

# Firm Risk and Leverage Based Business Cycles <sup>\*</sup>

Sanjay K. Chugh <sup>†</sup>

Boston College

Kiel Institute for the World Economy (Institut für Weltwirtschaft)

First Draft: October 2009

This Draft: March 2, 2013

## Abstract

I characterize cyclical fluctuations in the cross-sectional dispersion of firm-level productivity in the U.S. manufacturing sector. Using the estimated dispersion, or “risk,” stochastic process as an input to a baseline DSGE financial accelerator model, I assess how well the model reproduces aggregate cyclical movements in the financial conditions of U.S. non-financial firms. In the model, risk shocks calibrated to micro data induce large and empirically-relevant fluctuations in leverage, a financial measure typically thought to be closely associated with real activity. In terms of aggregate quantities, however, pure risk shocks account for only a small share of GDP fluctuations in the model, less than one percent. Instead, it is standard aggregate productivity shocks that explain virtually all of the model’s real fluctuations. These results reveal a dichotomy at the core of a popular class of DSGE financial frictions models: risk shocks induce large financial fluctuations, but have little effect on aggregate quantity fluctuations.

**Keywords:** leverage, second-moment shocks, time-varying volatility, credit frictions, financial accelerator, business cycles

**JEL Classification:** E10, E20, E32, E44

---

<sup>\*</sup>I thank participants in seminar presentations at Georgetown University, the Federal Reserve Bank of Cleveland, London Business School, Boston University, Boston College, the Federal Reserve Bank of Dallas, the Federal Reserve Bank of Boston, the Kiel Institute, the IZA Institute, Drexel University, the Federal Reserve Bank of Philadelphia, the University of Virginia, the IMF Institute, Emory University, Johns Hopkins University, and the Fall 2010 NBER Workshop on Methods and Applications for DSGE Models at the Federal Reserve Bank of Atlanta for helpful comments. I thank John Haltiwanger for sharing data, and, for helpful comments and discussions at various stages of this project, David Arseneau, S. Boragan Aruoba, Rudi Bachmann, Larry Ball, Susanto Basu, Christian Bayer, Charles Carlstrom, Chris Carroll, Pablo D’Erasmus, Timothy Fuerst, Simon Gilchrist, Francois Gourio, John Haltiwanger, Cosmin Ilut, Andre Kurmann, and Stephanie Schmitt-Grohe.

<sup>†</sup>email address: sanjay.chugh@bc.edu.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Risk Fluctuations</b>	<b>5</b>
2.1	Productivity Risk . . . . .	7
2.2	Average Productivity . . . . .	8
<b>3</b>	<b>Balance Sheet Fluctuations</b>	<b>9</b>
<b>4</b>	<b>Model</b>	<b>13</b>
4.1	Households . . . . .	13
4.2	Firms . . . . .	14
4.2.1	Firm Financing and Contractual Arrangement . . . . .	15
4.2.2	Operating Profits and Asset Evolution . . . . .	17
4.2.3	Profit Maximization . . . . .	18
4.2.4	Aggregation . . . . .	19
4.3	Private Sector Equilibrium . . . . .	20
<b>5</b>	<b>Quantitative Analysis</b>	<b>21</b>
5.1	Computational Strategy . . . . .	21
5.2	Calibration . . . . .	22
5.3	Long-Run Dispersion and Long-Run Equilibrium . . . . .	23
5.4	Business Cycle Dynamics . . . . .	24
5.4.1	Risk Shocks in General Equilibrium . . . . .	24
5.4.2	Risk Shocks in Partial Equilibrium . . . . .	26
5.4.3	Both Risk Shocks and Average Productivity Shocks . . . . .	26
5.4.4	Economic Interpretation . . . . .	27
<b>6</b>	<b>Conclusion</b>	<b>27</b>
<b>A</b>	<b>Basic Mechanism in Partial Equilibrium</b>	<b>33</b>
<b>B</b>	<b>Aggregate Productivity Shocks</b>	<b>34</b>
<b>C</b>	<b>Bundled Shocks: Productivity-Induced Risk Fluctuations</b>	<b>36</b>

# 1 Introduction

In this paper, I modify a widely-used class of general equilibrium financial accelerator models in a way that leads to empirically relevant fluctuations in firms’ leverage ratios, along with other measures of their financial conditions. Specifically, I show that dispersion, or “risk,” shocks can usefully be deployed in a simple DSGE agency-cost model to explain aggregate financial fluctuations. Such shocks, through their effects on leverage, also have the potential to cause fluctuations in aggregate macroeconomic quantities, independently from standard productivity and other “first-moment shocks” common in macro models. However, risk shocks are not treated as a free parameter. The empirical discipline brought to bear on the model relates to and contributes to a distinct recent literature that has studied how time variation in the *cross-sectional distribution of firm-level outcomes* — “risk shocks” — may in and of themselves drive business cycles. I find that fluctuations in cross-sectional dispersion of firm-level productivity are large. When input as an exogenous process to a simple agency-cost model, they generate empirically meaningful aggregate financial fluctuations; however, they induce very small macroeconomic quantity fluctuations.

More precisely, there are four main results, two from empirical work and two from the theoretical model that quantifies the link between the main empirical findings. First, I characterize business cycle fluctuations in firm-level dispersion using U.S. micro data for the period 1974-1988. Specifically, based on data constructed by Cooper and Haltiwanger (2006), I characterize the time variation in the cross-sectional dispersion of firm-level productivity. This time variation is identified in this paper as risk fluctuations.<sup>1</sup> This measure of firm risk is strongly countercyclical with respect to GDP, consistent with the micro-level evidence of Bloom, Floetotto, and Jaimovich (2010) and Bachmann and Bayer (2010). Firm risk is quite volatile over the business cycle: measured by the ratio of the standard deviation of *innovations* in risk to average risk, the volatility of annual firm risk is 17 percent. By this metric, volatility of firm risk is similar to that measured by Bloom, Floetotto, and Jaimovich (2010), but substantially larger than that measured by Bachmann and Bayer (2010). Comparisons must be made with caution, because the U.S. micro data I examine are different from the U.S. micro data examined by Bloom, Floetotto, and Jaimovich (2010), which in turn are different from the German micro data examined by Bachmann and Bayer (2010). Nevertheless, the evidence I present complements these and other emerging empirical measures of firm-level risk. The estimated risk shock process is used as an input to the theoretical model.

Second, using Compustat data, I construct cyclical measures of the aggregate leverage ratio in the U.S. non-financial business sector, which constitutes a large share of the demand side of credit markets. Using non-financial firms selected from Compustat, I find that cyclical fluctuations

---

<sup>1</sup>This identification does not imply that “risk fluctuations” are exclusively about disturbances to cross-sectional variances that affect equilibrium outcomes; rather, it is the aspect of risk on which this paper focuses.

in aggregate leverage were much larger during 1989-2009 than during 1974-1988: the volatility of leverage relative to that of GDP doubled from about two to four.<sup>2,3</sup> On the other hand, the relative volatilities of the underlying debt and, especially, equity measures rose much less sharply between the two time periods. Regardless of sample period, leverage is at least moderately countercyclical with respect to GDP. The cyclical properties of leverage, and, more informally, those of debt and equity separately (which are also mildly countercyclical), provide benchmark metrics against which the theoretical model’s endogenous financial dynamics are broadly assessed.<sup>4</sup>

The other two contributions of the paper are theoretical. The first main result from the model is that empirically relevant risk shocks drive virtually all of the business cycle volatility of the model’s financial aggregates. The quantitative fit of the model is especially tight in its predictions regarding fluctuations in leverage, which are often thought to play a central role in connecting financial and real activity. In the model, leverage fluctuations have the potential to drive, or at least be associated with, real fluctuations. Such “leverage-based business cycles” could arise through fluctuations in firms’ balance sheet conditions that are induced by risk shocks. The transmission channel that the model emphasizes and tests is thus explicitly financial: if there were no financial frictions, there is no channel by which risk shocks could affect real fluctuations at all.<sup>5</sup> This aspect of the model is similar to the qualitative business cycle model of Williamson (1987) and the quantitative model of Dorofeenko, Lee, and Salyer (2008).

However, the second main result from the theoretical model is that pure risk shocks, in which average productivity is held constant, lead to very small fluctuations of macro aggregates such as GDP. The volatility of GDP conditional on risk shocks alone is less than one percent of GDP volatility conditional on shocks to average productivity alone. Thus, risk shocks and the leverage-mediated fluctuations they have the potential to generate do *not* seem to be an important phenomenon when

---

<sup>2</sup>1974-1988 is chosen as a (sub-)sample period for the analysis of financial fluctuations because it matches the period for which the firm-level risk analysis is conducted.

<sup>3</sup>I define the leverage ratio as total end-of-quarter book value of debt to total end-of-quarter book value of equity for all non-financial firms in Compustat.

<sup>4</sup>A prominent recent study that uses Compustat evidence to examine the balance sheet conditions of non-financial firms is Covas and den Haan (2011). An important point that emerges from Covas and den Haan (2011) is that cyclical balance sheet fluctuations of the largest firms heavily influences cyclical movements in aggregates, and are fairly different from cyclical movements of financing conditions of firms outside the top few percentiles of the book-value-of-assets distribution. I nonetheless analyze only aggregates. For the model-based purposes of this study, however, the aggregates are constructed in a sufficiently different manner so as to make this point not directly applicable. Further discussion appears in Section 3. Besides Covas and den Haan (2011), other empirical studies that also use Compustat data to document firms’ balance sheet positions are Levin, Natalucci, and Zakrajsek (2004), Korajczyk and Levy (2003), Hennessy and Whited (2007), and Levy and Hennessy (2007). The latter three are broadly micro-finance oriented, while Levin, Natalucci, and Zakrajsek (2004) is macro oriented.

<sup>5</sup>This is not to deny that there may be other channels by which risk shocks could affect real outcomes; for clarity in this paper, other channels are not being tested.

viewed through the lens of a baseline agency cost model calibrated to firm-level data. This result emerges despite the fact that the underlying risk shocks in the model are, as noted above, fairly large compared to other micro evidence on risk fluctuations.

The economic intuition for the lack of meaningful macro volatility arising from risk shocks is the lack of a tight quantitative connection running *from* the “partial equilibrium” financial sector of the model *to* the rest of the general equilibrium model. More precisely, if the primitive shock directly affects the financial sector, which is the model’s view of risk fluctuations, it leads to very small real-side effects even though it generates quite empirically-meaningful financial market effects. In contrast, if the primitive shock directly affects the real side of the economy, which is a natural and widespread view of average productivity fluctuations, it leads to empirically-meaningful real-side effects, but, interestingly, also leads to empirically descriptive financial market effects. Thus, it is not just the connection between the macro and finance sectors of the model that matters; the location of the primitive shocks matters. These points are established by quantitative comparison of the cyclical fluctuations from the partial equilibrium financial model with the fluctuations in the full general equilibrium. Taken together, the results suggest a type of dichotomy present at the core of a widely-used class of DSGE accelerator models: risk shocks lead to large and empirically relevant financial fluctuations, but these are largely isolated from macro fluctuations.

In studying the joint business cycle dynamics of real and financial outcomes, this paper contributes to a large emerging literature. For example, Jermann and Quadrini (2010) document the cyclical properties of flows of firms’ equity and debt issuance. However, they do not report the cyclical behavior of the debt-to-equity (leverage) ratio, which is one point of focus of this paper.<sup>6</sup> The medium-scale monetary policy model of Christiano, Motto, and Rostagno (2009) also employs the risk shock highlighted in this paper, but they estimate the parameters of the stochastic process based on aggregate macro and financial data, rather than using direct firm-level evidence. In terms of main results, while I find that a miniscule share of GDP fluctuations can be attributed directly to risk shocks, Christiano, Motto, and Rostagno (2009) find that nearly 20 percent of GDP fluctuations arise from risk shocks. Much of the difference in results seems due to their much larger macro-estimates of risk fluctuations than micro evidence indicates.<sup>7</sup> Some of the difference may also be due to the host of nominal rigidities, real rigidities, and “news shock” events present in their model, from which I abstract in order to isolate the role of pure risk shocks.

It is clear that in order to consider fluctuations in cross-sectional dispersion, the model must

---

<sup>6</sup>Jermann and Quadrini (2010) use financial data from the Flow of Funds Accounts of the Federal Reserve Board, whereas I use Compustat data.

