Sharing rule identification for general collective consumption models*

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Abstract

We propose a method to identify bounds (i.e. set identification) on the sharing rule for a general collective household consumption model. Unlike the effects of distribution factors, it is well known that the level of the sharing rule cannot be uniquely identified without strong assumptions on preferences across households of different compositions. Our new results show that, though not point identified without these assumptions, narrow bounds on the sharing rule can be obtained. We get these bounds by applying revealed preference restrictions implied by the collective model to the household's continuous aggregate demand functions. We obtain informative bounds even if nothing is known about whether each good is public, private, or assignable within the household, though having such information tightens the bounds. We apply our method to US 1999-2009 PSID data. This application obtains tight sharing rule bounds that yield useful conclusions regarding the effects of income and wages on intrahousehold resource sharing, and on the prevalence of individual (as opposed to household level) poverty.

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1 Introduction

The collective model has become increasingly popular for analyzing household consumption behavior. Becker (1973, 1981) first considered collective household models, in which the household is characterized as a collection of individuals with well defined objective functions that interact to generate household level decisions. In a consumption setting, the model assumes that expenditures on each good and service the household buys are the outcome of multi-person decision making, in which each individual household member is characterized by her or his own rational preferences. Following Chiappori (1988, 1992), "rational" group consumption is defined as any Pareto efficient outcome of a within-group bargaining process. This collective approach contrasts with the conventional unitary approach, which models households as if they were single decision makers.[1]

An intrinsic feature of the collective model is the so-called sharing rule, which governs the within-household distribution of household resources. This sharing rule is often interpreted as an indicator of the bargaining power of individual household members. Unlike other measures of power such as Pareto weights, an attractive feature of the resource sharing rule is that it is expressed in monetary terms. The sharing rule is also useful for recovering information about the economic well-being of household members. For example, Lise and Seitz (2011) use sharing rule estimates to recover the population distribution of income across individuals rather than across households; Browning, Chiappori and Lewbel (2013) combine the sharing rule with other information to recover "indifference scales" that measure the welfare implications of changes in household composition; and Dunbar, Lewbel and Pendakur (2013) use sharing rule estimates to back out rates of child poverty. See also Browning, Bourguignon, Chiappori and Lechene (1994), Chiappori, Fortin and Lacroix (2002), Blundell, Chiappori and Meghir (2005), Lewbel and Pendakur (2008), Couprie, Peluso and Trannoy (2010), Bargain and Donni (2012) and Cherchye, De Rock and Vermeulen (2012a) for various applications of the collective consumption model that make use of the sharing rule concept.

In empirical analyses, the sharing rule is generally not observed. Typically, the only information available is total household expenditures on each good or service the household buys, along with general household characteristics, like demographic composition, information on wages, income, holdings of durables, and wealth measures. Even when detailed intrahousehold consumption data are available, the sharing rule can

[1] Chiappori (1988) only assumes Pareto efficiency of the intra-household allocation of resources. In this sense, he generalized the work of Manser and Brown (1980) and McElroy and Horney (1981) who focused on the empirical implications of specific bargaining rules such as Nash bargaining.

still be hard to measure because of shared consumption, public goods, and externalities within the household.

Information regarding the sharing rule is often obtained by making use of "distribution factors", in the terminology of Browning, Bourguignon, Chiappori, and Lechene (1994) or Bourguignon, Browning and Chiappori (2009).[2] Distribution factors are observed household characteristics that affect Pareto weights in a household's optimization model but not the preferences of individual household members or the household budget set. A well known result in the literature is that changes in the sharing rule, resulting from changes in distribution factors, can be identified given household level demand functions. However, the level of the sharing rule is not itself identified. See, for example, Chiappori and Ekeland (2009) for a general statement and proof of this result.[3]

This nonidentification is unfortunate, because many of the uses of sharing rule estimates, such as calculation of poverty lines, indifference scales, and distributions of income and welfare, depend on the level of the sharing rule. A few different responses to this nonidentification result have been proposed. The commonest response is to ignore the problem, and only report estimates of the impact of distribution factors on the sharing rule (see, e.g., Browning, Bourguignon, Chiappori and Lechene, 1994, Chiappori, Fortin and Lacroix, 2002, or Blundell, Chiappori and Meghir, 2005). A second approach is to try to collect more information on the consumption of individual household members (see Cherchye, De Rock and Vermeulen, 2012a). This method is inherently limited by the difficulty of measuring the fraction of shared goods that are consumed by each individual. A third response is to make additional identifying assumptions. These assumptions hypothesize that some features of individuals' preferences remain the same across households of different compositions (first proposed in Browning, Chiappori and Lewbel, 2013; see also Lewbel and Pendakur, 2008, and Bargain and Donni, 2012).

In this paper, we follow a completely different approach. We return to the standard Chiappori framework, where all that can be observed is household level demand functions, and no additional assumptions are made. We combine this standard framework with a revealed preference approach in the tradition of Samuelson (1938) and Houthakker (1950). We show that, although the sharing rule cannot be point identified, the household's demand functions do provide information regarding sharing, and that information can be used to calculate highly informative bounds on the sharing rule. In short, we show that the sharing rule can be usefully set identified. Moreover, in contrast to the rest of the literature, this set identification does not require distribution factors, or the availability of detailed intrahousehold allocation data, or additional

[2] These distribution factors also roughly correspond to what McElroy (1990) calls extra-environmental parameters.

[3] An additional obstacle to the use of distribution factors for identification is that one needs to assume that a proposed factor really only affects bargaining power and not preferences for goods. For example, relative ages of the members could affect bargaining power, but ages might also affect members' tastes for goods, and would therefore not be valid as distribution factors.

identifying assumptions.

We provide a practical method for calculating upper and lower bounds on the resource shares of each individual in a household, consistent with the collective consumption model. The method allows for the presence of both public and private goods within the household, and does not require the public or private nature of any good to be specified a priori. However, if a subset of goods is known to be private, then we can use this information to tighten the bounds.

Our method begins by estimating the observable demand functions of households. These estimates can embody restictions that are implied by rational collective household behavior, such as the absence of money illusion and the SR1 condition discussed below. Sharing rule bounds are then obtained by combining the information given by a household's demand function with inequality restrictions implied by revealed preference theory as applied to individual household members. Typically revealed preference restrictions are applied to a finite number of observed demand bundles, but we apply them to the entire demand function. Much of what makes our bounds usefully narrow is both that they make use of entire demand functions, and that these demand functions themselves incorporate useful information like the SR1 restrictions.

Other papers that combine estimated demand responses with revealed preference restrictions are Blundell, Browning and Crawford (2003, 2008), though both the goals and methodology of these papers differ from ours. These authors assume a unitary rather than collective model of consumption behavior, so their model does not contain a sharing rule and cannot be used to analyze intrahousehold allocation issues. Their goal is to obtain bounds on demand functions that are consistent with the unitary model, when faced with a limited number of price regimes. They apply revealed preference restrictions to estimated Engel curves, rather than to demand curves. They estimate demands as functions of total expenditures separately in each of a limited number of (observed) price regimes, and they impose revealed preference restrictions (assuming the household behaves as a single utility maximizing consumer) to bound demand functions. In contrast, we estimate household demands as functions of both total expenditures and prices, and then impose revealed preference restrictions at the level of individual household members to obtain bounds on the sharing rule in a nonunitary setting. What the two approaches have in common is that they both use estimated demand functions instead of observed data points to tighten bounds associated with revealed preference restrictions.

While not actually identifying the sharing rule, many papers propose tests (or checks) of whether household demands are consistent with the Chiappori model of rational, Pareto efficient group consumption. Browning and Chiappori (1998) provide a differential characterization of the general collective consumption model.[4] They find that household behavior is consistent with this model only if there exists a household

[4]The term 'differential' refers to the fact that the characterization is obtained by differentiating the functional specifications of the fundamentals of the model (e.g. the utility functions or demands of the household members) as in the calculation of Slutsky matrices.

pseudo-Slutsky matrix that can be decomposed as the sum of a symmetric negative semi-definite matrix and a matrix of rank 1 (in the case of two household members, i.e. the so-called SR1 condition). Chiappori and Ekeland (2006) show that this condition, together with homogeneity and adding up, is also (locally) sufficient for the existence of individual utility functions and Pareto weights that reproduce the observed household behavior. Following in the Afriat (1967), Diewert (1973) and Varian (1982) tradition, Cherchye, De Rock and Vermeulen (2007, 2010, 2011) work with discrete sets of price and quantity bundles reflecting households' expenditure choices. They derive revealed preference characterizations of several versions of the collective model, including the general version that we consider here. In particular, Cherchye, De Rock and Vermeulen (2011) show how some information regarding the sharing rule can be recovered using this pure revealed preference characterization, but only with discrete sets of price and quantity bundles, and under the assumption that the public or private nature of the goods is known. In contrast, the present paper exploits the greater information that is available in continuous demand functions for a general collective model.

To illustrate the practical usefulness of our method, we apply it to a sample of American households drawn from the 1999-2009 Panel Study of Income Dynamics (PSID). We find that our methodology yields quite narrow bounds. For example, we obtain less than a five percentage point difference between upper and the lower bounds on individual resource shares for more than 75% of the couples in our sample. We apply these results to draw a variety of meaningful and robust conclusions regarding the effects of household income and relative wages on the intrahousehold resource sharing, and on the prevalence of individual poverty. For example, 11% of our sample of couples have incomes below a two-person poverty line, but taking the individual allocations of resouces within households into account, our bounds show that 16% to 20% of individuals (comprising all the couples in our sample) are below the corresponding poverty line for individuals. This shows that our bounds are tight enough to reveal a substantial impact of intrahousehold resource share allocations on poverty measures.

The rest of this paper is organized as follows. Section 2 introduces the general collective consumption model and the corresponding sharing rule representation. Section 3 discusses revealed preference restrictions. Section 4 considers sharing rule identification. We also introduce extensions yielding tighter bounds in settings where the private consumption of some goods is assignable to individual household members. Section 5 presents our empirical results, and Section 6 concludes. The Appendix contains the proofs of our main results.

2 The collective model and the sharing rule

This section formally presents the collective household model, and introduces the sharing rule representation that will be used in the following sections.

2.1 A general collective model

We consider a household with two individuals (1 and 2) who consume a set of goods $N = \{1, ..., |N|\}$. We focus on two-member households only for notational convenience. All results can be generalized towards households with any finite number of members, and in particular our methods could be applied to households with children, where the children have their own preferences and weights within the household, as in Bargain and Donni (2012) or Dunbar, Lewbel and Pendakur (2013) (our methods can alternatively be applied treating expenditures on children as public goods in the parents' utility functions as in, e.g., Blundell, Chiappori and Meghir, 2005). Let $\mathbf{q} = \mathbf{g}(\mathbf{p}, y)$ denote a household demand function that defines a quantity bundle $\mathbf{q} \in \mathbb{R}_+^{|N|}$ as a function of prices $\mathbf{p} \in \mathbb{R}_{++}^{|N|}$ and income $y \in \mathbb{R}_{++}$.[5]

We focus on the most general version of the collective model discussed by Browning and Chiappori (1998) and Chiappori and Ekeland (2006, 2009). This model allows every good to have both private and publicly consumed components, so

$$\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H \text{ with } \mathbf{q}^c \in \mathbb{R}_+^{|N|} \ (c = 1, 2, H),$$

with $\mathbf{q}^1$ and $\mathbf{q}^2$ the privately consumed quantities of individuals 1 and 2, and $\mathbf{q}^H$ the publicly consumed quantities. For example, if the first good in bundle $\mathbf{q}$ is gasoline, the first elements of $\mathbf{q}^1$ and $\mathbf{q}^2$ would be the quantities used by household members 1 and 2, respectively, while driving alone, and the first element of $\mathbf{q}^H$ would be the quantity gasoline they used while riding in their car together.

