International Medium of Exchange: Privilege and Duty*

Ryan Chahrour
Boston College

Rosen Valchev
Boston College

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Abstract

The United States enjoys an "exorbitant privilege" that allows it to borrow at especially low interest rates. Meanwhile, the dollarization of world trade appears to shield the U.S. from international disturbances. We provide a new theory that links dollarization and exorbitant privilege through the need for an international medium of exchange. We consider a two-country world where international trade happens in decentralized matching markets, and must be collateralized by assets — a.k.a. currencies — issued by one of the two countries. Traders have an incentive to coordinate their currency choices and a single dominant currency arises in equilibrium. With small heterogeneity in traders' information, the model delivers a unique mapping from economic conditions to the dominant currency. Nevertheless, the model delivers a *dynamic* multiplicity: in steady-state either currency can serve as the international medium of exchange. The economy with the dominant currency enjoys lower interest rates and the ability to run current account deficits indefinitely. Currency regimes are stable, but sufficiently large shocks or policy changes can lead to transitions, with large welfare implications.

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*Contact: ryan.chahrour@bc.edu, valchev@bc.edu.

1 Introduction

As the world's primary supplier of safe liquid assets, the United States enjoys a so-called "exorbitant privilege" that allows it to borrow at especially low interest rates. For example, Gourinchas and Rey (2007) estimate that the United States earns an excess return on its foreign assets over foreign liabilities of about 3% a year, which in principle allows it to fund a sizable trade deficit indefinitely. At the same time, authors such as Goldberg (2011) and Gopinath (2015) have observed that a disproportionate share of international trade in both real goods and financial assets is denominated in dollars. The privilege and trade dollarization phenomena have motivated numerous empirical studies that emphasize the United States' special position in international markets. Nevertheless, to our knowledge, there is no model where both key aspects of this special status arise endogenously.

In this paper we provide such a theory, emphasizing the link between exorbitant privilege and the desire for a common medium of international exchange. We develop a dynamic two-country framework in which international trade happens in decentralized, bilateral markets with limited contract enforceability. This requires trading firms to collateralize their cross-country transactions with safe assets which guarantee eventual payment and delivery of goods. Trading firms are free to choose the currency in which they transact — and thus the currency of denomination of the needed collateral — but face a small "currency mismatch" cost if their counter-party uses a different currency. This gives traders incentives to coordinate their currency choices, which gives rise to potentially multiple asymmetric equilibria in which a single dominant currency arises. Moreover, because traders prefer to use a means of exchange that is plentiful in their home country, due to its higher availability, there is a feedback between the currency choice of trading firms and the asset holdings of domestic households. As a result the model features multiple steady-states, each corresponding to a different currency serving as the dominant international medium of exchange.

The economy with the dominant currency enjoys a number of benefits, including lower interest rates and the ability to run current account deficits indefinitely. Foreign demand for the dominant currency generally leads the supplier of that asset to have a negative net foreign asset position, but to earn a positive return on it, similar to the experience of the United States. Importantly, this exorbitant privilege is not exogenously fixed, but is an endogenous equilibrium outcome. Maintaining the dominant currency position requires the country to perform its "duty" to responsibly supply and support the asset that serves as international medium of exchange. Switches between dominant currencies are relatively rare, but sufficiently large shocks or changes in macroeconomic policy or conditions can push the world economy from one to the other, which gives rise to interesting non-linear dynamics. Thus, while rare, such switches can have profound effects on welfare.

We begin our analysis by studying a static coordination game between foreign and domestic firms that engage in international trade. Firms in each country begin the period uncertain as to whether they will be importing or exporting and, conditional on that outcome, who their foreign trading partner will be. When trade occurs, firms encounter their trading counterparts in a bilateral, decentralized, search and matching market. Because firms are not able to fully enforce contracts internationally, they must collateralize a portion of their trade in order to complete their transactions, consistent with the evidence of Amiti and Weinstein (2011) and Ahn (2015).

Firms acquire collateral in their domestic financing market, which operates at the beginning of each period. They may freely choose what type of financing they seek, either in the form of domestic or foreign safe assets, but know that they face a cost of "currency mismatch" if they eventually trade with a counterpart who holds a different type of asset. Given the uncertain identity of their eventual trading partners, and the costs of an ex-post currency mismatch, firms have an incentive to seek collateral funding in the same currency as that held by the foreign firms with whom they are likely to trade. The coordination incentives of trading firms in both countries thus give rise to a discrete choice global game in the spirit of Morris and Shin (1998), albeit with two agent types, foreign firms and domestic firms, rather than one.

Under complete information, the coordination game played by trading firms typically exhibits multiple equilibria: firms will be happy to acquire whatever type of funding they anticipate their eventual trading partners will hold. To resolve the indeterminacy of the model's predictions, we introduce a finite supply of safe assets. Firms therefore have an incentive not only to coordinate their currency choice with foreign firms, but also to choose a currency that is readily available in their home country financing markets. By introducing a small amount of uncertainty in the relative supply of the safe assets à la Morris and Shin (1998), we then recover a unique equilibrium currency choice that depends, crucially, on the availability of both assets in each country. In particular, trading firms tend to coordinate on the asset that is more readily available in *both* countries, as they anticipate that their trading partners would also have an easier time obtaining this type of funding.

We embed our model of currency choice and international trade within an otherwise standard dynamic two-country model of the world economy.[1] Households produce domestic goods and use the proceeds from production to acquire both domestic consumption goods and the foreign consumption goods sold domestically by importing firms. While final goods markets are segmented across countries, a common primary good (oil) is traded freely across countries and serves as the numeraire. Moreover, households freely trade both the domestic and foreign bonds issued by the two countries' governments.

Crucially, household bond holdings return not only the typical principal plus interest, but also earn an extra liquidity return due to the demand for collateral stemming from international trade transactions. Each period, households offer to lend their bonds (intra-period) to domestic trading firms to use as collateral, which happens in a funding market mediated by a search and matching friction. A bond offered and matched with a trading firm earns a fee, which the trading firms pay for the right to borrow the bond and use it as collateral, increasing the effective return of that bond.[2] Thus, households have an incentive *ceteris paribus* to hold bonds that are widely sought by their country's trading firms, as doing so increases the return the bond earns from being used as intra-period collateral. We call this additional return the "liquidity premium" of the asset, because it is a consequence of the asset's usefulness in facilitating international exchange. In equilibrium, the price of an in-demand bond rises, depressing the interest rate paid by the country that issues it.

This environment leads to a strong feedback effect between firms' currency choice and the long-run asset position of households in the two countries. To see this, suppose for example that households in both countries expect trading firms will coordinate on using U.S. dollar assets. If the supply of those U.S. assets is initially primarily held within the U.S., then foreign households have a strong incentive to acquire U.S. bonds, as any bond they do acquire is virtually certain to find a match in their domestic funding market, and earn the associated premium. U.S. households, however, have no corresponding incentive to acquire foreign bonds. To clear the market, the price of U.S. bonds must rise relative to the price of the foreign bond, as does the amount of U.S bonds held by foreigners relative to the quantity of foreign bonds held by U.S. households. In turn, from the perspective of the trading firms, the asset allocation described above — in which U.S. assets are relatively available in all countries — supports an equilibrium in which trading firms coordinate on seeking funding in those assets. Thus, this equilibrium asset allocation and coordinated

[1]Our approach of embedding a static coordination game within a dynamic economy is inspired by Schaal and Taschereau-Dumouchel (2015), who use a similar strategy to study the potential for two absorbing states in an economy with demand externalities.

[2]The intra-period funding essentially functions as a repurchasing agreement.

currency choice reinforce each other.

As a result of this feedback mechanism, the dynamic economy has a multiplicity of steady-states. This is true despite the fact that the economy exhibits a unique equilibrium conditional on the state variables – the households' last period's bond holdings. In other words, the economy exhibits a dynamic multiplicity – it features determinate equilibrium paths with more than one absorbing state. We find that at our benchmark calibration there exist two distinct steady-states where the U.S. asset is the dominant medium of exchange, another two mirror image steady-states where the foreign asset is dominant, and also a symmetric steady-state where both assets are used equally in trade. We find that at one of the dollar dominant steady-states, the U.S. has a negative net foreign asset position, but earns a significant premium on its foreign assets, allowing it to run a permanent trade deficit despite its net debtor position. Moreover, that steady-state also features significant home bias in asset holdings, despite there being no frictions in asset trade. Qualitatively, this situation describes the status quo in the data remarkably well.

While the sluggish adjustment of bond holdings (partially due to optimal consumption smoothing) implies that this situation is potentially sustainable for long periods, it is not necessarily permanent. In our benchmark calibration, we find that all five steady-states are dynamically stable, and feature non-trivial attraction regions. Shocks or policy changes can tip the balance towards or away from a particular steady-state. The attraction regions of the different steady-states are not all connected to each other, however, suggesting that some transitions are more likely than others. For example, the empirically relevant dollar-dominant steady-state, in which the U.S. has a negative net foreign asset position, can transition directly only to either the symmetric steady-state or to the foreign currency dominant steady-state in which the U.S. continues to be a large net debtor. On the other hand, transitions to the other two coordinated steady-states (either dollar or foreign currency dominant), where the U.S. enjoys a positive net foreign asset position, require first passing through the region of attraction that belongs to the symmetric steady-state.

The reason for these patterns is intuitive. A change in currency coordination must be supported by an appropriate adjustment in the portfolio holdings of both households; for a coordinated currency choice to be an equilibrium, the chosen asset must be relatively available in both countries, so that both sides of the market can acquire the needed financing. A portfolio re-alignment that implies a change in the composition of domestic and foreign assets is relatively easy. Hence, the economy can transition smoothly from the dollar-centric steady-state with negative U.S. net foreign assets, to the foreign currency centric steady-state with low U.S. net foreign assets, since it involves primarily a re-shuffling of holdings from U.S. assets to foreign assets, but not big changes in the overall size of the households' portfolios. On the other hand, a direct transition to the dollar-centric steady-state where the U.S. has a positive NFA requires the U.S. to first undergo a large wealth accumulation. This is not optimal from consumption smoothing perspective and is not an equilibrium path.

The above analysis highlights why, through the lens of our model, the current dominant position of the dollar can be described not only as one of privilege, but also as one of duty. The U.S. enjoys the privilege of supplying the world with the dominant medium of exchange, which gives it access to significant excess returns on its foreign assets and allows it to run a permanent trade deficit. Nevertheless, it runs the danger of falling into the worst steady-state situation, where it both remains indebted to the rest of the world, and loses its currency hegemony and the associated low interest rate on its debts. At our benchmark calibration, such transition entails a very significant welfare loss equivalent to 6% of steady-state consumption. Moreover, the danger of transitioning to that steady-state is increasing in the overall indebtedness of the U.S. economy – while it can run a moderate negative NFA position indefinitely, the larger the deficit becomes, the more likely it is that a shock will indeed transition the world economy to the dangerous, foreign currency coordinated steady-state. Thus, we find that in its current position the U.S. also has the "duty" to be a relatively responsible steward to the international financial system, and to not exploit its access to excess returns, by falling deeper and deeper into debt to the rest of the world.

Our work relates to several different strands of the literature. Motivated by some of the same empirical facts cited above, authors have sought to provide explanations for the special position of the U.S. focusing on the three canonical roles of money. In this paper, we provide a model where the dominant position of U.S. assets in the international monetary system is due to their (endogenous) role as the main *medium of exchange*. A number of previous papers instead explore the store of value and unit of account properties of money.

A large portion of the related literature focuses on the capacity of countries to generate good store-of-value assets. Caballero et al. (2008) argue that the United States' superior ability — relative to developing countries — to produce store-of-value assets can explain the U.S. experience of persistent trade deficits, falling interest rates, and rising portfolio share of U.S. assets in developing countries. Maggiori (2017) models the endogenous emergence of a dominant reserve currency, in which dominance emerges because a higher level of financial development in the dominant country lead it to provide insurance vis-a-vis the rest of the world. Gourinchas et al. (2010) propose a different insurance framework, where the U.S.

households are assumed to have lower risk-aversion than foreign households (perhaps due to an endogenous mechanism like the one in Maggiori, 2017) and, thus, in equilibrium end up taking on most of the world's risky assets in their portfolio. Meanwhile, Farhi and Maggiori (2016) consider the different positive and welfare implications of having a single dominant reserve asset or a multi-polar system; both of which can emerge endogenously in the equilibrium in our economy. He et al. (2016) also use a global-game approach to model how, in a world with two ex ante identical assets, it may be that a single safe asset emerges in equilibrium.

