# Demand Estimation Under Incomplete Product Availability*

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**Abstract**

Incomplete product availability is an important feature of many markets; ignoring changes in availability may bias demand estimates. We study a new dataset from a wireless inventory system installed on 54 vending machines to track product availability every four hours. The data allow us to account for product availability when estimating demand, and provides a valuable source of variation for identifying substitution patterns. We develop a procedure that allows for changes in product availability even when availability is only observed periodically. We find significant differences in demand estimates, with the corrected model predicting significantly larger impacts of stock-outs on profitability.

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# 1 Introduction

Incomplete product availability is a common and important feature of markets where products are perishable, seasonal, or have storage costs. For example, retail markets, sporting and concert events, and airlines face important capacity constraints that often lead to stock outs. Not surprisingly, firms in such industries identify inventory management as a critical strategic decision, and consumers cite product availability as a major concern.[1] In these settings, the failure to account for product availability not only ignores a useful source of variation for identifying demand parameters, but can also lead to biased estimates of demand. The first source of bias is the *censoring* of demand estimates. If a product sells out, the actual demand for a product (at given prices) may be greater than the observed sales, leading to a negative bias in demand estimates. At the same time, during periods of reduced availability of other products, sales of available products may increase. This *forced substitution* overstates demand for these goods conditional on the full choice set being available. As a result, failing to account for product availability leads to biased estimates of demand substitution patterns, typically making products look more substitutable than they really are. This bias can potentially undermine the reliability of many important applications of demand estimates for markets with incomplete product availability, such as simulating the welfare implications of mergers or new product introductions, or applying antitrust policy. Identifying unbiased demand estimates in these markets is also a critical step in evaluating optimal capacity choices of firms.

In this paper, we provide evidence that failing to appropriately account for periods of product unavailability can result in a substantial bias in demand estimates, and we develop a method for correcting this bias. To accomplish this, we collected a new and extensive dataset with detailed inventory and sales information. The dataset covers one of the first technological investments for wirelessly managing inventory: a wireless network installed on a set of 54 vending machines, providing updates on elapsed sales and inventory status every four hours. The data from the vending network provide extremely granular information on sales and inventory levels over a period of a year. Using this dataset, we develop and implement estimation methods to provide corrected estimates even when some choice sets are latent, and analyze the impact of stockouts for firm profitability in the short run. We find evidence of important biases on demand parameters and predicted sales in models that do not account correctly for stock-out events in this market. For example, under some specifications of the uncorrected model, we estimate demand parameters that are not consistent with utility maximization. In terms of the short-run impacts of stock-outs on profitability, the corrected model estimates that the negative profit impacts of stock-outs are 8 - 12 percent larger than the uncorrected model predicts.

Although not estimated here, the model we develop is also necessary for any examination of supply-side decisions over the long run. For example, estimation of optimal capacity

[1] Relatedly, firms throughout the economy are currently making large investments in technologies for tracking inventory and capacity information. For example, Walmart now requires many suppliers to use Radio Frequency Identification (RFID) technology, and many other firms have recently adopted related technology, such as wireless communication and networked data centers.

choices, restocking decisions or inventory policies in markets where stock-outs matter relies on a static demand model that accounts correctly for stock-out events as our model does.[2] Demand estimates that correctly account for product availability are also important for understanding the macroeconomic implications of inventories. Indeed, firms' abilities to manage inventories have been proposed as an agent for dampening recessions, a factor affecting vertical relationships, and a strategic variable affecting price competition.[3]

The wireless vending network provides actual stock-out events, which randomly change the set of products available at some locations for a period of time. This variation provides an attractive source of identification for estimating demand models, because although the probability of a stock-out event can be targeted with different inventory choices, the occurrence of any particular event is not chosen by the firm. Thus, stock outs generate exogenous "short-run" variation in choice sets, in addition to the long-run variation that is more typically the source of identification for structural models of demand. Many other data sources have the potential to generate similar sources of variation in availability, which makes the exercise here potentially quite broadly applicable. Any market for which the data contain repeated draws of demand and we observe capacity generates similar variation in choice sets. This variation comes from periods in which demand exceeds capacity constraints, and we investigate and example of this phenomenon with respect to vending data.

When discussing inventory systems we use the standard language established by Hadley and Whitman (1963). The first of two types of inventory systems is called a 'perpetual' data system. In this system, product availability is known and recorded when each purchase is made. Thus for every purchase, the retailer knows exactly how many units of each product are available.[4] The other type of inventory system is known as a 'periodic' inventory system. In this system, inventory is measured only at the beginning of each period. After the initial measurement, sales take place, but inventory is not measured again until the next period. Periodic inventory systems are problematic in analyses of stock-outs because inventory (and thus the consumer's choice set) is not recorded with each transaction. While perpetual inventory systems are becoming more common in retailing environments, most retailers still do not have access to such systems. Sampling inventory more frequently helps to mitigate limitations of the periodic inventory system. However, an additional goal of this paper is to provide consistent estimates of demand not only for perpetual inventory systems but for periodic ones as well.

In fact, despite the extremely detailed information from this dataset, we observe stock-

[2]Such supply-side problems require an additional focus on firm costs and dynamic inventory decisions, and we analyze such a model in a companion paper with Uli Doraszelski (in progress).

[3]McCarthy and Zakrajsek (forthcoming) reviews the literature on the effect of inventory management technology on business cycles, and provides empirical evidence on the theory. Narayanan and Raman (2004) examines the assignment of stocking rights in vertical settings theoretically, and Mortimer (2008) provides empirical evidence on the effect of inventory monitoring technology for vertical contracting in the video rental industry. Balachander and Farquhar (1994), Carlton (1978), Dana (2001), and Deneckere and Peck (1995), among others, address the impact of product availability on price or service competition.

[4]Note that if sales are recorded in the order they happen, this would be sufficient to construct an almost perpetual inventory system (assuming consumers do not hold goods for long before purchasing an item). This system is also known as 'real-time' inventory.

out events only periodically (every four hours). Some stock-out events occur in the middle of an observed four-hour time period, meaning that for these observations, the choice set of an individual consumer is latent. Discarding data from these periods would select on sales levels and lead to biased estimates. Thus, we develop a method for incorporating these observations that uses the well-known EM algorithm from the statistics literature (Dempster, Laird, and Rubin 1977) to estimate the allocation of sales across the unobserved choice set regimes.

As technologies like the one we study continue to become more prevalent, firms and researchers can expect to gain access to better data (i.e., more detailed information on sales and inventory/capacities) with which to analyze markets. Note that new technologies are not always necessary to observe such information; capacity data provide the same functionality for many markets, such as airlines, performance or sporting events, hospitals, etc. When such data are available, researchers gain valuable information on short-run choice set variation. Our results in this paper indicate that accounting for that choice set variation can substantially reduce potential biases in standard estimates for some markets, and that researchers should take on the responsibility to adjust for the effects of product availability in demand estimation when possible.

*Relationship to Literature*

The demand estimation literature in IO has primarily focused on cases in which all products are assumed to be available to all consumers ((Berry, Levinsohn, and Pakes 1995, Nevo 2001, Berry, Levinsohn, and Pakes 2004)). Dynamic models of demand ((Nevo and Hendel 2007)) have modeled consumer inventories or stockpiling, but do not incorporate data on availability at the time of purchase. Data on capacities/inventories have been used primarily in this literature in order to analyze supply-side behavior–particularly with respect to promotions ((Aguirregabiria 1999)).[5]

The marketing literature examines problems that arise when data are aggregated over the choices of consumers facing different choice sets, which is another way of viewing the problem of product availability. This literature points out some of the same biases we examine here. For example, Gupta, Chintagunta, Kaul, and Wittink (1995) review biases in city-level demand that arise when different stores carry different products and have different promotional conditions. In another approach, Bruno and Vilcassim (2008) have data that are aggregated across choice sets and assume additional structure on the marginal distributions of availability across retail outlets within a market to probability weight to different choice sets.

Stock-outs are frequently analyzed in the context of optimal inventory policies in operations research. In fact, an empirical analysis of stock-out based substitution has been addressed using vending data before by Anupindi, Dada, and Gupta (1998) (henceforth ADG). ADG use an eight-product soft-drink machine and observe the inventory at the beginning of each day. The authors assume that products are sold at a constant Poisson distributed rate (cans per hour). The sales rates of the products are treated as independent from one

[5]While we explicitly account for retailer inventories in our model, dynamic supply-side behavior does not arise because the retailer does not have the ability to dynamically alter the price or product mix.

another, and eight Poisson parameters are estimated. When a stock-out occurs, a new set of parameters is estimated with the restriction that the new set of parameters are at least as great as the original parameters. This means that each choice set requires its own set of parameters (and observed sales). If a Poisson rate was not fitted for a particular choice set, then only bounds can be inferred from the model. However, because of the lack of a utility-based framework for demand, the ADG method cannot be used to make out-of-sample predictions about alternative policies or their welfare impacts. More recently, Musalem, Olivares, Bradlow, Terwiesch, and Corsten (2008) study the problem of stock-outs by imputing the entire sequence of sales in a Markov chain Monte Carlo approach. The MLE approach that we use is more efficient and faster to implement, particularly for markets in which the outside good has a relatively large share and the total number of products stocked-out during period of latent availability is relatively small.

The economics literature has also studied Bayesian models of discrete choice consumer demand, for example, Athey and Imbens (2007). While our paper uses a common Bayesian technique to address missing data, it is not a fully Bayesian model. Relatedly, Fox (2007) examines semiparametric estimation of multinomial discrete choice models using a subset of choices, and proposes a maximum score estimator. His focus includes settings with very large choice sets, for which estimation is computationally burdensome, as well as choice subsets that arise due to data limitations or consideration sets. However, this methodology does not allow for computation of market shares, which limits the ability to analyze the welfare effects of stockouts.

The paper proceeds as follows. Section 2 provides the model of demand that adjusts for changes in product availability in the data under both perpetual and periodic inventory systems. In section 3 we provide estimation details and discuss identification of the model. Section 4 describes the data from the wireless vending route and provides correlations and regression results from the data. Section 5 reports results from estimating the model using the vending data, section 6 provides counterfactual experiments on the effect of stockouts on firm profitability, and section 7 concludes.

## 2 Model

Consider the probability that consumer $i$ chooses product $j$. This choice probability may result from a random utility maximization (RUM) problem such as in McFadden (1974), or Berry (1994), or from a simpler form such as a Poisson model as is common in the Operations Research literature. The choice probability is assumed to be a function of observable characteristics of the choice scenario, $x_i$, the set of available products, $a_i$, and parameters of the choice model, $\theta$. For example:

$$p_{ij} = p_j(x_i, a_i, \theta). \tag{1}$$

In the case of discrete-choice demand, sales are distributed multinomially. With individual data, where $y_{ij} = 1$ if consumer $i$ buys product $j$ and 0 otherwise, the likelihood of observed sales is:

$$L(\theta) = \prod_{\forall i,j} p_{ij}(x_i, a_i, \theta)^{y_{ij}} = \prod_{\forall_{k,j}} p_{k,j}(x_k, a_k, \theta)^{\sum_{i:(x_i,a_i)=(x_k,a_k)} y_{ij}} \tag{2}$$

As equation 3 shows, one need not observe the choices of each consumer $i$ in order to calculate the likelihood. The aggregate sales under each $(a, x)$ tuple (or $y_{kj} = \sum_{i:(x_i,a_i)=(x_k,a_k)} y_{ij}$) are the minimal sufficient statistics for the likelihood. Thus, aggregate data may be used in place of individual data without loss of generality.

Now consider aggregate data in the case where total sales of each product are observed for a given market or period $t$, and denote sales of product $j$ in market $t$ as $y_{jt}$. The likelihood is well defined for aggregate data when all consumers in a period face the same set of available products $a_t$, and the same observable market characteristics $x_t$.[6] Specifically:

$$\begin{aligned} L(\mathbf{y_t}|\theta, a_t, x_t) &= \left( \frac{(\sum_{j=0}^{J} y_{jt})!}{y_{0t}! y_{1t}! y_{2t}! \dots y_{Jt}!} \right) p_{0t}(a_t, x_t, \theta)^{y_{0t}} p_{1t}(a_t, x_t, \theta)^{y_{1t}} \dots p_{Jt}(a_t, x_t, \theta)^{y_{Jt}} \\ &= C(\mathbf{y_t}) p_{0t}(a_t, x_t, \theta)^{y_{0t}} p_{1t}(a_t, x_t, \theta)^{y_{1t}} \dots p_{Jt}(a_t, x_t, \theta)^{y_{Jt}} \\ &\propto p_{0t}(a_t, x_t, \theta)^{y_{0t}} p_{1t}(a_t, x_t, \theta)^{y_{1t}} \dots p_{Jt}(a_t, x_t, \theta)^{y_{Jt}}. \end{aligned} \tag{3}$$

Aggregate data imply that the order of sales is unobserved; therefore the likelihood contains a multinomial coefficient $C(\mathbf{y_t})$, which depends only on the data and not the parameters. Thus, we do not need to consider the order of individual sales so long as we observe the sufficient statistics (i.e., sales for each product under each $(a, x)$ tuple).

In order to characterize how variation in the set of available products $a_i$ may be used to estimate $\theta$, it is helpful to consider three types of markets. In the first type, $a_i$ is fixed over all consumers. This is the standard assumption maintained in much of the differentiated products literature. In type 2 markets, $a_i$ varies across consumers, but is fully observed. In the third type of market, $a_i$ varies across consumers, but is only partially observed. In the case of stock-outs, a type 3 market is one in which we observe that a product (or products) has stocked out, but we do not observe which sales took place before or after the stockout occurred. When this happens, the demand model implies a distribution for the stockout time, and the uncertainty over choice sets can be integrated out. Perpetual collection of inventory data generates only type 1 and type 2 markets. Periodic collection of inventory data generates all three types of markets.

[6]This need not imply that all consumers within a market are identical, but rather that they are ex-ante identical (e.g., because choice-relevant characteristics of consumers are unobserved). One could specify $p_{jt} = p_j(x_t, a_t, \theta) = \int p_j(x_i, a_t, \theta) f(x_i|x_t)$, in which uncertainty regarding the individual type is integrated out. Random coefficients are an example of this type of specification.

## 2.1 Fully Observed Availability / Perpetual Inventory

In the first type of market, the contribution to the likelihood for each value of $x_t$ can be written as:

$$L(y_t|x_t,\theta) = \prod_j \prod_i p_j(a_i, x_t, \theta)^{y_{ij}} = \prod_j p_j(a, x_t, \theta)^{\sum_i y_{ij}} = \prod_j p_j(a, x_t, \theta)^{y_{jt}}. \tag{4}$$

When there is no variation in the set of available products, identification of the choice probabilities comes from changes in other observable characteristics (e.g., changes in prices or product characteristics over time).[7]

In type 2 markets, availability is observed for all sales (the case of perpetual inventory or sequential sales data with constant capacity). We relax the assumption that $a_t$ (the set of available products) is constant during period $t$. However, availability $a_i$ corresponding to each sale $y_{ij}$ is observed. In this case, the contribution to the likelihood corresponding to each $x_t$ can be divided into two smaller periods of constant availability, denoted $(a_t, a_{t'})$ and we recompute the sufficient statistics as follows:

$$\begin{aligned} L(y_t|x_t,\theta) \propto \prod_{i,j} p_j(a_i, x_i, \theta)^{y_{ij}} &= \prod_{j \in a_t} p_j(a_t, x_t, \theta)^{\sum_{\forall i:(a_i,x_i)=(a_t,x_t)} y_{ji}} \cdot \prod_{j \in a_{t'}} p_j(a_{t'}, x_t, \theta)^{\sum_{\forall i:(a_i,x_i)=(a_{t'},x_t)} y_{ji}} \\ &= \prod_{j \in a_t} p_j(a_t, x_t, \theta)^{y_{jt}} \cdot \prod_{j \in a_{t'}} p_j(a_{t'}, x_t, \theta)^{y_{jt'}}. \end{aligned} \tag{5}$$

The ability to observe exogenous variation in choice sets generated by stock-out events means that type 2 markets provide multiple observations for each $x_t$. This short-run variation can be used to identify the distribution of tastes and substitution across products. This approach is different from most of the literature, because rather than being motivated by continuous variation in product characteristics, identification comes from quasi-experimental variation in availability.

Note that while firms still choose which products to stck and how often to restock, firms do not directly control which products are available to individual consumers. The set of products that are available at the time of an individual purchase depends on the random order in which consumers arrive; thus, stock-out based variation is exogenous. The embedded assumption is that consumer choice probabilities depend only on the products available to consumers at the time of purchase, and consumers take the set of available products as given.

[7] Recent work by Berry and Haile (2008) and Fox and Gandhi (2008) provide formal identification results for the latent utilities using continuous full-support variation in product characteristics similar to the special regressor econometric literature.

## 2.2 Partially Observed Availability / Periodic Inventory

In many market settings, firms only observe inventories or sales periodically. This presents additional challenges, because availability is known only at the beginning and the end of each period. As in the case of perpetual inventory, we denote the set of available choices at the beginning of period $t$ by $a_t$, and the set remaining at the end of $t$ by $a_{t'}$. One would like to assign the sales in period $t$ to each $(a_t, a_{t'})$ regime, but the $a_i$ corresponding to each $y_{ij}$ is no longer observed.

