Intersectoral Linkages, Diverse Information and Aggregate Dynamics*

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Abstract

What are the consequences of information frictions for aggregate dynamics? We address this question in a multi-sector real business cycle model with an arbitrary input-output structure. When information is exogenously dispersed, incomplete information slows adjustment to real shocks. When firms learn from local market-based information, however, the effects of incomplete information are dramatically reduced. We characterize cases where either sectoral or aggregate dynamics are exactly those of the full-information model. Sectoral irrelevance occurs when market-based information is sufficient to reveal optimal choices. Even without sectoral irrelevance, general equilibrium conditions may constrain average expectations such that aggregate dynamics match those under full-information. Calibrating the model to sectoral data from the United States, we show that the conditions for irrelevance of information are not met in practice but aggregate dynamics remain nearly identical to the model with full-information.

Keywords: Imperfect information, Information frictions, Dispersed information, Sectoral linkages, Strategic complementarity, Higher-order expectations

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1 Introduction

This paper asks whether information frictions, and in particular the sluggish movement of higher-order expectations, can provide an alternative framework for capturing delayed adjustment of macroeconomic quantities to real shocks. We assess this question in the context of a neoclassical model with sectoral input-output linkages in which each firm's output is potentially an input in the production of other firms. A longstanding research agenda explores how intermediate production structures influence the propagation of sectoral shocks to the aggregate economy.[1] The goal of this paper is to study how intermediate production structures affect the flow of information.

A finding that information frictions deliver sluggish responses to real shocks would satisfy the demands of Occam's razor in two ways. First, it would offer a unified and potentially more realistic microfoundation for the real adjustment costs and other frictions that the DSGE literature has adopted in order to match the movements of real variables observed in the data. Second, it suggests the potential for a unifying explanation of slow responses to both real and nominal shocks; the same friction that has proven successful in capturing the real effects of nominal shocks might also explain a different set of macroeconomic observations.[2]

The addition of a realistic input-output structure introduces two forces that typically lead incomplete information to have important consequences. First, intersectoral linkages create an environment of strategic interdependence, since the optimal choices in one sector depend on the actions of firms in other sectors. Specifically, our model of intersectoral linkages generates strategic complementarity in investment: if a firm's intermediate input suppliers engage in more investment, then the firm should also invest more as it will face relatively low marginal costs next period and therefore earn higher returns on its investment.[3] Second, the typical sparseness of input-output relations creates a situation in which firms in different sectors may have access to different pieces of information. Thus, sparse input-output structures may give rise to a dispersion of information across the economy. These two features, dispersed information and strategic complementarity,

[1] Papers in this line include Long and Plosser (1983), Horvath (1998), Dupor (1999), Horvath (2000), and Acemoglu et al. (2012).

[2] Among other findings, the literature on new-Keynesian models with dispersed information has found that information frictions can generate realistic hump-shaped dynamics of output in response to monetary shocks (Woodford, 2002); that mistaken expectations about aggregate productivity can have an impact akin to that of demand disturbances (Lorenzoni, 2009); and that the data favor dispersed information relative to other forms of price stickiness (Melosi, 2014).

[3] Authors including Basu (1995), Nakamura and Steinsson (2010), and Carvalho and Lee (2011) have examined the complementarities among price-setting decisions of firms generated by intersectoral linkages.

underly the new-Keynesian literature with imperfect information.

In order to study the role of *market-generated* informational asymmetries, we consider an environment with intersectoral linkages in which the information of firms is directly linked to the production structure of the economy. In particular, we assume that firms observe their own productivity, which is idiosyncratic to their sector, the price of their output, and the prices of those goods that are inputs in their production.[4] If firms use only a small subset of all intermediate inputs, as is realistically the case, then they will have only a limited local set of information about the situation of firms throughout the supply chain. In this context, information will also be dispersed: the firms in a given sector will have information that does not fully overlap with the information of firms in other sectors.[5] The endogenous nature of information in the economy, however, introduces richer possibilities for information transmission than the typical environment with exogenous private signals. Even if firms in a given sector use a small fraction of all intermediate goods, and therefore learn directly from relatively few intermediate prices, the prices of one sector's suppliers necessarily depend on the prices of its suppliers' suppliers, and so on.

Our central finding is that it is extremely difficult to generate a substantial impact of information frictions when firms observe and learn from their relevant market prices. Theoretically, we characterize cases where imperfect information is completely irrelevant at both the sectoral and aggregate level. We then show quantitatively that, even in the cases not covered by our theorems, the importance of incomplete information is typically very small. In short, information propagation through prices is extremely powerful regardless of the pattern of input-output linkages.

We begin our analysis with a standard sectoral model in which sectoral productivity shocks are the only shocks hitting the economy and investment choices are made with incomplete information. In our first three propositions, we characterize cases in which incomplete information has no consequence for either aggregate or sectoral quantities. With appropriate functional form restrictions for preferences and technology, irrelevance at the sectoral level can be recovered regardless of the sectoral structure of the economy. These propositions reveal that local sectoral prices have a remarkable ability to transmit the information relevant for optimal investment choices, even when those choices depend on all shocks hitting the economy.

[4] This assumption follows the suggestion of Hellwig and Venkateswaran (2009) and Graham and Wright (2010) that firms should condition their actions *at least* on the prices directly relevant to their own choices.

[5] The term "dispersed information" is often used to describe the situation of atomistic agents, each of whose contribution to the aggregate is negligible. When necessary to distinguish between that situation and the current one in which there is a finite set of agent types with different information, we will call the latter "diverse information."

In our fourth and fifth propositions, we characterize a notion of symmetry for which the linearized model with incomplete information delivers aggregate dynamics that are identical to the full-information economy This aggregate irrelevance may hold even when sectoral dynamics are quite different than under full information. The key requirement for aggregate irrelevance, beyond symmetry, is that agents observe a variable that reveals the aggregate state of the economy. Since we assume that firms know the structure of the economy, including market clearing conditions, firms use this knowledge to back out an aggregate view of the economy and they know others do the same thing. Since a sector interacts with only a subset of other sectors in the economy, this aggregate information is not sufficient to determine the optimal investment choice of that particular sector, but it is enough to ensure that in equilibrium deviations from full information actions made by one sector are offset by corresponding deviations of reverse sign in other sectors: *mistakes always cancel out*. Whether market-based information perfectly reveals the aggregate state, thereby ensuring aggregate irrelevance, depends on modeling details that we explore in the text. But even when aggregate learning is imperfect, it remains close-to-perfect and aggregate dynamics are almost exactly those of the full-information economy.

After establishing these analytical results, we extend the model to a full-fledged multi-sector model. We show that if firms make investment choices based on an exogenous information structure consisting of their own productivity and a noisy local signal about average productivity, then the economy indeed delivers realistic gradual hump-shaped responses of aggregate investment to shocks. When then show that, once firms are free to condition their investment choices on the information embedded in their local markets, the irrelevance results of the earlier sections reemerge very robustly. Despite this, sector-level responses are generally different and individual sectors are not able to determine the sectoral distribution of shocks. Moreover, sectoral information dispersion persists for long-periods of time even as aggregate responses exactly reproduce full-information responses.

Finally, we calibrate our model to match the empirical input-output structure of the United States economy, and solve the model using processes for aggregate and sectoral TFP estimated to match the Jorgenson et al. (2013) sectoral data. This version of the model simultaneously violates many of the conditions required for aggregate irrelevance and delivers substantial heterogeneity of expectations about both sectoral and aggregate shocks. Despite this, aggregate dynamics of this more realistic version of the model remain remarkably similar to those of the corresponding full-information model.

Our result that input-output structure does not matter for the consequences of information frictions contrasts with Acemoglu et al. (2012), who find that the propagation of sector-level shocks depends heavily on the nature of such linkages. Our results also contrast with the analogous new-Keynesian literature, which argues that strategic interactions among information constrained agents have important consequences for aggregate dynamics. A relatively small literature using more neoclassical "island" economies, such as Baxter et al. (2011) and Acharya (2013), also finds important consequences of similar information frictions.

Our results are, however, closely related to the findings of Hellwig and Venkateswaran (2014), who study the *Hayekian benchmark* in which market-based information leads to complete irrelevance of incomplete information. They show that deviations from this benchmark occur when firms face dynamic choices or strategic complementarities in their price setting decision. Our contribution relative to this paper is twofold. First, we characterize several instances where irrelevance of incomplete information holds, despite the fact that the investment choice of firms is both dynamic and strategically related to the investment choice of other sectors. Second, we demonstrate the potential for *aggregate* irrelevance, in which information frictions still matter at the sectoral level but cancel out exactly in the aggregate, even in the finite sector economy.

This paper proceeds as follows. In section 2, we describe the model environment. In section 3, we establish a set of analytical results characterizing cases where information is irrelevant for either the aggregate or sectoral outcomes. Section 4 performs a series of numerical experiments to demonstrate the importance of deviations from the assumptions underlying the analytical results. Section 5 calibrates the input-output structure and exogenous processes of the economy to match US data, and examines the consequences of incomplete information. Finally, section 6 concludes.

2 A Multi-Sector Model

We consider a discrete-time, island economy in the vein of Lucas (1972). The economy consists of a finite number of islands, each corresponding to a sector of the economy. On each island/sector resides a continuum of identical consumers and identical locally-owned firms. Consumers derive utility from consumption and experience disutility from supplying labor. The output of firms in each sector is supplied either as an intermediate input for other sectors or an input for a single final-good sector, exactly as in Long and Plosser (1983) and subsequent literature. The final goods sector does not employ any labor or capital, and its output is usable both as consumption and as the capital good in intermediate production. Since the price of the aggregate final good is observed by all islands, it is common knowledge and we treat final good as the numeraire and normalize its price P_t to 1 for all t.

2.1 Households

The representative household on island $i \in \{1, 2, ...N\}$ orders sequences of consumption and labor according to the per-period utility function, $u(C, L)$. Household income consists of wages paid to labor and the dividend payouts of the firms in sector i. Workers move freely across firms within their island but cannot work on other islands. Thus, the budget constraint of household on island i in period t is given by

$$C_{i,t} \leq W_{i,t} L_{i,t} + D_{i,t}, \tag{1}$$

where $C_{i,t}$ and $L_{i,t}$ are island-specific consumption and labor respectively for time t, and $W_{i,t}$ and $D_{i,t}$ are the sector-specific wage and dividend paid by firms for time t, denominated in terms of the final (numeraire) good.

The household maximizes

$$\max_{\{C_{i,t}, L_{i,t}\}_{t=0}^{\infty}} E_t^i \sum_{t=0}^{\infty} \beta^t u(C_{i,t}, L_{i,t})$$

subject to the budget constraint in (1). The expectation operator $E_t^i[V]$ denotes the expectation of a variable V conditional on the information set, Ω_t^i, for island i at time t. The first-order (necessary) conditions for the representative consumer's problem are

$$u_{c,t}(C_{i,t}, L_{i,t}) = \lambda_{i,t} \tag{2}$$

$$-u_{l,t}(C_{i,t}, L_{i,t}) = \lambda_{i,t} E_t^i[W_{i,t}], \tag{3}$$

where $\lambda_{i,t}$ is the (current-value) Lagrange multiplier for the household's budget constraint for period t. Under the assumption of market-consistent information, which we describe presently and maintain throughout this paper, consumers will observe both the aggregate price and their wage, so that the first order condition (3) always holds *ex post* (i.e. without the expectation operators) as well as *ex ante.*

2.2 Production Sector

Output in each sector $i \in \{1, 2, ..., N\}$ is produced according to the production function

$$Q_{i,t} = \Theta_{i,t} F(K_{i,t}, L_{i,t}, \{X_{ij,t}\}; \{a_{ij}\}), \tag{4}$$

where $\Theta_{i,t}$ is the total factor productivity of the representative firm on island i, $K_{i,t}$ and $L_{i,t}$ are the amounts of capital and labor used, and $X_{ij,t}$ denotes the quantity of intermediate good j used by the sector-i firm. The time-invariant parameters $\{a_{ij}\}$ describe the technology with which goods are transformed into output in sector i. We will use the convention that $a_{ij} = 0$ whenever good j is irrelevant to sector i's production. We summarize the input-output structure of the economy with the $N \times N$ matrix, IO, whose (i, j)'th entry is α_{ij}, where α_{ij} denotes the share of good j in sector i's output. Note that $\alpha_{ij} = 0$ whenever $a_{ij} = 0$ and visa-versa.

Firms in sector i take prices as given and choose all inputs, including next period's capital stock, so as to maximize the consumers' expected present discounted value of dividends, where expectations are with respect to the island-i information set. We assume a standard capital accumulation relation

$$K_{i,t+1} = I_{i,t} + (1-\delta)K_{i,t}, \tag{5}$$

where $I_{i,t}$ is the investment by the representative firm in sector i. Firm i's profit maximization problem is therefore

$$\max_{\{L_{i,t}, X_{ij,t}, I_{i,t}, K_{i,t+1}\}_{t=0}^{\infty}} E_t^i \sum_{t=0}^{\infty} \beta^t \lambda_{i,t} \left(P_{i,t}Q_{i,t} - W_{i,t}L_{i,t} - \sum_{j=1}^{N} P_{j,t}X_{ij,t} - I_{i,t} \right)$$

subject to equations (4) and (5).[6] Here, $P_{i,t}$ denotes the (relative) price of goods produced in sector i.

We assume that firms always observe the current period price of their inputs and output, an assumption we discuss below. Thus, the firm sets the marginal value product of labor and the relevant intermediate inputs equal to their price, yielding the following intratemporal optimality conditions:

$$W_{i,t} = P_{i,t} \frac{\partial Q_{i,t}}{\partial L_{i,t}}, \tag{6}$$

$$P_{j,t} = P_{i,t} \frac{\partial Q_{i,t}}{\partial X_{ij,t}}, \qquad \forall j \text{ s.t. } a_{ij} > 0. \tag{7}$$

Finally, firm i's first order conditions with respect to investment and future capital combine to yield

$$P_t = \beta E_t^i \left[\frac{\lambda_{i,t+1}}{\lambda_{i,t}} \left(P_{i,t+1} \frac{\partial Q_{i,t+1}}{\partial K_{i,t+1}} + P_{t+1}(1-\delta) \right) \right], \tag{8}$$

where again $P_t = P_{t+1} = 1$ denotes the price of the aggregate good used for investment.

[6]Our assumption that firms, rather than consumers, choose future capital contrasts with typical practice in the RBC literature. This assumption is for expositional reasons only. In our baseline model, firms on each island have the same information as consumers and therefore make capital accumulation decisions that are optimal from the consumers' perspective as well.