Next, define general (strongly concave and differentiable) utility functions U^1 and U^2 for the members, i.e.

$$U^1\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) \text{ and } U^2\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right).$$

These utility functions allow for (positive) externalities associated with the privately consumed goods, e.g., $\mathbf{q}^2$ appears in U^1 because member 1 can derive utility from having member 2 consume $\mathbf{q}^2$, and vice versa.[6] Finally, the model assumes a Pareto efficient intrahousehold allocation, i.e. there exists a Pareto weight $\mu(\mathbf{p}, y) \in \mathbb{R}_{++}$ such

[5] We refer to y as income for short, but more precisely y is the total amount of household resources that are devoted to purchasing goods and services in a given time period. This may differ from actual income due to saving or borrowing, or from implicitly buying flows of consumption from durables, and may include foregone labor income if leisure is included among consumed goods and services.

[6] Beckerian caring preferences are a special case of the general preferences we consider here. Specifically, caring preferences correspond to the model $U^m\left(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H\right) = F^m(V^1(\mathbf{q}^1, \mathbf{q}^H), V^2(\mathbf{q}^2, \mathbf{q}^H))$, where F^m is an increasing function and V^i are standard utility functions. See also Chiappori (1988, 1992) for a more detailed discussion.

that $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H = \mathbf{g}(\mathbf{p}, y)$ for

$$(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H) = \arg\max_{\mathbf{x}^1,\mathbf{x}^2,\mathbf{x}^H} [U^1\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) + \mu(\mathbf{p}, y)\, U^2\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) \text{ s.t.} \qquad (1)$$
$$\mathbf{p}'\left(\mathbf{x}^1 + \mathbf{x}^2 + \mathbf{x}^H\right) \leq y,\ \mathbf{x}^c \in \mathbb{R}_+^{|N|}\ (c = 1, 2, H)].$$

The Pareto weight function μ represents the "bargaining power" of member 2 relative to member 1, in the sense that the larger the weight, the greater is the extent to which household resources are allocated based on member 2's preferences. The Pareto weight can vary with prices, household income and (like member utility functions) could also depend on other exogenous variables such as demographic characteristics of the household members. Although not necessary for our identification theorems, in our later empirical application we assume that the household does not suffer from money illusion, which implies that $\mu(\mathbf{p}, y)$ is homogeneous of degree zero in $\mathbf{p}$ and y.

As noted in the Introduction, some identification results are achieved in the collective household model literature by assuming the existence of distribution factors, that is, additional exogenous variables that affect Pareto weights but do not affect the members' utility functions or the budget set. Our model permits but does not require or assume the existence of distribution factors.

2.2 Sharing rule representation

We now describe a sharing rule representation of household demand behavior that is consistent with the model introduced above.[7] Intuitively, this representation provides a decentralized interpretation of the household's optimization problem as defined in (1). Specifically, it represents Pareto efficient household behavior as equivalent to the outcome of a two-step allocation procedure. In the first step, the so-called *sharing rule* distributes the aggregate household income y across the group members, defining individual income shares y^1 and y^2 such that $y = y^1 + y^2$. In the second step, each individual maximizes her/his own utility function subject to her/his own income share and evaluated at her/his own Lindahl prices associated with privately and publicly consumed quantities. The Lindahl prices represent each individual's marginal willingness to pay for the different quantities. We are not assuming that households literally allocate resources using this procedure. Rather, the sharing rule representation simply states that the outcome of the household's allocation process defined in (1) is mathematically equivalent to this two step allocation procedure.

To formalize the sharing rule representation, for each individual m $(m = 1, 2)$ consider an individual demand function $\mathbf{g}^m$ that defines a quantity bundle $\widetilde{\mathbf{q}}^m \in \mathbb{R}_+^{|N|}$ as a function of individual prices $\mathbf{p}^{m,1} \in \mathbb{R}_+^{|N|}$, $\mathbf{p}^{m,2} \in \mathbb{R}_+^{|N|}$, $\mathbf{p}^{m,H} \in \mathbb{R}_+^{|N|}$ and individual income

[7] See Chiappori (1988, 1992) for a detailed discussion on the equivalence between the characterizations of Pareto efficient consumption behavior in (1) and (2). Chiappori concentrated on a simplified setting with privately consumed quantities without externalities. However, extending his argument to our setting is relatively straightforward.

$y^m \in \mathbb{R}_{++}$. Specifically let $\widetilde{\mathbf{q}}^m = \mathbf{g}^m(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ where $\widetilde{\mathbf{q}}^m = \widetilde{\mathbf{q}}^{m,1} + \widetilde{\mathbf{q}}^{m,2} + \widetilde{\mathbf{q}}^{m,H}$ such that

$$(\widetilde{\mathbf{q}}^{m,1}, \widetilde{\mathbf{q}}^{m,2}, \widetilde{\mathbf{q}}^{m,H}) = \arg \max_{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H} [U^m \left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) \text{ s.t.} \tag{2}$$
$$\left(\mathbf{p}^{m,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}^{m,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}^{m,H}\right)' \mathbf{x}^H \leq y^m, \ \mathbf{x}^c \in \mathbb{R}_+^{|N|} \ (c = 1, 2, H)].$$

Consider $\mathbf{q}^1$, $\mathbf{q}^2$ and $\mathbf{q}^H$ (with $\mathbf{q} = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$) that solve (1). The same quantities solve (2) (and thus $\mathbf{q} = \widetilde{\mathbf{q}}^m$) if

$$\mathbf{p}^{m,c} = U^m_{\mathbf{q}^c} / \lambda^m, \tag{3}$$

where $U^m_{\mathbf{q}^c}$ is the gradient of the function U^m defined at $\mathbf{q}^c$ $(c = 1, 2, H)$, λ^1 is the optimizing value of the Lagrange multiplier for the budget constraint in (1), and $\lambda^2 = \lambda^1/\mu$. In this set-up, each vector $\mathbf{p}^{m,c} \in \mathbb{R}_+^{|N|}$ represents individual m's marginal willingness to pay for $\mathbf{q}^c$. Pareto efficiency implies $\mathbf{p}^{1,c} + \mathbf{p}^{2,c} = \mathbf{p}$ $(c = 1, 2, H)$ by construction, so $\mathbf{p}^{m,c}$ equals the Lindahl price vector for individual m associated with $\mathbf{q}^c$. Thus, under these prices the maximization program (2) corresponds to the second step of the two-step procedure described above (given y^1 and y^2 defined in the first step).

As noted in the Introduction, the sharing rule is often used in place of the Pareto weight as a bargaining power measure, where the higher the relative income share y^m/y of member m is, the greater is his/her bargaining power in the household.[8] The sharing rule concept is often a more useful measure than the Pareto weight μ in empirical applications, because it is independent of cardinal representations of preferences. In addition, since y^1 and y^2 are measured in the same units as income y, they can be interpreted as measures of the wealth or poverty of individual household members.

3 Revealed preferences

Typical household data sets provide enough information to empirically estimate the household demand function $\mathbf{g}(\mathbf{p}, y)$, but not the individual demand functions $\mathbf{g}^m$ or income shares y^m $(m = 1, 2)$. Our primary goal is then to recover information regarding the sharing rule that defines the individual incomes y^1 and y^2 given $\mathbf{g}(\mathbf{p}, y)$. In this section we provide a revealed preference characterization of the collective consumption model. The inequalities obtained by this characterization will then be used to identify our bounds on the sharing rule.

3.1 Basic concepts

We start by defining some standard concepts in the revealed preference literature, as applied to an individual's demand function $\mathbf{g}^m$. For given $\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}$ and y^m,

[8] See, for example, Browning, Chiappori and Lewbel (2013) for a detailed discussion on the relation between relative income shares y^m/y in (2) and the bargaining Pareto weight μ in (1).

define the budget set

$$B\left(\mathbf{p}^{m,1},\mathbf{p}^{m,2},\mathbf{p}^{m,H},y^m\right)=\{\mathbf{x}\in\mathbb{R}_+^{|N|}|\mathbf{x}=\mathbf{x}^1+\mathbf{x}^2+\mathbf{x}^H,$$
$$\left(\mathbf{p}^{m,1}\right)'\mathbf{x}^1+\left(\mathbf{p}^{m,2}\right)'\mathbf{x}^2+\left(\mathbf{p}^{m,H}\right)'\mathbf{x}^H\leq y^m\}.$$

We now define the concept of the revealed preference relation associated with the demand function $\mathbf{g}^m$, denoted by $R_o^{\mathbf{g}^m}$.

Definition 1 (direct revealed preference) *Let $\mathbf{g}^m$ be an individual demand function. The direct revealed preference relation associated with $\mathbf{g}^m$ is defined by: for all $\mathbf{x}$, $\mathbf{z}\in\mathbb{R}_+^{|N|}$: $\mathbf{x}R_o^{\mathbf{g}^m}\mathbf{z}$ if there exist $\mathbf{p}^{m,c}\in\mathbb{R}_+^{|N|}$ $(c=1,2,H)$ and $y^m\in\mathbb{R}_{++}$ such that $\mathbf{x}=\mathbf{g}^m\left(\mathbf{p}^{m,1},\mathbf{p}^{m,2},\mathbf{p}^{m,H},y^m\right)$ and $\mathbf{z}\in B(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ with $\mathbf{x}\neq\mathbf{z}$.*

In words, quantity bundle $\mathbf{x}$ is revealed preferred to another bundle $\mathbf{z}$ if $\mathbf{z}$ belonged to the budget set $B(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$ under which $\mathbf{x}$ was chosen, i.e. $\mathbf{x}=\mathbf{g}^m(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m)$. Here, $\mathbf{x}R_o^{\mathbf{g}^m}\mathbf{z}$ because the individual could have afforded to chose $\mathbf{z}$ but did not do so, choosing $\mathbf{x}$ instead. The revealed preference concept in Definition 1 differs from the standard one only in that, for our collective setting, revealed preferences are defined at the level of an individual household member m, while in a standard unitary context they are defined at the level of the aggregate household. Correspondingly, we consider preferences that pertain to decomposed quantity bundles, which are evaluated at individual prices $\mathbf{p}^{m,1}$, $\mathbf{p}^{m,2}$ and $\mathbf{p}^{m,H}$ for individual m. The associated quantity decomposition appears from the definition of the budget set $B\left(\mathbf{p}^{m,1},\mathbf{p}^{m,2},\mathbf{p}^{m,H},y^m\right)$.

We can now define the Weak Axiom of Revealed Preference (WARP; after Samuelson, 1938) for an individual m. Our sharing rule identification method will exploit the empirical implications of WARP in the context of the collective household consumption model.