A standard approach for dealing with unobservable heterogeneity is to integrate it out. The only aspect of the order of sales that affects the likelihood is whether or not a sale takes place before or after a stockout. In the case of a single stockout, there are only two regimes $(a_t, a_{t'})$, which implies the following likelihood contribution for each $x_t$:

$$L(y_t|x_t,\theta) = \prod_i \prod_j \prod_{a\in\{a_t,a_{t'}\}} p_j(a_i,x_t,\theta)^{y_{ij}} Pr(a_i = a|x_t,\theta) \tag{6}$$

Stockouts represent an important case of unobservable choice-set heterogeneity, in which the demand model specifies the distribution over possible availability regimes without the need for additional assumptions. A stockout of product $k$ occurs only when the sales of product $k$, $y_{kt}$, exceed its initial inventory $\omega_{kt}$. For the sake of exposition, consider a single stockout with $M_t$ consumers in a period of observation.[8]

The question implied by $Pr(a_i = a|x_t,\theta)$ is, "How many consumers were required before $\omega_{kt}$ units of product $k$ were sold under availability set $a_t$?" This defines a negative binomial distribution, which describes the number of failures $r_t$ (sales of all other products) until $\omega_{kt}$ successes (sales of the focal product) are observed. This depends only on the choice probability for the product that stocked out, $p_k(a_t,x_t,\theta)$. In the context of stockouts, additional information is provided because we know that the stockout happened before $M_t$ consumers arrived. Thus, the stockout defines a negative binomial, conditional on $r_t + \omega_{kt} \le M_t$. The negative binomial distribution defined by its p.m.f. is:

$$f(r_t;\omega_{kt},p_k) = \frac{(\omega_{kt}+r_t-1)!}{r_t!(\omega_{kt}-1)!} p_k^{\omega_{kt}} (1-p_k)^{r_t}$$

And the conditional negative binomial defined by its p.m.f. is:

$$h(r_t|\omega_{kt},p_k(a_t,x_t,\theta)) \sim \frac{NegBin(\omega_{kt},p_k)}{NegBinCDF(M_t - \omega kt, \omega_{kt}, p_k)} = \frac{f(r_t,\omega_{kt},p_k)}{\sum_{m=1}^{M} f(r_t,\omega_{kt},p_k)} \tag{7}$$

Thus, $(M_t - \omega_{kt})$ consumers bought some product other than the one that stocked out, and the number of those consumers in the market before the stockout occurred $(r_t)$ has a

[8]We provide the extension to the case of multiple stockouts in section A.2 of the appendix.

conditional negative binomial distribution.[9]

There are a few potential ways in which we could proceed. One approach would be to combine (6) and (7) and directly maximize the resulting joint likelihood. Depending on the particular form for $p_j(\cdot)$, such a problem could be difficult to estimate. Our approach is to work directly with the sufficient statistics. If we knew that $r_t$ consumers arrived before the stockout and bought a product other than the focal product, then $(M_t - \omega_{kt} - r_t)$ consumers arrived after the stockout. This implies that the sales of product $j$ before the stockout are distributed binomially, and allows us to compute the expected sales before and after the stockout as follows:

$$\begin{aligned} y_{jt}^{a_t}|r_t &\sim Bin\left(y_{jt}, \frac{r_t \cdot p_j(a_t, x_t, \theta)}{r_t \cdot p_j(a_t, x_t, \theta) + (M_t - \omega_{kt} - r_t) \cdot p_j(a_{t'}, x_t, \theta)}\right) \\ E[y_{jt}^{a_t}|y_{jt}, \theta] &= y_{jt} \sum_{0 \le r_t \le M_t - \omega_{kt}} \frac{r_t \cdot p_j(a_t, x_t, \theta)}{r_t \cdot p_j(a_t, x_t, \theta) + (M_t - \omega_{kt} - r_t) \cdot p_j(a_{t'}, x_t, \theta)} h(r_t|\omega_{kt}, p_k(a_t, x_t, \theta)) \\ E[y_{jt}^{a_{t'}}|y_{jt}, \theta] &= y_{jt} - E[y_{jt}^{a_t}] \end{aligned} \tag{8}$$

While computing the expected likelihood is difficult and requires considering $\binom{M_t}{y_{1t} \cdots y_{Jt}}$ potential orderings, computing the expected sufficient statistics requires just one single-dimensional integral for each product, which implies that the sufficient statistics approach is several algorithmic orders of magnitude easier ($O(M_t)$ vs. $O(M_t!)$).[10]

## 2.3 An EM Algorithm

Another advantage of this formulation is that it is possible to use the Expectation-Maximization (EM) algorithm for estimation. The convergence of the EM Algorithm was established broadly by Dempster, Laird, and Rubin (1977), but much earlier for the multinomial (Hartley 1958).

For members of the exponential family, the EM algorithm is an iterative procedure that alternates between computing the expected (log)-likelihood and maximizing the expected

[9] This is different from other approaches taken in the growing literature on product availability, for example, Bruno and Vilcassim (2008) essentially replace $P(a_i = a|x_t, \theta)$ with the unconditional marginal probability that product $j$ is available. Such an approach is motivated by data limitations, but quite clearly does not address the importance of other products in the choice set, nor the particular way in which stockouts affect product availability.

[10] If there are $n$ stockouts the complexity of the naive approach grows as $O(M^n)$ which is still substantially better than $O(M_t!)$. Note that this is for exact computation of the integral (evaluating at every point of support). This integral is well behaved and amenable to quadrature based approaches for integration which are quite simple for cases where $M_t$ becomes large. Such approaches are presented in more detail in the appendix.

(log)-likelihood:

$$\begin{aligned}
\text{E-Step:} \qquad Q(\theta|\theta^l) &= E_{Z|y,\theta^l}\left[\log p(Z|\theta)\right] = \int_{Z(y)} \log p(z|\theta) p(z|y,\theta^l) dz \\
\text{M-Step:} \qquad \widehat{\theta^{l+1}} &= \arg\max_{\theta} Q(\theta|\theta^l)
\end{aligned}$$

For the stockout problem, the E-step is established by plugging the expected sufficient statistic from (8) into the log-likelihood. This works because the log-likelihood is linear in the sufficient statistics (shown here for a single $x_t$).

$$E[l(y_t|a_t, x_t, \theta^l)] = E\left[\sum_{j,t} \sum_{a \in \{a_t, a_{t'}\}} y^a_{jt} \ln p_j(\theta, a, x_t)|y_t, \theta^l\right] = \sum_{j,t} \sum_{a \in \{a_t, a_{t'}\}} E[y^a_{jt}|y_t, \theta^l] \ln p_j(\theta, a, x_t) \tag{9}$$

For the M-step, one maximizes the expected log-likelihood function:

$$\hat{\theta}^{l+1} = \arg\max_{\theta} \sum_{j,t} \sum_{a \in \{a_t, a_{t'}\}} E[y^a_{jt}|y_t, \theta^l] \ln p_j(\theta, a, x_t) \tag{10}$$

$$\tag{11}$$

Each iteration of the EM algorithm is guaranteed to improve the log-likelihood. When the stopping condition (i.e., $|\theta^{l+1} - \theta^l| < \varepsilon$) is met, the value of $\hat{\theta}$ represents a valid maximum of $E[l(y_t|\theta)]$.

There is no additional integration in the M-Step, so optimization routines treat the imputed values as if they were data. Thus, researchers can use 'off-the-shelf' routines for ML (or MSL, depending on the specification of $p_j(\cdot)$). Furthermore, the expectations over the missing data do not enter the optimization procedure, thus avoiding computation of the derivatives. Similarly, one need not worry about the combinatorics of the problem growing, since one only evaluates the expectations once per major iteration, rather that at each likelihood evaluation.

In general there are some criticisms of EM-type procedures. The first is that EM algorithms are slow to converge (that is, they require many major iterations). This is true, the rate of convergence depends on the amount of data that are missing, and how much the likelihood is affected by the missing data. For many cases involving stockouts, both of these tend to be small. That is, the bulk of the data is nonmissing (we don't always observe stockouts), and for cases where we do observe stockouts, the choice probabilities before and after the stockout are generally not wildly different. The other problem that EM-type procedures generally have is that they are local optimizers (as are almost all nonlinear search routines used by economists) and require good starting values. This is true, and EM is a slightly more dependent on starting values than other algorithms, but this is a problem that is endemic to the entire class of extremum estimators.

### 2.4 An Alternative Approach

In general, many alternative approaches require tracking the entire order of sales, rather than just the stockout time and expected sales, because different orderings can imply different likelihood values. One such alternative approach is an indirect inference estimator, where, for a guess of $\theta$, one records the initial inventory and number of consumers for each market, and then simulates individual consumer choices, repeating for each of the consumers in the market.[11] One would repeat this process some large number of times for each market and compute the average purchase probability for each product, denoting it as $\hat{p}_{jt}(\theta)$.[12] If each consumer's utility was determined by IID draws of random tastes and idiosyncratic preferences, then this would be a less efficient version of our estimator. On the other hand, the simulation approach allows the distribution of tastes to vary over time, even in a way that allows for correlation with stockouts. In fact, this indirect-inference type approach could be utilized for any scenario where one can write down a series of functions that allowed for simulation of consumer purchases. After computing the choice probabilities from a simulation procedure, one would simply evaluate the likelihood using MSL.[13]

Even for an infinite number of trials, a simulation approach is not as efficient as the EM procedure. The major efficiency loss arises because the procedure is not restricted to availability regimes that occur in the data. In other words, it does not condition on the full set of available data, and instead averages over all possible availability regimes encountered by simulation.[14] We provide further details on this type of approach in the appendix (section A.3), and refer other researchers with different applications to that information.

[11]Simulating the product choice for one consumer requires 4 steps: (1) draw a consumer type given $\theta$, (2) compute choice probabilities given the type, (3) simulate a consumer from those choice probabilities and record a purchase, (4) update the inventory and number of consumers remaining in the market.

[12]Leslie (2004) employs a similar strategy for handling seat capacities in an analysis of theater demand.

[13]One could also use an MSM procedure, as in Berry, Levinsohn, and Pakes (1995), to allow for price instruments or additional moment restrictions.

[14]Musalem, Olivares, Bradlow, Terwiesch, and Corsten (2008) show that it is possible to avoid this problem. They describe an MCMC procedure that assumes an ordering of sales, and then considers permutations of the ordering via Metropolis-Hastings steps. However, such an approach still requires considering (probabilistically) the full permutation of $M_t!$ possible orderings of the sales vector, whereas our approach only requires that we consider $M_t^n$ orderings (where $n$ is the number of unobserved stockous). Our advantage comes from directly working with the sufficient statistics and exploiting the fact that the likelihood is only affected by when sales take place with respect to a stockout.

# 3 Estimation

## 3.1 Parametrizations

In this section we present several familiar choices for specifying a functional form for $p_j(\theta, a_t, x_t)$, and we show how they can be adapted into our framework. In any discrete model, when $M$ is large and $p_j$ is small, the Poisson model becomes a good approximation for the sales process of any individual product. The simplest approach would be to parameterize $p_j(\cdot)$ in an semi-nonparametric way:

$$p_j(\theta, a_t, x_t) = \lambda_{j,a_t}.$$

Then, the maximum likelihood (ML) estimate is essentially the mean conditional on $(a_t, x_t)$. This type of approach is taken by Anupindi, Dada, and Gupta (1998). The advantage is that it avoids placing strong parametric restrictions on substitution patterns, and the M-Step is easy. The disadvantage is that it requires estimating $J$ additional parameters for each choice set that is observed. It also means that forecasting is difficult for choice sets that are not observed in the data or are rarely observed. Furthermore, the lack of a utility-based framework means that out-of-sample predictions about alternative policies cannot be made.

A typical solution from the differentiated products literature for handling these sorts of problems is to write down a nested-logit or random-coefficients logit form for choice probabilities. This still has considerable flexibility for representing substitution patterns, but avoids estimating an unrestricted covariance matrix. This family of models is also consistent with random utility maximization (RUM). Assume that consumer $i$ has utility $u_{ijt}(\theta)$ for product $j$ in market $t$, and chooses a product to solve:

$$\begin{aligned} d_{ijt} &= \arg\max_j u_{ijt}(\theta) \\ u_{ijt}(\theta) &= \delta_{jt}(\theta_1) + \mu_{ijt}(\theta_2) + \varepsilon_{ijt}, \end{aligned}$$

where $\delta_{jt}$ is the mean utility for product $j$ in market $t$, $\mu_{ijt}$ is the individual specific taste, and $\varepsilon_{ijt}$ is the idiosyncratic logit error. It is standard to partition the parameter space $\theta = [\theta_1, \theta_2]$ between the linear (mean utility) and non-linear (random taste) parameters. This specification produces the individual choice probability, and the aggregate choice probability

$$Pr(k|\theta, a_t, x_t) = \frac{\exp[\delta_k(\theta_1) + \mu_{ik}(\theta_2)]}{1 + \sum_{j \in a_t} \exp[\delta_j(\theta_1) + \mu_{ij}(\theta_2)]}$$

This is exactly the differentiated products structure found in many IO models (Berry 1994, Berry, Levinsohn, and Pakes 1995, Goldberg 1995). These models have some very nice properties. The first is that any RUM can be approximated arbitrarily well by this "logit" form (McFadden and Train 2000). This also means that the logit ($\mu_{ijt} = 0$) and the nested logit can be nested in the above framework. For the random coefficients logit of Berry, Levinsohn, and Pakes (1995), $\mu_{ijt} = \sum_l \sigma_l x_{jl} \nu_{il}$, where $x_{jl}$ represents the $l$th characteristic of product $j$ and $\nu$ is standard normal.

The second advantage of these parametrizations is that it is easy to predict choice probabilities as the set of available products changes. If a product stocks out, we simply adjust the $a_t$ in the denominator and recompute. A similar technique was used by Berry, Levinsohn, and Pakes (1995) to predict the effects of closing the Oldsmobile division and by Petrin (2002) to predict the effects of introducing the minivan. The parsimonious way of addressing changing choice sets is one of the primary advantages of these sorts of parameterizations, particularly in the investigation of stockouts.

We estimate the model using two specifications for the choice probabilities: a nested-logit model (with either a single nesting parameter or category-specific nesting parameters), and a random-coefficients model. For the nested-logit model, we denote product-specific effects as $d_j$, and the nesting parameter as $\lambda$. We use Full Information Maximum Likelihood, and reparameterize the model such that $\gamma_j = d_j/\lambda$. This gives the choice probabilities as

$$\hat{p}_{jt}(\theta) \quad = \quad \frac{e^{\gamma_j}(\sum_{k \in g_l} e^{\gamma_j})^{\lambda - 1}}{\sum_{\forall l'}(\sum_{k \in g'_l} e^{\gamma_j})^{\lambda}}.$$

For the random-coefficients logit form, we estimate via Maximum Simulated Likelihood (MSL). For the multinomial choice model, the MSL estimator begins with some random or quasi-random normal draws $v_{ik}$ for each $t$ in the dataset. For a given $\theta$ we can compute the average choice probability across draws and then plug this in to our likelihood function. The average choice probabilites are given by:

$$\hat{p}_{jt}(\theta) \quad = \quad \frac{1}{ns}\sum_{i=1}^{ns} \frac{\exp[d_j + \sum_l \sigma_l v_{il} x_{jl}]}{1 + \sum_{j \in a_t} \exp[d_j + \sum_l \sigma_l v_{il} x_{jl}]} \text{ where } v_{il} \sim N(0,1).$$

Finally, for either specification, the likelihood is simply:

$$l(\theta) \quad = \quad \sum_t \sum_{j \in a_t} y_{jt} \ln \hat{p}_{jt}(\theta).$$

## 3.2 Heterogeneity

Thus far, choice probabilities have been conditional on $x_t$. This is useful for showing that our result holds for the case of conditional likelihood, but it is also of practical significance to applied problems. It is likely that choice probabilities may vary substantially across periods in retail datasets, particularly if they are short. Over long periods of time (such as annual aggregate data) this variation is averaged out. The distribution of tastes over a long period is essentially the combination of many short-term taste distributions, and this is often the basis of estimation (ie., in the case of limited data we would estimate the long-run distribution). With high-frequency data, we can address this additional heterogeneity by conditioning on $x_t$, which could include information such as the time of day, day of the week, or local market identifiers. Depending on the application, failing to account for this additional heterogeneity may place a priori unreasonable restrictions on the data.

We can model dependence on $x_t$ in several ways. One is to treat $p(\cdot|x_t)$ as a different function for each $x_t$. Another is to require that all markets face the same distribution of consumers, but allow that distribution to vary with $x_t$. Another option is to fix some parameters across $x_t$, and allow others to vary with $x_t$. For example, allow mean tastes to differ across markets but assume that the correlation of tastes is constant. In addition, one can parameterize market size, $M$, as a function of $x_t$ (i.e., allow market size to vary across periods without affecting the choice probabilities). Parameterizing $M$ with auxiliary data has a long history in the literature (Berry 1992), and is done offline prior to parameter estimation.

### 3.3 Identification

In this section we address practical aspects of identification of the choice probabilities $p_j(\theta, a_t, x_t)$, when the underlying data generating process is multinomial. Denote $y_j(a, x)$ as sales of product $j$ when $(a_t = a, x_t = x)$, and $M_{(a,x)}$ as the corresponding market size. Since sales are distributed multinomially, the semi-nonparametric estimator for $\hat{p_j}$ is just the conditional mean, or the fraction of consumers facing $(a, x)$ who chose product $j$. This is:

$$\hat{p_j}(a,x) = \frac{y_j(a,x)}{M_{(a,x)}}$$

with variance:

$$\begin{aligned} Var(\hat{y_j}(a,x)) &= M_{(a,x)} p_{j(a,x)}(1 - p_{j(a,x)}) \\ Var(\hat{p_j}(a,x)) &= \frac{1}{M^2_{(a,x)}} \cdot Var(y_{j,(a,x)}) = \frac{p_j(a,x)(1-p_j(a,x))}{M_{(a,x)}} \end{aligned}$$

Thus, the variance of nonparametric estimators for $p_j(a, x)$ will go to zero as $M_{a,x} \to \infty$. This variance is typically referred to as "measurement error in the choice probabilities,"[15] and it declines almost uniformly across share sizes.[16]

[15]For ML estimators, "measurement error" is an efficiency issue. However, for GMM approaches, this can create problems with consistency as well, so we typically assume that $M_{a,x} \to \infty$. While this might be reasonable for annual data at a national level, it becomes more problematic in the analysis of high-frequency data.

[16]Thus we don't need more observations to correctly estimate smaller shares than we need to estimate larger shares.