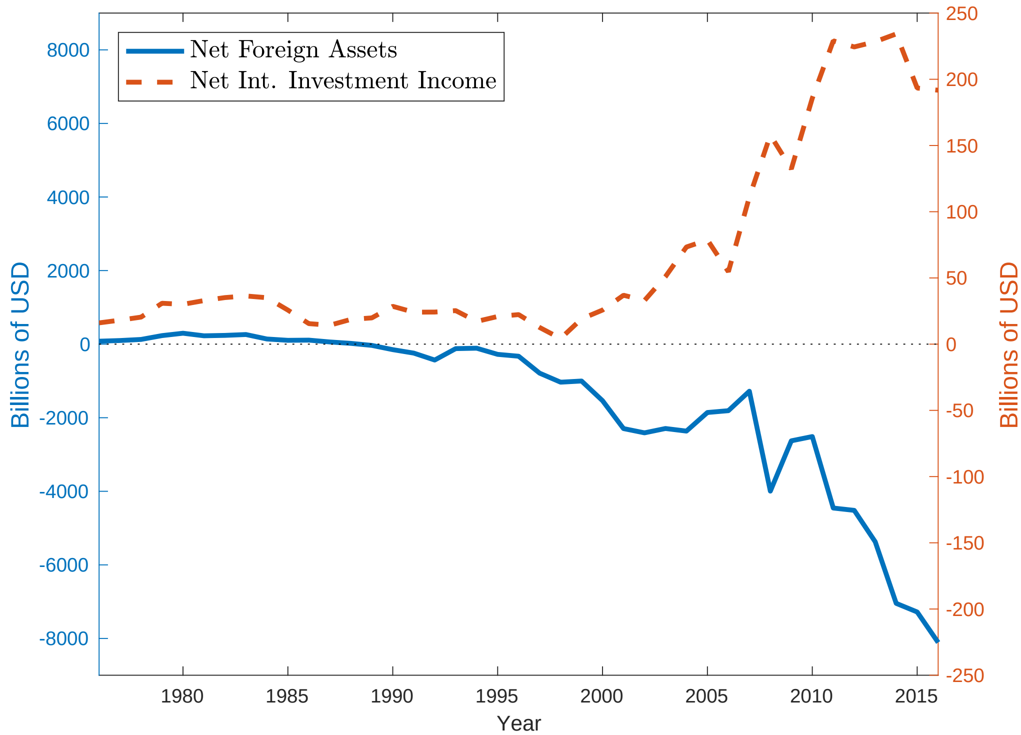
There is also work that instead explores the unit of account property of money. That literature is highlighted by Casas et al. (2016), which considers the effects of near universal dollar-denomination on shock pass-through and expenditure switching when nominal prices are sticky. The choice of unit of account is not endogenous, but is an exogenous parameter that the paper varies in a number of comparative statics exercises. The main focus is not necessarily on the reason for coordinating on a single unit of account, but on the implications, especially for monetary policy spillovers, if there is such a coordination.

2 Empirical Motivation

The U.S. has a unique external position. It is the world's largest debtor country, with its foreign liabilities exceeding foreign assets by more than $8 trillion dollars (roughly 40% of GDP). Yet, its net international investment income is *positive*, meaning that in effect it is making *negative* payments on its large net debt to the rest of the world. The U.S. is the only country in the world that has been able to sustain both a a negative net foreign assets position (NFA), and a positive net international investment income flow. Moreover, Figure 1 shows that as its NFA position has been steadily worsening over the last couple of decades, its net international income has actually been increasing. The net income is also quite significant in absolute terms – at about $250 billion, it accounts for about a third to a half of recent U.S. trade deficits. And in fact, once capital gains and other valuation changes are also taken into account (the NII measure if simply income flow, i.e. dividends and interest rate), the net return of the U.S. grows even bigger.

Gourinchas and Rey (2007) and Gourinchas et al. (2010) provide a fresh and updated look on the U.S.'s external position and its returns on it from 1952 to 2009. They show that the U.S. earns a significant excess return of about 3% on its foreign assets over its foreign liabilities, a premium that is primarily driven by differential returns within asset

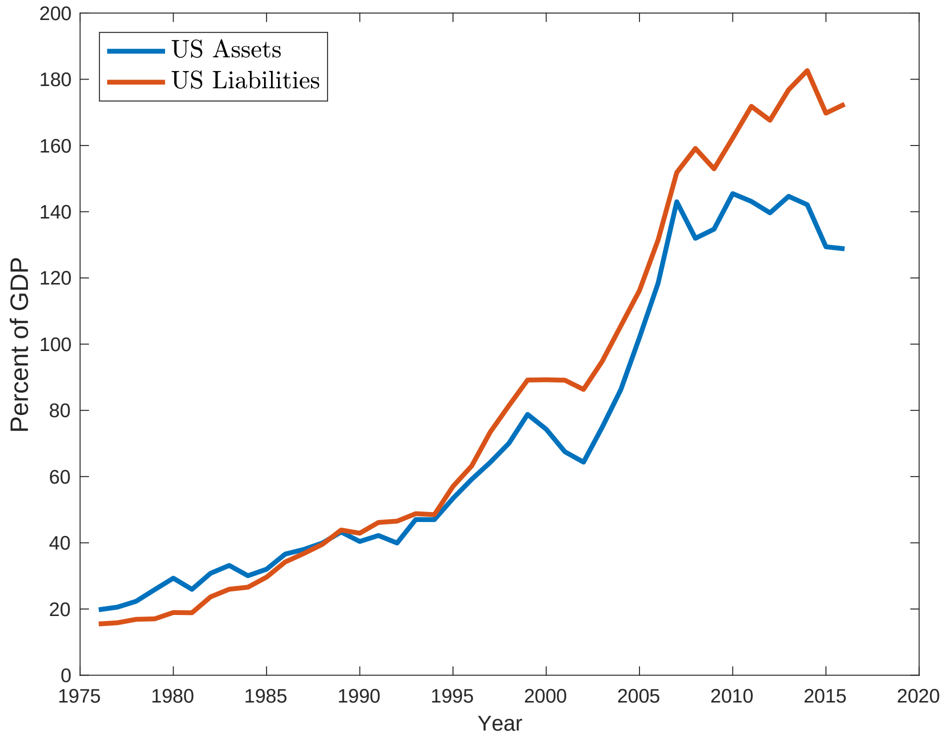
Figure 1: Evolution of U.S. External Position



classes rather than different portfolio composition in assets versus liabilities. The differential returns on debt, equity and foreign direct investment are all found to be between roughly 3% to 4%. Interestingly, the "exorbitant privilege" has increased dramatically in the period after Bretton Woods – the excess return during the Bretton Woods era was only about 1.3% a year, while this differential increases all the way to 3.5% for the years 1973 to 2009. Additional corroboration is provided by Habib (2010), who studies a large panel of 49 countries over the time period 1981 to 2007, and confirms the 3% excess return earned by the U.S., and also documents that the U.S. is the only country that earns a persistently positive differential. Moreover, he also finds that the composition of the portfolio, and in particular the U.S.'s levered foreign asset position, did not contribute to its exorbitant privilege.

This clearly represents a large real transfer from the rest of the world to the U.S., and hence the usual description of it being an "exorbitant privilege". A back of the envelope calculation, based on the current gross positions of the U.S. and the post-Bretton Woods returns indicates that it could help sustain a trade deficit of up to 2.5% of GDP indefinitely. This benefit in terms of absolute dollar amounts is sensitive to the gross external asset and liability positions of the U.S.. The larger the positions on which the differential return is

Figure 2: Evolution of U.S. External Position



applied, the larger the extra income generated. Thus, given an ever increasing financial globalization (see for example Figure 2 for the increase in the gross foreign asset and liability positions of the U.S.), the benefit to the U.S. economy could potentially become even bigger.

In terms of goods and services trade, Gopinath (2015) shows that the U.S. dollar invoices almost half of global trade, and the share of transactions in dollars is five times larger than total U.S. trade as a share of world trade. She also shows that no other currency comes anywhere close to the dominant position of the dollar in terms of intermediating third party transactions. The closest is the EUR which only intermediates 1.2 times the share of Euro-area trade. The case of Japan is particularly striking. Despite the role of the yen as a global reserve currency in its own right, 50% of all Japanese exports, and 71% of its exporters are denominated in U.S. dollars, and all the rest is in yen.

In terms of trade in financial assets, Goldberg (2011) shows that 85% of the bilateral transactions in foreign exchange markets involve the USD, testifying to its important role as a vehicle currency. Moreover, the 2015 ECB report on the international role of the euro shows that 60% of international securities are issued in U.S. dollars, and just 20% in euro. And recent work by Maggiori, Neiman and Schreger (in progress) has shown that U.S. debt is the only type of foreign debt that international investors like to buy denominated in its local currency. Strikingly, most other international debt investments are made into securities denominated in the investor's domestic currency, not the destination country's local currency. The one big exception is the U.S., which is able to sell debt securities issued in U.S. dollars to the rest of the world in large quantities, and hence most of the foreign investment into the U.S. (and hence its foreign liabilities) is indeed made up of U.S. dollar securities. Moreover, over the last 15 years there has been a significant uptick in the international investor preferences towards USD debt, and away from debt denominated in other currencies, which coincides with the apparent rise in the exorbitant privilege.

In the rest of the paper, we put these two facts, of the dominance of U.S. assets as a medium of exchange and the exorbitant privilege of the U.S.'s external position, together and develop a model where a single currency emerges as the predominant international medium of exchange in equilibrium, which leads to an endogenous exorbitant privilege being bestowed to the country that can issue it. We micro-found the need for the medium of exchange by modeling international trade markets as bilateral, anonymous search and matching markets. Our assumptions here are consistent with emerging facts in the international trade literature, which report on the intricate bilateral nature of import-export relationships (see for examples Eaton et al., 2008, 2016). The theoretical international trade literature has also recently considered incorporating such frictions, as exemplified by Eslava et al. (2015), Eaton et al. (2016) and Eslava et al. (2015).

3 Model Framework

There are two countries, which we label "US" and "EU". Each country is populated by a standard representative household, productive firms, government and a continuum of import-export firms (i.e. international trade firms) with endogenously determined mass $\mu_{c,t}$, $c \in \{US, EU\}$. Households consume both domestic and foreign goods, supply labor to the domestic productive firms and face incomplete financial markets where they can buy only home and foreign bonds. Firms combine labor and an internationally-traded commodity, call it oil, to produce a differentiated domestic good. Governments fund their expenditures through taxes and issuing debt, thus supply the bonds traded by the households. The commodity and asset trade happens in frictionless Walrasian markets, and all financial transactions are settled in the frictionlessly traded commodity (oil), which we take as the numeraire.

The key novelty of the model is that international trade in real goods happens in decentralized, bilateral trade markets. Each country features an import/export sector which intermediates all goods trade between countries; no trade in real goods happens in frictionless markets. Exporters source the domestic good from local producers and search for a foreign importer to whom they may sell the good. However, contracts are not perfectly enforceable across borders, leading to the need for collateralization of international transactions (i.e. medium of exchange). Hence, in order to operate firms first need to obtain some safe assets from the domestic households' holdings of US and EU bonds. Trading firms are free to choose the currency denomination of the funding (i.e. safe asset) they seek, but face an additional "mismatch" cost if they end up trading with a counterpart holding a different currency.

All economic agents are described in detail in the following sections. We describe the setup of the US economy, with the understanding that the foreign EU economy is symmetric.

3.1 International Trade in Goods: The Import-Export Sector

International trade in real goods happens in decentralized search and matching markets with exporting firms from one country look to match with importers from the other. Given a successful match, the exporting firm buys goods from the local producers, and sells them to the foreign importer, who then sells the good to the foreign household. The exporter-importer match then splits the surplus via Nash bargaining. The surplus is positive in equilibrium due to endogenous markups across countries, that are determined in equilibrium by a zero profit condition determining entry into the import-export sector.

Transactions in these decentralized international markets must be collateralized, and hence before import-export firms can trade they must first be "funded" by the domestic households. This funding takes the form of the household lending some of its safe assets, either home or foreign bonds, to a firm so that it can use it as collateral guaranteeing its international transactions. In return, the trading firm pays a fee to the household for this service. After the trading firm completes its transaction and receives its share of the resulting surplus, it pays the funding fee and returns the bond to the household. Thus, these funding markets function essentially as within period repurchasing agreements. We view this framework as a representation of the Letters of Credit that importers and exporters must obtain in practice before they sign international trade contracts. Such letters of credit guarantee either delivery of goods or payment, and are widely used in international trade due to the difficulty of enforcing contracts across borders. We model this by requiring the trading firms to obtain appropriate collateral for any transaction they would want to do.

Each period, new import-export firms are formed, operate, return any profits to the local household, and then are disbanded. The problem of these firms is static, but there are three stages to the life of each firm. In stage one, prospective firms choose whether or not to pay a fixed cost ϕ and become operational this period. Moreover, at this stage the firm does not know for sure whether it is going to be an importer or an exporter, but can optimally choose the probability of being one or the other.[3] Intuitively, the firm does not know whether in the current period it will get to have an importing or an exporting opportunity (those arrive stochastically), but can choose how hard to look for one versus the other.

In stage two, the firm looks for funding and has the choice of whether to seek funding in either domestic or foreign currency (i.e. to seek either domestic or foreign bonds as collateral). The firm faces a search friction in obtaining funding, hence the probability of being funded is not one. We assume that the firms look for a fixed amount of funding, which we normalize to one unit of the numeraire. If successful in obtaining funding, the firm proceeds to stage three, where it discovers whether it is an importer or an exporter, and then searches for an appropriate foreign trading partner, either an exporter or an importer respectively. If that search is successful, a trading match is formed, and the importer buys goods from the exporter to sell in his domestic market, and the resulting surplus is split between the two. This transaction must be collateralized, i.e. to trade firms require a medium of exchange, and this is where the initial funding of the companies matters. If the two counter-parties have chosen to operate in different currencies, the firms must pay a transaction cost before the trade is settled. Currency mismatch is costly because in that case before the transaction is settled, the importer must exchange its chosen funding in terms of another currency.

We solve the problem of these firms starting with stage three and working backwards.