Definition 2 (WARP) *Let $\mathbf{g}^m$ be an individual demand function. This function $\mathbf{g}^m$ satisfies WARP if the relation $R_o^{\mathbf{g}^m}$ is asymmetric.*

As shown by Houthakker (1950), utility maximization generally implies that the Strong Axiom of Revealed Preference (SARP) holds. SARP extends WARP by also exploiting transitivity of preferences. We could readily reformulate Proposition 2 below, showing how to include SARP restrictions in our identification bounds in theory. However, we only exploit WARP and not SARP in our application because it would be much more difficult if not completely intractable to fully operationalize transitivity when empirically implementing our identification procedure. It might be possible in practice to incorporate some but not all of the additional restrictions implied by SARP, by only considering the transitivity conditions implied by revealed preference sequences of a given fixed maximum length. See, for example, Echenique, Lee and Shum (2011), who apply a fixed length SARP procedure, albeit in a simpler context than ours. In their application, they found that considering SARP with sequences of fixed maximum length instead of WARP did not substantively affect their empirical results. Finally, Uzawa (1960) derived minimal regularity conditions for the function $\mathbf{g}^m$ that make the WARP condition equivalent to the SARP condition (see also Bossert, 1993, for discussion). Clearly, the loss of information in only considering WARP rather than SARP will be zero under these conditions.

3.2 Characterizing the collective model

We have described revealed preference relations in terms of individual demand functions $\mathbf{g}^m$, which are not observed. We now define the concept of "admissible" individual demand functions. Roughly, this concept captures all possible specifications of the (unknown) individual demand functions that are consistent with the (known) household demand function. We will then apply revealed preference relations to admissible demand functions to obtain sharing rule bounds.

Definition 3 (admissible individual demands) *For a given household demand function $\mathbf{g}$, the pair of individual demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ are admissible if, for all $\mathbf{p}$ and y,*

$$\mathbf{g}(\mathbf{p}, y) = \mathbf{g}^1\left(\mathbf{p}^{1,1}, \mathbf{p}^{1,2}, \mathbf{p}^{1,H}, y^1\right) = \mathbf{g}^2\left(\mathbf{p}^{2,1}, \mathbf{p}^{2,2}, \mathbf{p}^{2,H}, y^2\right),$$

for some $\mathbf{p}^{m,c}\,(m = 1, 2;\ c = 1, 2, H)$ and y^m such that

$$y^1 + y^2 = y \text{ and } \mathbf{p}^{1,c} + \mathbf{p}^{2,c} = \mathbf{p}, \text{ with } y^m \in \mathbb{R}_{++} \text{ and } \mathbf{p}^{m,c} \in \mathbb{R}_+^{|N|}.$$

Let $Q(\mathbf{g})$ represent the collection of all admissible individual demands $\mathbf{g}^1$ and $\mathbf{g}^2$, i.e.

$$Q(\mathbf{g}) = \{(\mathbf{g}^1, \mathbf{g}^2) \mid \mathbf{g}^1 \text{ and } \mathbf{g}^2 \text{ are admissible for the household demand function } \mathbf{g}\}.$$

We can now define the condition for a collective rationalization that we will use for identification. Basically, this condition states that for the given household demand function $\mathbf{g}$, there must exist at least one specification of admissible individual demand functions that solves (2).

Definition 4 (collective rationalization) *Let $\mathbf{g}$ be a household demand function. A pair of utility functions U^1 and U^2 provides a collective rationalization of $\mathbf{g}$ if there exist admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ such that, for each m,*

$$\mathbf{g}^m\left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m\right) = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$$

for

$$\begin{aligned}(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H) = \arg\max_{\mathbf{x}^1,\mathbf{x}^2,\mathbf{x}^H} [U^m\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) \text{ s.t.}\\ \left(\mathbf{p}^{m,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}^{m,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}^{m,H}\right)' \mathbf{x}^H \leq y^m].\end{aligned}$$

We have the following result, which establishes a revealed preference characterization of the collective consumption model under consideration.

Proposition 1 *Consider a household demand function* $\mathbf{g}$*. There exists a pair of utility functions* U^1 *and* U^2 *that provides a collective rationalization of* $\mathbf{g}$ *only if there exist admissible individual demand functions* $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ *that both satisfy WARP.*

In theory, we can construct bounds by enumerating every element of $Q(\mathbf{g})$ and evaluating every possible sharing rule consistent with each of these elements. However, such an enumeration is not tractable, so the aim now is to construct a computationally feasible procedure for obtaining these bounds.

4 Sharing rule identification

Consider a household demand function $\mathbf{g}$. Suppose that we evaluate a situation E that is characterized by prices $\mathbf{p}_E$ and household income y_E. The household's demand in this particular situation is observed and equals $\mathbf{g}(\mathbf{p}_E, y_E)$. The (set) identification question asks for bounds on the individual incomes (y_E^1 and y_E^2) that are consistent with a collective rationalization of the observed household demand $\mathbf{g}$. Our procedure will start from the characterization given in Proposition 1. Essentially, we define lower bounds y_E^{l1} and y_E^{l2} and upper bounds y_E^{u1} and y_E^{u2} so that

$$y_E^{l1} < y_E^1 < y_E^{u1} \text{ and } y_E^{l2} < y_E^2 < y_E^{u2}. \tag{4}$$

By construction, these bounds will simultaneously apply to all possible specifications of admissible individual demand functions.

4.1 Identification in theory

To sketch the basic idea of our approach, suppose for now that the individual demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ were known (in addition to knowing the household demand function $\mathbf{g}$). Then we would also know the individual prices $\mathbf{p}^{m,c}$ ($m = 1, 2;\ c = 1, 2, H$) and income y^m that imply $\mathbf{g}(\mathbf{p}, y) = \mathbf{g}^m(\mathbf{p}^{m,1},\ \mathbf{p}^{m,2},\ \mathbf{p}^{m,H},\ y^m)$ for each $\mathbf{p}$ and y. See Definition 3. So if the demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ were known, then the income shares would be known.

Although in practice we do not know the true $\mathbf{g}^1$ and $\mathbf{g}^2$, what we do know is that they must be consistent with a collective rationalization of $\mathbf{g}$. Let $\mathbf{q}_E = \mathbf{g}(\mathbf{p}_E, y_E) = \mathbf{g}^m(\mathbf{p}_E^{m,1},\ \mathbf{p}_E^{m,2},\ \mathbf{p}_E^{m,H},\ y_E)$. A necessary condition for a given pair of admissible demand functions $\mathbf{g}^1$ and $\mathbf{g}^2$ to achieve such consistency is

$$\begin{aligned} y_E^1 &\leq \inf_{\mathbf{x}_1^1, \mathbf{x}_1^2, \mathbf{x}_1^H} [\left(\mathbf{p}_E^{1,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_E^{1,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_E^{1,H}\right)' \mathbf{x}_1^H | \mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E] \text{ and} \\ y_E^2 &\leq \inf_{\mathbf{x}_2^1, \mathbf{x}_2^2, \mathbf{x}_2^H} [\left(\mathbf{p}_E^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_E^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_E^{2,H}\right)' \mathbf{x}_2^H | \mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E]. \end{aligned} \tag{5}$$

This necessary condition for a collective rationalization of $\mathbf{g}$ follows immediately from the WARP conditions in Proposition 1. The right hand sides of the inequalities in (5) define upper bounds y_E^{u1} and y_E^{u2} for the income shares y_E^1 and y_E^2, i.e. $y_E^1 < y_E^{u1}$ and $y_E^2 < y_E^{u2}$. Lower bounds then follow from $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$.

Since we do not know the functions $\mathbf{g}^1$ and $\mathbf{g}^2$, we obtain bounds by considering all possible admissible individual demands $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that are consistent with a collective rationalizatioin of $\mathbf{g}$. The tightest bounds will be obtained if the (positive) differences $y_E^{u1} - y_E^{l1}$ and $y_E^{u2} - y_E^{l2}$ are as small as possible. Substituting $y_E - y_E^{u2}$ for y_E^{l1} in $y_E^{u1} - y_E^{l1}$ (or substituting $y_E - y_E^{u1}$ for y_E^{l2} in $y_E^{u2} - y_E^{l2}$) obtains that these tightest bounds correspond to the smallest value of $y_E^{u1} + y_E^{u2} - y_E$. Since y_E is a constant, we will aim at minimizing the sum $y_E^{u1} + y_E^{u2}$ in what follows. (In this respect, also observe that minimizing $(y_E^{u1} + y_E^{u2})$ is equivalent to maximizing $(y_E^{l1} + y_E^{l2})$.)

Summarizing, the upper bounds y_E^{u1} and y_E^{u2} need to solve

$$\sup_{(\mathbf{g}^1,\mathbf{g}^2)\in Q(\mathbf{g})} \inf_{\substack{\mathbf{x}_1^1,\mathbf{x}_1^2,\mathbf{x}_1^H \\ \mathbf{x}_2^1,\mathbf{x}_2^2,\mathbf{x}_2^H}} (y_E^{u1} + y_E^{u2}) \tag{P.0}$$

$$\text{s.t.}$$

$$\begin{gathered} y_E^{u1} = \left(\mathbf{p}_E^{1,1}\right)' \mathbf{x}_1^1 + \left(\mathbf{p}_E^{1,2}\right)' \mathbf{x}_1^2 + \left(\mathbf{p}_E^{1,H}\right)' \mathbf{x}_1^H, \\ y_E^{u2} = \left(\mathbf{p}_E^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_E^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_E^{2,H}\right)' \mathbf{x}_2^H, \\ \mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E, \\ \mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E. \end{gathered}$$

The supremum operator in this objective makes that the upper bounds y_E^{u1} and y_E^{u2} apply to any possible specification of $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$. Corresponding lower bounds are defined as $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$. It directly follows that a collective rationalization of the data is possible only for individual income shares y_E^1 and y_E^2 that meet (4).

It follows from our discussion preceding program P.0 that, if the solution value of program P.0 does not exceed y_E, then it is impossible to specify income shares y_E^1 and y_E^2 that meet (4). If this happens then the demand function $\mathbf{g}$ cannot be collectively rationalized, meaning that the collective model is rejected for the given household demand function $\mathbf{g}$. Analogous results apply to the programs that we present below.

4.2 Identification in practice

The program P.0 described above is not directly useful in practice, because it requires direct consideration of all the (infinitely many) elements of $Q(\mathbf{g})$. In this section, based on program P.0 we provide a program describing bounds that can be applied in practice. Our procedure begins by characterizing the bundles $\mathbf{x}_1$ and $\mathbf{x}_2$ that satisfy $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ in program P.0, in terms of the observed household demand function $\mathbf{g}$.

