Costly Information Processing and Income Expectations*

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Abstract

Do individuals use all information at their disposal when forming expectations about future events? In this paper we present an econometric framework to answer this question. We show how individual information sets can be characterized by simple nonparametric exclusion restrictions and provide a quantile based test for costly information processing. In particular, our methodology does not require individuals' expectations to be rational, and we explicitly allow for individuals to have access to sources of information which the econometrician cannot observe. As an application, we use microdata on individual income expectations to study the information agents employ when forecasting future earnings. Consistent with models where information processing is costly, we find that individuals' information sets are coarse in that valuable information is discarded. To quantify the utility costs, we calibrate a standard consumption life-cycle model. Consumers would be willing to pay 0.04% of their permanent income to incorporate the econometrician's information set in their forecasts. This represents a lower bound on the costs of information processing.

1 Introduction

Individuals' expectations about uncertain events are a key aspect of modern economics. Knowing what expectations individuals hold is therefore crucial to understand and predict behavior (Manski, 2004). A key ingredient in the process of expectation formation is the information

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set agents employ. In this paper we estimate the content of information sets using micro data on income expectations. We first show that individuals' information sets can be characterized without further assumption about the agents' structural model. In particular, we do not have to assume rational expectations. More precisely, we show that agents' beliefs can be expressed as a non-separable model, taking agents' information sets as argument. As long as one is not interested *how* information enters in individuals' process of expectation formation but only *whether* their beliefs are measurable with respect to particular information sets, we can learn about agents' information sets through simple nonparametric exclusion restrictions. In our application we find that individuals use rather coarse information to predict their future income. In particular, we are not able to reject that agents only use their (log) of current income, age, occupational status and local labor market conditions to predict future income growth. In contrast, neither their educational status nor their sector of employment are contained in their predictions.

After establishing which information individuals use when forming expectations, we test if these information sets are consistent with costless information processing, i.e. whether agents are able to productively use information as long as it is available to them. We first show that information processing costs cannot be identified without further restrictions on the structural model individuals use. Intuitively, if individuals were to think that some information is not useful to predict the outcome of interest, they will not use it despite information processing to not be costly. We then show that under a weak restriction on the agents' model, which in its essence assumes a minimum degree of consistency between the agents' model and the objective data generating process, we can test for costly information processing. In our application, we can comfortably reject that information processing is costless. Hence, agents might be constrained in the amount of information they can be attentive to as claimed in the literature on rational inattention (see e.g. Sims (2003); Mackowiak and Wiederholt (2000)) and costly information processing more general (see e.g. Reis (2004); Mankiw and Reis (2002))

While our methodology is applicable in a wide range of situations, we quantify the utility costs of costly information processing within a particular model, namely the canonical life-cycle model of consumption. We view this as a natural benchmark, as we analyze individual's income expectations, which are important for optimal consumption behavior. In particular, we use a standard life-cycle model with uninsurable labor income risk (Carroll, 1997; Deaton, 1991; Gourinchas and Parker, 2002). Through the lens of the model, consumers' information sets affect the agents' perceived environment in that they determine how much of the income process is predictable and how much has to be attributed to permanent and transitory shocks. Using the information sets as estimated from the microdata, we find that households overestimate the variance of transitory shocks (compared with the econometrician) and slightly underestimate the predictable rate of income growth. This misconception of the income process they face will change individual behavior. At the estimated parameters, the utility loss of excluding information from their information sets is small in that the average willingness to pay for the econometricians' information set amounts to roughly 0.04% of agents' permanent income. Hence, the information processing costs can be quite low for individuals to rationally choose to not incorporate different sources of information in their income predictions. The reason is that - in the model - occupational characteristics and age do a good job to decompose the observed time-series of income in the micro-data into predictable components and transitory and permanent shocks. With the individuals' model being close to the income process, the utility consequences are relatively small as individuals are quite well insured.

Related literature: Empirical studies of individuals' expectations in general and their information sets in particular, have a long tradition in economics. First of all, there is a large empirical literature that tests the rational expectations hypothesis (Lovell, 1986; Keane and Runkle, 1990; Brown and Matial, 1981). This literature has often tested for "informational efficiency", which is similar to our concept of costless information processing and hence closely related to our specification test. Secondly, there are numerous contributions that explicitly study subjective expectation data (Dominitz, 1998; Dominitz and Manski, 1997; Hurd and McGarry, 1995). While data on subjective expectations has often been met with skepticism, Manski (2004) provides evidence that such data is helpful to predict choices and argues that it should be used more often given its wide availability. In this light, **?** use response patterns across various questions on probabilistic expectations to infer about individuals' rounding behavior. Finally there is an extensive literature on forecasting, that models agents' forecasts as the solution of a well-defined maximization problem for given preferences and information sets (Pesaran and Weale, 2006; Machina and Granger, 2006).

Recently, expectations data have also been explicitly used for particular applications. Guiso et al. (1996) use agents' self-reported income uncertainty in a study of portfolio choice, Carroll (2003) exploits expectations on future inflation and unemployment rates to estimate a structural model of expectation formation, Jappelli and Pistaferri (2000) provide tests for consumption excess sensitivity when explicitly controlling for individuals' income expectations and Coibion and Gorodnichenko (forthcoming) use data on household inflation expectations to explain the missing disinflation during the Great Recession. Finally, Cunha et al. (2005) show how individual information sets can be recovered from a structural model of college choice in a life-cycle framework. While not focusing on the precise content of individual information sets, Coibion and Gorodnichenko (2012) also use expectations data to provide evidence in favor of informational rigidities.

Regarding our application, the life-cycle of model of consumption is the workhorse model to analyze consumption behavior and has been tested extensively (see e.g. Hall and Mishkin (1982); Hall (1978); Attanasio and Weber (1995) and Browning and Lusardi (1996) for a review). While the robust finding that the observed consumption sensitivity to income shocks exceeds the one predicted by the standard model of perfect foresight (or its certainty-equivalent version with quadratic utility) and that changes in consumption are positively correlated with anticipated income shocks (the "excess sensitivity puzzle"), have often been interpreted as evidence against the life-cycle model, this conclusion has been challenged in the last decade. In particular, neither of these findings is inconsistent with the life-cycle theory once uninsurable income uncertainty and risk-aversion is allowed for (Carroll, 2001, 1997). The importance of the precautionary savings motive to reconcile the empirical evidence with the life-cycle theory of consumption already suggests two crucial ingredients for individuals' consumption behavior. The first concerns the income process itself, i.e. what are the statistical properties of the income process consumers face? In the context of the life-cycle model, many recent contributions use both consumption and income data simultaneously to learn about the structure of individual income (Gourinchas and Parker, 2002; Blundell et al., 2008; Krueger and Perri, 2011; Guvenen, 2007). The second one concerns consumers' information sets when forecasting future income. As the amount of information used when forecasting future income determines consumers' income uncertainty, the size of consumers' information will affect consumption behavior. To use microdata on income expectations to learn about consumers' information sets is the objective of the paper. We then gauge the utility consequences within a simple life-cycle model as one particular application.

The structure of the paper is as follows. In the next section we will present our methodology to characterize information sets and give conditions for identification. In section three we apply our econometric technique to microdata on income expectations and measure what information individuals use when forecasting future income. In section four we quantify the economic importance of agents' information on consumption behavior in the context of a standard life-cycle model. Section five concludes. All tables and figures are relegated to the appendix. Additional empirical results can be found in the supplementary material.

2 Characterizing Information Sets

We consider the following economy. There is a continuum of agents, which we model as realizations of an underlying random vector W. In particular, let $W = [X, U]$, where X is observable to the econometrician and U is unobservable. We are interested in agents' expectations about individual income Y, which is given by the structural relationship

$$Y = \psi(W) = \psi(X, U). \tag{1}$$

Individual (x, u) therefore earns income $y = \psi(x, u)$. Examples for $[X, U]$ are individual characteristics like education, experience or the match quality between the individual and the employer and aggregate characteristics like relative skill supplies in the individuals' local labor market. However, U may also contain deeper concepts, like preferences, abilities etc., which we collectively refer to as "types". In sum, the objective distribution of income is fully determined by the structural relationship $\psi(.)$ and the underlying distribution of observables and types, $F_{X,U}$.

Individuals in this economy try to forecast future income. This need to forecast income arises because some realizations of $[X, U]$ might be unknown to the individual. This might for example be the quality of the future match if there is a chance that the current employment relationship might be terminated. The structural model agents have in mind is given by

$$Y = \psi^I(W) = \psi^I(X, U), \tag{2}$$

where ψ^I does not necessarily equal ψ. Note that it is without loss of generality to define both ψ and ψ^I over $[X, U]$ as we can always have either ψ or ψ^I to be trivial in the respective argument. It is in this sense that the function ψ^I could depend on individual specific variables: they could form part of U, and the objective function ψ could not depend on them.

To forecast their income, individuals use information, which we model as a set of random variables Q. More precisely, the information is $\sigma(Q)$, where σ denotes the sigma-algebra spanned by Q, and we also denote the information set sometimes as $\mathcal{F}_Q$. However, for simplicity, we will mostly refer to Q as the information individuals hold.

Given this information, we denote the objective conditional distribution of $[X, U]$ given Q by $F_{X,U|Q}$. Analogously to above, the joint distribution of $[X, U]$ given Q as perceived by the agents is denoted $F^I_{X,U|Q}$; the same remark about heterogeneity applies, i.e. individuals do not have to hold rational expectations so that $F^I_{X,U|Q}$ and $F_{X,U|Q}$ do not have to be equal. We denote the respective densities by $f^I_{X,U|Q}$ and $f^I_{X,U|Q}$. Given Q and their "view of the world" encapsulated in $(F^I_{X,U|Q}, \psi^I)$, agents' subjective distribution of future income is given by

$$G^I_{Y|Q}(y, q) = \int_{(x,u):\psi^I(x,u)\leq y} f^I_{X,U|Q}(x, u; q)dxdu. \tag{3}$$

Hence, $G^I_{Y|Q}$ represents the joint mental act of having a structural relationship for Y given the characteristics (X, U) (i.e. ψ^I), and some process of learning about (X, U) from Q (i.e. $F^I_{X,U|Q}$). It is important to realize that, by observing realizations of $G^I_{Y|Q}(y, Q)$ which is a random variable in the cross-section, we cannot separately identify ψ^I and $F^I_{X,U|Q}$, at least not without further restrictions. Hence, we directly focus on $G^I_{Y|Q}$, and define this subjective distribution agents hold as agents' *forecasting model*, or *model.*

Definition 1. *A model is a conditional distribution function of Y given information Q, i.e. a model is*

$$G^I_{Y|Q}(y, q) = P[Y \leq y|Q = q; \psi^I, F^I] = \int_{-\infty}^{y} g^I_{Y|Q}(\eta; q)d\eta, \tag{4}$$

where g^I is derived from (3). In particular, G is a primitive of the individual's problem and is defined for all possible Q.

Similarly to (4), the econometricians beliefs about future income given the information Q (the econometrician's model) are given by

$$G_{Y|Q}(y,q) = P[Y \leq y|Q=q;\psi,F] = \int_{(x,u):\psi(x,u)\leq y} f_{X,U|Q}(x,u;q)dxdu. \tag{5}$$

Equation (4) and (5) illustrate that one may think of a forecasting model as a production function. It generates outputs (beliefs about future events) upon usage of inputs (information) without direct reference to the true underlying relationship between information affecting beliefs $\left(F^I_{X,U|Q}\right)$ and between characteristics and outcomes $\left(\psi^I\right)$. For concreteness consider the following example:

Example 1. *Suppose that individuals perceive the model ψ^I, which is given by*

$$Y = \psi^I(X,U) = a^I_X X + a^I_U U, \tag{6}$$

where X and U are scalars. Individuals' prior about X is given by $X \sim \mathcal{N}(\mu_X, \frac{1}{\rho_X})$ and they receive a signal $Q_X = X + \epsilon_X$, where $\epsilon_X \sim \mathcal{N}(0, \frac{1}{\eta_X})$. The information structure for U is analogous, and (X,U) and (ϵ_X, ϵ_U) are independent. The model G^I is then given by

$$G^I_{Y|Q}(y,(q_X,q_U)) \quad = \quad P[Y \leq y|Q=q;\psi^I,F^I] = \Phi\left(\frac{y - \sum_{i=X,U} a_i \frac{\rho_i\mu_i + \eta_i q_i}{\rho_i+\eta_i}}{\sqrt{\sum_{i=X,U} \frac{a_i^2}{\rho_i+\eta_i}}}\right). \tag{7}$$

Hence, the parameters of the agents' structural relationship ψ^I (i.e. $[a_i]_{i=X,U}$) and the parameters of the learning process (i.e. $[(\mu_i,\rho_i,\eta_i)]_{i=X,U}$) are not separably identified from data on individual expectations $G^I_{Y|Q}$.