Stage 3: Trading Round and Profits

In the final stage, firms discover whether they are importing or exporting this period, and then search for an appropriate foreign counterpart. Due to the search-and-matching nature of the decentralized markets and the possibility that there could be more potential exporters in country j than importers in country i, not all exporters are going to be matched with an

[3]This assumption is convenient, but not necessary to our mechanism. If we assumed that firms know whether they will import or export at the time of currency choice, we would need to analyze a four-way, rather than a two-way, coordination game but the coordination incentive would work similarly.

importer. For each sub-market, we assume that the total number of successful matches is given by the den Haan et al. (2000) matching function $M^T(u,v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}}+v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$ with elasticity parameter ε_T.

Let $\tilde{\mu}_{us,t}$ and $\tilde{\mu}_{eu,t}$ be the mass of *funded* trading firms in the US and EU respectively, and $p^{im}_{us,t}$ and $p^{im}_{eu,t}$ be the probabilities of being an importer, as chosen optimally by the firms in Stage 1. We thus have that the probability of a country j importer matching with a country k exporter is:

$$p^{ie}_{jt} = \frac{\tilde{\mu}_{kt}(1-p^{im}_{kt})}{\left[(\tilde{\mu}_{kt}(1-p^{im}_{kt}))^{1/\xi_T} + (\tilde{\mu}_{jt}p^{im}_{jt})^{1/\xi_T}\right]^{\xi_T}}$$

and the probability that a country j exporter matches up with a country k importer is

$$p^{ei}_{jt} = \frac{\tilde{\mu}_{kt}p^{im}_{kt}}{\left[(\tilde{\mu}_{kt}p^{im}_{kt}))^{1/\xi_T} + (\tilde{\mu}_{jt}(1-p^{im}_{jt}))^{1/\xi_T}\right]^{\xi_T}}.$$

Given a successful match, the two parties split the surplus that emerges from their trade. Here, we compute this value in the case of a successful match between a US exporter and an EU importer; the remaining three possibilities can be computed in parallel. For each good that the two exchange, they can realize a maximum surplus equal to the difference in the price of the US good in the US, $P^{us}_{us,t}$, and its price in the EU, $P^{us}_{eu,t}$. If there is a currency mismatch between the two counter-parties, however, the trading surplus is reduced by an additional transaction cost $\chi > 0$.[4] This cost gives firms an *ex ante* incentive to coordinate their currency choices.

We assume that the resulting surplus is split via Nash bargaining, with a weight α for the importer, thus the effective transaction price is:

$$P^{EX}_{us,t} = P^{us}_{us,t} + (1-\alpha)(P^{us}_{eu,t} - P^{us}_{us,t})$$

This is the wholesale price of US exports – the price at which the EU importer purchases the good from the US exporter. In turn, the EU importer then sells the good at its equilibrium EU retail price $P^{us}_{eu,t}$. In equilibrium, $P^{us}_{eu,t} > P^{EX}_{us,t}$ and hence there is a markup and an associated positive surplus to sustain trading.

[4]The mismatch cost could capture the cost of transforming collateral to be consistent with the counterparty. Or, the costs of liquidity management associated with clearing a payment in an unexpected currency.

The expected profits of a US exporter holding US safe assets as collateral is thus:

$$\pi^{\$,ex}_{us,t} = \frac{(1-\alpha)}{P^{EX}_{us,t}} \left[\tilde{X}_{eu,t}(P^{us}_{eu,t} - P^{us}_{us,t}) + (1-\tilde{X}_{eu,t})(P^{us}_{eu,t} - P^{us}_{us,t} - \chi) \right],$$

where $\tilde{X}_{eu,t}$ is the fraction of the EU trading sector that is funded with dollars, and $1-\tilde{X}_{eu,t}$ the fraction funded with euros. The term in the square brackets represents the expected surplus per unit of goods exchanged. This is divided by the wholesale price of exports since the importer has limited funding (normalized to 1 unit) and can only collateralize, and thus execute, $\frac{1}{P^{EX}_{us,t}}$ units of trade, and multiplied by $(1-\alpha)$, the exporter's share of the surplus, to arrive at the payoff of the US exporter.

In the general, the profits of a country j exporter that meets a country k importer are:

$$\pi^{\$,ex}_{jt} = \frac{(1-\alpha)}{P^{EX}_{jt}} \left[\tilde{X}_{kt}(P^{j}_{kt} - P^{j}_{jt}) + (1-\tilde{X}_{kt})(P^{j}_{kt} - P^{j}_{jt} - \chi) \right]$$

$$\pi^{€,ex}_{jt} = \frac{(1-\alpha)}{P^{EX}_{jt}} \left[(1-\tilde{X}_{kt})(P^{j}_{kt} - P^{j}_{jt}) + \tilde{X}_{kt}(P^{j}_{kt} - P^{j}_{jt} - \chi) \right]$$

depending on whether he is funded by dollars or euro respectively. We can similarly derive the profits of the importers by multiplying by $\frac{\alpha}{1-\alpha}$.

$$\pi^{\$,im}_{jt} = \frac{\alpha}{P^{EX}_{kt}} \left[\tilde{X}_{kt}(P^{k}_{jt} - P^{k}_{kt}) + (1-\tilde{X}_{kt})(P^{k}_{jt} - P^{k}_{kt} - \chi) \right]$$

$$\pi^{€,im}_{jt} = \frac{\alpha}{P^{EX}_{kt}} \left[(1-\tilde{X}_{kt})(P^{k}_{jt} - P^{k}_{kt}) + \tilde{X}_{kt}(P^{k}_{jt} - P^{k}_{kt} - \chi) \right]$$

Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. Seeking funding occurs in a matching market as well, hence it is not guaranteed that all firms will be funded each period. In each country there is one market for looking for funding in terms of US safe assets and one market for funding in EU safe assets, and the firms choose whether to search in one or the other. We also equivalently refer to funding with US safe assets as "dollar" funding, and funding with EU safe assets as "euro" funding. On one side of these markets are the domestic households, who can lend their bond holdings of US and EU bonds, and on the other side is the fraction of firms that chooses to search for one or the other. Let $X_{us,t}$ be the fraction of US trading firms choosing to seek US bonds, and $X_{eu,t}$ be the fraction of EU importers-exporters seeking US bonds, so that the total mass of US trading firms searching the domestic US bond market is $\mu_{us,t}X_{us,t}$.

Bonds promise payment of one unit of the issuing country's consumption good. Thus, the total value of the US bonds available for lending in country j at time t is given by $P_{us,t}B^{\$}_{jt}Q^{\$}_t$, where $B^{\$}_{jt}$ are the holdings of US bonds in the country j household's portfolio, $P_{us,t}$ is the price of the US consumption basket, and $Q^{\$}_t$ is the real price of the bond that pays off one unit of US consumption tomorrow. Lastly, we assume that the effective supply of dollar and euro bonds available for lending is subject to an iid global liquidity shock θ_t with pdf $\phi(\theta)$ that changes the relative availability of dollar versus euro funding. In particular, the effective availability of dollar funding in country j is $P_{us,t}B^{\$}_{jt}Q^{\$}_t \exp(\theta_t)$ and the effective supply of euro funding is given by $P_{eu,t}B^{€}_{jt}Q^{€}_t \exp(-\theta_t)$, so that a positive θ_t increases the effective supply of dollar funding relative to euro funding, and thus the ease with which dollar funding can be obtained, relative to euro funding. The shock θ_t is unknown to firms and introduces uncertainty in the funding matching probabilities.

The other side of the country j dollar funding market is the number of firms that choose to seek dollar funding, which is equal to $X_{jt}\mu_{jt}$. The total number of matches is given according to the constant returns to scale matching function $M^F(u, v)$, so that the probability that a country j trading firm seeking US bonds finds a suitable supplier is

$$p^{\$}_{jt} = \frac{M^F\left(\mu_{jt}X_{jt}, P_{us,t}B^{\$}_{j,t}Q^{\$}_t exp(\theta_t)\right)}{\mu_j X_{jt}},$$

where $M^F(u, v)$ capture the matching function describing search in the funding market. Similarly, the probability that a country j trading firm seeking EU bonds finds a match is

$$p^{€}_{jt} = \frac{M^F\left(\mu_{jt}(1 - X_{jt}), P_{eu,t}B^{€}_{j,t}Q^{€}_t exp(-\theta_t)\right)}{\mu_j(1 - X_{jt})}.$$

In the event that the trading firms find the funding they are seeking, they pay a fee $r^{\$}_j$ and $r^{€}_j$ for the funding services of dollars or euros respectively. In parallel with the labor match and searching literature, these prices could be fixed exogenous parameters — so long as they fall within the surplus range of the trading firms — or they could be endogenously determined by assuming a bargaining paradigm, like Nash bargaining.

In order to make their currency choice, firms compare their expected profits conditional on either being funded with dollars or euros. At this stage, they do not yet know whether they will be importers or exporters, or whether they will be able to find a successful trading matches in the next stage. Hence, they form expectations over the trading profits that they would receive, conditional on choosing one type of funding over the other.

The expected profit of a country j trading firm funded with US assets is

$$\tilde{\Pi}^{\$}_{jt} = p^{im}_{jt} p^{ie}_{jt} \pi^{\$,im}_{jt} + (1 - p^{im}_{jt}) p^{ei}_{jt} \pi^{\$,ex}_{jt}.$$

The first of the two terms in the above sum equals the expected profit of being a dollar-funded importer. It equals the probability of being an importer, times the probability of then finding a successful match with a foreign exporter, times the resulting profits from that match. The second component is the expected profit of being a dollar-funded exporter. The corresponding expected profits of a country j trading firm funded with EU assets instead is:

$$\tilde{\Pi}^{€}_{jt} = p^{im}_{jt} p^{ie}_{jt} \pi^{€,im}_{jt} + (1 - p^{im}_{jt}) p^{ei}_{jt} \pi^{€,ex}_{jt}.$$

Thus, the expected profit of a country j firm seeking dollar funding is given by

$$\Pi^{\$}_{jt} = p^{\$}_{jt} (\tilde{\Pi}^{\$}_{jt} - r^{\$}_{j}), \tag{1}$$

which is simply the probability of obtaining dollar funding, $p^{\$}_{jt}$, times the expected profit net of the dollar funding costs, $(\tilde{\Pi}^{\$}_{jt} - r^{\$}_{j})$. Similarly, we can compute that the expected profit of a country j firm seeking EUR funding:

$$\Pi^{€}_{jt} = p^{€}_{jt} (\tilde{\Pi}^{€}_{jt} - r^{€}_{j}). \tag{2}$$

Lastly, we need to determine the funding fees $r^{\$}_{j}$. As mentioned earlier, they can be equivalently specified in terms of a Nash bargaining process, and thus be given by

$$r^{\$}_{j} = \alpha_f \int (p^{ie}_{j} \Pi^{\$,im}_{j} + (1 - p^{im}_{j}) \Pi^{\$,ex}_{j} \phi(\theta)$$
$$r^{€}_{j} = \alpha_f \int (p^{ie}_{j} \Pi^{€,im}_{j} + (1 - p^{im}_{j}) \Pi^{€,ex}_{j} \phi(\theta),$$

or we can assume that the funding rates are fixed exogenous parameters. For simplicity, we do the latter, and check that the funding rates are such that it is always the case that both the lender and the borrower receives positive surplus.

Stage 1: Firm Formation

At stage one, the firm optimizes over the probability of being an importer, which leads to the following optimality condition:

$$\frac{\partial \max\{\Pi_{jt}^{\$}, \Pi_{jt}^{€}\}}{\partial p_{jt}^{im}} = 0. \tag{3}$$

The solution to equation (3) above determines the share of importers and exporters in equilibrium.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi_{jt}^{\$}, \Pi_{jt}^{€}\} - \phi = 0,$$

is satisfied, where $\Pi_{jt}^{\$}$ and $\Pi_{jt}^{€}$ are evaluated at the optimal choice of p_{jt}^{im}. This will determine the equilibrium size of the import-export sector in each country – μ_{jt}.

Currency Choice Equilibrium

The payoff structure described above incorporates a strong force for strategic complementarity in currency choice across countries. In particular, trading firms in the US have an incentive—whether they expect to be importers or exporters—to hold the same currency that (they believe) trading firms in the EU are holding. These payoffs also include a strategic substitutability of currency choice among firms in the same country; firms choosing to be funded with high-demand currency face a decreasing probability of finding the funding they desire. Finally, all trading firms have a desire *ceteris paribus* to hold a currency with a relatively large asset base, as a large supply of the asset to the market increases the likelihood of finding suitable funding.

The resulting equilibrium depends on the information set of the trading firms. If the firms knew the value of θ_t, then the strength of the complementarity in currency choice described by the pay-off functions above means that for certain (intermediate) values of θ_t, the economy under full information exhibits a multiplicity of equilibria: trading firms will typically find it desirable to hold a currency that they expect other trading firms to also select. Since in this paper we are primarily interested in studying potential dynamic multiplicity, i.e. multiple steady-states of the overall macro model, we follow Morris and

Shin (1998) in using an information friction to pin down a unique equilibrium in this game. This equilibrium shares a lot of the appealing non-linear features of the multiple equilibria perfect information game.