Proposition 2 *Let* $\mathbf{g}$ *be a household demand function. Then, we have* $\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E$ *and* $\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E$ $(m, l \in \{1,2\}$, $m \neq l)$ *for all admissible individual demand functions* $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ *that satisfy WARP if* $\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1)$ *and* $\mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2)$ *such that*

$$y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2) \text{ and } y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1). \tag{C}$$

This proposition shows that, as long as condition C holds, we can conclude that $\mathbf{x}_1 R_o^{\mathbf{g}^m} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^l} \mathbf{q}_E$. This result makes it possible to compute the upper bounds y_E^{u1} and y_E^{u2} (and corresponding lower bounds $y_E^{l1} = y_E - y_E^{u2}$ and $y_E^{l2} = y_E - y_E^{u1}$) through the following programming problem:

$$\min_{\mathbf{p}_1, \mathbf{p}_2, y_1, y_2} \left(y_E^{u1} + y_E^{u2}\right) \tag{P.1}$$

$$\text{s.t.}$$

$$y_E^{u1} = \mathbf{p}_E' \mathbf{x}_1,\ y_E^{u2} = \mathbf{p}_E' \mathbf{x}_2, \tag{P.1-1}$$

$$y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2),\ y_2 \geq \mathbf{p}_2' (\mathbf{q}_E + \mathbf{x}_1), \tag{P.1-2}$$

$$\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1),\ \mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2). \tag{P.1-3}$$

To see how this program works, observe first that, as in program P.0, the objective minimizes the sum $(y_E^{u1} + y_E^{u2})$ by suitably selecting $\mathbf{x}_1$ and $\mathbf{x}_2$, as defined by $\mathbf{p}_1$, $\mathbf{p}_2$, y_1 and y_2 using (P.1-3). The lower the value of the objective function, the tighter are the sharing rule bounds. Next, because of Proposition 2, the constraint (P.1-2) implies $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ or $\mathbf{x}_1 R_o^{\mathbf{g}^2} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^1} \mathbf{q}_E$. Without loss of generality, we assume $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$. Because $\mathbf{x}_m R_o^{\mathbf{g}^m} \mathbf{q}_E$, we need

$$y_E^m < \left(\mathbf{p}_E^{m,1}\right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2}\right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H}\right)' \mathbf{x}_m^H \tag{6}$$

as can be seen by comparing this with (5) above. By construction, we have $\left(\mathbf{p}_E^{m,1}\right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2}\right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H}\right)' \mathbf{x}_m^H \leq \mathbf{p}_E' \mathbf{x}_m$ for any specification of $\mathbf{p}_E^{m,c}$ and $\mathbf{x}_m^c$ $(c = 1, 2, H)$.

A feature of this program is that we avoid having to specify $\mathbf{p}_E^{m,c}$ and $\mathbf{x}_m^c$, by instead using just the upper bound $y_E^{um} = \mathbf{p}_E' \mathbf{x}_m$ in (P.1-1). This parallels the fact that we also do not need to consider particular specifications of the individual functions $\mathbf{g}^m$ in the program P.1. This is what makes the program P.1 empirically tractable. We compute the bounds y_E^{u1}, y_E^{u2}, y_E^{l1}, and y_E^{l2} by applying nonlinear programming techniques to this program. The application of these numerical methods are simplified by the fact that both the objective and most of the constraints in program P.1 are linear. The only nonlinear constraint is (P.1-3), since a household demand function $\mathbf{g}$ is generally nonlinear.

4.3 Extensions

In empirical applications, it is often reasonable to assume that a subset of goods is privately consumed without externalities, while the nature of the other goods is unknown. This type of information is often exploited to aid in the identification of collective household models. Particular examples of privately consumed goods include assignable goods and exclusive goods. An assignable good is one where information about who consumes various quantities of the good is known to the researcher. An example may be clothing. If information is collected about whether purchased clothing is male or female, then that information may be used to assign these clothing expenditures to the husband or wife. Closely related are exclusive goods, which are goods that are only consumed by one household member. For example, it is often assumed that an individual's leisure in a labor supply setting is exclusively consumed by that individual.[9] If assignable or exclusive goods do not entail externalities, then they exclusively benefit the utility of individual household members, which aids in identification (see Bourguignon, Browning and Chiappori, 2009, for more discussion). This section considers including such information on privately consumed goods to tighten sharing rule bounds.

4.3.1 Private goods without externalities

Let N_A be a subset of private goods without externalities and let N_B be the subset of other goods, so $N = N_A \cup N_B$. For any good $n \in N_A$, we can add the following condition to the set of collective rationalization conditions in Definition 4. Letting $(\mathbf{a})_n$ denote the nth entry of vector $\mathbf{a}$, for every $n \in N_A$,

$$\left(\mathbf{p}^{1,2}\right)_n = 0 \text{ (or } \left(\mathbf{p}^{1,1}\right)_n = (\mathbf{p})_n \text{) and } \left(\mathbf{p}^{2,1}\right)_n = 0 \text{ (or } \left(\mathbf{p}^{2,2}\right)_n = (\mathbf{p})_n \text{)}. \tag{7}$$

Intuitively, this condition says that, because there are no consumption externalities for good n, the willingness to pay of household member m for member l's consumption of good n is zero. Formally, condition (7) can be obtained directly from the definition of $(\mathbf{p}^{m,c})_n$ in (3).

[9] It is sometimes claimed that assignable goods have the same price, while exclusive goods have a different price (see Bourguignon, Browning and Chiappori, 2009).

Using (7), we can immediately reformulate program P.1 as follows:

$$\min_{\mathbf{p}_1,\mathbf{p}_2,y_1,y_2} \left(y_E^{u1}+y_E^{u2}\right) \tag{P.2}$$

$$\text{s.t.}$$

$$y_E^{u1} = \sum\nolimits_{n\in N_A} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_1^1\right)_n + \sum\nolimits_{n\in N_B} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_1\right)_n, \tag{P.2-1}$$

$$y_E^{u2} = \sum\nolimits_{n\in N_A} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_2^2\right)_n + \sum\nolimits_{n\in N_B} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_2\right)_n, \tag{P.2-2}$$

$$y_1 \geq \mathbf{p}_1' \left(\mathbf{q}_E+\mathbf{x}_2\right), \; y_2 \geq \mathbf{p}_2' \left(\mathbf{q}_E+\mathbf{x}_1\right), \tag{P.2-3}$$

$$\left(\mathbf{x}_k\right)_n = \left(\mathbf{x}_k^1\right)_n + \left(\mathbf{x}_k^2\right)_n \;\; \left(k=1,2;\; n\in N_A\right), \tag{P.2-4}$$

$$\mathbf{x}_1 = \mathbf{g}\left(\mathbf{p}_1,y_1\right), \; \mathbf{x}_2 = \mathbf{g}\left(\mathbf{p}_2,y_2\right).$$

Similar to before, the constraint (P.2-3) implies $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$. Thus, we again get the condition (6). In this case, we have

$$\left(\mathbf{p}_E^{m,m}\right)_n \left(\mathbf{x}_m^m\right)_n = \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_m^m\right)_n \text{ and } \left(\mathbf{p}_E^{m,l}\right)_n \left(\mathbf{x}_m^m\right)_n = 0 \; (m\neq l) \text{ if } n\in N_A,$$

while

$$\left(\mathbf{p}_E^{m,1}\right)_n \left(\mathbf{x}_m^1\right)_n + \left(\mathbf{p}_E^{m,2}\right)_n \left(\mathbf{x}_m^2\right)_n + \left(\mathbf{p}_E^{m,H}\right)_n \left(\mathbf{x}_m^H\right)_n \leq \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_m\right)_n \text{ if } n\in N_B.$$

Therefore, we can use

$$\left(\mathbf{p}_E^{m,1}\right)' \mathbf{x}_m^1 + \left(\mathbf{p}_E^{m,2}\right)' \mathbf{x}_m^2 + \left(\mathbf{p}_E^{m,H}\right)' \mathbf{x}_m^H \leq \sum\nolimits_{n\in N_A} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_m^m\right)_n + \sum\nolimits_{n\in N_B} \left(\mathbf{p}_E\right)_n \left(\mathbf{x}_m\right)_n,$$

which obtains (P.2-1) and (P.2-2) instead of (P.1-1), as before. Note that for $n \in N_A$ the privately consumed quantities $\left(\mathbf{x}_k^1\right)_n$ and $\left(\mathbf{x}_k^2\right)_n$ are not given a priori and are therefore defined within program P.2 (subject to the constraint (P.2-4)). Like the previous program, we can solve program P.2 by nonlinear programming techniques.

4.3.2 Assignable and exclusive goods

So far, we have used the fact that goods $n \in N_A$ only affect the utility of one household member, but we have not exploited assignability of these goods to individual household members. To make use of assignability, let $N_{Am} \subseteq N_A$ represent the set of goods that are assignable (or exclusive) to member m. Then, we get

$$\left(\mathbf{x}_k^m\right)_n = \left(\mathbf{x}_k\right)_n \text{ if } n\in N_{Am}.$$

Using this, we obtain the following extension of Proposition 2.

Proposition 3 *Let* $\mathbf{g}$ *be a household demand function. Then, we have* $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ *and* $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ *for all admissible individual demand functions* $\left(\mathbf{g}^1,\mathbf{g}^2\right) \in Q\left(\mathbf{g}\right)$ *that satisfy*

WARP if $\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1)$ *and* $\mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2)$ *such that one of the following conditions holds:*

$$\begin{aligned} y_1 &\geq \mathbf{p}_1'(\mathbf{q}_E + \mathbf{x}_2) \text{ and} \\ \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n &\geq \mathbf{p}_2'\mathbf{x} - \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n \text{ for } \mathbf{x} = \mathbf{x}_1, \mathbf{q}_E, \end{aligned} \tag{C.1}$$

$$\begin{aligned} y_2 &\geq \mathbf{p}_2'(\mathbf{q}_E + \mathbf{x}_1) \text{ and} \\ \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_1)_n (\mathbf{x}_1)_n &\geq \mathbf{p}_1'\mathbf{x} - \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_1)_n (\mathbf{x})_n \text{ for } \mathbf{x} = \mathbf{x}_2, \mathbf{q}_E, \end{aligned} \tag{C.2}$$

or

$$\begin{aligned} \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_1)_n (\mathbf{x}_1)_n &\geq \mathbf{p}_1'\mathbf{q}_E - \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_1)_n (\mathbf{q}_E)_n \text{ and} \\ \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n &\geq \mathbf{p}_2'\mathbf{q}_E - \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{q}_E)_n . \end{aligned} \tag{C.3}$$

The essential difference between this result and Proposition 2 is that, in contrast to condition C, the new conditions C.1, C.2 and C.3 "assign" preference relations to individual household members (i.e. we get $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$), based on the assignable goods.

Using Proposition 3, we must consider three nonlinear programs (in addition to program P.2) to define y_E^{u1}, y_E^{u2}, y_E^{l1}, y_E^{l2}. Each program has the same structure as P.2, except that the condition C (in the constraint (P.2-3)) is replaced by one of the conditions C.1-C.3. The best upper and lower bounds then correspond to minimum and maximum values defined over the three programs.

4.4 A stylized example

We end this theoretical section with a stylized example that demonstrates the mechanics of program P.1. This example, though rather unrealistic, shows that it is possible for our method to yield arbitrarily narrow bounds on the income shares, even when no good is specified as public or private a priori.

For this particular example, assume we have three goods so $|N| = 3$ and let the member utility functions (unknown to the researcher) be given by

$$U^1 = (\delta A) \cdot (\mathbf{q})_1 + (B - (\delta - 1)A) \cdot (\mathbf{q})_2 + C \cdot (\mathbf{q})_3 \text{ and} \tag{8}$$
$$U^2 = (B - (\delta - 1)A) \cdot (\mathbf{q})_1 + \delta A \cdot (\mathbf{q})_2 + C \cdot (\mathbf{q})_3, \tag{9}$$

where δ, A, B and C are positive real numbers specified below. Non-negative consumption externalities require $(B - (\delta - 1)A)$ to be positive. Intuitively, one may think of goods 1 and 2 as leisure of the first and the second household member, while good 3 represents other household consumption.