The population of individuals and their accompanying income expectations $G^I_{Y|Q}$ are therefore induced by realizations of the underlying random variable Q. From the point of view of the econometrician, it is precisely these observations of the random variable $G^I_{Y|Q}(Q;y)$, which we observe for all y. The two questions we ask are: (1) Which information do individuals use when forecasting their future income? (2) If we were to conclude that individuals do not use all available information, can we conclude that information processing is costly?[1] While the first question is a purely empirical one, which we can answer without any theoretical restrictions, the second one requires further identifying assumptions. We will address both of them in turn.

[1]We will define costly information processing formally below. Intuitively, we will think of information processing being costly if individuals face costs of using Q as an input into the production of their beliefs.

2.1 What information do individuals use?

In our setup, the question of what information individuals use is formalized by asking which Q that span the information set $\sigma(Q)$ individuals *use*. We emphasize the *usage* of information, precisely because individuals might not use all the information they in principle have when information processing is costly. In our application, we will employ information from survey data, i.e. individuals have in principle access to all the information we have. The questions is whether they also condition their forecasts on that information. The formal definition of what it means for information to be used is contained in the following definition.

Definition 2. *Let $Q = [Q_1, Q_2]$. We say that the information in Q_2 is used conditional on Q_1, whenever for (y, q) with positive probability*[2]

$$G^I_{Y|Q}(y; (q_1, q_2)) \neq G^I_{Y|Q}(y; (q_1, q_2'))$$

In words, information is actively used, whenever it affects the beliefs of some individuals in the population. We especially want to stress that the usage of information is a property of both the structural relationship for Y and the learning process that maps realizations of Q into beliefs about $[X, U]$. This can already be expected given the construction of $G^I_{Y|Q}$ in (3). It is also clearly seen in Example 1. According to Definition 2, the example implies that the information in Q_U is used conditional on Q_X, if for some y (see 7)

$$\Phi\left(\frac{y - a_X\frac{\rho_X\mu_X+\eta_X q_X}{\rho_X+\eta_X} - a_U\frac{\rho_U\mu_U+\eta_U q_U}{\rho_U+\eta_U}}{\sqrt{\sum_{i=X,U}\frac{a_i^2}{\rho_i+\eta_i}}}\right) \neq \Phi\left(\frac{y - a_X\frac{\rho_X\mu_X+\eta_X q_X}{\rho_X+\eta_X} - a_U\frac{\rho_U\mu_U+\eta_U q_U'}{\rho_U+\eta_U}}{\sqrt{\sum_{i=X,U}\frac{a_i^2}{\rho_i+\eta_i}}}\right),$$

with positive probability. This is the case if $\frac{a_U\eta_U}{\rho_U+\eta_U} \neq 0$. Hence, Q_U is *used* by individuals if *both* the information is considered informative ($\eta_U > 0$) and the factor it is predicting is part of the structural relationship ψ_I, i.e. $a_U \neq 0$. In contrast, Q_U is not used if either knowing Q_U does not help in predicting U (i.e. $\eta_U = 0$) or U is thought to be unrelated to income ($a_U = 0$).

When trying to characterize individuals information sets, we want to allow for the fact that individuals might use information, which is unobservable to the econometrician. We will of course only be able to make statements about variables, which are observable to us. As we will show now, we can in fact perfectly control for such unobserved information on the agent's behalf by focusing on different quantiles of the distribution of income forecasts. To do so, we have to introduce additional notation. Note first that we can always write individual information sets as

$$Q = \pi(Z, V), Z \perp V \tag{8}$$

[2] We say "(y, q) with positive probability", when all (y, q) form a set $\mathcal{Y}_1 \times \mathcal{Q}_1 \subseteq \mathcal{Y} \times \mathcal{Q}$, with $P[\mathcal{Y}_1 \times \mathcal{Q}_1] > 0$, and analogously throughout this paper.

where Z is observed and V is unobserved. This construction of V being orthogonal to Z is exactly the right construction to characterize informational *content*. This does not mean that individuals actually use the observable variables Z but as long as there is a positive correlation between the observable variable Z and information they use, the information contained in Z is contained in the forecast. Suppose for example that income growth was *only* a function of individual ability, which is unobservable to the econometrician but used by individuals to forecast future earnings. Now suppose we were to ask whether individuals use their information on educational attainment. If ability and education are correlated, we would correctly find that the information in education is reflected in individual forecasts. We as econometricians cannot say whether there is a Mincerian skill premium or if such skill premium is purely spurious and driven by the correlation between income and ability (in this example obviously the latter is the case). But to measure informational content we are not interested in the underlying structural model. For us it is only important if the information contained in education is reflected in individuals' forecasts. This example also makes clear why Definition 2 makes only *conditional* statements. If we were to observe ability, then we would correctly conclude that educational information is not used once ability is controlled for.

To analyze individual predictions more formally, assume that (Y, Z, V) are jointly continuously distributed. Individuals' beliefs about their future income can therefore be written as nonseparable model, i.e.,

$$G^I_{Y|Q}(y;q) = G^I_{Y|\pi(Z,V)}(y;z,v) = \int_{(x,u):\psi^I(x,u)\leq y} f^I_{X,U|\pi(Z,V)}(x,u;z,v)dxdu \tag{9}$$

$$\equiv \varphi(z,v;y). \tag{10}$$

We emphasize here that we think of y as a fixed index, and of Z and V as the actual argument of the function, i.e., $\varphi(Z,V;y)$ denotes the (random) conditional probability that $Y < y$, which is induced by individual information sets $Q = (Z, V)$. Since we do not aim to identify the structural relationship, we can not only choose the unobservable V to be independent of Z (see (8)), but we can also let v to enter $\varphi(z,\cdot;y)$ strictly monotonically (for any (y,z)). Following Matzkin (2003) it is w.l.o.g. to let $V \backsim U[0,1]$, and then conclude that there is a family of quantile regressions indexed by y, i.e.,

$$\varphi(z,v;y) = k^v_{G^I_y|Z}(z), \tag{11}$$

where $k^v_{G^I_y|Z}(z)$ denotes the v quantile of $G^I_{Y|Q}(y;Q)$.

In words, we can consider the cross-sectional quantiles of the individual predictions of probabilities for any value of interest y as a tool to evaluate what information they use. The v quantile of $G^I_{Y|Q}(y;Q)$ given $Z = z$ gives then the prediction of an individual with observable information $Z = z$, and unobservable reduced form information $V = v$. As people with different realizations of V given Z make different predictions for their income growth, $k^V_{G^I_y|Z}(z)$ is a random variable (given z) and the unobserved heterogeneity in the population is encapsulated in the random variable V. Basically: When we are not interested in the structural model, we can fully control for the unobserved heterogeneity by equating it to the quantiles of the reduced form distribution. To see how and why this construction works, consider the continuation of our example above.

Example (continued) *Suppose for simplicity that $\mu_X = \mu_U = 0$ and that the individual observes two signals, where $Q_X = Z$ and $Q_U = V$, i.e. only the first information is observed by the econometrician. Then (7) implies that*

$$G^I_{Y|Z,V}(y; z, v) = \Phi\left(\frac{y - \theta_z z - \theta_v v}{\delta}\right)$$

where $\delta = \sqrt{\sum_{i=X,U} \frac{a_i^2}{\rho_i + \eta_i}}$ and $\theta_i = \frac{a_i \eta_i}{\rho_i + \eta_i}$. Now define $M = \Phi(\frac{V}{\sigma_V})$ so that $M \sim U[0, 1]$ and M is observationally equivalent to V. Hence,

$$G^I_{Y|Z,M}(y; z, m) = \Phi\left(\frac{y - \theta_z z - \theta_v \sigma_V \Phi^{-1}(m)}{\delta}\right).$$

The α quantile of $G^I_{Y|Q}$ given $Z = z$, $k^\alpha_{G^I_y|Z}(z)$, is then defined by

$$\begin{aligned} \alpha &= P\left[\Phi\left(\frac{y - \theta_z z - \theta_v \sigma_V \Phi^{-1}(M)}{\delta}\right) \leq k^\alpha_{G^I_y|Z}(z)|Z = z\right] \\ &= 1 - \Phi\left(\frac{y - \theta_z z - \delta\Phi^{-1}(k^\alpha_{G^I_y|Z}(z))}{\sigma_V \theta_v}\right), \end{aligned}$$

as M is uniform and independent of Z. Hence, solving for $k^\alpha_{G^I_y|Z}(z)$ yields

$$\begin{aligned} k^\alpha_{G^I_y|Z}(z)) &= \Phi\left(\frac{y - \theta_z z - \theta_v \sigma_v \Phi^{-1}(1-\alpha)}{\delta}\right) = G_{Y|Z,V}(y; z, \sigma_v \Phi^{-1}(1-\alpha)) \\ &\equiv \varphi(z, \alpha; y), \end{aligned} \tag{12}$$

which is exactly the form of (11). Hence, for any v there is a specific α such that the conditional distribution of $G^I_{Y|Z,V}$ given $[Z, V]$ is equal to the α-quantile of the conditional distribution of G^I_y given Z.

This construction suggests how we can test for the informational content of individuals' forecasts, i.e. to answer the question if there is a positive measure of people paying attention to some information. In particular, let $Z = [Z_1, Z_2]$. Then, individuals do not use the information contained in Z_2 conditional on $[Z_1, V]$, if for all (y, z_1, v) we have

$$k^v_{G^I_y|Z_1,Z_2}(z_1, z_2) = k^v_{G^I_y|Z_1}(z_1) \tag{13}$$

If this was the case, individuals receiving the signal $q = (z_1, z_2, v)$ report the same income expectation as individuals receiving the signal $q = (z_1, v)$, i.e. individuals do not incorporate information contained in Z_2 once $[Z_1, V]$ is controlled for. As (13) contains our first testable restriction, we state it in the following proposition.

Proposition 1. *Consider the model above. Let individuals' information sets be given by $Q = \pi(Z, V)$. Let $Z = [Z_1, Z_2]$. Then, individuals use Z_2 conditional on $[Z_1, V]$ in the sense of Definition 2 if and only if*

$$k^v_{G^I_y|Z_1,Z_2}(z_1, z_2) \neq k^v_{G^I_y|Z_1}(z_1)$$

for z with positive probability.

Proof. Follows directly from Definition 2, (10) and (11). □

To see why Proposition 1 is indeed the correct test for the usage of information, consider again the example.

Example (continued) *Recall from* (12) *that*

$$k^v_{G^I_y|Z_1,Z_2}(z_1, z_2) = \Phi\left(\frac{y - \theta_{z_1} z_1 - \theta_{z_2} z_2 - \theta_v \sigma_v \Phi^{-1}(1 - v)}{\delta}\right).$$

$k^v_{G^I_y|Z_1}(z_1)$ *however, is implicitly defined by*

$$1 - v = E\left[\Phi\left(\frac{y - \theta_{z_1} z_1 - \theta_{z_2} Z_2 - \delta\Phi^{-1}(k^v_{G_y|Z_1}(z_1))}{\sigma_V \theta_v}\right) |Z_1 = z_1\right]$$

For these to be equal for all (y, v, z_1), we need that $\theta_{z_2} = \frac{a_{z_2}\eta_{z_2}}{\rho_2 + \eta_{z_2}} = 0$, which is exactly the condition that Z_2 is not used by individuals.

Hence, using the quantile exclusion restriction contained in Proposition 1, we can exactly characterize which information affects individuals' beliefs. In particular, we can do so without assuming whether or not individuals have rational expectations and without any restrictions on which additional information individuals use.