2.2.1 Final Goods Sector

Competitive firms in the final goods sector aggregate intermediate goods using a standard CES technology,

$$Y_t = \left\{ \sum_{i=1}^{N} a_i^{\frac{1}{\zeta}} Z_{i,t}^{1-\frac{1}{\zeta}} \right\}^{\frac{1}{1-1/\zeta}}, \tag{9}$$

where $\sum_{i=1}^{N} a_i = 1$ and Y_t is the output of the final good, $Z_{i,t}$ is the usage of inputs from industry i, and $\{a_i\}_{i=1}^{N}$ represent exogenous, time-invariant weights in the CES aggregator. Input demands are given by

$$Z_{i,t} = a_i \left(\frac{P_{i,t}}{P_t} \right)^{-\zeta} Y_t. \tag{10}$$

2.3 Equilibrium

The equilibrium of the economy is described by equations (2) through (10), exogenous processes for $\Theta_{i,t}$, and the island-specific market clearing conditions and resource constraints,

$$Q_{i,t} = Z_{i,t} + \sum_{j=1}^{N} X_{ji,t} \tag{11}$$

$$P_{i,t}Q_{i,t} = C_{i,t} + I_{i,t} + \sum_{j=1}^{N} P_{j,t}X_{ij,t}. \tag{12}$$

By Walras' law, we have ignored the aggregate market clearing condition $Y_t = \sum_{i=1}^{N} C_{i,t} + \sum_{i=1}^{N} I_{i,t}$. Thus, we have 1+9N+N^2 equations in the same number of unknowns: Y_t, $\{P_{i,t}\}_{i=1}^{N}$, $\{W_{i,t}\}_{i=1}^{N}$, $\{C_{i,t}\}_{i=1}^{N}$, $\{\lambda_{i,t}\}_{i=1}^{N}$, $\{Q_{i,t}\}_{i=1}^{N}$,$\{Z_{i,t}\}_{i=1}^{N}$, $\{L_{i,t}\}_{i=1}^{N}$, $\{I_{i,t}\}_{i=1}^{N}$, $\{K_{i,t}\}_{i=1}^{N}$, and $\{X_{ij,t}\}_{i,j=1}^{N}$. Depending on the number of the zeros in the input-output matrix, some of the unknown $X_{ij,t}$ and corresponding first-order conditions in equation (7) will drop out reducing the size of the system.

2.4 Information

In this paper, we follow the suggestion of Graham and Wright (2010) that agents should learn about the economy based on "market-consistent" information. That is, a firm's information set should include, as a minimal requirement, those prices that are generated by the markets it trades in. In our context, this means that firms will observe and learn from the prices of their output and all inputs with a positive share in their production.

In addition to these prices, we also take as a baseline assumption that firms observe their own productivity.[7] The following definition makes this assumption precise:

Definition 1. *The* ***market consistent*** *information set of agents in sector i, denoted by $\Omega_t^{i,MC}$, is given by full histories*

$$\{\Theta_{i,t-h}, P_{i,t-h}, P_{j,t-h}, \forall j \;\; s.t. \;\; \alpha_{ij} > 0\}_{h=0}^{\infty} \tag{13}$$

We also consider a linear approximation to the model above in later sections. For those cases, $\hat{\Omega}_t^{i,MC}$, is defined analogously to contain log-level deviations of the same variables.

Our key observation is that, under the assumption of market-consistent information, the nature of intersectoral trade will be a crucial determinant of the information available to firms. In particular, the existence of a relatively sparse input-output structure, which is the empirically relevant case, implies that firms have direct observations on a very small portion of the overall economy. The macroeconomic literature on intersectoral linkages has traditionally focused on how the nature of intersectoral linkages affects the economy-wide propagation of sectoral shocks; our goal is to study how such linkages affect the broader propagation of information.

Assumptions about information are susceptible to the "Lucas critique" because what agents choose to learn about may be influenced by policy and other non-informational features of the economic environment. The assumption of market-consistent information represents a compromise between assuming an exogenous fixed information structure (as much of the previous literature on information frictions does) and the assumption that agents endogenously design an optimal signaling mechanism according to a constraint or cost on information processing (as suggested by the literature on rational inattention initiated by Sims, 2003.) Because agents form expectations based on prices, the information content of which depends on agents' actions, there is scope for an endogenous response of information to the fundamental parameters governing the environment. Thus, the assumption of market-consistent information offers at least a partial response to the critique: if agents face a discretely lower marginal cost of learning from variables which they must anyways observe in their market transactions, then comparative statics for small changes in parameters may be valid.

[7]Since firms know that they are identical, observing any endogenous island-specific variable (firm profits or the local wage, for examples) would be sufficient to infer own productivity.

3 Irrelevance Results

In this section, we develop several propositions that provide important benchmarks cases when information frictions cannot matter for the dynamics of the model. The first proposition establishes conditions on preferences and technology that guarantee that market consistent information leads to the full-information (and therefore optimal) allocations in the economy. The remaining propositions follow the tradition in the RBC and information-friction literature and focus on a log-linear approximation of the economy. These propositions establish conditions under which incomplete information either (1) affects neither sectoral nor aggregate outcomes, or (2) potentially affects sectoral outcomes but has no effect on aggregate outcomes.

3.1 Irrelevance in the Non-linear Model

The first proposition establishes conditions under which market-consistent information is sufficient to ensure that allocations at both the sectoral and aggregate levels are those of the full-information model.

Proposition 1 (Long and Plosser (1983) equilibrium)**.** *Suppose that capital depreciates fully each period, that the intermediate production function is Cobb-Douglas in all inputs, and that the time-separable utility function is given by*

$$u(C, L) = \log(C) + v(L). \tag{14}$$

Then the model with market consistent information replicates the full-information equilibrium of the economy.

Proof. Under Cobb-Douglas production and log utility, the optimality conditions of the firm in equations (7) and (8) become

$$P_{j,t}X_{ij,t} = \alpha_{ij}P_{i,t}Q_{i,t} \tag{15}$$

$$\frac{K_{i,t+1}}{C_{i,t}} = \beta E_t^i \left[\frac{\alpha_{ik}}{C_{i,t+1}} P_{i,t+1}Q_{i,t+1} \right] \tag{16}$$

where α_{ij} and α_{ik} represent the share of good j and the share of capital, respectively, in the production of good i.

For each intermediate sector i, combine the constant share result from equation (15) with the island resource constraint (12) to find

$$P_{i,t}Q_{i,t} = \frac{1}{\left(1 - \sum_{j=1}^{N} \alpha_{ij}\right)} \left(C_{i,t} + K_{i,t+1}\right). \tag{17}$$

Substituting expression (17) into the intertemporal condition of the firm gives

$$\frac{K_{i,t+1}}{C_{i,t}} = \beta \frac{\alpha_{ik}}{1 - \sum_{j=1}^{N} \alpha_{ij}} E_t^i \left[1 + \frac{K_{i,t+2}}{C_{i,t+1}} \right]. \tag{18}$$

Recursively substituting, the law of iterated expectations and the transversality condition yield

$$\frac{K_{i,t+1}}{C_{i,t}} = \frac{1 - \sum_{j=1}^{N} \alpha_{ij}}{1 - \beta\alpha_{ik} - \sum_{j=1}^{N} \alpha_{ij}}, \tag{19}$$

which is independent of the information assumption we made. □

Under the conditions of Proposition 1 incomplete information has no impact on either aggregate *or* sectoral quantities or prices. Market consistent information is all that is needed for the firm to back out its own optimal action. This is true even though firms may not know (and indeed generally have a very inaccurate perceptions of) what is happening in other sectors. The conclusion that information frictions do not matter follows from a "bottom-up" logic: aggregate outcomes are the same as under full information because individual choices themselves do not depend on the missing information.

This proposition bears a close relationship to the finding of Long and Plosser (1983), which is further discussed by King et al. (1988). These authors show that, under full-information and the conditions on preferences and technology above, income and substitution effects cancel so that the capital choice becomes essentially static and is disconnected from the stochastic nature of the underlying shock. This unravelling of the dynamic choice leads our result to closely resemble the first proposition in Hellwig and Venkateswaran (2014). Those authors show in a static model of monopolistic price-setting that market-generated information is sufficient for firms to infer their own (full-information) optimal pricing choice. When this is true, the full-information outcome must be an equilibrium of the partial information model. The same reasoning applies here as well, because agents who can infer their optimal choice under full information have no incentive to do otherwise if other agents also behave as they would under full information.

An important difference arises, however, because in our model the investment choice becomes static only after imposing market-clearing at *all future dates.* It is only because the firm knows the model and, in particular, knows that the future choices will be also be based on the relevant prices, that it can infer its current optimal choice. Thus, although the optimal action is independent of expectations *ex-post*, this result remains fundamentally driven by the formation of rational expectations about future firm choices.

3.2 Irrelevance in the Linearized Model

Outside of the special case discussed in Proposition 1, it is impossible to make generic statements about the consequences of information for the non-linear model. We hereafter focus on a linearized version of the model in which labor is supplied inelastically, intermediate production is Cobb-Douglas in all inputs, sectoral weight in final good production are symmetric, capital depreciates fully each period, and preferences take a CRRA-form with an elasticity of intertemporal substation equal to τ. While linearization is important, none of the results in this section depend on inelastic labor or the rate of depreciation.

For the remainder of this section, we will also assume the process for $\Theta_{i,t}$ is independent and identically distributed across firms according to an AR(1) process in logs with symmetric autoregressive parameter,

$$\hat{\theta}_{i,t+1} = \rho\hat{\theta}_{i,t} + \sigma\epsilon_{i,t+1}, \tag{20}$$

where the iid shocks $\epsilon_{i,t}$ have unit variance. Furthermore, we use the convention that for any variable V_t, $\hat{v}_t$ denotes its log-deviation from steady-state.

The linearized first order condition of the consumer in sector i is

$$\hat{c}_{i,t} = -\tau\hat{\lambda}_{i,t}. \tag{21}$$

Intermediate production is characterized by the linearized production function

$$\hat{q}_{i,t} = \hat{\theta}_{i,t} + \alpha_{ik}\hat{k}_{i,t} + \sum_{j=1}^{N}\alpha_{ij}\hat{x}_{ij,t}, \tag{22}$$

where the parameters α_{ik} denote the capital share of output in sector i and α_{ij} the share of good j in the output of sector i. We assume that $\alpha_{ik} + \sum_{j=1}^{N}\alpha_{ij} = 1 - \phi_l < 1$, so that the share of inelastically-supplied labor is positive (or equivalently, that the economy exhibits decreasing returns to scale.) To simplify summation statements, we adopt the normalization that $\hat{x}_{ij,t} = 0$ whenever $\alpha_{ij} = 0$.

The firm's optimal choice of input $\hat{x}_{ij,t}$ is given by

$$\hat{p}_{j,t} = \hat{p}_{i,t} + \hat{q}_{i,t} - \hat{x}_{ij,t}. \tag{23}$$

Linearizing the intertemporal equation of the firm, and using the consumer's first order condition to substitute out $\hat{\lambda}_t$ yields

$$-\frac{1}{\tau}E_t^i\left[\hat{c}_{i,t} - \hat{c}_{i,t+1}\right] = E_t^i\left[\hat{p}_{i,t+1} + \hat{q}_{i,t+1} - \hat{k}_{i,t+1}\right]. \tag{24}$$

Final goods aggregation with symmetric weights implies that

$$\hat{y}_t = \frac{1}{N}\sum_{i=1}^{N}\hat{z}_{i,t}, \tag{25}$$

with $\hat{z}_{i,t}$ demanded according to

$$\hat{z}_{i,t} = \hat{y}_t - \zeta\hat{p}_{i,t}. \tag{26}$$

Sectoral market clearing implies

$$\hat{q}_{i,t} = s_{iz}\hat{z}_{i,t} + (1 - s_{iz})\sum_{j=1}^{N}\eta_{ji}\hat{x}_{ji,t} \tag{27}$$

$$\hat{p}_{i,t} + \hat{q}_{i,t} = s_{ic}\hat{c}_{i,t} + s_{ik}\hat{k}_{i,t} + (1 - s_{ic} - s_{ik})\sum_{j=1}^{N}\omega_{ij}(\hat{p}_{j,t} + \hat{x}_{ij,t}) \tag{28}$$

where s_{iz} is the steady-state share of sector i output devoted to final good production, s_{ic} and s_{ik} are the shares of gross value of sectoral output dedicated to consumption and investment respectively, η_{ji} is the fraction of sector i intermediate usage devoted to sector j, and ω_{ij} the fraction of intermediate payments from sector i going to sector j.

Equations (20) through (28) fully characterize the linearized model.

3.2.1 Sectoral Irrelevance in the Linearized Model

Despite linearity, typically very little can be said about dynamic models of incomplete information without resorting to numerical solution methods. However, under the assumptions for the linearized model outlined earlier, we can establish some important properties of the model without fully solving the firm's inference problem. We assume throughout this section that the model is parameterized so that it has a unique stationary equilibrium under full-information.

Before proceeding to the propositions, it is convenient to define the concept of *action informative* information.

Definition 2. *An information set* $\hat{\Omega}^i$ *is* **action informative** *for agents of type* i *if, in the full-information economy, it is a sufficient statistic for type* i*'s optimal action.*

The concept of action informativeness is the key behind the observation of Hellwig and Venkateswaran (2014) that observation of own price and quantity lead to an irrelevance of incompleteness of information in the standard monopolistic competition model. More generally, whenever all agents in an economy have access to an action informative information set, then there exists an equilibrium of the economy with outcomes that are identical to the full-information economy. To see that this must be the case, consider the choice of an individual with an action informative information set when all other agents in the economy behave according to the prescriptions of the full-information economy. By the definition of action informative, the individual's information must reveal her optimal action. By construction, however, they can do no better than to take that action and the same applies to all other agents in the economy; the conjectured equilibrium replicating full information is sustained.

Proposition 2 and 3 each characterize cases in which market-consistent information is always action informative. Proposition 2 establishes that, with the additional restriction of no intermediate production interlinkages, market consistent information is sufficient to reproduce the full-information equilibrium of the model.

