructed as the log-change in real absorption minus the log changes in capital and labor, each weighted by its income share out of absorption. Data on aggregate production inputs are provided by EU-KLEMS. The capital stock is constructed by applying the perpetual inventory method to investment data. Labor input is the total amount of hours worked by employed persons. To obtain per-capita quantities, we divide absorption, capital, and labor by total population. We assume that economic profits are zero in the steady-state so that we can recover the gross (tax unadjusted) share of capital as one minus the labor share.

In order to compare welfare across countries, we combine data from the Penn World Tables with hours data from EU-KLEMS dataset. Specifically, our basic measure of real absorption is constructed from the Penn World Tables as the weighted average of PPP-converted log private consumption, log gross investment and log government consumption, using as weights their respective shares of absorption in the reference country; as in the within-country case, we use shares that are averaged across the twenty years in our sample.

To construct the modified log productivity residual for each country, we subtract share weighted log capital and labor from log real absorption. The shares are the compensation of each input out of absorption in the reference country, also in this case kept constant at their average values. The stock of capital in the economy is constructed using the perpetual-inventory method on the PPP-converted investment time series from the Penn World Tables. Labour input is total hours worked, from EU-KLEMS.

For the comparative welfare calculations in the 63 country sample, absorption, capital, and the factor shares are constructed exactly as before. Since consistent data on hours of work are

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[26]The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009). We are unable to include Germany in the sample, since official data for unified Germany are available only since 1995 in EU-KLEMS.

not available for most countries, we use, as an imperfect proxy for total labor input, aggregate employment from the ILO's Key Indicators database. In this broader sample, we included all countries for which sufficient data were available to reconstruct TFP for at least 20 years (1985-2005).

For the empirical exercises including human capital, we construct per-capita human capital stocks as in Caselli (2005, p. 685-686), using the Barro and Lee (2010) data on average years of schooling of the population over 25 years of age. The source reports data at 5-year intervals; we use log-linear interpolation to obtain annual data.

Finally, to take into account of distortionary taxation, we use data on average tax rates on capital and labor provided by Boscá et al. (2005).

4.2   Within-Country Results

We construct country specific indexes of welfare change over time for our seven benchmark countries. Since the change in welfare over time depends on the expected present discounted value of TFP growth and its revision, as shown by equation (24), we need to construct forecasts of future TFP. To keep our empirical illustration simple and uniform across countries, we estimate univariate time-series models using annual data over 1985-2005. We leave for future work the extension to a multivariate forecasting framework.

For each country, we estimate simple AR processes for log TFP. We focus on three measures of aggregate TFP suggested by our theory. In all cases, our output concept is absorption rather than GDP, so our TFP indexes are appropriate for measuring welfare in open economies. The first two measures correspond to the case in which government purchases are wasteful, while the third assumes that government purchases are optimal. In the former case, we construct absorption by aggregating consumption and investment only, using shares that sum to $(1 - s_{c_G})$. In the latter, output is the sum (in logs) of consumption, investment and government purchases, aggregated with shares that sum to one. In the first case we assume that taxes are lump-sum, so we do not need to adjust the factor income shares. In the second and third cases, government spending is assumed to be financed with distortionary taxes, and the capital and labor shares employed in the calculation of TFP are corrected for both indirect and income taxes. Since taxes are distortionary in the real world, and it is likely that a major portion of public spending is valued by consumers, we take the latter scenario of distortionary taxes and optimal spending as our benchmark case, but retain in this section the other two scenarios for comparison purposes.

Estimation results (available in the working-paper version) show that, for all countries and TFP measures, the log level of TFP is well described by either an AR(1) or AR(2) stationary process around a linear trend. In all cases, the null of a unit root in the log TFP process can be rejected.

We construct our welfare indexes using the estimated AR processes to form expectations of future levels or differences of TFP. We use equation (24) to express the average welfare change per year in each country in terms of changes in equivalent consumption. Given the time-series processes for TFP in each country, we can readily construct the first two terms in equation (24), the present value of expected TFP growth and the change in expectations of that quantity. The third term, involving the change in the capital stock, is constructed from the data setting the

Table 1: Annual Average Log Change in Per-Capita Equivalent Consumption and its Components

	Wasteful Spending Lump-Sum Taxes			Wasteful Spending Distortionary Taxes			Optimal Spending Distortionary Taxes		
	Total	Fraction due to:		**Total**	Fraction due to:		**Total**	Fraction due to:	
		TFP	*Capital*		*TFP*	*Capital*		*TFP*	*Capital*
Canada	**0.013**	0.445	0.555	**0.021**	0.658	0.342	**0.023**	0.690	0.310
France	**0.026**	0.830	0.170	**0.026**	0.827	0.173	**0.031**	0.857	0.143
Italy	**0.018**	0.659	0.341	**0.021**	0.707	0.293	**0.023**	0.724	0.276
Japan	**0.018**	0.429	0.571	**0.023**	0.559	0.441	**0.030**	0.661	0.339
Spain	**0.021**	0.512	0.488	**0.030**	0.663	0.337	**0.040**	0.747	0.253
UK	**0.032**	0.816	0.184	**0.036**	0.833	0.167	**0.039**	0.848	0.152
USA	**0.025**	0.830	0.170	**0.029**	0.852	0.148	**0.030**	0.858	0.142

Note: Sample period: 1985-2005 (except Canada: 1985-2004).

composite discount rate, β, equal to 0.95 for all countries.[27]

The resulting average welfare growth rates for the period 1985-2005 are shown in Table 1. for several fiscal scenarios.[28] Consider first the case of the US, given in the last row. With wasteful government expenditure and lump-sum taxes, as assumed in the first column, the average annual growth rate of welfare in the US is equivalent to a permanent annual increase in consumption of about 2.5 percent. With distortionary taxes, it rises to 2.9 percent. Welfare growth rises because the after-tax shares of capital and labor sum to less than one, so tax-adjusted TFP growth is higher. Maintaining the assumption of distortionary taxes but with optimal government spending, US welfare growth is marginally higher, 3.0 versus 2.9 percent. This result does *not* mean that the US consumer is indifferent between wasteful and optimal government spending. The level of welfare is surely much lower when the government wastes 20 percent of GDP. However, our results imply that the difference in welfare between the two cases is almost entirely a *level* difference rather than a *growth rate* difference.

As in the US, welfare growth in most countries only shows modest variation across the scenarios. The biggest differences arise in countries with a high rate of growth of factor inputs and government purchases per capita, such as Spain and, to a lesser extent, Japan. For Spain the differences are large, with growth rates basically doubling from the first column to the third, which shows that assumptions about fiscal policy can matter significantly for welfare. With wasteful government spending, the UK leads our sample of countries in terms of welfare growth; with optimal spending, it is basically tied with Spain. In both cases, Italy and Canada lag behind the rest of countries.

Table 1 also shows the relative contribution of the two components of welfare–TFP growth and capital accumulation–to the growth rate of welfare in each country. For this decomposition, we treat the expectation-revision term as part of the contribution of TFP. In virtually all cases,

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[27]We construct our measure of β following the method of Cooley and Prescott (1995), who find $\beta = 0.947$. Note that for the cross-country welfare comparisons we need to calibrate only the β for the US, which we also set to 0.95.

[28]We omit the scenario of optimal spending and lump sum taxes for reasons of space. See our (2012) working paper for those results.

two-thirds or more of the welfare gains are attributable to TFP. Notice further that viewing public spending as optimal rather than wasteful has the effect of raising the growth rate of output, and thus TFP growth and its relative contribution to welfare.

4.3 Cross-Country Results

We now turn to measuring welfare differences across the countries in our sample. For each country and time period, we calculate the welfare gap between that country and the US, as defined in equation (26). Recall that this gap is the loss in welfare of a representative US household that is moved permanently to country i starting at time t, expressed as the log gap between equivalent consumption at home and abroad. In this hypothetical move, the household loses the per-capita capital stock of the US, but gains the equivalent capital stock of country i. From time t on, the household faces the same product and factor prices and tax rates, and receives the same lump-sum transfers and government expenditure benefits as all the other households in country i. In a slight abuse of language, below we refer to the incremental equivalent consumption as "the welfare difference" or "the welfare gap." Note that these gaps are all from the point of view of a US household. Hence, all the shares in (26), even those used to construct output and TFP growth in country i, are the *US* shares. This naturally raises the question whether our results would be quite different if we took a different country as our baseline. We return to this issue after presenting our basic set of results.

We present numerical results in Table 2 for our benchmark case of distortionary taxation and optimal government spending.[29] Since the magnitudes of the welfare gaps vary over time, we present results at the beginning and end of our sample, along with the average over the sample period. Moreover, we continue to break down the gaps into the fraction due to the TFP terms, and that due to the initial capital stock per worker. Finally, we plot in Panel A of Figure 1 the welfare gap for all the countries and time periods in our sample. Note that by definition the gap is zero for the US, since the US household neither gains nor loses by moving to the US at any point in time. The vertical axis shows, therefore, the gain to the US household of moving to any of the other countries at any point in the sample period, expressed in log points of equivalent consumption.

It is instructive to begin by focusing on the beginning and end of the sample. At the beginning of the sample, expected lifetime welfare in both France and the UK was about 20 percent lower than in the US (gaps of 16 and 19 percent, respectively). This relatively small gap reflects both the long-run European advantage in leisure and the fact that in the mid-1980s the US was still struggling with its productivity slowdown, while TFP in the leading European economies was growing faster than in the US. Capital accumulation was also proceeding briskly in those countries. By the end of the sample, the continental European economies, Canada and Japan are generally falling behind the US, because they had not matched the pickup in TFP growth and investment experienced in the US after 1995. Italy experiences the greatest relative "reversal of fortune," ending up with a welfare gap of nearly 50 percent relative to the US. The results for France are qualitatively similar, but less extreme. France starts with a welfare gap

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[29] For results using diferent assumptions about taxes and government spending see the working paper version of our paper.