While the quantile function $k^v_{G^I_y|Z}$ is exactly the right statistic to test for informational content, we can also look at the conditional mean function. Doing so delivers an intuitive but weaker test for informational usage. In particular, given agents' information Q and view of the world $(\psi^I, f^I_{X,U|Q})$, their future expected income is given by

$$\begin{aligned} E^I[Y|Q=q] = \int y g^I_{Y|Q}(y,q)dy &= \int \psi^I(w) f^I_{W|Q}(w;q)dw = \int \psi^I(w) f^I_{W|\pi(Z,V)}(w;(z,v))dw \\ &\equiv m(z_1,z_2,v), \end{aligned} \tag{14}$$

where again $Z=[Z_1,Z_2]$. If Z_2 is not used conditional on $[Z_1,V]$, then $m(z_1,z_2,v)=m(z_1,v)$, i.e. $m(z_1,z_2,v)$ is trivial in z_2, where m is defined in (14). Note that this is only an "if" statement. However, it is an "if and only if" statement under the regularity condition that changes in the subjective density $g^I_{Y|Q}(y,q)$ do not average out once we integrate over y. Hence, Proposition 1 is stronger because it focuses on this subjective distribution directly. Looking at the exclusion restriction contained in (14) is still useful in that is has less data requirements and is easier to implement.[3]

2.2 Do individuals face costs of information processing?

While Proposition 1 delivers a simple non-parametric test for the size of individual information sets, it is not helpful in interpreting *why* individuals might exclude some information from their forecast. Hence, we now ask in what sense the finding that some variable is not part of individual information sets is evidence for costly information processing. In this setup, this can be rephrased as saying: Would someone endowed with the model $G^I_{Y|Q}$ but no information processing costs have chosen to use this information? Hence, the essence of costly information processing is that there is a demand for information, but that the marginal value falls short of the marginal processing costs. To test for costly information processing, we therefore have to define the value of (or demand for) information.

Definition 3. *Consider the setup described above. Let $Q=[Q_1,Q_2]$. We say that the information contained in Q_2 is valuable given the model G^I and the information Q, whenever*

$$G^I_{Y|Q}(y,(q_1,q_2)) \neq G^I_{Y|Q}(y,(q_1,q_2')) \tag{15}$$

with positive probability. For notational simplicity we will say that Q_2 is $(G^I,[Q_1,Q_2])-$valuable if (15) holds true.

[3]In our empirical part, we are going to focus on the restriction embedded in (14) as our data is not rich enough to estimate a non-parametric exclusion restriction on different quantiles of the subjective expectation data. We are going to come back to this in our empirical part below.

Hence, according to Definition 3, additional information is valuable whenever it changes the individuals' posterior in some states of the world. While we believe this definition to be natural in our setup, we also want to stress that in general the demand for information obviously also depends on the preferences of the individual. If no decision depends on the beliefs about personal income (and the decision maker does not experience any utility loss from ambiguity aversion or other behavioral aspects), the demand for information is obviously zero as the individual does not care about her posterior beliefs about income. In Definition 3 we do not consider these possibilities, i.e. we only care about cases, where the individual actually cares about the beliefs she ends up with, before decisions have to be taken.

Given this definition of information being valuable and our definition of information usage, we can also give a precise definition of what we are looking for in order to find costly information processing.

Definition 4. *Consider the setup described above. We say that individuals are characterized by costly information processing with respect to Q_2, whenever Q_2 is not used conditional on Q_1, despite Q_2 being $(G^I, [Q_1, Q_2])$-valuable.*

Hence, whenever some information Q_2 would have changed individuals' posterior (given their model and their information) but individuals decide to not use Q_2, we will conclude that their expectation formation process is subject to costly information processing. The important aspect of Definition 4 is precisely the dependence of the value of information on G^I *and* on Q_1 - both of which are unobserved by the econometrician. Therefore the question is: Can we detect occurrences of costly information processing given data on income expectations without further restrictions on G^I and Q_1? The answer is no. The reason is simply that we can always find an agent's model $(\psi^I, f^I_{X,U|Q})$ such that the excluded information is not $\big(G^I, [Z_1, Z_2, V]\big)$-valuable. Intuitively, if the model agents are using is such that Z_2 is considered noise, Z_2 would not have been used even without processing costs. Hence, in order to give the hypothesis of costly information processing empirical content, we impose the following restriction on the relationship between the agents' and the objective model.

Assumption 1. *For all Q_1, Q_2, if Q_2 is $(G, [Q_1, Q_2])$-valuable, then Q_2 is also $(G^I, [Q_1, Q_2])$-valuable.*

Assumption 1 requires a minimum amount of consistency between the agents' view of the world and the structural model of the economy. Hence, we refer to Assumption 1 as an assumption of *weak rationality.* Intuitively, it requires the following: whenever the econometrician with $Q = [Q_1, Q_2]$ at his disposal would not discard Q_2, we require that individuals would not do so either. Individuals could disagree with the econometrician how Q_2 enters and how important it is, they could disagree about the structural model, or they could disagree about the distribution of all the variables. But they have to agree that Q_2 determines the distribution of income conditional on Q_1 in some way. We consider Assumption 1 to be very weak and it turns out that it is sufficient to detect costly information processing in the data.

To develop a test for costly information processing, note that observing realizations of $G_{Y|Q}(y;Q)$ for different y is the most we can learn about the information individuals have. In particular, let $G^I_{Y|Q}(y;Q) = h_y(Q)$, which is indexed by y. For simplicity, assume that y takes J finite values and that we can (in principle) observe $\{h_{y_j}(Q)\}_{j=1}^J$, for instance if individuals report their expectations in income bins as is the case in our data. Using (10), we can, for every y_j with $j = 1,..,J$, write $h_{y_j}(Q) = \varphi(Z, V_j; y_j)$ with $Z = [Z_1, Z_2]$ and V_j independent of Z for every $j = 1,...,J$.[4] For a fixed grid of values $y_1,...,y_J$, this simply means that we have a collection of known functions $\varphi_1,...,\varphi_J$. Rather than constructing a test in terms of $\varphi_1,...,\varphi_J$, it will turn out convenient for our application to use $V_1,...,V_J$ instead, which can be derived from the former as $V_j = \varphi^{-1}_{y_j}(Z, h_j(Q))$ for each $j = 1,...,J$.[5] Hence, we can observe the following conditional probability

$$P\left[Y < y_j | Z_1, Z_2, V_1, .., V_J\right] \equiv \gamma(y_j, \{V_k\}_{k=1}^J, Z_1, Z_2). \tag{16}$$

The function γ will be the crucial object to test whether information processing is costly.

A test for costless information processing, i.e. for whether individuals should have used Z_2 conditional on Z_1 and $V_1,...,V_J$, can now be based on observables as follows:

Proposition 2. *Consider the setup described above and let Assumption 1 hold true. Let γ be defined in (16). If Z_2 is not used conditional on $\left[Z_1, \{V_k\}_{k=1}^J\right]$, i.e. $Z_2 \notin \mathcal{F}$ conditional on Z_1 and $\{V_k\}_{k=1}^J$, then, information processing is costly whenever for every $j = 1,...,J$,*

$$\gamma(y_j, \{v_k\}_{k=1}^J, z_1, z_2) \neq \gamma(y_j, \{v_k\}_{k=1}^J, z_1, z_2^{'})$$

with positive probability.

Proof. By assumption Z_2 is excluded from the individuals' information sets. First, recall that for every $j = 1,...,J$ and $\{z_1, z_2\}$

$$\begin{aligned}
\gamma(y_j, \{v_k\}_{k=1}^J, z_1, z_2) &= P\left[Y < y_j | Z_1 = z_1, Z_2 = z_2, V_1 = v_1, .., V_J = v_J\right] \\
&= P\left[Y < y_j | \{h_{y_k}(Q) = h_{y_k}(q)\}_{k=1}^J, Z_2 = z_2\right] \\
&\equiv \hat{\gamma}(y_j, \{h_{y_k}(Q)\}_{k=1}^J, z_2),
\end{aligned}$$

[4] In what follows, we will denote, with slight abuse of notation, $\varphi(Z, V_j; y_j)$ by $\varphi_j(Z, V_j)$.

[5] Note that, together with Z as a conditioning argument, we will retain the same information when conditioning on $V_1,...,V_J$ as when conditioning on $h_{y_1},...,h_{y_J}$.

which allows us to work with $\hat{\gamma}(y_j, \{h_{y_k}(Q)\}_{k=1}^J, z_2)$ instead of $\gamma(y_j, \{v_k\}_{k=1}^J, z_1, z_2)$. Next, note that

$$\begin{aligned}\hat{\gamma}(y_j, \{h_{y_k}(q)\}_{k=1}^J, z_2) &= E[P[Y \leq y_j|Q, Z_2]|\{h_{y_k}(Q) = h_{y_k}(q)\}_{k=1}^J, Z_2 = z_2] \\ &= E[G_{Y|Q,Z_2}(y_j; Q, Z_2)|\{h_{y_k}(Q) = h_{y_k}(q)\}_{k=1}^J, Z_2 = z_2].\end{aligned}$$

Again we express agents' information set as

$$Q = \pi(\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S) \quad \text{with } S \perp \{h_{y_k}(Q)\}_{k=1}^J, Z_2. \tag{17}$$

Then

$$\begin{aligned}&E[G_{Y|Q,Z_2}(y_j; Q, Z_2)|\{h_{y_k}(Q) = h_{y_k}(q)\}_{k=1}^J, Z_2 = z_2] \\ =\ & E[G_{Y|\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S}(y_j; \{h_{y_k}(Q)\}_{k=1}^J, Z_2, S)|\{h_{y_k}(Q) = h_{y_k}(q)\}_{k=1}^J, Z_2 = z_2] \\ =\ & \int G_{Y|\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S}(y_j; \{h_{y_k}(q)\}_{k=1}^J, z_2, s) f_{S|\{h_{y_k}(Q)\}_{k=1}^J, Z_2}(s; \{h_{y_k}(q)\}_{k=1}^J, z_2) ds \\ =\ & \int G_{Y|\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S}(y_j; \{h_{y_k}(q)\}_{k=1}^J, z_2, s) f_S(s) ds. \end{aligned} \tag{18}$$

If γ is not trivial in z_2, then (18) implies that

$$G_{Y|\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S}(y_j; \{h_{y_k}(q)\}_{k=1}^J, z_2, s) \neq G_{Y|\{h_{y_k}(Q)\}_{k=1}^J, Z_2, S}(y_j; \{h_{y_k}(q)\}_{k=1}^J, z_2', s) \tag{19}$$

for some (z_2, z_2') with positive probability and every $j = 1, ..., J$. According to Definition 3 this implies that Z_2 is $(G, [Q, Z_2])$-valueable. Under Assumption 1 this also implies that Z_2 is $(G^I, [Q, Z_2])$-valuable. As $Z_2 \notin \mathcal{F}$, information processing is costly. $\square$

The essence of Proposition 2 is the following: Since conditioning on Z and $V_1, ..., V_J$ retains the same information as conditioning on $h_1, ..., h_J$, whenever we as econometricians would consider some information Z_2 valuable conditional on agents' forecasts, the assumption of weak rationality implies that agents should consider that information valuable as well, even though they may disagree about the exact way it enters. If they should use it, and do in fact choose not to, we obtain and inconsistency with costless information processing.

Also note the key distinction between the test of Proposition 2, where we test for the exclusion of Z_2 by looking at

$$P\left[Y < y_j|Z_1, Z_2, V_1, .., V_J\right],$$

and tests based on

$$P\left[Y < y_j|Z_1, Z_2\right].$$

Conditioning on individuals' unobserved information $V_1, ..., V_J$ is crucial, as it reflects a reduced form measure of unobserved individual information heterogeneity. Hence, conditional on V is akin to a control function in standard regression analysis. Observe in particular that even if we find that

$$P\left[Y < y_j | Z_1, Z_2\right] = P\left[Y < y_j | Z_1\right],$$

for all $j = 1, ..., J$, this does not imply that

$$P\left[Y < y_j | Z_1, Z_2, V_1, .., V_J\right] = P\left[Y < y_j | Z_1, V_1, ..., V_J\right],$$

as the latter depends on the objective conditional density of U, the structural error, given Z, V, as the influence of Z_2 may average out in $P\left[Y < y_j | Z_1, Z_2\right]$.