Proposition 2. *Suppose that the share of intermediates is zero, $\alpha_{ij} = 0, \forall i, j$, and that the information set of firms is $\hat{\Omega}_t^{i,MC}$. Then the equilibrium of the full-information model is also an equilibrium of the diverse-information model.*

Proof. In this case, $\hat{z}_{i,t} = \hat{q}_{i,t} = -\zeta \hat{p}_{i,t} + \hat{y}_t$, implying that observations of sector i's own price and output are sufficient to determine aggregate output $\hat{y}_t$ in each period. Under the full-information equilibrium, the history of $\hat{y}_t$ is sufficient to infer $\hat{\theta}_t$ and $\hat{k}_t$, and therefore to optimally predict future $\hat{y}_t$. But the forecast of $\hat{y}_t$ is the only piece of non-local information that is required to forecast $\{\hat{p}_{i,t+h}\}_{h=1}^{\infty}$. If aggregate dynamics follow the full-information path, forecasts of future $\hat{p}_{i,t+h}$ are equivalent to full-information forecasts. Each sector can therefore infer its optimal investment choice under full information and sectoral allocations are consistent with the full-information equilibrium. □

Proposition 2 is analogous to the second proposition in Hellwig and Venkateswaran (2014) which considers the choice of price-setters who must take into account future, as well as current, conditions and characterizes conditions under which market-generated information leads to an equilibrium identical to that under full-information. Because demand and aggregate output are integrally linked, market consistent information is a powerful force for learning about aggregates, pushing the model towards its full-information equilibrium.

Proposition 3 provides additional restrictions on preferences (τ) and the final-goods aggregator (ζ) for the linearized model such that market consistent information is sufficient to reproduce the full-information equilibrium *regardless of the input-output structure.*

Proposition 3. *Suppose that $\tau = \infty$, $\zeta = 1$, and the information set of firms is $\hat{\Omega}_t^{i,MC}$.*

Then the equilibrium of the full-information model is also an equilibrium of the diverse-information model.

Proof. See Appendix A. □

While fairly involved, the proof of Proposition 3 proceeds by showing that, under full information, relative prices are always a sufficient statistic for forecasting their own evolution. Because of this, prices today combined with own productivity are all that is required to forecast the future value of a unit of capital, and therefore to determine today's optimal investment choice.

The proposition provides an important benchmark for assessing the importance of information frictions under the assumption of market consistent information: information transmission of payoff relevant states is complete and does not depend on the sparsity, balance, or degree of linkages. In this respect, the theorem contrasts with the finding of Acemoglu et al. (2012) that the pattern of intersectoral linkages is crucial for understanding the transmission of sectoral shocks to the aggregate economy. It also suggests that the degree of substitutability in the consumption aggregator will be an important determinant of the impact of information frictions when we turn to quantifying it in the more general model.

3.2.2 Aggregate Irrelevance in the Linearized Model

Propositions 2 and 3 establish aggregate irrelevance from the "bottom-up," by showing the existence of equilibria in which all firms take the same actions as they would under full information. The proposition in this section, in contrast, proceed via a "top-down" logic by showing that equilibrium conditions can impose restrictions on aggregates independently of what they imply for sector-level dynamics. Proposition 4 shows that the linearized economy may have a representation in which aggregates quantities can be determined without reference to sector-specific variables. It applies to cases where the economy is symmetric in the sense defined by Dupor (1999). Propsition 5 shows that the same type of symmetry ensures that beliefs, and therefore aggregate dynamics, must be consistent with full-information aggregate dynamics, regardless of the inferences drawn for sectoral-level disturbances.

Definition 3. *The input-output matrix IO is* **circulant** *if its rows can be rearranged to* *take the following form*

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_N \\ \alpha_2 & \alpha_3 & & \alpha_1 \\ \vdots & & \ddots & \vdots \\ \alpha_N & \alpha_1 & \dots & \alpha_{N-1} \end{pmatrix} \tag{29}$$

Proposition 4. *Suppose the economy has a circulant input-output structure. Then, aggregates in the economy are identical to a representative agent economy whose equilibrium condition are given by*

$$\hat{y}_t = (1-\alpha_x)^{-1}\hat{\theta}_t + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t \tag{30}$$

$$\hat{y}_t = (1-\alpha_x)^{-1}\alpha_l\hat{c}_t + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t \tag{31}$$

$$-\frac{1}{\tau}E_t^f\left[\hat{c}_t - \hat{c}_{t+1}\right] = E_t^f\left[\hat{y}_{t+1} - \hat{k}_{t+1}\right] \tag{32}$$

$$\hat{\theta}_{t+1} = \rho\hat{\theta}_t + \epsilon_{t+1} \tag{33}$$

where α_x is the row sum of the IO matrix, $\alpha_k = 1 - \alpha_l - \alpha_x$ is the capital share of the sectoral economy, and $\hat{v}_t \equiv \frac{1}{N}\sum_{i=1}^N \hat{v}_{i,t}$ for any sectoral variable $\hat{v}_{i,t}$.

Proof. Proved in Appendix A. □

Proposition 4 closely resembles a result of Dupor (1999). Equations (30) through (33) are simply the linearized first-order conditions of a standard signle-sector RBC model. Only equation (32) is potentially affected by imperfect information of the kind we consider here; the remaining equations (30), (31), and (33) always hold under the market-consistent information assumption.

When the economy is circulant, the relevant exogenous state for aggregates is simply the mean of sector-level shocks. We call $\hat{\theta}_t$ the *notional aggregate state* of the economy. In Dupor (1999), this form of symmetry was also shown to ensure that sector-level shocks decayed quickly (root-N) with the degree of disaggregation. It turns out that the same symmetry is the essential ingredient for the aggregate irrelevance result of Proposition 5. Before proceeding to the proposition, however, it is helpful to define an aggregate analogue to an action-informative information set.

Definition 4. *For an economy with a circulant input-output structure, an information set $\hat{\Omega}^i$ is* **aggregate informative** *if, in the full-information economy, it is a sufficient statistic for the notional aggregate state.*

Notice that being action informative is neither necessary nor sufficient for an information set to be aggregate informative. Corollary 1 shows, nonetheless, that market information satisfies both requirements in the special case of Proposition 3.

Corollary 1. *Suppose that $\tau = \infty$, $\xi = 1$, and the input-out structure is circulant. Then, market-consistent information is also aggregate informative.*

Corollary 31 follows directly from the proof of Proposition 3: in a circulant economy satisfying the prerequisites of Proposition 3, market-consistent information is simultaneously action informative and aggregate informative. The coincidence of these two features of market-consistent information will be instructive for interpreting our numerical results as we move away from the postulates of the theorem.

Proposition 5 considers the consequences of incomplete information in a circulant input-output economy, when agents have access to an aggregate informative variable. It is proved in Appendix A.

Proposition 5. *Suppose that the input-output matrix is circulant and that*

$$\hat{\Omega}_t^i = \left\{ \hat{\Omega}_t^{i,MC}, \{\hat{\theta}_{t-h}\}_{h=0}^{\infty} \right\}. \tag{34}$$

Then any symmetric equilibrium of the diverse information economy has the same aggregate dynamics as the full information equilibrium.

Proof. Proved in Appendix A. □

Corollary 2. *Any equilibrium of the model with $\hat{\Omega}_t^i = \left\{ \hat{\Omega}_t^{i,MC}, \{\hat{\theta}_{t-h}\}_{h=0}^{\infty} \right\}$ is also an equilibrium of the model with $\hat{\Omega}_t^i = \left\{ \hat{\Omega}_t^{i,MC}, \{\hat{v}_{t-h}\}_{h=0}^{\infty} \right\}$, where $\{\hat{v}_{t-h}\}_{h=0}^{\infty}$ is aggregate informative.*

Proposition 5 is a bit more startling given earlier results in the literature. First, the presence of complementarities in decisions means that higher-order expectations matter for the decisions of individual firms. In the context of price-setting firms, such complementarity typically leads to large aggregate consequences of information frictions, and increased persistence in particular. Second, the result on aggregates holds even though sectoral expectations and choices can be substantially different under market-consistent information. Sectoral mistakes cancel each other out, despite the fact that *no law of large numbers is being invoked*, nor does any apply in our economy.

Technically, the key to the results above is that agents have some means of inferring the average state of productivity from their information set either directly, as in Proposition 5, or indirectly, as in Corollary 2. When they do, agents can track aggregates in the economy quite independently of their ability to track the idiosyncratic conditions relevant to their choices. Since average expectations must then be consistent with the common knowledge aggregate dynamics, expectational mistakes, and therefore mistakes in actions, must cancel out; the economy exhibits a disconnect between what is happening in aggregate and what is happening at the sectoral level.

To see the logic of the proof in more detail, observe that price aggregation under symmetry requires that (log) prices sum to zero. Therefore, in a linear equilibrium, average actions can depend only on the aggregate state, since any dependency on the sectoral prices in the information set must cancel out. Moreover, sectoral prices themselves *cannot* depend on the aggregate state. If they did, the average of sectoral prices would be non-zero, violating price aggregation. Since firms are assumed to observe the notional aggregate state, their expectations of their own price must also be orthogonal to the aggregate state, so that average beliefs of any sectoral variable, $\frac{1}{N}\sum E^i[\hat{v}_{i,t}]$, must depend only on the aggregate state. But, if aggregate actions and average beliefs depend only on the aggregate, rational expectations requires average beliefs must be equal to full information expectations of the aggregate, $E^f[\hat{v}_t] = \frac{1}{N}\sum E^i[\hat{v}_{i,t}]$. When this is true, summing the Euler equation in equation (24) yields the Euler equation of the aggregate representation of the full-information economy in equation (32).

As the proof of Proposition 5 makes clear, the inclusion of market consistent information is not essential to this result; other symmetric information structures that also reveal the notional aggregate state deliver the same aggregate irrelevance. In principal, these results permit very large implications of limited information at the sectoral level while perfectly imitating the aggregate dynamics of the full-information model. Generating examples which demonstrate such a large disconnect is rather easy as we show in section 4. However, in practice we will find that it is hard to do so for realistic calibrations and specifications of the information structure.

Conversely, while the ability to forecast aggregates is essential for the exact results in Proposition 5, in practice the consequences of removing the aggregate informative variable $\hat{v}_t$ from the market-consistent information set is small. In the next section, we show that relative prices, in conjunction with the observation of own-sector productivity, do a nearly perfect job at revealing the aggregate state despite the fact that, with intermediate inputs, the firm can no longer use its prices and market clearing condition in its sector to determine aggregate output. Any movement in relative prices must be explained by a change in overall productivity in the economy. While many constellations of idiosyncratic shocks can lead to same observed relative price movements (among those prices observed by a given sector) they all share roughly the same overall change in average productivity.

Finally, notice that Propositions 2 through 5 do not establish uniqueness of the equilibria they describe. Numerical experimentation, however, has consistently confirmed for us that the equilibria in all cases are unique, so long as the full-information economy also displays uniqueness.

4 Beyond Irrelevance

We now examine the degree to which the analytical results derived above apply to the more general model outlined in Section 2. We therefore relax the restrictions of section 3 and reinstate partial depreciation of capital and labor-leisure choice in the model. Moreover, we work with more general functional forms for the utility and production functions and calibrate the associated parameters to realistic values.

We begin our analysis with a version of the model with only identical and independent sectoral productivity shocks, as in equation (20), and a (symmetric) circulant input-output structure. In this version of the model, we show that, when information is *not* market consistent, incomplete information can have substantial impact on the dynamics and may lead to slow aggregate responses to sectoral productivity shocks. The physical environment of the economy *is consistent* with an important role for information.

We then turn to information sets that include market consistent information, and demonstrate that the result in Proposition 5 holds to numerical precision in this more general model. That is, the aggregate dynamics under market-consistent information plus an aggregate informative variable are the same as those with full information, while sectoral dynamics are different. Moreover, the impact of excluding the aggregate informative variable from agents' information is extremely small. In the more general model, market-consistent information is remarkably close to being aggregate informative.

Next, we add a common aggregate productivity shock to sectoral productivity and show that, when the persistence of this common aggregate component is different than that of the sectoral component, aggregate dynamics under market-consistent information are somewhat different than under full-information. However, these differences have nothing to do whatsoever with dispersion of information. Instead, while agents remain uncertain about the decomposition between sectoral and aggregate realizations, their beliefs about this decomposition are both common and common knowledge.

4.1 Functional Forms and Calibration

For our quantitative analysis, we use the per-period utility function

$$u\left(C,L\right)=\frac{\left(C(1-L)^{\varphi}\right)^{1-\frac{1}{\tau}}-1}{1-\frac{1}{\tau}}, \tag{35}$$

Table 1: Baseline parameterization of the model.

Parameter	Concept (Target)	Vaue
N	Number of sectors	6.00
δ	Capital depreciation	0.05
κ	Capital-labor elasticity	0.99
ξ	Elasticity among intermediates (when used)	0.33
σ	Elasticity between composite inputs	0.20
ζ	Final goods elasticity	1.50
Φ_x	Share of intermediate inputs (when used)	0.00
Φ_k	Capital share of value-added	0.34
β	Discount factor	0.99
τ	Intertemporal elasticity	0.50
φ^{-1}	Implied Frisch elasticity = 1.9	15.00
ρ_ς	AR coeff. sectoral prod. shocks	0.90
ρ_d	AR coeff. sectoral demand shocks (when used)	0.00
ρ_A	AR coeff. agg shock (when used)	0.95

where, τ, as is again the elasticity of intertemporal substitution and the Frisch elasticity of labor supply is given by $\frac{1-\bar{L}}{\bar{L}}\frac{1}{1+\varphi(1-\tau)}$, where $\bar{L}$ is the average fraction of overall hours dedicated to production.

On the firm side of the economy, we assume that the production function $F(\cdot)$ takes the form of a nested-CES technology:

$$F\left(K_{i,t}, L_{i,t}, \{X_{ij,t}\}\right) = \left[b_{i1} \left\{ \sum_{j=1}^{N} a_{ij} X_{ij,t}^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} + b_{i2} \left\{ a_{il} L_{i,t}^{1-\frac{1}{\kappa}} + a_{ik} K_{i,t}^{1-\frac{1}{\kappa}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}} \right]^{\frac{1}{1-1/\sigma}} \tag{36}$$

where ξ is the elasticity of substitution between intermediate inputs, κ is elasticity of substitution between capital and labor, and σ is the elasticity of substitution between the composite intermediate input and the composite capital-labor input. Finally, a_{il},a_{ik}, b_{i1} and b_{i2} are production parameters that are set to match the (cost) shares of various inputs. Without loss of generality, we normalize $b_{i1} = b_{i2} = 1$.

The procedure for calibration is outlined Appendix B and the calibrated parameters with their associated targets are summarized in Table 1. For this stylized example, we take the number of sectors to be six. Although this is a relatively small number, none of our qualitative results depend on this choice. A few other parameters choices warrant special attention. First, we calibrate the elasticity between the two composite inputs, $\sigma = 0.20$, well below unity. This value is in line with the estimates discussed in the working-paper version of Moro (2012). We calibrate the share of intermediate inputs to be 0.6, which is the value suggested by Woodford (2003). These two choices are crucial in determining the degree of complementarity in the model, as we discuss in the next section. Additionally, we set the final goods elasticity $\zeta = 1.5$, which is higher than the value used in Horvath (2000) and somewhat less than what is typically assumed in the new-Keynesian literature (which instead focuses on the markups generated by imperfect competition). We take $\varphi = 15$, which implies a frisch-elasticity in our model of slightly under two. Finally, capital depreciation rate is set to a standard value and we begin by assuming that sectoral shocks follow symmetric, independent, AR(1) structure.