Table 2: Welfare Gap Relative to the US and its Components; Optimal Government Spending and Distortionary Taxes

	PANEL A year=1985			PANEL B year=2005			PANEL C average 1985-2005		
	Total	Fraction due to:		**Total**	Fraction due to:		**Total**	Fraction due to:	
		TFP	*Capital*		*TFP*	*Capital*		*TFP*	*Capital*
Canada	**-0.295**	0.945	0.055	**-0.451**	0.910	0.090	**-0.373**	0.925	0.075
France	**-0.165**	0.987	0.013	**-0.213**	0.788	0.212	**-0.181**	0.879	0.121
Italy	**-0.437**	1.006	-0.006	**-0.664**	0.976	0.024	**-0.529**	0.999	0.001
Japan	**-0.471**	1.099	-0.099	**-0.526**	1.067	-0.067	**-0.468**	1.122	-0.122
Spain	**-0.414**	0.885	0.115	**-0.362**	0.888	0.112	**-0.375**	0.895	0.105
UK	**-0.189**	0.683	0.317	**-0.059**	-0.233	1.233	**-0.116**	0.443	0.557
USA	**0.000**	-	-	**0.000**	-	-	**0.000**	-	-

Note: estimates for Canada refer to 2004 and 1985-2004.

of 16 percent, and slowly slips further behind, ending with a gap of 21 percent. In continental Europe, only Spain shows convergence to the US in terms of welfare: it starts with a gap of 41 percent, and ends with a gap of 36 percent. However, after 1995 Spain holds steady relative to the US, but does not gain further.

The only economy in our sample that exhibits convergence to the US throughout the period of analysis is the UK. Although the welfare level of the UK is always below that of the US, the UK shows strong convergence, slicing two-thirds off the welfare gap in two decades. This result is interesting, because the UK experienced much the same lack of TFP growth in the late 1990s and early 2000s as the major continental European economies. However, the UK had very rapid productivity growth from 1985 to 1995. The other "Anglo-Saxon" country in our sample, Canada, had a welfare level about 30 percent below that of the US in 1985, but the welfare gap had grown by an additional 50 percent by the end of the sample. This result is due primarily to the differential productivity performance of the two countries: TFP in Canada actually fell during the 1990s, and rose only slowly in the early 2000s.

One of the most striking comparison is between the US and Japan. Even in 1985, when its economic performance was the envy of much of the world, Japan would have been the least attractive country in our sample for a US household contemplating emigration; such a household would give up nearly 50 percent of consumption permanently in order to stay in the US instead of moving to Japan. However, like the UK and Spain, Japan was closing the gap with the US until the start of its 'lost decade' in 1991. The relative performance of the three countries changes dramatically from that point: unlike the UK, which continues to catch up, and Spain, which holds steady, Japan begins to fall behind the US, first slowly and then more rapidly. Having closed to within 43 percent of the US welfare level in 1991, Japan ends our sample 53 percent behind. Even more striking is the continued divergence of Italy from the US since the early 1990s, reflecting Italy's dismal TFP performance.

In Table 2, we also investigate whether the cross-country welfare gaps are driven mostly by the TFP gap or by differences in capital per worker. We focus on the results averaged over the full sample period. We find that for five of the six countries, TFP is responsible for the vast majority of the welfare gap relative to the US. Indeed, for Japan TFP accounts for more
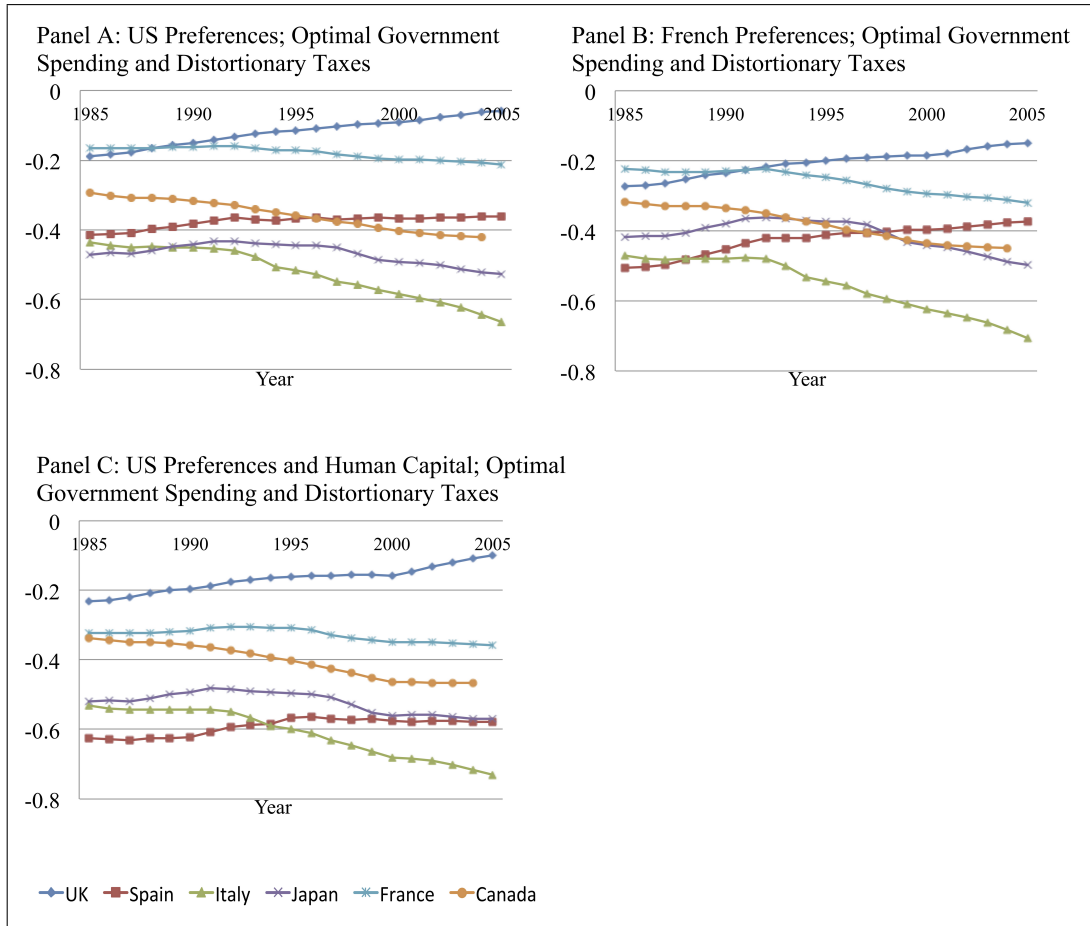
Figure 1: Cross-Country Welfare Comparisons (log Equivalent Consumption relative to the US)

than 100 percent of the gap (meaning that Japan has generally had a higher level of capital per person than the US). Thus we arrive at much the same conclusion as Hall and Jones (1999), although our definition of TFP is quite different from the one they used, and we do not focus only on steady-state differences. The exception to this finding is the UK. The average welfare gap between the US and the UK is driven about equally by TFP and by capital. Panel A shows that by the end of the sample, the UK had surpassed the US in "welfare-relevant TFP," and the difference in per-capita capital between the two economies accounted for more than 100 percent of the welfare gap.

We now check the robustness of these results along three dimensions. First, we check whether our welfare ranking is sensitive to the choice of the reference country. We redo the preceding exercises taking France as the baseline country. France is the largest and most successful continental European economy in our sample, and by revealed preference French households place much higher weight on leisure than do US ones. We summarize the results for our baseline case of optimal spending with distortionary taxes in Panel B of Figure 1. For ease of comparison with the preceding cross-country figures, we still normalize the US welfare level to zero throughout, even though the comparison is done from the perspective of the French household and is based on French shares. Reassuringly, the qualitative results are unchanged. France and the UK start closest to the US in 1985, but even they are well behind the US level of welfare. The UK converges towards the US welfare level and so, from a much lower starting

point, does Spain. All the other economies, including France, fall steadily farther behind the US over time. Interestingly, from the French point of view almost all the other countries are shifted down *vis-a-vis* the US relative to the ranking from the US point of view.

In our second robustness check, we bring human capital into the analysis. From the derivation in section 3.6, this may change our welfare results for two reasons. First, countries may differ in their initial human capital stocks. Second, the series for labor input is now adjusted for human capital. Notice that the first of these two factors comes into play only to the extent that the hypothetical move of the household from the reference country (the US) to country i entails losing her initial human capital stock and acquiring the human capital capital stock of country i. In principle, there is no compelling reason why the thought experiment should be framed in this way, rather than allowing the US household to retain the US human capital stock when moving to country i. However, for comparability with the development accounting literature, which assigns a prominent role to cross-country differences in human capital stocks, we opt for assuming that the US household does not retain its human capital when moving to country i.

The results from repeating the previous exercises bringing human capital into our framework are shown in Panel C in Figure 1, for our baseline case of optimal spending with distortionary taxes. Comparison with Panel B reveals that the welfare gap with respect to the US is now wider for all countries. However, the magnitude of the change varies across countries. It is especially large for Spain and France, and more modest for the other countries. Further comparison with Panel B also shows that the slopes of the various lines are fairly similar across the two figures, so the widening of welfare gaps is roughly the same with or without human capital. At the beginning of the sample period, Spain now shows the largest welfare gap relative to the US, and the UK the smallest one (when ignoring human capital, Japan and France respectively assumed those roles). At the end of the sample period, Spain is behind all countries except Italy, while the relative ranking of the remaining countries is the same as in the case without human capital.