To proceed, collect the full set of aggregate states of the economy in the vector ω_t. Then the profit functions in equations (1) and (2) can be written as $\Pi^{\$}_{jt}(\theta_t, X_{us,t}, X_{eu,t}, \omega_t)$ and $\Pi^{€}_{j}(\theta, X_{us,t}, X_{eu,t}, \omega_t)$ respectively, with parallel constructions — and the same arguments — for the US and EU respectively. We assume that θ_t is unknown to firms and $\theta_t \sim iidN(0, \sigma^2_\theta)$. The strategic uncertainty generated by the shock θ_t is enough to break the equilibrium multiplicity characteristic of the full-information economy.[5]

Each period trading firms also observe the private signal

$$s_{it} = \theta_t + \epsilon_{it}$$

where the noise term $\epsilon_{it} \sim iidN(0, \sigma^2_\epsilon)$ is independent across agents and time periods. In this context, agent's posterior belief about θ is Normal, and centered around

$$E[\theta_t | s_{it}] = \beta s_{it}$$

where $\beta = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\epsilon}$ is the signal-to-noise ratio of the private signal, and with posterior-variance

$$E[(\theta_t - \beta s_{it})^2 | s_{it}] = (1-\beta)^2 \sigma^2_\theta + \beta^2 \sigma^2_\epsilon = \frac{\sigma^2_\theta \sigma^2_\varepsilon}{\sigma^2_\theta + \sigma^2_\varepsilon} \equiv \tilde{\sigma}^2_\theta.$$

Given that an increase in the (relative) availability of US bonds increases the expected payoff of seeking dollar funding to all agents, we restrict our analysis to the space of monotone strategies in which trading firms from both countries adopt the US asset so long as their signal exceeds country-specific thresholds $\bar{s}_{us,t}$ and $\bar{s}_{eu,t}$, respectively. Thus, given a particular value for each of these cutoffs, the resulting fractions of each country's trading firms adopting the US asset are

$$X_{us,t}(\bar{s}_{us,t}) = \int_{\bar{s}_{us,t}-\theta_t}^{\infty} f(\epsilon_i) d\epsilon_i = 1 - \Phi\left(\frac{\bar{s}_{us,t} - \theta_t}{\sigma_\epsilon}\right),$$

[5] This formulation treats the shock θ_t as a shock to *relative* liquidity, i.e. liquidity shocks are perfectly negatively correlated across countries. Nothing substantial changes if we assumed imperfectly correlated or independent liquidity disturbances across countries.

$$X_{eu,t}(\bar{s}_{eu,t}) = \int_{\bar{s}_{eu,t}-\theta_t}^{\infty} f(\epsilon_i)d\epsilon_i = 1 - \Phi\left(\frac{\bar{s}_{eu,t} - \theta_t}{\sigma_\epsilon}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

Given the conjectured monotonicity, the equilibrium cutoff $\bar{s}_{us,t}$ is value of the signal that leaves the importer-exporter indifferent between choosing one asset or the other, given everyone else's strategy. Denoting with $\phi(\theta_t|s_{it})$ firm i's posterior distribution over θ_t, the equilibrium cutoff values solve:

$$\int_{-\infty}^{\infty} \left[\Pi^{\$}_{us,t}(\theta_t, X_{us}(\bar{s}_{us,t}), X_{eu}(\bar{s}_{eu,t}), \omega_t) - \Pi^{€}_{us,t}(\theta, X_{us}(\bar{s}_{us,t}), X_{us}(\bar{s}_{eu,t}), \omega_t)\right] \phi(\theta_t|\bar{s}_{us,t})d\theta_t = 0$$

and

$$\int_{-\infty}^{\infty} \left[\Pi^{\$}_{eu,t}(\theta_t, X_{us}(\bar{s}_{us,t}), X_{eu}(\bar{s}_{eu,t}), \omega_t) - \Pi^{€}_{eu,t}(\theta_t, X_{us}(\bar{s}_{us,t}), X_{us}(\bar{s}_{eu,t}), \omega_t)\right] \phi(\theta_t|\bar{s}_{eu,t})d\theta_t = 0$$

This defines a system of two equations in the two unknown cutoffs $\bar{s}_{us}$ and $\bar{s}_{eu}$, whose solution describes the equilibrium of the asset choice game.

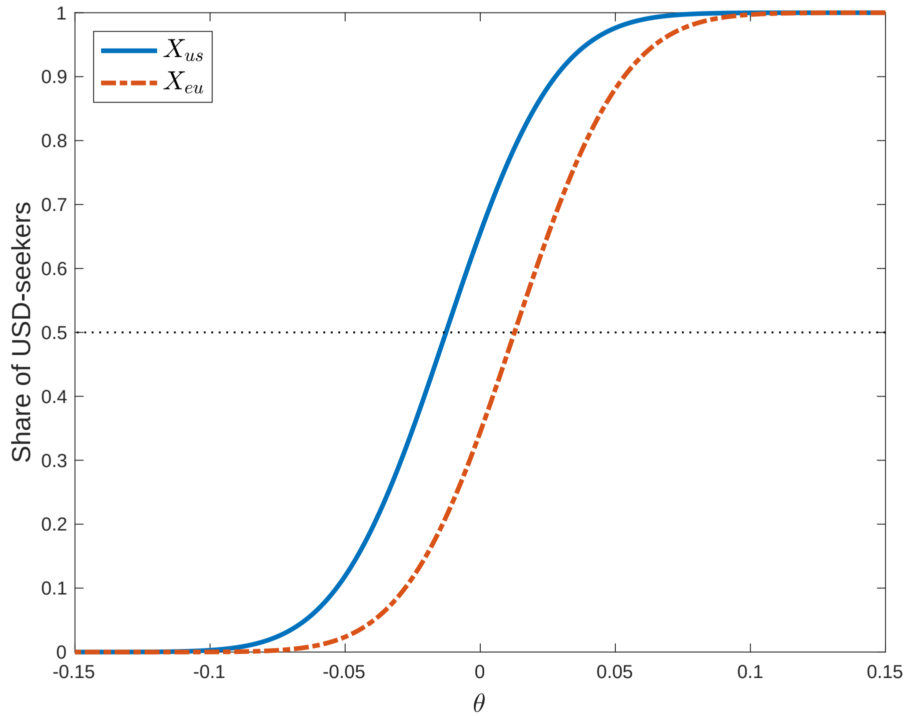
Comparative Statics

To fix some intuition for this currency choice game, we present some comparative statics taking the aggregate prices and bond quantities of the underlying macroeconomy as given. In the next section we endogenize the aggregate quantities, and analyze the implications for the fully dynamic, general equilibrium economy.

We start by describing the fundamentally non-linear nature of the equilibrium by considering a comparative static in terms of θ_t, the shock to relative liquidity. Figure 3 plots the share of trading firms choosing to search for dollar funding in the two countries, as a function of θ_t. The actual value of θ_t is unknown to the firms, but since they receive unbiased signals, on average they respond to it. Thus, we see that the proportion of firms that choose dollar funding is increasing in θ_t, since for high values of θ_t a larger mass of firms receive signals indicating a higher availability of dollar assets and thus choose to seek dollar funding.

Importantly, the relationship is non-linear. For high values of θ_t almost all firms in *both* countries tend to choose to work with dollars and the choice becomes very unresponsive to further movements in θ_t – the lines flatten out as the share of dollar seekers quickly asymptote to 1. Similarly, for low values of θ_t we have the symmetric case where almost all firms in both countries choose euros, and as they approach that point their currency choice becomes

Figure 3: Comparative Static θ_t

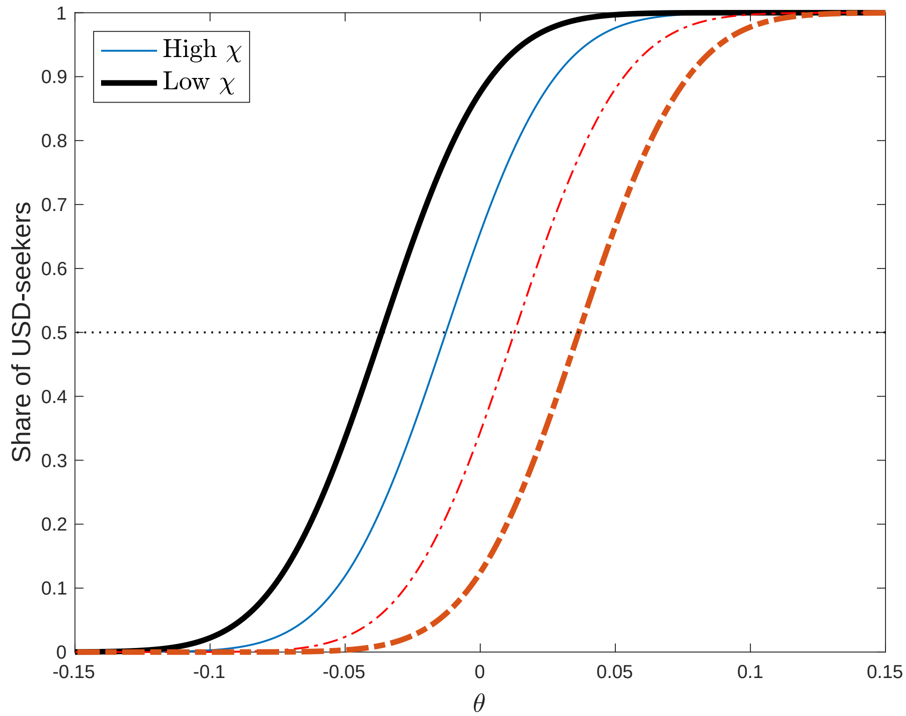


unresponsive to the actual value of θ_t. On the other hand, in the middle region the currency choice is highly sensitive to θ_t, and thus the proportion of firms choosing dollars tends to rise quickly from its one flat spot close to zero to the other flat spot close to one.

This non-linearity comes about due to the strategic complementarity in the currency choice. If the supply of both currencies is on equal footing ($\theta_t = 0$), then there's a relatively even distribution of firms choosing dollars versus euros (although the currency choice is still slightly home-biased, as we explain below). However, if the liquidity shock starts favoring the dollar for example ($\theta_t > 0$), then the marginal firms will re-allocate towards dollars because of the easier time obtaining that type of funding. This initial round of re-allocation, however, shifts the incentives of all other firms towards dollars as well, because now they are relatively more likely to meet a dollar-funded counter-party. We can see the complementarity in the fact that the two lines, depicting currency choice in the US and EU respectively, follow each other very closely. If US firms tend to use more dollars, then EU firms also find it optimal to use more dollars. As a result, small changes in macro-economic conditions could lead to large shifts in the currency choice. This potential sensitivity of the currency choice to aggregate state variables is essential for generating the multiple steady-states in the full model.

The strategic complementarity underlying all of this is due to the currency mismatch cost

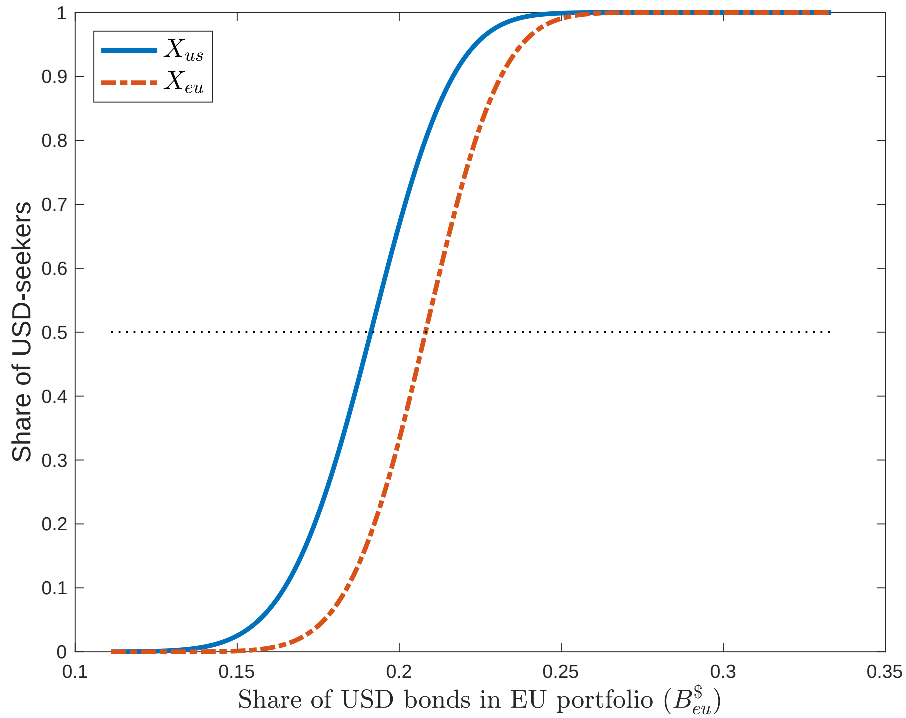
Figure 4: Comparative static χ



χ. To further illustrate how this affects the problem of our firms, in Figure 4 we consider varying the value of χ. The thinner lines show the benchmark relationship plotted above in Figure 3, and the thicker lines consider a smaller χ. As we can see, this lower currency mismatch cost serves to pull the two lines depicting US and EU firms currency choice away from one another. Thus, now for any given value of θ_t, the US and the EU currency choices are less similar. For example, we can see that it would take an 90% increase in the dollar use in the US (X_{us}), going from near zero to almost 90%, before the use of the dollar by EU firms goes above 10%.