Next, assume the (unknown to the researcher) Pareto weight specification is

$$\mu = 0 \text{ if } (\mathbf{p})_1 > (\mathbf{p})_2,\ \mu = \infty \text{ if } (\mathbf{p})_2 > (\mathbf{p})_1 \text{ and } \mu = 1 \text{ if } (\mathbf{p})_1 = (\mathbf{p})_2. \tag{10}$$

In the example where goods 1 and 2 are leisure, this corresponds to a model where the household member with the higher wage exercises complete control over the household's consumption allocation, while both members have equal bargaining weight if their wages are equal.

For given ϵ, assuming $0 < \epsilon < 1$, specify δ, A, B and C such that

$$C < \delta A < A + B < C\frac{1+2\epsilon}{1+\epsilon} \text{ and } (A+B)\frac{2+2\epsilon}{2+3\epsilon} < \delta A.$$

Then, for $(B - (\delta - 1)A)$ positive but sufficiently small, it is easily verified that the model specification in (8) and (10) generates the following results.

For

$$\mathbf{p}_E = (0.5+\epsilon, 0.5+\epsilon, 1+\epsilon) \text{ and } y_E = 1$$

the household demand function $\mathbf{g}$ generated by this model is

$$\mathbf{q}_E = \mathbf{g}(\mathbf{p}_E, y_E) = \left(0, 0, \frac{1}{1+\epsilon}\right),$$

while for

$$\begin{aligned} \mathbf{p}_1 &= \left(1+\epsilon, \frac{\epsilon}{2}, 1+\epsilon\right) \text{ and } y_1 = 1+\epsilon, \\ \mathbf{p}_2 &= \left(\frac{\epsilon}{2}, 1+\epsilon, 1+\epsilon\right) \text{ and } y_2 = 1+\epsilon, \end{aligned}$$

the household demand function is

$$\mathbf{x}_1 = \mathbf{g}(\mathbf{p}_1, y_1) = (1,0,0) \text{ and } \mathbf{x}_2 = \mathbf{g}(\mathbf{p}_2, y_2) = (0,1,0).$$

Note that

$$\begin{aligned} y_1 &= 1+\epsilon,\ \mathbf{p}_1'\mathbf{x}_2 = \frac{\epsilon}{2},\ \mathbf{p}_1'\mathbf{q}_E = 1, \\ y_2 &= 1+\epsilon,\ \mathbf{p}_2'\mathbf{x}_1 = \frac{\epsilon}{2},\ \mathbf{p}_2'\mathbf{q}_E = 1, \end{aligned}$$

so that constraint (P.1-2) is indeed satisfied.

Applying our program P.1 to this household demand function yields bounds

$$\begin{gathered} y_E^{u1} = \mathbf{p}_E'\mathbf{x}_1 = 0.5+\epsilon,\ y_E^{u2} = \mathbf{p}_E'\mathbf{x}_2 = 0.5+\epsilon, \\ y_E^{l1} = y_E - y_E^{u2} = 0.5-\epsilon,\ y_E^{l2} = y_E - y_E^{u1} = 0.5-\epsilon. \end{gathered}$$

These upper and lower bounds become arbitrarily tight as ϵ gets arbitrarily small.

This example generated household demand functions in which, for some values of prices, only one of the three goods was demanded in nonzero quantities. This was done only for mathematical simplicity. It would be possible, with more complicated models, to generate examples where quantities are always nonzero while bounds are still arbitrarily narrow.

This example shows that it is possible in theory to obtain arbitrarily narrow bounds without any information on the privateness or assignability of any good. However, in practice such information will typically be helpful in tightening the sharing rule bounds. For example, this will be the case for the empirical application that we present in the next section.

5 Application

5.1 Set-up

We apply our method to a model of couples' joint labor supply and consumption decisions. Specifically, at observed individual wage rates and market prices we consider the allocation of a household's full income to both spouses' leisure and to a set of three types of nondurable consumption goods. Here full income is defined as the sum of both spouses' maximum possible labor income and nonlabor income (excluding savings and expenditures on durables). While we rely on the standard assumption of separability between the modeled commodities and durable goods, we explicitly allow for nonseparability between the spouses' leisure and nondurables consumption. This joint consumption-labor supply setting contains substantial price and wage variation, which is useful for obtaining informative sharing rule bounds. Also, this application allows us to consider various assumptions regarding the nature of the different goods. For example, we examine how much the bounds tighten when we treat each individual's leisure as an exclusive good without externalities. Lise and Seitz (2011) similarly use labor supply to identify resource shares, but for identification their results depend on strong functional form assumptions as well as restrictions across households.

Some implementations of collective consumption models treat wages as distribution factors, thereby assuming that wages only affect Pareto weights. These models implicitly assume that consumption goods are separable from leisure in utility functions, and only deal with the consumption component of utility. In our application, we use wages as prices (of leisure), which may therefore both affect individual demand functions via the budget constraint and appear directly as arguments in the household's Pareto weights. Our methodology does not require existence of any distribution factors, though if any are available they can be readily incorporated into the analysis (i.e. as covariates in the estimation of household demand functions; see also Section 6).

5.2 Data

We apply our sharing rule identification method to a sample of households drawn from the 1999-2009 Panel Study of Income Dynamics (PSID). This widely used data set began in 1968 with a representative sample of over 18,000 individuals living in 5,000 households in the United States. The data set contains information about employment, income, wealth and socio-demographic variables of these individuals and their descendants. Moreover, since 1999 the panel also contains expenditures on a detailed set of consumption categories (see Blundell, Pistaferri and Saporta-Eksten, 2012, for more elaborate information on the data).

Our selected households are couples where both adult members participate in the labor market, so both are on an intensive margin regarding their demand for labor. The self-employed are excluded to avoid issues regarding the imputation of wages and the separation of consumption from work-related expenditures. While our general methodology allows for any number of household members, we only consider childless couples to avoid dealing with the question of whether children should be modeled as having separate bargaining power and utility functions within the household, as in Bargain and Donni (2012), Cherchye, De Rock and Vermeulen (2012a) or Dunbar, Lewbel and Pendakur (2013). Finally, we deleted households with important missing information (mostly, incomplete information on expenditures) and trimmed out a few households with extremely high or low expenditures or wages. We do not exploit the panel aspect of this dataset, because the number of periods for which we have detailed consumption data is limited and yields a very unbalanced panel. Our resulting sample has 865 observations.

We analyze the allocation of the households' full income to both spouses' leisure and to three categories of consumption: food (which includes food at home and food outside the home), housing (rent or rent equivalent, home insurance and utilities) and other goods (health and transportation). These five categories of goods provide sufficient relative price and wage variation to obtain significantly informative bounds, while avoiding modeling issues associated with zero consumption of some goods, which can arise at more disaggregate levels of consumption.[10]

Table 1 provides summary statistics on the relevant data for the sample at hand. Wages are net hourly wages. Leisure is measured in hours per year. To compute leisure we assume that an individual needs 8 hours per day for nonmarket labor requirements like sleeping (and so could work at a job for at most 16 hours per day), and that she works for 50 weeks a year, which is standard in these types of studies. Leisure per year, defined as time that could potentially have been spent on market labor but was not, therefore equals 112 available hours per week minus average hours worked per week,

[10]Zeros arising from corner solutions (such as nonparticipation on the labor market or nonconsumption of some goods) do not interfere with the revealed preference characterization of the collective model that underlies our bounds identification method. However, such zeros complicate the estimation of household demand as a function of prices and income. To deal with these issues, one might proceed along the lines of Donni (2003) or Blundell, Chiappori, Magnac and Meghir (2007).

all multiplied by 50 weeks. Full income and consumption expenditures are measured in nominal dollars per year. The prices of our three nondurable goods are region-specific consumer price indices that have been constructed by the Bureau of Labor Statistics.

	Mean	Std.dev.	Min.	Max.
Female leisure	4,109.2	502.8	2,077.6	5,771.2
Male leisure	3,611.5	503.1	327	5537
Expenditures on food	6,095.6	4,550.3	120	60,000
Expenditures on housing	18,295.9	13,110.1	2,620	99,540
Expenditures on other goods	15,048.3	15,389.2	240	140,816
Full income	235,855.4	117,377.7	75,620.37	716,813.6
Female wage	22.61	14.26	3.13	113.90
Male wage	28.43	18.82	3.43	140.77
Price food	190.38	18.79	159.87	222.27
Price housing	197.42	26.63	153.08	253.06
Price other goods	204.96	32.31	148.92	243.45
Age female	48.7	8.1	25	64
Age male	50.2	8.0	25	65
Grade education female	13.9	2.0	6	17
Grade education male	13.7	2.2	6	17

Table 1: Descriptive statistics; prices and quantities for the PSID sample

5.3 Demand estimation

A crucial ingredient to our identification method is $\mathbf{g}(\mathbf{p}, y)$, the household's vector-valued demand function with respect to both spouses' leisure and the three non-durables, which we need to estimate. We consider three different demand system estimates: a fully nonparametric demand system, a flexible parametric demand system, and the some parametric system but now imposing (differential based) parameter restrictions that are implied by the collective consumption model.

The function $\mathbf{g}$ is a five element vector, but only four elements need to be estimated because the fifth element is determined given the others by the budget constraint $\mathbf{p}'\mathbf{g} = y$, where $\mathbf{p}$ is the vector of individual wages and prices and y is full income. This is known in the consumer demand literature as the "adding up" constraint. For our nonparametric demand system, we estimate the first four elements of $\mathbf{g}$ (i.e. spouses' leisure and consumption of food and housing) as the fitted values of nonparametrically regressing the houshold's quantities of these goods on both members' deflated wages, on the deflated prices of food and housing, and on the deflated household's full income. Each of these regressors was obtained by dividing nominal wages, prices and full income by the price of the third nondurable (other goods). This deflation by the price of the remaining good in the system imposes the homogeneity of demand implied by the absence of money illusion. The nonparametric regressions we use are Nadaraya-Watson kernel estimators with a Gaussian product kernel.

The two parametric demand systems we estimate are versions of Banks, Blundell and Lewbel's (1997) Quadratic Almost Ideal Demand System (QUAIDS). Thanks to its flexibility, QUAIDS is one of the most widely used demand systems in empirical analyses that are based on the unitary model. However, as demonstrated by Browning and Chiappori (1998), it can also be used to analyze collective household behavior. In particular, these authors derived restrictions that the QUAIDS parameters must satisfy if household demand is to be consistent with the collective model. In our application, we estimated QUAIDS both with and without these restrictions. This will allow us to assess the impact on sharing rule bounds of imposing collective model restrictions in the demand estimation step.