Given a perfect data set, this is what we suggest be performed. In our dataset, however, the observations on the events $h_{y_1}(Q), \ldots, h_{y_J}(Q)$ are quite poor (see next section). Therefore, we only use a single h function, namely the conditional mean, i.e., $E^I[Y|Q]$, which we denote by $\tilde{h}(Q)$. Note however that the above logic is still valid, and hence we are able to use

$$P\left[Y < y_j | Z_1, Z_2, V\right],$$

for a grid $y_1, ..., y_J$, where V is now, in abuse of notation, the scalar nonseparable residual in $\tilde{h}(Q) = \tilde{\varphi}(Z, V)$, i.e., $V = F_{E^I[Y|Q]|Z}(E^I[Y|Q]; Z)$. Finally, instead of looking at $P\left[Y < y_j | Z_1, Z_2, V\right]$ for a grid $y_1, ..., y_J$, we now look at a (roughly equivalent) set of quantiles

$$k^{\alpha_j}\left(Y | Z_1, Z_2, V\right),$$

for a grid $\alpha_1, ..., \alpha_J$ of quantiles of Y given Z and V.

Propositions 1 and 2 show that we can test for costly information processing in a two-step procedure. First we focus solely on the data on individual expectations to identify the size of individual information sets, i.e. estimate which information is not used by individuals. Then we ask whether we, as econometricians, would have used the information individuals chose to discard. Under the identifying assumption of weak rationality, we can conclude that information processing has to be costly whenever the discarded information has predictive power for the econometrician's model conditional on the information individuals do use, in particular their unobserved information V, which we estimate as the residual V in the first stage.

3 Empirical Analysis

In this section we will apply this framework to cross-sectional micro-data on individuals' income expectations. As in the theory laid out in Section 2, we will first measure the content of individual information sets and then ask if the micro-data is consistent with models of costly information processing.

3.1 Data Sources

The data we use is from the 'Survey of Household Income and Wealth' (SHIW), collected by the Bank of Italy.[6] The SHIW provides detailed information on individual characteristics, sources of income, and financial assets for about 8000 households (roughly 24.000 individuals). In 1991, the survey included a question on individual income and inflation expectations. The same data was also used in Jappelli and Pistaferri (2000), who use the expectation data as an instrument for consumption growth in a standard Euler equation framework and in Guiso et al. (1996), who show that income expectations are helpful in explaining portfolio choices. The survey does not only elicit point estimates on respondents' expectations (say about their mean income growth) but asks individuals about their entire subjective distribution about future income growth. More precisely, the question about individual income expectations has the following wording: "Think about your entire working income or pension payments. On this card you see several possible categories of growth rates. Which possibilities concerning your income change do you rule out? Assume you could distribute 100 points on the remaining categories: how many points would you give to each category?".[7]

Besides the expectation data, the SHIW survey also contains data on realized income growth Y_i and on various economic characteristics. It will be those characteristics for whose exclusion we will test. Note especially that the entire data is self-reported, i.e. our analysis does not suffer from the problem that individuals might not have access to the information the researcher tests for. So if we conclude that some variable Z is not included in the income expectations, we can rule out the case that Z was not known to the individuals. They clearly knew Z but decided to not use it when forming their income expectations. This aspect of the data is important because it allows us to exclusively focus on the aspect the literature on rational inattention focuses on - in principle individuals have access to a wide range of information but they optimally choose to be inattentive to parts of it.

From an economic point of view we are interested in the capacity of individuals' to forecast their labor income. Hence, we focus only on working males, which are between 20 and 65 years old.

3.2 Descriptive Statistics and Reduced Form Results

Before turning to the nonparametric test, we take a reduced form look at the data to gauge the validity of the reported income expectations. In Table 1 we regress the realized income growth (for both labor and capital income) on individuals' expectations and other characteristics. We

[6] The data and all the programs used to generate the results of this paper are available on our website.

[7] Overall, there are 12 categories with the 10 inner intervals spanning a range between 0% and 25% and the boundary intervals being wider than the inner ones.

see that there is a robust positive correlation between expected and realized income growth for labor income.[8] We view the results in Table 1 as reassuring that individuals' reported income expectations are not merely noise but in fact do have predictive power for realized growth rates. Additionally, the results also show that individuals seem to predict their labor income and not their capital income. In the last two columns we regress the growth rate of capital income on individuals' expectations and do not find any discernible pattern, because the coefficients are very imprecisely estimated. The last column focuses only on individuals reporting non-zero capital income growth. The standard errors decline substantially and the estimated coefficient is statistically zero.

Now consider a first pass to measure the informational content of individual information sets. In Table 2 we report the results of simple OLS regression of individuals' expected income growth on various characteristics. This provides us with a reduced form sense, which information individuals do pay attention to and which not. While current (log) income, local labor market conditions (which are captured by the area dummies) and occupational characteristics are highly significant and therefore not excluded from individuals' income expectations, age and education are not part of individual information sets. In the following we will test these hypothesis non-parametrically as required by the theory.

3.3 Testing for Informational Content

We now turn to the test of individuals' information sets. This could in principle be done by testing the quantile exclusion restrictions outlined in Proposition 1 since, for any y in the support of Y, individuals use Z_2 conditional on $[Z_1, V]$ if and only if:

$$k^v_{G^I_y|Z_1,Z_2}(z_1, z_2) \neq k^v_{G^I_y|Z_1}(z_1) \tag{20}$$

for z_1 with positive probability. Unfortunately, however, data limitations impede us from implementing a test of this exact format. More specifically, we observe that the vast majority of people in our data set heap all available points into one category making it almost impossible to estimate different quantiles of the heterogeneity distribution across different $y_1, \ldots, y_J$. In what follows, we will therefore focus on a functional of that distribution, namely the conditional mean of individuals' expectations, i.e. $\tilde{h}(Q) = E^I[Y|Q]$, and test for the informational content of the distribution of $E^I[Y|Q]$ as a proxy of the test in Proposition 1. As outlined at the end

[8] In its purest form, Table 1 could be considered as a test for the rational expectations hypothesis, according to which we would expect a coefficient of unity on individuals' expected income growth and a coefficient of zero on other characteristics. However, as stressed by Keane and Runkle (1990), this requires the assumption that there are no aggregate shocks, which is unlikely to be the case for our application. In any case, we are *not* testing for the rationality of individuals' expectations but are only concerned with the size of their information sets.

of Section 2.1, this requires us to strengthen the assumption setup and we will assume that changes in the subjective density do not average out when averaging over y.

Let $k^{\alpha_j}_{\tilde{h}(Q)}(Z)$ denote the conditional quantile function of individuals' income expectations for a set of α_j's with $j = 1, \ldots, J$ where each $\alpha_j \in (0, 1)$.[9] Moreover, with slight abuse of notation, we will denote observations of individuals' mean functions of their income expectations by $E^I[Y_i|Q]$. The test is then implemented using the following procedure:

1. We first estimate the conditional quantile functions $k^{\alpha_j}_{\tilde{h}(Q)}(Z_1 = z_1)$ and $k^{\alpha_j}_{\tilde{h}(Q)}(Z_1 = z_1, Z_2 = z_2)$ using a (semi-)linear specification for $j = 1, \ldots, J$.

2. Given the estimates $\hat{k}^{\alpha_j}_{\tilde{h}(Q)}(z_{1,i})$, we generate the residuals

$$\hat{\varepsilon}_{\alpha_j,i} = E^I[Y_i|Q] - \hat{k}^{\alpha_j}_{\tilde{h}(Q)}(z_{1,i}). \tag{21}$$

3. With these residuals at hand, we construct B bootstrap samples with $E^I[Y_i|Q]^\star = \hat{k}^{\alpha_j}_{\tilde{h}(Q)}(z_{1,i}) + \hat{\varepsilon}^\star_{\alpha_j,i}$, where $\hat{\varepsilon}^\star_{\alpha_j,i}$ are the bootstrap residuals, which have been constructed on the basis of $\hat{\varepsilon}_{\alpha_j,i}$ using the wild bootstrap method of Haerdle and Mammen (1993), with a simple adjustment to suit the asymmetric loss function in quantile estimation as suggested by Feng et al. (2011). Crucially, note that $E^I[Y_i|Q]^\star$ is generated under the null, i.e. using the model where the exclusion restriction is imposed.

4. Next, we compute the empirical equivalent of the α_j-th quantile test statistic:

$$\tau^{\alpha_j} = \int [k^{\alpha_j}_{\tilde{h}(Q)}(z_1) - k^{\alpha_j}_{\tilde{h}(Q)}(z_1, z_2)]^2 \omega(z_1, z_2) dz_1 dz_2$$

as:

$$\hat{\tau}^{\alpha_j} = \frac{1}{n} \sum_i [\hat{k}^{\alpha_j}_{\tilde{h}(Q)}(z_{1,i}) - \hat{k}^{\alpha_j}_{\tilde{h}(Q)}(z_{1,i}, z_{2,i})]^2 \omega(z_{1,i}, z_{2,i}), \tag{22}$$

where $\omega(z_{1,i}, z_{2,i})$ is a suitable weighting function.[10]

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[9] We use α_j to avoid confusion with ν, the quantiles of the heterogeneity distribution across different $y_1, ..., y_J$.

[10] In practice, we take the weighting function

$$\omega(z_1) = \begin{cases} 1 & \text{if } z_i \leq q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right) \\ 0 & \text{if } z_i > q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right) \end{cases},$$

where $q^{95}\left((z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)\right)$ is the 95%-quantile of $(z_1 - \bar{z}_1)' \Sigma^{-1}_{Z_1} (z_1 - \bar{z}_1)$ with $\bar{z}_1$ and Σ_{Z_1} denoting the sample mean and covariance of z_1, respectively.

5. Using the B bootstrap samples we then estimate the distribution of τ, say $\hat{H}_\tau$, on the basis of Equation (22).

6. We conclude that Z_2 is excluded from the information sets of the individuals (conditional on Z_1) if $\hat{\tau}^{\alpha_j}$ does not exceed the 95% quantile of $\hat{H}_\tau$.

Before turning to the actual test results, we provide an overview of the variation in the income expectations and in the realized income growth data in Table 3. Notice that in particular around the 0.15 and the 0.25 quantiles, the expected income variable displays very little variation owed to the heaped reporting of individuals, which suggests considerable point mass around the value 0.015.[11] As a consequence, we will - for the main body of the test - focus on quantiles for our test above 0.25, namely the 0.35, the 0.5, and the 0.65 quantiles (Tables for the full set of quantiles can be found in the supplementary material).

For each test, we use three different specifications for the semi-linear conditional quantile function:

$$k^{\alpha_j}_{\tilde{h}(Q)}(z) = g_{\alpha_j}(ln\,(y)\,, a) + w'\beta_{0,\alpha_j},$$

where $g(\cdot)$ is a nonlinear function, $ln\,(y)$ denotes the natural logarithm of income, a age in years, w a vector that contains other covariates such as occupation, area, sector or education dummies, and $z = (y, a, w')$. The first specification is of a linear form and taken to be $k^{\alpha_j}_{\tilde{h}(Q)}(y, a, w) = \gamma_{0,\alpha_j} + \gamma_{1,\alpha_j} ln\,(y) + \gamma_{2,\alpha_j} a + w'\beta_{0,\alpha_j}$. The second and third specifications are nonlinear, where $g(\cdot, \cdot)$ is either modeled as:

$$g_{\alpha_j}(lny, a) = \sum_{i=1}^{K} \gamma_{i,\alpha_j} p_i(ln\,(y)) + \delta_{1,\alpha_j} a + \delta_{2,\alpha_j} a^2,$$

or

$$g_{\alpha_j}(lny, a) = \sum_{i=1}^{K} \gamma_{i,\alpha_j} p_i(a) + \delta_{1,\alpha_j} ln\,(y) + \delta_{2,\alpha_j} ln\,(y)^2\,,$$

and $\sum_{i=1}^{K} \gamma_{i,\alpha_j} p_i(\cdot)$ denotes a linear combination of base functions of a fourth order (cubic) B-spline (the inner knots are chosen to be the $\{0.25, 0.5, 0.75\}$ quantiles of the data).