<sup>7</sup>To be clear, the magnitude of risk fluctuations I find in the Cooper and Haltiwanger (2006) micro data is *large* compared to the *micro* evidence of studies such as Bloom, Floetotto, and Jaimovich (2010) and Bachmann and Bayer (2010), but it is *small* compared to the *macro* evidence of studies such as Christiano, Motto, and Rostagno (2009).

have some notion of heterogeneity and cannot be a strict representative-agent economy. In the Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999) agency-cost class of models on which I build, the heterogeneity is in borrowers' idiosyncratic ability to repay their loans, which in turn stems from idiosyncratic productivity. This feature is central in these models because with no cross-sectional heterogeneity of borrowers' ability to repay, there is no risk at all from the point of view of lenders, and hence no financial friction.<sup>8</sup> In typical quantitative analysis of these models, parameters for the distribution are chosen based on evidence on long-run risk premia or other financial measures, but then the distributional aspect of the model invariably fades into the background.

I instead place the distributional aspect of the model in the foreground by emphasizing the time variation in cross-sectional dispersion of firms' productivity, using firm-level evidence as discipline. Fluctuations in firm-level risk presents lenders with time-varying risk of their overall loan portfolios, and hence leads them to extend more or less credit to borrowers — i.e., extend more or less leverage. In the model, the *only* way for risk shocks to possibly transmit into fluctuations of GDP and other macro aggregates is through changes in debt and hence in leverage. While risk shocks turn out to account quite well for financial fluctuations in the model, risk-induced financial fluctuations are almost completely isolated from real fluctuations. Dorofeenko, Lee, and Salyer (2008) also find this dichotomy result in a closely related study. These results are perhaps unsettling because the agency-cost setup is a common building block of richer DSGE models of financial frictions, whose development is a very active area of research. The results obtained here suggest that in quantitative agency-cost models that *do* find important connections between financial fluctuations and real fluctuations, the linkages are not driven by the basic agency-cost friction itself, but rather by empirically questionable risk fluctuations or other features of the model that interact with the agency friction.

In terms of broader motivation, a widespread recent view is that the cyclical behavior of leverage may be important to both empirical and theoretical understanding of how financial and real outcomes co-move along the business cycle. Geanakoplos (2009), Adrian and Shin (2008), and others have stressed the cyclical behavior of leverage in the financial sector. Mimir (2010) tabulates the cyclical properties of leverage in the financial sector using standard business cycle filtering tools. Given recent events, a focus on leverage in the financial sector is natural. However, a long tradition in both macro and finance has emphasized leverage in the non-financial corporate sector as being important for aggregate fluctuations, which is the channel studied in this paper.<sup>9</sup>

---

<sup>8</sup>A bit more precisely, it is the combination of idiosyncratic risk and the fact that idiosyncratic realizations are unknown at the time of financial contracting that leads to imperfect outcomes in financial markets.

<sup>9</sup>Bernanke, Campbell, and Whited (1990) is an early empirical study suggesting the importance of non-financial sector leverage in aggregate fluctuations.

Finally, two points regarding modeling approach are in order. First, as should be clear from the discussion so far, the idea of “risk shocks” in this paper is variations over time in the cross-sectional standard deviation of firm-level productivity, holding constant average (aggregate) productivity. This is the same notion of idiosyncratic “second-moment shocks” that Bloom, Floetotto, and Jaimovich (2010), Bachmann and Bayer (2010), Christiano, Motto, and Rostagno (2009), and Dorofeenko, Lee, and Salyer (2008) study. However, it is distinct from an aggregate notion of “second-moment shocks” emphasized by Justiniano and Primiceri (2008), Fernández-Villaverde and Rubio-Ramirez (2007), Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2010), Gourio (2011), Basu and Bundick (2011), and others, in which the standard deviation of the innovations affecting aggregate driving processes such as productivity, real interest rates, and monetary disturbances vary over time. Crucial in this latter group of studies is that they are all representative-agent economies, so there is no meaningful concept of cross-sectional dispersion. Focusing on the cross section is the main idea in Bloom, Floetotto, and Jaimovich (2010), Bachmann and Bayer (2010), Christiano, Motto, and Rostagno (2009), Dorofeenko, Lee, and Salyer (2008), and this paper. Second, echoing the well-articulated argument in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2010), I treat cross-sectional risk fluctuations as exogenous. How to endogenize such fluctuations is an interesting question, but because the empirical evidence in this area is so new and fast moving, I adopt the view that this is not the first natural question to consider. Instead, I focus on the consequences of such fluctuations as mediated through agency-cost frictions, which sharply focuses attention on the transmission mechanism between the financial and real sectors in the model. Regarding terminology, I use the terms “risk shocks,” “firm-level risk,” “second-moment shocks,” and “dispersion shocks” interchangeably.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on firm-level risk and its business cycle properties. This evidence serves as quantitative input to the model. Section 3 then documents the business cycle behavior of an aggregate measure of the leverage ratio, along with the underlying debt and equity measures, in the U.S. non-financial business sector. This evidence provides part of the metrics against which the output of the model is evaluated. Section 4 presents the baseline model, in which shocks to average productivity and risk shocks are independent exogenous processes. Section 5 presents quantitative results, which are supplemented by further analysis in the Appendix. Section 6 concludes.

## 2 Risk Fluctuations

The main goal of this section is to document the properties of business cycle fluctuations in firm-level dispersion. The analysis is based on a balanced panel, constructed by Cooper and Haltiwanger (2006), from the Longitudinal Research Database (LRD). The data are annual observations of plant-

level measures such as revenue, materials and labor costs, and investment at approximately 7,000 large U.S. manufacturing plants over the period 1974-1988. The starting point for my analysis is Cooper and Haltiwanger’s (2006) measures of plant-level profitability residuals from this panel.<sup>10</sup>

Briefly, Cooper and Haltiwanger (2006) compute for each plant  $i$  in year  $t$  a residual  $A_{it}$  that reconciles exactly the observations of plant  $i$ ’s profits and capital stock in year  $t$  when described by a profit function that depends only on the capital stock.<sup>11</sup> The year-specific aggregate residual  $\omega_{mt}$  is computed as the mean of  $A_{it}$  across firms in year  $t$ . Plant  $i$ ’s profit function in year  $t$  is viewed as being shifted by both the aggregate shock  $\omega_{mt}$  and an idiosyncratic shock  $\omega_{it} \equiv A_{it}/\omega_{mt}$ . In each year, there is thus a cross-sectional distribution of  $\omega_{it}$ . Denote by  $\sigma_t^\omega$  the cross-sectional standard deviation in year  $t$  of the idiosyncratic component of profitability  $\omega_{it}$ . I make three identifying assumptions regarding  $\omega_{it}$  and thus the interpretation of its cross-sectional dispersion  $\sigma_t^\omega$ . These assumptions align the analysis of the data with the model into which they will be an input.

First, although  $\sigma_t^\omega$  measures cross-firm *dispersion*, I treat it as measuring true cross-firm *risk*.<sup>12</sup> The two concepts are identical only if each firm’s idiosyncratic component  $\omega_{it}$  has zero persistence. Cooper and Haltiwanger (2006, p. 622-623) estimate an AR(1) coefficient of the idiosyncratic component of 0.885, hence  $\omega_{it}$  is actually quite persistent (recall the data are annual). However, it is computationally very difficult to handle persistent idiosyncratic shocks in the theoretical model developed below, so the model assumes *iid* idiosyncratic shocks.<sup>13</sup> To align the empirical analysis of  $\sigma_t^\omega$  with its role in the model, I thus proceed by assuming zero idiosyncratic persistence. There are both advantages and drawbacks of this approach. An advantage is that the dispersion of firm-level *outcomes* in the model are thus calibrated to the data. An obvious drawback is that  $\sigma^\omega$  is thus an overestimate of firm-level risk, which, when input as an exogenous process to the model, in principle gives risk shocks the largest possible role in driving the model’s fluctuations. As the results in Section 4 show, however, even though this overestimate of risk enables the model to explain financial fluctuations fairly well, risk shocks turn out to have little role in driving real-side fluctuations.

The second identifying assumption is that firm-level *profitability* shocks are true *productivity* shocks. Because plant-level price deflators are unavailable in the dataset, it is impossible to distinguish cost shocks from revenue shocks, so the  $\omega_{it}$  residuals mix both supply and demand shifts (hence the term “profitability” shocks).<sup>14</sup> As an identifying assumption for the theoretical model, I simply interpret these profitability shocks as true productivity shocks. A model-based justification

<sup>10</sup>I thank John Haltiwanger for providing their aggregative data on profitability residuals.

<sup>11</sup>The Appendix in Cooper and Haltiwanger (2006) describes in detail the construction of the data and the residuals.

<sup>12</sup>Which is the basis for my interchangeable references to firm-level “dispersion” and firm-level “risk.”

<sup>13</sup>To my knowledge, no DSGE models based on the agency-cost framework have been solved assuming persistent idiosyncratic shocks.

<sup>14</sup>More precisely, they are available only at five-year intervals, too low a frequency for business cycle analysis.



for this is that the relative price of all goods in the model is always unity due to perfect competition in goods markets. Thus, one can think of this aspect of the data analysis as also being conducted strictly through the lens of the model.

Third, when deploying the evidence documented here in the model, I identify “plants” as “firms,” abstracting from the fact that a non-negligible share of plant-level output in the LRD represents output of multi-plant firms. With these three identifying assumptions, I characterize the business cycle behavior of both  $\omega_{mt}$  and of  $\sigma_t^\omega$ , aspects of the data not studied by Cooper and Haltiwanger (2006).

## 2.1 Productivity Risk

I first compute the cross-sectional coefficient of variation of productivity (profitability) for each of the 15 years of the sample. Cross-sectional coefficients of variation are used because the residually-computed aggregate mean level of productivity ( $\omega_{mt}$ ) is not unity in the data, but it is normalized to unity in the model below. The time-averaged mean of the cross-sectional coefficient of variation is 0.156, hence I normalize long-run dispersion in the model to  $\bar{\sigma}^\omega = 0.156$ . Given the discussion above, true long-run “risk” is smaller than  $\bar{\sigma}^\omega = 0.156$ . Specifically, taking a stationary AR(1) view of idiosyncratic productivity and using the Cooper and Haltiwanger (2006, p. 622-623) estimate of idiosyncratic persistence of 0.885, true long-run firm-level risk is  $\sqrt{1 - 0.885^2} \bar{\sigma}^\omega = 0.0726$ . Aligning the empirical analysis with the model thus overstates firm-level risk by roughly a factor of two.

Figure 1 plots the time series  $\sigma_t^\omega$ , which suggests a modest upward trend in dispersion. Figure 2 displays the HP-filtered components of  $\sigma_t^\omega$  and GDP over the period 1974-1988. A clear negative cyclical correlation between the two series is apparent — the contemporaneous correlation between the two series is -0.83, hence expansions are associated with a decrease in dispersion of firms’ idiosyncratic productivity, and recessions are associated with an increase in dispersion of firms’ idiosyncratic productivity. Strongly countercyclical firm-level risk is also a robust finding in the micro evidence of Bachmann and Bayer (2010) and Bloom, Floetotto, and Jaimovich (2010) (hereafter, BB and BFJ, respectively). In terms of volatility, the standard deviation of the cyclical component of  $\sigma_t^\omega$  is 3.15 percent over the sample period. With an innocuous abuse of notation, I hereafter use  $\sigma_t^\omega$  to denote the cyclical component of cross-sectional dispersion.

In the model presented below, I suppose that  $\sigma_t^\omega$  follows the exogenous AR(1)

$$\ln \sigma_{t+1}^\omega = (1 - \rho_{\sigma^\omega}) \ln \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_{t+1}^{\sigma^\omega}, \quad (1)$$

with  $\epsilon^{\sigma^\omega} \sim N(0, \sigma_{\sigma^\omega})$ . Given  $\bar{\sigma}^\omega = 0.156$ , the point estimate (using OLS) of the AR(1) parameter is  $\rho_{\sigma^\omega} = 0.48$ , with a t-statistic of 1.93. With this estimate of  $\rho_{\sigma^\omega}$  and the standard deviation of  $\sigma_t^\omega$  of 3.15 percent, the standard deviation of the (annual) *innovations* to the cross-firm dispersion process

can be computed to be 0.0276. This implies a coefficient of variation (with respect to the mean dispersion  $\bar{\sigma}^\omega = 0.156$ ) of 17.7 percent, which can be directly compared to the empirical evidence reported by BB and BFJ. Computed in a variety of ways, BB find a coefficient of variation of innovations to firm-level productivity for their entire sample of German firms between two and three percent. However, because the Cooper and Haltiwanger (2006) analysis is of large manufacturing plants, the most comparable result in BB is their finding for the largest (ranked by employment) five percent of firms in their sample. For this sample, BB find a coefficient of variation of firm-level innovations of 5.5 percent (see their Table 8). The 17.7 percent coefficient of variation of plant-level innovations in the Cooper and Haltiwanger (2006) sample is thus substantially larger than the largest firms in BB’s sample. However, this degree of volatility of firm risk lines up much better with the evidence of BFJ, who document using a variety of cross-sectional measures that dispersion of firm outcomes rises very sharply during recessions.