Solving the model poses a technical challenge because agents must "forecast the forecasts of others" as in (Townsend, 1983) and because they must condition these expectations on the information embodied in endogenous variables. Appendix C summarizes our approach to numerically solving the model.

4.2 Intersectoral Linkages and Complementarities

Before proceeding to our numerical results, it is helpful to understand the sources and the strength of the strategic interactions generated by the introduction of an intermediate production structure. In new-Keynesian environments, strategic complementarities pertain to the static price-setting decision of firms.[8] In contrast, here they arise from investment decisions which are inherently dynamic, complicating any discussion of complementarities. In order to maintain tractability, we therefore consider the strategic interactions in investment occurring in steady-state in a symmetric two-sector version of the model from Section 3. Specifically, we consider the steady-state investment choice of sector one, and examine sector one's response to a percentage deviation, Δ, of sector two investment from its steady-state equilibrium value.[9] In Appendix D, we show that theresulting investment choice of sector one is given by

$$\hat{k}_1^* = \hat{k}_{1,ss} + \frac{\phi_k}{1+\phi_k}\Delta. \tag{37}$$

The parameter $\phi_k > 0$ is therefore the relevant measure of strategic complementarity in the model.

[8]This is true even if prices are sticky, as in Angeletos and La'O (2009), as the optimal price can be viewed as a weighted average of future target prices.

[9]The decentralized first-order conditions of sector one can be interpreted as the first order conditions of a price-taking planner who maximizes that island's welfare.

In order to do simple comparative statics for ϕ_k vis-a-vis various parameters of the model, it is helpful to specialize, for the time being, to the Cobb-Douglas production function with a fixed supply of labor. Specifically, assume that

$$F(K, L, X) = K^{\tilde{\alpha}_k(1-\alpha_x)} L^{(1-\tilde{\alpha}_k)(1-\alpha_x)} X^{\alpha_x}. \tag{38}$$

In this formulation, $\tilde{\alpha}_k = \alpha_k/(1-\alpha_x)$ represents the shares of capital in *value-added* in the economy and α_x is the economy-wide share of intermediates in production. In this special case, we have that

$$\phi_k = \frac{1}{2}\left(\frac{\tilde{\alpha}_k}{1-\tilde{\alpha}_k}\right)\left(\frac{(1+\alpha_x)^2}{(1-\alpha_x)^2\zeta + 4\alpha_x}\right). \tag{39}$$

It follows immediately that complementarity increases in the model when (1) the share of capital in value added ($\tilde{\alpha}_k$) is very large (2) the share of intermediates (α_x) is large and (3) the input elasticity (ζ) in the final-goods sector is relatively low. Notice, in particular, the contrast of comparative static (3) relative to standard new-Keynesian environments where higher elasticities lead to greater, rather than smaller, pricing complementarities.

Since complementarity is increasing in α_x, the limit as $\alpha_x \to 1$ delivers an upper bound on the degree complementarity:

$$\lim_{\alpha_x \to 1} \phi_k = \frac{1}{2}\left(\frac{\tilde{\alpha}_k}{1-\tilde{\alpha}_k}\right). \tag{40}$$

Thus, under a standard calibration with a capital share of one-third, a one-percentage exogenous increase in sector two's capital choice can deliver no more than a $\frac{1/3}{1+1/3} = 0.25$-percentage increase in sector ones own capital choice, a relatively weak complementarity by the standard of the new-Keynesian literature.

In the more general version of the model, the steady-state investment complementarity may differ from the value in the fixed-labor, Cobb-Douglas version of the model discussed above. Figure 1 plots the value of $\frac{\phi_k}{1+\phi_k}$ against the share of intermediates under the baseline calibration of the model. Although the comparative statics derived above are robust, the bound derived under Cobb-Douglas production turns out to be quite conservative. This difference is driven primarily by the introduction of an endogenous labor choice and our calibration of a much-lower-than-one elasticity of substitution between the intermediate good and the capital-labor composite. Under our baseline calibration of an intermediate share of 0.6, the value of the this complementary is roughly $\frac{\phi_k}{1+\phi_k} = 0.78$. Though slightly lower than the standard new-Keynesian calibration[10], this value of complementarily is sufficient to generate a strong role for higher-order expectations in equilibrium dynamics, as we demonstrate shortly.

[10] See Woodford (2003) for a detailed discussion how this parameter has been calibrated in new-Keynesian models.

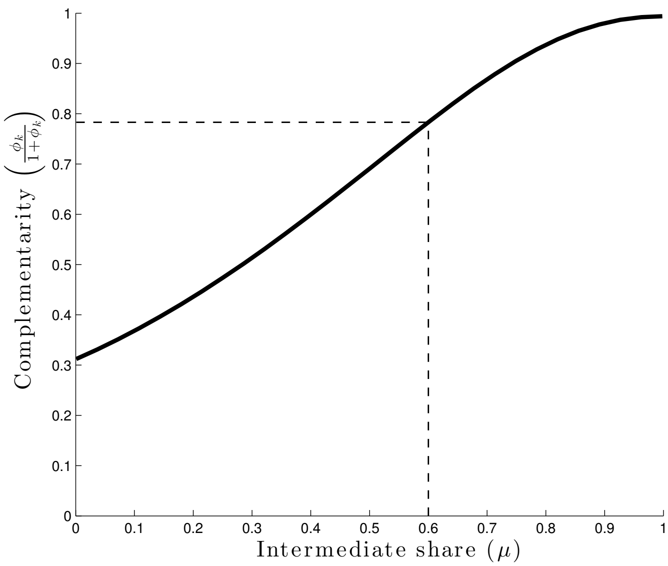


Figure 1: Steady-state complementarities for the general model.

Table 2: Relative standard deviations for circle production structure with sectoral shocks only.

	Output	**Cons.**	**Inv.**	**Hours**	**Sect. Inv.**
Full Information	1.00000	0.69781	1.88524	0.35229	1.999461
Market-consistent + GDP	1.00000	0.69781	1.88524	0.35229	1.999506
Market-consistent	1.00001	0.69781	1.88527	0.35230	1.999512
Own-price only	1.03151	0.70732	1.98956	0.38088	2.046435
Exogenous	0.77456	0.64663	1.16956	0.17471	1.262126

4.3 Sectoral Shocks Only

We begin by considering the model with only sectoral shocks and a stylized symmetric circle production structure given by

$$IO^{cir} = .6 \times \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

This structure is notable because, while it is circulant, it is also extremely sparse and thus corresponds to an especially restricted set of observable prices within the market-consistent information set.

Recall that since the version of the model we are considering has a finite number of sectors, sectoral shocks always have aggregate implications. The first row of Table 2 summarizes the aggregate moments of the full-information model. The model does a relatively good job of capturing the relative variances of output, consumption, and investment. The model shows somewhat low volatility of hours, which is a well known challenge for the basic neoclassical model. However, we are primarily concerned with how the information friction may change the dynamics of the model, in particular responses over time to shocks, and to this question we turn now.

4.3.1 Exogenous Information

Before turning to the case of market-consistent information, we first examine the consequences of the information friction based on an exogenous information set, which corresponds most closely to the typical assumption made by the new-Keynesian literature, for example Woodford (2002). In particular, we assume that investment choices are based on the information set

$$\hat{\Omega}_t^i = \{\hat{\theta}_{i,t-h}, \hat{s}_{i,t-h}, \}_{h=0}^{\infty} \tag{42}$$

where $\hat{s}_{i,t} = \frac{1}{N}\sum \hat{\theta}_{i,t} + \nu_{i,t}$ is a signal on average productivity in the economy.[11]

Figure 2 shows impulse responses of investment to a productivity shock hitting sector one for the exogenous information and full-information economy under different assumptions about the share of intermediates in the economy. Under exogenous information, other sectors learn gradually about the shock hitting sector one. However, as the right-panel shows, the average sector has nearly completely learned the nature of the shock after five quarters. With low intermediate share, and therefore relatively weak complementarities, the dynamics of these first-order expectations essentially determine the investment response; investment adjustment is slowed only to the extent that agents gradually learn about the realization of the shock. As the intermediate share increases, however, sluggish higher-order expectations take an increasingly important role. With an intermediate share of 0.9, complementarities lead to an extremely muted and gradual response of investment to the shock

Figure 3 shows that both output and labor supply inherit the hump-shaped dynamics of investment, while consumption does not. Consistent with these impulse responses, Table 2 shows that overall volatility is much lower in the baseline model. In short, the model with exogenous information generates very different dynamics than the full-information

[11] In order to ensure that markets clear, we maintain the assumption that static optimality conditions continue to hold *ex post*. Models with price-setting firms avoid this complication, since firms are required to meet demand regardless of whether so producing is optimal ex post.

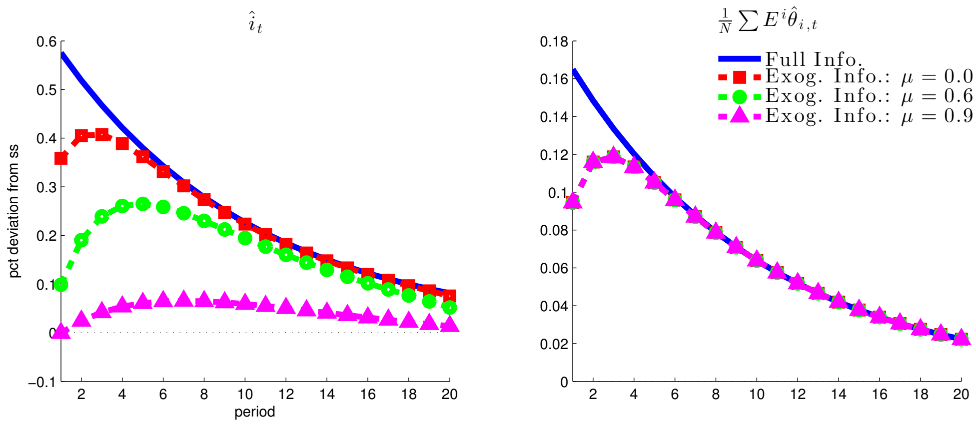


Figure 2: Investment and expectations responses to technology shock in sector one under exogenous information.

model and realistically hump-shaped responses for at least investment, output and labor. These results establish that agent beliefs, and higher-order expectations in particular, are at least *potentially* important for determining the paths of aggregate variables.

We, therefore, see that when dispersed information is exogenous (and not market consistent) it interacts with strategic complementarity to introduce delay and persistence in aggregate dynamics in ways that is well known in the existing literature. However, as we show next, in the presence of market-consistent information, these responses change significantly, virtually or entirely eliminating the effect on aggregate dynamics of informational frictions arising from dispersed information.

4.3.2 Market-Consistent Information

We now return to a version of the model in which agent's information contains market-based information. In particular, we consider two cases. In the first, we assume that firms observe not only their own productivity and relevant market prices but also aggregate GDP. Consistent with our theoretical results in Proposition 5, we find that aggregate dynamics are identical to full information under this restricted information assumption. This result is an exact result—it is true to the numerical tolerances we set in the algorithm—and it holds regardless of the number of periods for which we assume information remains dispersed. Despite this result, sectoral dynamics are not exactly the same under the restricted information assumption. Table 2 shows, as an example, that sectoral investment is different at the fifth decimal place. While this difference is tiny in our example, it highlights the point that, theoretically, sectoral dynamics can be different under market-consistent information without any impact at all on aggregate dynamics.

What explains these results? Figure 4 shows the inference of a firm in sector three

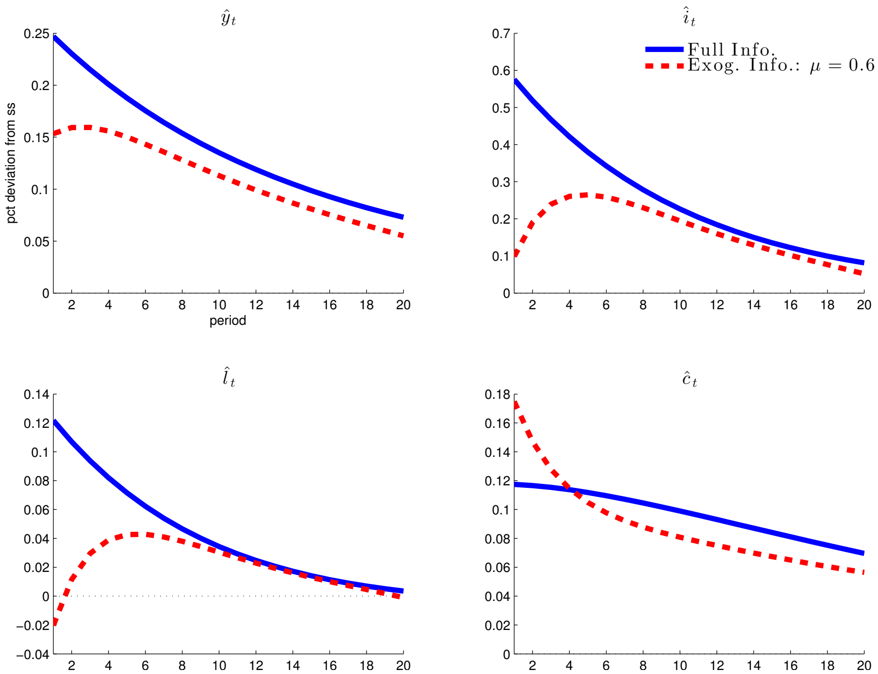


Figure 3: Impulse responses to technology shock in sector one under exogenous information.