Conversely, unless every $(a, x)$ pair in the domain is observed with a substantial number of consumers, the semi-nonparametric representation of $p_j(a, x)$ will not be nonparametrically identified. This problem is well known, and the standard approach is to assume a functional form for the $p_j(.)$'s, which may also specify an underlying utility function. One then determines what variation is needed in the data to identify this function. There is a growing literature on identification results for this class of estimators starting with Matzkin (1992). For example, Ackerberg and Rysman (2005) use continuous variation in product characteristics (such as price) to obtain some derivative-based identification arguments for choice probabilities in the nested logit. More recent work by Berry and Haile (2008) and Fox and Gandhi (2008) provide formal identification results for the latent utilities using continuous full-support variation in product characteristics similar to the special regressor econometric literature.

Our approach is different, because it is not motivated by continuous full support $x$ variation. Instead stock-outs or capacity constraints generate quasi-experimental variation among available choice sets. We use this short-run variation to identify the distribution of tastes and substitution across products. The underlying intuition is to identify substitution patterns by exogenously removing products from the choice set, and recording the sales.

If we observe only a single choice set, then we have only a single observation in our data. However, we could identify product dummies for every product that was available for the logit form from this single observation (in the absence of any random coefficients). Identification of nonlinear parameters in a random-coefficients specification comes from the fact that the sales of two products $j$ and $k$ will be differentially affected by a stockout of product $l$ depending on how similar $j$ and $k$ are to $l$ in product space. Specifically, for each consumer type $i$ we inflate the probability of buying good $j$ by a factor proportional to the probability that type $i$ bought the stocked-out good $l$. Thus, we can think about a stockout as providing information not only about the level of $p_{jt}$, but also the ratio of the choice probabilities before and after a stockout.[17]

When type 3 markets are observed in a dataset, we need to identify both the distribution of the latent availability regimes and the choice probabilities. As discussed in the model section, the distribution of the latent availability regimes is specified by the multinomial model of sales. Separate identification of these two components (the choice probabilities and the distribution of availability regimes) arises from two sources of variation: first, variation in choice sets across all market types (including type 1 and type 2 markets), and second, variation in starting inventory levels in type 3 markets. The latter variation helps to pin down the distribution of latent availability regimes. In principle, one could identify the model from either source of variation, although the presence of type 1 and type 2 markets is clearly helpful. If working with data from only type 3 markets, one would want extensive variation in starting inventory levels in order to separately identify the two components of the model.

In practice, much empirical work uses a characteristics-based approach, and relies on discrete changes in stocking decisions, or non-price product characteristics, and a finite

[17] We provide this ratio and its derivation in section A.4 of the appendix.

amount of (possibly small) price variation. As Ackerberg and Rysman (2005) point out, it's important to observe variation in price, and not just discrete or non-price characteristics, in order to identify price sensitivity. And, furthermore, small price variation across a tight range will generally provide less information than well-spaced price variation. In our vending application, we don't observe price variation, and we do not attempt to identify a price coefficient. In general, one would want variation across prices in addition to changes in the choice set in order to reliably estimate a price coefficient.[18]

In summary, our model presents a different way to interpret variation in choice sets, in which availability varies as products stock-out, rather than with potentially endogenous changes in the long-term product mix or pricing decisions. This variation is exogenous because choice sets are realizations of the stochastic choices of consumers, and as a result, firms and consumers both take these short-run changes in choice sets as given. While firms choose inventory or capacity to affect the probability that a product stocks out, once we condition on that probability, the occurrence of any particular stockout is random.

# 4 Industry Description, Data, and Reduced-form Results

## 4.1 The Vending Industry

The vending industry is well suited to studying the effects of product availability in many respects. Product availability is well defined: goods are either in-stock or not (there are no extra candy bars in the back, on the wrong shelf, or in some other customer's hands). Likewise, products are on a mostly equal footing (no special displays, promotions, etc.). The product mix, and layout of machines is relatively uniform across the machines in our sample, and for the most part remains constant over time. Thus most of the variation in the choice set comes from stockouts, which are a result of stochastic consumer demand rather than the possibly endogenous firm decisions to set prices and introduce new brands.[19]

Typically, a location seeking vending service requests sealed bids from several vending companies for contracts that apply for several years. The bids often take the form of a two-part tariff, which is comprised of a lump-sum transfer and a commission paid to the owner of the property on which the vending machine is located. A typical commission ranges from $10 - 25\%$ of gross sales. Delivery, installation, and refilling of the machines are the

[18] Our estimation method allows for inclusion of a price instrument by either the addition of distributional assumptions to the ML problem, or the use of a GMM procedure. Draganska and Jain (2004) develops a method for including IVs and supply-side restrictions into an ML estimator, assuming a normal distribution on the unobservable product attributes. For a GMM procedure, Berry (1994) and Berry, Levinsohn, and Pakes (1995) can be used in the M step, because these estimators improve the likelihood at each step. However, these methods rely on an assumption that individual markets are large ($M_t \to \infty$), which might be problematic in very granular datasets. For the most 'extreme case' of granularity, Chintagunta, Dube, and Goh (2005) provide a method to extend the Berry (1994) method with price IVs to cases with individual-level data. Finally, Allenby, Chen, and Yang (2003) provide a Bayesian estimator when IVs are required.

[19] In this sense, our setup is substantially simpler than that of Nevo (2001), Goldberg (1995), or Berry, Levinsohn, and Pakes (1995) where new brands and prices are substantial sources of identification.

responsibility of the vending company. The vending company chooses the interval at which to service and restock the machine, and collects cash at that interval. The vending company is also responsible for any repairs or damage to the machines. The vending client will often specify the number and location of machines.

Vending operators may own several "routes" each administered by a driver. Drivers are often paid partly on commission so that they maintain, clean, and repair machines as necessary. Drivers often have a thousand dollars worth of product on their truck, and a few thousand dollars in coins and small bills by the end of the day. These issues have motivated advances in data collection, which enable operators to not only monitor their employees, but also to transparently provide commissions to their clients and make better restocking decisions.

Machines typically collect internal data on sales. The vending industry standard data format (called Digital Exchange or DEX) was originally developed for handheld devices in the early 1990's. In a DEX dataset, the machine records the number and price of all of the products vended; these data are typically transferred to a hand-held device by the route driver while he services and restocks the machine. The hand-held device is then synchronized with a computer at the end of each day.

### 4.2 Data Description

In order to measure the effects of stock-outs, we use data from 54 vending machines on the campus of Arizona State University (ASU). This is a proprietary dataset acquired from North County Vending with the help of Audit Systems Corp (later InOne Technologies, now Streamware Inc.). The data were collected from the spring semester of 2003 and the spring semester of 2004. The ASU route was one of the first vending routes to be fully wireless enabled and monitored through Audit System's (now Streamware's) software. The wireless technology provides additional inventory observations between service visits, when the DEX data are wirelessly transmitted several times each day (approximately every four hours).

The dataset covers snack and coffee machines; we focus on the snack machines in this study. Throughout the period of observation, the machines stock chips, crackers, candy bars, baked goods, gum/mints, and a few additional products. Some products are present only for a few weeks, or only in a few machines. Of these products, some of them are non-food items[20] or have insubstantial sales (usually less than a dozen total over all machines).[21] In our analysis, we exclude these items in addition to gum/mints, based on an assumption that these products are substantially different from more typical snack foods (and rarely experience stockouts). For a few brands of chips, we observe rotation over time in the same slot of the machine, and for these goods, we create two composite chip products (Misc Chips

[20] While often sold alongside of snacks in vending machines, condoms are poor substitutes for potato chips.

[21] Products dropped for insubstantial sales are: Grandma's Lemon Cheese, Grandma's Chocolate Croissant, and Nestle 100 Grand.

1 and Misc Chips 2).[22] Finally, we combined two different versions of three products.[23] The 44 products in the final dataset are listed in Table 1.

Sales at each machine are observed at four-hour intervals. Retail prices are constant over time, machines, and across broad groups of products as shown in Table 1. Baked goods typically vend for $1.00, chips for $0.90, cookies for $0.75, and candy bars for $0.65. As compared to typical studies of retail demand and inventories (which often utilize supermarket scanner data), there are no promotions or dynamic price changes as in Nevo and Hendel (2007) or Aguirregabiria (1999). This means that once most product characteristics (and certainly product or category dummies) are included, price effects are not identified. The method we present will work fine in cases where a price coefficient is identified, but in our particular empirical example, we have no variation for identifying a price effect.

In addition to the sales, prices, and inventory of each product, we also observe product names, which we link to the nutritional information for each product in the dataset. For products with more than one serving per bag, the characteristics correspond to the entire contents of the bag.

The dataset also contains stockout information and marginal cost data (the wholesale price paid by the firm) for each product. The stockout percentage is the percentage of time in which a product is observed to have stocked-out. We report both an upper and a lower bound for this estimate. The lower bound assumes that the product stocked out at the very end of the 4-hour period we observe, and the upper bound assumes that it stocked out at the very beginning of the 4-hour period of observation. For most categories and products, this ranges from two to three percent, with larger rates of stockouts for pastry items. The marginal cost data are consistent with available wholesale prices for the region. There is slight variation in the marginal costs of certain products, which may correspond to infrequent re-pricing by the wholesaler. The median wholesale prices for each product are listed in Table 1. Table 1 also allows one to calculate markups of the products. Markups tend to be lowest on branded candy bars (about 50%), and high on chips (about 70%). The product with the highest markup is Peter Pan crackers, which has a markup of 84%.

Other costs of holding inventory are also observed in the data, including spoilage/expiration and removal from machines for other reasons (e.g., ripped packaging, contamination, etc.). Spoilage does not constitute more than 3% of most products sold. The notable exceptions are the Hostess products, which are baked goods and have a shorter shelf life than most products (approximately 2 weeks vs. several months). For our static analysis of demand, we assume that the costs associated with such events are negligible.

[22]Misc Chips 1 rotates: Cool Ranch, Lays Kettle Jalapeno, Ruffles Baked Cheddar, and Salsa Dorito. Misc Chips 2 rotates: Frito Jalapeno, KC Masterpiece BBQ, Lays Baked Potato, Lays Wisconsin Cheese, Rubbles Hearty Chili, and Frito Chili Cheese. Product characteristics for the goods that are combined are very similar; for the composite good, we use the average of the characteristics of the individual products.

[23]These were: combine Gardetto's with Gardettos Snackems, combine Nestle Crunch with Caramel Nestle Crunch, and combine Nutter Butter with Nutter Butter Bites. Product characteristics in the first two combinations are identical. In the last combination, the product characteristics differ slightly, and in that case, we use the characteristics from Nutter Butter Bites.

### 4.3 Reduced-form Results

Before applying the estimation procedure described above to the dataset, we first describe the results of two simple reduced-form analyses of stockouts. In table 2, we compute the profits for each four-hour wireless time period at each individual machine and regress this on the number of products stocked out. The first specification (Column 1) estimates the four-hour profit loss to be about \$0.90 per product stocked out. Column 2 allows the effect of a stockout to differ based on the number of stockouts in the category with the most stockouts, in order to capture the fact that substitution to the outside good may increase when multiple products are unavailable in the same category (ie., missing one candy bar and one brand of chips is different from missing two brands of chips). We estimate the effect of a stockout in the category with the most products missing to be about \$1.90 per four-hour period, and the base effect of a product stocking out to be \$0.44. In column 3, we include the number of stockouts in each separate category. These results estimate the costs per stockout at around \$1.45 for chips to \$3.85 for candy on top of a base effect of \$0.41. The results are robust to the inclusion of machine fixed effects, which explain an additional 20 percent of the variation in profits. All of these regressions are clearly endogenous, and may be picking up many other factors, but they suggest some empirical trends that can be explained by the full model. Namely, stockouts decrease hourly profits as consumers substitute to the outside good, and multiple stockouts among similar products causes consumers to substitute to the outside good at an increasing rate.

Table 3 reports the results of a regression of stockout rates on starting inventory levels. An observation is a service visit-product pair. We report results for Probit and OLS (Linear Probability Model) with and without product and machine fixed effects. We find that an additional unit of inventory at the beginning of a service period reduces the chance of a stockout in that product by about 1%. A full column of candy bars usually contains 20 units. This means that the OLS (fixed effects) probability of witnessing a stockout from a full machine in a 3-day period is $.238 - .0101 * 20 = 3.6\%$. For a product at a machine with a starting inventory of five units, the predicted chance of a stockout is about one in five.

## 5 Empirical Results

We estimate nested logit and random-coefficients logit demand specifications using three different treatments for stock-out events. In the first treatment we assume full availability in all periods, including those periods in which a stockout was observed. Choice set variation in this specification is generated by the introduction or removal of products over time, and, to a lesser extent, from selective stocking of products in different machines. We refer to this as the 'Full Availability' model, and it is the standard method of estimation in the literature. In the second treatment we account for stock-outs that were fully observed, but ignore data that were generated during periods in which the timing of a stockout was ambiguous. We call this the 'Ignore' model. In the third treatment, we account for fully-observed stock-out events, and use the EM algorithm to estimate which sales occurred under the various stock-out regimes within any ambiguous period.[24] This is the 'EM-corrected' model.

Overall sales levels in the data vary across vending machines and time periods (such as time-of-day or day-of-week indicators), but relative choice probabilities are remarkably similar, and so we accommodate heterogeneity through $M$.[25] We divide machines into three tiers based on overall sales levels, and multiply a base rate of 360 people per day per machine by one-third and 3 for the smallest and largest tiers respectively. In addition, the rate of arrivals is reduced by a factor of one-third at night and one-fifth on weekends. Thus, for a machine in the middle tier, we have 250 consumers in a weekday, and about 75 consumers on a weekend day, for a total of 1400 consumers per week on average. For machines in the lowest tier, this is roughly 467 consumers per week on average, and for a machine in the highest tier, this is about 4200 consumers per week on average.[26] Due to data limitations, we do not allow choice probabilities to vary across locations or time periods. The most important limitation for the data is that we want to be sure there are enough potential consumers in any particular availability set. The more conditioning we do, the fewer consumers we have for any given availability set. For example, if we allowed choice probabilities to differ freely for each machine, we could only use the observations generated by that machine for each availability set that that machine encountered.

In addition to the results reported here, which use the complete dataset, we also estimated the model after aggregating the data to the daily level. We did this to insure that the EM correction does not perform poorly when 'ambiguous' periods comprise a larger portion of our dataset. At the four-hourly level, approximately 17 percent of the data come from periods during which a stock-out occurred, versus 35 percent when the data are aggregated at the daily level. We estimate very similar results for both the disaggregated and aggregated data.[27]

Table 4 reports the number of choice sets, market size, log likelihood, and nonlinear parameters from estimation of nested-logit and random-coefficients specifications under each of the three treatments of stock-out events.[28] The first two panels report estimates from

[24]The data contain a small number of observations (less than one percent) in which three or more products stock out. Based on conversations with the vendor, we assume such events occur at the very end of any ambiguous period for the estimates of demand reported here. The results are robust to omitting these observations, which the vendor believed may contain coding errors, or indicate removal or replacement of a machine. While inclusion of these observations–were we to believe the data from them fully–is possible for estimation, the simpler treatment of them here eases the computational burden in our application. For settings in which large numbers of products stock out within a period of observation, refer again to the methods in section A.3 of the appendix on alternative computational methods, which avoid integration of the exact distribution.

[25]Thus, it is assumed that the choice probabilities (the $p_j(\cdot)$ functions) are stable across markets as discussed in the estimation section.

[26]We use some additional conditions to prevent sales from exceeding the marketsize, particularly in very short periods, but in general these are not binding.

[27]All daily-level results are available upon request from the authors.

[28]All results were obtained by using the KNITRO optimization package. All reported values satisfy first and second order conditions for valid optima. A number of different starting values were used in estimation, and the best optimum value was reported in each case. These results have also been checked against standard MATLAB packages (fminsearch, fminunc).

two nested-logit specifications, the first using a single nesting parameter, and the second using five category-specific nesting parameters. Both nested-logit models are estimated by full information maximum likelihood methods. We report the nesting parameter $\lambda$ from McFadden (1978) rather than the $\sigma$ correlation parameter from Cardell (1997) or Berry (1994).[29] Roughly speaking, $\lambda \approx (1 - \sigma)$ such that $\lambda = 1$ is the simple logit and $\lambda = 0$ is perfect correlation within group. In general a $\lambda > 1$ is allowed, but is not necessarily consistent with random utility maximization (McFadden and Train 2000).

In the first column of the first panel (under the 'Full Availability' model), 238 choice sets contribute to the estimation of the ML problem, and total market size is about 5.7 million consumer visits over the two semesters. The correlation within nest is approximately $(1-\lambda)$, or 0.48 under an assumption of full availability. The second column (the 'Ignore' model) uses 2649 unique choice sets and a market size of about 5.3 million consumer visits. The estimate of $\lambda$ in this specification, 1.08, is not consistent with utility maximization, highlighting the extent of the bias from ignoring periods in which stock-outs occurred. The likelihood improves, but is not directly comparable to the likelihood in the other columns because it applies only to a subset of the data. The last column ('EM') reports the results after using the EM correction to assign sales to different availability regimes in the ambiguous periods. The number of choice sets in this model is 3966, which incorporates probabilistic choice sets, as well as those that were only encountered as an intermediary between the beginning-of-period and end-of-period availability in the case of multiple stockouts. The EM-corrected estimate of $\lambda$, 0.77, implies a within-nest correlation of 0.23. The EM-corrected model has a superior log-likelihood to the Full Availability model, although due to the biased estimates of $\theta$ in the Full Availability model, the likelihoods of the two models are not directly comparable.[30]

Standard errors are provided for all estimates. Standard errors for the Full Availability and Ignore models are readily available. The EM-corrected model requires a correction to the usual standard errors to account for the fact that sales in periods of latent availability regimes are estimated. We provide this correction in section A.6 of the appendix. In general, all parameters are estimated fairly precisely, with significant differences between the

[29] We should reiterate the reason we use FIML rather than the simpler least squares estimator for the nested logit is that we worry not only about the endogeneity of $\ln(s_{j|g})$ the within group share, and the lack of potential instruments, but for many small markets the only within-group sales are the sales of product $j$ – the case of extreme measurement error.