Specifically, the second demand system in our analysis is QUAIDS without any restrictions imposed other than the same adding-up and homogeneity constraints that are imposed in the nonparametric demand system. Under QUAIDS, the budget share of commodity i $(i = 1, ..., 5)$ takes the parametric form

$$w_i = \alpha_i + \beta_i \ln \left[\frac{y}{a(\mathbf{p})} \right] + \frac{\lambda_i}{b(\mathbf{p})} \left\{ \ln \left[\frac{y}{a(\mathbf{p})} \right] \right\}^2 + \sum_{j=1}^{5} \gamma_{ij} \ln p_j,$$

where p_j is the j'th element of $\mathbf{p}$ and where

$$\begin{aligned} \ln a(\mathbf{p}) &= \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln p_i \ln p_j, \\ b(\mathbf{p}) &= \prod_{i=1}^{5} p_i^{\beta_i}, \\ \lambda(\mathbf{p}) &= \sum_{i=1}^{5} \lambda_i \ln p_i. \end{aligned}$$

The parameters α_i, β_i, λ_i and γ_{ij} $(\forall i, j)$ are to be estimated. We follow Banks, Blundell and Lewbel (1997) by setting the parameter α_0 to a value just below the lowest value of $\ln y$ in the data. The adding-up constraint implies that $\sum_i \alpha_i = 1$, $\sum_i \beta_i = 0$, $\sum_i \lambda_i = 0$ and $\sum_i \gamma_{ij} = 0$ $(\forall j)$, while homogeneity requires $\sum_j \gamma_{ij} = 0$ $(\forall i)$. Adding-up is satisfied by construction. We impose homogeneity by estimating the system in terms of deflated prices and deflated full income (in line with the nonparametric system discussed above). Assuming additive errors, the system is estimated by means of the Generalized Method of Moments (GMM), where the wages, prices and full income serve as their own instruments. The fitted value of the quantity consumed of a particular good is obtained by multiplying its fitted value of the budget share by full income divided by the good's own price.

Our third demand system estimator exploits the (differential) behavioral implications of the collective model by imposing restrictions on the above QUAIDS. As discussed in the Introduction, Browning and Chiappori (1998) showed that a necessary condition for the household demand function $\mathbf{g}$ to be consistent with the collective model is that it satisfies SR1, i.e. that the associated pseudo-Slutsky matrix, defined as $\mathbf{S} = \frac{\partial \mathbf{g}(\mathbf{p},y)}{\partial \mathbf{p}'} + \frac{\partial \mathbf{g}(\mathbf{p},y)}{\partial y} \mathbf{g}(\mathbf{p},y)'$, can be decomposed into the sum of a symmetric negative semi-definite matrix and a matrix of rank 1. They also show that in the QUAIDS model, this SR1 condition holds if and only if the same decomposition applies to the matrix $\Gamma = (\gamma_{ij})$, which equivalently requires that the rank of the (real) antisymmetric matrix $\mathbf{M} = \Gamma - \Gamma'$ is at most 2. Let m_{ij} denote an element of $\mathbf{M}$. Lemma 3 of Browning and Chiappori shows that this rank condition holds if and only if, for all (i,k) such that $k > i > 2$,

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}} \tag{11}$$

where, without loss of generality, m_{12} is assumed to be different from zero. To obtain our SR1-restricted QUAIDS parameters, we estimated the above described budget share equations by means of GMM while imposing the equality restrictions in (11).

When constructing bounds based on these estimated demand systems, we imposed the restriction that male and female leisure are each private, assignable, and do not generate externalities. We did not place any restriction on the nature of our three nondurable goods, which allows the consumption of each of these goods to be public, private (with or without externalities), or both.[11] We coded both the demand estimation step and the bounds calculations in MATLAB, using TOMLAB/SNOPT to solve our nonlinear programs.

5.4 Empirical results

5.4.1 Comparisons of sharing rule bounds

We assumed leisure is private and assignable, so the value of a household member's leisure is a lower bound on that member's share of full income. This "naive" bound assigns all of the household's nondurable consumption to the other household member. Similarly, a naive upper bound gives a household member his/her leisure and all of the household's nondurable consumption. These naive bounds do not make use of any RP restrictions associated with the collective consumption model.

We now have four different estimators of sharing rule bounds: naive bounds, and bounds based on each of the three demand system estimators described in the previous section. The results are summarized in Table 2, where the columns RP1, RP2, and RP3 refer to the nonparametric demand system, QUAIDS without SR1 imposed, and

[11] We experimented with estimates that allowed all goods to be public, private, or both, and allowed all to have externalities, but we found that this most general specification generally did not provide useful bounds. Our theory shows that it is possible for this general model to yield informative bounds, so the empirical results with this specification are likely due to the limited size of our available data set.

QUAIDS with SR1 imposed, respectively. Each entry of this table peratins to the difference between the upper and lower bound on the female relative income share, $y_1/(y_1+y_2)$ (where member 1 is the female).

Comparing the naive and RP1 columns of Table 2 shows that our RP based method provides a substantial improvement over naive bounds, even with fully nonparametric demand function estimates. The average difference between the upper and lower naive bounds is about 17.5 percentage points, which narrows to about 11 percentage points using the nonparametric RP1 estimates.

The RP1 and RP2 bounds are quite similar, showing that our parametric QUAIDS model yields results similar to nonparametric demand functions. In contrast, comparing the RP2 and RP3 results shows that imposing the SR1 condition (thereby exploiting both the differential and RP restrictions implied by the collective model) considerably tightens the bounds, narrowing them all the way down to below 3.5 percentage points between the upper and lower bound on average.

The fact that the RP1 and RP2 bounds are similar while the RP3 bounds imply a substantial improvement over the RP2 bounds is important for two reasons. First, this shows that the parametric QUAIDS functional form does not impose undue restrictions on the data, since it yields roughly the same results as the nonparametric alternative. Second, this shows that the gains in going from RP1 to RP3 are almost entirely due to the SR1 restrictions and not to the use of a parametric functional form for imposing SR1. The SR1 restrictions, i.e. the differential restrictions implied by the collective household model, do not by themselves suffice to identify the level of the sharing rule (i.e. the nonidentification result that we cited in the Introduction). However, as the RP3 column shows, combining these differential restrictions with the RP restrictions of the collective model yields very informative bounds. The third quantile row of Table 2 shows that, with RP3, for more than 75% of the couples the difference between the upper and lower bounds is less than 5 percentage points. This means that we can make quite precise predictions about the spouses' resource shares for a substantial majority of households.[12]

	Naive bounds	RP1 bounds	RP2 bounds	RP3 bounds
Mean	17.52	11.33	9.72	3.44
Minimum	3.28	0.01	0.00	0.00
First quartile	12.12	6.54	3.93	1.49
Median	15.78	11.67	8.91	2.78
Third quartile	21.68	15.13	14.53	4.74
Maximum	64.03	52.86	43.04	60.64

Table 2: Percentage point differences between upper and lower bounds on the female relative income share

[12] Also, the smallest RP3 based bounds are zero to two decimal places, showing that for some households we come extremely close to point identification.

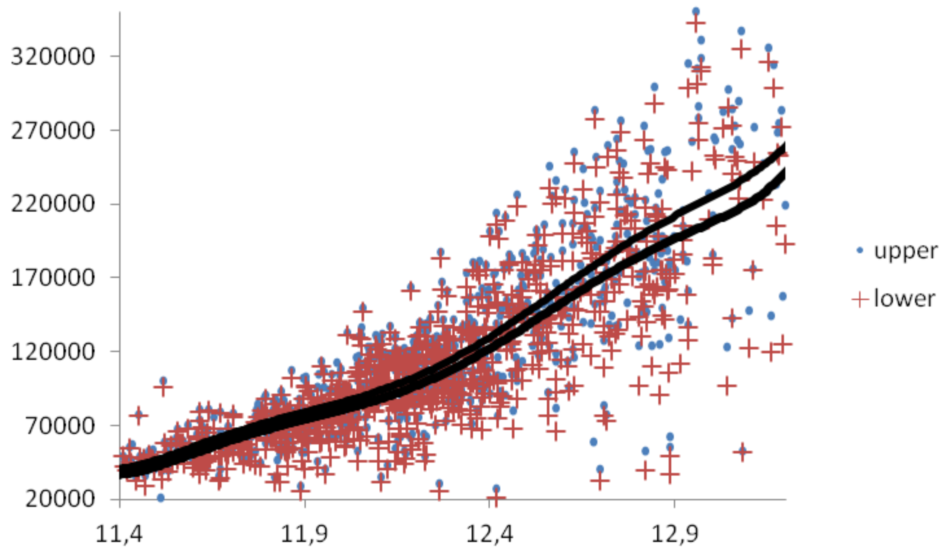

Figure 1: Absolute sharing rule bounds (Y-axis) and the logarithm of full income (X-axis)

Overall, the results in Table 2 show that we obtain quite narrow and hence informative sharing rule bounds. In what follows, we investigate the implications of these bounds in more detail. For compactness and to focus our discussion, we restrict attention to the results using RP3 bounds. However, we found that the other two demand systems yielded qualitatively similar results.

5.4.2 The relation between sharing rule bounds, total income, and relative wages

We next focus on the relation between our RP based bounds and various individual and household characteristics. Figure 1 shows the relation between the RP based bounds on the female income share in dollars, y_1, and the logarithm of the household's full income, $\ln(y)$ (where $y = y_1 + y_2$). Each $\bullet$ and $+$ sign on the figure represents the upper and lower bound for a given household in our sample. To help visualize the results, we included trendlines showing local sample averages (i.e. nonparametric regressions) of the estimated upper and lower bounds.

Figure 1 shows that the bounds are mostly very narrow, reflecting the results in Table 2. The trendlines are upward sloping as one would expect, showing that female income share is a normal good (as opposed to an inferior good) for the household.

Figure 2 plots the relative female income share, y_1/y, against $\ln(y)$. The trendlines are quite close to horizontal (with perhaps a small decline at high incomes). This finding lends empirical support to the assumption that relative income shares do not vary with total income, which Lewbel and Pendakur (2008) and Bargain and Donni

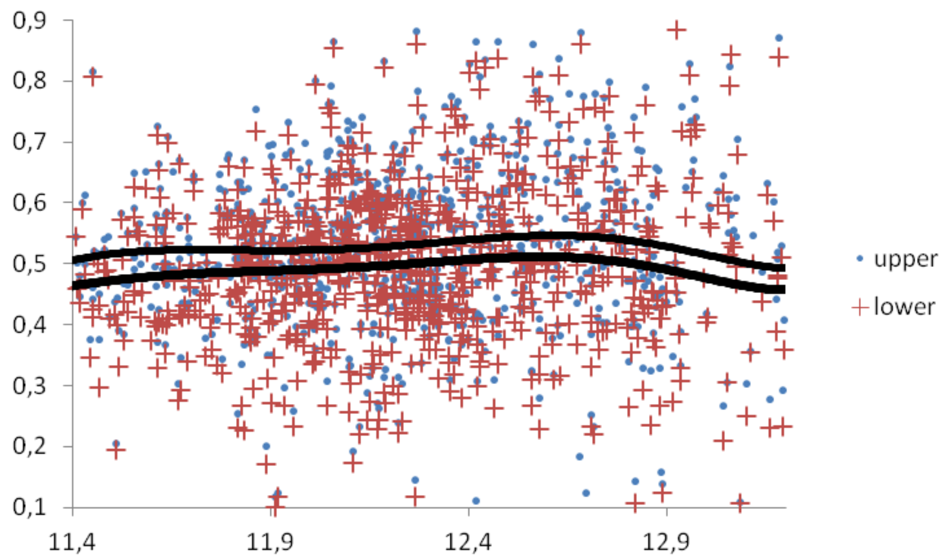

Figure 2: Relative sharing rule bounds (Y-axis) and the logarithm of full income (X-axis)

(2012) used to help point identify resource shares. These results lend even stronger support for the assumption used by Dunbar, Lewbel and Pendakur (2013) to help obtain point identification, which is that relative income shares not vary with total income at low income levels.[13]

The trendlines in Figure 2 give an average upper bound hovering around 51% and an average lower bound around 48%. These results suggests that the income shares of females and males are not far from equal on average. This is confirmed by the average lower and upper bound for all the households in the sample, which equal 48.4% and 51.7% respectively. However, the figure also shows that some households divide their full income very unequally, e.g., there exist some households that have upper and lower bounds of the relative female income share less than 15%, and there exist other household where these bounds are both above 80%. Since leisure is assumed to be private, and is priced at an individual's own wage level, many of these extreme households are likely to be ones where the spouses have very unequal wage levels. In such households a sizeable fraction of total household nondurable consumption would need to be given to the low wage spouse to roughly equalize income shares.