Step 1: Which information do individuals use when forecasting future income Using this procedure, we can now test for the exclusion of different pieces of information using our results in proposition 1. Our tests will always be based on the reasoning laid out above, i.e. we will test for the exclusion of some information Z_2 via the test "Z_1 vs $[Z_1, Z_2]$". We

[11] In fact, there is a discrete jump between the 0.05 and the 0.06 quantile from -0.0175 to 0.015.

start out from a natural benchmark, namely the case where individuals only perceive a rudimentary life-cycle profile, i.e. use only their age and their current income to predict future income growth. Hence, $Z_1 = [ln(y), age]$. Given Z_1, we then test for the exclusion of *all* other individual characteristics we consider, namely the information contained in individuals' regional, occupational, sectoral and educational characteristics, i.e. $Z_2 = [area, occ, sec, educ]$. The results are contained in Table 4. The first column contains the actual specification for the conditional quantile function, the second column the test statistic (calculated as in Equation (22)), the third column the critical value, i.e. the 95% quantile of the distribution of the test statistic. The last column finally contains the p-value. The first table therefore shows that we can confidently reject this hypothesis across specifications for all three selected quantiles. Hence, individuals use information in addition to their income and age to predict future income growth. We will now decompose which information individuals pay attention to. We begin by considering $Z_1 = [ln(y), age, occ, area)]$ and $Z_2 = [sec, educ]$, i.e. we formally test the null hypothesis that individuals do not condition on their educational and sectoral characteristics, once Z_1 is controlled for. The first part of Table 5 shows indeed that we cannot reject this null hypothesis. For all specifications and all quantiles, the test-statistic is below the critical value at conventional levels of significance.[12] This contrasts with the second part of Table 5, where we test $Z_1 = [ln(y), age, sec, educ]$ and $Z_2 = [occ, area]$ and clearly reject the null hypothesis at all conventional levels, implying that occupational affiliation and regional characteristics play a role even after conditioning on Z_1.

To confirm that it is actually the information contained in both the regional and occupational characteristics that enter individuals' information sets besides current income and age, we conduct two further robustness tests for the expectation data. First we test $Z_1 = [ln(y), age, occ]$ against against $[Z_1, Z_2]$, where $Z_2 = [area, sec, educ]$. Then we reverse the role of localities and occupation and test $Z_1 = [ln(y), age, area]$ against $[Z_1, Z_2]$, where $Z_2 = [occ, sec, educ]$. Both of these exercises are contained in Table 6 and both of these show that we comfortably reject either of these alternatives. Hence, there is useful information in both variables, which individuals use when forming their expectations about future income.

Finally, we test whether age, which was found to be insignificant in the reduced form case (see Table 2), actually plays a role in the quantile context. Thus, we test $Z_1 = [ln(y), occ, area)]$ against $[Z_1, Z_2]$, where $Z_2 = [age]$.[13] While for individuals around the 0.35 quantile age does not

[12]To check whether any of these two variables matters individually, we repeated this test against the alternatives $Z_2 = [educ]$ and $Z_2 = [sec]$ individually. As expected, in both cases the null hypothesis could not be rejected at any reasonable significance level across all quantiles (an exception being the 0.35 quantile for the linear specification) leading us to the conclusion that neither educational attainment nor sectoral affiliation seem to enter the individuals' information sets once occupation, area, age, and income were controlled for.

[13]Notice that when conducting a test against omission of age in the nonlinear case, we test either against a linear and quadratic (second specification) or a fourth order B-spline (third specification) term of age.

appear to play an important role when forming their predictions, we observe that it does clearly matter for the 0.5 and the 0.65 quantiles (see Table 7). The latter result holds irrespective of whether we test against a linear or a nonlinear term of age. As the different quantiles absorb individual heterogeneity in unobserved information and hence refer to different types in the population, Table 7 shows that at least a subgroup of individuals in the population actively uses the information in age to predict their future income growth. This underlines the importance of allowing for nonlinear specifications in our setup.

In sum, the four variables log income, age, occupation and area provide a sufficient description of the individuals' information sets in our data and we cannot reject that educational status and sectoral affiliation does not predict individual income expectations, once the former characteristics are controlled for. In the next step, we are now going to ask whether there is evidence that the omission of these variables is due information processing costs, or whether the information in these variables is indeed redundant in the sense that even a decision maker without processing costs had decided to discard this information.

Step 2: Testing for costly information processing In a second step, we now try to infer the causes behind the omission of information. To gauge whether information processing is actually costly, we implement a test on the basis of Proposition 2. More specifically and as outlined at the end of Section 2, we will do so by examining $k^{\alpha_j}(Y|Z_1, Z_2, V)$ for our selected quantiles, where $V = F_{E^I[Y|Q]|Z}(E^I[Y|Q]; Z)$. However, while V is in principle constructed using $Z = \{Z_1^{'}, Z_2^{'}\}$, our data constraints force us to construct V in two different ways, namely as a function of Z_1 and, for comparison reasons, of $\{Z_1^c, Z_2\}$, where Z_1^c denotes the continuous elements of Z_1. The former vector consists of $[ln(y), age, occ, area]$ and the latter one contains $[ln(y), age, sec, educ]$. The V's are then constructed as conditional quantile ranks with ten types (0-10%, 10-20%, etc.) for each conditioning set.[14] Also, notice that to conduct this test, we restrict our sample to individuals with observations on both expected and realized growth, which reduces the sample size to 1418.[15]

Turning to the first part of results in Table 8, we observe that we can clearly reject the null hypothesis of costless information processing across all quantiles and specifications at conventional levels of significance. This test outcome is confirmed when examining the second set of results for the case where V is constructed as a function of Z_2 as well. Hence, the conclusion from this table is twofold: firstly, it appears that the construction of V, albeit not fully in line with the theoretical setup, does have very little influence on the actual test outcome

[14] To construct the conditioning sets for the continuos variables, we used a kernel function $K(u) = \mathbb{I}\{|u| < h\}$ with $u = (z_1^c - Z_{1i}^c)$ and bandwidth $h = std(Z_{1i}^c)$, where $std(\cdot)$ denotes the standard deviation.

[15] Unreported results show that the key results of testing $Z_1 = [ln\,(y)\,, age, occ, area)]$ against against $[Z_1, Z_2]$, where $Z_2 = [sec, educ]$ are not affected by this reduction of the sample size.

leading us to conjecture that the misspecification is rather innocuous in our case. Secondly and more importantly, we can clearly reject the hypothesis of costless information processing since the fact that we as econometricians consider $Z_2 = \{educ, sec\}$ to be valuable conditional on knowing individuals' mean income expectations implies that agents should consider that information valuable as well (by weak rationality).[16]

As a final remark, notice that all hypotheses above have been tested independently. This decision, albeit theoretically somewhat questionable, appears relatively innocuous in our case as most of the fundamental conclusions in this section have been drawn on the basis of test statistics that lay very far from either side of the corresponding critical value. For instance, applying the Holm-Bonferroni adjustment method with a significance level of $\alpha = 0.05$ to the test sequence above would not alter any of the test conclusions.[17]

4 The Value of Information

The fact consumers seems to exclude information from their information set, which we as econometricians would include, is consistent with the presence of information processing costs as stressed in the literature on rational inattention (Sims (2003); Mackowiak and Wiederholt (2000); Luo (2008); Reis (2004)). In this section, we are going to quantify such processing costs within the realms of a standard life-cycle model of consumption. This model is not only a natural starting point to analyze the value of information, but it also follows very natural from our econometric application: predicting future income if precisely the crucial forecasting problem, individuals have to perform.

Our approach is the following. We consider a standard life-cycle problem, where individual face income risk and markets are exogenously incomplete in that only a risk-less bond is available. There are no other constraints on borrowing. Parametrizing the income process requires us to distinguish between the predictable component of future income and the perceived innovation. It is at this point, where differences in the agents' information set come in. Given the same microdata on income realizations, variations in the information set used to predict future income growth, will lead to different decompositions of the income process into its predictable

[16] As a final check for the usefulness of the discarded information, we also replicated the entire analysis in Step 1 using the data on realized income growth as a dependent variable (see Tables 19 - 24 in the supplementary material for complete results). For instance, as seen in Table 9, we can comfortably reject that the information in educational attainment or sectoral affiliation is not predictive for income realizations, once the remaining observable information is controlled for. Note that these tests are different than the one reported in Table 8: in the latter we explicitly control for the estimated unobserved information V individuals employ to predict income growth.

[17] If we also account for the test sequence described in footnote 12, only some of the conclusions on the 0.35 and the 0.5 quantile for the two nonlinear specifications are altered.

and unpredictable components and to different behavior as encapsulated in the policy function. To estimate the willingness to pay for information, we will therefore first solve for the optimal consumption and savings policies under the individuals' information set. We will then simulate life-cycle profiles using these policy functions but having income evolve under the law of motion, which we as econometricians could infer from the data. These simulated life-cycle profiles allow us to estimate the utility loss of "operating" under a misspecified information set and the willingness to pay for the econometricians' information set.

4.1 The Environment

We consider a parametrization of the life-cycle model that is standard in the literature and for example used by Carroll (1997); Gourinchas and Parker (2002) and Deaton (1991). An infinitely lived consumer chooses consumption to maximize expected utility

$$U = E\left[\sum_{t=1}^{\infty} \beta^t u\left(C_t\right)\right], \tag{23}$$

subject to the per-period budget constraint

$$A_{t+1} = R\left(A_t + Y_t - C_t\right), \tag{24}$$

where Y_t denotes personal income at time t and A_t are the individuals' savings between t and $t+1$. Given an initial condition A_0 and the No-Ponzi condition, (23) and (24) fully characterize the agents' optimal consumption choices for any particular income process $\{Y_t\}_t$ the consumer perceives. We parametrize $\{Y_t\}_t$ in the standard way as

$$Y_t = P_t V_t, \tag{25}$$

where P_t denotes permanent income and V_t is a transitory income shock. The stochastic process for permanent income is given by

$$P_t = G_t P_{t-1} N_t, \tag{26}$$

where G_t denotes the predictable growth in permanent income and N_t is a shock to permanent income. (25) and (26) provide a very parsimonious parametrization of the income process, which nevertheless has been shown to capture salient features of individual income data reasonably well (see e.g. Gourinchas and Parker (2002)). Individuals only need to know the distribution of shocks V_t and N_t and the predictable growth process $\{G_t\}_t$ to know the entire joint distribution of their income process. In particular, suppose that V_t and N_t were log-normally distributed with parameters (μ_V, σ_V^2) and (μ_N, σ_N^2). Then, $(\mu_V, \sigma_V^2, \mu_N, \sigma_N^2, \{G_t\}_t)$ fully characterizes the income process. The concept of permanent income implies that $E[Y_t|P_t] = P_t$, so that (25) requires $\mu_V = -\frac{1}{2}\sigma_V^2$. Similarly, we can always normalize $\mu_N = -\frac{1}{2}\sigma_N^2$ and adjust G_t accordingly.[18]

How would the agents *in this model* predict $(\sigma_V^2, \sigma_N^2, \{G_t\}_t)$? We assume that they follow the rationale of econometricians and hence follow the approach laid out in Carroll and Samwick (1997). Letting $y_t \equiv ln(Y_t)$ (and for the other variables analogously), the growth rate of income is given by

$$y_{t+1} - y_t \;\; = \;\; p_{t+1} + v_{t+1} - p_t - v_t = g_{t+1} + n_{t+1} + v_{t+1} - v_t. \tag{27}$$

Similarly, the h-step difference is

$$r_{h,t} \equiv y_{t+h} - y_t = \sum_{m=1}^{h} g_{t+m} + \sum_{m=1}^{h} n_{t+m} + v_{t+h}^i - v_t^i. \tag{28}$$

According to the logic of the model, g_{t+1} is the predictable component of income growth, i.e. given *their* information set, the agents would estimate

$$E\left[y_{t+1} - y_t|\mathcal{F}^I\right] = g_{t+1} - \frac{1}{2}\sigma_N^2. \tag{29}$$

From (28) and (29), individuals could then calculate the residual

$$\omega_{h,t} \equiv r_{h,t} - \sum_{m=1}^{h} E\left[y_{t+m} - y_{t+m-1}|\mathcal{F}^I\right] = \sum_{m=1}^{h}\left(n_{t+m} + \frac{1}{2}\sigma_N^2\right) + v_{t+h}^i - v_t^i \sim \mathcal{N}\left(0, h\sigma_N^2 + 2\sigma_V^2\right).$$

Hence, given more than 2 observations of income (i.e. a sufficiently long panel), σ_N^2 and σ_V^2 can be estimated from $\left\{\omega_{h,t}^2\right\}_h$.