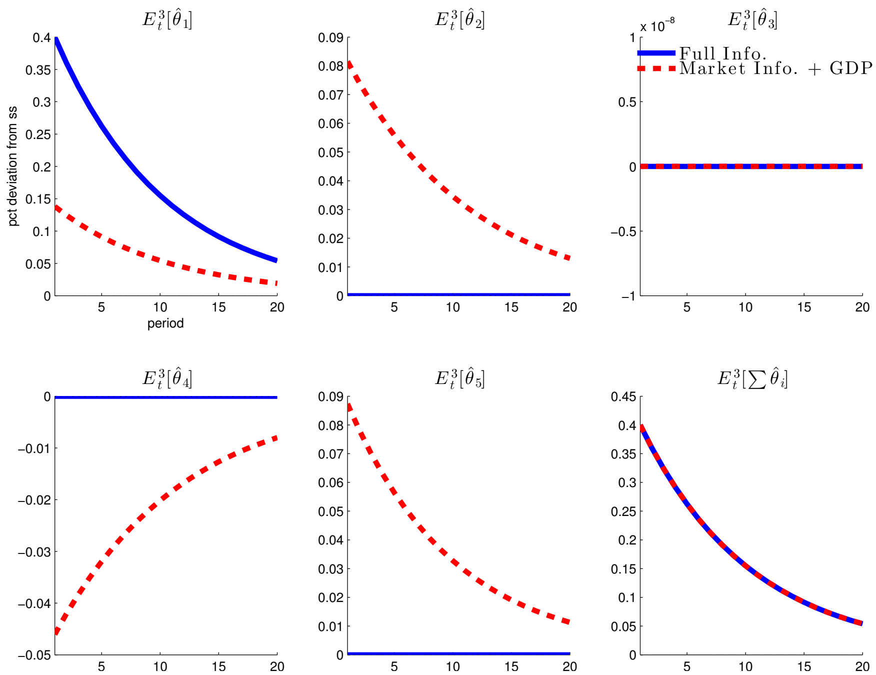


Figure 4: The inference of sector three in response to a sector-one productivity shock under the market consistent + GDP information assumption.

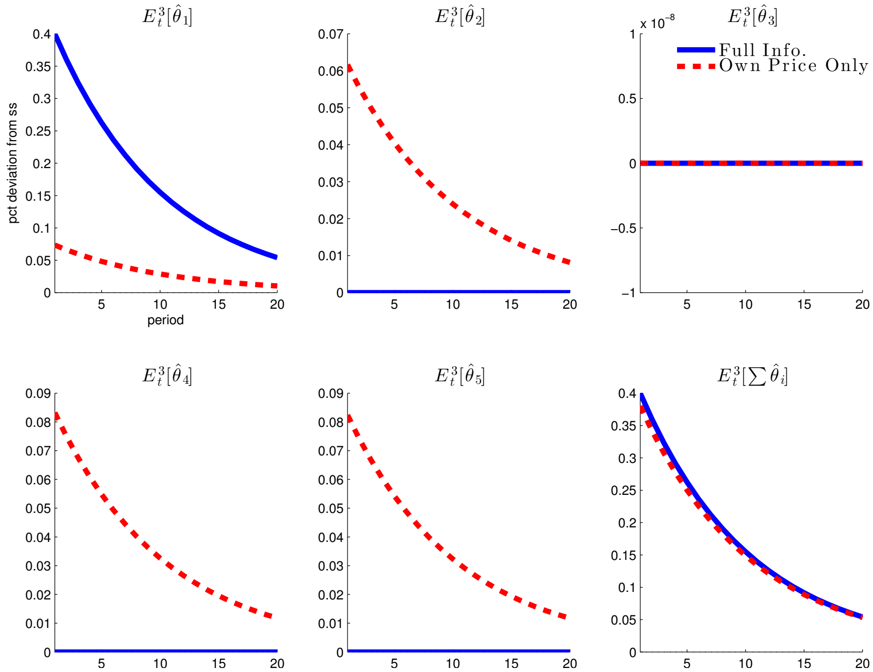


Figure 5: The inference of sector three in response to a sector-one productivity shock when only own price and productivity are observed.

to the shock in sector one. While the firm's inference about the sectoral shocks faced by other sectors is imperfect (and indeed quite so!) it has perfectly inferred the movement in average productivity in the economy (the last panel.) All other firms have done the same, leaving no room for any dynamics induced by higher-order expectations (or indeed any sort of imperfect information) in the aggregate.

Next, we consider the consequence of removing GDP from the information set of firms, so that firms learn only from the relevant sectoral prices and their own productivity. Table 2 shows that moments, both aggregate and sectoral, are very little changed. Despite somewhat larger sectoral mistakes, agents back out the average change in productivity so well that their inference (not reported in the figures) is visually identical to that in the full-information case. Sectoral variables do a remarkably good job at revealing the aggregate state of the economy. The nearly-full revelation of aggregates, in turns, leads the aggregate consequences of the information friction to remain negligible.

In order to better understand how restricted the information must be to deliver substantial consequences, we consider the case where firms observe only the price of their own good, and not that of their supplier's good. Recall that in the case of the model without sectoral linkages this price is enough to infer the aggregate state, and therefore to generate both aggregate and sectoral irrelevance. In this case, restricted information is not

Table 3: Relative standard deviations for circle production structure with sectoral shocks and lagged information.

	Output	**Cons.**	**Inv.**	**Hours**	**Sect. Inv.**
Full Information	1.000	0.698	1.885	0.352	1.999
Lagged M-C + GDP	1.000	0.698	1.885	0.352	2.565
Lagged M-C	0.939	0.660	1.850	0.277	2.700

enough to infer the aggregate state exactly, but it still does a very good job at revealing it, as demonstrated by Figure 5. Thus, while the demand market clearing condition is no longer available to directly infer the aggregate state of productivity in the economy, the combination of relative price and own productivity remains immensely informative about aggregates.

Finally, to demonstrate that aggregate informative information, even without current market information, may deliver aggregate irrelevance, we consider the (perhaps unrealistic) case where firms observe their own market consistent information with a one-period lag, while observing GDP contemporaneously and compare this to the case that GDP is not observed. Table 3 shows that the addition of the GDP, which again is a sufficient statistic for the state of aggregate productivity, once again generates aggregate moments that are identical to the full-information economy. In this case, however, sectoral quantities are dramatically different as demonstrated by the much-greater volatility of sectoral investment in the table. This result highlights the disconnect that can occur in the economy between aggregate outcomes, and the sectoral movements that generate them.

4.4 Disentangling Aggregate and Sectoral Productivity

So far we have followed the earlier literature on sectoral interlinkages in explicitly excluding aggregate productivity shocks from our consideration. Indeed the goal of most of the literature has been to argue that such linkages allow the RBC model to explain aggregate fluctuations without recourse to (implausibly large) aggregate shocks. In contrast, much the of literature on the consequences of information frictions emphasizes the difficulty agents may face in disentangling aggregate and idiosyncratic shocks. For some examples, see Lorenzoni (2009), Graham and Wright (2010) and Acharya (2013). While we are sympathetic to the goal of explaining aggregate fluctuations without aggregate shocks, we now turn to the question of whether adding such shocks might "reinstate" the importance of the information friction in our model.

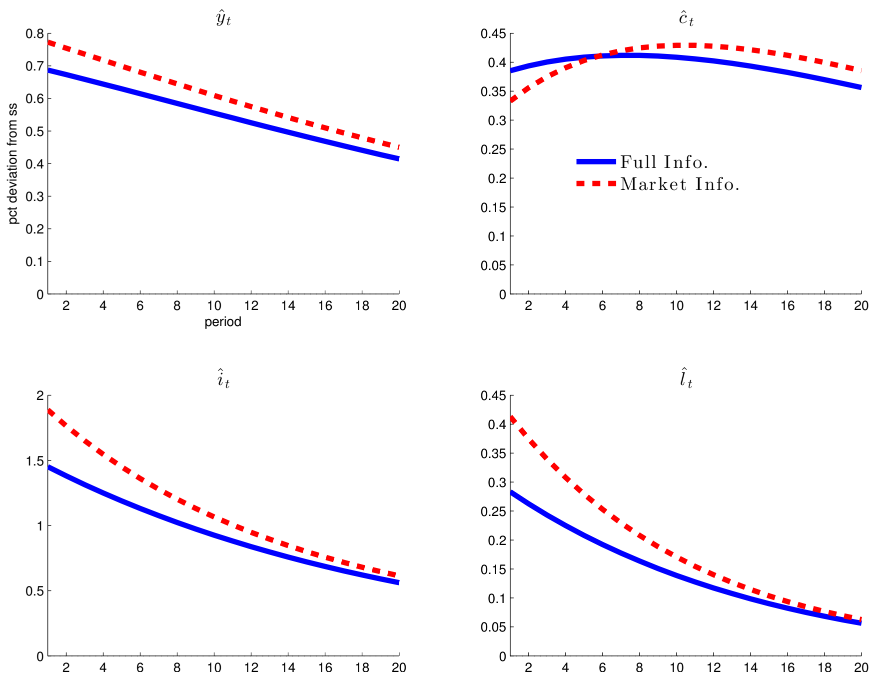


Figure 6: Aggregate impulse responses to an aggregate technology shock.

To do this, we decompose the process for $\Theta_{i,t}$ into aggregate and sectoral components, A_t and $\varsigma_{i,t}$, according to the log-level processes

$$\hat{\theta}_{i,t} = \hat{a}_t + \hat{\varsigma}_{i,t}. \tag{43}$$

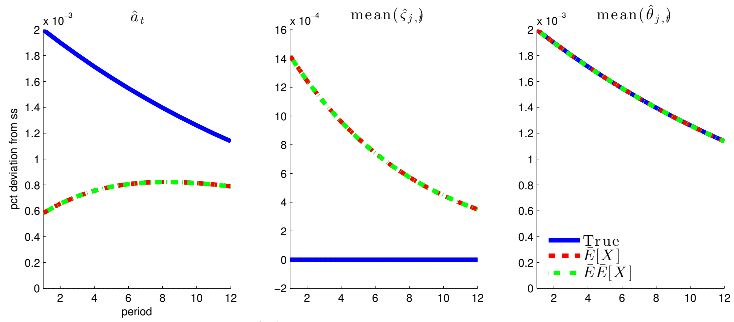
We assume that each component follows an AR(1) process with potentially different persistence

$$\hat{a}_{t+1} = \rho_A \hat{a}_t + \sigma_A \epsilon_{t+1} \tag{44}$$

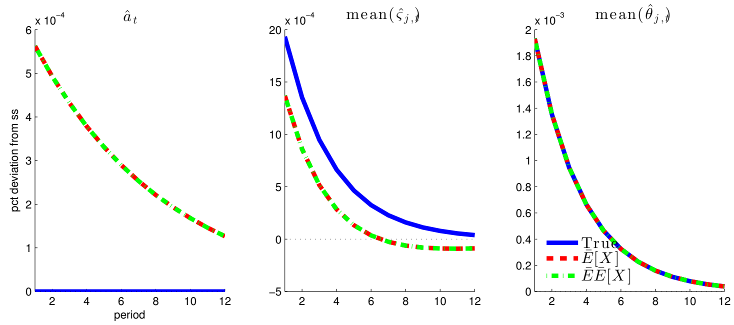
$$\hat{\varsigma}_{i,t+1} = \rho_\varsigma \hat{\varsigma}_{i,t} + \sigma \epsilon_{i,t+1} \tag{45}$$

where the shocks ϵ_t and $\epsilon_{i,t}$ each have unit variances. We calibrate the aggregate shock so that it is somewhat more persistent than the idiosyncratic shock ($\rho_\varsigma = 0.70, \rho_A = 0.95$) and accounts for around 50% of aggregate fluctuations in the economy. As an aside, note that if we assume that aggregate and idiosyncratic shocks had identical persistence, as do Graham and Wright (2010), we will once again recover the result that the information assumption has zero consequence for aggregate dynamics. Following the proof of Proposition 4, it easy to see that in this case aggregate dynamics of the model are driven by a single aggregate state with persistence parameter ρ.

Figure 6 shows that restriction to market-based information assumption has a modest



(a) Aggregate shock



(b) Sectoral shock

Figure 7: Expectations responses to aggregate and sectoral productivity shocks.

effect on aggregate dynamics, at least in response to the aggregate shock.[12] But this effect is precisely the opposite effect one might expect using the intuition from a model with exogenous information. In fact, the investment response is greater than the full-information investment response for a natural reason and one that is not linked to the dispersion of information at all. Since each sector sees prices they can once again infer average productivity in the economy. However they are uncertain about whether that average productivity is driven by a coincidence of (more temporary) sectoral shocks or by a (more permanent) aggregate shock. As the model is calibrated, short lived shocks lead to a relatively greater increase in optimal investment due to the standard permanent income logic. To the extent that agents perceive the aggregate shock as more temporary than it really is, they will tend to overreact to the shock leading to a larger initial change in investment.

Moreover, note that the presence of price information in the information set has completely killed any role for higher-order expectations in this version of the model. Panel (a) of Figure 7 shows that in response to the aggregate shock, first-order and higher-order expectations of the shock are perfectly aligned, i.e. there is no disagreement about the

[12] In fact, overall moments change very little, since the "over reaction" in response to aggregate shocks is offset somewhat by "under reaction" to sectoral shocks.

aggregate in the economy. As a consequence, the aggregate quantities in the economy look identical to the quantities delivered by a representative agent model in which productivity has two components, one with higher persistence than the other, which agents distinguish only over time by following the realizations of total TFP. Panel (b) of Figure 7 shows that, in response to a sector-specific shock, agreement is once again achieved regarding the aggregate state in the economy. For sectoral shocks, disagreement about the sectoral distribution changes (not reported in the figure) lead to large difference in higher-order expectations with respect to first-order expectations. In short, prices transmit all aggregate information, but can leave behind substantial residual disagreement about the distribution of sectoral disturbances. Without disagreement about aggregates, however, dispersed information plays no role.

5 Information Transmission in Model Calibrated to US Data

In this section, we calibrate the model to match US data on the sectoral input-output structure and the empirical measures of sectoral total-factor productivity. In doing so, we relax all the symmetry assumptions regarding production shares, the input-output matrix, and the shock processes that we have maintained up to this point. The assumptions underlying Propositions 1 through 5 are strongly violated, giving the potential for information frictions to play a substantially larger role in explaining aggregate dynamics. However, our results show that aggregate dynamics in the calibrated diverse-information model are remarkably close to those under complete information.

We start by calibrating the intermediate shares of each sector in the economy to match the empirical input-output tables for the US economy. The raw data for these tables come from 2002 detailed benchmark table available from the Bureau of Economic Analysis, available from http://www.bea.gov/industry/iedguide.htm#io. At this fine level of disaggregation, in which the US economy is divided into roughly 450 different sectors, the input output-output table is quite sparse, with less than 2% of entries being non-zero. Ideally, we would proceed with this completely disaggregated input-output structure. However, this is not possible both because numerical limitations prevent us from solving the model at such a disaggregated level, and because no analysis of sectoral productivity exists at such a refined level.