In Table 3 we compare our welfare results to those based on traditional measures, namely PPP-adjusted GDP and consumption per capita. Focusing on Panel A, for the final year of our sample, we see that the three measures sometimes give identical results. For example, the US is atop the world rankings by all three measures, although the gap between the US and the second-ranked country is much smaller in percentage terms for welfare (6 percent) than it is for the other two variables (18 or 19 percent). On the other hand, the differences can be striking. For example, Canada, which leads Spain by 20 percent or more in terms of consumption and GDP per capita, is overtaken by Spain and France in our welfare comparison. Indeed, Spain is last within our group of countries in terms of the conventional metrics of consumption and GDP, but ranks fourth in welfare terms, trailing only the US, UK and France. For the other countries, the welfare measure is not so kind. Japan trails the US by only 26 percent in GDP per capita, but double that—52 percent—in terms of welfare. Similarly, Italy has more than 60 percent of the per-capita GDP of the US, but only about one-third the welfare level. On the other hand, France trails the US by 40 percent in consumption per-capita, but by only half that amount in terms of welfare. Thus, our measure clearly provides new information on welfare differences among countries.

Table 3: Per-Capita GDP, Consumption and Equivalent Consumption relative to the US

	Consumption	GDP	Equiv. Cons. Opt Gov, Dist Tax	Equiv. Cons. Opt Gov, Dist Tax (US dynamics)	Equiv. Cons. Opt Gov, Dist Tax (No Leisure)
PANEL A: 2005					
Canada	-0.324	-0.177	-0.451	-0.353	-0.455
France	-0.401	-0.317	-0.213	-0.226	-0.240
Italy	-0.501	-0.370	-0.664	-0.553	-0.719
Japan	-0.456	-0.261	-0.526	-0.487	-0.531
Spain	-0.527	-0.419	-0.362	-0.423	-0.376
UK	-0.190	-0.219	-0.059	-0.203	-0.061
USA	0.000	0.000	0.000	0.000	0.000
PANEL B: average 1985-2005					
Canada	-0.248	-0.163	-0.373	-0.259	-0.377
France	-0.338	-0.265	-0.181	-0.166	-0.203
Italy	-0.396	-0.278	-0.529	-0.304	-0.579
Japan	-0.391	-0.176	-0.468	-0.403	-0.460
Spain	-0.537	-0.447	-0.375	-0.405	-0.397
UK	-0.220	-0.251	-0.116	-0.121	-0.120
USA	0.000	0.000	0.000	0.000	0.000

Note: estimates for Canada refer to 2004 and 1985-2004.

Relative to the traditional indexes of country performance, two new factors we highlight are dynamics (the expectation of future productivity change) and the treatment of leisure. To show their importance for cross-country differences in welfare, we have recalculated our basic index assuming, first, that future levels of productivity in all countries are expected to grow at the US rate, and second by assuming that leisure does not enter the utility function. The results appear in the last two columns of Table 3. When we assume that future productivity in each country is expected to grow at the US rate, the significant changes occur for Italy, Canada, and the UK: the performance of Canada and Italy, whose productivity growth trails that of the US, improves, while the performance of the UK, with faster productivity growth than the US, worsens. When, instead, differences in leisure are not taken into account in the calculation of the index, Italy, France and Spain, with much lower average hours of work per capita than the US, worsen even more in terms of welfare relative to the US.

4.4 A Broader Cross-Country Sample

The empirical exercises so far are limited to a small set of large advanced countries, determined by the availability of the requisite data. However, it may be interesting to assess welfare gaps across countries in a broader sample, including both advanced and developing countries. Thus, as a final empirical exercise, we extend our cross-country results to a large set of countries for the year 2005. As noted, however, consistent information on hours worked is almost completely unavailable for countries outside the OECD. Hence, as already mentioned, we measure

the aggregate labor input using total employment rather than total hours. The immediate consequence is that cross-country differences in work hours per person are ignored in the calculation of cross-country differences in the productivity residual, and thus also in the calculation of differences in welfare.[30]

As with the smaller country sample, we calculated the welfare gaps (always from the perspective of the US household) under different assumptions regarding public spending and taxation, and both including human capital and excluding it.[31] In each case, we estimated country-specific autoregressive models and used them to project the future path of the relevant version of the modified productivity residual. For the sake of space, we only report the results of our baseline specification – optimal government expenditure and distortionary taxes – with the addition of human capital. Except for the different measurement of the labor input, the exercise is therefore the same as that reported in Panel C of Figure 1.

Results appear in Table 4. The first column reports the log differences in per capita welfare relative to the US for the year 2005. Only Luxembourg ranks ahead of the US, and all the industrial countries in the sample rank above the median, with Italy bringing up the rear in 30th place. Among the six advanced countries in our earlier exercises, relative ranks are the same as those shown in Figure 1-C, with the only exception being that Spain and Japan trade places, although their respective welfare gaps are numerically similar (as in Figure 1-C).

In turn, most developing countries exhibit fairly large welfare gaps relative to the US. Translating the log-differences shown in the table to percentage terms, we find that a US household moving to South Korea would suffer a welfare loss equivalent to 40 percent of her permanent consumption. Moving instead to Mexico or China would raise the loss above 85 percent of consumption. In addition, inspection of 2005 per capita GDP data (not shown in the table) reveals that, of the 61 sample countries that trail the US in terms of welfare, the vast majority (57 of them) lag further behind in terms of welfare than in terms of per-capita GDP. For these countries, the median difference between the two gaps (in percentage terms) equals 10 percent. In addition, for the six advanced non-US countries in our earlier exercises, the 2005 welfare gaps in Table 8 are in all cases larger than those shown in Figure 1-D. We conjecture that ignoring cross-country differences in hours of work – as we are effectively doing in the enlarged country sample – may lead, for most countries, to an overestimation of the present value of their labor input relative to the US, and thereby to an overestimation of their respective welfare lag *vis-a-vis* the US.[32]

As before, we may ask how these welfare-based country comparisons would relate to those obtained on the basis of per capita consumption or GDP. The answer is that the resulting country ranking would show visible differences – for example, Norway had higher PPP GDP per capita than the US in 2005, but, according to the results in Table 4, lower welfare. In contrast,

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[30] Recall that, aside from absorption and labor and capital aggregates for the countries involved, numerical comparisons of welfare across countries only require information on factor shares and tax rates for the reference country (which continues to be the US for this exercise).

[31] The fact that many of the countries in the broader sample are quite far from the US steady state could raise concerns about the accuracy of the first-order approximation underlying the calculations. However, Appendix B suggests that the approximation should remain quite reliable even in this case.

[32] For the countries with data, the correlation between total employment and total hours of work (both in logs) is 0.53, which confirms that the former is a fairly noisy proxy for the latter.

Table 4: Welfare Gap Relative to the US and its Components in 2005. Benchmark Case (Optimal Government Spending and Distortionary Taxes) with Human Capital

	Welfare Gap	Fraction due to TFP	Capital		Welfare Gap	Fraction due to: TFP	Capital
Luxembourg	0.507	1.035	-0.035				
USA	0.000	—	—	Iran	-1.835	0.701	0.299
Australia	-0.150	0.830	0.170	Mexico	-1.965	0.777	0.223
Singapore	-0.162	0.195	0.805	China	-1.970	0.694	0.306
Norway	-0.191	1.317	-0.317	Jamaica	-2.048	0.823	0.177
UK	-0.226	-0.013	1.013	Malaysia	-2.185	0.837	0.163
Sweden	-0.450	0.837	0.163	Costa Rica	-2.238	0.791	0.209
Belgium	-0.468	0.851	0.149	Thailand	-2.258	0.747	0.253
Finland	-0.476	0.735	0.265	Panama	-2.470	0.809	0.191
Netherlands	-0.496	0.822	0.178	Tunisia	-2.484	0.750	0.250
South Korea	-0.509	0.829	0.171	Argentina	-2.578	0.842	0.158
Cyprus	-0.518	0.443	0.557	Guatemala	-2.706	0.680	0.320
Denmark	-0.529	0.790	0.210	Egypt	-2.728	0.696	0.304
Austria	-0.538	0.773	0.227	Venezuela	-2.744	0.780	0.220
France	-0.542	0.704	0.296	Sri Lanka	-2.746	0.770	0.230
Canada	-0.565	0.855	0.145	Brazil	-2.920	0.804	0.196
Hong Kong	-0.589	0.823	0.177	India	-2.978	0.673	0.327
Spain	-0.590	0.730	0.270	Indonesia	-3.071	0.748	0.252
Japan	-0.614	0.964	0.036	Peru	-3.152	0.834	0.166
Israel	-0.617	0.835	0.165	El Salvador	-3.335	0.798	0.202
Switzerland	-0.648	0.926	0.074	Syria	-3.452	0.744	0.256
Portugal	-0.701	0.558	0.442	Ecuador	-3.480	0.844	0.156
New Zealand	-0.728	0.870	0.130	Colombia	-3.683	0.833	0.167
Ireland	-0.779	0.887	0.113	Honduras	-3.764	0.799	0.201
Romania	-0.936	0.644	0.356	Philippines	-3.841	0.822	0.178
Italy	-1.068	0.846	0.154	Pakistan	-4.034	0.760	0.240
Trin. & Tob.	-1.142	0.754	0.246	Bolivia	-4.267	0.834	0.166
Uruguay	-1.516	0.672	0.328	Lesotho	-4.336	0.795	0.205
Chile	-1.604	0.759	0.241	Morocco	-4.417	0.807	0.193
Barbados	-1.630	0.890	0.110	Uganda	-4.426	0.727	0.273
Turkey	-1.662	0.622	0.378	Togo	-4.565	0.760	0.240
Botswana	-1.767	0.791	0.209	Paraguay	-5.418	0.874	0.126