## 2.2 Average Productivity

For further consistency in the way the firm-level data are used as an input to the model, I also characterize the time-series behavior of  $\omega_{mt}$ , the average productivity (profitability) residual. In the model, this measure will correspond conceptually to the standard notion of aggregate productivity (i.e., the first moment of the productivity distribution). Figures 3 and 4 display the actual series, its HP trend, and the cyclical component of average productivity.<sup>15</sup>

The cyclical component of  $\omega_{mt}$  is highly correlated with the cyclical component of GDP, as Figure 4 shows — the contemporaneous correlation between the two is 0.87. The volatility of the cyclical component of  $\omega_{mt}$  is 1.26 percent (at an annual horizon). Again with an innocuous abuse of notation, I hereafter use  $\omega_{mt}$  to denote the cyclical component of average productivity.

In the model presented below, I suppose that  $\omega_{mt}$  follows the exogenous AR(1)

$$\ln \omega_{mt+1} = \rho_{\omega_m} \ln \omega_{mt} + \epsilon_{t+1}^{\omega_m}, \quad (2)$$

with  $\epsilon^{\omega_m} \sim N(0, \sigma_{\omega_m})$ . Estimation gives a point estimate  $\rho_{\omega_m} = 0.47$ , with a t-statistic of 1.84.<sup>16</sup> With this estimate of  $\rho_{\omega_m}$  and the standard deviation of  $\omega_{mt}$  of 1.26 percent, the standard deviation of the (annual) *innovations* to the average productivity process can be computed to be 0.0111. Finally, the cyclical correlation between average productivity and the dispersion of productivity (i.e., the concept of firm risk) is -0.97; this extremely strong negative correlation is part of the motivation of the “bundled-shock” model extension considered in Appendix C.

<sup>15</sup>As noted above, long-run average productivity is normalized to unity in the model, so the vertical scale in Figure 3 is arbitrary. In the empirical analysis of Cooper and Haltiwanger (2006), mean productivity was not normalized.

<sup>16</sup>This differs from Cooper and Haltiwanger’s (2006, p. 623) estimate of the persistence of mean productivity because they do not detrend; the AR(1) coefficient of the unfiltered  $\omega_{mt}$  series is 0.76.

In the model developed below, I pursue a quarterly calibration, rather than an annual calibration, because the leverage evidence documented in Section 3 is quarterly. Because the evidence presented in this section is from annual data, I use persistence parameters of  $\rho_{\sigma^\omega} = 0.48^{0.25} = 0.83$  and  $\rho_{\omega_m} = 0.48^{0.25} = 0.83$ , which assumes smoothness in the processes during the year. How this inference of quarterly persistence from annual estimates affects the model calibration of the innovation parameters  $\sigma_{\sigma^\omega}$  and  $\sigma_{\omega_m}$  is discussed in Section 5.2.

One final note is helpful: a concern may be the slight downward trend in “productivity” in the manufacturing sector during 1974-1988. Keep in mind, however, that what is being measured is actually profitability residuals. If the relative prices of the inputs, capital and labor, trended during this period, this would show up as a trend in profitability.<sup>17</sup> As the results above show, the final AR(1) stochastic process that describes average profitability/productivity is very similar to a simple RBC model’s average productivity process. So, if one prefers, one can think of the AR(1) process as an illustrative, off-the-shelf RBC-style process, the precise parameter settings for which are not crucial to the main conclusions of the paper.

### 3 Balance Sheet Fluctuations

In this section, I present simple quarterly business cycle statistics for aggregate measures of the leverage ratio, along with their debt and equity components, of U.S. non-financial businesses over the past 25 years. As noted in the introduction, Covas and den Haan (2011) perform an exhaustive analysis of how financing conditions vary over the cycle, stratified by book value of firms’ assets. One of their main findings is that large firms (defined as firms with either the top one percent or top five percent of book value of assets) handle their debt and equity differently than do firms outside the top few percentiles. An implication of this is that examining only aggregates may cause misleading inference regarding the quantitative performance of structural models meant to portray a positive description of firms’ financing conditions.

The aggregates I analyze are constructed in a different way from Covas and den Haan (2011), which makes the previous point, though clearly an important one, not directly applicable to the comparison between model and data in Section 5. Specifically, this section documents only aggregates for three reasons. First, providing a very granular positive description of real and financial outcomes via a model requires a rich set of features such as decreasing returns to scale, fixed and variable costs of adjusting factor inputs, and so on. Providing such a fine view of micro outcomes is not the goal of the parsimonious aggregative model, one that is familiar in the DSGE literature, described in Section 4. Second, given the simplicity of the model used to try to reconcile risk shocks

---

<sup>17</sup>I thank Larry Ball and Chris Carroll for raising these points.

and real and financial fluctuations, it does not generate a firm-size distribution that influences how any given firm handles its financing. These two points lead directly to the third main reason for looking at only aggregates in this study, which is that, given the popularity of the simple agency cost model as a basic building block of many recent medium- and large-scale DSGE models, it is important to know how risk fluctuations transmit into both macro and financial aggregates in the model vis-a-vis the data.

Given this perspective on the empirical analysis, the data examined are quarterly Compustat data on publicly-traded non-financial U.S. firms. The sample period analyzed is 1974:Q1 — 2009:Q1, as well as the subsamples 1974:Q1 — 1988:Q4 and 1989:Q1 — 2009:Q1 separately. The former subsample corresponds to the time period of the Cooper and Haltiwanger (2006) data analyzed in Section 2. The latter time period, although beginning a few years later than the commonly-accepted dating of the beginning of the Great Moderation, corresponds roughly to the Great Moderation period.<sup>18</sup> For each quarter of the sample, every non-financial firm in Compustat that has data recorded for debt, equity, and revenue (an item used as a proxy that a firm is indeed active) is selected.<sup>19</sup> The measure of debt is the book value of firms' total debt, and the measure of equity is the book value of total shareholder equity.<sup>20</sup> In each quarter, aggregate debt and aggregate equity are computed as the simple sums of debt and equity over all firms selected in that quarter. The aggregate leverage ratio is then defined as the ratio of aggregate debt to aggregate equity in each quarter. The empirical debt and equity series whose statistics are reported below are the aggregates divided by aggregate revenues of all the firms selected in each quarter, which render the debt and equity measures stationary over the time period.<sup>21</sup> The precise interpretation of the statistics reported below for debt and equity is thus on a per-unit-of-revenue basis.

For the entire time period and the two subsamples separately, Figures 5 and 6 plot the time series of aggregate leverage, the HP trend components (computed using HP smoothing parameter 1,600), and the cyclical components.<sup>22</sup> In constructing the cyclical components, HP trends were extracted separately for each of the three time periods analyzed. Figure 5 shows that leverage was virtually stationary from the mid-1970's through the mid-1980's, and has trended upward

---

<sup>18</sup>For convenience, I thus sometimes refer to the latter subsample as the Great Moderation period.

<sup>19</sup>That is, a firm-quarter observation for which *any* of these three data were missing was dropped. Thus, the data are not a panel.

<sup>20</sup>In using book values and in measuring each of equity and debt in total terms (rather than extracting some components of either or both), the approach is the same as in Covas and den Haan (2011).

<sup>21</sup>In particular, the number of firms in the sample jumps up in late 1979, a jump that is reversed in mid-1984. Scaling by revenue thus achieves stationarity of debt and equity over this time period and still allows me to use the full sample of firms.

<sup>22</sup>The data were first seasonally adjusted because the Compustat data are not adjusted; a single seasonal adjustment was done for the entire time period. Seasonal filtering was performed with the X12 ARIMA algorithm implemented on the econometrics software package gretl.

since then, with two marked jumps in the late 1980's and early 2000's.<sup>23,24</sup> Figure 6 shows that the volatility of aggregate leverage increased as the Great Moderation took hold, both in absolute terms and even more dramatically relative to the volatility of GDP.

Figure 7, which presents the cyclical components of the aggregate debt and aggregate equity components separately, shows that underlying the change in magnitude of leverage cycles were interesting changes in comovements between debt and equity. Pre-Great Moderation, non-financial firms' debt and equity tracked each other a bit more closely than during the Great Moderation. Moreover, the business cycle volatility of debt and equity financing were each larger (relative to the volatility of GDP) during the Great Moderation period than before, although the increases in relative volatilities are not as sharp as for leverage. Changes in financial regulations along with other shifts that occurred in the economy since the 1980's evidently permitted and encouraged non-financial firms to manage their debt and equity financing differently by the mid-1980's than they had previously.<sup>25</sup>

Tables 1, 2, and 3 provide more quantitative detail on the observations that emerge from Figures 5, 6, and 7 by documenting standard business cycle statistics for aggregate leverage, aggregate debt, and aggregate equity during, respectively, 1974:Q1-1988:Q4, 1989:Q1-2009:Q1, and the entire sample. A couple of main features are worth highlighting, which reinforce the impressions left by Figures 5, 6, and 7. First, and perhaps counter to conventional wisdom, the contemporaneous correlation of leverage in the non-financial business sector with GDP is, depending on time period, moderately to strongly countercyclical. Non-financial firms do not seem to load up on leverage during expansions; in fact, somewhat the opposite. This finding is consistent with that in Levy and Hennessy (2007), who show that leverage ratios in highly-constrained firms are countercyclical, while leverage ratios in less-constrained firms are acyclical. Table 4 shows that leverage is also moderately countercyclical with respect to leads of GDP, but essentially uncorrelated with lags of

---

<sup>23</sup>This latter aspect of the leverage ratio I construct differs from Levin, Natalucci, and Zakrajsek (2004), who show in their Figure 3 that the leverage ratio displays a downward trend during the period 1988-2000, which is not evident here. Some differences may be definitional ones (for example, they use the market value of common equity as their measure of equity, in contrast to my metric of total shareholder equity) and some may be sample selection and construction issues (for example, they use a sales-weighted average of firm-level leverage ratios, whereas I focus directly on an aggregative measure of leverage, ignoring the cross-sectional dimension of leverage).

<sup>24</sup>I also note that the level of the leverage ratio I compute is substantially larger than that computed by Levy and Whited (2007, Table 1), which may be at least partly, and perhaps almost entirely, attributable to the different sample selection methods employed. Yet another (early) point of comparison for the results presented in Figures 5 and 6 is Bernanke, Campbell, and Whited (1990), who computed aggregate non-financial sector leverage in the late 1980's of about 0.4; as Figure 5 shows, I find that it was about 0.7 in the late 1980's.

<sup>25</sup>I do not speculate further on the nature or sources of these shifts, which is part of the topic of the literature on the Great Moderation.

GDP.<sup>26</sup>

A second main feature is that the volatility of leverage rose from 3.4 percent in the pre-Great Moderation period to 4.6 percent during the Great Moderation; relative to the volatility of GDP, it rose more sharply, nearly doubling from 2.2 to 4.1. Associated with this were more modest increases in the relative volatility of debt and equity, and a slight weakening of their contemporaneous positive correlation (from 0.78 during the pre-Great Moderation period to 0.68 during the Great Moderation).

This latter observation perhaps most clearly highlights that the aggregate financial measures analyzed in this section may be immune to the Covas and den Haan (2011) caution against examining aggregates. The strong positive cyclical correlation between debt and equity is consistent with their finding for firms outside the top few percentiles. In contrast, in Covas and den Haan (2011), this result is blurred in the aggregate due to the presence of the largest firms.<sup>27</sup>

Tables 1, 2, and 3 also present business cycle moments for standard macro aggregates, which display the decline in volatility of aggregate quantities in the later sample period. Of note for the calibration of the model presented below is the fact that total hours worked displays virtually the same cyclical volatility as GDP regardless of sample period. This fact is consistent with, for example, Cooley and Prescott's (1995, Table 1.1) finding that total hours worked is roughly as volatile as GDP.

For the purposes of the rest of this paper, I focus on the facts presented in Table 1 because they align with the time period of the risk analysis of Section 2. I thus take the following as stylized facts for the period 1974 – 1988: the volatility of leverage relative to that of GDP was roughly two, the volatility of debt and equity relative to GDP was each about three, and leverage, debt, and equity were all at least moderately countercyclical. The idea of the model analysis in Sections 4 and 5 is to assess the role the risk fluctuations documented in Section 2 can play in broadly explaining these joint financial and macro fluctuations.