In order to proceed, we aggregate the IO tables to correspond with the thirty Jorgenson

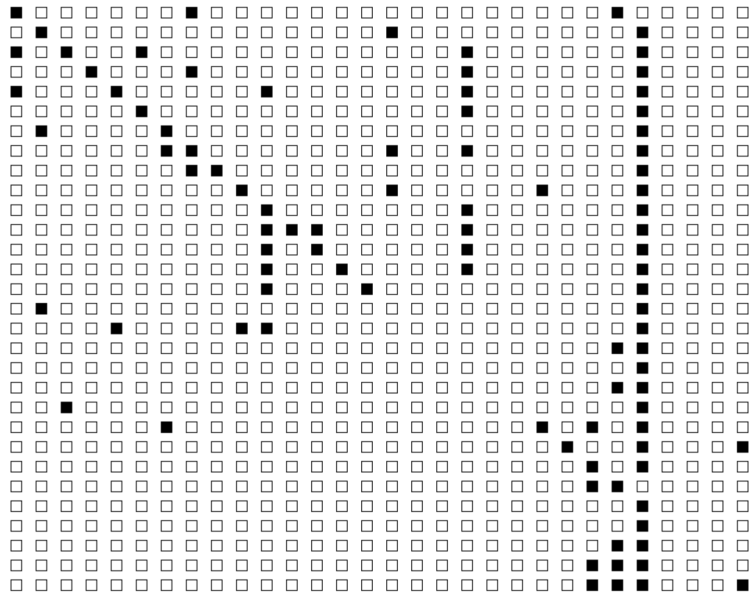


Figure 8: Sparsity of the US input-output table in the 30 Jorgenson et al. (2013) sectors.

et al. (2013) industries, according to correspondences provided by those authors.[13] Very few entries of the resulting partially-aggregated IO matrix are strictly zero, however many entries remain relatively very small. Thus, in our calibration, we treat as zero any input that accounts for less than 4% of gross output in a particular industry, reallocating that share proportionally to inputs with larger initial shares to keep the total intermediate share constant. Figure 8 visually represents the structure of the resulting-input output matrix. Roughly 10% of all entries are non-zero, and the matrix is highly diagonal: off-diagonal sparsity is substantially higher. The matrix is also highly asymmetric, with the sector "renting of machine and equipment, and other business services" constituting a non-trivial input in nearly every other industry. In short, the input-output matrix is very different from the stylized symmetric formulation used in our earlier examples.

In order to calibrate the process for the aggregate and idiosyncratic TFP shocks, we proceed by estimating a simple factor model in which sectoral TFP depends on idiosyncratic shocks and a single aggregate factor. Specifically, we assume that

$$\hat{\theta}_{i,t+1} = \mu_i \hat{a}_t + \hat{\varsigma}_{i,t+1} \tag{46}$$

$$\hat{a}_{t+1} = \rho_A \hat{a}_t + \sigma_A \epsilon_{t+1} \tag{47}$$

$$\hat{\varsigma}_{i,t+1} = \rho_{\varsigma,j} \hat{\varsigma}_{i,t} + \sigma_i \epsilon_{i,t+1}. \tag{48}$$

[13] Jorgenson et al. (2013) describe thirty-two sectors. However two of those sectors, that of home production and non-comparable imports, do not map well into model. For these reasons, we exclude them from our calibration.

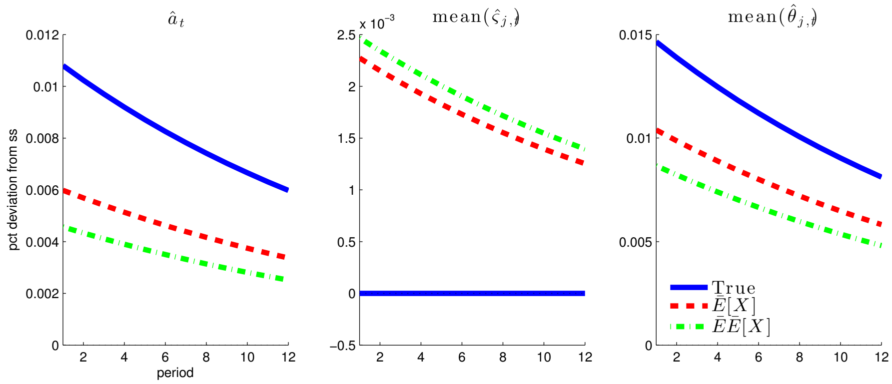


Figure 9: Expectations responses to aggregate productivity shock in the model calibrated to US sectoral data.

This process for TFP generalizes the process in equations (43) - (45) in three respects. First, it allows for sectoral differences in the autocorrelation coefficient of the sectoral shocks. Second, it allows for differences in the variances of the shock to sectoral productivity. Finally, through sector-specific loadings μ_i's, it allows sectoral productivities to correlate more or less strongly with the aggregate component of TFP.

Using the sectoral TFP measurements of Jorgenson et al. (2013), we treat equation (46) as a measurement equation, with $\hat{\varsigma}_{i,t}$ and $\hat{a}_t$ as unobserved components, and estimate the parameters $\{\rho_a, \rho_{\varsigma,i}, \mu_i, \sigma_i\}$ using Bayesian methods. Table 4 reports the estimated autocorrelation coefficients, showing that indeed there is substantial sectoral heterogeneity in the persistence of shocks. Despite this, however, the average estimate of sectoral persistence is quite close (identical to two decimals) to the estimated persistence of the aggregate component, suggesting that even the need to disentangle aggregate and idiosyncratic shocks may have little aggregate consequence. For completeness, the remaining columns show estimated sectoral variances and the corresponding weights on the aggregate component. Both sets of estimated values also show substantial heterogeneity across sectors. We set all parameters not related to the input-output structure and sectoral productivity processes at their baseline values in Table 1.

Figure 9 shows that, in this asymmetric environment, endogenous information does, on average, a rather poor job of revealing the arrival of an aggregate shock to firms in the economy. Even twelve quarters after the shock, firms mistakenly attribute more than half of the shock to idiosyncratic rather than aggregate changes in productivity. Moreover, there is substantial dispersion of information about the aggregate, which can be seen by noticing the relatively sluggish response of second-order expectations (green line) relative to first-order expectations (red line.) Asymmetry in the production structure and the processes for sectoral shocks clearly reduces the ability of market-based information to

Table 4: Estimated parameters for sectoral TFP factor model.

	ρ_i	σ_i	μ_i
aggregate tfp	0.95	0.01	
sectoral mean	0.95	0.03	1.27
agriculture, hunting, forestry	0.87	0.05	0.63
mining and quarrying	0.98	0.04	2.32
food , beverages and tobacco	0.96	0.04	1.14
textiles, textile , leather an	0.92	0.03	-0.18
wood and of wood and cork	0.97	0.03	-0.66
pulp, paper, paper , printing	0.98	0.02	1.67
chemical, rubber, plastics and	0.85	0.02	5.92
coke, refined petroleum and nu	0.96	0.15	13.85
chemicals and chemical product	0.98	0.03	4.31
rubber and plastics	0.90	0.03	1.97
other non-metallic mineral	0.86	0.03	1.60
basic metals and fabricated me	0.95	0.02	1.09
machinery, nec	0.98	0.04	0.80
electrical and optical equipme	1.00	0.04	-0.41
transport equipment	0.92	0.04	1.84
manufacturing nec; recycling	0.96	0.03	1.07
post and telecommunications	0.97	0.02	-0.46
construction	1.00	0.02	0.58
sale, maintenance and repair o	0.92	0.04	1.60
wholesale trade and commission	0.94	0.03	1.16
retail trade, except of motor	0.93	0.03	1.40
hotels and restaurants	0.99	0.02	0.27
transport and storage	0.94	0.02	0.51
post and telecommunications	0.97	0.02	-0.44
financial intermediation	0.98	0.03	-0.45
real estate, renting and busin	0.98	0.01	-0.13
real estate activities	0.97	0.02	-0.30
renting of m&eq and other busi	0.94	0.02	0.08
public admin and defence; comp	0.97	0.02	-0.23
education	0.98	0.02	0.16
health and social work	0.97	0.02	-0.20
other community, social and pe	0.98	0.01	0.11

Note: Table provides posterior median estimates for each parameter. Aggregate refers to the parameters of the aggregate TFP process. Sectoral mean provides the mean over median posterior values of all sectors.

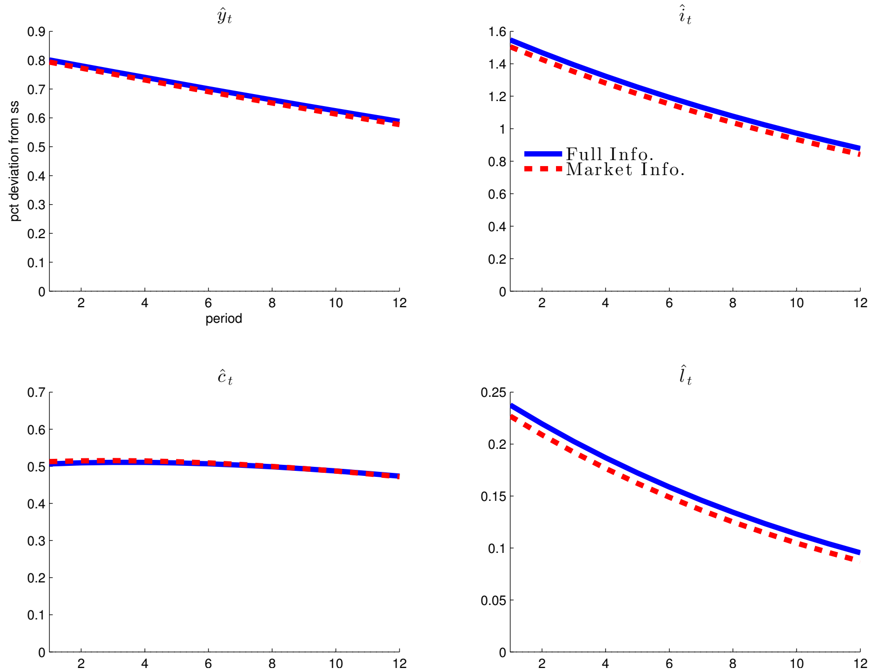


Figure 10: Impulse responses to an aggregate technology shock for the full and market-consistent information models calibrated to US data.

coordinate expectations in response to the aggregate shock, contrasting with our results on the more stylized symmetric economy.

Despite the presence of dispersed information, however, Figure 10 demonstrates that the impulse responses of the realistically calibrated economy are almost entirely unaffected by the presence of incomplete information. Indeed, the similarity here is even stronger than that in the stylized version of the model with an aggregate and idiosyncratic component to productivity. This result is driven primarily by the close alignment between the average persistence of idiosyncratic shocks and the persistence of the aggregate component of productivity in the estimated process for TFP. Even though agents disagree over long periods about the cause of the price changes they see in their own markets, on average those changes will last the same amount of time *regardless of their source.* As long as firms detect the persistence of these change correctly on average, average choices will align quite closely with the full information economy, despite both disagreement about the nature of the shock hitting the economy and the relatively large "mistakes" that occur from the sectoral perspective.

6 Conclusions

Here we have explored an environment of dispersed information and strategic interactions among firms in which exogenously dispersed information leads to large consequences for aggregate dynamics, but learning through market prices virtually eliminates their effect. This is true even though sectoral dynamics can change, sometimes substantially, and no law of large numbers is available. In one respect, this paper makes the cautionary point that informational asymmetries and strategic interdependence, the two key ingredients in nearly all the related literature, do not guarantee an important role for information. We believe that the key assumption driving this difference—that firms condition their investment choices on their market-based information—is realistic. More generally, we have argued that general equilibrium places important restrictions on expectations conditioned on endogenous information, many of which are independent of the precise details of the agents' information set. Our analytical results offer some avenues for "breaking" these results, and thereby generate an important role for information frictions. However, our quantitative results suggest even when exact irrelevance fails to hold, the plausible quantitative consequences are quite small.

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A Proofs of Propositions

A.1 Proof of Proposition 3: Sectoral Irrelevance

Proof. The proof proceeds by demonstrating that the set of market consistent information is *action informative.* To do this, we need only describe model dynamics under full information. Using equation (23), we can derive an expression for the $\hat{x}_{ij,t}$,

$$\hat{x}_{ij,t} = \hat{p}_{i,t} + \hat{q}_{i,t} - \hat{p}_{j,t}. \tag{49}$$

Substitute this expression into the linearized production in equation (22) delivers the following matrix representation of that equation,

$$\mathbf{q}_t = \boldsymbol{\theta}_t + (\Phi_x - IO)\mathbf{p}_t + \Phi_x\mathbf{q}_t + \Phi_k\mathbf{k}_t, \tag{50}$$

where bold-face type represent vector $\mathbf{x}_t \equiv [\hat{x}_{1,t}, \hat{x}_{2,t}, ..., \hat{x}_{N,t}]'$ for any variable $\hat{x}_{i,t}$, Φ_x is a diagonal matrix with the row-sum of IO along the diagonal, and Φ_k is a diagonal matrix with entries α_{ik}. Equation (50) can be rearranged to provide an explicit expression for $\mathbf{q}_t$,

$$\mathbf{q}_t = (I - \Phi_x)^{-1}\boldsymbol{\theta}_t + (I - \Phi_x)^{-1}(\Phi_x - IO)\mathbf{p}_t + (I - \Phi_x)^{-1}\Phi_k\mathbf{k}_t. \tag{51}$$

Similarly, plugging equation (49) into the market clearing condition in equation (28) and solving for $\mathbf{q}_t$ yields

$$\mathbf{q}_t = \Psi(\mathbf{z}_t + \mathbf{p}_t) - \mathbf{p}_t, \tag{52}$$

where $\Psi \equiv (I - (I - \Phi_z)\Gamma')^{-1}\Phi_z$ and Φ_z is a diagonal matrix with $\{s_{iz} = Z_i/Q_i\}_{i=1}^N$ on the diagonal. In the above, Γ contains the entries η_{ij} and it is worth observing that, by construction, the row-sums of Γ' are unity. Combining final demand and aggregation equations (25) and (26) yields

$$\mathbf{p}_t + \mathbf{z}_t = A_v\mathbf{z}_t \tag{53}$$

where the matrix A_v equals $1/N$ times an $N \times N$ unit matrix and replicates the column averages of any conformable matrix that it premultiplies. For future reference, rearranging this equation yields

$$\mathbf{p}_t = (A_v - I)\mathbf{z}_t. \tag{54}$$

Finally, we have the equation describing intertemporal choice in the economy,

$$\mathbf{k}_{t+1} = E_t[\mathbf{p}_{t+1} + \mathbf{q}_{t+1}]. \tag{55}$$

We proceed by a method of undetermined coefficients. We suppose that the policy function for $\mathbf{k}_{t+1}$ is

$$\mathbf{k}_{t+1} = \Lambda\boldsymbol{\theta}_t \tag{56}$$

where, importantly, the matrix Λ has constant (all identical) rows. The conjecture, thus, includes the presumption that the investment choice in each sector depends on the very same linear combination of the shocks. Plug this conjecture into the period $t+1$ version of equation (51), set equal to equation (52), substitute out $\mathbf{p}_{t+1}$ using equation (54) and solve for $\mathbf{z}_{t+1}$:

$$\mathbf{z}_{t+1} = H^{-1}(I - \Phi_x)^{-1}\left(\boldsymbol{\theta}_{t+1} + \Phi_k\Lambda\boldsymbol{\theta}_t\right) \tag{57}$$

where

$$H \equiv \left[\Psi - (A_v - I) - (I - \Phi_x)^{-1}(\Phi_x - IO)(A_v - I)\right]. \tag{58}$$

We can now use equation (52) to solve for

$$\mathbf{p}_{t+1} + \mathbf{q}_{t+1} = \Psi A_v H^{-1}(I - \Phi_x)^{-1}\left(\boldsymbol{\theta}_{t+1} + \Phi_k\Lambda\boldsymbol{\theta}_t\right). \tag{59}$$

Finally, taking expectations, we have

$$\mathbf{k}_{t+1} = E_t[\mathbf{p}_{t+1} + \mathbf{q}_{t+1}] = \Psi A_v H^{-1}(I - \Phi_x)^{-1} (\rho I + \Phi_k \Lambda) \boldsymbol{\theta}_t \tag{60}$$

To verify the conjecture, we must show that fixed point

$$\Lambda = \Psi A_v H^{-1}(I - \Phi_x)^{-1} (\rho I + \Phi_k \Lambda) \tag{61}$$

indeed has constant rows. By virtue of the definition of A_v, any matrix pre-multiplied by A_v will have this property. Moreover, by construction, the row-sums of Ψ are constant and equal to one. Thus, pre-multiplication by Ψ only rescales the rows of the constant-row matrix. Thus, the rows of Λ are constant and the conjecture is sustained.