Luxembourg ranks ahead of the US in terms of welfare, but not in terms of consumption per capita (at PPP prices) in 2005. On the whole, however, there is broad agreement among the three measures. Indeed, the correlation coefficients between the log differences in welfare shown in Table 8, and the log differences in per capita PPP GDP or consumption (both *vis-a-vis* the US) observed in 2005, equal 0.93 and 0.90, respectively. One factor behind this high correlation is probably the lack of data on hours worked for this larger sample, which forces us to ignore the variation in average hours per employee across countries.[33]

Lastly, we examine the extent to which the 2005 cross-country welfare gaps shown in the first column of Table 8 are driven by differences in TFP and by differences in initial capital (both physical and human) per worker. The relevant decomposition is shown in the second and third columns of Table 8. On the whole, TFP accounts for the bulk of the welfare differences. In 56 out of the 61 countries that trail the US in terms of welfare, TFP accounts for over two-thirds of the welfare gap. Across countries, its median contribution equals 79 percent. An extreme case is Norway, whose welfare gap is entirely due to TFP, with part of the gap offset by an initial capital stock above that of the US. At the other extreme, the UK is the only country whose welfare gap relative to the US is fully due to its lower initial capital stock.

5 Relationship to the Literature

Measuring welfare change over time and differences across countries using observable national income accounts data has been a long-standing challenge for economists. We note here the similarities and differences between our approach and others found in previous literature. We also suggest ways in which our results might be useful in other fields of economics, where the same question arises in different contexts.

Nordhaus and Tobin (1972) originated one approach, which is to take a snapshot of the economy's flow output at a point in time and then go "beyond GDP," by adjusting GDP in various ways to make it a better flow measure of welfare. Nordhaus and Tobin found that the largest gap between flow output and flow welfare comes from the value that consumers put on leisure. Their result motivated us to add leisure to the utility function in our model, which is standard in business-cycle analysis but not in growth theory. Nordhaus and Tobin's approach has been followed and extended recently by Jones and Klenow (2010) who add other corrections, notably for life expectancy and inequality. Their basic approach is to assume that a consumer with US preferences is assigned the consumption and leisure of another country. The US household is not allowed to optimize given the new conditions (prices, endowments, etc.) found in the other country. By contrast, we allow the household of a reference country to re-optimize dynamically when faced with the paths of exogenous variables of another country, and then compare the difference between the optimized values of utility at home and abroad. Our thought experiment gives the "irreducible" difference in welfare between the two countries, while the approach of assigning choices that are sub-optimal for the assumed preferences and new conditions will produce an upward-biased measure of welfare gaps, unless preferences are

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[33] For the sample of seven advanced countries, where we have both hours and employment data, the correlation of GDP per capita with equivalent consumption, correctly computed using total hours, is 0.59. If equivalent consumption is computed instead using employment data, the correlation rises to 0.77.

identical across countries.

Our approach echoes the methods used in the literature started by Weitzman (1976) and analyzed in depth by Weitzman (2003), with notable contributions from many other authors.[34] This literature also relates the welfare of an infinitely lived representative agent to observables; for example, Weitzman (1976) linked intertemporal welfare to net domestic product. The results in these papers are derived using a number of strong restrictions on the nature of technology, product market competition, and the nature of exogenous shocks to the economy. Most of the analysis in the literature applies to a closed economy where growth is optimal. By contrast, we derive all our results based only on first-order conditions from household optimization, which allows for imperfect competition in product markets of an arbitrary type and for a vast range of production possibilities and shocks, and for the economy to be closed or open.

Our findings shed light on a variety of issues that bedevil the measurement of productivity and allocative efficiency. For example, Baker and Rosnick (2007), reasoning that the ultimate object of growth is consumption, make the reasonable conjecture that one should deflate nominal productivity gains by a consumption price index to create a measure they call "usable productivity." We begin from the assumption that consumption (and leisure) at different dates are the only inputs to economic well-being, but nevertheless show that output should be calculated in the conventional way, rather than being deflated by consumer prices.

Our work clarifies and unifies several results in other literatures, especially international economics. Kohli (2004) shows in a static setting that terms-of-trade changes can improve welfare in open economies even when technology is constant. Kehoe and Ruhl (2008) prove a related result in a dynamic model with balanced trade: opening to trade may increase welfare, even if it does not change TFP. In these models, which assume competition and constant returns, technology is equivalent to TFP. We generalize and extend these results, and show that in a dynamic environment with unbalanced trade welfare can also change with the quantity of net foreign assets and their rates of return.[35] In general, we show that there is a link between observable aggregate data and welfare in an open economy, which is the objective of Bajona, Gibson, Kehoe and Ruhl (2010).

While we agree with the conclusion of these authors that GDP is not a sufficient statistic for uncovering the effect of trade policy on welfare, we show that one can construct such a sufficient statistic by considering a relatively small number of other variables. Our results also shed light on the work of Arkolakis, Costinot and Rodriguez-Clare (2012). These authors show that in a class of modern trade models, which includes models with imperfect competition and micro-level productivity heterogeneity, one can construct measures of the welfare gain from trade without reference to micro data.[36] Our results imply that this conclusion actually holds in a much larger class of models, although the exact functional form of the result in Arkolakis et al. (2012) may not. Finally, since changes in net foreign asset positions and their rates of

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[34] A far from exhaustive list includes Asheim (1994), Arronson and Löfgren (1995), Mino (2004), Sefton and Weale (2006), Basu, Pascali, Schiantarelli and Serven (2009), and Hulten and Schreyer (2010). Reis (2005) analyses the related problem of computing a dynamic measure of inflation for a long-lived representative consumer.

[35] The result that openness does not change TFP may be fragile in models with increasing returns. If opening to trade changes factor inputs, either on impact or over time, then TFP as measured by Solow's residual will change as well, which we show has an effect on welfare even holding constant the terms of trade.

[36] Atkeson and Burstein (2010) come to similar conclusions in a related model.

return are extremely hard to measure, we show that one can measure welfare using data only on TFP and the capital stock, even in an open economy, provided that TFP is calculated using absorption rather than GDP as the output concept.

Our work provides a different view of a large and burgeoning literature that investigates the reasons for output differences across countries. If we specialize our cross-country result to the lump sum taxes-optimal spending case, we obtain something closely related to the results produced by the "development accounting" literature. We show that in that case, (the present value of) the log differences in TFP levels emphasized by the developing accounting literature need to be supplemented with only one additional variable, namely log level gaps in capital per person (also a focus of that literature), in order to serve as a measure of welfare differences among countries.

A number of recent papers suggest that countries can increase output and TFP substantially by allocating resources more efficiently across firms. Our work implies that the literature is correct to focus on the connection between reallocation and aggregate TFP. An increase in aggregate TFP due to reallocation is as much of a welfare gain for the representative consumer as a change in technology with the same magnitude and persistence. This result implies immediately that estimates of TFP losses due to allocative inefficiency (e.g., Hsieh and Klenow, 2009) can be translated to estimates of welfare losses.

6 Conclusions

We show that aggregate TFP, appropriately defined, and the capital stock can be used to construct sufficient statistics for the welfare of a representative consumer. To a first order approximation, welfare is measured by the expected present value of aggregate TFP and by the initial capital stock. This result holds regardless of the type of production technology and the degree of product market competition, and applies to closed or open economies with or without distortionary taxation. Crucially, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable. Finally, in an open economy, the change in welfare will also reflect present and future changes in the returns on net foreign assets and in the terms of trade. However, these latter terms disappear if absorption rather than GDP is used as the output concept for constructing TFP, and TFP and the initial capital stock are again sufficient statistics for measuring welfare in open economies. Most importantly, these variables also suffice to measure welfare level differences across countries, with both variables computed as log level deviations from a reference country. Notably, our approach allows us to perform these welfare comparisons without having to assume identical preferences across countries and without having to make assumptions about the value of preference parameters.

We extend the existing literature on intertemporal welfare measurement by deriving all our results from household first-order conditions alone. The generality of our derivation allows us to propose a new interpretation of TFP that sheds new light on several distinct areas of study. For instance, we show that measures of cross-country TFP differences akin to those produced by the "development accounting" literature are crucial for calculating welfare differences among

countries. In general, our results imply that all changes in the time path of the Solow residual, whatever their source (for example, technology, increasing returns, or reallocation) are equally important for welfare. While our approach allows us to make historical comparisons, counter-factual policy experiments would require a full general equilibrium model. Thus, our method allows researchers to measure actual welfare using standard data under very general assumptions, while DSGE models can answer "what if" questions, at the cost of making many more assumptions. We view the two approaches as complementary.

We illustrate our results by using national accounts data to measure welfare growth rates and gaps across countries. Our evidence suggests that expectations about future productivity and the presence of leisure in the utility function are important determinants of welfare rankings. Importantly, in the vast majority of cases, the bulk of the welfare gap relative to the US, our welfare leader among large countries, is due to the productivity gap, rather than the gap in the physical or human capital stock.

Our analysis has been confined to the case of a representative agent, which automatically rules out distributional issues and, more generally, household heterogeneity. Our theoretical framework can be extended relatively easily to the case of heterogeneous households, since mathematically the case of heterogeneous households within a country is no different from the case that we have already analyzed, of heterogeneous countries within the world. The main barrier to pursing the study of heterogeneity is data availability. For instance, it would require data on the productivity residual at a sufficient number of points of the income distribution and Markov transition matrices for income mobility between those points. While the challenges are great, the payoff is compelling, and thus this major extension is the subject of our ongoing research. We view the present paper as the first step along this road.