Next, we want to illustrate the feedback between the portfolio position of households and the resulting equilibrium currency choice, which is at the heart of the dynamic multiplicity of the model. We showcase this with Figure 5 which plots the equilibrium currency choice against different levels of US bonds holdings, as a share of the EU household's portfolio. We choose to illustrate with this comparative static, because as will become clear, the crucial feedback works through the household's holdings of *foreign* bonds. The higher is the appetite of households for foreign assets, the more abundantly available is foreign currency funding to the domestic import-export firms, and hence the more likely are these firms to tilt their currency choice towards the foreign currency. We can see this effect clearly in the figure,

Figure 5: Comparative Static B_{eu}^{us}

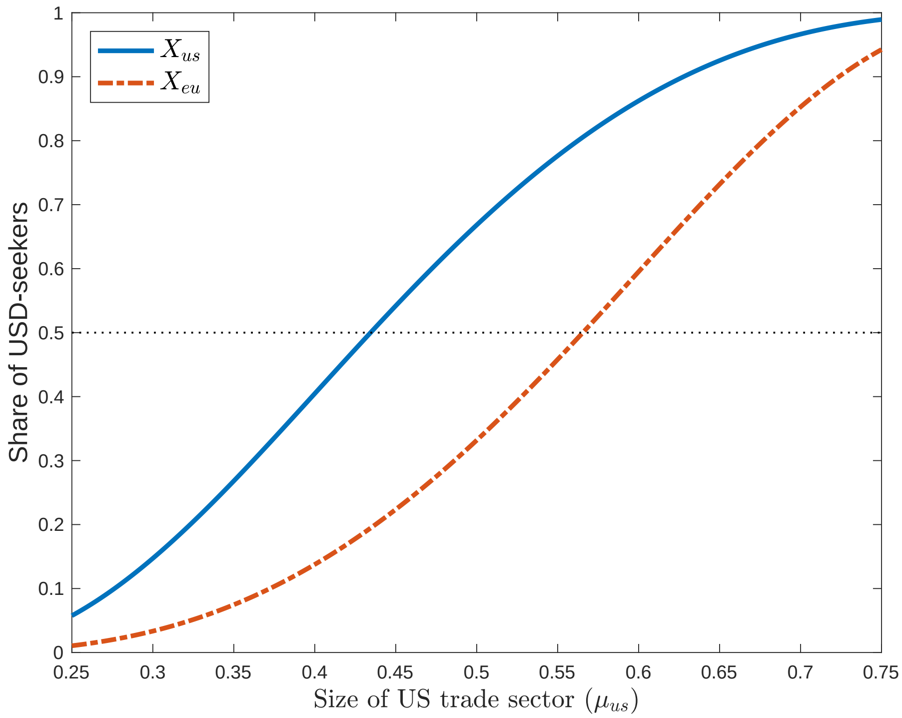


which shows that the equilibrium currency choice is strongly increasing in the holdings of US bonds by the EU household. As the total holdings of US assets increases by just 10%, from 15% to 25%, we can move from a euro-centric to a dollar-centric equilibrium.

This illustrates the strong feedback effect from the households' portfolio choice to the firms' currency choice. In the next section, we describe how the firms' currency choice in turn has a strong reinforcing effect on the households' portfolio choice as well. Intuitively, the more firms are using a currency, say dollars, the higher is the liquidity premium earned on dollar denominated assets, and hence the larger the incentive for households to hold these assets. But as we can see, as US bond holdings abroad increase, this leads to an equilibrium with an even higher dollar-use in trade, and hence a higher liquidity premium and so on.

Finally, we consider comparative statics concerning the size of the trading sector $\mu_{us,t}$ in the US. There are two things we want to highlight here. First, while there is a strategic complementarity in currency choice across countries, there is a strategic *substitutability* in currency choice *within* a country. The reason is that there is a congestion effect in funding markets, where if more firms are seeking funding in the same currency the increased market tightness lowers the probability of obtaining it. This effect is strongest for the relatively scarce foreign currency. Hence, as we increase the size of the trading sector in a given country,

Figure 6: Comparative Static μ_{us}



i.e. $\mu_{us,t}$, this gives firms higher incentives to seek the more available domestic currency funding. But as more of the domestic firms look for domestic currency, this naturally leads to the foreign firms to also prefer this type of funding.

Second, increasing $\mu_{us,t}$ also increases the congestion on the US side of trading markets, and thus US trading firms face a smaller probability of finding a trading match. This decreases the expected profits from the trading round, and also makes them less sensitive to costs of currency mismatch, since now firms are less likely to find someone to trade with, and with whom they may experience a currency mismatch. Thus, in addition to the strategic substitutability within countries outlined above, an increase in the size of the domestic trade sector tends to also weaken the strategic complementarity force across countries. These two effects combine to generate the positive slope we see in Figure 6 – as the size of the US trade sector increases, the equilibrium currency choice shifts more and more towards the dollar. Among other things, this shows that trade policies that either stimulate or contract the international trade sector within a country can also affect that country's currency's position in international markets.

3.2 Domestic Sector

We embed our model of the import-export sector within a standard two-country open economy model. The domestic sector of the economy is summarized below.

Households

The household sector in country c consists of a representative consumer-worker, who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{c,t}^{1-\sigma}}{1-\sigma} - \frac{L_{c,t}^{1+\nu}}{1+\nu} \right)$$

where the consumption basket $C_{c,t}$ is a CES aggregator of US (C_{ct}^{us}) and EU (C_{ct}^{eu}) goods:

$$C_c(C_{c,t}^{us}, C_{c,t}^{eu}) = \left(a_c^{us\,\frac{1}{\eta}} C_{c,t}^{us\,\frac{\eta-1}{\eta}} + a_c^{eu\,\frac{1}{\eta}} C_{c,t}^{eu\,\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{4}$$

with the price index

$$P_{c,t} = \left(a_c^{us} (P_{c,t}^{us})^{1-\eta} + a_c^{us} (P_{c,t}^{eu})^{1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{5}$$

In addition, the household chooses how to allocate its savings among US and EU bonds, which are risk-free in the units of their denomination. Since trade in the differentiated US and EU products happen in decentralized markets, as described above, the bond payments are settled in terms of the international numeraire, which is the only real good that is frictionlessly traded across countries. As a result, the bonds do not promise delivery of real goods, but rather enough resources to purchase the promised consumption good. Thus, payments on the US bond are indexed to the price of US consumption – one unit of US bonds purchased at time $t-1$ yields a payment of $P_{us,t}$ – and the EU bond similarly returns $P_{eu,t}$. Lastly, each period the households of both countries are also endowed with one unit of oil.

Thus, a household faces the following budget constraint (with prices expressed in terms of the numeraire),

$$\begin{aligned} P_{c,t}C_{c,t} + (1-\Delta_{c,t}^{\$})P_{us,t}Q_t^{\$}B_{c,t}^{\$} + (1-\Delta_{c,t}^{€})P_{eu,t}Q_t^{€}B_{c,t}^{€} + P_{us,t}Q_t^{\$}\frac{\tau_c^{\$}}{2}(B_{c,t}^{\$} - B_{c,t-1}^{\$})^2 + P_{eu,t}Q_t^{€}\frac{\tau_c^{€}}{2}(B_{c,t}^{€} - B_{c,t-1}^{€})^2 \\ = P_{us,t}B_{c,t-1}^{\$} + P_{eu,t}B_{c,t-1}^{€} + W_{c,t}L_{c,t} + R_t + \Pi_{c,t} + T_{c,t}, \end{aligned} \tag{6}$$

where $\Pi_{c,t}$ is the total profit of country c's trading sector, which is returned lump-sum to the representative household, $R_t \equiv 1$ represents the common price of the unit endowment of the numeraire good which we normalize to one, $Q_t^{\$}$ and $Q_t^{€}$ are the prices of the US and the EU bonds respectively, and $\Delta_{c,t}^{\$}$ and $\Delta_{c,t}^{€}$ represent the endogenous liquidity premia (i.e. lending fees) earned by the bonds lent within the period to trading firms. Lastly, there are also quadratic portfolio adjustment costs. Adjustment costs are parameterized to be zero in steady-state, and serve to prevent excess volatility of capital flows, without affecting the average level of bond holdings. We will consider model specifications with and without portfolio adjustment costs, and the main qualitative implications do not differ.

The first order conditions of the household yield the Euler equations:

$$1 = \beta E_t \left[\left(\frac{C_{c,t+1}}{C_{c,t}} \right)^{-\sigma} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{us,t+1}}{P_{us,t}} \frac{1}{Q_t^{\$}(1 - \Delta_{c,t}^{\$} + \tau_c^{\$}(B_{c,t+1}^{\$} - B_{ct}^{\$}))} \right] \tag{7}$$

$$1 = \beta E_t \left[\left(\frac{C_{c,t+1}}{C_{c,t}} \right)^{-\sigma} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{eu,t+1}}{P_{eu,t}} \frac{1}{Q_t^{€}(1 - \Delta_{c,t}^{€} + \tau_c^{€}(B_{c,t+1}^{€} - B_{ct}^{€}))} \right] \tag{8}$$

Domestic Production

All output in the economy is tradable, and is produced by competitive domestic firms according to the production function

$$Y_{c,t} = A_{c,t} L_{c,t}^{\alpha_l} \tilde{K}_{c,t}^{1-\alpha_l}. \tag{9}$$

where $\tilde{K}_{c,t}$ represents the firms' use of international numeraire good in production. Standard first-order condition imply that

$$W_{c,t} = P_c^c \alpha_l \left(\frac{L_{c,t}}{\tilde{K}_{c,t}} \right)^{\alpha_l - 1} \tag{10}$$

$$1 = R_t = P_c^c (1 - \alpha_l) \left(\frac{L_{c,t}}{\tilde{K}_{c,t}} \right)^{\alpha_l}. \tag{11}$$

Notice that in general $\tilde{K}_{c,t} \neq 1$, i.e. that domestic use of the commodity input good need not equal the country's unit endowment.

Government

The government faces expenditures fixed in proportion to domestic production:

$$G_{c,t} = \phi_{c,g} Y_{c,t}. \tag{12}$$

For simplicity, we begin by assuming that the government levies lump-sum taxes so at keep the stock of debt $\bar{B}^c > 0$ constant.

Market Clearing

Market clearing in the goods market requires that domestic production is either consumed at home or exported abroad via the export sector. Letting the measures of household in the US and EU be μ and $1 - \mu$ respectively, we have

$$(1 - \phi_{us,g}) Y_{us,t} = C^{us}_{us,t} + \frac{(1-\mu)}{\mu} C^{us}_{eu,t} \tag{13}$$

$$(1 - \phi_{eu,g}) Y_{eu,t} = \frac{\mu}{(1-\mu)} C^{eu}_{us,t} + C^{eu}_{eu,t} \tag{14}$$

Market clearing for the frictionlessly traded input good (oil) is

$$\mu \left(\tilde{K}_{us,t} + \Delta^k_{us,t} \right) + (1-\mu) \left(\tilde{K}_{eu,t} + \Delta^k_{eu,t} \right) + \bar{\chi} = 1. \tag{15}$$

where $\Delta^k_{us,t}$ and $\Delta^k_{eu,t}$ represent the capital resources devoted to establishing trading firms (i.e. covers the fixed cost of entry),

$$\Delta^k_{ct} = \phi \mu_{ct},$$

where μ_{ct} is the equilibrium measure of trading firms that enter the market. The parameter $\bar{\chi}$ denotes the total amount of resources used up in transaction costs due to currency mismatch.

Bond market clearing requires that the foreign and domestic holding of bonds combine to equal the fixed aggregate supply of government debt.

$$B^{\$}_{us,t+1} + \frac{(1-\mu)}{\mu} B^{\$}_{eu,t+1} = \bar{B}^{\$} \tag{16}$$

$$\frac{\mu}{(1-\mu)} B^{€}_{us,t+1} + B^{€}_{eu,t+1} = \bar{B}^{€} \tag{17}$$

Finally, note that because of the frictions in cross-border trade, the law of one price does not hold across countries. Specifically,

$$P^{eu}_{us,t} = \Delta_{us,t} P^{eu}_{eu,t} \tag{18}$$

$$P^{us}_{eu,t} = \Delta_{eu,t} P^{us}_{us,t} \tag{19}$$

where the wedge $\Delta_{c,t}$ is an equilibrium object pinned down by free entry and the equilibrium of the coordination game played by traders.

Given processes for $\left\{A_{c,t}, \Delta^{\$}_{c,t}, \Delta^{€}_{c,t}, \Delta_{us,t}, \Delta_{eu,t}\right\}$, an equilibrium in the domestic sector is described by the aggregate prices, $\left\{Q^{\$}_t, Q^{€}_t\right\}$, the set of country specific prices $\left\{P_{c,t}, P^{us}_{c,t}, P^{eu}_{c,t}, W_{c,t}\right\}$, and the set of country specific allocations $\left\{C_{c,t}, C^{us}_{c,t}, C^{eu}_{c,t}, B^{\$}_{c,t}, B^{€}_{c,t}, L_{c,t}, Y_{c,t}, G_{c,t}, \tilde{K}_{c,t}\right\}$ that satisfy equations (4) through (19). Accounting for equations that appear for country c and c', this yields a total of 28 equations in 28 unknowns.

4 Steady-state

While our baseline economy is dynamic, many of the interesting features of environment are already apparent from an examination of the non-stochastic steady-state of the economy. To begin, rearrange the steady-state versions of the Euler equations (7) and (8) to obtain:

$$\frac{1}{\beta} = \frac{1}{Q^{\$}(1-\Delta^{\$}_c)} \tag{20}$$

$$\frac{1}{\beta} = \frac{1}{Q^{€}(1-\Delta^{€}_c)}. \tag{21}$$

These conditions equate the inverse of the time discount parameter, β, to the effective returns on the bonds, which include both the interest rate $\frac{1}{Q^{\$}_t}$ and the extra return from repo-ing the bonds to the trading firms $\frac{1}{1-\Delta^{\$}_c}$, i.e. the liquidity premium. If $\Delta^{\$}_c$ and $\Delta^{€}_c$ are fixed exogenously then the only possible interior solutions, in which each country holds a positive quantity of both types of bonds, are such that $\Delta^{\$}_{us} = \Delta^{\$}_{eu}$ and $\Delta^{€}_{us} = \Delta^{€}_{eu}$. This is because if the premium on the same asset is not equalized across countries, then (20) and (21) cannot be simultaneously satisfied for both countries, leading one household to take an infinitely positive position, while the other takes an infinitely negative one. Conversely, when equations (20) and (21) do hold with equality, the two assets become perfect substitutes and the steady-state portfolio position is indeterminate, since agents can shift their portfolios around without affecting the asset returns. This is a well known issue in international models with incomplete markets.