[30] One could also imagine estimating a model in which sales during periods of unknown availability are arbitrarily assigned–for example, by assuming that all stocked-out products stock out either at the very end or the very beginning of any ambiguous period. One might expect that the likelihood from such an exercise would be improved by the application of the EM algorithm, and in that sense, provide a further check of the EM-corrected method used here. While this is true for cases in which the unobserved data do not depend on $\mathbf{y}$ (see Tanner and Wong (1987)), it need not hold in the case of stock-outs, where the missing data include sales. Put another way, consistent estimation of demand implies a distribution for the missing sales, as detailed in section 2. Substituting arbitrary distributions instead will not give consistent estimates of $\theta$, because the demand model implies a specific distribution as a function of observed sales. A similar argument applies for comparing the likelihood from the Full Availability case with the likelihood from the EM-corrected estimator, as these are not comparable for the same reason. Essentially, any such exercise injects data that are known to be false into the estimate of $\theta$, making the resulting likelihood incomparable to the likelihood for the true model. We provide additional detail on this point in section A.5 of the appendix.

three models of stock-outs. The EM-corrected model produces the most efficient results by incorporating many more choice sets in estimation (in spite of the correction to account for estimating sales during periods of latent availability).

The second panel of table 4 reports similar patterns in the nested-logit model with five nesting parameters. In this specification, the estimated correlation between products in the same nest is negative for some nests in both the Full Availability and Ignore models, again highlighting the bias from ignoring stock-outs or dropping periods in which they occur. The EM-corrected model shows sensible correlation patterns, with within-nest correlation highest for chocolate $(1-\lambda) = 0.54$, and lowest for pastry $(1-\lambda) = 0.17$.

The third panel of table 4 reports the same set of estimates using a random-coefficients logit model for demand in which random coefficients are estimated for each of three observable product characteristics: fat, salt, and sugar.[31] The random-coefficients specification has a higher likelihood than either of the nested-logit specifications under the 'Full Availability' assumption. However, the likelihood under both of the nested-logit specifications exceeds the random-coefficients specification under the EM-corrected model. In the case of Ignore, the random-coefficients specification favors no correlation in taste for the three observable characteristics (i.e., estimation reduces to a 'plain vanilla' logit model).[32]

Using the EM-corrected estimates from table 4, table 5 lists the best substitute for the 35 most commonly-held products according to the category-specific nested-logit and the random-coefficients specifications. In both cases, we get sensible substitution patterns, with the two specifications predicting the same best substitute in many cases (particularly among chips and chocolate bars). When the predicted best substitutes differ, we see the trade-offs between the two demand specifications. For products that are 'harder to categorize,' such as Oreos (in the candy category), the random-coefficients model gives more intuitive results, whereas for products with less helpful characteristics, such as PopTarts, the nested-logit model seems more sensible.

## 6 Estimated Sales and the Impact of Stockouts

In this section, we use the results from the three estimated models to predict sales and the impact of stock-out events on firm profitability. These predictions give an indication of how important the corrections to the demand system are likely to be. They also lie at the heart of supply-side decisions about capacity and restocking efforts, and play a fundamental role in determining welfare calculations on the impacts of mergers, the value of new products,

[31] We have one additional product characteristic that is continuous: calories. We found no effect on correlation in tastes from this variable, so it was excluded from the set of non-linear parameters. We observe two additional discrete product characteristics: cheese and chocolate dummies. These are excluded from estimation because they were not identified after the inclusion of product dummies. We believe the non-identification of these parameters in our particular setting is due to the lack of additional product characteristics that vary continuously, such as price. Such a characteristic is a key assumption more generally for identification (see Berry and Haile (2008)).

[32] We report estimates of the linear parameters (i.e., the product dummies) and the results of second-stage regressions of product dummies on characteristics in section A.7 of the appendix.

or the application of antitrust policy. Table 6 shows predicted weekly sales for a fully-stocked 'typical' machine under the category-specific nested-logit and random-coefficients specifications. A typical machine is defined as one carrying the 35 most widely-carried products (measured across machines and over time) out of the full set of 44 products in the data, for which we simulate the arrival of 4500 consumers.[33] Comparing Ignore to Full Availability for this typical machine shows that predicted sales levels under Ignore are substantially lower for all products except three (Chocolate Donuts, Ding Dong, and PopTart–under random coefficients). This highlights the bias that results from excluding data on periods in which sales exceed inventory. The most interesting comparison is between the Full Availability and EM-corrected models, and we report the difference between these models in the fourth and eighth columns of the table. Looking across categories, pastry and chip products generally have higher sales under the EM-corrected model. These are the two categories with the lowest capacities and highest average rates of stock-outs in the data (see table 1). Within category, we also see some sensible patterns. For example, among chocolate bars, three products have higher estimated sales under the EM-corrected model: Snickers, M&M Peanut, and M&M's. These three products have the highest rate of stock-out events in the chocolate category in the data (not including Babyruth, which was not carried by the 'typical' machine, and so was excluded from the simulation). Overall, the EM-corrected model predicts more people purchasing an inside good (total sales of 250, compared to 245 or 229 under Full Availability or Ignore).

In order to demonstrate how the different demand estimates affect the impact of stock outs on profitability, we conduct an experiment in which we consider weekly sales at the same typical machine, and compare them to a machine where the two best-selling products in each category are unavailable.[34] We report the results of these stockout experiments in Table 7. For each of the products stocked out, we report the number of forgone sales predicted by the two demand specifications. This ranges from roughly 10 in the pastry and chips categories, 6 or 7 in the cookie and candy categories, and 20 in the chocolate category.[35]

The different treatments of stock-out events give substantially different predictions for sales as the set of available products changes. For both demand specifications, the Ignore model predicts much lower sales of available products across the board than either the Full Availability or the EM-corrected models. This reflects lower estimated correlation in tastes (the exception being the chocolate category in the nested-logit specification), but also highlights the bias that results from dropping periods of high demand from estimation.

In the case of the category-specific nested logit, the $\lambda$'s that are greater than one for the

[33]We chose 35 products because this is the number of product facings in a single machine. The market size of 4500 consumers is the number of consumers assumed to pass by a relatively high-volume machine over the course of one week in our demand model.

[34]We simulate the removal of the following products: Chocolate Donuts, Strawberry Frosted PopTarts, Grandma's Oatmeal Raisin Cookie, Chips Ahoy Cookies, Rold Gold Pretzels, Sunchips Harvest Cheddar, Snickers, Twix, Starburst, and Kar Nut's Sweet & Salty Mix.

[35]As an interesting supply-side comparison, the forgone sales of these products match up in a sensible way against the capacity of the machine, which is visited once a week or slightly more often. Capacities in the various categories are: 9 to 11 for pastry and most chips, 15 for cookie and candy, and 20 for most chocolate bars.

Full Availability and Ignore models (i.e., the pastry category in both models and the chips category in the Ignore model) lead to the prediction that fewer consumers buy other products in the category when the top-selling products are stocked out. Thus, for example, a PopTart appears to be a complement to Ding Dongs and other pastry items in these specifications. For the remaining categories, the EM-corrected model generally predicts lower correlation in tastes (higher $\lambda$'s), and thus fewer additional sales for the available substitutes (e.g., Peanuts sells 2.31 additional units according to the Full Availability model, but only 1.81 additional units under the EM-corrected model). The exception is the chocolate category, in which the EM-corrected model predicts higher within-nest correlation of tastes (a lower $\lambda$), and correspondingly higher rates of substitution to the available substitutes within the nest (e.g., 5.16 units of M&M Peanut are sold according to the EM-corrected model, compared to 2.96 units under the Full Availability model).

In the random-coefficients specification, the EM-corrected estimates predict less substitution to available products than the Full Availability model across the board. This demonstrates the censoring and forced-substitution effects, because the products that were stocked-out are the best-selling products in each category, and they also stock-out more frequently than other products. (One exception to this rule is Twix, which suffers fewer stock-outs in the data.) The EM-corrected model, under both demand specifications, generally predicts demand that is stronger for the best-selling, frequently stocked-out products (giving larger negative numbers of forgone sales for those products), and weaker for the remaining available products.

In table 8, we report the overall impact of the stockouts. The Full Availability and Ignore models predict lower levels of forgone sales compared to the EM-corrected model under both demand specifications. The Full Availability model predicts higher increased sales of substitutes than the EM-corrected results under the random-coefficients specification, while the Ignore model predicts ridiculously low levels of substitution to the available products. Correspondingly, the Ignore model predicts a much lower percentage of consumers purchasing an inside good when their first-choice product is unavailable (between 3 and 9 percent), while the Full Availability and EM-corrected models are closer to each other, with the EM-corrected model predicting fewer consumers staying inside (25 - 36 percent vs. 30 - 35 percent for Full Availability). Gross profit is thus lower under the EM-corrected model, with a loss from the stockouts of roughly \$36 to \$40 compared to \$33 to \$36 for the Full Availability model. In percentage terms, this difference is 7.9% in the nested-logit specification, and 11.6% in the random-coefficients specification. However, the predictions in each category are wildly off, with the change in gross profit differing by between 7% to 30% in the random-coefficient specification, and by even more in the nested-logit specification. For example, the nested logit model predicts only a \$2.41 loss in profits in the Chips category in the Full Availability model, vs. a \$10.32 loss in the EM-corrected results. For an industry with profit margins of less than 4%, in which category-level sales are important, these are significant differences.[36]

[36]Companies with over \$1 million in revenue have a 4.3% profit margin on average, while companies with less than \$1 million in revenue (75% of all vending operators, by count) have an average profit margin of

As a final exercise, we quantify the expected change in sales for substitute products, given that a focal product stocks out. Doing this for each of the products stocked in a typical machine produces 35 graphs with 35 sales effects in each one. The first bar in these graphs is the closest substitute, the second bar is the next closest substitute, and so on. Figure 7 shows the median change in the sales of substitutes by rank across all 35 products using the demand parameters from the EM-corrected random-coefficients model.[37] The median effect shows that the closest substitute experiences about a three percent increase in its sales when the focal product stocks out. The second closest substitute has a median sales increase of two percent.

# 7 Conclusion

Incomplete product availability is a common and important feature of many markets. This paper demonstrates that failing to account for product availability correctly can lead to biased estimates of demand, which can give misleading estimates of sales and the welfare impacts of stock outs. We show that the welfare impact of stockouts in vending machines has a substantial effect on firm profits, indicating that product availability may be an important strategic and operational concern facing firms and driving investment decisions. Furthermore, biases that result from the incorrect treatment of stock-out events can potentially undermine the reliability of many important applications of demand estimates for markets with incomplete product availability, such as simulating the welfare implications of mergers or new product introductions, applying antitrust policy, constructing price indices, or evaluating the optimal capacity choices of firms.

A failure to account for product availability also ignores a useful source of variation for identifying demand parameters. Rather than examining the effect of changing market structure (entry, exit, new goods, mergers, etc.) on market equilibrium outcomes, stock outs allow us to examine the effect that temporary changes to the consumer's choice set have on producer profits and our estimators. Standard demand estimation techniques have used long-term variations in the choice set as an important source of identification for substitution patterns, and this paper demonstrates that it is also possible to incorporate data from short-term variations in the choice set to identify substitution patterns, even when the changes to the choice set are not fully observed.

We collect and analyze a dataset in which a new wireless technology allows for quite detailed information on sales and inventories. However, the method we describe can be used in any setting in which periodic information on sales is available with inventory or capacity data. For example, hospitals, airlines, and sporting or concert events often have fixed and/or observable capacities, and many retail markets collect periodic inventory data. When such data are available, researchers gain valuable information on short-run choice set variation. Our results in this paper indicate that accounting for that choice set variation can substantially reduce potential biases in standard estimates for some markets, and that

-2.5% (www.vending.org 2008).

[37]The set of 35 individual graphs are available upon request from the authors.

researchers should take on the responsibility to adjust for the effects of product availability in demand estimation when possible.

Table 1: Summary of Products and Markups

| Product | Category | % SO (Low/Up) | | p | c | Share | ADS | Mach. |
|---|---|---|---|---|---|---|---|---|
| PopTart | Pastry | 5.26 | 6.51 | 1.00 | 0.35 | 3.79 | 0.93 | 49 |
| Choc Donuts | Pastry | 18.43 | 21.45 | 1.00 | 0.46 | 3.28 | 0.79 | 53 |
| Ding Dong | Pastry | 13.71 | 15.97 | 1.00 | 0.46 | 3.14 | 0.72 | 54 |
| Banana Nut Muffin | Pastry | 7.07 | 8.28 | 1.00 | 0.40 | 2.94 | 0.65 | 52 |
| Rice Krispies | Pastry | 1.15 | 1.39 | 1.00 | 0.31 | 2.22 | 0.50 | 54 |
| Pastry | Pastry | 15.72 | 18.68 | 1.00 | 0.46 | 0.92 | 1.02 | 23 |
| Gma Oatmeal Raisin | Cookie | 1.19 | 1.48 | 0.75 | 0.23 | 3.10 | 0.68 | 52 |
| Chips Ahoy | Cookie | 0.67 | 0.89 | 0.75 | 0.25 | 2.80 | 0.60 | 53 |
| Nutter Butter Bites | Cookie | 0.19 | 0.25 | 0.75 | 0.26 | 1.97 | 0.45 | 51 |
| Knotts Raspberry Cookie | Cookie | 0.34 | 0.43 | 0.75 | 0.19 | 1.96 | 0.43 | 52 |
| Gma Choc Chip | Cookie | 1.45 | 1.84 | 0.75 | 0.22 | 1.50 | 0.80 | 45 |
| Gma Mini Cookie | Cookie | 5.01 | 5.59 | 0.75 | 0.21 | 0.52 | 0.54 | 46 |
| Gma Caramel Choc Chip | Cookie | 4.32 | 4.92 | 0.75 | 0.23 | 0.23 | 0.61 | 50 |
| Rold Gold | Chips | 6.77 | 8.15 | 0.90 | 0.27 | 4.56 | 0.96 | 54 |
| Sunchip Harvest | Chips | 5.35 | 6.45 | 0.90 | 0.27 | 4.36 | 0.92 | 54 |
| Cheeto Crunchy | Chips | 3.86 | 4.60 | 0.90 | 0.27 | 3.82 | 0.80 | 54 |
| Dorito Nacho | Chips | 1.74 | 2.13 | 0.90 | 0.27 | 3.71 | 0.80 | 54 |
| Gardetto Snackens | Chips | 0.93 | 1.08 | 0.75 | 0.23 | 2.71 | 1.04 | 54 |
| Ruffles Cheddar | Chips | 3.79 | 4.69 | 0.90 | 0.27 | 1.98 | 0.65 | 48 |
| Fritos | Chips | 2.23 | 2.53 | 0.90 | 0.27 | 1.93 | 0.44 | 51 |
| Lays Potato Chip | Chips | 2.43 | 2.78 | 0.90 | 0.27 | 1.83 | 0.40 | 54 |
| Misc Chips 2 | Chips | 1.74 | 2.00 | 0.90 | 0.28 | 1.15 | 0.37 | 54 |
| Dorito Guacamole | Chips | 2.27 | 2.67 | 0.90 | 0.28 | 0.93 | 0.47 | 46 |
| Munchies | Chips | 5.12 | 5.59 | 0.90 | 0.25 | 0.60 | 0.50 | 42 |
| Misc Chips 1 | Chips | 5.43 | 5.87 | 0.90 | 0.26 | 0.50 | 0.42 | 42 |
| Munchies Hot | Chips | 0.37 | 0.43 | 0.75 | 0.25 | 0.47 | 0.53 | 40 |
| Snickers | Chocolate | 0.61 | 0.82 | 0.75 | 0.33 | 9.21 | 1.92 | 54 |
| Twix | Chocolate | 0.52 | 0.67 | 0.75 | 0.33 | 6.88 | 1.43 | 54 |
| M&M Peanut | Chocolate | 1.38 | 1.75 | 0.75 | 0.33 | 5.22 | 1.14 | 52 |
| Reese's Cup | Chocolate | 0.61 | 0.72 | 0.75 | 0.33 | 2.70 | 0.57 | 54 |
| Kit Kat | Chocolate | 0.47 | 0.58 | 0.75 | 0.33 | 2.47 | 0.52 | 54 |
| Caramel Crunch | Chocolate | 0.41 | 0.50 | 0.75 | 0.33 | 2.44 | 0.51 | 54 |
| Hershey Almond | Chocolate | 0.28 | 0.33 | 0.75 | 0.33 | 1.71 | 0.40 | 50 |
| M&M | Chocolate | 3.59 | 3.98 | 0.75 | 0.33 | 0.55 | 0.50 | 48 |
| Babyruth | Chocolate | 3.56 | 3.97 | 0.75 | 0.28 | 0.22 | 0.29 | 52 |
| Kar Nut Sweet/Salt | Candy | 1.21 | 1.45 | 0.75 | 0.22 | 3.23 | 0.69 | 53 |
| Snackwell | Candy | 0.35 | 0.40 | 0.75 | 0.28 | 1.89 | 0.41 | 54 |
| Skittles | Candy | 0.72 | 0.95 | 0.75 | 0.34 | 1.56 | 0.79 | 47 |
| Payday | Candy | 0.35 | 0.44 | 0.75 | 0.35 | 1.27 | 0.53 | 54 |
| Oreo | Candy | 0.10 | 0.12 | 0.75 | 0.22 | 1.05 | 0.25 | 49 |
| Peter Pan (Crck) | Candy | 0.54 | 0.63 | 0.75 | 0.12 | 0.83 | 0.42 | 47 |
| Peanuts | Candy | 1.07 | 1.18 | 0.75 | 0.26 | 0.81 | 0.44 | 45 |
| Starburst | Candy | 2.85 | 3.62 | 0.75 | 0.33 | 0.79 | 0.80 | 47 |
| Hot Tamales | Candy | 4.58 | 5.33 | 0.75 | 0.27 | 0.27 | 0.46 | 54 |



Product and Category provided by the vending company, % SO (Low/Up) reports lower and upper bounds on stock-out frequencies, p is price charged at vending machines, c is wholesale cost, Share is 'inside good' marketshare, ADS is average daily sales across all machines that carried a product, and Mach. is number of machines that carried a product.