As a following exercise, we look at the relationship between the bounds on the relative female income share and the relative wage (defined as female wage divided by male wage). Figure 3 shows the household specific upper and lower bounds and

[13]Note however that these authors define publicness of goods differently than we do, and their empirical applications focus on a different definition of full income, since they do not include leisure in their consumption models. See also Menon, Pendakur, and Perali (2012), who provide additional empirical evidence supporting this assumption.

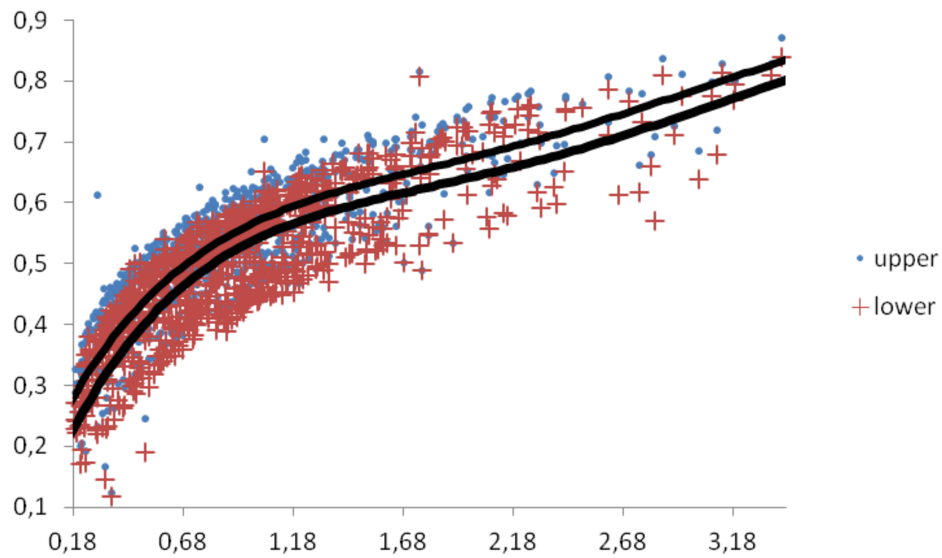

Figure 3: Relative sharing rule bounds (Y-axis) and the relative wage (wage female/wage male) (X-axis)

the corresponding trendlines. In line with our prior expectations, the bounds tend to increase when the relative wage of females goes up. This result, which we obtained through our RP based approach, confirms earlier evidence found in the literature, which shows that a household member's bargaining power generally increases with her/his wage (see Chiappori, Fortin and Lacroix, 2002, Blundell, Chiappori, Magnac and Meghir, 2007, and Oreffice, 2011, among many others). However, as noted above, this result may also be in part an artifact of including leisure in total income and assuming leisure is private.

We next consider the statistical precision of our sharing rule upper and lower bound estimates, by means of a bootstrap procedure.[14] We construct 100 data sets by drawing randomly (with replacement) 865 households from our original data set of 865. For each of these new datasets, we estimated QUAIDS with SR1 imposed and used the resulting parameter estimates to obtain RP3 sharing rule bounds. To summarize the results, we estimate bounds for nine combinations of the spouses' wages: female wage equal to the first, second or third quartile of the female wage distribution combined with male wage equal to the first, second or third quartile of the male wage distribution (with full income each time adjusted accordingly).

Table 3 shows the means and standard deviations across the bootstrap replications of the lower bounds (LB) and the upper bounds (UB) for each of the nine scenarios. All

[14] We do not attempt to derive formal limiting distribution theory for our estimates of bounds. While some asymptotic results exist for inference on set identified objects, our estimators are rather more elaborate than those to which existing theory can be applied. These bootstrap results should therefore only be interpreted informally.

the bounds appear to be rather precisely estimated, with standard deviations between 3 and 5 percent. If we assume normality, we can reject the assumption that spouses with very unequal wages share income equally, but we cannot reject equal sharing for spouses with similar wages. Finally, Table 3 provides more evidence that spouses' shares of income increase when their relative wages increases.

Female wage/Male wage	Mean LB	Std.dev. LB	Mean UB	Std.dev. UB
Quartile 1/Quartile 1	48.02%	3.90%	51.69%	3.35%
Quartile 2/Quartile 1	53.12%	3.62%	57.63%	4.47%
Quartile 3/Quartile 1	58.58%	3.95%	66.16%	4.05%
Quartile 1/Quartile 2	41.56%	4.32%	47.11%	3.60%
Quartile 2/Quartile 2	47.02%	4.00%	52.88%	4.11%
Quartile 3/Quartile 2	52.49%	3.97%	59.57%	4.20%
Quartile 1/Quartile 3	32.51%	4.20%	40.35%	3.72%
Quartile 2/Quartile 3	40.12%	4.26%	46.63%	3.25%
Quartile 3/Quartile 3	46.17%	3.90%	52.17%	3.67%

Table 3: Bootstrap summary statistics on the female income share

5.4.3 Poverty analysis

A unique advantage of models that focus on the intrahousehold allocation of resources is that they allow one to conduct welfare analyses directly at the level of individuals rather than at the level of households (see Chiappori, 1992, Blundell, Chiappori and Meghir, 2005, and Browning, Chiappori and Lewbel, 2013, for further discussion, and Lise and Seitz, 2011, Cherchye, De Rock and Vermeulen, 2012b, and Dunbar, Lewbel, and Pendakur, 2013, for examples of such analyses).

Here we use our RP based bounds to conduct a poverty analysis at the level of individuals. Unlike previous studies that are based on point identified sharing rules, for our analysis we do not impose any assumptions regarding similarity of preferences across individuals, or functional forms at the level of individual's demand functions, or functional restrictions on the sharing rule.[15] We also do not require or assume the availability of distribution factors.

Table 4 summarizes the results of our poverty analysis. The first column contains the poverty rate calculated in the usual way, i.e. the povery rate is defined as the percentage of households having full income that falls below a poverty line, which we define as 60% of the median full income in our sample of households. Note that while 60% of median income is a standard measure of relative poverty (e.g. used in the definition of OECD poverty rates), in our case the poverty rate is calculated on the

[15]For example, Browning, Chiappori and Lewbel (2013) and Lise and Seitz (2011) assume similarity of preferences of (fe)male singles and preferences of (fe)male individuals in couples, while Dunbar, Lewbel and Pendakur (2013) assume restrictions upon individual preferences.

basis of full income instead of (the more commonly used bases of) earnings or total expenditures. Also, our data set consists of couples where both spouses participate in the labor market, and so our poverty line will be higher than a line based on data that includes households containing an unemployed, retired or disabled spouse.

The second column of Table 4 shows the incidence of poverty at the level of individuals in our sample, based on our RP income share estimates. For this and the remaining columns in Table 4, an individual is labeled as poor if her/his income share falls below the individual poverty line (defined as half of the poverty line for couples used in column 1). Using our estimated income share bounds, we compute upper and lower bounds for the individual poverty rates. If all couples split income perfectly equally, then these poverty rates would equal those in column 1. However, despite our earlier finding that many couples have close to equal divisions of income, we obtain lower and upper poverty rate bounds of 15.90% and 20.69%, respectively, compared to the household rate of 11.33%.

Our results imply that, due to unequal sharing of resources within households, the fraction of individuals living below the poverty line is about 4.5 to 9 percentage points greater than the fraction obtained by standard measures that ignore intrahousehold allocations. Thus, the incidence of poverty at the individual level may be considerably higher than is indicated by standard measures based on household level income.

The remaining columns of Table 4 provide separate bounds on poverty rates for females and males. We find that the lower bound on the poverty rate is a bit lower for males than for females, while the opposite holds for the corresponding upper bounds. Overall, these results suggest that poverty is not primarily a female or male phenomenon in our particular data set.

	Households	All individuals	Females	Males
Household poverty rate	11.33%	-	-	-
Lower bound	-	15.90%	16.18%	15.64%
Upper bound	-	20.69%	20.58%	20.81%

Table 4: Poverty rates

6 Conclusion

It has long been known that, under the general collective household model, the income sharing rule is not identified. Past responses to this result have been to focus on features of the model that are identified (like the impacts of distribution factors), or to add strong additional assumptions on preferences or behavior to obtain point identification. In contrast, we show that, given just household level demand functions, bounds on the sharing rule can be obtained by imposing inequality restrictions that revealed preference theory implies for the (unobserved) demand functions of individual household members. We show that informative bounds are possible even when nothing is known about the privateness or assignability of any of the goods being consumed by household members, and when no distribution factors are observed. We show how these bounds can be implemented using standard programming methods, employing household level demand functions that are estimated by standard parametric and nonparametric regression methods. We also show how bounds can be tightened by using additional information that may be available on the privateness and assignability of some goods, and by the use of household level demand functions that satisfy the SR1 differential restrictions associated with the collective household model.

At the practical level, we demonstrate that our identification methods are empirically tractable and yield usefully narrow bounds when applied to a data set of American households drawn from the PSID. We show the usefulness of these bounds by providing empirical analyses of the effects of household characteristics like income and relative wages on income shares, and we provide a distributional analysis of the incidence of poverty at the level of individuals rather than at the level of observed households.

In our analysis, the household demand functions are estimated, and we have only to a limited extent taken into account estimation errors. In our empirical application the sample size is relatively small, which would in any case limit the applicability of asymptotic distribution theory. Some results exist on inference for set identified objects, including Manski (2003), Chernozhukov, Hong and Tamer (2007), Beresteanu and Molinari (2008), Galichon and Henry (2009), Hoderlein and Stoye (2013), Kitamura and Stoye (2012) and Henry and Mourifié (2013). An area for future research would be to develop sufficient extensions of these methods to permit their application to our context. Similar issues arise in other applications that combine demand estimation with revealed preference restrictions, such as Blundell, Browning and Crawford (2008) and Blundell, Kristensen and Matzkin (2011).

For simplicity, our analysis was based on couples without children. However, our methods immediately extend to handle more than two consumers per household, and therefore could be used to estimate bounds on children's resource shares as well. In such extensions, children can be treated as additional consumers with their own utility functions and Pareto weights in the collective model, as in Bargain and Donni (2012) or Dunbar, Lewbel, and Pendakur (2013), but without the restrictions on preferences that these authors require.[16]

Finally, it would be straightforward to add covariates such as demographic characteristics in the empirical analysis. One could include additional covariates in the estimation of the demand functions using standard parametric or nonparametric regression techniques. Our inequalities analysis could then be performed on the resulting conditional demand functions, conditioning on whatever values of the covariates are of interest.