In contrast to many standard environments, however, the endogenous premia terms in our economy help deliver a determinate distribution of world assets. To make the intuition concrete, consider the asset choice of the US household, initially taking as given the size of the trading sectors and their currency choices. In this case, an increase in US holdings of dollar bonds decreases the likelihood that the marginal bond will find a match in the trading sector, thereby decreasing the expect return of the dollar bonds for the US household. This decreases the attractiveness of dollar holdings, and this mechanism tends to moderate shifts in portfolio positions. Essentially, shifts in the aggregate US portfolio will affect the equilibrium asset returns, which prevents the indeterminacy mentioned above.

4.1 Steady-state multiplicity

While the equilibrium in the model is determinate, meaning that conditional on state variables there is a unique equilibrium path, there are multiple steady-states. In other words, the model features a dynamic multiplicity – there are several distinct points on the distribution of world assets that are absorbing states. The reason is that the endogenous liquidity premia terms are functions not only of the household's choices, but also of the trading firms' currency choices, and there is an interaction between both of those choices.

The equilibrium liquidity premium is simply the funding fees that the household charges the firms per unit of funding, $r_c^{\$}$ and $r_c^{€}$, times the probability that a household finds someone to lend its bonds to:

$$\Delta_c^{\$} = p_c^{\$,h} r_c^{\$}$$
$$\Delta_c^{€} = p_c^{€,h} r_c^{€}$$

where $p_c^{\$,h}$ and $p_c^{€,h}$ are the probabilities of a household finding a lending opportunity. In our calibration we assume that $M^F(\bar{c}_c, u, v) = \bar{c}_c u^{\xi_f} v^{1-\xi_f}$, where the scaling constant $\bar{c}_c$ is chosen to ensure that matching probabilities are below one. This form implies that

$$p_c^{\$,h} = \frac{M^F\left(\mu_c X_c, P_{us} B_c^{\$} Q^{\$}\right)}{P_{us} B_c^{\$} Q^{\$}},$$

$$p_c^{€,h} = \frac{M^F\left(\mu_c(1-X_c), P_{eu}B_c^{€}Q^{€}\right)}{P_{eu}B_c^{€}Q^{€}}.$$

Hence, increasing the stock of available US assets ($B_c^{\$}$), ceteris paribus, lowers the probability of lending out the marginal US bond, and thus lowers the dollar liquidity premium. The less scarce are the dollar safe assets, the lower is their liquidity premium. However, in addition to this supply effect, there is also potentially a demand effect as captured by the term $\mu_c X_c$. If the number of firms that choose to seek dollar funding increases, this increases the tightness of the funding markets and raises the probability of lending the marginal bond out, thus increasing the liquidity premium on dollar assets.

Importantly, there is a feedback between the household's portfolio choice and resulting bond positions $B_c^{\$}$ and $B_c^{€}$, and the firms' currency choice decisions. From the perspective of a trading firm, a large domestic stock of an asset increases the desirability of using that asset as its source of liquidity. Why? First, because the large stock of that asset increases, other things equal, the likelihood that the firm will successfully find appropriate funding for its activity. And second, because if the other domestic firms end up coordinating on using the same asset, that will cause a smaller congestion externality, and thus have a smaller effect on an individual firm's ability to find the funding it seeks. In short, a large asset base creates both a direct and a strategic incentive to coordinate on using that asset.

As a result, an increase in the US bond holdings in country c, $B_c^{\$}$, would lead to more firms to select into searching for dollar funding, pushing up the probability of dollar bonds being lent and thus increasing their liquidity premium. In other words, an increase in the supply of available liquidity of a certain kind can increase the demand for that liquidity as well. Consequently, increased demand for US bonds will increase their liquidity premium $\Delta_c^{\$}$ and thus incentivize households to hold more of them. But as the country's holdings of US bonds increase, more domestic firms will choose to search for dollar funding and so on. Hence, there are strong reinforcing feedback effects between the currency choice of domestic firms and the portfolio positions of the domestic households.

In addition, there is also a reinforcing effect coming from the currency choice of foreign firms. Due to the strategic complementarity in currency choice, if domestic firms shift towards use of dollars, then foreign firms' incentive to use dollars also goes up. And as they shift towards seeking dollar funding, the domestic firms have even stronger incentive to look for dollar funding over euro funding and so on. Similarly, the increase in dollar funding abroad increases the foreign household's demand for US bonds. Overall, this will increase the volume of trade intermediated in dollars, and the demand for US bonds across the world.

Of course, the basic symmetry of the environment means that the same logic can work when applied to the euro, instead of the dollar. Hence, if the feedback effects are strong enough, there could be a multiplicity of steady states where one country experiences (i) high asset price, (ii) large foreign demand for their asset, and (iii) coordination of trade around the domestic asset.

4.2 Coordinated Steady-State Characteristics

In all, there are two types of potential steady-states – one where trade is coordinated on mostly using a single currency, giving rise to a dominant currency paradigm, and a symmetric one where both currency see equal use. Moreover, there are two steady-states of the first kind, one where the dollar emerges as the dominant currency, and one where the euro does so. And naturally, there is only a single symmetric steady-state. The symmetric steady-state is rather standard – both countries face exactly the same demand for their assets, and have identical equilibrium allocations. On the other hand, the coordinated steady-states are the main novelty of the model, hence we focus on analyzing their characteristics.

We will discuss the features of the dollar-centric steady-state, with the understanding that since the model is symmetric everything applies, in mirror, to the euro-centric steady-state. The dollar coordinated steady-state has a number of key features, but it all starts with the fact that most firms in both countries are using dollars as their funding currency, and hence most of the trade is intermediated in dollars. This is arguably a good representation of the actual invoice currency distribution we see in the data (e.g. Gopinath (2015)). And the reason this is a steady-state is that, as discussed above, when trading firms coordinate on using dollars as their main medium of exchange, this drives up the dollar liquidity premium in both countries, hence increasing households' incentives to hold dollars. As a result, dollar assets are relatively available in both countries, which makes funding in dollars easier than funding in euros for firms in both countries, and hence reinforces their choice to operate in dollars.

Beyond the trading sector, the coordination on the dollar gives rise to a liquidity premium on US safe assets over EU safe assets. This operates through the fact that when X_c is high there is little demand for EU bonds in the repo markets, hence $\Delta^{€}$ is small, while $\Delta^{\$}$ is relatively large. As a result, the US bonds enjoy a high price and hence lower interest rates. This is best seen by re-arranging the steady-state Euler equations to arrive at:

$$\frac{Q^{\$}}{Q^{€}} = \frac{1 - \Delta^{€}}{1 - \Delta^{\$}}$$

Note that we have dropped the country index and simply write, $\Delta^{\$}$, because in steady-state the liquidity premium on each type of asset is the same in both countries (but differs across assets). Thus, higher dollar liquidity premium ($\Delta^{\$}$) leads to a high price of the US bonds, and hence a lower effective interest rate ($\frac{1}{Q^{\$}}$). This gives rise to a positive excess return of EU bonds over US bonds – essentially, since the EU bonds lack the liquidity premium of the US bonds, they are compensated with excess financial returns. In particular, we can further re-arrange the Euler equation to express the excess bond return as:

$$\frac{1}{Q^{€}} - \frac{1}{Q^{\$}} = \frac{\Delta^{\$} - \Delta^{€}}{\beta}$$

Moreover, note that this excess return is something the US enjoys on their foreign assets ($B^{€}_{us}$), relative to their foreign liabilities ($B^{\$}_{eu}$). In other words, the US economy is able to borrow at lower interest rates from foreigners, than the EU economy, because the US assets play a special role in the international financial system, and hence their price is bid up.

This difference in the effective interest rates of the two assets gives rise to a transfer in real resources from the EU economy to the US economy each period, and this transfer is the reason why such excess return has been referred to as an "exorbitant privilege". We can see that most clearly by expressing the trade balance of the US as the negative of the capital account, to obtain:

$$\underbrace{B^{\$}_{eu}(1 + R^{\$}) - B^{€}_{us}(1 + R^{€})}_{\text{US Financial Account Deficit}} = \text{ US Trade Balance}$$

Since the US bonds pay a lower interest rate, the US incurs lower expenditures on its foreign debts, and hence could potentially sustain a negative trade balance indefinitely. The bigger is the interest rate differential at steady-state, the larger the potential deficit that is sustainable indefinitely. Essentially, in this coordinated steady-state, the EU households have strong incentives to acquire US bonds, and hence they are willing to transfer resources every period to the US economy in the form of a US trade deficit. Thus, in this economy, persistent trade deficits are not a sign of economic stress or potential balance of payments issues, but in fact a sign of the dominant position that the domestic currency enjoys in the international financial system.

As a result of the ability of the US to sustain permanent trade deficits, the US households enjoy higher permanent incomes and hence have a higher consumption, lower work effort and overall higher welfare than the EU households. In conclusion, the coordinated steady-states are ones where the country with the dominant currency enjoys (i) high asset prices, (ii) excess returns on its foreign assets, (iii) a permanent trade deficit and (iv) higher consumption and welfare.

5 Calibration and Numerical Results

Next, we calibrate our model and quantify its steady-state and dynamic properties. The chosen parameters are listed in Table 1. The top half of the table consists of parameters that are standard in the literature, and those are calibrated to commonly used values. To highlight a few, we set the household's consumption preference parameters such that there is no exogenously hardwired bias in consumption – i.e. we set $\alpha_c^{us} = \alpha_c^{eu} = 0.5$. In this case, there is no fundamental reason for households to prefer consumption of one good over that of another, and the only real exchange rate movements will come from the frictions in trade markets. Such real exchange rate movements are due to two forces – law of one price deviations due to the costly international search markets, and also the equilibrium consumption baskets of the two economies will naturally differ, as domestic goods are generally cheaper. We pick a elasticity of substitution between domestic and foreign goods equal to 5, in line with micro level estimates in the trade literature. Lastly, we fix the supply of both types of assets to 250% of GDP in the symmetric steady-state. We view the assets not simply as government bonds, but generally as short-term high quality obligations of a country, including demand deposits for example. In any case, little changes if we were to view the assets as strictly representing government bonds – a calibration where we set that supply to 70% of GDP yields similar results.[6]

The second panel of the table lists the new parameters specific to our mechanism. We calibrate the currency mismatch cost χ to 0.5% of the transaction value of a matched import-export pair at the symmetric steady-state, which we see as a relatively small value. We assume that all funding fees are equal across assets and countries, $r_c^{\$} = r_c^{€} = r$, and we pick them to imply an exorbitant privilege excess return of 2.4% which matches the estimates in Gourinchas et al. (2010). Moreover, the numbers imply a 4% funding cost, which is

[6]We use the symmetric steady-state as the calibration benchmark because we want to calibrate both economies symmetrically, but in any case the GDP differences across steady states are only on the order of 1-2%, hence it does not make a big difference.

Table 1: Calibration Parameters

Parameter	USD Coord., low NFA	Value
$A_{us} = A_{eu}$	Productivity	1
$a_c^{us} = a_c^{eu}$	Pref. for domestic vs foreign goods	0.5
η	Elasticity of substitution between consumption goods	5
α_l	Labor share in production	0.4
β	Time preference	0.99
γ	Risk Aversion	2
ν	Labor elasticity parameter	2
μ	Country size	0.5
$\phi_{us,g} = \phi_{eu,g}$	Government spending	0
$\frac{\bar{B}^{\$}}{\bar{Y}_{us}} = \frac{\bar{B}^{€}}{\bar{Y}_{eu}}$	Supply of liquid assets as fraction of GDP	2.5
	New Parameters	
χ	Currency mismatch cost	0.0137
α	Exporters bargaining parameter	0.5
ϕ	Fixed cost of entry into trading sector	0.009
ξ_T	Elasticity of trade matching function	0.2
ξ_f	Elasticity of funding matching function	0.9
$r^{\$} = r^{€}$	Funding fees	0.04
σ_θ^2	Firm's prior variance of θ	0.01
σ_ε^2	Variance of noise in signals	0.0001

relatively low and in line with the data on letters of credit cost. We pick the fixed cost ϕ to imply a markup of about 13% on domestic goods sold abroad. We pick the parameters of the matching functions so that they imply high steady-state finding probabilities. The probability of matching with a trading partner is 87% and the probability of finding funding is 72%, at the symmetric steady-state. Note that we calibrate using symmetric steady-state moments in order to have a consistent symmetric calibration. However, the implied numbers are similar across all steady-states, with finding probabilities ranging from 70% to 95% across the different steady-states. Lastly, we make both the prior beliefs of the firms about the unknown liquidity shock θ_t and their signals s_{it} very precise. We set the prior variance of θ_t equal to just 0.01, and the signal to noise ratio in the firms' signals is 80. Thus, we are very close to the full information benchmark, and the model has just enough heterogeneity in information sets to recover a unique equilibrium in the static currency choice game.