Table 2: Regression of Profit on Stock-Out Variables

| | (1) | (2) | (3) |
|---|---|---|---|
| # Products Stocked Out | -0.895*** | -0.437*** | -0.408*** |
| | (0.017) | (0.021) | (0.023) |
| (max) SO's, category | | -1.896*** | |
| | | (0.053) | |
| # SO, Pastry | | | -2.273*** |
| | | | (0.062) |
| # SO, Cookie | | | -2.637*** |
| | | | (0.21) |
| # SO, Chips | | | -1.450*** |
| | | | (0.069) |
| # SO, Chocolate | | | -1.611*** |
| | | | (0.15) |
| # SO, Candy | | | -3.847*** |
| | | | (0.24) |
| Constant | 7.425*** | 7.727*** | 7.743*** |
| | (0.049) | (0.050) | (0.050) |
| Observations | 111195 | 111195 | 111195 |
| $R^2$ | 0.0238 | 0.0349 | 0.0367 |

An observation is a four-hour period at an individual machine (recorded wirelessly).

Table 3: Regressions of Stockout Rates on Starting Inventory

| | OLS (1) | OLS (2) | OLS (3) | Probit (1) | Probit (2) | Probit (3) |
|---|---|---|---|---|---|---|
| Beginning Inv | -0.0168*** | -0.00872*** | -0.0101*** | -0.0142*** | -0.00681*** | -0.00893*** |
| | (0.00025) | (0.00042) | (0.00055) | (0.00022) | (0.00035) | (0.00036) |
| Hours | 0.00203*** | 0.00217*** | 0.00278*** | 0.00169*** | 0.00174*** | 0.00212*** |
| | (0.00011) | (0.00010) | (0.00011) | (0.000092) | (0.000081) | (0.000084) |
| Constant | 0.339*** | 0.225*** | 0.238*** | | | |
| FE | - | Product | Prod x Mach | - | Product | Prod + Mach |
| Observations | 98900 | 98900 | 98900 | 98900 | 98900 | 98900 |
| $R^2$ | 0.0486 | 0.1326 | 0.2092 | 0.0585 | 0.1561 | 0.1788 |

An observation is an individual product at a service visit for an individual machine.

Table 4: Non-linear Parameter Estimates

| | Full Availability | Ignore | EM |
|---|---|---|---|
| Single Nesting Parameter ($\lambda$) | | | |
| Category | 0.525 | 1.075 | 0.768 |
| | (0.029) | (0.020) | (0.0002) |
| Neg LL | 2,279,524 | 1,906,749 | 2,268,233 |
| Category-Specific Nesting Parameter ($\lambda$) | | | |
| Pastry | 1.550 | 1.131 | 0.833 |
| | (0.101) | (0.026) | (0.021) |
| Cookie | 0.518 | 0.677 | 0.520 |
| | (0.044) | (0.051) | (0.039) |
| Chips | 0.202 | 1.465 | 0.805 |
| | (0.068) | (0.044) | (0.035) |
| Chocolate | 0.673 | 0.767 | 0.465 |
| | (0.108) | (0.055) | (0.041) |
| Candy | 0.332 | 0.492 | 0.475 |
| | (0.048) | (0.055) | (0.047) |
| Neg LL | 2,279,496 | 1,906,631 | 2,268,096 |
| Random Coefficients | | | |
| Fat | 0.566 | 0.000 | 0.306 |
| | (1.409) | (0.048) | (0.793) |
| Salt | 2.851 | 0.000 | 2.523 |
| | (0.045) | (0.047) | (0.010) |
| Sugar | 5.638 | 0.000 | 4.822 |
| | (0.021) | (0.251) | (0.007) |
| Neg LL | 2,279,544 | 1,906,756 | 2,267,203 |
| Choice Sets | 238 | 2649 | 3966 |
| Marketsize | 5,786,721 | 5,319,851 | 5,786,721 |

Full Availability assumes that all products stocked in a machine are available to all consumers (ie., it ignores stockout events). Ignore adjusts for stockouts during periods in which all sales and availability regimes are observed, but ignores (discards) periods in which stockouts happened at an unknown point in time. EM adjusts for all stockout events, regardless of whether the timing of a stockout was fully observed in the data. Standard errors are reported in parentheses. The nesting parameter $\lambda$ is the parameter specified in McFadden (1978), rather than the paramater $\sigma$ specified in Cardell (1997) or Berry (1994), where $\lambda \approx (1 - \sigma)$.

Table 5: Best Substitutes

| Product | Category-Specific Nested Logit | Random Coefficients |
|---|---|---|
| PopTart | Choc Donuts | Snickers |
| Choc Donuts | PopTart | Snickers |
| Ding Dong | Choc Donuts | Snickers |
| Banana Nut Muffin | Choc Donuts | PopTart |
| Rice Krispies | Choc Donuts | Snickers |
| Gma Oatmeal Raisin | Gma Choc Chip | Banana Nut Muffin |
| Chips Ahoy | Gma Choc Chip | Snickers |
| Nutter Butter Bites | Gma Choc Chip | Snickers |
| Knotts Raspberry Cookies | Gma Choc Chip | Snickers |
| Gma Choc Chip | Gma Oatmeal Raisin | Snickers |
| Rold Gold | Sunchip Harvest | Cheeto Crunchy |
| Sunchip Harvest | Rold Gold | Rold Gold |
| Dorito Nacho | Rold Gold | Rold Gold |
| Cheeto Crunchy | Rold Gold | Rold Gold |
| Ruffles Cheddar | Rold Gold | Rold Gold |
| Fritos | Rold Gold | Rold Gold |
| Lays Potato Chip | Rold Gold | Rold Gold |
| Munchies Hot | Rold Gold | Rold Gold |
| Misc Chips 2 | Rold Gold | Rold Gold |
| Munchies | Rold Gold | Rold Gold |
| Dorito Guacamole | Rold Gold | Rold Gold |
| Snickers | Twix | Twix |
| Twix | Snickers | Snickers |
| M&M Peanut | Snickers | Snickers |
| Reeses | Snickers | Snickers |
| Kit Kat | Snickers | Snickers |
| Caramel Crunch | Snickers | Snickers |
| M&M | Snickers | Snickers |
| Hershey Almond | Snickers | Snickers |
| Starburst | Skittles | Skittles |
| Kar Nut Sweet/Salt | Starburst | Snickers |
| Snackwell | Starburst | Snickers |
| Skittles | Starburst | Starburst |
| Oreo | Starburst | Snickers |
| Peanuts | Starburst | Rold Gold |

Reports the closest substitute for each of the 35 most commonly-held products, based on EM-corrected demand estimates.

Table 6: Predicted Weekly Sales

| | Category-Specific Nested Logit | | | | Random Coefficients Logit | | | |
|---|---|---|---|---|---|---|---|---|
| | Full | Ignore | EM | Diff | Full | Ignore | EM | Diff |
| PopTart | 9.08 | 8.92 | 9.49 | 0.41 | 8.75 | 8.8 | 9.21 | 0.46 |
| Choc Donuts | 8.75 | 9.85 | 10.63 | 1.88 | 8.50 | 9.77 | 10.47 | 1.96 |
| Ding Dong | 7.72 | 8.11 | 8.82 | 1.09 | 7.38 | 8.01 | 8.60 | 1.22 |
| Banana Nut Muffin | 6.67 | 6.57 | 7.04 | 0.37 | 6.54 | 6.47 | 6.91 | 0.37 |
| Rice Krispies | 4.99 | 4.57 | 4.90 | -0.08 | 4.97 | 4.49 | 5.00 | 0.04 |
| Gma Oatmeal Raisin | 6.71 | 6.21 | 6.84 | 0.13 | 6.85 | 6.31 | 6.83 | -0.02 |
| Chips Ahoy | 5.84 | 5.29 | 5.89 | 0.06 | 6.03 | 5.42 | 6.06 | 0.03 |
| Nutter Butter Bites | 4.28 | 3.88 | 4.27 | -0.01 | 4.39 | 3.95 | 4.38 | -0.01 |
| Knott's Raspberry Cookie | 4.18 | 3.83 | 4.18 | 0.00 | 4.26 | 3.90 | 4.24 | -0.02 |
| Gma Choc Chip | 8.22 | 7.24 | 8.21 | -0.01 | 7.75 | 6.93 | 7.68 | -0.07 |
| Rold Gold | 9.89 | 9.60 | 10.69 | 0.80 | 10.06 | 9.55 | 10.87 | 0.81 |
| Sunchip Harvest | 9.43 | 8.92 | 9.97 | 0.54 | 9.37 | 8.86 | 9.99 | 0.62 |
| Dorito Nacho | 8.22 | 7.43 | 8.20 | -0.02 | 8.17 | 7.36 | 8.24 | 0.07 |
| Cheeto Crunchy | 8.21 | 7.60 | 8.41 | 0.20 | 8.24 | 7.54 | 8.48 | 0.24 |
| Ruffles Cheddar | 6.91 | 6.25 | 6.89 | -0.03 | 6.90 | 6.19 | 6.94 | 0.04 |
| Fritos | 4.56 | 4.06 | 4.55 | -0.01 | 4.53 | 4.02 | 4.58 | 0.05 |
| Lays Potato Chip | 4.15 | 3.79 | 4.17 | 0.02 | 4.13 | 3.75 | 4.20 | 0.07 |
| Munchies Hot | 7.82 | 7.42 | 7.89 | 0.07 | 7.92 | 7.22 | 7.96 | 0.05 |
| Misc Chips 2 | 3.42 | 3.16 | 3.50 | 0.08 | 3.43 | 3.11 | 3.53 | 0.10 |
| Munchies | 5.36 | 4.81 | 5.44 | 0.08 | 5.28 | 4.80 | 5.46 | 0.18 |
| Dorito Guacamole | 4.71 | 4.10 | 4.60 | -0.11 | 4.61 | 4.10 | 4.61 | 0.01 |
| Snickers | 20.14 | 18.5 | 20.23 | 0.09 | 19.72 | 18.57 | 19.73 | 0.01 |
| Twix | 14.99 | 13.48 | 14.97 | -0.02 | 14.70 | 13.54 | 14.68 | -0.03 |
| M&M Peanut | 11.35 | 10.43 | 11.50 | 0.16 | 11.13 | 10.47 | 11.30 | 0.17 |
| Reese's Cup | 5.71 | 5.18 | 5.70 | -0.02 | 5.67 | 5.20 | 5.66 | -0.01 |
| Kit Kat | 5.23 | 4.72 | 5.21 | -0.02 | 5.06 | 4.74 | 5.05 | 0.00 |
| Caramel Crunch | 5.19 | 4.74 | 5.16 | -0.03 | 5.12 | 4.77 | 5.11 | -0.01 |
| M&M | 5.76 | 5.28 | 5.82 | 0.07 | 5.65 | 5.26 | 5.66 | 0.01 |
| Hershey Almond | 3.99 | 3.64 | 3.97 | -0.03 | 3.96 | 3.66 | 3.96 | -0.01 |
| Starburst | 8.67 | 7.98 | 8.83 | 0.16 | 8.62 | 8.28 | 8.68 | 0.07 |
| Kar Nut Sweet/Salt | 6.45 | 5.95 | 6.60 | 0.15 | 6.84 | 6.28 | 6.88 | 0.04 |
| Snackwell | 3.81 | 3.53 | 3.92 | 0.11 | 4.07 | 3.74 | 4.12 | 0.04 |
| Skittles | 8.08 | 7.31 | 8.12 | 0.04 | 8.11 | 7.21 | 8.02 | -0.09 |
| Oreo | 2.33 | 2.14 | 2.39 | 0.05 | 2.49 | 2.27 | 2.49 | 0.00 |
| Peanuts | 4.50 | 4.11 | 4.50 | 0.00 | 4.39 | 4.02 | 4.41 | 0.02 |
| Total | 245.34 | 228.59 | 251.53 | 6.19 | 243.59 | 228.56 | 250.00 | 6.41 |

Reports predicted weekly sales for each of the 35 most commonly-held products under the Full Availability, Ignore, and EM-corrected models. Diff reports the difference in weekly sales between the EM-corrected and Full Availability estimates for each of the two specifications (Category-specific Nested Logit and Random Coefficients Logit).

Table 7: Weekly Sales Impact of Simulated Stockout

| | Category-Specific Nested Logit | | | Random Coefficients Logit | | |
|---|---|---|---|---|---|---|
| | Full | Ignore | EM | Full | Ignore | EM |
| PopTart | -9.08 | -8.92 | -9.49 | -8.75 | -8.80 | -9.21 |
| Choc Donuts | -8.75 | -9.85 | -10.63 | -8.50 | -9.77 | -10.47 |
| Ding Dong | -2.13 | -0.54 | 1.21 | 3.11 | 0.17 | 2.97 |
| Banana Nut Muffin | -1.84 | -0.44 | 0.97 | 4.02 | 0.14 | 3.60 |
| Rice Krispies | -1.37 | -0.31 | 0.67 | 0.56 | 0.10 | 0.49 |
| Gma Oatmeal Raisin | -6.71 | -6.21 | -6.84 | -6.85 | -6.31 | -6.83 |
| Chips Ahoy | -5.84 | -5.29 | -5.89 | -6.03 | -5.42 | -6.06 |
| Nutter Butter Bites | 1.41 | 0.88 | 1.42 | 0.56 | 0.09 | 0.48 |
| Knott's Raspberry Cookie | 1.38 | 0.87 | 1.40 | 0.94 | 0.08 | 0.77 |
| Gma Choc Chip | 2.71 | 1.64 | 2.74 | 1.83 | 0.15 | 1.49 |
| Rold Gold | -9.89 | -9.60 | -10.69 | -10.06 | -9.55 | -10.87 |
| Sunchip Harvest | -9.43 | -8.92 | -9.97 | -9.37 | -8.86 | -9.99 |
| Dorito Nacho | 2.46 | -0.91 | 0.67 | 0.44 | 0.16 | 0.41 |
| Cheeto Crunchy | 2.45 | -0.93 | 0.69 | 0.83 | 0.16 | 0.72 |
| Ruffles Cheddar | 2.07 | -0.76 | 0.57 | 0.44 | 0.13 | 0.39 |
| Fritos | 1.36 | -0.50 | 0.37 | 0.20 | 0.09 | 0.19 |
| Lays Potato Chip | 1.24 | -0.46 | 0.34 | 0.19 | 0.08 | 0.18 |
| Munchies Hot | 2.34 | -0.91 | 0.65 | 0.47 | 0.16 | 0.42 |
| Misc Chips 2 | 1.02 | -0.39 | 0.29 | 0.15 | 0.07 | 0.15 |
| Munchies | 1.60 | -0.59 | 0.45 | 0.42 | 0.10 | 0.38 |
| Dorito Guacamole | 1.41 | -0.50 | 0.38 | 0.16 | 0.09 | 0.15 |
| Snickers | -20.14 | -18.50 | -20.23 | -19.72 | -18.57 | -19.73 |
| Twix | -14.99 | -13.48 | -14.97 | -14.70 | -13.54 | -14.68 |
| M&M Peanut | 2.96 | 2.00 | 5.16 | 2.77 | 0.23 | 2.32 |
| Reese's Cup | 1.49 | 0.99 | 2.56 | 1.21 | 0.11 | 0.99 |
| Kit Kat | 1.37 | 0.90 | 2.34 | 1.67 | 0.10 | 1.38 |
| Caramel Crunch | 1.36 | 0.91 | 2.32 | 1.18 | 0.10 | 0.97 |
| M&M | 1.50 | 1.01 | 2.61 | 1.94 | 0.11 | 1.61 |
| Hershey Almond | 1.04 | 0.70 | 1.78 | 0.66 | 0.08 | 0.54 |
| Starburst | -8.67 | -7.98 | -8.83 | -8.62 | -8.28 | -8.68 |
| Kar Nut Sweet/Salt | -6.45 | -5.95 | -6.60 | -6.84 | -6.28 | -6.88 |
| Snackwell | 1.94 | 1.34 | 1.58 | 0.76 | 0.08 | 0.63 |
| Skittles | 4.10 | 2.78 | 3.27 | 4.51 | 0.16 | 3.91 |
| Oreo | 1.19 | 0.81 | 0.96 | 0.69 | 0.05 | 0.56 |
| Peanuts | 2.29 | 1.56 | 1.81 | 0.12 | 0.09 | 0.12 |

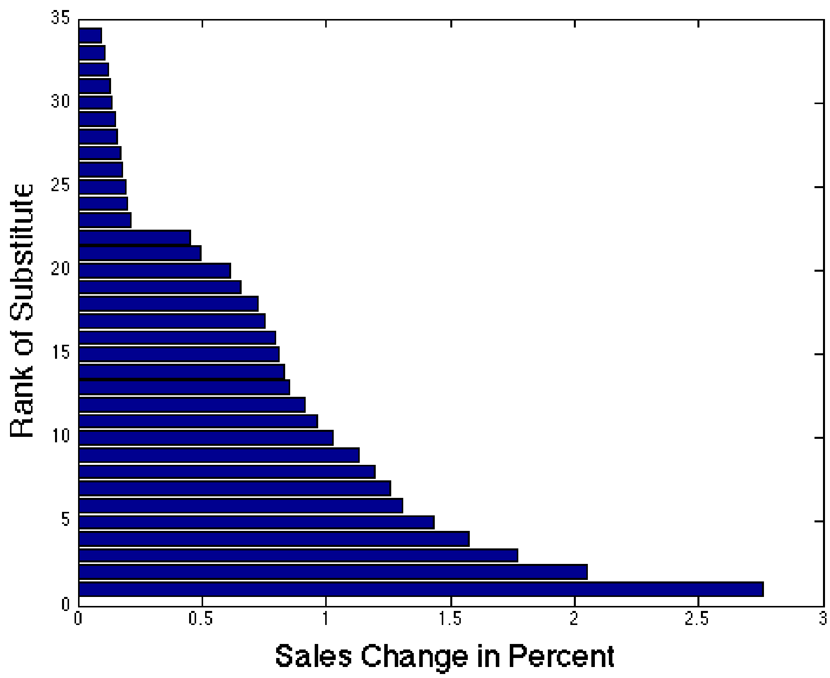
Reports predicted weekly sales impact on the 35 most commonly-held products of a simulated stockout of the top two selling products in each category (Chocolate Donuts, Strawberry Frosted PopTarts, Grandma's Oatmeal Raisin Cookie, Chips Ahoy Cookies, Rold Gold Pretzels, Sunchips Harvest Cheddar, Snickers, Twix, Starburst, and Kar Nut's Sweet & Salty Mix).