---

<sup>26</sup>Note that the evidence of Adrian and Shin (2008), who document procyclicality of leverage amongst the five large U.S. investment banks leading up to the most acute phase of the financial crisis in September 2008, is for the supply side of the credit markets — lenders. The evidence I present is for the demand side of credit markets — (corporate) borrowers. Hence there is no inconsistency between these findings and Adrian and Shin (2008). In fact, my finding of moderate countercyclicality of non-financial sector leverage is consistent with the one piece of evidence Adrian and Shin (2008) document for non-financial firms: their Figure 2.3 also displays mild countercyclicality of non-financial sector leverage (although note that their notions of cyclicity are with respect to market asset values, rather than with respect to GDP). See Mimir (2010) for a standard business cycle accounting of financial-sector balance-sheet conditions.

<sup>27</sup>There are two data definitional issues that seem to prevent such blurring from occurring in the measures reported here. First, debt and equity are both defined on a per-revenue basis, for stationarity reasons described above. Second, it is the stocks of each, rather than the flows of issuance of each, being examined. Examining stocks is appropriate given the nature of the financial sector of the model presented in Section 4.

## 4 Model

As described in the introduction, the model is based on the agency-cost frameworks of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999). The model is most directly based on the “output model” of Carlstrom and Fuerst (1998), in which all prices are flexible, a homogenous final good is used for both consumption and investment purposes, firms require short-term working capital (formally, intraperiod) to finance their production costs, and there are no other rigidities or frictions whatsoever. This provides the cleanest model to evaluate the role of empirically-relevant shocks to firm risk, so I refer to the Carlstrom and Fuerst (1998) — henceforth, CF — output model as “the” underlying model, recognizing that it is meant to capture an entire literature of work. In a study with a very similar motivation, Dorofeenko, Lee, and Salyer (2008) study the role of risk shocks in the Carlstrom and Fuerst (1997) “investment model,” in which it is only capital-goods producers that are subject to financing constraints. Besides this difference in the applicability of agency frictions, Dorofeenko, Lee, and Salyer (2008) parameterize the risk process in an illustrative way, rather than calibrating it to micro data as I do.

As an aid to the ensuing description of the model, Figure 8 illustrates the timing of events in the model. Because the model is virtually identical to the CF output model, with only a couple of modifications made to align the model with the data analysis in Sections 2 and 3, readers familiar with the CF model may prefer to skip to the analysis beginning in Section 5.

### 4.1 Households

A representative household maximizes expected lifetime discounted utility over streams of consumption  $c_t$  and labor  $n_t$ ,

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(n_t)], \quad (3)$$

subject to the sequence of flow budget constraints

$$c_t + k_{ht+1} = w_t n_t + k_{ht} [1 + r_t - \delta] + \Pi_t. \quad (4)$$

The functions  $u(\cdot)$  and  $v(\cdot)$  are standard strictly-increasing and strictly-concave subutility functions over consumption and labor, respectively. The rest of the notation is as follows. The household’s subjective discount factor is  $\beta \in (0, 1)$ ,  $k_{ht}$  denotes the household’s capital holdings at the start of period  $t$ ,  $w_t$  is the real wage that is taken as given,  $r_t$  is the market rental rate on capital that is also taken as given, and  $\delta$  is the per-period depreciation rate of capital. The capital good and consumption good are identical and thus have a unit relative price. The household also receives aggregate dividend payments  $\Pi_t$  from firms as lump-sum income, the determination of which is

described below.<sup>28</sup>

Emerging from household optimization is a completely standard labor supply condition

$$-\frac{v'(n_t)}{u'(c_t)} = w_t, \quad (5)$$

and a standard capital Euler condition

$$u'(c_t) = \beta E_t \{u'(c_{t+1}) [1 + r_{t+1} - \delta]\}, \quad (6)$$

which follow as usual from the household's period- $t$  first-order conditions with respect to  $c_t$ ,  $n_t$ , and  $k_{ht+1}$ . The one-period-ahead stochastic discount factor is defined as  $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$ , with which firms, in equilibrium, discount profit flows.

## 4.2 Firms

There is a continuum of unit mass of firms, each of which produces output by operating a constant-returns technology. Firms are heterogenous in their productivity. Firm  $i$  produces output using the technology  $\omega_{it}F(k_{it}, n_{it})$ :  $k_{it}$  is the firm's purchase of physical capital on spot markets,  $n_{it}$  is the firm's hiring of labor on spot markets, and  $\omega_{it}$  is a firm-specific productivity realization.

Each period, firm  $i$ 's idiosyncratic productivity is a draw from a distribution with cumulative distribution function  $\Phi(\omega)$ , which has a time-varying mean  $\omega_{mt}$ , a time-varying standard deviation  $\sigma_t^\omega$ , and associated density function  $\phi(\omega)$ , all of which are identical across firms. Time-variation in  $\omega_{mt}$  corresponds to the usual notion of aggregate productivity shocks, in the sense of exogenous variation in the mean of firms' technology. The time-varying volatility  $\sigma_t^\omega$  is the key innovation in the model compared to CF. Given the first and second moments  $\omega_{mt}$  and  $\sigma_t^\omega$  common across firms, idiosyncratic productivity for a given firm is i.i.d. over time, an assumption made for tractability.<sup>29</sup>

---

<sup>28</sup>I could also introduce shares in order to directly price streams of dividends paid by firms to households; but this extra detail is unnecessary for the main points, so it is omitted.

<sup>29</sup>The assumption of zero persistence of the idiosyncratic component of a firm's productivity was noted in Section 2, and it greatly simplifies the computation of the model because the firm sector essentially can be analyzed as a representative agent. This point is discussed further below when I come to the aggregation of the model. This simplification still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to large fluctuations in aggregate leverage and possibly, in turn, to fluctuations in economic activity. In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the CF and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the simple debt contracts of these models could not be improved upon by the contracting parties by, say, multi-period contracts. Sidestepping this issue is yet another reason to assume no persistence in realized idiosyncratic productivity. Note, however, that assuming persistence in shocks to  $\sigma_t^\omega$ , as the empirical results in Section 2 indicate, does not pose any of these problems; indeed, shocks to  $\sigma_t^\omega$  really are aggregate shocks.



Firms are owned by households, and the objective of firms is to maximize the expected present discounted value of dividends remitted to households. Denote by  $\Pi_{it}$  the dividend payment made by firm  $i$  to households. For descriptive convenience, I decompose  $\Pi_{it}$  into a “non-retained earnings” component  $\Pi_{it}^e$  and an “expected operating profit” component  $E_\omega \Pi_{it}^f$ ; the notation  $E_\omega$  indicates an expectation conditional on the period- $t$  aggregate state but before idiosyncratic realizations are revealed to any firm.<sup>30</sup> Thus,  $\Pi_{it} \equiv \Pi_{it}^e + E_\omega \Pi_{it}^f$ . As described below, the component  $E_\omega \Pi_{it}^f$  essentially corresponds to static profits as in a simple RBC model.

Because they are owned by households, firms apply the representative household’s stochastic discount factor (the one-period-ahead discount factor is  $\Xi_{t+1|t}$ , as defined above) to their intertemporal optimization problem. However, firms are also assumed to be “more impatient” than households by the factor  $\gamma < 1$ , which can be thought of as an principal-agent problem that prevents perfect alignment of the firms’ objectives with households’ intertemporal preferences. At a technical level,  $\gamma < 1$  ensures that firms cannot accumulate enough assets to become self-financing, which would render irrelevant the financial frictions described below. This device for avoiding self-financing outcomes is common in models of financial frictions.

The intertemporal objective function of firm  $i$  is thus

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left[ \Pi_{it}^e + E_\omega \Pi_{it}^f \right]. \quad (7)$$

The firm problem is now further developed and analyzed.

#### 4.2.1 Firm Financing and Contractual Arrangement

This subsection describes the financial arrangements of the model, which I refer to below as the “partial equilibrium” financial sector of the model.<sup>31</sup> To facilitate comparing the general equilibrium to the partial equilibrium results below, the following intuitive descriptions of state variables is helpful. From a general equilibrium perspective, financial outcomes are contingent on the aggregate state  $(\omega_{mt}, \sigma_t^\omega)$  of the economy. From a partial equilibrium perspective, financial outcomes also take as given net worth  $nw_{it}$  and the markup  $p_t$ , each of which is determined in other markets (which in turn are contingent on the aggregate state  $(\omega_{mt}, \sigma_t^\omega)$ ).

In period  $t$ , total operating costs of firm  $i$ , which are the sum of capital rental costs and wage payments, are

$$M_{it} = w_t n_{it} + r_t k_{it}. \quad (8)$$

---

<sup>30</sup>As Figure 8 indicates, firm decisions are made in the first “subperiod” of period  $t$ , before idiosyncratic shocks have been realized but after aggregate shocks have been realized, hence the need for  $E_\omega$ .

<sup>31</sup>Especially in the quantitative analysis in Section 5 that parses results into partial vs. general equilibrium.

As in CF and as shown in Figure 8, the firm is assumed to commit to all of its input costs after observing the aggregate exogenous state  $(\omega_{mt}, \sigma_t^\omega)$ , but before observing its idiosyncratic realization  $\omega_{it}$  and thus before any output or revenue are created.

Part of the financing of the firm’s costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each firm accumulates is rented on spot markets to (other) firms, just like households rent their capital on spot markets. Firm  $i$ ’s capital holdings at the start of period  $t$  are  $k_{it}^e$ . Thus, note that  $k_{it}^e$ , which reflects the firm’s *savings* decisions, is distinct from  $k_{it}$ , which reflects the firm’s *capital demand* decisions for production purposes.

However, the firm’s internal funds (which I refer to interchangeably as its net worth or its equity) are insufficient to cover all input costs. To finance the remainder, a firm borrows short-term — formally, intraperiod — working capital. A firm requires external financing because of the assumption that it is more impatient than households, as described above.<sup>32</sup> By acquiring external funds, the firm is able to leverage its net worth in period  $t$ ,

$$nw_{it} = k_{it}^e [1 + r_t - \delta] + e_t, \quad (9)$$

into coverage of its operating costs  $M_{it}$ . Total borrowing by the firm is thus  $M_{it} - nw_{it}$ . The component  $e_t$  of net worth is a small amount of “endowment income” that each firm receives to ensure its continued operations in the event that it was unable to repay its debt and thus had to undergo costly reorganization in the previous period. In closing the model, this endowment is absorbed into the payout  $\Pi_{it}$  the firm pays to its owners, which is the representative household. The payout  $\Pi_{it}$  is thus interpreted as net of the endowment  $e_t$ .<sup>33</sup>

I describe only briefly the outcome of the contracting arrangement between borrowers (firms) and lenders (households) because it is standard in this class of models.<sup>34</sup> The financial contract is a debt contract, which is fully characterized by a reorganization threshold  $\bar{\omega}_t$  and a loan size  $M_{it} - nw_{it}$ . A firm must be “reorganized” if its realized productivity  $\omega_{it}$  is below the contractually specified threshold  $\bar{\omega}_t$ . Below this threshold, the firm does not have enough resources to fully repay its loan. In that case, the firm is declared insolvent and receives nothing, while the lender must pay

---

<sup>32</sup>As noted above, this is a standard assumption in this class of models and avoids the self-financing outcome. See, for example, Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999).

<sup>33</sup>Thus, equivalently,  $e_t$  can be interpreted as a lump-sum transfer of “startup funds” provided by households to firms, as in Gertler and Karadi (2011). By allowing a “firm’s” operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the “costs of bankruptcy” in the model are more properly interpreted as “costs of reorganization” without any disruption of its output-producing activities (i.e., bringing in new management to oversee ongoing operations).

<sup>34</sup>In the context of general equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and Faia and Monacelli (2007). In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).

reorganization costs that are proportional to the total output of the firm and receives, net of these reorganization costs, all of the output of the firm. Note that all firms, regardless of whether or not they end up requiring reorganization, do produce output up to their full (idiosyncratic) capacity.

Define by  $f(\bar{\omega}_t)$  the expected share of idiosyncratic output  $\omega_{it}F(k_{it}, n_{it})$  the borrower (the firm) keeps after repaying the loan, and by  $g(\bar{\omega}_t)$  the expected share received by the lender.<sup>35</sup> These expectations are conditional on the realization of the time- $t$  aggregate state, but before revelation of a firm's idiosyncratic productivity  $\omega_{it}$ . The contractually-specified loan size is characterized by a zero-profit condition on the part of lenders,

$$M_{it} = \frac{nw_{it}}{1 - p_t g(\bar{\omega}_t)}, \quad (10)$$

and the contractually-specified liquidation threshold is characterized by

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \quad (11)$$

in which  $p_t > 1$  is a “markup” on input costs that arises solely from the external financing needs of the firm.<sup>36</sup> Thus, for each unit of capital the firm rents, the cost, inclusive of financing costs, is  $p_t r_t$ , not just  $r_t$ . The same holds for payment of labor.