With this parameterization we find five steady-states, all of which turn out to be stable.[7]

[7]We discuss the stability of the steady-states in our analysis of the dynamic properties of the model in the next section

One symmetric, two distinct steady-states where the dollar emerges as the dominant currency, and two mirror image steady-states where the euro emerges as the dominant currency. Below we discuss the characteristics of the symmetric steady-state and the two dollar-centric steady-states, with the understanding that the euro-centric steady-states are mirrors of those.

It is interesting to observe that there is not a single dollar-coordinated steady-state, but two of them. It turns out that the key difference between the two is in the overall net foreign asset position of the US, and otherwise the two dollar-coordinated steady-states are very similar in terms of the extent of the dollar use in trade and excess return on foreign assets. One of these steady-states is such that the US has a steady-state negative Net Foreign Asset (NFA) position, and in the other one it has a positive NFA position. Hence we will refer to the first one as the dollar coordinated steady-state where the US is "poor", in the sense of having low stock of assets and a net debtor positions, and the latter as the one where the US is relatively "rich", and has a net creditor positions. The key implied moments of all three types of steady-states are listed in Table 2.

Table 2: Numerical Results

Moments	USD Coord., low NFA	USD Coord., high NFA	Symmetric
Consumption	0.906	0.926	0.896
$C_{us} - C_{eu}$	0.020	0.059	0
Trade Balance	-1.79%	-5.27%	0%
$400(r^{\$} - r^{€})$	2.43%	2.43%	0%
Net Foreign Assets (as % of GDP)	-38%	35%	0%
Gross Foreign Assets ($B_{us}^{€}$)	91%	171%	125%
Gross Foreign Liab. ($B_{eu}^{\$}$)	129%	136%	125%
Home Bias in Portolio	0.19	-0.14	0
X_{us}	0.97	0.95	0.5
X_{eu}	0.95	0.97	0.5

Comparing the two dollar coordinated steady-states together, we see that the main difference comes in the stock of foreign bonds held by the US household. In the low NFA steady-state, the US has foreign assets worth only 91% of GDP, and a significantly home biased overall portfolio position. To measure the portfolio home bias we use the usual index

$$\text{Home Bias} = 1 - \frac{\text{Share of Foreign Bonds in HH Portfolio}}{\text{Share of Foreign Bonds in World Supply}}$$

In the low NFA steady-state, the US portfolio displays a significant home bias of 0.19 which reflects the fact that it has relatively few foreign assets, while the EU households have significant US asset holdings. The EU households have large holdings of US bonds because those are needed to fund their domestic trading firms which have coordinated on using dollars as their primary source of funding. As a result, in this steady-state the holdings of US assets is relatively equally distributed across the world, with both US and EU households holding about half of the available supply of US bonds each. This relatively equal distribution of US assets plays a crucial role in supporting the coordination on the dollar as the main medium of international exchange, as it ensures that it is a viable funding source for firms in both countries. If firms in one of the countries were not able to acquire US bonds collateral sufficiently easily, then the coordination on the dollar would not be possible.

Still, not every single firm seeks dollar funding – as we can see from the last two rows of the table, 3% of US firms and 5% of EU firms operate in euros. While both of these are small numbers, the important thing is that there are about twice as many EU firms using euros, as US firms using euros. As a result, there is a relatively bigger demand for euros in the EU, and hence in steady-state most of the EU assets are being held by the EU household. In this way, we arrive at a steady-state where the US is significantly indebted to the rest of the world, because while the US has relatively little incentive to own foreign assets, US assets are highly sought after in international markets. In particular, the model implies an empirically realistic NFA position of negative 38% of GDP.

Crucially, the US is able to sustain a negative NFA position in steady-state thanks to the excess return it earns on its foreign assets relative to its foreign liabilities. This excess return comes about as a result of the liquidity premium that the US bonds earn relative to EU bonds, because firms across the world have coordinated on using dollars as their main funding source. This generates a significant liquidity premium on US bonds, and results in an annualized excess return of US assets over EU assets of 2.4% (row four of the table), which matches the empirical evidence in Gourinchas et al. (2010). In fact, this excess return is not only enough to sustain a negative NFA position indefinitely, but the US is even able to run a trade deficit at the same time! So even though the US is essentially running a positive credit card balance with the rest of the world, it is effectively paying a negative interest rate on it. Coupled with large gross asset positions, which are in line with those in the data, the 2.4%

excess return is able to generate a steady-state trade deficit of almost 1.8% of GDP. This is a substantial figure, and represents a significant transfer of resources from the rest of the world to the US. This can also be seen from the fact that the US's steady-state consumption is more than 2% higher than that of the EU.

Moreover, note that this steady-state arguably captures many features of international financial positions as found in the data, even though they were not directly targeted by the calibration. The model was designed to generate an excess return on US foreign assets, but beyond that, it also implies that the US could sustain both a persistent trade deficit and a significantly negative NFA position, as we have seen happen over the last few decades. Moreover, this steady-state implies not only large gross capital flows across countries (with magnitudes similar to those found in the data), but also features non-trivial amount of portfolio home bias. While the table only lists the US positions, the EU households are also home biased, despite their relatively large holdings of dollar assets. The reason that EU households also display home bias is the low US holdings of foreign assets, which leaves EU households holding most of the supply of EU safe assets.

It is interesting to also briefly consider how the high NFA dollar coordinated equilibrium differs from the one we just discussed. As we can see, that equilibrium features the same excess return on euro assets over dollar assets, which is underpinned by the same liquidity premium differential. The main difference is in the gross foreign holdings of the US household, and in the relative use of euros across both countries. In this steady-state, the US households are "rich" and have large EU bond holdings, which drive a significantly positive NFA position. Interestingly, the foreign liabilities of the US are virtually the same, and remain around 130% of GDP. The reason is that in order to sustain the dollar as the main currency of choice of trading firms in both countries, dollar funding needs to be relatively available in both. As a result, US assets are relatively equally distributed across countries, in a way very similar to the other dollar coordinated steady-state. However, the euro use and distribution is the mirror image of the other steady-state – in this case, euros are relatively more heavily used in the US than in the EU, which leads to a relatively large holdings of EU assets by US households. Thus, in this steady-state the US households are relatively rich and have a substantially positive NFA position. This is clearly a better steady-state for the US households to be in. They both face high excess returns on their foreign assets and have a substantially positive NFA position (and thus higher net wealth than the EU households). In other words, they are both rich and enjoy high excess returns. As a result, the steady-state trade deficit the US is able to run is very high, over 5% of GDP, and the US's consumption is more than 6% higher than that of the EU household in steady-state.

Lastly, the symmetric steady-state is the easiest one to understand. This is a steady-state where both assets are perfectly symmetrically distributed across the world, hence both households' portfolios are perfectly diversified and feature no home bias. Moreover, firms use both currencies equally in their trading activities, hence there is no exorbitant privilege, and no excess returns on either asset.

Clearly the different steady-states have differing welfare implications. To quantify those, we compute the corresponding compensating consumption differential, as a fraction of the symmetric steady-state consumption. We do this analysis from the perspective of the US household, but the table can also be read in reverse for the EU household's perspective. The results are presented in Table 3. The second column of the table reports how much higher or lower should steady-state consumption be in the different steady-states, to leave the US household at the same level of utility as in the symmetric one.

Table 3: Welfare Comparison

Steady-State	Consumption Equivalent rel. to symm.
Dollar Coordination, high US NFA	1.0477
Dollar Coordination, low US NFA	1.0162
Symmetric	1
Euro Coordination, high US NFA	0.9841
Euro Coordination, low US NFA	0.9539

Unsurprisingly, the best steady-state for the US household is the one where both the USD is the dominant currency *and* the US households are rich, hence the US has a positive NFA position. In that case, the US households are quite wealthy and their welfare improvement over the symmetric steady-state is equal to 4.77% of steady-state consumption. While this represents a very large effect, arguably we have not seen this steady-state in the data. On the other hand, as we have just argued, the dollar coordinated steady-state where the US runs a negative NFA position appears to be quite empirically relevant, and there the US also enjoys welfare gain of 1.6% of steady-state consumption. This is a very significant number, and shows the large potential importance of the exorbitant privilege.

Another relevant comparison is between the welfare in the low NFA dollar steady-state, and its mirror image – the euro coordinated steady-state, where the US has a significantly positive NFA position. The welfare gap between those two cases is more than 3.2% of consumption, implying that if the US loses its dominant currency position, and the world swings to this different single currency situation, the US stands to lose a very substantial amount in terms of consumption equivalent. Moreover, if the switch is instead to the worst equilibrium, where the US loses both its dominant position and remains indebted to the rest of the world (a more likely scenario as we discuss below), then the US will experience a welfare loss equivalent to 6.2% of steady-state consumption.

6 Dynamics

A natural question for our environment is what could lead to changes in the world currency regime. To better understand which potential switches are most likely and why, we now turn to analyzing the dynamic properties of our economy. For now, we focus on the perfect foresight dynamics and transition paths.[8]

In Figure 7 we plot the attraction regions of all five distinct steady-states, filled-in in different colors. The figure has two axes which represent the two relevant state variables of the model – the home and foreign bond positions with which a household enters a given period. We plot the picture from the view point of the US household, with its holdings of EU bonds on the X-axis, and its holdings of US bonds on the Y-axis. Then we construct the figure by starting the dynamic economy from a fine grid in this two-dimensional space, and simulating for many periods. We then color code each initial bond holdings position according to according to the steady-state to which it converges. This gives us a picture of the overall attraction regions of the steady-states. The solid black dots within each region correspond to the steady-state of their respective regions.

A number of interesting features emerge. First, we see that all five steady-states are stable and have non-trivial attraction regions.[9] Thus, if we know that the economy is close to one steady-state at time t, we would expect that it stays in that neighborhood for a significant amount of time. Small shocks are not enough to cause a switch from one steady-state to another. The stability of the steady-states is due to the endogenous response of the liquidity premia $\Delta_c^{\$}$ and $\Delta_c^{€}$. In response to small shocks, those liquidity premia adjust in such a way as to incentivize the households to choose bond positions that will bring the economy back to the local steady-state. We explain this process through an example further below.

[8]The dynamic model is highly non-linear, so characterizing dynamics requires a global solution. We solve the model using a parameterized expectations approach. We parameterize expectations in our procedure using multilinear projection over a very fine grid of bond holdings.

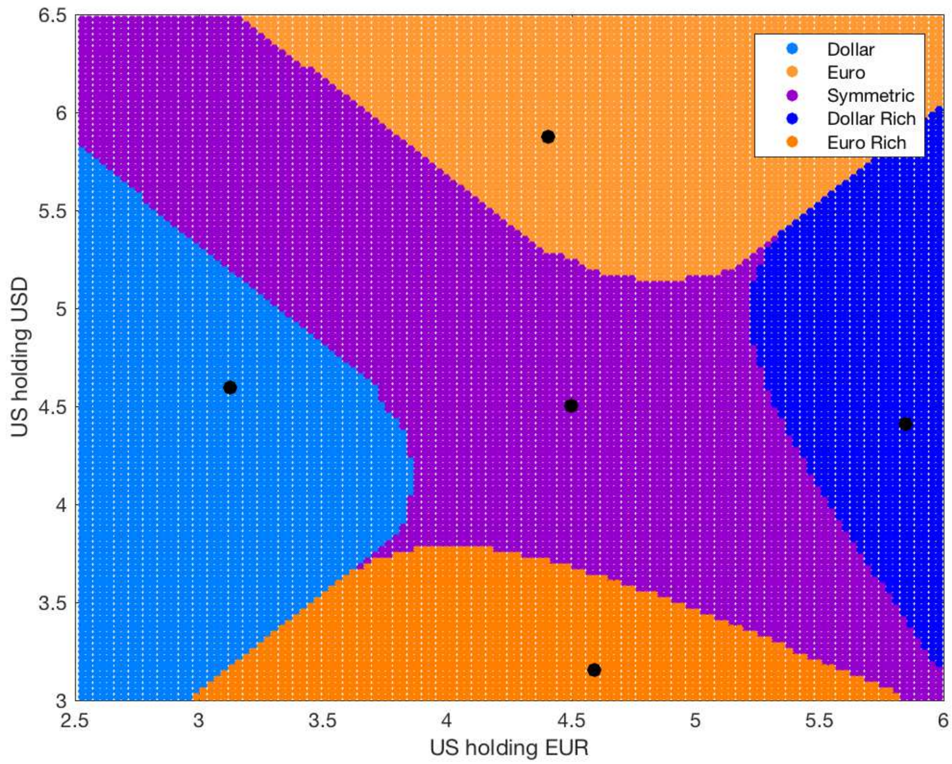
[9]This is also confirmed by solving the linear approximations to the model at each steady-state, and then computing the implied eigenvalues of the linear system.

But before we get to that, it is interesting to first analyze the positions of the attraction regions of the different steady-states relative to each other. First, we note that all four coordinated steady-states tend to have attraction regions of a relatively similar size, with perhaps the coordinated steady-states where the dominant county has a negative NFA position having somewhat larger attraction regions, and are thus relatively more likely to occur.[10] Second, the symmetric steady-state itself has a relatively large region of attraction, but it has a very specific shape and position. Essentially, it acts as a separating barrier between the two coordinated steady-states in the lower left part of the graph, where the US runs a negative NFA position, and the coordinated steady-states in the upper right, where the US has a positive NFA position. Hence, it is not easy to transition from the bottom left part of the graph directly to the upper right portion. Such a transition is likely to end up in the symmetric steady-state in the middle. Hence, being the currency hegemon that has a relatively low level of assets (leftmost steady state in case of the US) is a double edged sword. Such an economy enjoys benefits as long as it remains in the neighborhood of that steady-state, but it also runs the risk of going to a significantly worse situation that is likely to persist. As a result, we see the excess returns arising from currency hegemony as both a privilege and a duty.