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A   Derivations

A.1   Proposition 1

Assume that in the steady state aggregate per capita variables grow at a constant rate g. Thus, they are proportional to some variable $X_t = X_0(1 + g)^t$. Rewrite the utility function, the budget constraint and the capital accumulation, equations (1), (2) and (3), in normalized form by dividing by X_t :

$$v_t = \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \overline{L} - L_{t+s}) \tag{A.1}$$

$$k_t + b_t = \frac{(1 - \delta) + p_t^K}{(1 + g)(1 + n)} k_{t-1} + \frac{(1 + r_t)}{(1 + g)(1 + n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \tag{A.2}$$

$$k_t = \frac{(1 - \delta)}{(1 + g)(1 + n)} k_{t-1} + i_t \tag{A.3}$$

where: $v_t = \frac{V_t}{X_t^{(1-\sigma)}}$, $c_t = \frac{C_t}{X_t}$ $k_t = \frac{K_t}{X_t}$, $b_t = \frac{B_t}{P_t^I X_t}$, $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_t^C = \frac{P_t^C}{P_t^I}$, $(1 + r_t) = \left(1 + i_t^B\right) \frac{P_{t-1}^I}{P_t^I}$, $\pi_t = \frac{\Pi_t}{P_t^I X_t}$ and $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$.

The first order conditions for this problem are:

$$U_{c_t} - \lambda_t p_t^C = 0 \tag{A.4}$$

$$U_{L_t} + \lambda_t p_t^L = 0 \tag{A.5}$$

$$-\lambda_t + \beta E_t \frac{(1 - \delta) + p_{t+1}^K}{(1 + g)(1 + n)} \lambda_{t+1} = 0 \tag{A.6}$$

$$-\lambda_t + \beta \frac{1}{(1 + g)(1 + n)} E_t (1 + r_{t+1}) \lambda_{t+1} = 0 \tag{A.7}$$

where λ_t is the Lagrange multiplier associated with the budget constraint. Define with $\widehat{x} = \frac{x_t - x}{x}$ the percent deviation from the steady-state of a variable (x is the steady-state value of x_t). Taking a first order approximation of the Lagrangean (which equals the value function along the optimal path), one obtains:

$$v_t - v =$$

$$E_t[\sum_{s=0}^{\infty} \beta^s (U_c c \widehat{c}_{t+s} + U_L L \widehat{L}_{t+s}$$

$$+ \lambda p^L L \widehat{L}_{t+s} - \lambda p^C c \widehat{c}_{t+s} - \lambda k \widehat{k}_{t+s} - \lambda b \widehat{b}_{t+s})$$

$$+ \sum_{s=0}^{\infty} \beta^{s+1} (\lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s})$$

$$+ \sum_{s=0}^{\infty} \beta^s \widehat{\lambda}_{t+s} \, \lambda (-k - b + \frac{(1-\delta) + p^K}{(1+g)(1+n)} k + \frac{(1+r)}{(1+g)(1+n)} b$$

$$+ p^L L + \pi - p^C c)]$$

$$+ \sum_{s=0}^{\infty} \beta^s \lambda (p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \, \widehat{p}_{t+s}^K - p^C c \widehat{p}_{t+s}^C + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s})$$

$$+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} \tag{A.8}$$

Using the first-order conditions, the first five lines equal zero:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \, \widehat{p}_{t+s}^K - p^C c \widehat{p}_{t+s}^C + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \right]$$

$$+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} \tag{A.9}$$

Linearize the budget constraint and the law of motion for capital:

$$k \, \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^L L \widehat{p}_t^L - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_t^K$$

$$- \frac{rb}{(1+g)(1+n)} \widehat{r}_t - \pi \widehat{\pi}_t + p^C c \widehat{c}_t + p^C c \widehat{p}_t = 0 \tag{A.10}$$

$$k \widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t-1} + i \widehat{i}_t \tag{A.11}$$

Using these two equations and the steady-state version of the FOC for capital in (A.9) gives us:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1}$$

$$+ \lambda \sum_{s=0}^{\infty} \beta^s \left[b \widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s} \right] \tag{A.12}$$

Using the FOC and the transversality condition for bonds, the last line in the equation above equals zero. Hence we get:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \right]$$

$$+ \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \tag{A.13}$$

Take the difference between the expected level of intertemporal utility v_t as in (A.13) and v_{t-1} and use the fact that $\frac{x_t - x}{x} \simeq \log x_t - \log x$ for positive x_t. After adding and subtracting $E_t x_{t+s}$ for each variable x_{t+s}, we obtain:

$$\Delta v_t = E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \Delta \log c_{t+s} + i \Delta \log i_t - p^L L \Delta \log L_{t+s} - \frac{p^K k}{(1+g)(1+n)} \Delta \log \ k_{t+s-1}]$$

$$+ \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \left(E_t \log c_{t+s-1} - E_{t-1} \log c_{t+s-1} \right) + i \left(E_t \log i_{t+s-1} - E_{t-1} \log i_{t+s-1} \right)$$

$$- p^L L \left(E_t \log L_{t+s-1} - E_{t-1} \log L_{t+s-1} \right) - \frac{p^K k}{(1+g)(1+n)} \left(E_t \log k_{t+s-2} - E_{t-1} \log k_{t+s-2} \right)]$$

$$+ \lambda \frac{1}{\beta} k \Delta \log k_{t-1} \tag{A.14}$$

Define domestic absorption growth (in normalized form) as:

$$\Delta \log y_t = s_C \Delta \log c_t + s_I \Delta \log i_t \tag{A.15}$$

Inserting (A.15) into (A.14) and dividing both terms by $\lambda p^Y y$ one obtains:

$$\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1}] \tag{A.16}$$

$$+ \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1})$$

$$- s_L \left(E_t \log L_{t+s-1} - E_{t-1} \log L_{t+s-1} \right) - s_K \left(E_t \log k_{t+s-2} - E_{t-1} \log k_{t+s-2} \right)]$$

$$+ \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log k_{t-1}$$

The definition of equivalent consumption in terms of normalized variables is:

$$v_t = \frac{1}{(1-\sigma)(1-\beta)} c_t^{*1-\sigma} \nu(\overline{L} - L) \tag{A.17}$$

Expanding the right hand side of (A.17) in terms of $\log c_t^*$ and using the FOC for consumption, it follows that:

$$\frac{v}{\lambda p^Y y} \frac{\Delta v_t}{v} = \frac{s_C}{1-\beta} \Delta \log c_t^* \tag{A.18}$$

Using the result above in equation (A.16) one obtains, to a first-order approximation:

$$\frac{s_C}{1-\beta}\Delta\log c_t^* \ = \ E_t\sum_{s=0}^{\infty}\beta^s[\Delta\log y_{t+s} - s_L\Delta\log L_{t+s} - s_K\Delta\log\ k_{t+s-1}] \tag{A.19}$$

$$+\sum_{s=0}^{\infty}\beta^s\lambda[(E_t\log y_{t+s-1} - E_{t-1}\log y_{t+s-1})$$

$$-s_L\left(E_t\log L_{t+s-1} - E_{t-1}\log L_{t+s-1}\right) - s_K\left(E_t\log k_{t+s-2} - E_{t-1}\log k_{t+s-2}\right)]$$

$$+\frac{1}{\beta}\frac{k}{p^Y y}\Delta\log k_{t-1}$$

Using the fact that $\Delta\log y_t = \Delta\log Y_t - g,\ \Delta\log k_t = \Delta\log K_t - g, \Delta\log c_t^* = \Delta\log C_t^* - g,$ equation (A.19) can be rewritten as:

$$\frac{s_C}{1-\beta}\Delta\log C_t^* \ = \ E_t\sum_{s=0}^{\infty}\beta^s\Delta\log PR_{t+s}$$

$$+\sum_{s=0}^{\infty}\beta^s\left[E_t\log PR_{t+s-1} - E_{t-1}\log PR_{t+s-1}\right]$$

$$+\frac{1}{\beta}\frac{k}{p^Y y}\Delta\log K_{t-1} \tag{A.20}$$

$$+\frac{s_C}{1-\beta}g - \frac{1}{(1-\beta)}g(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y}g$$

where $\log PR_{t+s}$ is defined in (6). Using equations (A.6) and (A.3) evaluated at the steady-state, one can easily show that $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y} = 0$, so that the last line in equation (A.20) equals zero. Multiplying both sides of (A.20) by $\frac{1-\beta}{s_C}$ yields equation (7) in Proposition 1 in the main text.