It is clearly seen from (29) how differences in the information set $\mathcal{F}^I$ will lead to different interpretations of the same data $\{y_{i,t}\}_{i,t}$. Not only will the predictable component of income growth be different, but the backed out residual $\omega_{h,t}$ will also have different statistical properties, which will lead the decision maker to arrive at different estimates for the variance of transitory and permanent shocks.

Table 10 reports the results of this exercise for the two different information sets we estimated. In the first row we report the parameters of the process individuals perceive by only using regional and occupational information to forecast their future income. Given their information set, they conclude that transitory shocks had a variance of 0.0552 and permanent shocks one of 0.0145. In the second row we report the implied model of the econometrician,

[18] Suppose the true process has $ln(N_t) \sim \mathcal{N}(\mu, \sigma^2)$. Then $ln(P_t) | \sim \mathcal{N}(g_t + \mu + p_{t-1}, \sigma_N^2)$. As μ is known to the agent, we can always incorporate in the predictable component g_t and normalize $\mu_N = -\frac{1}{2}\sigma_N^2$.

who also realizes that educational and sectoral information is valuable. By incorporating these sources of information, the perceived variance of both shocks decline. However, Table 10 also suggests that the the differences induced by variations in the information set are not very large. In how far these differences in agents' model environment translate into utility difference is the subject of the next section.

4.2 Optimal Consumption Behavior

Our empirical results indicate that individuals' information sets are well described by including only age, occupational characteristics and education. Hence, the relevant data to solve the life-cycle problem is contained in row three of Table 10, which together with (23), (24), (25) and (26) fully describe the individuals' problem. As usual, it is convenient to write the problem recursively. Conditional on permanent income P_t, the only additional state variable is cash-on-hand $X_t = A_t + Y_t$. This yields the recursive formulation

$$\begin{aligned} V(X,P) &= \max_{A'} \left\{ u\left(X - \frac{1}{R}A' \right) + \beta E^I \left[V(X',P') \,|P \right] \right\} \\ \text{s.t. } & X' = A' + Y' \\ & Y' = GPN'V', \end{aligned} \tag{30}$$

where E^I denotes the expectations taken over the perceived joint distribution of N' and V'. We assume that u takes the CRRA form $u(c) = \frac{c^{1-\theta}}{1-\theta}$. (30) can then be solved numerically in a straightforward manner to yield policy functions π_c^I and π_a^I, where the superscript "I" stresses that these policies are contingent on the individuals' information set. To solve this model, we take standard parameter values, which are displayed in Table 11 below.

The properties of the solution of this problem are well known. As in Carroll (1997), the consumer displays buffer stock behavior. For low value of cash-on-hand (relative to permanent income), the marginal propensity to consume is high and cash-on-hand will grow on average. Once a "target level" of cash-on-hand is reached, where cash-on-hand is expected to stay constant, the marginal propensity to consume declines substantially and is similar to the one of certainty equivalence consumers for high values of cash-on-hand. In particular, the consumption function is concave, as is the value function.

4.3 The Willingness to Pay for Information

By how much would consumers do better if they were to use a more complete information set? Table 10 shows the consequences of these coarse information sets - because individuals use too little information, they erroneously assign variations in their income process to transitory shocks, even though such changes could be predicted based on sectoral, educational and regional information. Given this reasoning, we are going to measure the willingness to pay for this improved forecast by the following criterion: how much would a consumer be willing to pay, if she could use the full information set to estimate $(G, \sigma_N^2, \sigma_V^2)$ instead of the one observed in the data.

To answer this question, we are going to adopt the following procedure. Let π_c^I and π_a^I be the policy functions of a consumer with too small an information set and call $\left\{Y_t^I\right\}_t$ the income process implied by this information set. In contrast, let $\left\{Y_t^F\right\}_t$ be the income process under the full information set, i.e. when using all the valuable information to estimate the predictable component of income growth g_t. In our application, this refers to the last row of Table 10. Now suppose a consumer were to base his behavior on $\left(\pi_c^I, \pi_a^I\right)$ when facing the income process $\left\{Y_t^F\right\}_t$. How much would he be willing to pay to be able to use the policy functions $\left(\pi_c^F, \pi_a^F\right)$, which are the solution to the life-cycle problem, when the income process is indeed perceived to be $\left\{Y_t^F\right\}_t$. We think of this willingness to pay as a lower bound on agents' information processing costs.

To calculate these welfare losses numerically, we are simulating M life-cycle profiles using the income process $\left\{Y_t^F\right\}_t$, but behavior based on $\left(\pi_c^I, \pi_a^I\right)$.[19] Hence: consumers face an income process, which has slightly *less* transitory uncertainty than they thought when they made their consumption and savings plans. With N and V being both entirely idiosyncratic shocks, this corresponds exactly to the empirical distribution of future histories, a consumer could experience.

To measure the willingness to pay for superior information, we then redo this analysis for behavior based on $\left(\pi_c^F, \pi_a^F\right)$, i.e. for the policy functions derived under the correct income process. The difference in ex-ante values of these two scenarios is exactly the utility loss of using a coarse information set. Formally, let $V_F^I(x)$ and $V_F^F(x)$ be the value of facing the income process $\left\{Y_t^F\right\}_t$ with behavior governed by $\left(\pi_c^I, \pi_a^I\right)$ and $\left(\pi_c^F, \pi_a^F\right)$ at a level of cash-on-hand x. We then define the willingness to pay for information $\Delta^{I,F}(x)$ implicitly by

$$V_F^F\left(x\left(1 + \Delta^{I,F}(x)\right)\right) = V_F^I(x). \tag{31}$$

Hence, $\Delta^{I,F}(x)$ is the required relative change in cash-on-hand, which would make an informed consumer equally well off as the less informed consumer. By construction we have $V_F^I(x) < V_F^F(x)$ so that $\Delta^{I,F}(x) < 0$. The results of this exercise in our application are contained in Table 12, which reports $\Delta^{I,F}(x)$ for the different quantiles of the stationary distribution of cash-on-hand.

It is clearly seen that the utility loss form coarse information is small. On average, consumers would be willing to pay roughly 0.04% of their cash-on-hand (relative to permanent income).

[19]In practice we take $M = 50.000$.

Hence, the utility loss is very minor as consumers are sufficiently well self-insured to not be materially affected by their slight overestimate of uncertainty. In fact: precisely because they consider the world as more risky, they will accumulate a bigger buffer stock of savings compared to the well-informed counterpart. Hence, uninformed consumers hold slightly "too much" assets, which however does not have large utility consequences.

5 Conclusion

What information do individuals use when they form expectations about future events? In this paper we present an econometric framework to answer that question and apply our methods to the case of individuals' income expectations. Using micro-data on agents' beliefs about wage growth, we show that information sets are relatively coarse: while individuals do incorporate occupational characteristics, their age (or their labor market experience) and local labor market conditions in their income forecasts, we do not find evidence for educational characteristics or sectoral affiliation or to matter. As this information is self-reported, i.e. in principle available, we interpret this informational coarseness as being consistent with costly information processing. To gauge the utility consequences of this behavior, we calibrate a standard consumption life-cycle model using consumers' information sets from the micro-data. On average consumers would be willing to pay 0.04% of their permanent income to have access to the information set of the econometrician. This represents a lower bound on the costs of information processing.

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Tables

		Realized Growth of Labor Income				Realized Growth of Capital Income	
Exp. Growth	0.517**	0.637**	0.504**	0.336**	0.112**	4.080	-0.224
	(0.114)	(0.170)	(0.120)	(0.101)	(0.0491)	(4.197)	(0.353)
ln(wages)		-0.233*	-0.382**	-0.482**	-0.145**	-0.187	-0.0600
		(0.139)	(0.194)	(0.226)	(0.0113)	(0.387)	(0.0572)
age		0.0195*	0.0266**	0.00594	0.0112**	0.181**	0.00974
		(0.0110)	(0.00706)	(0.00832)	(0.00221)	(0.0919)	(0.0123)
age^2		-0.000259**	-0.000329**	-0.0000389	-0.000122**	-0.00230**	-0.000106
		(0.000127)	(0.0000867)	(0.000106)	(0.0000260)	(0.00109)	(0.000137)
N	2075	2075	2075	1827	1665	1841	1571
R^2	0.004	0.066	0.115	0.168	0.132	0.005	0.015

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$

Notes: Robust standard errors in parentheses. Specifications 3 to 7 control for a full set of education, industry, occupation and area fixed effects.

Table 1: Predictive Power of Income Expectations

	(1)	(2)	(3)	(4)	(5)	(6)
	Exp. Growth	Exp. Growth	Exp. Growth	Exp. Growth	Exp. Growth	Exp. Growth
log(wages)	0.00611**	0.00342	0.00112	0.00777**	0.00657**	0.00270
	(0.00243)	(0.00281)	(0.00295)	(0.00253)	(0.00255)	(0.00317)
age	0.000332	0.000420	0.000452	0.000201	0.000348	0.000350
	(0.000604)	(0.000606)	(0.000602)	(0.000604)	(0.000603)	(0.000608)
age^2	-0.00000796	-0.00000860	-0.00000945	-0.00000688	-0.00000811	-0.00000841
	(0.00000706)	(0.00000706)	(0.00000705)	(0.00000705)	(0.00000706)	(0.00000708)
	1	2	3	4	5	6
F-Test: Education		1.66 (0.155)				0.46 (0.76)
F-Test: Occupation			6.34 (0.00)			4.16 (0.01)
F-Test: Region				7.56 (0.00)		7.16 (0.00)
F-Test: Sector					1.74 (0.156)	3.23 (0.0214)
N	3196	3196	3196	3196	3196	3196
R^2	0.008	0.010	0.014	0.017	0.009	0.025

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$

Notes: Robust standard errors in parentheses. Specifications 2 to 5 control for education, occupation, area, and sectoral fixed effects, respectively. Specification 6 controls for all different types of fixed effects

Table 2: Individual Information Sets: Reduced Form Estimates

α-Quantile:	$E^I[Y_i\|Q]$	Y_i
0.05	-0.0175	-0.3267
0.15	0.015	-0.1582
0.25	0.015	-0.0965
0.35	0.0175	-0.0495
0.45	0.0377	-0.0157
0.55	0.04	0.0149
0.65	0.0475	0.0529
0.75	0.0587	0.1048
0.85	0.075	0.1756
0.95	0.1145	0.3845
# of obs.	3196	1755

Table 3: Quantiles of Individuals' Expectations $E^I[Y_i|Q]$ and Realizations Y_i

Restricted Model:	Income, Age			
Excluded Information:	Occupation, Area, Education, Sector			
Model Specification: $k^{\alpha_j}_{\tilde{h}(Q)}(z)$	Quantile	Statistic	95% CV	P value
	0.35	5.744	1.735	0.000
$= \gamma_{0,\nu} + \gamma_{1,\nu} y + \gamma_{2,\nu} a + w'\beta_{0,\nu}$	0.5	89.646	5.332	0.000
	0.65	4.086	1.557	0.000
	0.35	8.212	1.968	0.000
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(y) + \delta_{1,\nu} a + \delta_{2,\nu} a^2 + w'\beta_{0,\nu}$	0.5	14.224	1.878	0.000
	0.65	3.628	1.612	0.000
	0.35	4.893	1.694	0.000
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(a) + \delta_{1,\nu} y + \delta_{2,\nu} y^2 + w'\beta_{0,\nu}$	0.5	10.227	2.361	0.000
	0.65	3.376	1.602	0.000