Now, using time t and $t+1$ versions of equation (57), we can find that

$$\mathbf{z}_{t+1} - \mathbf{z}_t = H^{-1}(I - \Phi_x)^{-1} (\boldsymbol{\theta}_{t+1} + (\Phi_k \Lambda - I)\boldsymbol{\theta}_t - \Phi_k \Lambda \boldsymbol{\theta}_{t-1}) \tag{62}$$

Using equation (54), the expected change in prices is therefore

$$E_t[\mathbf{p}_{t+1} - \mathbf{p}_t] = (A_v - I)H^{-1}(I - \Phi_x)^{-1} [\{(\rho - 1)I + \Phi_k \Lambda\} \boldsymbol{\theta}_t - \Phi_k \Lambda \boldsymbol{\theta}_{t-1}] \tag{63}$$

Meanwhile, we have that

$$\mathbf{p}_t = (A_v - I)H^{-1}(I - \Phi_x)^{-1}(\boldsymbol{\theta}_t + \Lambda \boldsymbol{\theta}_{t-1}). \tag{64}$$

Inspection of equations (63) and (64) reveals that the minimal market-consistent information set is indeed action informative. To see this, observe that in order to make optimal investment decisions, the firm has only to forecast its own productivity next period as well the relative prices it will face in the markets in which it participates. Current own-sector productivity is a sufficient statistic for the full-information forecast of own-productivity next period. Second, since it observes the relevant prices today, the firm need only forecast the change in prices between today and tomorrow. If the firm enters the period knowing the values of $\Lambda\boldsymbol{\theta}_{t-1}$, then current prices reveal exactly the linear combination of the shocks the firm needs to predict the change in prices and therefore make the optimal investment choice. Moreover, the optimal investment choice itself reveals the linear combination $\Lambda\boldsymbol{\theta}_t$ that is needed to infer the necessary linear combination of $\boldsymbol{\theta}_t$ in the subsequent period. □

A.2 Proof of Proposition 4: An Aggregate Representation with Circulant IO Structure

Proof. Define $\hat{q}_t = \frac{1}{N}\sum_{i=1}^{N} q_{i,t}$ and define $\hat{c}_t$, $\hat{k}_t$ and $\hat{x}_t$ analogously. Let $h = \frac{1}{N}[1 \quad 1 \quad \dots \quad 1]$ be the $1 \times N$ row vector that deliver the vector of column means of any matrix it premultiplies. Observe that hA is a constant vector for any matrix with constant column sums, including circulant matrices.

From equations (95) and (94), it follows that diagonal matrix Φ_z contains constant, non-zero values and so can be treated as a scalar in matrix multiplication. The circulant nature of IO similarly implies Φ_x and Φ_k may also be treated as scalars α_x and α_k. Using these results, multiply equation (51) by h to find

$$\hat{q}_t = (1-\alpha_x)^{-1} + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t \tag{65}$$

where we use the result that $hIO\mathbf{p}_t = h\mathbf{p}_t = \hat{p}_t = 0$ by the assumption of the numeraire.

Next, observe that given the assumption of a circulant matrix IO, $\Gamma = \Gamma' = IO$. From equation (52), we therefore have that

$$\mathbf{q}_t = \mathbf{z}_t, \tag{66}$$

while from market clearing it follows that $\mathbf{z_t} = \mathbf{y}_t$. Combining yields equation (30) in the text.

Next, use the intermediate good optimality condition in (23) to eliminate x_{ij} from equation (28):

$$\hat{p}_{i,t} + \hat{q}_{i,t} = s_{ic}\hat{c}_{i,t} + s_{ik}k_{i,t+1} + (1 - s_{ic} - s_{ik})\sum_{j=1}^{N}\omega_{ij}(\hat{p}_{i,t} + \hat{q}_{i,t}). \tag{67}$$

Rewrite equation (67) in matrix form using the fact that $\omega_{ij} = \epsilon_{ij}$ and $s_{ic} = \alpha_l$ and $s_{ik} = \alpha_k$.

$$\mathbf{p}_t + \mathbf{q}_t = \alpha_l\mathbf{c}_t + \alpha_k\mathbf{k}_{t+1} + \alpha_x(\mathbf{p}_t + \mathbf{q_t}). \tag{68}$$

Multiplying by h, and solving for $\hat{q}_t = \hat{y}_t$ yields

$$\hat{y}_t = (1-\alpha_x)^{-1}\alpha_l\hat{c}_t + (1-\alpha_x)^{-1}\alpha_k\hat{k}_t, \tag{69}$$

which corresponds to equation (31) in the main text.

Finally, equation (32) follows directly from summing the log-linear Euler equation (24).

□

A.3 Proof of Proposition 5: Aggregate Irrelevance

Lemma 1. *Relative prices do not depend on the aggregate shock θ_t.*

Proof. In any equilibrium, the price of the good in sector i can be written as a weighted sum of past sectoral shocks:

$$\begin{aligned}\hat{p}_{i,t} &= \sum_{\tau=0}^{\infty}\sum_{j=1}^{N}\gamma_{ij,\tau}\hat{\theta}_{j,t-\tau} \\ &= \sum_{\tau=0}^{\infty}\sum_{j=1}^{N}\gamma_{ij,\tau}(\hat{\theta}_{j,t-\tau} - \hat{\theta}_{t-\tau}) + \sum_{\tau=0}^{\infty}\hat{\theta}_{t-\tau}\left(\sum_{i=1}^{N}\gamma_{ij,\tau}\right),\end{aligned} \tag{70}$$

where the coefficients $\gamma_{ij,\tau}$ are generic coefficients in the MA representation of $\hat{p}_{i,t}$. Summing this expression across the (symmetric) sectors and dividing by N yields

$$\begin{aligned} 0 \equiv \hat{p}_t &= \sum_{\tau=0}^{\infty}\sum_{j=1}^{N}(\hat{\theta}_{j,t-\tau} - \hat{\theta}_{t-\tau})\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_{ij,\tau}\right) + \sum_{\tau=0}^{\infty}\hat{\theta}_{t-\tau}\left(\frac{1}{N}\sum_{j=1}^{N}\gamma_{ij,\tau}\right) \\ &= 0 + \sum_{\tau=0}^{\infty}\hat{\theta}_{t-\tau}\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_{ij,\tau}\right) \end{aligned} \tag{71}$$

where the second line follows from the fact that, by symmetry, $\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_{ij,\tau}\right)$ is constant for all j and from the definition of $\hat{\theta}_t$. Since the last equation must hold for any sequence of $\theta_{t-\tau}$, however, it immediately follows that $\left(\frac{1}{N}\sum_{i=1}^{N}\gamma_{ij,\tau}\right) = 0, \forall\tau$, so that $p_{i,t}$ may only depend only the deviations of productivity from the average, $\theta_{j,t-\tau} - \theta_{t-\tau}$ and not independently on the average. □

Corollary 3. *Suppose that the information set of firms in sector i consists of market consistent information and $\hat{\theta}_t$. Then, sector i's expectations of any price at any future horizon must be a function only of the histories of $(\hat{\theta}_{i,t} - \hat{\theta}_t), p_{i,t}$, and $\{p_{j,t}, \forall j \text{ s.t } a_{ij} > 0\}$.*

Proof. This holds because relative prices and aggregate outcomes are orthogonal at all horizons. □

Corollary 4. *Suppose that the information set of firms in sector i consists of market consistent information and $\hat{\theta}_t$. Then the average expectations regarding any future price are zero, i.e.* $\sum_{i=1}^{N} E_t^i[\hat{p}_{i,t+\tau}] = \sum_{i=1}^{N} E_t^i[\hat{p}_{i+1,t+\tau}] = \sum_{i=1}^{N} E_t^i[\hat{p}_{i+2,t+\tau}] = ... = 0, \forall\tau$.

Proof. Since expectations of future prices depend symmetrically on a set mean-zero objects, sums of those expectations must be zero. □

We now prove Proposition 5:

Proof. Our goal is to prove that

$$\frac{1}{N}\sum_{i=1}^{N} E_t^i[\hat{x}_{i,t+1}] = E_t^f[\hat{x}_{t+1}], \tag{72}$$

for any variable $\hat{x}_{i,t+1}$. If this is true, then individual Euler equations can be summed to yield the aggregate full-information Euler in equation (32) and the conclusion follows.

The action of a firm in sector j can be written

$$\hat{x}_{i,t} = \sum_{\tau=0}^{\infty}\left(\tilde{\varphi}_{1,\tau}\hat{\theta}_{j,t-\tau} + \tilde{\varphi}_{2,\tau}\hat{\theta}_{t-\tau} + \sum_{k=0}^{N-1}\tilde{\nu}_{k,\tau}\hat{p}_{i+k,t-\tau}\right) \tag{73}$$

where $\tilde{\nu}_{k,\tau} = 0$ for all k such that $a_{i(i+k)} = 0$. Since we have assumed a circulant matrix, $a_{i(i+k)} = 0$ will be true for all i if it is true for any i.

Summing across sectors, the final term in the summation cancels due to symmetry. Thus, the average action is given by

$$\hat{x}_t = \sum_{\tau=0}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{2,\tau}\right) \hat{\theta}_{t-\tau}. \tag{74}$$

and the one-period ahead full information expectation is given by

$$E_t^f[\hat{x}_{t+1}] = \left(\sum_{\tau=1}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{2,\tau}\right) \hat{\theta}_{t+1-\tau}\right) + \left(\tilde{\varphi}_{1,0} + \tilde{\varphi}_{2,0}\right) \rho \hat{\theta}_t. \tag{75}$$

The one period ahead expectation of a firm in sector i is given by $E_t^i[\hat{x}_{i,t+1}]$ is then given by

$$E_t^i[\hat{x}_{i,t+1}] = \left(\sum_{\tau=1}^{\infty} \tilde{\varphi}_{1,\tau}\hat{\theta}_{j,t+1-\tau} + \tilde{\varphi}_{2,\tau}\hat{\theta}_{t+1-\tau} + \sum_{k=0}^{N-1} \tilde{\nu}_{k,\tau}\hat{p}_{i+k,t+1-\tau}\right) + \\ \tilde{\varphi}_{1,0}\rho\hat{\theta}_{j,t} + \tilde{\varphi}_{2,0}\rho\hat{\theta}_t + \sum_{k=1}^{N-1} \tilde{\nu}_{k,0} E_t^i[p_{i+k,t+1}] \tag{76}$$

Averaging across sectors yields and use the result in Corollary 4 to eliminate terms depending on prices to get

$$\frac{1}{N}\sum_{j=i}^{N} E_t^i[\hat{x}_{i,t+1}] = \left(\sum_{\tau=1}^{\infty} \left(\tilde{\varphi}_{1,\tau} + \tilde{\varphi}_{2,\tau}\right) \hat{\theta}_{t+1-\tau}\right) + \left(\tilde{\varphi}_{1,0} + \tilde{\varphi}_{2,0}\right) \rho \hat{\theta}_t = E_t^f[\hat{x}_{t+1}]. \tag{77}$$

□

B Calibration of the Model

With a nested-CES production structure, the mapping between long-run sector shares and the parameters of production is non-trivial. In this appendix, we describe in detail the steps required to infer these parameters. Recall that we take $p = 1$ to be the numeraire in the economy. In steady state, the following sector-specific equations must hold for each sector j:

$$\lambda_i = c_i^{-\frac{1}{\tau}}(1-l_i)^{\varphi(1-\frac{1}{\tau})} \tag{78}$$

$$\lambda_i w_i = \varphi c_i^{1-\frac{1}{\tau}}(1-l_i)^{\varphi(1-\frac{1}{\tau})-1} \tag{79}$$

$$w_i = p_i F_{l,i} \tag{80}$$

$$p_j = p_i F_{x_{ij},i} \quad \forall i \text{ s.t. } a_{ij} > 0 \tag{81}$$

$$1 = \beta(p_j F_{k,j} + 1 - \delta) \tag{82}$$

$$z_i = a_i p_i^{-\zeta} y \tag{83}$$

$$q_i = z_i + \sum_j x_{ji} \tag{84}$$

$$p_i z_i = c_i + i_i \tag{85}$$

$$q_i = F(k_i, l_i, \{x_{ij}\}) \tag{86}$$

$$i_i = \delta k_i \tag{87}$$

where

$$F(k_i, l_i, \{x_{ij}\}) = \left[\left\{\sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} + \left\{a_{il} l_i^{1-\frac{1}{\kappa}} + a_{ik} k_i^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}}\right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{88}$$

and

$$F_{l,i} = q_i^{\frac{1}{\sigma}} \left\{a_{il} l_i^{1-\frac{1}{\kappa}} + a_{ik} k_i^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{il} l_i^{-\frac{1}{\kappa}} \tag{89}$$

$$F_{k,i} = q_i^{\frac{1}{\sigma}} \left\{a_{il} l_i^{1-\frac{1}{\kappa}} + a_{ik} k_i^{1-\frac{1}{\kappa}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{ik} k_i^{-\frac{1}{\kappa}} \tag{90}$$

$$F_{x_{ij},i} = q_i^{\frac{1}{\sigma}} \left\{\sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}}\right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}-1} a_{ij} x_{ij}^{-\frac{1}{\xi}}. \tag{91}$$

Moreover, the following aggregate conditions must also hold

$$y = \left\{\sum_{i=1}^{N} a_i^{\frac{1}{\zeta}} z_i^{1-\frac{1}{\zeta}}\right\}^{\frac{1}{1-\frac{1}{\zeta}}} \tag{92}$$

We proceed by fixing the share of good i in final production, the capital share of *value added output* in sector i, and the share of sector i's revenue dedicated to purchasing inputs from sector j. Additionally, we normalize the steady-state prices of all intermediate goods to $p_i = 1$. Call these shares ψ_{iy}, ψ_{ik}, and ψ_{ij} respectively. Note that $\sum_{i=1}^{N} \psi_{iy}$ must equal one. These values, along with the normalization of aggregate output, $y = 1$, fix the production parameters $a_i, a_{ij}, a_{ik}, a_{il}$. Since we have little a priori guidance on the value of φ, we calibrate φ to match a value for the steady-state Frisch elasticity.