Table 8: Weekly Profit Impact of Simulated Stockout

| | Category-Specific Nested Logit | | | Random Coefficients Logit | | |
|---|---|---|---|---|---|---|
| | Full | Ignore | EM | Full | Ignore | EM |
| Foregone Sales of Stockout | | | | | | |
| Pastry | -17.83 | -18.77 | -20.12 | -17.25 | -18.58 | -19.68 |
| Cookie | -12.55 | -11.50 | -12.74 | -12.88 | -11.73 | -12.89 |
| Chips | -19.32 | -18.52 | -20.66 | -19.44 | -18.41 | -20.87 |
| Chocolate | -35.13 | -31.98 | -35.20 | -34.42 | -32.11 | -34.41 |
| Candy | -15.13 | -13.93 | -15.44 | -15.46 | -14.56 | -15.56 |
| Total | -99.95 | -94.69 | -104.15 | -99.45 | -95.39 | -103.4 |
| Increased Sales of Substitutes | | | | | | |
| Pastry | -5.34 | -1.29 | 2.85 | 7.69 | 0.41 | 7.06 |
| Cookie | 5.51 | 3.38 | 5.56 | 3.33 | 0.32 | 2.73 |
| Chips | 15.95 | -5.95 | 4.41 | 3.30 | 1.04 | 2.98 |
| Chocolate | 9.73 | 6.50 | 16.77 | 9.43 | 0.74 | 7.81 |
| Candy | 9.51 | 6.49 | 7.62 | 6.06 | 0.37 | 5.22 |
| Total | 35.35 | 9.14 | 37.20 | 29.81 | 2.88 | 25.81 |
| Change in Total Sales | | | | | | |
| Pastry | -23.17 | -20.05 | -17.27 | -9.56 | -18.16 | -12.62 |
| Cookie | -7.04 | -8.12 | -7.17 | -9.55 | -11.41 | -10.16 |
| Chips | -3.37 | -24.46 | -16.25 | -16.14 | -17.37 | -17.88 |
| Chocolate | -25.40 | -25.48 | -18.44 | -25.00 | -31.37 | -26.6 |
| Candy | -5.61 | -7.44 | -7.81 | -9.39 | -14.19 | -10.34 |
| Total | -64.6 | -85.55 | -66.95 | -69.65 | -92.50 | -77.59 |
| % Staying Inside | | | | | | |
| Pastry | 29.97 | 6.86 | 14.15 | 44.6 | 2.21 | 35.89 |
| Cookie | 43.88 | 29.42 | 43.68 | 25.83 | 2.73 | 21.20 |
| Chips | 82.55 | 32.10 | 21.33 | 16.95 | 5.65 | 14.29 |
| Chocolate | 27.69 | 20.33 | 47.63 | 27.38 | 2.30 | 22.7 |
| Candy | 62.88 | 46.60 | 49.38 | 39.23 | 2.56 | 33.55 |
| Total | 35.37 | 9.66 | 35.72 | 29.97 | 3.02 | 24.96 |
| Change in Gross Profit | | | | | | |
| Pastry | -13.72 | -11.86 | -10.27 | -6.04 | -10.76 | -7.75 |
| Cookie | -3.49 | -4.08 | -3.56 | -4.80 | -5.82 | -5.12 |
| Chips | -2.41 | -15.30 | -10.32 | -10.22 | -10.96 | -11.32 |
| Chocolate | -10.67 | -10.70 | -7.74 | -10.50 | -13.18 | -11.17 |
| Candy | -2.72 | -3.54 | -3.73 | -4.62 | -6.64 | -5.04 |
| Total | -33.01 | -45.48 | -35.62 | -36.19 | -47.36 | -40.40 |

Reports predicted weekly profit impact of a simulated stockout of the top two selling products in each category (Chocolate Donuts, Strawberry Frosted PopTarts, Grandma's Oatmeal Raisin Cookie, Chips Ahoy Cookies, Rold Gold Pretzels, Sunchips Harvest Cheddar, Snickers, Twix, Starburst, and Kar Nut's Sweet & Salty Mix). Uses the same group of 35 most commonly-held products in sales and sales impact tables.

Figure 1: Median Change in Sales of Substitutes by Rank




Reports the median change in sales of substitute by rank using the demand parameters from the EM-corrected random-coefficients specification.

# A Appendix

## A.1 Latent Types

As discussed in section 2.4, one might worry that our method relies too heavily on the assumption of conditional independence in some settings. For example, suppose we had daily data and we knew that there were very different consumer segments making purchases in the morning and the afternoon. If we knew a stockout happened sometime during the day, we might not expect the same distribution of consumer types before and after the stockout. This would generate latent consumer types, which are not addressed in our baseline model.

In this section, we discuss a method for using finite mixtures to incorporate latent types. We then show that when we add latent types into a model with latent stockout events, we require information on the joint distribution of the two sets of mixing parameters: those that apply to the latent types, and those that apply to the latent stockout events. We discuss the implication of this for alternative approaches that use simulation.

First consider how one might incorporate latent types in the absence of stock-out events. Suppose there are two consumer segments, type $A$ and type $B$, with different tastes for a characteristic $x$. Each type has its own mean and standard deviation for its taste for $x$, denoted $\theta = [\mu_A, \mu_B, \sigma_A, \sigma_B]$. In addition, a mixing parameter, $\gamma$ indicates the share of segment $A$ in the population.[38] If we consider two products with different prices, and two consumer segments varying in their distaste for price, then the following choice probabilities apply for each type (assume non-subscripted elements are the full vector, and a random-coefficients logit specification).

$$
\begin{aligned}
p_{jtA}(a_t, x, d, \mu_A, \sigma_A) &= \int \frac{\exp(\beta_i x_{jt} + d_j)}{1 + \sum_{k \in a_t} \exp(\beta_i x_{kt} + d_k)} \phi(\beta_i | \mu_A, \sigma_A) \\
p_{jtB}(a_t, x, d, \mu_B, \sigma_B) &= \int \frac{\exp(\beta_i x_{jt} + d_j)}{1 + \sum_{k \in a_t} \exp(\beta_i x_{kt} + d_k)} \phi(\beta_i | \mu_B, \sigma_B)
\end{aligned}
$$

The resulting population shares would then be a mixture of the two:

$$
p_{jt}(x, \theta) = \gamma \cdot p_{jtA}(a_t, x, d, \mu_A, \sigma_A) + (1 - \gamma) \cdot p_{jtB}(a_t, x, d, \mu_B, \sigma_B)
$$

We could estimate this model using a variety of techniques, including ML, where $\theta = [\delta, \mu_A, \mu_B, \sigma_A, \sigma_B, \gamma]$. When $p_{jt}$ is computed via simulation, then we use MSL.

[38] In many datasets, one observes aggregate annual sales, and uses changes in average annual prices as the primary source of variation in choice sets. Standard models assume that all consumers face the same choice set in a particular year, and that they are exchangeable (or IID). It is easy to generate exceptions to this. For example, imagine an iPhone that costs \$600 for the first six months, and \$400 for the next six months. Annual sales are reported with an average annual price of \$500, and we cannot recover the relevant structural parameters for the early vs. late purchasers.

It may also seem appealing to include moments regarding observed substitution after stockouts and use this to improve our estimates. Recall the multinomial is a semi-parametric form; once we fix a parameterization for $p_{jt}(\theta)$, our focus is on pinning down the mapping, not fitting the cell counts. We are already using all of the stockout information in identification of the choice probabilities, and ML estimation achieves the semi-parametric efficiency bound.[39]

Now we add latent stockouts to the estimation. Suppose we observe a machine at 4pm and at 8pm. There are still two types of customers: students and administrative staff, with different distributions of preferences for snack foods. Each $\beta_i$ is drawn from either $\beta_i \sim (b_S, W_S)$ or $\beta_i \sim (b_A, W_A)$ for students and administrators respectively. We know that the staff all leave at 5pm sharp, and the students are around until at least 8pm. This scenario is a bit problematic, because we do not expect the same proportions of students and staff before and after the stockout, and they are thus non-exchangeable.

In cases where such phenomena are important, we could specify the random coefficients choice probabilities for students and staff before the stockout as $p_j(a_t, \theta_S)$, and $p_j(a_t, \theta_A)$ respectively, and after the stockout as $p_j(a_s, \theta_S)$, and $p_j(a_s, \theta_A)$. For a guess of $\theta = [\theta_A, \theta_S]$ we know all four choice probabilities. Now we have a four-element mixture with three mixing parameters; $\alpha$ governs before and after the stockout and $\gamma$'s indicate the share of each subpopulation before and after the stockout. Market shares are now given by:

$$p_{jt}(\theta, \alpha, \gamma) = \alpha\left[\gamma_t p_j(a_t, \theta_S) + (1-\gamma_t) p_j(a_t, \theta_A)\right] + (1-\alpha)\left[\gamma_s p_j(a_s, \theta_S) + (1-\gamma_s) p_j(a_s, \theta_A)\right]$$

The same two approaches we used before are available. We can work with the "mixed" probability or we can impute the sales (sufficient statistics) for each of the four cases and use an EM-type procedure. In the baseline case without latent types, the model implies a distribution on $\alpha$ (through the conditional negative binomial distribution on sales), which is easy to integrate out. We could use a similar technique if we fixed:

$$\begin{aligned}
\tilde{p}_j(a_t, \theta, \gamma) &= \gamma_t p_j(a_t, \theta_S) + (1-\gamma_t) p_j(a_t, \theta_A) \\
\tilde{p}_j(a_s, \theta, \gamma) &= \gamma_s p_j(a_s, \theta_S) + (1-\gamma_t) p_j(a_s, \theta_A)
\end{aligned}$$

[39] Relatedly, random-coefficients demand estimators like that in Berry, Levinsohn, and Pakes (1995) use a mapping between parameters and data to produce choice probabilities. Those estimators include an error in the space of latent utilities rather than in the space of choice probabilities (i.e., the product-specific unobservables, $\xi_{jt}$'s), and require that the MSM condition on choice probabilities holds exactly. This is essentially the same problem we face: under the "true" model, each consumer has her own $\beta_i$, but the data do not include information about this latent consumer type. Stockouts are another example where data relevant to the choice probability computation is unobserved, except now it is the sales under different availability regimes. Thus, the two types of estimators represent two ways to handle these sorts of problems. One possibility is to impute the missing sufficient statistics throughout the dataset (this is how we handle stockouts). The other is to specify a more complicated mixture form for choice probabilities, which integrates out the latent variable (this is how BLP-style estimators deal with random coefficients). Just as we can (and do) formulate the stockout problem as a mixture of choice probabilities across availability regimes, we could in fact write the random coefficients model as a (finite-mixture) missing data problem if we discretized the type-space. An example of a paper that does this (although not explicitly as such) is Bajari, Fox, and Ryan (2006).

While this procedure would work, we would lose smoothness in the choice probabilities, which makes optimization difficult.

Whether we choose to impute sales for all four cases, or work with the mixture likelihood, we need to make an assumption on the joint distribution of $(\alpha, \gamma)$. While we knew the marginal distribution of $\alpha$, we do not know this joint distribution without more data or assumptions. One could assume that all administrative staff leave at 5pm sharp, which would imply a joint distribution. Alternatively, one could add free parameters to the model. However, doing this several times for each stockout scenario would dramatically increase the parameter space. It should also be clear that simulating consumer utilities to generate probabilities is not going to solve this, because we still need to specify the joint distribution of $(\alpha, \gamma)$.

## A.2 Multiple Unobserved Stockouts

Addressing the case of multiple unobserved stockouts is quite similar to the single stockout case. The rest of the estimation procedure proceeds just as it did in the case of a single unobserved stockout, with the exception of the E-step (where the missing data are imputed). Conditional on the imputed values for the missing data, the M-step remains unchanged.

Suppose we denoted the single stockout case as a problem in which we integrated over the fraction of consumers facing a particular availability regime. Thus there are two availability regimes, which apply to fractions $\alpha$ and $(1-\alpha)$ of the population, and one would write:

$$E[y^s_{jt}|\theta] = y_{jt} \int \frac{\alpha p_{jt}(\theta, a_s, x_t)}{\alpha p_{jt}(\theta, a_s, x_t) + (1-\alpha) p_{jt}(\theta, a_t, x_t)} g(\alpha|\theta, \mathbf{y_t}) \partial \alpha$$

When moving to $K$ unobserved stockouts, there are $2^K$ possible availability regimes, and $(2^K - 1)$ parameters $\boldsymbol{\alpha} = [\alpha_0, \alpha_A, \alpha_B, \ldots, \alpha_{AB}, \ldots]$ where subscripts denote stocked out products:

$$E[y^s_{jtl}|\theta] = y_{jt} \int \frac{\alpha_l p_{jt}(\theta, a_l, x_t)}{\sum_{\forall s} \alpha_s p_{jt}(\theta, a_s, x_t)} g(\boldsymbol{\alpha}|\theta, \mathbf{y_t}) \partial \boldsymbol{\alpha}$$

Once again we can evaluate the expectation exactly, by evaluating at every $\alpha$ in the domain, but this is now computationally much more difficult. If we think about the dimension of the problem, the order of the stockouts now matters (since different orders imply different choice probabilities). There are $K!$ ways to order $K$ stockouts. Once we assume an ordering we must divide up $M_t$ consumers among the $K$ availability regimes. This means that the summation would require $\binom{M_t}{K}$ elements for each possible ordering of stockouts or $K! \times \binom{M_t}{K} = \frac{M_t!}{M_t - K!}$ elements overall. For $M_t$ large and $K$ small this is (roughly) approximated as $M^K$. If a week's worth of data contains $M_t = 1000$ and $K = 5$, this implies $10^{15}$ elements. In this case, approximate methods must be used to compute the expectation.

Suppose we try to compute $g(\boldsymbol{\alpha}|\cdot)$ for a single value of $\boldsymbol{\alpha}$. An important property to notice is that each $\boldsymbol{\alpha}$ vector implies a unique sequence of stockouts. If we had $K = 2$ products then $\boldsymbol{\alpha} = [\alpha_0, \alpha_A, \alpha_B, \alpha_{AB}]$. In this case, we know that $[0.2, 0.3, 0, 0.5]$ implies that product $A$ stocked out first, and then product $B$. Likewise $[0.5, 0, 0.1, 0.4]$ tells us that product $B$ stocked out first and then product $A$. We might think about $\boldsymbol{\alpha} = [0.3, 0.2, 0.2, 0.3]$, but this is impossible since we could not have observed a sequence where $A$ was available when $B$ was not AND also observed $B$ available when $A$ was not. The probability of such a sequence is zero.

We define $\boldsymbol{\omega} = [\omega_1, \omega_2, \ldots, \omega_K]$ as the beginning of period inventory for the $K$ products that stocked out, where $(\boldsymbol{\omega}, \boldsymbol{\alpha})$ are arranged in the order of the stockouts. It is also helpful to write $a(\alpha^{(k)}) = a_k$ which denotes the availability set corresponding to the $k^{th}$ component of $\boldsymbol{\alpha}$, and to define $\boldsymbol{q}$ as the ordered sales vector for all stocked-out products, and $\boldsymbol{p}(\theta, a(\alpha), x_t)$ as the vector of choice probabilities with $(K+1)$ elements where the first $K$ elements are $p_k(\theta, a(\alpha), x_t)$ for the $K$ products which stockout, and $p_{k+1} = \sum_{j \in a(\alpha)} p_j(\theta, a(\alpha), x_t)$, or the sum of the choice probabilities of all other goods (including the outside good). Finally we define an operator $h(\cdot)$ which takes a $K$ dimensional vector and returns the vector of the last $K-1$ elements.

Now we can define a finite recursive relationship for $g(\boldsymbol{\alpha}, \boldsymbol{\omega}, \theta, M_t)$, for the case where $K > 1$ in terms of the multinomial generalization of the negative binomial, the negative multinomial (defined below):

$$
\begin{aligned}
g(\boldsymbol{\alpha}, \boldsymbol{\omega}, \theta, M_t) &= \sum_{\forall h(\mathbf{q}): h(\mathbf{q} < \boldsymbol{\omega})} NMult\left(\alpha^{(1)} M_t, \omega^{(1)}, h(\mathbf{q}), \mathbf{p}(a(\alpha^{(1)}), \theta)\right) \cdot g(h(\boldsymbol{\alpha}), h(\boldsymbol{\omega} - \mathbf{q}), \theta, M_t) \\
&= \sum_{q^{(2)}=0}^{\omega^2 - 1} \sum_{q^{(3)}=0}^{\omega^3 - 1} \cdots \sum_{q^{(K)}=0}^{\omega^K - 1} NMult\left(\alpha^{(1)} M_t, \omega^{(1)}, h(\mathbf{q}), \mathbf{p}(a(\alpha^{(1)}), \theta)\right) \cdot g(h(\boldsymbol{\alpha}), h(\boldsymbol{\omega} - \mathbf{q}), \theta, M_t)
\end{aligned}
$$

And for the base case $K = 1$ (all arguments scalar), it is identical to the single stockout case:

$$
g(\alpha, \omega, M_t) = \frac{NegBin(\alpha M_t - \omega, \omega, p_j(a(\alpha), x_t, \theta))}{NegBinCDF(M_t, \omega, p_j(a(\alpha), x_t, \theta))}
$$

In words, to evaluate the density at a vector of weights for different availability regimes, we "pop" the first element of the availability regime and inventory off of our stack, and compute the negative multinomial probability times the function $g(\cdot)$ applied to the remaining stack. Each time we call $g(\cdot)$ we must evaluate the sum over all possible sales configurations for the products that did not stock out during that particular availability regime, but eventually stocked out. This can still be computationally burdensome (and for some problems infeasible). This represents a dramatic savings because we need not worry about the ordering of products that did not stock out, thus there are only $K < J$ sums to evaluate.