The loan size  $M_{it} - nw_{it}$  is firm-specific. However, the reorganization threshold  $\bar{\omega}_t$  is not because idiosyncratic productivity has zero persistence. Condition (11) thus implies  $p_t$  is also identical across firms, which is the key result that makes aggregation in the model simple, which justifies omission of firm- $i$  indexes for the variables  $p$  and  $\bar{\omega}$ .

CF interpret  $p_t$  as a “markup” that drives a wedge between factor prices and marginal products, which the analysis below shows. Another informative interpretation of  $p_t$  is as an external finance premium. For every unit of cost firms incur for their inputs, they must pay  $p > 1$  units inclusive of borrowing costs. Thus,  $p$  naturally has an interpretation as an external finance premium.

#### 4.2.2 Operating Profits and Asset Evolution

Firms take as given contractual outcomes when maximizing profits. The *expected* operating profit of firm  $i$  in period  $t$  is

$$E_{\omega} \Pi_{it}^f = \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}]. \quad (12)$$

<sup>35</sup>Formally,  $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_i - \bar{\omega}_t) \phi(\omega_i) d\omega_i = \int_{\bar{\omega}_t}^{\infty} \omega_i \phi(\omega_i) d\omega_i - [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t$  is the share received by the firm, and  $g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} (\omega_i - \mu) \phi(\omega_i) d\omega_i + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t \phi(\omega_i) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t - \mu \Phi(\bar{\omega}_t)$  is the share received by the lender.

<sup>36</sup>The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into lending is costless. Formally, the two conditions characterizing the optimal contract result from maximizing (the firm's share of) the return on the financial contract (because the firm, if it remains solvent, is the residual claimant on output),  $p_t f(\bar{\omega}_t) M_{it}$ , subject to the zero profit condition of the lender,  $p_t g(\bar{\omega}_t) M_{it} = M_{it} - nw_{it}$ .

As discussed above, this is an *expected* profit because it is measured before the realization of firm-specific idiosyncratic productivity but after the realization of the aggregate period- $t$  state of the economy,  $(\omega_{mt}, \sigma_t^\omega)$ . Because the mean of  $\omega_{it}$  is  $\omega_{mt}$ , ex-ante revenue of the firm is  $\omega_{mt}F(k_{it}, n_{it})$ . The idiosyncratic risk  $\omega_{it}$  and associated financing costs implied by it are captured by the inclusion of the external finance premium  $p_t$  in the above expression.<sup>37</sup> Firms take as given the competitively-determined factor prices  $w_t$  and  $r_t$ .

Regarding the dynamic aspect of firms, firm  $i$  begins period  $t$  with assets  $k_{it}^e$ , whose beginning-of-period- $t$  market value determines the firm's net worth  $nw_{it}$ , as shown in (9). The firm borrows  $M_{it} - nw_{it}$  against the value of these assets, and it expects to keep  $p_t f(\omega_t)M_{it}$  after repaying its loan.<sup>38</sup> Of these "excess" resources, the firm can either accumulate assets or make payments to households. That is,

$$\Pi_{it}^e + k_{it+1}^e = p_t f(\bar{\omega}_t) M_{it}, \quad (13)$$

which highlights that  $k_{it+1}^e$  can be thought of as retained earnings. Substituting the contractually-specified quantity of borrowing,  $M = \frac{nw}{1-p_t g(\bar{\omega})}$ , this can be re-written as

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} nw_{it}. \quad (14)$$

Further substituting the definition of net worth from (9), the firm's asset evolution is described by

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_{it}^e [1 + r_t - \delta] + e_t). \quad (15)$$

Finally substituting (12) and (15) into (7), the dynamic profit function of the firm is

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left\{ \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_{it}^e [1 + r_t - \delta] + e_t) - k_{it+1}^e + \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}] \right\}. \quad (16)$$

### 4.2.3 Profit Maximization

Maximization of (16) with respect to capital rental  $k_{it}$  and labor hiring  $n_{it}$  gives rise to the capital demand condition

$$r_t = \frac{\omega_{mt} F_k(k_{it}, n_{it})}{p_t} \quad (17)$$

and the labor demand condition

$$w_t = \frac{\omega_{mt} F_n(k_{it}, n_{it})}{p_t}. \quad (18)$$

---

<sup>37</sup>As is common in macro models, writing, for example,  $p_t$ , is shorthand for the state-contingent equilibrium function  $p(\omega_{mt}, \sigma_t^\omega)$ . If the distribution of  $\omega$  were degenerate — that is, if there were no idiosyncratic component of technology — then we would have  $p_t = 1 \forall t$ , which simply has the interpretation that financing issues are irrelevant as in, say, a baseline RBC model.

<sup>38</sup>This is because, as noted in footnote 28, the firm keeps the entire (expected) surplus from the contractual arrangement. Hence, in expectation, the firm is left with  $p_t f(\omega_t)M_{it}$  after the sequence of borrowing, renting factors of production, producing output, and repaying its loan.

In (17) and (18), the effective payments per unit of each factor are  $p_t r_t$  for capital rental and  $p_t w_t$  for labor, reflecting firms' need for external financing. Financing costs drive an endogenous time-varying wedge between prices and marginal returns in factor markets, which, as noted above, leads CF to refer to  $p_t$  as a "markup." As discussed above,  $p_t$  can also usefully be interpreted as the model's external finance premium. That the external finance premium drives an endogenous time-varying wedge between prices and marginal returns in neoclassical factor markets is a key feature of the model. Note that, although firms may differ in their levels of factor usage, each firm chooses an identical capital-labor ratio because the market prices  $r_t$  and  $w_t$  and the external premium  $p_t$  are identical for all firms and the production technology  $F(\cdot)$  is constant-returns.

Maximization of (16) with respect to asset accumulation  $k_{it+1}^e$  yields the capital Euler equation for firms,

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} [1 + r_{t+1} - \delta] \right\}, \quad (19)$$

which, note, is independent of firm- $i$  conditions.

#### 4.2.4 Aggregation

Firms are heterogenous with respect to their net worth and differ (only) in size — a firm with a larger net worth receives a proportionately larger loan and so produces more output. However, the size distribution of firms is irrelevant for computing prices and hence aggregates in the economy, which makes the agency-cost framework tractable in a DSGE setting. This irrelevance is also the justification for examining only financial aggregates in Section 3. The production side of the economy can thus be analyzed as if there were a representative firm that held the average quantity of net worth and hired the average quantity of labor and capital for production. The specific assumptions and results behind this aggregation result are: the constant-returns nature of the production function  $F(\cdot)$ ; the linearity of the monitoring technology (in the quantity monitored); and, crucially, the result that the prices  $w_t$ ,  $r_t$ , and  $p_t$  are identical for all firms.<sup>39</sup>

The stand-in representative firm has a profit function identical to (16) (with firm indices dropped), which clearly gives rise to the same optimality conditions (17), (18), and (19). The (aggregate) profits that get transferred to households are thus

$$\begin{aligned} \Pi_t &= \Pi_t^e + \Pi_t^f = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_t^e [1 + r_t - \delta] + e_t) - k_{t+1}^e + \omega_{mt} F(k_t, n_t) - p_t [w_t n_t + r_t k_t] \\ &= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_t^e [1 + r_t - \delta] + e_t) - k_{t+1}^e + \omega_{mt} F(k_t, n_t) - \omega_{mt} F_n(k_t, n_t) n_t - \omega_{mt} F_k(k_t, n_t) k_t \end{aligned}$$

---

<sup>39</sup>The result that  $p$  is identical for all firms is an implication of zero persistence of firms' idiosyncratic productivity, which, as described above, makes it impossible to condition the contractually-specified liquidation threshold  $\bar{\omega}$  on firm-specific variables. See also CF (1997, 1998) for further discussion. The result that  $w$  and  $r$  are identical for all firms follows simply from the assumption of perfectly-competitive rental markets for labor input and capital input.

$$= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_t^e [1 + r_t - \delta] + e_t) - k_{t+1}^e. \quad (20)$$

The second line makes use of the factor price conditions (17) and (18), and the third line follows because  $F(\cdot)$  is constant-returns. Thus, note that in this representative-firm foundation of aggregates, firms earn zero aggregate operating profits, so  $\Pi_t = \Pi_t^e$ . The capital Euler equation that arises from maximizing this representative-firm profit function with respect to aggregate entrepreneurial capital holdings  $k_{t+1}^e$  is clearly identical to (19).

Finally, the aggregate resource constraint of the economy is

$$c_t + k_{t+1} - (1 - \delta)k_t = \omega_{mt} F(k_t, n_t) [1 - \mu \Phi(\omega_t)], \quad (21)$$

in which  $k_t = k_{ht} + k_t^e$  is the equilibrium quantity of physical capital at the beginning of period  $t$ . Note that aggregate monitoring costs are a final use of output.

### 4.3 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes  $\{c_t, n_t, k_{ht+1}, k_{t+1}^e, k_{t+1}, \Pi_t^e, w_t, r_t, p_t, \bar{\omega}_t\}$  that satisfy the following conditions: the labor-supply condition

$$-\frac{v'(n_t)}{u'(c_t)} = w_t; \quad (22)$$

the labor-demand condition

$$w_t = \frac{\omega_{mt} F_n(k_t, n_t)}{p_t}; \quad (23)$$

the capital-demand condition

$$r_t = \frac{\omega_{mt} F_k(k_t, n_t)}{p_t}; \quad (24)$$

the representative household's Euler equation for capital holdings

$$1 = E_t \left\{ \Xi_{t+1|t} [1 + r_{t+1} - \delta] \right\}; \quad (25)$$

the (representative) firm's Euler equation for capital holdings

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} [1 + r_{t+1} - \delta] \right\}; \quad (26)$$

aggregate capital market clearing

$$k_t = k_{ht} + k_t^e; \quad (27)$$

the aggregate resource constraint

$$c_t + k_{t+1} - (1 - \delta)k_t = \omega_{mt} F(k_t, n_t) [1 - \mu \Phi(\bar{\omega}_t)]; \quad (28)$$

the contractually-specified loan size

$$M_t = \frac{nw_t}{1 - p_t g(\bar{\omega}_t)}, \quad (29)$$

in which expression (9) for  $nw_t$  is substituted in; the contractually-specified liquidation threshold

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \quad (30)$$

and the evolution of the aggregate assets of firms (equivalently, the assets of the representative firm)

$$\Pi_t^e + k_{t+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_t^e [1 + r_t - \delta] + e_t). \quad (31)$$

The private sector takes as given the stochastic process for  $\{\omega_{mt}, \sigma_t^\omega\}_{t=0}^\infty$ . To emphasize, and as noted above, conditions (29) and (30) characterize the partial-equilibrium financial outcomes, and hence can be viewed (in partial equilibrium) as taking  $p_t$  and net worth as given.

## 5 Quantitative Analysis

### 5.1 Computational Strategy

Changes in cross-sectional risk are aggregate, not idiosyncratic, shocks in the model economy. Because I track only aggregate outcomes and do not track any firm-specific outcomes, there is no reason to think that local approximation methods will misrepresent the model's aggregate dynamics.<sup>40</sup> To study the dynamics of the model, I nonetheless compute a second-order, rather than a first-order, approximation of the equilibrium.<sup>41</sup> Because the main interest is in business cycle fluctuations, such methods are likely to accurately portray the model's dynamic behavior, as the studies by Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) and Caldera, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2009) suggest. However, it is useful to note that the results reported below are virtually identical to those obtained from a linear approximation given the purely aggregate nature of shocks. This reinforces the point made by Dorofeenko, Lee, and Salyer (2008, p. 386) that linearization does not impose certainty equivalence on this type of second-moment (a cross-sectional variance) shock. The quantitative results reported below are thus fundamentally driven by the model's mechanism — changes in cross-sectional risk leading to changes in firms' leverage, which then potentially are transmitted to the real economy — rather than choice of approximation method.

---

<sup>40</sup>Recall the discussion above that, given the maintained assumptions of the model, aggregates in the model do not depend on distributions of outcomes at the firm level.

<sup>41</sup>The numerical algorithm is my own implementation of the perturbation method described by Schmitt-Grohe and Uribe (2004).

Before presenting the dynamic results, I complete the description of the calibration of the model (which was begun in Section 2) and briefly describe some of its long-run predictions.

## 5.2 Calibration

The novel aspect of the model calibration is the risk shock process using micro data, as described in Section 2. As described there, long-run dispersion of firm productivity is  $\bar{\sigma}^\omega = 0.156$ . This is about half the value used by CF (1998, p. 590) and Bernanke, Gertler, and Gilchrist (1999, p. 1368), which are calibrated to aggregate financial data, not firm-level data: the former set  $\bar{\sigma}^\omega = 0.37$ , and the latter set  $\bar{\sigma}^\omega = 0.28$ . Thus, direct micro evidence indicates less cross-sectional dispersion than standard macro calibrations of agency-cost models.