If we were to use real GDP rather than absorption as a measure of output, then the welfare-relevant residual can be written as the sum of a conventionally-defined productivity residual plus additional components that capture terms of trade and capital gains effects. Moreover, the initial conditions then should include the initial value of the net foreign asset stock. To show this, assume that the domestic economy buys imports $IM_{t+s}N_{t+s}$ at a price P_{t+s}^{IM} and sells domestic goods abroad $EX_{t+s}N_{t+s}$ at a price P_{t+s}^X. The current account balance can be written:

$$B_t N_t - B_{t-1}N_{t-1} = i_t B_{t-1}N_{t-1} + P_t^{EX}EX_t N_t - P_t^{IM}IM_t N_t \tag{A.21}$$

which can be re-written in a normalized form as:

$$b_t = \frac{(1+r_t)}{(1+g)\,(1+n)}b_{t-1} + p_t^{EX}ex_t - p_t^{IM}im_t \tag{A.22}$$

Linearizing the equation above, we get:

$$b\widehat{b}_t = \frac{(1+r)}{(1+g)\,(1+n)}b\widehat{b}_{t-1} + \frac{rb}{(1+g)\,(1+n)}\widehat{r}_t + p^{EX}ex\ \widehat{ex}_t + p^{EX}ex\ \widehat{p^{EX}}_t - p^{IM}im\ \widehat{im}_t - p^{IM}im\ \widehat{p^{IM}}_t$$
$$\tag{A.23}$$

Using equations (A.10), (A.11) and (A.23), and the steady-state version of the FOC for capital and bonds in (A.9) gives us:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} + p^{EX} ex \, \widehat{ex}_t - p^{IM} im \, \widehat{im}_t - \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} - p^L L \widehat{L}_{t+s} \\
&\quad + p^{EX} ex \, \widehat{p^{EX}}_t - p^{IM} im \, \widehat{p^{IM}}_t] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} + \lambda \frac{1}{\beta} b \widehat{b}_{t-1}
\end{aligned}
\tag{A.24}
$$

Define GDP growth (in normalized form) as:

$$
\Delta \log gdp_t = s'_C \Delta \log c_t + s'_I \Delta \log i_t + s'_X \Delta \log ex_t - s'_M \Delta \log im_t
\tag{A.25}
$$

where s'_C, s'_I, s'_X and s'_M are respectively the steady-state shares of consumption, investment, exports and imports out of GDP. Using the definition above and some algebra, equation (A.24) can be re-written as:

$$
\begin{aligned}
\Delta \log C_t^* &= \frac{(1-\beta)}{s_c} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PRTT_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PRTT_{t+s-1} \right. \\
&\quad \left. + \frac{1}{\beta} \left(\frac{P^I K}{P^Y Y} \right) \Delta \log K_{t-1} + \frac{1}{\beta} \left(\frac{P^I}{P^Y Y} \right) \Delta \frac{B_{t-1}}{P_{t-1}^I} \right]
\end{aligned}
\tag{A.26}
$$

where $\Delta \log PRTT_{t+s}$ is defined as in equation (12).

A.2 Proposition 2

Consider the maximization problem of a hypothetical household, which has the preferences of a household in country j and faces the prices and per-capita endowments of a household living in country i. It maximizes (with variables defined in normalized form):

$$
\widetilde{v}_t^i = E_t \sum_{s=0}^{\infty} \beta^s \frac{\widetilde{c}_{t+s}^i \, {}^{1-\sigma} \nu(\overline{L} - \widetilde{L}_{t+s}^i)}{1 - \sigma}
\tag{A.27}
$$

subject to:

$$
\widetilde{k}_{t+s}^i + \widetilde{b}_{t+s}^i = \frac{(1-\delta) + p_{t+s}^{iK}}{(1+g)(1+n)} \widetilde{k}_{t+s-1}^i + \frac{(1+r_t^i)}{(1+g)(1+n)} \widetilde{b}_{t+s-1}^i + p_{t+s}^{iL} \widetilde{L}_{t+s}^i + \pi_{t+s}^i - p_{t+s}^{iC} \widetilde{c}_{t+s}^i
\tag{A.28}
$$

where: $\widetilde{v}_t^i = \frac{\widetilde{V}_t^i}{X_t^i}$, $\widetilde{c}_{t+s}^i = \frac{\widetilde{C}_{t+s}^i}{X_{t+s}^i}$, $\widetilde{k}_t^i = \frac{K_t^i}{X_t^i}$, $\widetilde{b}_t^i = \frac{B_t^i}{P_t^{iI} X_t^i}$, $p_t^{iK} = \frac{P_t^{iK}}{P_t^{iI}}$, $p_t^{iL} = \frac{P_t^{iL}}{P_t^{iI} X_t^i}$, $p_t^{iC} = \frac{P_t^{iC}}{P_t^{iI}}$, $(1+r_t) = \left(1 + i_t^{B,i}\right) \frac{P_{t-1}^{iI}}{P_t^{iI}}$ and $\pi_t^i = \frac{\Pi_t^i}{P_t^{iI} X_t^i}$. A tilde denotes the (unobservable) quantities that the household from country j would choose when facing prices and initial conditions of country i. To simplify the notation in this proof, let the variables without the superscript i denote utility, preference parameters, quantities and prices in country j. Linearizing around country j's steady state and using the envelope theorem, one obtains:

$$\widetilde{v}_t^i = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^L L \widehat{p}_{t+s}^{iL} + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} + \pi \widehat{\pi}_{t+s}^i - p^C c \widehat{p}_{t+s}^{iC} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}^i]$$

$$+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1}^i + \lambda \frac{(1+r)}{(1+g)(1+n)} \widehat{b}_{t-1}^i \tag{A.29}$$

Linearize the budget constraint and the law of motion for capital:

$$k \, \widehat{\widetilde{k}}_{t+s}^i + b \widehat{\widetilde{b}}_{t+s}^i - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{\widetilde{k}}_{t+s-1}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{\widetilde{b}}_{t+s-1}^i - p^L L \widehat{\widetilde{L}}_{t+s}^i - p^L L \widehat{p}_{t+s}^{iL}$$

$$- \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}^i - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} - \pi \widehat{\pi}_{t+s}^i + p^C c \widehat{\widetilde{c}}_{t+s}^i + p^C c \widehat{p}_{t+s}^{iC} = 0 \tag{A.30}$$

$$k \widehat{\widetilde{k}}_{t+s}^i = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{\widetilde{k}}_{t+s-1}^i + i \widehat{\widetilde{i}}_{t+s}^i \tag{A.31}$$

Using the two equations above in equation (A.29), together with the fact that at $t-1$, $\widehat{k}_{t-1}^i = \widehat{\widetilde{k}}_{t-1}^i$ and $\widehat{b}_{t-1}^i = \widehat{\widetilde{b}}_{t-1}^i$, we obtain:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{\widetilde{c}}_{t+s}^i + i \widehat{\widetilde{i}}_{t+s}^i - \frac{p^K}{(1+g)(1+n)} k \, \widehat{\widetilde{k}}_{t+s-1}^i - p^L L \, \widehat{\widetilde{L}}_{t+s}^i \right]$$

$$+ \lambda \sum_{s=0}^{\infty} \beta^s \left[b \widehat{\widetilde{b}}_{t+s}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{\widetilde{b}}_{t+s-1}^i \right]$$

$$+ \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1}^i + \lambda \frac{(1+r)}{(1+g)(1+n)} \widehat{b}_{t-1}^i \tag{A.32}$$

Now consider the budget constraint of a household in country i and the law of motion for capital in country i, linearized around country j's steady state.

$$k \, \widehat{k}_{t+s}^i + b \widehat{b}_{t+s}^i - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s-1}^i - p^L L \widehat{L}_{t+s}^i - p^L L \widehat{p}_{t+s}^{iL}$$

$$- \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}^i - \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^{iK} - \pi \widehat{\pi}_{t+s}^i + p^C c \widehat{c}_{t+s}^i + p^C c \widehat{p}_{t+s}^{iC} = 0 \tag{A.33}$$

$$k \widehat{k}_{t+s}^i = \frac{(1-\delta)}{(1+g)(1+n)} k \widehat{k}_{t+s-1}^i + i \widehat{i}_{t+s}^i \tag{A.34}$$

where $k_t^i = \frac{K_t^i}{X_t}$, $b_t^i = \frac{B_t^i}{P_t^I X_t}$, $\pi_t^i = \frac{\Pi_t^i}{P_t^I X_t}$ and $p^{iL} = \frac{P_t^{iL}}{P_t^I X_t}$. Using the two budget constraints in equations (A.30) and (A.33) and the two laws of motion for capital in equations (A.31) and (A.34), we obtain:

$$p^C c \widehat{c}_{t+s}^i + i \widehat{i}_{t+s}^i - p^K k \widehat{k}_{t+s-1}^i - p^L L \widehat{L}_{t+s}^i + b \widehat{b}_{t+s}^i - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s-1}^i =$$

$$= p^C c \widehat{\widetilde{c}}^i_{t+s} + i \widehat{\widetilde{i}}^i_{t+s} - p^K k \widehat{\widetilde{k}}^i_{t+s-1} - p^L L \ \widehat{\widetilde{L}}^i_{t+s} + b \widehat{\widetilde{b}}^i_{t+s} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{\widetilde{b}}^i_{t+s} \qquad (A.35)$$

which implies that equation (A.32) can be re-written as:

$$
\begin{aligned}
v_t \ = \ & v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}^i_{t+s} + i \widehat{i}^i_{t+s} - \frac{p^K}{(1+g)(1+n)} k \widehat{k}^i_{t+s-1} - p^L L \ \widehat{L}^i_{t+s} \right] \\
& + \lambda \sum_{s=0}^{\infty} \beta^s \left[b \widehat{b}^i_{t+s} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}^i_{t+s-1} \right] \\
& + \lambda \frac{(1-\delta)+p^K}{(1+g)(1+n)} k \widehat{k}^i_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} \widehat{b}^i_{t-1} \qquad (A.36)
\end{aligned}
$$

or:

$$
\begin{aligned}
v_t \ = \ & v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}^i_{t+s} + i \widehat{i}^i_{t+s} - \frac{p^K}{(1+g)(1+n)} k \widehat{k}^i_{t+s-1} - p^L L \ \widehat{L}^i_{t+s} \right] \\
& + \lambda \sum_{s=0}^{\infty} \beta^s \left[b \widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s} \right] \\
& + \lambda \frac{(1-\delta)+p^K}{(1+g)(1+n)} k \widehat{k}^i_{t-1} \qquad (A.37)
\end{aligned}
$$