### Negative Multinomial

The negative multinomial is simply the multinomial generalization of the negative binomial. This entire family of distributions (binomial, multinomial, geometric, negative binomial, negative multinomial, etc.) are all just derived distributions for the Bernoulli process. We have results for multinomials, and geometrics, etc. because they frequently occur in applied problems, and these standard results are often incorporated in textbooks and statistical packages. The negative multinomial is a bit less common, and results are not as well known.

The negative multinomial is similar to the negative binomial in that it describes the probability of the number of failures $r_t$ before $\omega_{kt}$ successes of the first cell are observed. What makes it different from the negative binomial is that it also accounts for $q_2, \ldots, q_K$ successes of the next $K-1$ cells. The p.m.f. can be written:

$$NMult\left(r_t, \omega_{kt}, \mathbf{q}, \mathbf{p}(a, \theta)\right) = \frac{(r_t + \omega_{kt} + \sum_k q_{kt} - 1)!}{(r_t - \omega_{kt} - 1)! \omega_{kt}! \ldots q_K! q_0!} p_1^{\omega_{kt}} p_2^{q_2} \ldots p_k^{q_k} p_0^{r_t}$$

## A.3 Alternative Computational Methods

For up to three unobservable stockouts, the exact method we present in the text is generally computationally feasible. However, when there are many stockouts at once, integrating over the distribution $g(\boldsymbol{\alpha}|\cdot)$ at all values of the domain becomes prohibitive, and approximate methods must be used. Recall the form of the E-step:

$$E[y_{jt}^s] \quad = \quad \sum_{\forall s} y_{jt} \frac{\alpha_s p_j(\theta, a_s, x_t)}{\sum_{\forall r} \alpha_r p_j(\theta, a_r, x_t)} g(\boldsymbol{\alpha}|\theta, y_{jt})$$

This can be easily approximated by linear functions since it is smooth and of the form $\frac{f(z_i)}{\sum_i f(z_i)} \in [0, 1]$, and because many stockouts do not induce large changes in $p_j(\cdot)$ (a stockout of Doritos often has very little effect on sales of Snickers). This means that quadrature, Monte Carlo integration, and Quasi-Monte Carlo integration should work fine even with a small number of points at which the function is evaluated. One way to generate random draws from $g(\cdot)$ is to pick an ordering for stockouts and then successively draw from the negative multinomial distribution, and repeat this for all possible orderings of the stockouts.

For extremely high dimensional problems we might find that even drawing from $g(\cdot)$ becomes too burdensome, as there are an increasing number of potential orderings for stockouts. One could consider simulating consumer purchases (because we know the sales are distributed as a multinomial for a given set of parameters $\theta$). One only needs to track those products stocked out and an 'other' option. In this case, we simulate consumers until we've observed all of the stockouts. We can count the fraction of all consumers facing each availability set and compute $\alpha$ directly. One of the key benefits of our method is that it allows us to compute the expectation of the missing data by only evaluating over distributions of the stocked-out products, without resorting to computing the likelihood for every possible permutation of sales.

## A.4 Identification Details for Random Coefficients Specification

Identification of nonlinear parameters in a random-coefficients specification comes from the fact that the sales of two products $j, k$ will be differentially affected by a stockout of product $l$ depending on how similar their characteristics $x_j, x_k$ are to $x_l$. In the case where we have only one continuous, real-valued $x_{jt}$ characteristic with a random coefficient, we can consider the choice probabilities for good $j$ before and after the stockout of good $l$. That is availability regime $a_t' = \{a_t\} \setminus \{l\}$. The choice probabilities are:

$$
\begin{aligned}
p_{jt}(\theta) &= \frac{1}{ns}\sum_{i=1}^{ns} P_{ijt} = \frac{1}{ns}\sum_{i=1}^{ns} \frac{\exp[d_j + \sigma v_i x_j]}{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]} \\
\tilde{p}_{jt}(\theta) &= \frac{1}{ns}\sum_{i=1}^{ns} \frac{\exp[d_j + \sigma v_i x_j]}{1+\sum_{k\in a_t'}\exp[d_k + \sigma v_i x_k]} \\
&= \frac{1}{ns}\sum_{i=1}^{ns} \frac{\exp[d_j + \sigma v_i x_j]}{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]} \cdot \frac{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]}{1+\sum_{k\in a_t'}\exp[d_k + \sigma v_i x_k]} \\
&= \frac{1}{ns}\sum_{i=1}^{ns} \frac{P_{ijt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)}{1 - P_{ilt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)}
\end{aligned}
$$

Where the last equation follows because:

$$
\begin{aligned}
\frac{1+\sum_{k\in a_t'}\exp[d_k + \sigma v_i x_k]}{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]} &= \frac{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]}{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]} - \frac{\exp[d_l + \sigma v_i x_l]}{1+\sum_{k\in a_t}\exp[d_k + \sigma v_i x_k]} \\
&= 1 - P_{ilt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)
\end{aligned}
$$

Specifically, for each consumer type $i$ we inflate the probability of buying good $j$ by a factor proportional to the probability that type $i$ bought the stocked-out good $l$.[40] Thus, we can think about a stockout as providing information not only about the level of $p_{jt}$, but also about the ratio of the choice probabilities before and after a stockout.

$$
\frac{\tilde{p}_{jt}}{p_{jt}} = \frac{\sum_{i=1}^{ns} \frac{P_{ijt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)}{1 - P_{ilt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)}}{\sum_{i=1}^{ns} P_{ijt}(a_t, \mathbf{x}, \mathbf{d}, \sigma)}
$$

Thus, identification comes from the fact that the correlation between choice probabilities $(p_{jt}, p_{lt})$ is determined by $\sigma(x_j - x_l)$.

[40] In the case of the plain logit model, $P_{ilt}$ is constant across all types $i$, and we recover the IIA property so that all products are inflated by the same $\frac{1}{1-p_{lt}}$ factor.

### A.5 Treating the Stockout Time as a Free Parameter

In section 2, we show that the likelihood for the dataset can be written down as the sum of the fully-observed and partially-observed observations, where the partially-observed observations must be integrated over the unobservable stockouts.

$$\hat{\theta} = \arg\max_{\theta} \sum_{\forall (a,x)} \left( \sum_{\forall t \in T_{mis}} E[y^{mis}_{j,(a,x)} | \theta] \ln p_j(\theta, a, x) + \sum_{\forall t \in T_{obs}} y^{obs}_{j,(a,x)} \ln p_j(\theta, a, x) \right)$$

This is the expected likelihood of the observed data $(\mathbf{a}, \mathbf{x}, \mathbf{y})$ given the parameter $\theta$. Let $\alpha$ denote the fraction of consumers arriving before the stockout and $1 - \alpha$ denote the fraction of consumers arriving after the stockout. We could compute the expected sales for each product before and after the stockout:

$$\begin{aligned} E[y^{before}_{jt}] &= y_{jt} \int \frac{\alpha p_{jt}(\theta, a_t, x_t)}{\alpha p_{jt}(\theta, a_t, x_t) + (1-\alpha) p_{jt}(\theta, a_s, x_t)} f(\alpha) \partial \alpha \quad \forall j \\ E[y^{after}_{jt}] &= y_{jt} - E[y^{before}_{jt}] \end{aligned}$$

This is essentially what we do in the E-Step of our EM procedure, though with a slightly different parameterization for the stockout time. An alternative might be to consider the marginal data augmentation framework of Tanner and Wong (1987), in which we think of the stockout time $\alpha$ as the missing data and estimate it as an additional parameter. In general this approach works when the integral is single dimensional because the integrand is a convex combination of choice probabilities. That is, there might exist a $\hat{\alpha}$ such that:

$$\frac{\hat{\alpha} p_{jt}(\theta, a_t, x_t)}{\hat{\alpha} p_{jt}(\theta, a_t, x_t) + (1-\hat{\alpha}) p_{jt}(\theta, a_s, x_t)} = \int \frac{\alpha p_{jt}(\theta, a_t, x_t)}{\alpha p_{jt}(\theta, a_t, x_t) + (1-\alpha) p_{jt}(\theta, a_s, x_t)} f(\alpha) \partial \alpha$$

If this were true we could treat $\hat{\alpha}$ as an additional parameter to estimate. Unfortunately, we don't have a single equation, but rather a set of $(J-1)$ equations (one for each product that did not stock out), and only a single $\alpha$. Thus only in very special (degenerate) cases can a single $\hat{\alpha}$ satisfy all $(J-1)$ equations.[41] This highlights the importance of always letting the E-Step operate on the sufficient statistics for estimation, rather than some other quantity. In our case, the sufficient statistics are sales under each regime, rather than stockout times. Our approach does follow the marginal data augmentation framework of Tanner and Wong (1987), but it works by considering a model where we know sales under all availability sets (even though these aren't directly observed), rather than integrating the likelihood at each guess of the parameters.

Standard approaches do not solve (12), but rather assume a different $f(\alpha)$ other than the

[41]The independent poisson model as used by Anupindi, Dada, and Gupta (1998) is such a degenerate case.

one the stockout distribution implies. For example, in the case where we ignore the missing data it would be as if we set $f(\alpha) = 0$ everywhere that things are ambiguous (which is not a proper density distribution). Or in the case of full availability it would be as if we set $f(\alpha)$ to be a delta function that took on value 1 only at the full availability value of $a_t$. Assuming stockouts happen at the beginning or the end of the period places similar structure on $f(\alpha)$ (making it a delta function). The problem with this is that $f(\alpha)$ does not have any free parameters, but is completely specified by the demand parameters as a conditional negative binomial.

## A.6 Standard Errors

The covariance matrix is just the inverse Fisher Information matrix or the negative expected Hessian, and for maximum likelihood estimators we use the outer product of scores in place of the Hessian, denoted as $H(\theta)$:

$$\begin{aligned} H(\theta) &= -E\left[\frac{\partial^2 L(\theta|Z)}{\partial\theta^2}\right] = -E\left[\frac{\partial l(\theta|Z)}{\partial\theta}\frac{\partial l(\theta|Z)'}{\partial\theta}\right] = \left[\sum_i \frac{\partial l_i(\theta|Z)}{\partial\theta} \times \frac{\partial l_i(\theta|Z)'}{\partial\theta}\right] \\ l_i(\theta|y) &= y_{jt}(\theta)\ln p_{jt}(\theta) \end{aligned}$$

We obtain the score by differentiating the log-likelihood:

$$\begin{aligned} \frac{\partial l_i(\theta|y)}{\partial\theta} &= \tfrac{\partial \hat{y}_{jt}(\theta)}{\partial\theta}\ln p_{jt}(\theta) \quad + \underbrace{\hat{y}_{jt}(\theta)\frac{1}{p_{jt}(\theta)}\frac{\partial p_{jt}(\theta)}{\partial\theta}} \\ &= \textstyle\sum_{jt} \tfrac{\partial \hat{y}_{jt}(\theta)}{\partial\theta}\ln p_{jt}(\theta) \quad + \quad \frac{\partial l_i(\theta|\hat{y_{jt}})}{\partial\theta} \end{aligned}$$

This gradient can be composed into two parts. The gradient of the observed data $\hat{y}_{jt}$ is the sum of two components, the gradient where we assume we have the complete data, that is where $\hat{y}_{jt}$ is treated as the truth, and a correction for the fact that the sufficient statistics are not fixed observable quantities but rather random quantities containing some uncertainty. We get the score as the gradient of the log-likelihood at the final parameter values, so all we need to do is compute the correction. The easiest way to actually compute this is to numerically differentiate the imputed sufficient statistics $\frac{\partial y_{jt}(\theta)}{\partial\theta}$

Note that were we to compute $\frac{\partial \hat{y}_{jt}(\theta)}{\partial \theta}$, it would be:

$$\begin{aligned}
\frac{\partial \hat{y}_{jt}(\theta)}{\partial \theta} &= \frac{\partial}{\partial \theta}\left[\sum_{r_t \leq M_t} y_{jt} \frac{r_t \cdot p_{jt}(\theta)}{r_t \cdot p_{jt}(\theta) + (M_t - \omega_{kt} - r_t) \cdot p'_{jt}(\theta)} h(r_t|, \omega_{kt}, p_k(\theta))\right] \\
&= \frac{\widehat{y^0_{jt}}}{p_{jt}} \frac{\partial p_{jt}}{\partial \theta} + \widehat{y^0_{jt}} b_t \frac{\partial p_{lt}}{\partial \theta} - \left(\frac{\partial p_{jt}}{\partial \theta} - \frac{\partial p'_{jt}}{\partial \theta}\right) \frac{\widehat{y^2_{jt}}}{p_{jt}} - (M_t - y_{jt}) \frac{\partial p'_{jt}}{\partial \theta} y_{jt} p_{jt} E_r\left[\frac{r_t}{D(r_t)^2}\right] \\
&+ \frac{\partial p_{lt}}{\partial \theta} y_{jt} p_{jt} E_r\left[\frac{r_t^2}{D(r_t)}\right]
\end{aligned}$$

where

$$b_t = \left(\frac{2\omega_{kt}}{p_k} - \frac{M_t - \omega_{kt}}{1 - p_k}\right)$$

and

$$D(r_t) = r_t \cdot p_{jt}(\theta) + (M_t - \omega_{kt} - r_t) \cdot p'_{jt}(\theta)$$

This expression indicates that: (1) the quadratic term dies out as gradient of the choice probabilities becomes more similar before and after the stockout (which we could think about as the finite difference of the stockout), and (2) the other terms depend on how quickly the choice probabilities change in the parameters for the stocked-out product and the product of interest, roughly weighted by how much data is missing $y_{jt}$.[42]

### Delta Method Correction for Nested Logit Parameters

For the nested logit models, we require a correction when we report the standard errors for the product dummies, the $d_j$'s. We make the substitution $\tilde{\delta}_j = \frac{d_j}{\lambda_{j,k}}$ in estimation, where we define $\lambda_{j,k}$ as the nesting parameter which corresponds to the $j$th product. We write $\theta = \left[\tilde{\delta}_1, \ldots \tilde{\delta}_J, \lambda_1, \ldots, \lambda_K\right] \in \mathbb{R}^L$, and use the delta method to recover standard errors on the $d_j$'s. The asymptotic distribution for $\theta$ is:

$$\sqrt{n}(\hat{\theta} - \theta_0) \sim N\left(0, V(\hat{\theta})\right)$$

[42]For those interested, the denominator term $D(v_t)$ is an incomplete beta function and Stirling's formula can be used to approximate derivatives for large $M$ and small $\omega_{kt}$.

The asymptotic distribution of our normalized $d_j$'s is the asymptotic distribution of $g(\theta)$, which we define as:

$$g(\theta) = \left[\tilde{\delta}_1 \lambda_{1k}, \ldots, \tilde{\delta}_J \lambda_{Jk}, \lambda_1, \ldots, \lambda_K\right]'$$
$$\sqrt{n}\left(g(\hat{\theta}) - g(\theta_0)\right) \sim N\left(0, \left[\frac{dg(\theta)}{d\theta}\right]' V(\hat{\theta}) \left[\frac{dg(\theta)}{d\theta}\right]\right)$$

To recover the marginal distributions of $g(\theta)$ we consider the function $g_j(\theta) = \tilde{\delta}_j \lambda_{jk}$, which has the following derivative vector $\nabla g_j = [0, \ldots 0, \lambda_{jk}, 0 \ldots, 0, \tilde{\delta}_j, 0]$ where the nonzero elements are in position $j$ and the position of the corresponding $\lambda_k$.
When we expand out we get the following quadratic form:

$$\sqrt{n}\left(g_j(\hat{\theta}) - g_j(\theta_0)\right) \sim N\left(0, \lambda_{jk}^2 \sigma_{\delta_j}^2 + \delta_j^2 \sigma_{\lambda_{jk}}^2 + 2\lambda_{jk}\delta_j \sigma_{\delta_j, \lambda_{jk}}\right)$$

And the standard error is:

$$SE(d_j) = \sqrt{\lambda_{jk}^2 \sigma_{\delta_j}^2 + \delta_j^2 \sigma_{\lambda_{jk}}^2 + 2\lambda_{jk}\delta_j \sigma_{\delta_j, \lambda_{jk}}}$$

For the $\lambda$ parameters, no correction is necessary.

## A.7 Additional Results

Tables 9, 10 and 11 report the estimates of the product dummies from each of the models estimated on the base dataset. Table 12 reports results of a second-stage regression of the fitted coefficients on product dummies on observable product characteristics to provide the mean levels of the tastes for these characteristics. The $R^2$ from these regressions is relatively low in the case of the nested-logit models. This indicates that the size of the unobservable $\xi_j$ is large in our application, and highlights the need for product dummies. The $R^2$ in the single $\lambda$ case is about 0.25, but this doubles to about 0.50 when we use category-specific $\lambda$'s. In a second-stage regression that includes category dummies, the $R^2$ improves significantly. The random-coefficients model allows for greater variation in the fitted product dummies, and has a higher $R^2$ in the second-stage regression.