As also discussed in Section 2, I assume sufficient smoothness in the average productivity and risk processes so that I can set quarterly persistence parameters  $\rho_{\omega_m} = 0.83$  and  $\rho_{\sigma^\omega} = 0.83$ , even though the data on which the estimation is based are annual. This mismatch between (desired) model frequency and empirical frequency raises the question of the appropriate calibration of the standard errors of the quarterly innovations in the productivity and risk processes.<sup>42</sup> Given the quarterly frequency of the model and the annual frequency of the productivity data, I simply time aggregate the simulated data from the model, and set parameters  $\sigma_{\omega_m}$  and  $\sigma_{\sigma^\omega}$  so that the annualized volatilities of average productivity and dispersion of productivity in the model match their annual empirical counterparts. As documented in Section 2, the empirical volatilities are, respectively, 1.26 percent and 3.15 percent. This calibration procedure leads to  $\sigma_{\omega_m} = 0.008$  and  $\sigma_{\sigma^\omega} = 0.0033$ .<sup>43</sup>

Besides the calibration of the exogenous processes, Table 5 lists all functional forms used in the quantitative experiments, and Table 6 lists all baseline parameter settings. The preference and production parameters are standard in business cycle models. The Frisch labor supply elasticity is set fairly high,  $\varepsilon = 4$ , which is conventional in DSGE models. The agency cost parameter is set to  $\mu = 0.15$ , which is the same as the calibrated value in Covas and den Haan (2006) and in line with the estimate  $\mu = 0.12$  by Levin, Natalucci, and Zakrajsek (2004). The value for firms' "additional" discount factor is set to  $\gamma = 0.99$ , which allows the model to match a long-run annualized external finance premium of two percent. This value of  $\gamma$  is larger than the calibrations of CF and BGG (meaning firms appear "less impatient") and seems due to the lower calibrated value of  $\bar{\sigma}^\omega$  here.

<sup>42</sup>Recall from Section 2 that the point estimates for annual persistence are  $\rho_{\omega_m} = 0.47$  and  $\rho_{\sigma^\omega} = 0.48$ , and the standard deviation of the annual innovations in the average productivity and risk processes are, respectively, 0.0111 and 0.0276.

<sup>43</sup>It is interesting to note that  $\sigma_{\omega_m} = 0.008$  is quite similar to the calibration of the size of quarterly innovations in the aggregate productivity process in a baseline RBC model, in which a benchmark value is 0.007. Here, of course,  $\sigma_{\omega_m} = 0.008$  is computed directly from micro data.



### 5.3 Long-Run Dispersion and Long-Run Equilibrium

The long-run deterministic (steady-state) equilibrium is computed numerically using a standard nonlinear equation solver. The main comparative static exercise is presented in Figure 9, which plots long-run equilibria as a function of long-run cross-sectional dispersion  $\bar{\sigma}^\omega$ . All other parameters are held fixed at those presented in Table 6.

Figure 9 shows that the long-run response of the economy to changes in  $\bar{\sigma}^\omega$  is non-monotonic. For low dispersion of idiosyncratic productivity, GDP falls as dispersion rises, but for high dispersion, the comparative static result reverses. The nonmonotonicity is also evident in the long-run behavior of the finance premium (lower right panel) as well as other standard aggregate quantities such as gross investment and consumption (for brevity, the latter are not shown in Figure 9). This effect is not due to any nonmonotonicity of the contract terms, as debt (upper middle panel) is strictly decreasing in  $\bar{\sigma}^\omega$ , and the bankruptcy threshold  $\bar{\omega}$  (not shown) and hence bankruptcies (lower middle panel) are strictly increasing in  $\bar{\sigma}^\omega$ . The financial results are intuitive. When  $\sigma_t^\omega$  is allowed to fluctuate around the long-run dispersion  $\bar{\sigma}^\omega = 0.156$  during dynamic simulations of the model, dispersion never reaches as high as 0.40, hence the model's dynamics do not cover the inflection point Figure 9 reveals.<sup>44</sup> I leave to future investigation further study of the nonmonotonicity.

For the baseline calibration, the model's long-run leverage ratio is 1.8, which is larger than the leverage ratio at any point during the period 1974-2009, as comparison with Figure 5 shows. In the model, the conceptually most important determinant of long-run leverage is long-run dispersion,  $\bar{\sigma}^\omega$ . As dispersion shrinks to zero, which means that lenders face no risk whatsoever on their loans, leverage grows unboundedly, independent of all other parameter values. This effect is shown in the lower left panel of Figure 9.<sup>45</sup> Apparently, the empirically-relevant  $\bar{\sigma}^\omega = 0.156$  is small enough steady-state dispersion that the model overpredicts long-run leverage. To force the model to explain a long-run leverage ratio of, say, unity, requires  $\bar{\sigma}^\omega = 0.24$ , given the rest of the parameters. Indeed,  $\bar{\sigma}^\omega = 0.24$  is closer to typical macro calibrations of this class of models, such as CF and BGG. However, the overprediction of long-run leverage here is not a shortcoming of the analysis. Instead of treating  $\bar{\sigma}^\omega$  as a free parameter to match aggregate moments, as other agency-cost macro models do, it seems important to know that direct micro evidence on this parameter leads to perhaps substantially different long-run aggregate predictions.

It is useful to also highlight the long-run values implied by the model of two other financial variables of interest: the (annualized) finance premium and the bankruptcy rate. These are collected

---

<sup>44</sup>As Table 6 shows, the calibrated value of the standard error of the shocks to the dispersion process is  $\sigma_{\sigma^\omega} = 0.0027$ , which is sufficiently small that during simulations,  $\sigma_t^\omega = 0.40$  was never reached.

<sup>45</sup>That is, as  $\bar{\sigma}^\omega \rightarrow 0$ , lenders are willing to lend ever larger quantities. Alternatively, one could say that leverage is undefined because financial frictions do not matter and the model technically pins down neither loan amounts nor leverage.

in Table 7. The long-run bankruptcy rate is substantially lower than in the Dun & Bradstreet evidence cited by CF (1998, p. 590), while the finance premium is in line with most of the measures of premia presented in DeGraeve (2008).<sup>46</sup> The former result is again a reflection of a relatively low level of long-run risk, while the latter is the calibration target at which  $\gamma$  was aimed.

## 5.4 Business Cycle Dynamics

The presentation of the baseline model’s cyclical dynamics is divided in three parts. First, I document how the model responds to only risk shocks, with average productivity held constant at  $\omega_m = 1$ . Second, shocks to both average productivity and risk are assumed to simultaneously drive the economy. Finally, the effects of just risk shocks in the partial equilibrium financial phase of the model are considered. The partial equilibrium results show that, conditional on shocks emanating in the financial sector of the model, there is very little spillover *from* financial fluctuations *to* real fluctuations. Supporting analysis appears in Appendix B, which documents how macro and financial aggregates respond to only average productivity shocks, with cross-sectional dispersion of productivity held constant at  $\bar{\sigma}^\omega$ ; these experiments can be directly compared to CF’s experiments. In all simulation experiments, 1000 simulations are conducted, each 200 periods long. For each simulation, first and second moments are computed, a standard HP filter is applied (smoothing parameter 1,600), and the medians of relevant moments across the 1000 simulations are reported.

### 5.4.1 Risk Shocks in General Equilibrium

The first set of experiments conducted in the full general equilibrium model is dynamics driven by pure risk shocks. Figure 10 presents impulse responses to a one-time, one-standard deviation positive shock to the cross-sectional dispersion of firm productivity, holding constant average productivity. Firm-level productivity thus has larger idiosyncratic risk. Complementing this impulse-response analysis are the simulated business cycle statistics reported in Table 8. There are two main results from these experiments with pure risk shock .

First, as Figure 10 shows, a pure risk shock induces virtually no GDP response: the peak response of GDP is 1/200th of one percent! Thus, empirically-relevant risk shocks play virtually no role as an independent driver of aggregate macro fluctuations when mediated through a typical

---

<sup>46</sup>As discussed extensively by DeGraeve (2008), it is not clear what is the most relevant empirical counterpart to the model’s external finance premium. Many natural alternatives suggest themselves, such as the difference between the prime borrowing rate and the short-term T-bill rate, the interest spread between AAA-rated commercial paper and T-bills, the spread between BBB-commercial paper and T-bills, and so on. DeGraeve (2008) documents that these various empirical measures of “the external finance premium” behave differently enough over the business cycle that it remains an open question what the natural empirical counterpart of the model’s external finance premium is.

agency-cost friction.<sup>47</sup> The small pass-through of risk shocks to macro fluctuations arises despite the fact that the risk innovations documented in Section 2 are larger than found in other micro-level evidence. This negative result is also in line with the findings of Dorofeenko, Lee, and Salyer (2008).

However, Figure 10 also shows that financial variables do react extremely strongly to a risk shock: debt and hence leverage fall (sharply) in response to an increase in firm risk as lenders extend less credit. Bankruptcies simultaneously jump up sharply, although quickly recede back towards steady state. Viewed through the agency-cost lens, risk fluctuations clearly matter a great deal for the magnitude of financial fluctuations.

The quantitative power of pure risk shocks on financial outcomes is also starkly revealed by the simulation-based results in Table 8. Although smaller than in the data presented in Table 1, the cyclical volatilities of leverage, debt, and equity are at least at the same order of magnitude as the data. However, the virtual lack of GDP fluctuations renders relative volatilities uninterpretable. As shown next, this model prediction can be exploited to disentangle how risk shocks operate in the model.<sup>48</sup>

---

<sup>47</sup>This is also one of the main messages of the theoretical model of BB, even though their model does not situate financial frictions as part of the transmission channel for risk shocks. Examining just the role of financial frictions in the transmission channel leads to a broadly similar conclusion as BB. The result here is even starker than in BB, though, because I found innovations in firm risk to be five to ten times larger than found by BB, as discussed in Section 2. However, as described next, the aggregate financial fluctuations induced by risk shocks in the model are empirically meaningful, hence the overall conclusions are not as pessimistic.

<sup>48</sup>Although this is not the preferred interpretation of the results, if one did want to interpret the small real-side movements in the model as meaningful, the seemingly counterintuitive and clearly counterfactual result is that an increase in cross-sectional dispersion induces an *increase* in GDP. As shown in Figure 2 and as also documented by BB and BFJ, firm-level dispersion is clearly countercyclical in the data. This counterfactual result seems to arise in the model due to the “Hartman-Abel effect,” which also arises in the simplest version of the BFJ model that features a minimum of adjustment costs for capital and labor. The idea, as described by BFJ (p. 20), is that absent sufficient adjustment costs, a higher variance of productivity increases output because marginal revenue products are convex in productivity. While I do not model “adjustment costs” in the way the firm-level literature typically does, the entire agency cost/financial friction mechanism can be viewed broadly as a type of “adjustment cost.” However, it apparently is not strong enough to overturn the Hartman-Abel effect. Appendix C considers a modified version of the model, in which average productivity and risk fluctuations have non-zero correlation, that by construction delivers countercyclical firm risk. The leverage volatility result in this baseline model, though, carries over to the modified model, hence it is useful to understand how the baseline model works, both its successes and shortcomings. Nevertheless, one should take such model analysis with much caution given the extremely small magnitude of quantity fluctuations induced by risk shocks.

### 5.4.2 Risk Shocks in Partial Equilibrium

To achieve this disentangling, consider the same quantitative analysis, but now restricted to just the variables endogenous to the “partial equilibrium” financial model. Specifically, consider how conditions (29) and (30) generate fluctuations of the threshold level of idiosyncratic productivity,  $\bar{\omega}_t$ , and the quantity of credit extended,  $M_t - nw_t$ , in response to risk shocks. In this partial equilibrium analysis, both net worth ( $nw_t$ ) and the external finance premium ( $p_t$ ) are assumed constant over time at their deterministic steady-state general equilibrium values from the full model. Important to emphasize is that, in partial equilibrium, conditions (29) and (30) are purely static (within-period) conditions. Because the financial contract is of within-period duration, isolating away general equilibrium effects means financial outcomes can be solved exactly via nonlinear numerical solutions, rather than their dynamics requiring approximation.<sup>49</sup>

Table 9 presents the exactly-computed risk-induced dynamics of leverage (which are isomorphic to the dynamics of credit  $M_t - nw_t$  in this partial equilibrium model) and the bankruptcy probability  $\Phi(\bar{\omega}_t)$  (an economically-meaningful representation of the dynamics of  $\bar{\omega}_t$ ). For comparison, the general equilibrium volatility of  $\Phi(\bar{\omega}_t)$  (not shown in Table 8) induced by pure risk shocks is 18 percent. As a further guide for referencing to the results of the full model, the general equilibrium dynamics for GDP, which does not fluctuate in partial equilibrium, are reproduced in Table 9.