From equation (83) and the normalization $y = p_i = 1$ it immediately follows that

$$z_i = a_i = \psi_{iy}. \tag{93}$$

Substitute the shares of revenue devoted to intermediate intermediate inputs into the market clearing condition in (84), we have that

$$q_i = z_i + \sum_j \psi_{ji} \frac{p_j q_j}{p_i}. \tag{94}$$

Combining the N equations yields a matrix expression for the values of $p_i y_i$,

$$\mathbf{pq} = (I - IO')^{-1} \mathbf{pz} \tag{95}$$

where boldface letters represent the vector of sector values (e.g. $\mathbf{p} = [p_1, p_2, ..., p_n]'$) and IO is matrix of intermediate shares defined in the text. Having solved for the vector $\mathbf{pq}$, we can directly back out the values of sectoral production, q_i. It follows from the definition of $\psi_{ij} \equiv \frac{p_j x_{ij}}{p_i q_i}$ that

$$x_{ij} = p_i q_i \frac{\psi_{ij}}{p_j}. \tag{96}$$

Multiply the intermediate input first order condition in equation (81) by x_{ij}, and sum sectors i for which $a_{ij} > 0$ to get

$$\sum_j p_j x_{ij} = p_i q_i^{\frac{1}{\sigma}} \left\{ \sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}-1} \sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}} \tag{97}$$

$$= p_i q_i^{\frac{1}{\sigma}} \left\{ \sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}} \right\}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}}, \tag{98}$$

which can be easily solved for $\Omega_{1,i} \equiv \sum_j a_{ij} x_{ij}^{1-\frac{1}{\xi}}$. Plugging this value back into equation (81), yields a solution for a_{ij}

$$a_{ij} = \frac{p_j}{p_i} x_{ij}^{\frac{1}{\xi}} q_i^{-\frac{1}{\sigma}} \Omega_{1,i}^{1-\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}}. \tag{99}$$

Using a similar procedure, we can now solve a_{ik} and a_{il}. First, use the production function to solve for $\Omega_{2,i} \equiv a_{il} l_i^{1-\frac{1}{\kappa}} + a_{ik} k_i^{1-\frac{1}{\kappa}}$:

$$\Omega_{2,i} = \left(q_i^{1-\frac{1}{\sigma}} - \Omega_{1,i}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\xi}}} \right)^{\frac{1-\frac{1}{\kappa}}{1-\frac{1}{\sigma}}} \tag{100}$$

To back out k_i, note that

$$\psi_{ik} \equiv \frac{p_i F_{k,i} k_i}{p_i q_i - \sum_j p_j x_{ij}} \\ = \frac{F_{k,i} k_i / q_i}{1 - \sum_j \psi_{ij}}. \tag{101}$$

Rearranging equation (82) gives the following expression for capital in sector i:

$$k_i = \frac{p_i \psi_{ik} q_i}{\beta^{-1} - 1 + \delta} \left(1 - \sum_j \psi_{ij} \right). \tag{102}$$

Sectoral investment is now simply $i_i = \delta k_i$. To solve for a_{ik}, use the above result and the expression for $F_{k,i}$, to find

$$a_{ik} = \psi_{ik} \left(1 - \sum_j \psi_{ij} \right) q_i^{1-\frac{1}{\sigma}} \Omega_{2,i}^{1 - \frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}} k_i^{\frac{1}{\kappa}-1}. \tag{103}$$

From this, we can also easily determine

$$a_{il} l_i^{1-\frac{1}{\kappa}} = \Omega_{2,i} - a_{ik} k_i^{1-\frac{1}{\kappa}}. \tag{104}$$

Using island market clearing in equation (85), sectoral output and investment can be used to compute consumption on each island. Finally, to determine sectoral labor, use consumer equations (78) and (79) to derive the relation $\varphi = \frac{w_i}{c_j}(1 - l_i)$, which implies that

$$w_i = c_i \varphi + w_i l_i. \tag{105}$$

From the labor choice condition in equation (80) we have,

$$w_i l_i = p_i q_i^{\frac{1}{\sigma}} \Omega_{2,i}^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\kappa}}-1} a_{il} l_i^{1-\frac{1}{\kappa}}, \tag{106}$$

which can be plugged back into equation (105) to determine the wage. The steady-state value of l_i follows directly. Finally, equation (104) can be used to solve for a_{il} and consumer equations (78) can be used to determine λ_i.

C Solution Method

A substantial literature has arisen in recent years for solving models of dispersed information, including Kasa et al. (2004); Hellwig and Venkateswaran (2009); Baxter et al. (2011); Nimark (2011); Rondina and Walker (2012) and Huo and Takayama (2015). These techniques are not applicable here because they assume information symmetry across all agent types and/or a large number (or continuum) of agents. In these environments, agents are shown to care only about their own expectation of the states, the economy-wide average expectation of the same states, the average expectation of the average expectation, and so on. In contrast, with a finite number of sectors, we must keep track of a complete structure of each agent-type's expectation of other agent-type's expectation, *for each level of expectation.* Concretely, firms in sector one must follow the expectations of firms in sector two and firms in sector three separately, as the dependence of their optimal choice on these two sectors is not identical.

The linearized equations in our model can be rearranged to take the form

$$0 = \sum_{j=0}^{N} \left(\begin{bmatrix} \mathbf{A}_1^i & \mathbf{A}_2^i \end{bmatrix} E_t^j \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{y}_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1^i & \mathbf{B}_2^i \end{bmatrix} E_t^j \begin{bmatrix} \mathbf{x}_t \\ \mathbf{y}_t \end{bmatrix} \right). \tag{107}$$

where $j = 0$ denotes the full information set. The vector of endogenous choice variables, $\mathbf{y}_t$, has dimension $n_y \times 1$ and the vector of predetermined states, $\mathbf{x}_t$, is of dimension $n_x \times 1$. The state vector $\mathbf{x}_t$ is decomposed into a vector $\mathbf{x}_t^1$ of n_x^1 endogenous state variables and a vector $\mathbf{x}_t^2$ of n_x^2 exogenous state variables which follow the autoregressive process

$$\mathbf{x}_{t+1}^2 = \rho \mathbf{x}_t^2 + \tilde{\eta}\epsilon_{t+1} \tag{108}$$

where ρ is a square matrix of dimension n_x^2. The column vector of n_ϵ exogenous shocks ϵ_t is assumed to be i.i.d. with identity covariance matrix.

In general, the solution to such a model is an MA(∞) process. Atolia and Chahrour (2014) shows how to approximate the solution to such models as an ARMA(1,K) under the assumption that past shocks become common knowledge in period $K+1$.[14] This approach generalizes the one taken by Townsend (1983). Nimark (2011) discusses some theoretical requirements for a related approach to such approximations to be valid, although such theoretical details have yet to be fully expounded for our current environment. The (approximate) solution to the model can then be written as

$$\mathbf{x}_{t+1} = \mathbf{h}_x \mathbf{x}_t + \sum_{\kappa=0}^{K} \mathbf{h}_\kappa \epsilon_{t-\kappa} + \eta\epsilon_{t+1} \tag{109}$$

$$\mathbf{y}_t = \mathbf{g}_x \mathbf{x}_t + \sum_{\kappa=0}^{K} \mathbf{g}_\kappa \epsilon_{t-\kappa}. \tag{110}$$

Formulating the model solution in this way ensures that the matrices $\mathbf{h}_x$ and $\mathbf{g}_x$ do not depend on the information assumption: they are the transition and observation matrices

[14] Atolia and Chahrour (2014) also discusses an alternative "bounded rationality" assumption in which agents ignore observations that are more than K periods in the past.

implied by the solution to the (linearized) full information model. Thus the presence of incomplete (and heterogeneous) information is captured completely by the MA terms in equations (109) and (110). Atolia and Chahrour (2014) provides a numerical approach for finding the matrices $\mathbf{h}_\kappa$ and $\mathbf{g}_\kappa$, which we employ in our calibration exercises above.

Atolia and Chahrour (2014) also discuss an alternative approximation to such models in which agents have "finite recall," and include in their information sets only their observations for the most recent K periods. This alternative approach prevents agents' inference from putting arbitrarily large weights on shocks far in the past, and thus prevents agents from perfect inference when the observables are non-fundamental in the shocks, as in Graham and Wright (2010) and Rondina and Walker (2012). Our claim in the text that such non-fundamental such equilibria do not appear is based on the observation that the imperfect recall and delayed-but-complete revelation approaches to approximation converge to the same dynamics for sufficiently large horizons K.

D Derivation of Steady-State Investment Complementarities

In this section, we derive the expression for the steady-state complementarities in capital for the two sector model; we begin by relaxing the Cobb-Douglas assumption for intermediate production in Section 3 and then reimpose it later to get explicit expressions in terms of the production parameters. For the production function (38), in steady-state, equations (22), (23), (26), and (27) become respectively,

$$\hat{q}_i = \alpha_k \hat{k}_i + \alpha_x \hat{x}_{ij} \tag{111}$$

$$\hat{p}_j = \hat{p}_i + \alpha_k \hat{k}_i + (\alpha_x - 1)\, \hat{x}_{ij} \tag{112}$$

$$\hat{z}_i = -\zeta \hat{p}_i + \frac{1}{2} \sum_j \hat{z}_j \tag{113}$$

$$\hat{q}_i = \alpha_x \hat{x}_{ji} + (1 - \alpha_x) \hat{z}_i. \tag{114}$$

Moreover, since we are considering steady-state, consumption drops from the intertemporal relation in equation (24). Substituting out for the production functions yields

$$\hat{p}_i = -\left(\alpha_k - 1\right) \hat{k}_i - \alpha_x \hat{x}_{ij}. \tag{115}$$

Now, combine equations (112) and (113) to find,

$$\left(\hat{z}_i - \hat{z}_j\right) = \zeta \left(\alpha_k \hat{k}_i + (\alpha_x - 1)\, \hat{x}_{ij}\right). \tag{116}$$

Since the above equation holds for all i and j, we have that

$$2(\hat{z}_1 - \hat{z}_2) = \zeta \left[\alpha_k(\hat{k}_1 - \hat{k}_2) + (\alpha_x - 1)(\hat{x}_{12} - \hat{x}_{21})\right] \tag{117}$$

Equation (117) can be solved for the difference $(\hat{x}_{12} - \hat{x}_{21})$:

$$(\hat{x}_{12} - \hat{x}_{21}) = \frac{2}{\zeta(\alpha_x - 1)}(\hat{z}_1 - \hat{z}_2) - \frac{\alpha_k}{\alpha_x - 1}(\hat{k}_1 - \hat{k}_2). \tag{118}$$

Equations (111) and (114) can be combined to yield

$$\hat{z}_i = \frac{\alpha_k}{1 - \alpha_x}\hat{k}_i + \frac{\alpha_x}{1 - \alpha_x}(\hat{x}_{ij} - \hat{x}_{ji}), \tag{119}$$

which implies that

$$(\hat{z}_1 - \hat{z}_2) = \frac{\alpha_k}{1 - \alpha_x}(\hat{k}_1 - \hat{k}_2) + \frac{2\alpha_x}{1 - \alpha_x}(\hat{x}_{12} - \hat{x}_{21}). \tag{120}$$

Combine equations (118) and (120) to find

$$(\hat{z}_i - \hat{z}_j) = \phi_1(\hat{k}_i - \hat{k}_j), \tag{121}$$

where $\phi_1 \equiv \frac{\alpha_k(1+\alpha_x)}{(1-\alpha_x)^2 + 4\alpha_x/\zeta}$. Rearranging equation (113) yields

$$p_1 = -\frac{1}{2\zeta}(\hat{z}_1 - \hat{z}_2). \tag{122}$$

Plugging equation (121) back into equation (122) yields

$$\hat{p}_1 = -\frac{1}{2\zeta}\phi_1(\hat{k}_1 - \hat{k}_2). \tag{123}$$

Price aggregation requires that $p_1 = -p_2$. Using this result, equations (115) and (112) together implies

$$\hat{p}_1 = (1 - \alpha_k)\hat{k}_1 + \frac{\alpha_x}{1 - \alpha_x}(\hat{p}_2 - \hat{p}_1 - \alpha_k\hat{k}_1) \tag{124}$$

$$= (1 - \alpha_k)\hat{k}_1 + \frac{\alpha_x}{1 - \alpha_x}(-2\hat{p}_1 - \alpha_k\hat{k}_1) \tag{125}$$

$$\tag{126}$$

Solving for p_1 yields

$$p_1 = \phi_2 k_1. \tag{127}$$

where $\phi_2 \equiv \frac{1-\alpha_x-\alpha_k}{1+\alpha_x}$. Finally, combining equations (123) and (127), yields the expression,

$$\hat{k}_1 = \frac{1}{2\zeta}\frac{\phi_1}{\phi_2}(\hat{k}_2 - \hat{k}_1), \tag{128}$$

so that

$$\phi_k \equiv \frac{1}{2\zeta}\frac{\phi_1}{\phi_2} = \frac{1}{2}\frac{\alpha_k}{1 - \alpha_k - \alpha_x}\frac{(1 + \alpha_x)^2}{(1 - \alpha_x)^2\zeta + 4\alpha_x}. \tag{129}$$

Evaluating using the definition $\tilde{\alpha}_k$ yields expression (39).