Using the FOC and the transversality condition for bonds, the second line in the equation above equals zero. After some algebra, we get:

$$\frac{v}{\lambda p^y y} \frac{\widetilde{v}^i_t - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_C \widehat{c}^i_{t+s} + s_I \widehat{i}^i_{t+s} - s_L \ \widehat{L}^i_{t+s} - s_K \ \widehat{k}^i_{t+s-1} \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}^i_{t-1} \qquad (A.38)$$

Similarly, for the household from country j living in country j:

$$\frac{v}{\lambda p^y y} \frac{v_t - v}{v} = E_t \sum_{s=0}^{\infty} \beta^s \left[s_C \widehat{c}_{t+s} + s_I \widehat{i}_{t+s} - s_L \ \widehat{L}_{t+s} - s_K \ \widehat{k}_{t+s-1} \right] + \frac{1}{\beta} \frac{k}{p^y y} \widehat{k}_{t-1} \qquad (A.39)$$

Using the fact that $\frac{\widetilde{x}^i_t - x}{x} \simeq \log \widetilde{x}^i_t - \log x$ for a generic non-negative variable x, and subtracting equation (A.39) from equation (A.38), we obtain:

$$
\begin{aligned}
\frac{v}{\lambda p^y y} \frac{\widetilde{v}^i_t - v_t}{v} \ = \ & E_t \sum_{s=0}^{\infty} \beta^s [s_C (\log c^i_{t+s} - \log c_{t+s}) + s_I (\log i^i_{t+s} - \log i_{t+s}) \\
& - s_L \ (\log L^i_{t+s} - \ln L_{t+s}) - s_K (\log k^i_{t+s-1} - \log k_{t+s-1})] \\
& + \frac{1}{\beta} \frac{k}{p^Y y} (\log k^i_{t-1} - \log k_{t-1}) \qquad (A.40)
\end{aligned}
$$

Using equation (A.18) and the fact that $\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y} = 0$, we obtain, after some algebra, equation (14) in Proposition 2 in the main text, with the productivity terms defined by (15) and (16).

A.3 Extensions

A.3.1 Government Expenditure

Assume that utility depends upon private consumption and government spending on public consumption, as in equation (20). Assume government expenditure is financed through a lump-sum tax. Re-writing the household maximization problem in normalized variables and proceeding in a similar fashion as in the proof of proposition 1, we obtain:

$$
\begin{aligned}
v_t \;=\;& v + \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{U_{c_G} c_G \widehat{c}_{G,t+s}}{\lambda} + p^C c\widehat{c}_{t+s} + \widehat{ii}_{t+s} - p^L L \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] \\
& + \lambda \frac{1}{\beta} k \widehat{k}_{t-1}
\end{aligned}
\tag{A.41}
$$

where $c_{G,t} = \frac{C_{G,t}}{X_t}$. The log-change in per-capita domestic absorption, in normalized variables is defined as:

$$
\Delta \log y_t = s_C \Delta \log c_{t+s} + s_{c_G} \Delta \log c_{G,t} + s_I \Delta \log i_t
\tag{A.42}
$$

where s_{c_G} is the steady state value of $s_{c_G,t} = \frac{P_t^G C_{G,t}}{P_t^Y Y_t}$ and P^G is the public consumption deflator.

Using this result, after some algebra equation (A.41) can be rewritten as:

$$
\begin{aligned}
\frac{v}{\lambda p^Y y}\frac{\Delta v_t}{v} \;=\;& E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_{t+s} - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1} + \left(s_{c_G}^* - s_{c_G} \right) \Delta \log c_{G,t}] \\
& + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s-1} - E_{t-1} \log y_{t+s-1}) \\
& + \frac{1}{\beta}\frac{k}{p^Y y} \Delta \log k_{t-1}
\end{aligned}
\tag{A.43}
$$

where $s_{c_G}^*$ is the steady state value of $s_{c_G,t}^* = \frac{U_{c_G,t} c_{G,t}}{\lambda_t}$. From this point, the algebra is very similar to the benchmark case, and yields (21) and (22) in the main text.

A.3.2 Summing up

Using the extensions developed in A.3.1-A.3.2 and the others described in the text (distortionary taxes and multiple investment and consumption goods) we can state Proposition 1'. A parallel argument can be used to derive the generalization of Proposition 2, Proposition 2'.

A.4 Human capital

As in Lucas (1988), assume that non-leisure time can be used either to work or to accumulate human capital. The representative household maximizes intertemporal utility:

$$W_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \overline{L} - L_{t+s} - E_{t+s}) \tag{A.44}$$

where E_t denotes the amount of time devoted to human capital accumulation, under the following budget constraint:

$$
\begin{aligned}
P_t^I K_t N_t + B_t N_t &= (1-\delta) P_t^I K_{t-1} N_{t-1} + \left(1 + i_t^B\right) B_{t-1} N_{t-1} + P_t^L L_t H_{t-1} N_t + P_t^K K_{t-1} N_{t-1} \\
&\quad + \Pi_t N_t - P_t^C C_t N_t
\end{aligned}
\tag{A.45}
$$

where labor income now depends on the initial level of human capital H_{t-1}. The human capital accumulation equation is assumed to be as in (29):

$$(H_t - H_{t-1}) + \delta_H H_{t-1} = F(E_t) H_{t-1} \tag{A.46}$$

Linearizing the maximization problem as before, we get:

$$
\begin{aligned}
v_t - v &= E_t \sum_{s=0}^{\infty} \beta^s \lambda \left(\frac{p^L L h}{(1+g)} \widehat{p}_{t+s}^L + \frac{p^K k}{(1+g)(1+n)} \widehat{p}_{t+s}^K - p_t^C c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s} \right) \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \left[U_L \frac{1+g}{F'(E)h} + \frac{\lambda p^L L}{(1+g)} \right] h \widehat{h}_{t-1}
\end{aligned}
\tag{A.47}
$$

where $h_t = \frac{H_t}{X_t}$ and $p_t^L = \frac{P_t^L}{P_t^I}$. The FOCs for human capital and labor, in normalized terms, are:

$$
\begin{aligned}
0 &= -U_{L_t} \frac{1+g}{F'(E)h_{t-1}} \\
&\quad + \beta E_t \left[U_{L_{t+1}} \frac{(1+g)h_{t+1}}{F'(E)h_t^2} + \frac{L_{t+1} p_{t+1}^L}{(1+g)} \lambda_{t+1} \right]
\end{aligned}
\tag{A.48}
$$

and

$$0 = -U_{L_t} + \frac{\lambda_t p_t^L h_{t-1}}{(1+g)} \tag{A.49}$$

while the FOCs for consumption, physical capital and bonds are still defined by equations (A.4), (A.6) and (A.7).

Using the FOCs for physical capital and human capital evaluated in the steady state in (A.47) and the log-linearized version of the budget constraint, we obtain:

$$v_t - v = \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\left(p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} \right) - \left(\frac{p^L L h}{(1+g)} \left(\widehat{L}_{t+s} + \widehat{h}_{t+s-1} \right) + \frac{p^K}{(1+g)(1+n)} k \widehat{k}_{t+s-1} \right) \right]$$

$$+ \frac{\lambda}{\beta} k \widehat{k}_{t-1} + \left(\frac{1}{1-\beta} \right) \frac{\lambda p^L L h}{(1+g)} h \widehat{h}_{t-1} \tag{A.50}$$

The rest of the proof parallels subsection A.1 and yields equations (30) and (31) in the text.

B The quality of our approximation: some examples

A potential concern with our main results, as stated in Proposition 1' and 2', is that they are proved using first-order approximations. This approach may seem especially problematic for cross-country comparisons, where gaps in living standards are often large. We now use simple general-equilibrium models to investigate the quantitative error introduced by our use of approximations. We consider a set of workhorse models that are standard in the macroeconomic literature, solve them, and then compare the calculated welfare values to our approximated measures.[37]

Assume that there is a fixed number of identical household with an infinite time horizon. The representative household chooses consumption, leisure and investments in capital and bonds to maximize the following intertemporal utility function:

$$W_t = E_t \sum_{s=0}^{\infty} \beta^s C_{t+s}^{1-\sigma} (\overline{L} - L_{t+s})^{\frac{1-\gamma}{\gamma}(1-\sigma)}$$

subject to the following budget constraint:

$$I_t + B_t = \left(1 + i_t^B\right) B_{t-1} + P_t^L (1 - \tau_t) L_t + P_t^K (1 - \tau_t) K_{t-1} - \Pi_t - C_t$$

In this model public expenditure is pure waste and can be financed through an income tax (with tax-rate τ_t) or a lump-sum tax (Π_t). Thus, total public expenditure is:

$$C_{G,t} = P_t^L \tau_t L_t + P_t^K \tau_t K_{t-1} + \Pi_t$$

The law of motion for capital is:

$$K_t = (1 - \delta) K_{t-1} + I_t$$

There is a large number of firms which operate under perfect competition and are characterized by the same Cobb-Douglas function:

$$Y_t = E_t A_t L_t^{1-\alpha} K_{t-1}^{\alpha}$$

where A_t is the Harrod-neutral technology parameter while E_t measures an externality that arises from aggregate production:

$$E_t = Y_t^{1-\frac{1}{e}}$$

In equilibrium, the national income account equation holds:

$$Y_t = C_t + C_{G,t} + I_t$$

---

[37]We compare welfare across countries assuming that they are at their steady states. These calculations are exact solutions of the non-linear models. For the comparisons over time in within-country, dynamic settings, we solve the models using third-order approximations and compare our results based on first-order approximations to the third-order solutions. To check the third-order approximations, we also solved the models using fourth-order approximations and solved the simpler models using global methods. In both cases the results were barely distinguishable numerically from the third-order solutions, so we think these are a good baseline for the purposes of checking the first-order approximations.