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 4: Basic Income Profile

Restricted Model: Excluded Information:	Income, Age, Occupation, Area Sector, Education			
Model Specification: $k^{\alpha_j}_{\tilde{h}(Q)}(z)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\nu} + \gamma_{1,\nu} y + \gamma_{2,\alpha} a + w'\beta_{0,\nu}$	0.35	1.484	2.184	0.164
	0.5	1.197	1.665	0.244
	0.65	1.339	1.591	0.136
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(y) + \delta_{1,\nu} a + \delta_{2,\nu} a^2 + w'\beta_{0,\nu}$	0.35	0.926	1.866	0.460
	0.5	1.045	1.620	0.408
	0.65	1.136	1.503	0.312
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(a) + \delta_{1,\nu} y + \delta_{2,\nu} y^2 + w'\beta_{0,\nu}$	0.35	1.465	2.061	0.148
	0.5	1.014	1.707	0.400
	0.65	1.404	1.493	0.064
Restricted Model: Excluded Information:	Income, Age, Sector, Education Occupation, Area			
Model Specification: $k^{\alpha_j}_{y}(z)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\alpha} + \gamma_{1,\alpha} y + \gamma_{2,\alpha} a + w'\beta_{0,\nu}$	0.35	9.526	2.442	0.000
	0.5	5.333	2.041	0.000
	0.65	2.451	1.601	0.000
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(y) + \delta_{1,\nu} a + \delta_{2,\nu} a^2 + w'\beta_{0,\nu}$	0.35	9.996	2.154	0.000
	0.5	3.868	1.762	0.000
	0.65	2.349	1.598	0.000
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(a) + \delta_{1,\nu} y + \delta_{2,\nu} y^2 + w'\beta_{0,\nu}$	0.35	7.680	2.116	0.000
	0.5	3.193	1.655	0.000
	0.65	1.958	1.497	0.004

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 5: Sufficient Information Set

Restricted Model: Excluded Information:	Income, Age, Occupation Area, Education, Sector			
Model Specification: $k^{\alpha_j}_{\tilde{h}(Q)}(z)$	Quantile	Statistic	95% CV	P value
	0.35	14.667	2.365	0.000
$=\gamma_{0,\nu}+\gamma_{1,\nu}y+\gamma_{2,\nu}a+w'\beta_{0,\nu}$	0.5	5.963	1.713	0.000
	0.65	2.786	1.573	0.000
	0.35	12.411	2.145	0.000
$=\sum_{j=1}^{K}\gamma_{i,\nu}p_i(y)+\delta_{1,\nu}a+\delta_{2,\nu}a^2+w'\beta_{0,\nu}$	0.5	4.088	1.775	0.000
	0.65	2.580	1.648	0.000
	0.35	12.865	2.132	0.000
$=\sum_{j=1}^{K}\gamma_{i,\nu}p_i(a)+\delta_{1,\nu}y+\delta_{2,\nu}y^2+w'\beta_{0,\nu}$	0.5	6.063	2.448	0.000
	0.65	2.211	1.569	0.000
Restricted Model: Excluded Information:	Income, Age, Area Occupation, Education, Sector			
Model Specification: $k^{\alpha_j}_{\tilde{h}(Q)}(z)$	Quantile	Statistic	95% CV	P value
	0.35	3.791	1.651	0.000
$=\gamma_{0,\nu}+\gamma_{1,\nu}y+\gamma_{2,\nu}a+w'\beta_{0,\nu}$	0.5	4.847	1.912	0.000
	0.65	2.306	1.525	0.000
	0.35	3.242	1.793	0.004
$=\sum_{j=1}^{K}\gamma_{i,\nu}p_i(y)+\delta_{1,\nu}a+\delta_{2,\nu}a^2+w'\beta_{0,\nu}$	0.5	2.265	1.855	0.012
	0.65	1.841	1.526	0.004
	0.35	2.915	1.619	0.000
$=\sum_{j=1}^{K}\gamma_{i,\nu}p_i(a)+\delta_{1,\nu}y+\delta_{2,\nu}y^2+w'\beta_{0,\nu}$	0.5	2.335	1.663	0.012
	0.65	2.287	1.445	0.000

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 6: Testing Robustness of Final Information Set

Restricted Model: Excluded Information:	Income, Occupation, Area Age			
	Expected Income Growth			
Model Specification: $k^{\alpha_j}_{\tilde{h}(Q)}(z)$	Quantile	Statistic	95% Crit. Value	P value
	0.35	0.452	2.740	0.720
$= \gamma_{0,\nu} + \gamma_{1,\nu} y + \gamma_{2,\nu} a + w'\beta_{0,\nu}$	0.5	6.758	2.713	0.008
	0.65	2.261	2.011	0.020
	0.35	0.552	2.468	0.684
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(y) + \delta_{1,\nu} a + \delta_{2,\nu} a^2 + w'\beta_{0,\nu}$	0.5	3.717	2.543	0.008
	0.65	1.881	1.855	0.044
	0.35	0.680	2.705	0.572
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(a) + \delta_{1,\nu} y + \delta_{2,\nu} y^2 + w'\beta_{0,\nu}$	0.5	5.984	2.367	0.008
	0.65	2.360	1.692	0.000

Note: In the nonlinear specifications, we test with $Z_2 = [\text{age}, \text{age}^2]$ and $Z_2 = [g(\text{age})]$, where $g(\cdot)$ is a fourth order B-spline.

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 7: Nonlinearities in the Age Profile

Restricted Model: Excluded Information:	Income, Age, Occupation, Area, V Education, Sector			
$V = \vartheta(E^I[Y\|Q]; Z),\ Z = \{Z_1\}$				
Model Specification: $k_y^{\alpha_j}(lny, a, w, v)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\alpha_j} + \gamma_{1,\alpha_j} lny + \gamma_{2,\alpha_j} a + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	3.7248	3.8006	0.056
	0.5	9.4809	3.7235	0.000
	0.65	16.2833	3.3557	0.000
$= \sum_{j=1}^{K} \gamma_{i,\alpha_j} p_i(lny) + \delta_{1,\alpha_j} a + \delta_{2,\alpha_j} a^2 + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	9.0759	3.751	0.000
	0.5	8.8684	3.283	0.000
	0.65	6.3322	3.9692	0.008
$= \sum_{j=1}^{K} \gamma_{i,\alpha_j} p_i(a) + \delta_{1,\alpha_j} lny + \delta_{2,\alpha_j} lny^2 + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	5.3653	3.2124	0.016
	0.5	10.1637	4.1438	0.000
	0.65	8.0034	3.8275	0.004
$V = \vartheta(E^I[Y\|Q]; Z),\ Z = \{Z_1^c, Z_2\}$				
Model Specification: $k_y^{\alpha_j}(lny, a, w, v)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\alpha_j} + \gamma_{1,\alpha_j} lny + \gamma_{2,\alpha_j} a + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	5.2276	3.8588	0.012
	0.5	9.6453	4.6645	0.000
	0.65	13.1217	3.0385	0.000
$= \sum_{j=1}^{K} \gamma_{i,\alpha_j} p_i(lny) + \delta_{1,\alpha_j} a + \delta_{2,\alpha_j} a^2 + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	9.6582	3.7812	0.004
	0.5	8.2847	3.5995	0.004
	0.65	4.1463	3.4879	0.032
$= \sum_{j=1}^{K} \gamma_{i,\alpha_j} p_i(a) + \delta_{1,\alpha_j} lny + \delta_{2,\alpha_j} lny^2 + w'\beta_{0,\alpha_j} + v\beta_{1,\alpha_j}$	0.35	5.4381	3.7761	0.012
	0.5	8.7521	3.5148	0.004
	0.65	4.8307	3.7125	0.028

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Note: $Z_1 = \{income, age, occupation, area\}$, $Z_2 = \{sector, education\}$, and $Z_1^c = \{income, age\}$.

Table 8: Test for Costly Information Processing

Restricted Model: Excluded Information:	Income, Age, Occupation, Area Sector, Education			
Model Specification: $k_y^{\alpha_j}(z)$	Quantile	Statistic	95% CV	P value
$= \gamma_{0,\alpha} + \gamma_{1,\alpha} y + \gamma_{2,\alpha} a + w'\beta_{0,\nu}$	0.35	2.529	1.668	0.000
	0.5	3.948	1.674	0.000
	0.65	3.146	1.589	0.000
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(y) + \delta_{1,\nu} a + \delta_{2,\nu} a^2 + w'\beta_{0,\nu}$	0.35	2.926	1.531	0.000
	0.5	2.601	1.572	0.000
	0.65	2.498	1.564	0.004
$= \sum_{j=1}^{K} \gamma_{i,\nu} p_i(a) + \delta_{1,\nu} y + \delta_{2,\nu} y^2 + w'\beta_{0,\nu}$	0.35	2.756	1.570	0.000
	0.5	3.782	1.527	0.000
	0.65	2.718	1.492	0.000

Note: We use 250 bootstrap iterations and normalize the mean of the test statistic to unity.

Table 9: Results for the Exclusion in the data on Income Realizations

$\mathcal{F}$	σ_V^2	σ_N^2	$E[g_t]$
age, occupation, area	0.0552	0.0145	0.0346
age, occupation, area, education, sector	0.0547	0.0143	0.0345

Table 10: Perceived Income Processes as a Function of the Information Set

Parameter	Value
β	0.94
R	1.02
θ	1
$(G, \sigma_V^2, \sigma_N^2)$	see Table 10

Table 11: Parameter Values for Life-Cycle Problem

Quantile						Mean
0.1	0.25	0.35	0.5	0.65	0.75	
-0.0407	-0.0389	-0.0389	-0.0389	-0.0413	-0.0395	-0.0388

Table 12: Willingness to Pay for Information

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Table 13: Full Results of Table 4 - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age	sector, education, area, occupation	0.35	5.745	1.735	0.000
		0.45	3.902	1.531	0.000
		0.55	55.124	5.999	0.000
		0.65	4.086	1.557	0.000
		0.75	3.130	1.556	0.000
		0.85	3.111	1.697	0.000
		0.95	1.514	1.956	0.172
income, age	sector, education, area, occupation	0.35	8.212	1.968	0.000
		0.45	3.080	1.479	0.000
		0.55	20.027	2.655	0.000
		0.65	3.628	1.612	0.000
		0.75	2.796	1.655	0.000
		0.85	2.690	1.564	0.000
		0.95	2.264	2.036	0.016
income, age	sector, education, area, occupation	0.35	4.893	1.694	0.000
		0.45	4.054	1.536	0.000
		0.55	51.252	4.412	0.000
		0.65	3.376	1.602	0.000
		0.75	2.945	1.616	0.000
		0.85	3.190	1.637	0.000
		0.95	1.597	2.180	0.188

Table 14: Full Results of Table 5 (1st part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation, area	sector, education	0.35	1.484	2.184	0.164
		0.45	0.903	1.532	0.624
		0.55	1.536	1.708	0.092
		0.65	1.339	1.591	0.136
		0.75	1.376	1.592	0.108
		0.85	1.412	1.593	0.124
		0.95	1.099	2.058	0.328
income, age, occupation, area	sector, education	0.35	0.926	1.866	0.460
		0.45	0.745	1.520	0.804
		0.55	1.270	1.584	0.168
		0.65	1.136	1.503	0.312
		0.75	0.923	1.638	0.556
		0.85	1.175	1.551	0.252
		0.95	1.644	2.086	0.100
income, age, occupation, area	sector, education	0.35	1.465	2.061	0.148
		0.45	0.804	1.508	0.752
		0.55	1.241	1.651	0.208
		0.65	1.404	1.494	0.064
		0.75	1.072	1.578	0.368
		0.85	1.287	1.543	0.168
		0.95	0.894	2.211	0.468