Comparison of Table 9 with Table 8 shows that the amplitude of leverage fluctuations, measured as volatility relative to that of GDP, is similar to that in general equilibrium — extremely large! Its moderate positive autocorrelation is also similar to that in general equilibrium. These comparisons are especially stark because the general equilibrium results are computed using local approximation methods, rather than the exact solutions for the partial equilibrium results. They nonetheless generate qualitatively similar predictions: risk fluctuations, when calibrated to micro-level data, matter a lot for financial fluctuations, but, by themselves, not much for macro fluctuations.

### 5.4.3 Both Risk Shocks and Average Productivity Shocks

Based on the results so far, a reasonable view to consider is that fluctuations are due to *both* first-moment shocks *and* second-moment shocks. Assessing this fundamentally productivity-driven view of business cycles (albeit a richer view than in standard analysis) requires returning to the full general equilibrium model. Table 10 reports statistics when the model economy is hit by independent shocks to both average productivity and cross-firm dispersion.

Surprisingly, the combination of first-moment and second-moment shocks generates a relative volatility of leverage — 2.2 — virtually identical to the evidence presented in Table 1. This result is surprising because this particular second moment was not targeted by the selection of

---

<sup>49</sup>I thank Pablo D’Erasmus for pointing out the appropriateness of employing an exact solution.

any model parameters. The relative volatilities of debt and equity are only half as large in this joint-productivity version of the model than during the period 1974-1988. Nonetheless, given its parsimony, the basic agency-cost model generates financial fluctuations, conditional on shocks to a combination of average productivity and productivity dispersion, that are empirically relevant in both an absolute sense and, importantly, in a relative-to-GDP sense. A simultaneous battery of first- and second-moment shocks also enables the model to broadly reproduce basic business cycle stylized facts: the volatility of GDP is in line with data (1.88 percent in the model vs. 1.55 percent in the data), gross investment is four times as volatile as GDP, consumption is less volatile than GDP (although more dramatically so than in the data), and GDP, consumption, and investment are all highly persistent.

#### **5.4.4 Economic Interpretation**

Taken together, the preceding three sets of results lead to the following conclusions. If the primitive shock directly affects the financial sector, which is the natural view of how risk fluctuations affect the agency-cost model, it leads to miniscule real-side effects even though it generates quite empirically-meaningful financial market effects. In contrast, if the primitive shock directly affects the real side of the economy, which is a natural and widespread view of average productivity fluctuations, it leads to empirically-meaningful real quantity effects, but, interestingly, also leads to broadly empirically descriptive financial market effects (further quantitative analysis presented in Appendix B establishes this point even more rigorously).

To sum up, it is thus not just the connection per se between the macro and finance sides of an agency-cost model that matters for its predictions. The market location of the primitive shocks also matters quite critically. Overall, the results suggest a type of dichotomy present at the core of a widely-used class of DSGE financial accelerator models: pure risk shocks, in which average productivity does not change, lead to large financial fluctuations, but macro fluctuations are largely immune to them.

## **6 Conclusion**

This paper documented the business cycle properties of firm risk based on micro-level data, and of aggregate leverage, debt, and equity in the non-financial business sector. Firm risk is fairly volatile over the cycle and highly countercyclical; and, focusing on leverage, aggregate leverage is at least twice as volatile as GDP. Using a baseline quantitative financial accelerator model which is only two parameters removed from the frictionless RBC model, the main theoretical question was to assess the extent to which the former can explain the latter. Empirically-relevant risk shocks turn

out to explain quite well the observed volatility of leverage, and they also generate volatilities of the underlying debt and equity components that are also in the empirically-relevant range. However, in the model, the leverage fluctuations that risk shocks induce lead to only tiny fluctuations of real activity — GDP volatility conditional on risk shocks alone is less than one percent of GDP volatility conditional on shocks to average productivity alone.

To be clear, these results are about the transmission of risk fluctuations to aggregates through a model of financial frictions. Nonetheless, coupled with similar results by Dorofeenko, Lee, and Salyer (2008), they pose a challenge for DSGE agency-cost models in mediating the effects of such risk — or, more broadly, “financial” — shocks. The results of this paper show that, when calibrated in a way consistent with micro evidence, risk shocks have small effects on real activity, even though provide a good starting point for explaining financial fluctuations. An optimistic interpretation of these results is that they suggest that in richer agency-cost models that *do* find important transmission of risk shocks to real fluctuations, the linkages are not driven by the basic agency-cost friction per se, but rather by other features of the model that interact with the friction. Examples of this idea are Gourio (2011) and Gilchrist, Sim, and Zakrajsek (2011), which enrich the agency-cost model in interesting dimensions. Such model-based understanding of results is important as the literature on the joint modeling of financial and real dynamics expands.

More broadly, the results of this paper also optimistically reach an intermediate position in the active debate between Bloom, Floetotto, and Jaimovich (2010) and Bachmann and Bayer (2010) about the aggregate importance of risk shocks. The model analysis of the former predicts that risk fluctuations have large aggregate real-side consequences, while the analysis of the latter predicts that risk fluctuations have little aggregate real-side consequences. In terms of real-side consequences, the results of this paper are more in line with the latter, even though the calibration of micro-level risk shocks is more in line with the former. But this is perhaps a secondary point compared to the finding that risk shocks *do* generate very large model-based fluctuations in aggregate financial outcomes, from which both Bloom, Floetotto, and Jaimovich (2010) and Bachmann and Bayer (2010) abstract given the richness of other primitives of their models. One reading of these results is that risk shocks can indeed lead to large and empirically-relevant fluctuations in at least some aggregate outcomes. Conditional on this result, whether or how they transmit into large fluctuations of aggregate quantity outcomes is still an open issue.

Another broad idea that emerges is that understanding changes directly in the distribution of micro-level risk may be important for guiding the further development of business cycle models in which financial frictions are prominent. This paper has exploited second-moment disturbances. As noted by LNZ (2004, p. 33), fluctuations in third- or higher-order moments may also need to be considered for understanding some aspects of the financial data. This requires moving away from

the symmetry of normally-distributed (log) productivity standard in macro models. Given the robust evidence that firm-level outcomes are distributed non-normally, there seems reason to think that skewness and higher moments of the firm productivity distribution may be time-varying. Such “higher-moment shocks” would also be expected to affect leverage and so possibly real activity; the quantitative degree to which they do may be an interesting question.

## References

- ABEL, ANDREW B. 1983. "Investment Under Uncertainty." *American Economic Review*, Vol. 73, pp. 228-233.
- ADRIAN, TOBIAS AND HYUN SONG SHIN. 2008. "Liquidity and Leverage." *Journal of Financial Intermediation*. Forthcoming.
- ARUOBA, S. BORAGAN, JESUS FERNANDEZ-VILLAVERDE, AND JUAN F. RUBIO-RAMIREZ. 2006. "Comparing Solution Methods for Dynamic Equilibrium Economies." *Journal of Economic Dynamics and Control*, Vol. 30, pp. 2477-2508.
- BERNANKE, BEN S., JOHN Y. CAMPBELL, AND TONI M. WHITED. 1990. "U.S. Corporate Leverage: Developments in 1987 and 1988." *Brookings Papers on Economic Activity*, Vol. 1, pp. 255-286.
- BERNANKE, BEN S. AND MARK GERTLER. 1989. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*, Vol. 79, pp. 14-31.
- BERNANKE, BEN S., MARK GERTLER, AND SIMON GILCHRIST. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework. In *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, Vol. 1C. Elsevier.
- BACHMANN, RUEDIGER AND CHRISTIAN BAYER. 2010. "Uncertainty Business Cycles — Really?." University of Michigan.
- BUNDICK, BRENT AND SUSANTO BASU. 2011. "Uncertainty Shocks in a Model of Effective Demand." Mimeo, Boston College.
- BLOOM, NICHOLAS, MAX FLOETOTTO, AND NIR JAIMOVICH. 2010. "Really Uncertain Business Cycles." Stanford University.
- CALDERA, DARIO, JESUS FERNANDEZ-VILLAVERDE, JUAN F. RUBIO-RAMIREZ, AND WEN YAO. 2009. "Computing DSGE Models with Recursive Preferences." University of Pennsylvania.
- CARLSTROM, CHARLES T. AND TIMOTHY S. FUERST. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review*, Vol. 87, pp. 893-910.
- CARLSTROM, CHARLES T. AND TIMOTHY S. FUERST. 1998. "Agency Costs and Business Cycles." *Economic Theory*, Vol. 12, pp. 583-597.
- CARLSTROM, CHARLES T., TIMOTHY S. FUERST, AND MATTHIAS PAUSTIAN. 2009. "Optimal Monetary Policy in a Model with Agency Costs." Federal Reserve Bank of Cleveland.
- CHRISTIANO, LAWRENCE, ROBERTO MOTTO, AND MASSIMO ROSTAGNO. 2009. "Financial Factors in Economic Fluctuations." Northwestern University.
- COOLEY, THOMAS F. AND EDWARD C. PRESCOTT. 1995. "Economic Growth and Business Cycles. In *Frontiers of Business Cycle Research*, edited by Thomas F. Cooley. Princeton.



- COOPER, RUSSELL W. COOPER AND JOHN C. HALTIWANGER. 2006. "On the Nature of Capital Adjustment Costs." *Review of Economic Studies*, Vol. 73, pp. 611-633.
- COVAS, FRANCISCO AND WOUTER J. DEN HAAN. 2011. "The Cyclical Behavior of Debt and Equity Finance." *American Economic Review*, Vol. 101, pp. 877-899.
- DE GRAEVE, FERRE. 2008. "The External Finance Premium and the Macroeconomy: U.S. Post-WWII Evidence." *Journal of Economic Dynamics and Control*, Vol. 32, pp. 3415-3440.
- DOROFEEENKO, VICTOR, GABRIEL S. LEE, AND KEVIN D. SALYER. 2008. "Time-Varying Uncertainty and the Credit Channel." *Bulletin of Economic Research*, Vol. 60, pp. 375-403.
- FAIA, ESTER AND TOMASSO MONACELLI. 2007. "Optimal Interest Rate Rules, Asset Prices, and Credit Frictions." *Journal of Economic Dynamics and Control*, Vol. 31, pp. 3228-3254.
- FERNÁNDEZ-VILLAYERDE, JESÚS AND JUAN F. RUBIO-RAMIREZ. 2007. "How Structural are Structural Parameters?." NBER Macroeconomics Annual 2007.
- FERNÁNDEZ-VILLAYERDE, JESÚS, PABLO GUERRÓN-QUINTANA, JUAN F. RUBIO-RAMIREZ, AND MARTIN URIBE. 2010. "Risk Matters: The Real Effects of Volatility Shocks." *American Economic Review*. Forthcoming.
- GALE, DOUGLAS AND MARTIN HELLMIG. 1985. "Incentive-Compatible Debt Contracts: The One-Period Problem." *Review of Economics Studies*, Vol. 52, pp. 647-663.
- GEANAKOPOLOS, JOHN. 2009. "The Leverage Cycle." NBER Macroeconomics Annual 2009 (forthcoming).
- GERTLER, MARK AND PETER KARADI. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, Vol. 58, pp. 17-34.
- GILCHRIST, SIMON, JAE SIM, AND EGON ZAKRAJSEK. 2011. "Uncertainty, Credit Spreads, and Investment Dynamics." Mimeo, Boston University.
- GOMES, JOAO F., AMIR YARON, AND LU ZHANG. 2003. "Asset Prices and Business Cycles with Costly External Finance." *Review of Economic Dynamics*, Vol. 6, pp. 767-788.
- GOURIO, FRANCOIS. 2011. "Credit Risk and Disaster Risk." Mimeo, Boston University.
- HENNESSY, CHRISTOPHER A. AND TONI M. WHITED. 2007. "How Costly is External Financing? Evidence from a Structural Estimation." *Journal of Finance*, Vol. 52, pp. 1705-1745.
- JERMANN, URBAN AND VINCENZO QUADRINI. 2010. "Macroeconomic Effects of Financial Shocks." *American Economic Review*. Forthcoming.
- JUSTINIANO, ALEJANDRO AND GIORGIO E. PRIMICERI. 2008. "The Time-Varying Volatility of Macroeconomic Fluctuations." *American Economic Review*, Vol. 98, pp. 604-641.
- stdjournalref Korajczyk, Robert A. and Amnon Levy/ 2003/ Capital Structure Choice: Macroeconomic Conditions and Financial Constraints/ *Journal Financial Economics*/ 68/ 75-109.
- LEVIN, ANDREW T., FABIO M. NATALUCCI, AND EGON ZAKRAJSEK. 2004. "The Magnitude

- and Cyclical Behavior of Financial Market Frictions.” Finance and Discussion Series No. 70, Federal Reserve Board of Governors.
- LEVY, AMNON AND CHRISTOPHER A. HENNESSY. 2007. “Why Does Capital Structure Choice Vary with Macroeconomic Conditions.” *Journal of Monetary Economics*, Vol. 54, pp. 1545-1564.
- MIMIR, YASIN. 2010. “Financial Intermediaries, Leverage Ratios, and Business Cycles.” University of Maryland.
- SCHMITT-GROHE, STEPHANIE AND MARTIN URIBE. 2004. “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function.” *Journal of Economic Dynamics and Control*, Vol. 28, pp. 755-775.
- TOWNSEND, ROBERT M. 1979. “Optimal Contracts and Competitive Markets with Costly State Verification.” *Journal of Economic Theory*, Vol. 21, pp. 265-293.
- WILLIAMSON, STEPHEN D. 1987. “Financial Intermediation, Business Failures, and Real Business Cycles.” *Journal of Political Economy*, Vol. 95, pp. 1196-1216.