Table 9: $d_j$ Parameters, Single-Parameter Nested Logit

| | Full | (S.E.) | Ignore | (S.E.) | EM | (S.E.) |
|---|---|---|---|---|---|---|
| PopTart | -5.488 | 0.040 | -6.286 | 0.029 | -5.754 | 0.022 |
| Choc Donuts | -5.519 | 0.041 | -6.177 | 0.028 | -5.672 | 0.021 |
| Ding Dong | -5.582 | 0.045 | -6.388 | 0.031 | -5.811 | 0.023 |
| Banana Nut Muffin | -5.652 | 0.049 | -6.616 | 0.035 | -5.981 | 0.026 |
| Rice Krispies | -5.801 | 0.057 | -7.007 | 0.042 | -6.255 | 0.032 |
| Pastry | -5.576 | 0.047 | -6.452 | 0.038 | -5.860 | 0.028 |
| Gma Oatmeal Raisin | -5.751 | 0.041 | -6.623 | 0.030 | -6.058 | 0.023 |
| Chips Ahoy | -5.824 | 0.045 | -6.784 | 0.033 | -6.167 | 0.025 |
| Nutter Butter Bites | -5.987 | 0.054 | -7.127 | 0.040 | -6.413 | 0.030 |
| Knotts Raspberry Cookie | -5.999 | 0.054 | -7.140 | 0.040 | -6.429 | 0.030 |
| Gma Choc Chip | -5.646 | 0.039 | -6.535 | 0.034 | -5.959 | 0.024 |
| Gma Mini Cookie | -5.834 | 0.044 | -6.700 | 0.035 | -6.151 | 0.026 |
| Gma Caramel Choc Chip | -5.804 | 0.043 | -6.539 | 0.037 | -6.050 | 0.028 |
| Rold Gold | -5.120 | 0.057 | -6.248 | 0.039 | -5.510 | 0.030 |
| Sunchip Harvest | -5.144 | 0.059 | -6.328 | 0.041 | -5.563 | 0.031 |
| Dorito Nacho | -5.217 | 0.063 | -6.527 | 0.044 | -5.711 | 0.034 |
| Cheeto Crunchy | -5.217 | 0.063 | -6.501 | 0.044 | -5.691 | 0.033 |
| Gardetto Snackens | -4.961 | 0.049 | -6.026 | 0.036 | -5.348 | 0.027 |
| Ruffles Cheddar | -5.307 | 0.068 | -6.714 | 0.048 | -5.842 | 0.037 |
| Fritos | -5.525 | 0.080 | -7.178 | 0.057 | -6.155 | 0.043 |
| Lays Potato Chip | -5.574 | 0.082 | -7.252 | 0.058 | -6.221 | 0.044 |
| Munchies Hot | -5.238 | 0.064 | -6.545 | 0.046 | -5.742 | 0.035 |
| Misc Chips 2 | -5.674 | 0.088 | -7.452 | 0.063 | -6.353 | 0.047 |
| Munchies | -5.443 | 0.076 | -6.989 | 0.055 | -6.020 | 0.041 |
| Misc Chips 1 | -5.438 | 0.075 | -6.925 | 0.053 | -5.992 | 0.040 |
| Dorito Guacamole | -5.511 | 0.079 | -7.159 | 0.059 | -6.146 | 0.044 |
| Snickers | -4.746 | 0.036 | -5.533 | 0.026 | -5.029 | 0.020 |
| Twix | -4.900 | 0.045 | -5.872 | 0.032 | -5.255 | 0.024 |
| M&M Peanut | -5.046 | 0.053 | -6.148 | 0.037 | -5.454 | 0.028 |
| Reese's Cup | -5.406 | 0.072 | -6.901 | 0.052 | -5.983 | 0.039 |
| Kit Kat | -5.452 | 0.075 | -7.001 | 0.054 | -6.051 | 0.041 |
| Caramel Crunch | -5.456 | 0.075 | -6.994 | 0.053 | -6.057 | 0.041 |
| M&M | -5.400 | 0.072 | -6.893 | 0.053 | -5.975 | 0.040 |
| Hershey Almond | -5.593 | 0.083 | -7.280 | 0.059 | -6.256 | 0.045 |
| Babyruth | -5.625 | 0.084 | -7.272 | 0.062 | -6.262 | 0.047 |
| Starburst | -5.531 | 0.037 | -6.344 | 0.028 | -5.817 | 0.021 |
| Kar Nut Sweet/Salt | -5.683 | 0.045 | -6.638 | 0.033 | -6.031 | 0.025 |
| Snackwell | -5.959 | 0.060 | -7.195 | 0.044 | -6.422 | 0.033 |
| Skittles | -5.578 | 0.041 | -6.499 | 0.034 | -5.904 | 0.025 |
| Payday | -5.835 | 0.052 | -6.892 | 0.038 | -6.239 | 0.029 |
| Oreo | -6.215 | 0.074 | -7.733 | 0.055 | -6.795 | 0.041 |
| Peanuts | -5.885 | 0.058 | -7.129 | 0.048 | -6.352 | 0.035 |
| Peter Pan (Crck) | -5.986 | 0.060 | -7.201 | 0.045 | -6.451 | 0.034 |
| Hot Tamales | -5.809 | 0.052 | -6.674 | 0.042 | -6.100 | 0.032 |



Full assumes that all products stocked in a machine are available to all consumers (ie., it ignores stockout events). Ignore adjusts for stockouts during periods in which all sales and availability regimes are observed, but ignores (discards) periods in which stockouts happened at an unknown point in time. EM adjusts for all stockout events, regardless of whether the timing of a stockout was fully observed in the data. Standard errors are reported in parentheses.

Table 10: $d_j$ Parameters, Category-Specific Nested Logit

| | Full | (S.E.) | Ignore | (S.E.) | EM | (S.E.) |
|---|---|---|---|---|---|---|
| PopTart | -6.880 | 0.142 | -6.361 | 0.037 | -5.861 | 0.029 |
| Choc Donuts | -6.937 | 0.144 | -6.250 | 0.037 | -5.767 | 0.028 |
| Ding Dong | -7.126 | 0.156 | -6.469 | 0.041 | -5.923 | 0.031 |
| Banana Nut Muffin | -7.348 | 0.173 | -6.708 | 0.045 | -6.110 | 0.035 |
| Rice Krispies | -7.789 | 0.203 | -7.118 | 0.055 | -6.411 | 0.042 |
| Pastry | -7.192 | 0.167 | -6.539 | 0.047 | -5.982 | 0.036 |
| Gma Oatmeal Raisin | -5.743 | 0.063 | -6.066 | 0.071 | -5.732 | 0.054 |
| Chips Ahoy | -5.816 | 0.069 | -6.174 | 0.079 | -5.810 | 0.059 |
| Nutter Butter Bites | -5.976 | 0.083 | -6.385 | 0.095 | -5.978 | 0.072 |
| Knotts Raspberry Cookie | -5.989 | 0.084 | -6.393 | 0.096 | -5.988 | 0.073 |
| Gma Choc Chip | -5.638 | 0.060 | -5.962 | 0.071 | -5.637 | 0.052 |
| Gma Mini Cookie | -5.826 | 0.067 | -6.166 | 0.073 | -5.820 | 0.057 |
| Gma Caramel Choc Chip | -5.796 | 0.065 | -6.057 | 0.067 | -5.748 | 0.053 |
| Rold Gold | -4.474 | 0.137 | -7.004 | 0.087 | -5.609 | 0.067 |
| Sunchip Harvest | -4.483 | 0.140 | -7.112 | 0.090 | -5.665 | 0.069 |
| Dorito Nacho | -4.511 | 0.150 | -7.378 | 0.097 | -5.822 | 0.076 |
| Cheeto Crunchy | -4.511 | 0.150 | -7.346 | 0.097 | -5.801 | 0.075 |
| Gardetto Snackens | -4.410 | 0.116 | -6.714 | 0.080 | -5.437 | 0.060 |
| Ruffles Cheddar | -4.546 | 0.161 | -7.632 | 0.105 | -5.963 | 0.082 |
| Fritos | -4.630 | 0.190 | -8.265 | 0.125 | -6.296 | 0.096 |
| Lays Potato Chip | -4.649 | 0.196 | -8.364 | 0.127 | -6.367 | 0.099 |
| Munchies Hot | -4.521 | 0.153 | -7.381 | 0.097 | -5.854 | 0.077 |
| Misc Chips 2 | -4.688 | 0.209 | -8.634 | 0.136 | -6.507 | 0.105 |
| Munchies | -4.598 | 0.179 | -8.016 | 0.119 | -6.153 | 0.091 |
| Misc Chips 1 | -4.597 | 0.178 | -7.901 | 0.113 | -6.121 | 0.089 |
| Dorito Guacamole | -4.624 | 0.188 | -8.252 | 0.127 | -6.287 | 0.097 |
| Snickers | -4.935 | 0.138 | -5.144 | 0.069 | -4.664 | 0.052 |
| Twix | -5.134 | 0.170 | -5.387 | 0.087 | -4.804 | 0.064 |
| M&M Peanut | -5.322 | 0.200 | -5.583 | 0.101 | -4.927 | 0.075 |
| Reese's Cup | -5.784 | 0.274 | -6.120 | 0.139 | -5.253 | 0.104 |
| Kit Kat | -5.844 | 0.284 | -6.191 | 0.144 | -5.295 | 0.107 |
| Caramel Crunch | -5.848 | 0.284 | -6.187 | 0.144 | -5.299 | 0.108 |
| M&M | -5.779 | 0.276 | -6.104 | 0.140 | -5.243 | 0.103 |
| Hershey Almond | -6.025 | 0.313 | -6.390 | 0.159 | -5.422 | 0.118 |
| Babyruth | -6.059 | 0.314 | -6.410 | 0.157 | -5.437 | 0.119 |
| Starburst | -5.286 | 0.061 | -5.595 | 0.070 | -5.440 | 0.060 |
| Kar Nut Sweet/Salt | -5.384 | 0.075 | -5.740 | 0.085 | -5.574 | 0.074 |
| Snackwell | -5.559 | 0.100 | -5.997 | 0.113 | -5.813 | 0.098 |
| Skittles | -5.309 | 0.066 | -5.638 | 0.077 | -5.479 | 0.067 |
| Payday | -5.486 | 0.088 | -5.893 | 0.099 | -5.718 | 0.086 |
| Oreo | -5.721 | 0.123 | -6.245 | 0.141 | -6.041 | 0.122 |
| Peanuts | -5.503 | 0.094 | -5.923 | 0.109 | -5.749 | 0.095 |
| Peter Pan (Crck) | -5.583 | 0.102 | -6.037 | 0.115 | -5.848 | 0.100 |
| Hot Tamales | -5.470 | 0.087 | -5.814 | 0.090 | -5.637 | 0.078 |



Full assumes that all products stocked in a machine are available to all consumers (ie., it ignores stockout events). Ignore adjusts for stockouts during periods in which all sales and availability regimes are observed, but ignores (discards) periods in which stockouts happened at an unknown point in time. EM adjusts for all stockout events, regardless of whether the timing of a stockout was fully observed in the data. Standard errors are reported in parentheses.

Table 11: $d_j$ Parameters-Random Coefficients

| | Full | (S.E.) | Ignore | (S.E.) | EM | (S.E.) |
|---|---|---|---|---|---|---|
| PopTart | -12.697 | 0.038 | -6.185 | 0.010 | -11.354 | 0.012 |
| Choc Donuts | -8.946 | 0.033 | -6.080 | 0.011 | -8.111 | 0.006 |
| Ding Dong | -11.116 | 0.030 | -6.279 | 0.012 | -9.965 | 0.007 |
| Banana Nut Muffin | -14.200 | 0.072 | -6.492 | 0.012 | -12.615 | 0.019 |
| Rice Krispies | -7.760 | 0.002 | -6.858 | 0.014 | -7.507 | 0.001 |
| Pastry | -10.923 | 0.045 | -6.335 | 0.021 | -9.850 | 0.009 |
| Gma Oatmeal Raisin | -11.614 | 0.023 | -6.518 | 0.012 | -10.564 | 0.008 |
| Chips Ahoy | -8.374 | 0.007 | -6.669 | 0.013 | -7.940 | 0.002 |
| Nutter Butter Bites | -8.072 | 0.002 | -6.987 | 0.015 | -7.785 | 0.001 |
| Knotts Raspberry Cookie | -9.213 | 0.018 | -6.999 | 0.015 | -8.678 | 0.004 |
| Gma Choc Chip | -8.875 | 0.010 | -6.424 | 0.016 | -8.299 | 0.002 |
| Gma Mini Cookie | -7.552 | 0.003 | -6.603 | 0.022 | -7.276 | 0.001 |
| Gma Caramel Choc Chip | -10.735 | 0.022 | -6.450 | 0.027 | -9.746 | 0.006 |
| Rold Gold | -9.470 | 0.177 | -6.103 | 0.010 | -8.728 | 0.033 |
| Sunchip Harvest | -6.672 | 0.005 | -6.178 | 0.010 | -6.480 | 0.001 |
| Dorito Nacho | -6.964 | 0.009 | -6.363 | 0.011 | -6.790 | 0.003 |
| Cheeto Crunchy | -8.038 | 0.034 | -6.339 | 0.011 | -7.628 | 0.010 |
| Gardetto Snackens | -7.804 | 0.053 | -5.895 | 0.011 | -7.385 | 0.011 |
| Ruffles Cheddar | -7.589 | 0.026 | -6.537 | 0.012 | -7.317 | 0.007 |
| Fritos | -7.502 | 0.029 | -6.969 | 0.015 | -7.327 | 0.007 |
| Lays Potato Chip | -7.667 | 0.026 | -7.038 | 0.015 | -7.471 | 0.006 |
| Munchies Hot | -7.147 | 0.010 | -6.383 | 0.017 | -6.949 | 0.003 |
| Misc Chips 2 | -7.622 | 0.005 | -7.225 | 0.018 | -7.475 | 0.002 |
| Munchies | -8.026 | 0.020 | -6.792 | 0.018 | -7.706 | 0.005 |
| Misc Chips 1 | -7.369 | 0.006 | -6.736 | 0.019 | -7.147 | 0.002 |
| Dorito Guacamole | -7.196 | 0.005 | -6.950 | 0.021 | -7.109 | 0.001 |
| Snickers | -9.014 | 0.018 | -5.438 | 0.007 | -8.225 | 0.005 |
| Twix | -9.078 | 0.017 | -5.754 | 0.008 | -8.329 | 0.004 |
| M&M Peanut | -8.914 | 0.015 | -6.011 | 0.009 | -8.229 | 0.004 |
| Reese's Cup | -8.848 | 0.010 | -6.712 | 0.013 | -8.332 | 0.002 |
| Kit Kat | -10.826 | 0.025 | -6.804 | 0.013 | -9.942 | 0.006 |
| Caramel Crunch | -9.267 | 0.010 | -6.798 | 0.013 | -8.690 | 0.003 |
| M&M | -10.947 | 0.026 | -6.701 | 0.015 | -10.022 | 0.007 |
| Hershey Almond | -8.782 | 0.011 | -7.064 | 0.015 | -8.348 | 0.002 |
| Babyruth | -11.926 | 0.031 | -7.065 | 0.029 | -10.877 | 0.009 |
| Starburst | -12.602 | 0.050 | -6.247 | 0.011 | -11.301 | 0.015 |
| Kar Nut Sweet/Salt | -8.424 | 0.009 | -6.522 | 0.012 | -7.949 | 0.002 |
| Snackwell | -8.824 | 0.004 | -7.040 | 0.015 | -8.383 | 0.002 |
| Skittles | -15.042 | 0.083 | -6.384 | 0.015 | -13.346 | 0.026 |
| Payday | -8.970 | 0.009 | -6.768 | 0.018 | -8.452 | 0.003 |
| Oreo | -10.383 | 0.007 | -7.541 | 0.020 | -9.736 | 0.003 |
| Peanuts | -6.985 | 0.014 | -6.969 | 0.021 | -6.939 | 0.003 |
| Peter Pan (Crck) | -8.099 | 0.019 | -7.056 | 0.022 | -7.849 | 0.004 |
| Hot Tamales | -12.078 | 0.038 | -6.570 | 0.031 | -10.818 | 0.011 |



Full assumes that all products stocked in a machine are available to all consumers (ie., it ignores stockout events). Ignore adjusts for stockouts during periods in which all sales and availability regimes are observed, but ignores (discards) periods in which stockouts happened at an unknown point in time. EM adjusts for all stockout events, regardless of whether the timing of a stockout was fully observed in the data. Standard errors are reported in parentheses.

Table 12: $d_j$'s on Characteristics

| | Single Parameter Nested | | | Category-Specific Nested | | | Random Coefficients | | |
|---|---|---|---|---|---|---|---|---|---|
| | Full | Ignore | EM | Full | Ignore | EM | Full | Ignore | EM |
| Constant | -5.62 | -7.25 | -6.27 | -4.61 | -6.81 | -5.72 | -6.30 | -7.04 | -6.41 |
| | (0.19) | (0.27) | (0.21) | (0.41) | (0.37) | (0.20) | (0.39) | (0.25) | (0.35) |
| Calories | 5.09 | 6.55 | 5.73 | 7.13 | -3.03 | 2.15 | 7.85 | 6.35 | 7.71 |
| | (2.21) | (3.05) | (2.42) | (4.76) | (4.26) | (2.27) | (4.53) | (2.88) | (3.97) |
| Fat | -2.59 | -3.37 | -2.91 | -4.82 | 0.37 | -1.65 | -3.01 | -3.26 | -3.06 |
| | (1.09) | (1.50) | (1.19) | (2.34) | (2.09) | (1.12) | (2.29) | (1.42) | (1.95) |
| Sodium | 0.09 | 0.07 | 0.10 | -0.66 | 0.38 | 0.14 | -3.73 | 0.08 | -3.01 |
| | (0.41) | (0.56) | (0.45) | (0.88) | (0.79) | (0.42) | (0.84) | (0.53) | (0.73) |
| Carbs | -2.47 | -2.33 | -2.40 | -2.48 | 0.53 | -1.65 | -1.91 | -2.35 | -2.00 |
| | (1.23) | (1.70) | (1.35) | (2.66) | (2.38) | (1.27) | (2.53) | (1.61) | (2.21) |
| Sugar | -0.14 | -0.01 | -0.09 | -2.30 | 2.29 | 0.73 | -8.88 | -0.02 | -7.17 |
| | (0.50) | (0.69) | (0.55) | (1.08) | (0.96) | (0.51) | (1.02) | (0.65) | (0.90) |
| Chocolate | 0.17 | 0.21 | 0.19 | 0.27 | 0.45 | 0.39 | 0.84 | 0.20 | 0.75 |
| | (0.14) | (0.20) | (0.15) | (0.30) | (0.27) | (0.15) | (0.29) | (0.18) | (0.25) |
| Cheese | 0.18 | 0.17 | 0.18 | 0.51 | 0.02 | 0.17 | -0.19 | 0.17 | -0.12 |
| | (0.16) | (0.22) | (0.18) | (0.35) | (0.31) | (0.17) | (0.33) | (0.21) | (0.29) |
| $R^2$ | 0.218 | 0.253 | 0.219 | 0.525 | 0.608 | 0.480 | 0.917 | 0.244 | 0.900 |

Regresses the estimated linear parameters ($d_j$'s) on observed product characteristics for the Full, Ignore, and EM-corrected models. Standard errors are reported in parentheses.

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