There are three potential shocks in this economy to technology A_t, to the lump-sum tax, Π_t, and to the income tax rate, τ_t. The laws of motion of these variables are the following:

$$\log A_t = \rho_1 \log A_t + \varepsilon_{1t}$$

$$\Pi_t = (1 - \rho_2)\Pi^{SS} + \rho_2 \Pi_{t-1} + \varepsilon_{2t}$$

$$\tau_t = (1 - \rho_3)\tau^{SS} + \rho_3 \tau_{t-1} + \varepsilon_{3t}$$

The following table reports the calibration for the benchmark model:

β	0.987
σ	1.1
$\frac{1-\gamma}{\gamma}$	1.78
δ	0.012
α	0.4
e	2
$\rho_1 = \rho_2 = \rho_3$	0.95
Π^{SS}	0.16
τ^{SS}	0.29
$sd(\varepsilon_1) = sd(\varepsilon_2) = sd(\varepsilon_3)$	0.0075

We have considered four special cases of this model:

1. Ramsey model ($\gamma = 1; e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$)

2. RBC model ($e = 1; \Pi^{SS} = 0; \tau^{SS} = 0$)

3. RBC model with public expenditure financed by distortionary taxes ($e = 1; \Pi^{SS} = 0$)

4. RBC model with public expenditure financed by lump-sum taxes and externalities from production ($\tau^{SS} = 0$)

First, we discuss the quality of the approximation in a within-country analysis. Figure (B.1) reports the impulse response function of our measure of approximated welfare and compares it to welfare measures based on third-order approximations, in four standard macro models subject to different types of shocks. Panel (a) reports the impulse response of both equivalent consumption and our approximated measure of it following a one-standard deviation technology shock in a standard Ramsey growth model. The two lines are practically indistinguishable. On impact, equivalent consumption increases by 19.35% while its approximated value increases by 19.32%.[38] In the following periods, the approximated value converges monotonically to the exact one. The non-linearity of the utility function does not have a large effect on the quality of the approximation. In panel (b), we perform the same experiment as in panel (a) but using an extremely concave utility function: we raise the coefficient of relative risk aversion from a

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[38] Here and in the rest of the paper, we use percent (%) change to refer to differences in natural logs multiplied by 100.

common business-cycle value of 1.1 to 10. Following the technology shock, on impact equivalent consumption increases by 19.19% while its approximated value again increases by 19.32%.[39] As we would expect, the approximation error is larger when the utility function is more concave, but the magnitude of the difference is still quite small and converges to zero quickly.

To ensure that the quality of the approximation is not a peculiarity of the Ramsey model, in the following panels, we perform the same exercise in different theoretical frameworks. Panel (c) considers a technological shock in a Real Business Cycle (RBC) model with standard calibration. Panel (d) considers a tax shock in a RBC model with distortionary income taxes and wasteful public expenditure. Panel (e) considers a public expenditure shock in a RBC model with lump-sum taxes, wasteful public expenditure and production externalities. Note that in these last two cases, our welfare-relevant TFP differs from exogenous technology—in the first case due to taxes, and in the second case because of the externality. In all three cases, however, the first-order approximation gives results that are close to the calculated value for welfare.

We next evaluate the approximation in a cross-country setting and compare steady-state welfare differences between countries. Comparing steady states allows us to solve for the exact values of welfare in the two countries, and compare the gap to our approximated result. In our theoretical results, which we take to the data later in the paper, we allow for both steady-state and transitory welfare gaps between countries. To evaluate the approximation error for this type of comparison, one can take the approximation error for transitory shocks which we just discussed and add it to the approximation error for the steady-state differences.

We use a standard RBC model with distortionary income taxes to analyze welfare gaps between countries. Both countries are assumed to be in their respective steady state. We compute the change in equivalent consumption of a representative agent in a reference country who moves permanently to a different country characterized by different exogenous technology parameters or tax rates. We then compute the approximated change in equivalent consumption and see how it compares to the exact value. We conduct three different experiments, with the results in the three panels of Figure (B.2). First, in panel (a), we consider an increase in the capital elasticity parameter from 0.28 to 0.39 (thus moving from British to Canadian capital shares, the two extremes in our sample): it produces a steady-state increase in equivalent consumption of 73.77%, while the result from our approximation is 72.78%. Second, in panel (b), we consider an increase in the income tax rate from 30% to 40% (thus moving from the average US tax rate over 1985-2005 to the French average over the same period of time): it produces a reduction in equivalent consumption of 16.58 percent while the approximated change is 13.85 percent. Finally, in panel (c), we consider differences in technology. The figure illustrates the exact and the approximated change in equivalent consumption when the level of technology drop to a fraction x of its original level. Moving to a country with a level of productivity that is 50% (10%) that of the reference country implies a reduction in equivalent consumption of 69.31 (230.26%), while the approximated value is 65.90% (218.90%). It is interesting to note that the approximation error is largest for differences in tax rates. However, all the changes

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[39]It is not a coincidence that the approximated first-period welfare change is the same in the two models. Since both models are neoclassical, the time path of TFP is just the exogenous shocks to technology, which is the same in the two cases. Moreover, since we shock both models starting at their steady states, the period $t-1$ change in the capital stock is also the same in the two models—zero.

we consider are large ones. Relative to the large size of the welfare gaps we are considering, we believe the approximation errors are modest and quite acceptable.

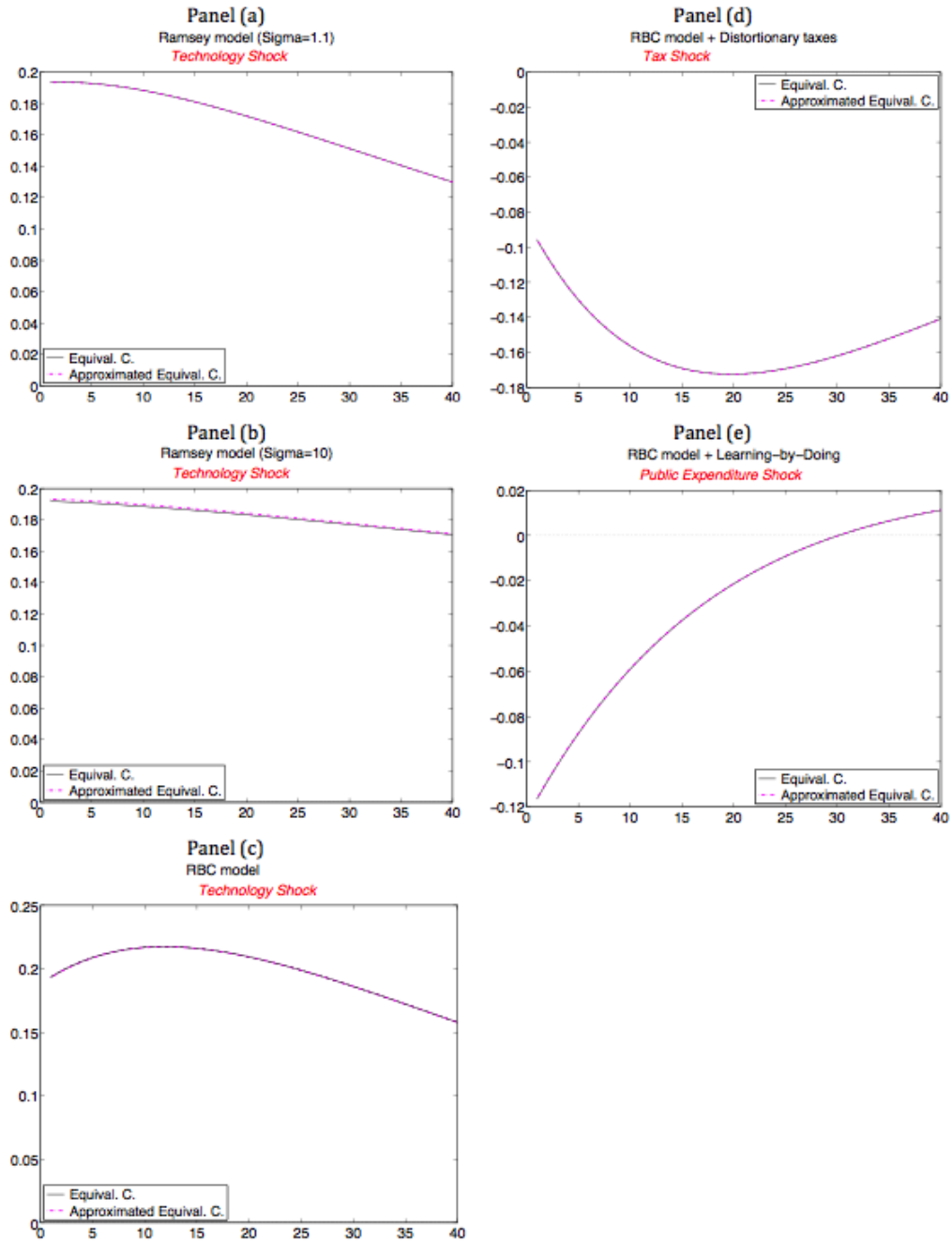Figure B.1: Numerical Experiments: Impulse Responses for log Equivalent Consumption

Figure B.2: Numerical Experiments: Cross-Country Gaps for log Equivalent Consumption

Panel (a)

Panel (c)

Panel (b)