## A Basic Mechanism in Partial Equilibrium

The analysis here investigates a bit further the “partial equilibrium” financial model considered in Section 5, and presents some of the arguments in a different way. While fairly simple, it is useful to consider analytically the intuition behind the model’s main mechanism, which lies in the partial equilibrium financial contract of the model. The analysis here does not formally prove the main results, which are quantitative in nature. But they shed light on the transmission mechanism, which is quantified in Section 5.

To begin this intuitive consideration, note that conditions (29) and (30), which characterize the terms of the financial contract (and what I refer to as the financial “partial equilibrium” model), can be combined to

$$M - nw = - \left( \frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^\omega) g(\bar{\omega}; \sigma^\omega)}{f(\bar{\omega}; \sigma^\omega) g_{\bar{\omega}}(\bar{\omega}; \sigma^\omega)} \right) nw. \quad (32)$$

I drop time indices here for ease of notation. The term in parentheses is the leverage ratio because it expresses a firm’s total debt obligation,  $M - nw$ , as a multiple of its net worth (its equity). Thus, define the leverage ratio as

$$\ell(\bar{\omega}; \sigma^\omega) \equiv - \frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^\omega) g(\bar{\omega}; \sigma^\omega)}{f(\bar{\omega}; \sigma^\omega) g_{\bar{\omega}}(\bar{\omega}; \sigma^\omega)}. \quad (33)$$

The expected share functions  $f(\cdot)$  and  $g(\cdot)$  and their derivatives depend on the cross-sectional dispersion  $\sigma^\omega$  of firm productivity, hence the leverage ratio also depends on  $\sigma^\omega$ . For this intuitive argument, I emphasize this dependence by explicitly noting it as an argument of these functions.

Figure 11 sketches why changes in the cross-sectional dispersion of firms’ TFP would be expected to cause changes in leverage. Suppose the solid black curve in Figure 11 is the pdf  $\phi(\omega)$  before a risk shock occurs. The liquidation threshold  $\bar{\omega}$  shown is for this initial distribution. Suppose there is an exogenous reduction in dispersion. If the liquidation threshold  $\bar{\omega}$  were to remain unchanged, fewer firms would draw an idiosyncratic  $\omega < \bar{\omega}$ , which lenders understand because the density  $\phi(\omega)$  is common knowledge. This in turn means that fewer firms are expected to be unable to repay their loans, which reduces lenders’ risk. Conditional on a value for  $\bar{\omega}$ , lenders would be willing to extend more credit, which implies higher leverage ratios for firms (borrowers).

In partial equilibrium,  $\bar{\omega}$  will of course also change, which can only be determined quantitatively. Table 11 presents the dynamics of leverage and, as further illustration of the model’s financial terms, the bankruptcy probability  $\Phi(\cdot)$ , conditional on the calibrated risk shock process described in Section 5. This partial equilibrium exercise holds both net worth ( $nw$ ) and the external premium ( $p$ ) constant at their deterministic steady state general equilibrium values from the full model.<sup>50</sup>

<sup>50</sup>Debt dynamics are also shown in Table 11, for symmetry with the treatment of the general equilibrium results in Section 5. In the partial equilibrium analysis here, leverage and debt have the same cyclical dynamics because net worth  $nw$  and the external premium  $p$  are by construction taken as given in the financial contracting phase. Inspection of the financial outcome conditions (29) and (30) makes this point clear.

As a rough guide for referencing to the results of the full model, basic general equilibrium dynamics for GDP, which in the partial equilibrium analysis here does not fluctuate, are also shown. Comparison of Table 11 with Table 8 shows that the amplitude of cyclical fluctuations in leverage, measured in terms of volatility relative to that of GDP, is similar to that in general equilibrium — extremely large. Its moderate positive autocorrelation is also quite similar to that found in the general equilibrium version of the model.

Important to emphasize is that, in partial equilibrium, conditions (29) and (30) are purely static (within-period) conditions. Because the financial contract is of one period duration, isolating away general equilibrium effects means the financial terms can be solved exactly via nonlinear numerical solutions, rather than their dynamics requiring approximation, and Table 11 presents these exact solutions.<sup>51</sup> Comparison of the financial variables’ dynamics in Table 11 with Table 8 is thus even more stark — the local approximation method used for the full model gives qualitatively similar answers.

Risk shocks thus generate similarly (very) large fluctuations in financial variables in both partial equilibrium and general equilibrium. This similarity of leverage dynamics, conditional on risk fluctuations, further highlights that the connections of financial outcomes to the rest of model do not depend very importantly on the general equilibrium dynamics of the model when risk fluctuations are calibrated to micro-level data.

## B Aggregate Productivity Shocks

The quantitative analysis in Section 5 focused on dynamics conditional on pure risk shocks. For completeness, this section demonstrates that the model’s predictions are in line with those obtained by CF when  $\sigma_t^\omega = \bar{\sigma}^\omega \forall t$  and it is only average productivity shocks that drive fluctuations. Figure 12 displays impulse responses to a one-time, one-standard deviation positive shock to average productivity, holding constant cross-sectional dispersion of firm productivity. The results are qualitatively in line with those documented in CF (1998, Figure 1) for their “output model,” although magnitudes differ due to different calibrations.

Figure 12 shows that leverage rises somewhat substantially, with the peak response about twice as large as the peak response of GDP. CF (1998, Figure 1) do not report the dynamics of leverage (nor is it reported in the related models of CF (1997) or BGG), so this result is new in the literature. Thus, in contrast to the conjecture in Carlstrom, Fuerst, and Paustian (2009, p. 8), the leverage ratio is not virtually constant, conditional on productivity shocks, in the basic agency-cost model.<sup>52</sup>

<sup>51</sup>I thank Pablo D’Erasmus for pointing out the appropriateness of employing an exact solution.

<sup>52</sup>More precisely, leverage is not virtually constant, conditional on productivity shocks, in the CF output model. This paper does not test the dynamics of leverage in the CF (1997) investment model.

However, a more informative metric may be the relative volatility of leverage with respect to GDP induced by productivity shocks.

To this end, Table 12 presents business cycle statistics from model simulations when the only exogenous process is fluctuations in average productivity.<sup>53</sup> While CF do not report simulation-based moments, the model reproduces basic business cycle stylized facts: for example, gross investment is nearly four times as volatile as GDP, consumption is less volatile than GDP (although more dramatically than in the data), and GDP, consumption, and investment are all highly persistent.

On financial measures, leverage, debt, and equity are all more volatile than GDP. In a relative volatility sense, the magnitude of leverage fluctuations is surprisingly close to the evidence documented in Table 1 — a relative volatility of 1.9 in the model versus 2.2 in the data. The relative volatilities of debt and equity, however, are only half as large in the model than during the period 1974-1988. Given the parsimony of the model, on balance, the basic agency-cost model generates financial fluctuations, conditional on shocks to average productivity, that at least reach the empirically-relevant range. Leverage fluctuations are certainly not miniscule, which is the impression left by Carlstrom, Fuerst, and Paustian (2009, p. 8), but debt and equity fluctuations are smaller than in the data.

---

<sup>53</sup>These business cycle statistics are generated by simulating the model 1000 times around the deterministic steady state equilibrium, with each simulation 1000 periods in length, and then computing the medians across simulations of standard deviations, correlations, etc.

## C Bundled Shocks: Productivity-Induced Risk Fluctuations

Countercyclicity of firm risk can be modeled by linking time-variation in average TFP directly to fluctuations in firm-level risk. Specifically, the cross-sectional dispersion of productivity across firms is now assumed to decline when average TFP improves. First-moment shocks are thus assumed to be bundled with second-moment shocks, and I refer to the entire bundle as an “aggregate shock.” The two processes are assumed to be linked according to

$$\sigma_t^\omega = \bar{\sigma}^\omega + \varphi \ln \omega_{mt}. \quad (34)$$

This condition replaces the exogenous law of motion (1) for  $\sigma_t^\omega$ , and the evolution of  $\omega_{mt}$  is still described by (2). The rest of the model is exactly the same as above. The parameter  $\varphi$  is clearly the key parameter of this version of the model, with  $\varphi < 0$  implying countercyclicity of firm-level risk.<sup>54</sup> In terms of correlation between average TFP and dispersion of TFP,  $\varphi < 0$  obviously implies a perfect negative correlation between the two, but this portrayal is not counterfactually stark compared to the data; recall from Section 2 that the contemporaneous cyclical correlation between average TFP and dispersion of TFP is -0.98.

Figure 13 illustrates why  $\varphi < 0$  leads to countercyclical firm risk. A positive shift in average TFP will, all else equal, increase GDP. If at the same time cross-sectional dispersion declines due to  $\varphi < 0$ , and supposing initially that the bankruptcy threshold  $\bar{\omega}$  were fixed, fewer firms would be expected to go bankrupt. This in turn would induce lenders to extend more credit, hence leverage rises for given net worth. Indeed, the second part of the intuitive argument is exactly the same as that underlying Figure 11. What is different from the baseline model is the event that now induces the change in dispersion. In the baseline model, the change in dispersion itself was the exogenous event, whereas here it is a positive shock to average TFP.

This bundled aggregate shock is of course a reduced-form construct. However, I bring the same empirical evidence presented in Section 5.2 to bear on the calibration of the crucial elasticity parameter  $\varphi$ . The calibration approach is to choose  $\varphi$  so that the model matches the observed time-series variation in cross-sectional dispersion. Section 5.2 documented that the time-series volatility in annual cross-sectional dispersion is 3.15 percent. Given this target and holding fixed all parameters in Table 6, this calibration procedure (with average TFP fluctuations now as the sole truly exogenous driving process) leads to  $\varphi = -1.43$ .

Figure 14 presents impulse responses to a positive bundled aggregate shock. The most salient comparison for these impulse responses are those presented in Figure 12, in which the same size first-moment shock is also the exogenous impulse except with no change in cross-firm dispersion.

---

<sup>54</sup>Clearly,  $\varphi > 0$  would deliver procyclical firm-level risk, and  $\varphi = 0$  would recover the baseline CF model in which there are never any changes in firm risk.

Comparing Figure 14 with Figure 12 shows that the bundled aggregate shock induces very similar dynamics in most variables as does the unbundled first-moment shock alone. The only difference compared to Figure 12 is that equity rises by much less in response to the bundled shock.

Finally, Table 13 presents simulation-based business cycle statistics. The first row shows that the volatility of leverage (and debt) carries over from the baseline model's results presented in Table 10. However, a shortcoming of the bundled-shock model is that leverage is extremely procyclical, at odds with the evidence presented in Section 3.

To summarize, the bundled-shock model by construction is consistent with the empirically-observed countercyclicality of cross-sectional firm risk (see the last two rows of the lower panel of Table 13), and it retains the volatility predictions of the baseline model driven by independent first-moment and second-moment shocks. However, it fails to reproduce the countercyclicality that leverage exhibits in the data. On the other hand, the baseline model driven by a complete set of independent, "unbundled," shocks performed well on the volatility dimension, but failed to capture the countercyclicality of firm-level risk. Although I do not take up this extension here, a conjecture is that a combination of bundled shocks along with independent, exogenous, shocks to firm risk may help in capturing all these dimensions of the data.<sup>55</sup>

---

<sup>55</sup>Of course, there are a host of other model features and/or shocks one could consider introducing to the model. Such analysis is left to future work.