Let's take for example the empirically relevant steady sate where the US has the dominant currency and has a negative NFA position. The only two destinations of a potential transition to a new steady state are the symmetric steady-state, or the euro-coordinated steady-state where the US remains a net debtor and has a significantly negative NFA position. The empirically relevant dollar-dominant steady-state is in the left part of the figure, and is colored in light blue, while the euro coordinated steady-state where the US has a negative NFA position is the orange area in the bottom part of the graph. In particular, it appears impossible for the economy to transition directly from one of the dollar coordinated steady-states to the other one. Hence, the model implies that even though the US enjoys a currency hegemonic position currently, it is not easy for it to go to the best possible steady-state, where it is both a hegemon and significantly wealthier than the rest of the world. Instead, it only runs the risk of losing is hegemonic position by falling into either the symmetric steady-state, or the bottom euro-coordinated steady-state. And moreover, the potential switch to a euro-coordinated world is not the mirror image one, where the US would at least end up accumulating a lot of assets and hence improve its NFA, but rather the worst possible one

[10] They also have a stronger attraction pull in the sense of lower maximum eigen values of their respective local system of linear approximation.

Figure 7: steady-state Attraction Regions



where it both remains a net debtor and loses its ability to borrow at low interest rates.

The reason for these possible transitions is quite intuitive. While there is a strategic complementarity in currency choice, the equilibrium cannot easily switch from a dollar coordinated world to a euro coordinated world because for the new coordination to be an actual equilibrium firms in both countries must be able to obtain sufficient financing in the other currency. Thus, households must also adjust their portfolios accordingly. However, what kind of portfolio adjustments are potential equilibrium paths depend on the optimal consumption smoothing incentive of the households. And naturally, it is unlikely that the steady-state switches from the dollar coordinated one where the US runs a negative NFA position, to the euro-coordinated one where the US runs a positive NFA position, because that would require a huge amount of saving and wealth accumulation in the US. The US will essentially have to completely reverse its negative NFA position, while at the same time losing the benefit of the excess return it enjoys as the currency hegemon. This is not an optimal consumption path for the US households, hence there is no equilibrium path that switches directly between those two steady-states. Instead, if there are shocks that move the US household to accumulate some wealth, the end result is more likely to end up in the symmetric steady-state region.

On the other hand, switching between the two potential coordinated steady-states where the US remains a net debtor is quite possible. In that case, the US does not need to experience a large wealth accumulation, but would essentially need to just shift its portfolio composition by tilting it towards foreign assets. Yet, while that transition is a lot more likely, it will also be even more painful for the US. This is because it would lose the excess return it enjoys on its assets *and* remain a net debtor, hence they will have to work extra hard and consume a lot less, in order to be able to service their debt at the new, higher interest rates. This potential dangerous transition to euro hegemonic position is why the US could be said to currently be not only enjoying an exorbitant privilege, but also a duty. It has the duty of shepherding the supply of the international medium of exchange relatively responsibly, so that it does not rely too strongly on its access to excess returns and get too indebted. Because if the US keeps getting deeper and deeper into debt to the rest of the world, combined with a falling home bias this will shift it further and further towards the bottom of the above figure. But the further down it goes, the more likely it is that a shock could move the global equilibrium into the attraction region of the euro-dominant bottom equilibrium, where the EU will then enjoy the best of both worlds – both an exorbitant privilege, and a large and positive net foreign asset position.

To illustrate all potential switches, and the likely transition paths, in Figure 8 we plot the transition paths from three points that are close to the intersection of the attraction regions of those three steady-states. In particular, we choose a point that is just inside the attraction region of the dollar-dominant steady-state where the US is a net debtor, another point that is just inside the attraction region of the symmetric steady-state, and one that is just inside the euro-dominant region where the US remains relatively poor. In the top left panel, we show the transition paths after just a few periods, and then proceeds clockwise, by adding more and more periods to the transition paths.

In the top left we can see how the paths diverge almost immediately. If the economy is within the dollar-dominant attraction region, the US household quickly decumulates EU assets and starts accumulating some more US assets, which strengthens the dominant role of the dollar in international transactions, increases its liquidity premium, and moves the economy closer to that steady-state. On the other hand, we see that in both of the other cases the US keeps its holdings of US assets relatively stable, but instead quickly increases its holdings of foreign assets. As a result, along both of those paths the dollar liquidity premium is falling, and the euro liquidity premium is rising. Essentially, the distribution of EU assets

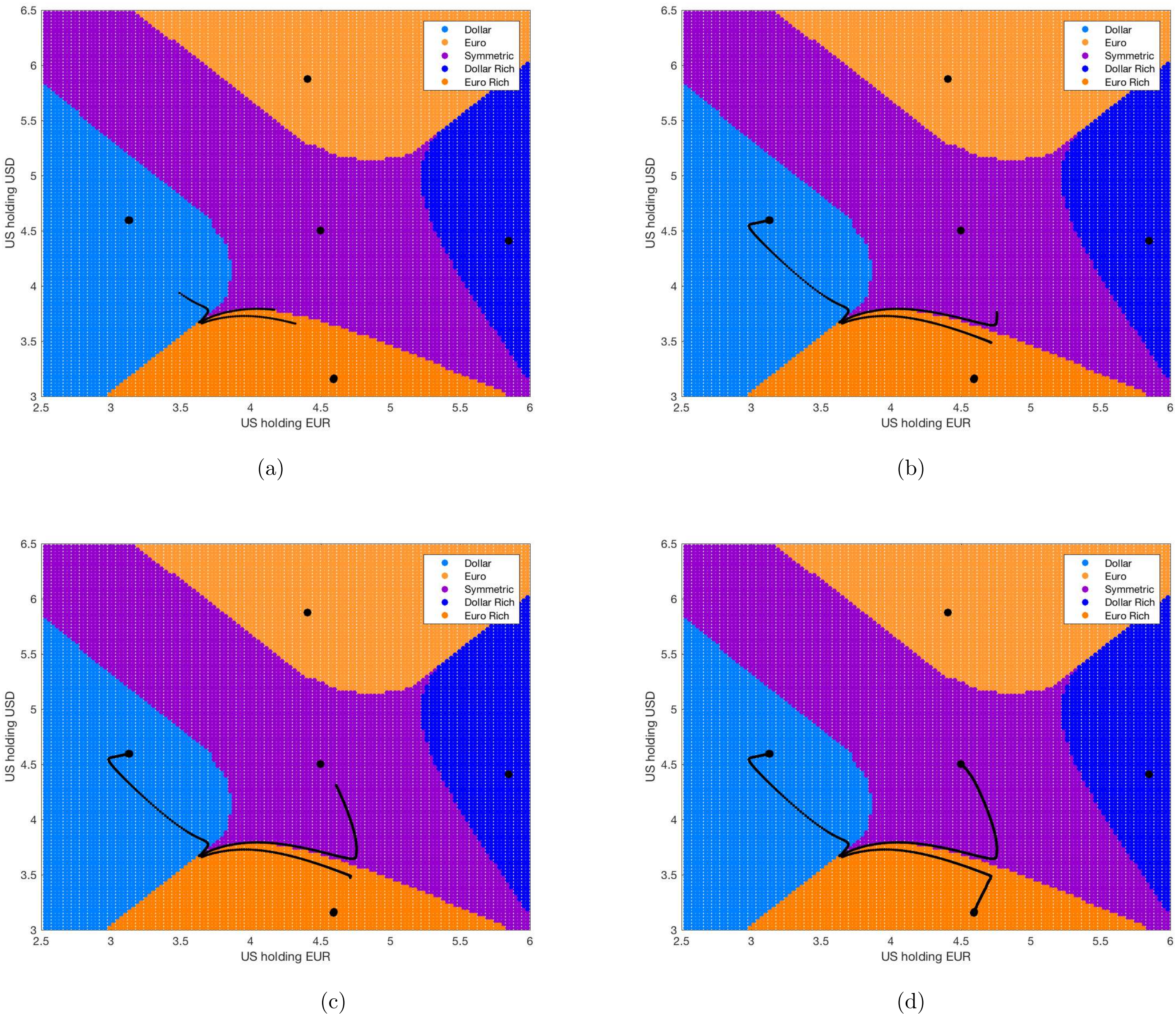
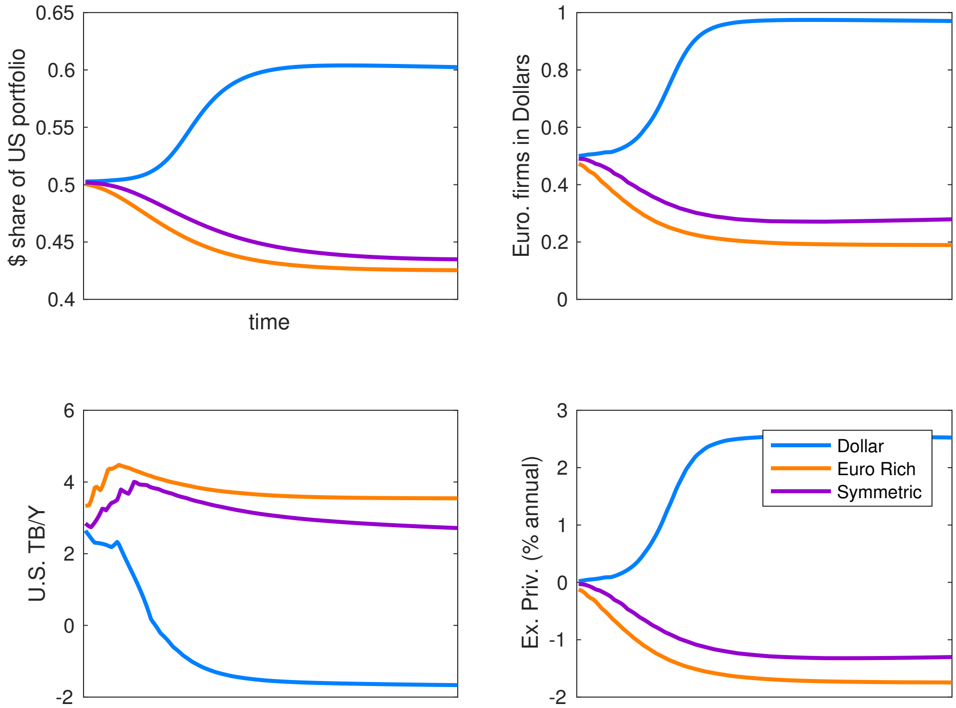


Figure 8: Transition Paths

Figure 9: Transition Paths



starts becoming more equal in both countries, hence an increasingly larger proportion of firms choose to seek euro funding.

Eventually, those two paths start diverging, and converge to two different steady-states. The reason is that the top line, which eventually goes to the symmetric steady-state, is an equilibrium path where the US has just enough US assets to start with, so that the dollar does not completely lose its role as an international medium of exchange, but rather the euro becomes similarly important. Along that path the share of dollar transactions slowly converges to 50%. On the other hand, along the bottom line, the US is slowly decumulating US assets, and eventually the increase in EU assets prompts more and more US firms to seek euro funding, because of its now greater availability. And since euros are already quite abundant abroad, as the US firms start shifting slightly towards euro assets, the EU firms are more than happy to follow, as they themselves also have easy access to euros. Thus, while the initial transition to the bottom steady-state is rather slow, eventually it builds up steam, and the economy goes through a rapid final transition to the bottom steady-state.

Finally, Figure 9 describes the evolution of a few key variables as the economy transitions from the nearly-indistinguishable starting points depicted in Figure 8 to the respective steady-states. The top left panel shows that the economies begin in all cases with nearly balanced portfolios. As time passes, the path that converges to the dollar-dominant steady-state exhibits increasing home bias in its portfolio holdings. Meanwhile, the top right panel shows that share of firms using dollars in their trade transactions grows quickly — in the case of the dollar-converging path — while the dollar share falls initially along the path towards the other two steady-states. The trade balance begins positive in all these cases, but falls into substantially negative territory in the case of the dollar-dominant path. It is positive to begin with, because at the border point the dollar liquidity premium is small, while the US is still heavily indebted to the rest of the world. Finally, the bottom-right panels show that exorbitant privilege grows over time for the dollar-dominant path, but falls for the other cases, as it closely mirrors the faction of European firms using dollars. From all the pictures it is evident that, although all the paths eventually converge to new steady-states, convergence along the path for dollar dominance is substantially faster than in the other cases. And in general, the transition paths exhibit interesting, non-linear dynamics.

7 Conclusions

This paper proposes a feedback mechanism between household portfolio choices and the funding choices made by firms engaged in trade. An asset that is widely used for liquidity purposes earns an excess return. The liquidity premium is higher in countries where the asset is relatively scarce, leading it to be relatively more broadly-held internationally. Conversely, a broadly-held asset is more likely to be chosen by trading firms to perform the liquidity function. Hence, a multiplicity of steady-states emerges corresponding to different dominant currencies and different long-run asset positions. A country issuing the dominant asset benefits from lower interest rates on its foreign asset position, corresponding to the liquidity premium earned by the assets it issues. Such a privilege can persist indefinitely, but it is not unconditional: changes in the environment or policy may lead the dominant country to lose it privilege, potentially falling into a long-lasting period of low wealth and low returns on its foreign asset position.

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