Table 15: Full Results of Table 5 (2nd part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, sector, education	area, occupation	0.35	9.527	2.442	0.000
		0.45	2.681	1.633	0.000
		0.55	6.160	1.991	0.000
		0.65	2.451	1.601	0.000
		0.75	1.948	1.584	0.008
		0.85	2.197	1.626	0.008
		0.95	1.837	2.224	0.096
income, age, sector, education	area, occupation	0.35	9.996	2.155	0.000
		0.45	2.623	1.505	0.000
		0.55	5.160	1.864	0.000
		0.65	2.349	1.598	0.000
		0.75	1.874	1.538	0.004
		0.85	1.892	1.483	0.008
		0.95	2.378	1.977	0.008
income, age, sector, education	area, occupation	0.35	7.680	2.117	0.000
		0.45	2.852	1.682	0.000
		0.55	4.674	1.775	0.000
		0.65	1.958	1.497	0.004
		0.75	1.826	1.496	0.008
		0.85	2.070	1.707	0.008
		0.95	1.227	2.154	0.288

Table 16: Full Results of Table 6 (1st part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation	area, education, sector	0.35	14.668	2.365	0.000
		0.45	3.402	1.702	0.000
		0.55	12.354	2.761	0.000
		0.65	2.786	1.573	0.000
		0.75	2.816	1.590	0.000
		0.85	2.601	1.565	0.000
		0.95	2.274	1.939	0.024
income, age, occupation	area, education, sector	0.35	12.411	2.145	0.000
		0.45	2.648	1.544	0.000
		0.55	9.387	2.211	0.000
		0.65	2.580	1.648	0.000
		0.75	2.295	1.540	0.000
		0.85	2.677	1.633	0.000
		0.95	2.710	2.057	0.008
income, age, occupation	area, education, sector	0.35	12.865	2.132	0.000
		0.45	2.721	1.597	0.000
		0.55	4.647	1.998	0.000
		0.65	2.211	1.569	0.000
		0.75	2.157	1.585	0.000
		0.85	2.417	1.542	0.000
		0.95	1.344	2.373	0.204

Table 17: Full Results of Table 6 (2nd part) - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, area	occupation, education, sector	0.35	3.792	1.660	0.000
		0.45	1.791	1.560	0.012
		0.55	2.981	1.664	0.000
		0.65	2.306	1.525	0.000
		0.75	1.805	1.566	0.020
		0.85	2.043	1.571	0.004
		0.95	1.242	2.558	0.232
income, age, area	occupation, education, sector	0.35	3.242	1.793	0.004
		0.45	1.521	1.545	0.064
		0.55	2.729	1.708	0.004
		0.65	1.841	1.526	0.004
		0.75	1.386	1.616	0.096
		0.85	1.623	1.556	0.036
		0.95	1.179	2.147	0.304
income, age, area	occupation, education, sector	0.35	2.915	1.619	0.000
		0.45	1.837	1.511	0.004
		0.55	2.263	1.561	0.004
		0.65	2.287	1.445	0.000
		0.75	1.619	1.597	0.044
		0.85	1.814	1.607	0.012
		0.95	1.009	2.178	0.392

Table 18: Full Results of Table 7 - Specifications are as in the paper

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, occupation, area	age	0.35	0.452	2.740	0.720
		0.45	2.122	2.277	0.060
		0.55	1.891	2.345	0.124
		0.65	2.261	2.011	0.020
		0.75	3.483	2.453	0.004
		0.85	2.331	2.067	0.016
		0.95	1.208	2.406	0.304
income, occupation, area	age, age^2	0.35	0.552	2.468	0.684
		0.45	2.249	1.891	0.012
		0.55	2.005	2.099	0.064
		0.65	1.881	1.855	0.044
		0.75	1.737	1.648	0.036
		0.85	1.219	1.696	0.240
		0.95	0.910	2.330	0.456
income, occupation, area	g(age)	0.35	0.680	2.705	0.572
		0.45	1.427	1.671	0.116
		0.55	2.784	1.730	0.000
		0.65	2.360	1.692	0.000
		0.75	2.725	1.741	0.000
		0.85	3.552	1.710	0.000
		0.95	4.767	2.612	0.000

Table 19: Full Results of Table 4 - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age	sector, area, education, occupation	0.05	1.627	2.059	0.144
		0.15	4.940	1.770	0.000
		0.25	5.199	1.706	0.000
		0.35	6.352	1.662	0.000
		0.45	8.245	1.551	0.000
		0.55	7.246	1.531	0.000
		0.65	7.132	1.557	0.000
		0.75	7.765	1.707	0.000
		0.85	6.938	1.819	0.000
		0.95	3.534	2.204	0.004
income, age	sector, area, education, occupation	0.05	1.556	2.063	0.156
		0.15	4.419	1.652	0.000
		0.25	4.305	1.571	0.000
		0.35	5.283	1.469	0.000
		0.45	7.577	1.600	0.000
		0.55	6.652	1.510	0.000
		0.65	6.163	1.566	0.000
		0.75	5.528	1.635	0.000
		0.85	4.538	1.724	0.000
		0.95	3.469	2.278	0.004
income, age	sector, area, education, occupation	0.05	1.600	2.158	0.148
		0.15	4.236	1.877	0.000
		0.25	4.716	1.615	0.000
		0.35	6.576	1.506	0.000
		0.45	7.449	1.609	0.000
		0.55	6.315	1.605	0.000
		0.65	5.858	1.503	0.000
		0.75	6.174	1.692	0.000
		0.85	5.533	1.808	0.000
		0.95	3.912	2.354	0.004

Table 20: Full Results of Table 5 (1st part) - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation, area	sector, education	0.05	1.142	2.393	0.364
		0.15	2.278	1.754	0.020
		0.25	2.786	1.851	0.004
		0.35	2.529	1.668	0.000
		0.45	3.202	1.678	0.000
		0.55	2.951	1.573	0.000
		0.65	3.146	1.589	0.000
		0.75	2.953	1.651	0.000
		0.85	3.148	1.806	0.000
		0.95	3.068	2.315	0.004
income, age, occupation, area	sector, education	0.05	1.371	2.276	0.256
		0.15	2.614	1.781	0.008
		0.25	2.925	1.634	0.000
		0.35	2.926	1.531	0.000
		0.45	2.969	1.604	0.000
		0.55	2.254	1.534	0.000
		0.65	2.498	1.564	0.004
		0.75	2.154	1.492	0.000
		0.85	2.560	1.610	0.000
		0.95	2.165	2.273	0.056
income, age, occupation, area	sector, education	0.05	1.962	2.112	0.076
		0.15	2.659	1.746	0.004
		0.25	2.557	1.595	0.000
		0.35	2.756	1.570	0.000
		0.45	2.779	1.550	0.000
		0.55	2.197	1.540	0.004
		0.65	2.718	1.492	0.000
		0.75	2.609	1.618	0.000
		0.85	3.046	1.537	0.000
		0.95	3.227	2.344	0.004

Table 21: Full Results of Table 5 (2nd part) - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, sector, education	occupation, area	0.05	0.876	2.283	0.480
		0.15	1.531	1.650	0.080
		0.25	1.990	1.669	0.008
		0.35	2.259	1.618	0.000
		0.45	2.047	1.587	0.000
		0.55	2.216	1.629	0.004
		0.65	2.605	1.682	0.000
		0.75	1.961	1.598	0.012
		0.85	1.588	1.621	0.064
		0.95	1.930	2.492	0.096
income, age, sector, education	occupation, area	0.05	0.688	2.240	0.636
		0.15	1.389	1.886	0.172
		0.25	1.703	1.617	0.040
		0.35	1.850	1.494	0.000
		0.45	2.160	1.723	0.000
		0.55	1.959	1.565	0.004
		0.65	1.604	1.555	0.032
		0.75	1.370	1.646	0.156
		0.85	0.917	1.602	0.548
		0.95	2.002	1.947	0.040
income, age, sector, education	occupation, area	0.05	0.536	2.014	0.796
		0.15	1.424	1.612	0.116
		0.25	1.673	1.541	0.032
		0.35	1.965	1.491	0.000
		0.45	2.402	1.521	0.000
		0.55	2.416	1.531	0.000
		0.65	1.792	1.496	0.020
		0.75	1.436	1.577	0.108
		0.85	1.362	1.592	0.120
		0.95	1.084	2.377	0.356

Table 22: Full Results of Table 6 (1st part) - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, occupation	sector, area, education	0.05	1.535	2.494	0.160
		0.15	3.085	1.791	0.000
		0.25	4.021	1.638	0.000
		0.35	4.038	1.549	0.000
		0.45	4.633	1.560	0.000
		0.55	4.211	1.653	0.000
		0.65	4.667	1.672	0.000
		0.75	3.871	1.639	0.000
		0.85	3.702	1.601	0.000
		0.95	2.581	2.319	0.032
income, age, occupation	sector, area, education	0.05	1.523	2.107	0.172
		0.15	3.295	1.713	0.000
		0.25	3.687	1.536	0.000
		0.35	4.273	1.617	0.000
		0.45	4.942	1.509	0.000
		0.55	3.125	1.499	0.000
		0.65	3.529	1.569	0.000
		0.75	3.095	1.562	0.000
		0.85	3.025	1.744	0.000
		0.95	2.753	2.017	0.008
income, age, occupation	sector, area, education	0.05	1.559	2.057	0.168
		0.15	2.895	1.888	0.000
		0.25	3.499	1.610	0.000
		0.35	4.271	1.487	0.000
		0.45	4.418	1.497	0.000
		0.55	3.025	1.532	0.000
		0.65	3.931	1.623	0.000
		0.75	3.481	1.543	0.000
		0.85	2.910	1.707	0.000
		0.95	1.980	2.108	0.080

Table 23: Full Results of Table 6 (2nd part) - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, age, area	sector, occupation, education	0.05	1.825	1.856	0.060
		0.15	4.839	1.656	0.000
		0.25	4.640	1.853	0.000
		0.35	5.938	1.656	0.000
		0.45	7.481	1.610	0.000
		0.55	6.953	1.601	0.000
		0.65	7.280	1.611	0.000
		0.75	8.044	1.794	0.000
		0.85	6.167	1.901	0.000
		0.95	4.557	2.649	0.004
income, age, area	sector, occupation, education	0.05	2.561	2.069	0.020
		0.15	4.233	1.801	0.000
		0.25	4.207	1.735	0.000
		0.35	5.043	1.635	0.000
		0.45	6.302	1.600	0.000
		0.55	6.031	1.590	0.000
		0.65	5.941	1.571	0.000
		0.75	6.356	1.623	0.000
		0.85	4.507	1.674	0.000
		0.95	3.374	2.075	0.012
income, age, area	sector, occupation, education	0.05	1.555	1.959	0.136
		0.15	4.378	1.780	0.000
		0.25	4.115	1.608	0.000
		0.35	5.679	1.566	0.000
		0.45	6.562	1.707	0.000
		0.55	6.172	1.521	0.000
		0.65	5.919	1.594	0.000
		0.75	6.775	1.583	0.000
		0.85	4.838	1.897	0.000
		0.95	1.538	5.625	0.164

Table 24: Full Results of Table 7 - Data on Income Realizations

Restricted Model	Excluded information	Quantile	Statistic	95% CV	P value
income, occupation, area	age	0.05	0.060	2.869	1.000
		0.15	0.291	1.907	0.968
		0.25	0.777	1.848	0.644
		0.35	0.599	1.885	0.796
		0.45	1.294	1.826	0.244
		0.55	1.436	1.877	0.164
		0.65	1.035	1.894	0.416
		0.75	0.906	1.752	0.516
		0.85	0.747	1.896	0.680
		0.95	2.514	2.744	0.056
income, occupation, area	age, age^2	0.05	0.137	2.449	1.000
		0.15	1.493	1.812	0.128
		0.25	2.796	1.693	0.000
		0.35	3.355	1.771	0.000
		0.45	3.411	1.679	0.000
		0.55	3.072	1.637	0.000
		0.65	2.432	1.763	0.000
		0.75	3.851	1.670	0.000
		0.85	3.642	1.679	0.000
		0.95	8.015	2.500	0.000
income, occupation, area	g(age)	0.05	1.599	2.427	0.144
		0.15	3.958	1.778	0.000
		0.25	4.488	1.765	0.000
		0.35	5.620	1.713	0.000
		0.45	7.405	1.708	0.000
		0.55	9.782	1.606	0.000
		0.65	8.217	1.771	0.000
		0.75	8.477	1.613	0.000
		0.85	9.335	1.798	0.000
		0.95	4.282	2.945	0.000