[16] Our empirical application included leisure, with prices given by wages, but our general method does not require this. Since the price of leisure (wages) is not observed for children, utility for children would need to be based only on consumption of goods.

Appendix

Proof of Proposition 1

This result follows from adapting the original reasoning of Samuelson (1938) to our collective setting. Specifically, Definition 4 states that a pair of utility functions U^1 and U^2 provides a collective rationalization of $\mathbf{g}$ if there exist admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ such that, for each m,

$$\mathbf{g}^m\left(\mathbf{p}^{m,1}, \mathbf{p}^{m,2}, \mathbf{p}^{m,H}, y^m\right) = \mathbf{q}^1 + \mathbf{q}^2 + \mathbf{q}^H$$

for

$$\begin{gathered}(\mathbf{q}^1, \mathbf{q}^2, \mathbf{q}^H) = \arg\max_{\mathbf{x}^1,\mathbf{x}^2,\mathbf{x}^H} [U^m\left(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^H\right) \text{ s.t.} \\ \left(\mathbf{p}^{m,1}\right)'\mathbf{x}^1 + \left(\mathbf{p}^{m,2}\right)'\mathbf{x}^2 + \left(\mathbf{p}^{m,H}\right)'\mathbf{x}^H \leq y^m].\end{gathered}$$

Thus, for each individual m there must exist a utility function U^m such that the function $\mathbf{g}^m$ solves the corresponding maximization problem for any prices $\mathbf{p}^{m,1}$, $\mathbf{p}^{m,2}$, $\mathbf{p}^{m,H}$ and income y^m. Samuelson's (1938) argument obtains that this is possible only if the function $\mathbf{g}^m$ satisfies the WARP condition in Definition 2.

Proof of Proposition 2

As a first step, we prove (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$(y_1 \geq \mathbf{p}_1'\mathbf{x}_2 \text{ and } y_2 \geq \mathbf{p}_2'\mathbf{x}_1) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^m}\mathbf{x}_2 \text{ and } \mathbf{x}_2 R_o^{\mathbf{g}l}\mathbf{x}_1 \ (l \neq m)\right). \tag{12}$$

To obtain this result, we note that $y_1 \geq \mathbf{p}_1'\mathbf{x}_2$ implies by construction

$$\begin{aligned} & \sum\nolimits_{m=1}^{2} \left(\mathbf{p}_1^{m,1}\right)'\mathbf{x}_1^1 + \left(\mathbf{p}_1^{m,2}\right)'\mathbf{x}_1^2 + \left(\mathbf{p}_1^{m,H}\right)'\mathbf{x}_1^H \\ \geq & \sum\nolimits_{m=1}^{2} \left(\mathbf{p}_1^{m,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_1^{m,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_1^{m,H}\right)'\mathbf{x}_2^H \end{aligned} \tag{13}$$

for all possible specifications of $\mathbf{p}_1^{m,c}$, $\mathbf{x}_1^c$ and $\mathbf{x}_2^c$ ($m = 1,2;\ c = 1,2,3$). The inequality (13) necessarily obtains

$$\left(\mathbf{p}_1^{m,1}\right)'\mathbf{x}_1^1 + \left(\mathbf{p}_1^{m,2}\right)'\mathbf{x}_1^2 + \left(\mathbf{p}_1^{m,H}\right)'\mathbf{x}_1^H \geq \left(\mathbf{p}_1^{m,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_1^{m,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_1^{m,H}\right)'\mathbf{x}_2^H$$

for $m = 1$ *or* 2, which can also be expressed as $\mathbf{x}_1 R_o^{\mathbf{g}^1}\mathbf{x}_2$ *or* $\mathbf{x}_1 R_o^{\mathbf{g}^2}\mathbf{x}_2$ for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$.

Now, without loss of generality, let us assume $\mathbf{x}_1 R_o^{\mathbf{g}^1}\mathbf{x}_2$ (i.e. $m = 1$ in (12)). Then, because the functions $(\mathbf{g}^1, \mathbf{g}^2)$ satisfy WARP, we must have

$$\left(\mathbf{p}_2^{1,1}\right)'\mathbf{x}_1^1 + \left(\mathbf{p}_2^{1,2}\right)'\mathbf{x}_1^2 + \left(\mathbf{p}_2^{1,H}\right)'\mathbf{x}_1^H > \left(\mathbf{p}_2^{1,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_2^{1,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_2^{1,H}\right)'\mathbf{x}_2^H. \tag{14}$$

In turn, because $y_2 \geq \mathbf{p}_2'\mathbf{x}_1$, this implies

$$\left(\mathbf{p}_2^{2,1}\right)'\mathbf{x}_1^1 + \left(\mathbf{p}_2^{2,2}\right)'\mathbf{x}_1^2 + \left(\mathbf{p}_2^{2,H}\right)'\mathbf{x}_1^H \leq \left(\mathbf{p}_2^{2,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)'\mathbf{x}_2^H,$$

or $\mathbf{x}_2 R_o^{\mathbf{g}^2}\mathbf{x}_1$. This proves (12).

As a second step, we show (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\left(\mathbf{x}_1 R_o^{\mathbf{g}^m}\mathbf{x}_2 \text{ and } y_2 \geq \mathbf{p}_2'\left(\mathbf{q}_E + \mathbf{x}_1\right)\right) \Rightarrow \left(\mathbf{x}_2 R_o^{\mathbf{g}^l}\mathbf{q}_E \ (l \neq m)\right). \tag{15}$$

To prove this result, we first observe that $y_2 \geq \mathbf{p}_2'\left(\mathbf{q}_E + \mathbf{x}_1\right)$ implies

$$\begin{aligned} & \sum\nolimits_{m=1}^{2} \left(\mathbf{p}_2^{m,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_2^{m,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_2^{m,H}\right)'\mathbf{x}_2^H \\ \geq \ & \sum\nolimits_{m=1}^{2} \left(\mathbf{p}_2^{m,1}\right)'\left(\mathbf{q}_E^1 + \mathbf{x}_1^1\right) + \left(\mathbf{p}_2^{m,2}\right)'\left(\mathbf{q}_E^2 + \mathbf{x}_1^2\right) + \left(\mathbf{p}_2^{m,H}\right)'\left(\mathbf{q}_E^H + \mathbf{x}_1^H\right), \end{aligned} \tag{16}$$

for all possible specifications of $\mathbf{p}_1^{m,c}$, $\mathbf{x}_1^c$, $\mathbf{x}_2^c$ and $\mathbf{q}_E^c$ $(m = 1, 2;\ c = 1, 2, H)$.

Without loss of generality, let us now assume $\mathbf{x}_1 R_o^{\mathbf{g}^1}\mathbf{x}_2$ (i.e. $m = 1$ in (15)). Like before, WARP consistency then requires (14), and combining this last inequality with (16) yields

$$\left(\mathbf{p}_2^{2,1}\right)'\mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)'\mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)'\mathbf{x}_2^H > \left(\mathbf{p}_2^{2,1}\right)'\mathbf{q}_E^1 + \left(\mathbf{p}_2^{2,2}\right)'\mathbf{q}_E^2 + \left(\mathbf{p}_2^{2,H}\right)'\mathbf{q}_E^H,$$

or $\mathbf{x}_2 R_o^{\mathbf{g}^2}\mathbf{q}_E$. This proves (15).

Now, a directly similar reasoning as the one leading up to (15) yields

$$\left(\mathbf{x}_2 R_o^{\mathbf{g}^l}\mathbf{x}_1 \text{ and } y_1 \geq \mathbf{p}_1'\left(\mathbf{q}_E + \mathbf{x}_2\right)\right) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^m}\mathbf{q}_E \ (m \neq l)\right). \tag{17}$$

Combining (12), (15) and (17) gives the wanted result: we have $\mathbf{x}_1 R_o^{\mathbf{g}^m}\mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^l}\mathbf{q}_E$ for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that satisfy WARP if $y_1 \geq \mathbf{p}_1'\left(\mathbf{q}_E + \mathbf{x}_2\right)$ and $y_2 \geq \mathbf{p}_2'\left(\mathbf{q}_E + \mathbf{x}_1\right)$. ■

Proof of Proposition 3

In what follows, we only give the proof for condition C.1. The arguments for the remaining conditions C.2 and C.3 are analogous.

As a first step, we prove that (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$ and for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\begin{aligned} \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n &\geq \mathbf{p}_2' \mathbf{x} - \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n \\ &\Rightarrow \left(\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1 \text{ and } \mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E \right) \end{aligned} \tag{18}$$

To obtain this result, we note that

$$\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H \geq \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n \tag{19}$$

for any possible specification of $\mathbf{p}_2^{2,c}$ and $\mathbf{x}_2^c$ $(c = 1, 2, H)$. Indeed, $(\mathbf{x}_k^2)_n = (\mathbf{x}_k)_n$ and $\left(\mathbf{p}_2^{2,2}\right)_n = (\mathbf{p}_2)_n$ for the assignable goods $n \in N_{A_2}$, which yields

$$\left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 \geq \sum\nolimits_{n \in N_{A_2}} (\mathbf{p}_2)_n (\mathbf{x}_2)_n \,,$$

while $\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H \geq 0$ by construction.

Similarly, we have (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$)

$$\mathbf{p}_2' \mathbf{x} - \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n \geq \left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}^H. \tag{20}$$

To see this, we can use an analogous argument as before to get

$$\left(\mathbf{p}_2^{1,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{1,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{1,H}\right)' \mathbf{x}^H \geq \sum\nolimits_{n \in N_{A1}} (\mathbf{p}_2)_n (\mathbf{x}_1)_n \,,$$

and thus

$$\begin{aligned} & \mathbf{p}_2' \mathbf{x} - \sum\nolimits_{n \in N_{A_1}} (\mathbf{p}_2)_n (\mathbf{x})_n \\ \geq\ & \mathbf{p}_2 \mathbf{x} - \left[\left(\mathbf{p}_2^{1,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{1,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{1,H}\right)' \mathbf{x}^H \right] \\ =\ & \left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}^H. \end{aligned}$$

Combining (19) and (20) gives (for $\mathbf{x} = \mathbf{x}_1, \mathbf{q}_E$)

$$\left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}_2^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}_2^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}_2^H \geq \left(\mathbf{p}_2^{2,1}\right)' \mathbf{x}^1 + \left(\mathbf{p}_2^{2,2}\right)' \mathbf{x}^2 + \left(\mathbf{p}_2^{2,H}\right)' \mathbf{x}^H,$$

which yields $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$. This proves (18).

From this first step we conclude $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1$ under condition C.1. Next,we can use a similar argument as in Proposition 2 (for (15)) to obtain (for all $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$)

$$\left(\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{x}_1 \text{ and } y_1 \geq \mathbf{p}_1' (\mathbf{q}_E + \mathbf{x}_2)\right) \Rightarrow \left(\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E\right). \tag{21}$$

This gives the wanted result: we have $\mathbf{x}_1 R_o^{\mathbf{g}^1} \mathbf{q}_E$ and $\mathbf{x}_2 R_o^{\mathbf{g}^2} \mathbf{q}_E$ for all admissible individual demand functions $(\mathbf{g}^1, \mathbf{g}^2) \in Q(\mathbf{g})$ that satisfy WARP if condition C.1 holds.
■

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