Estimating (S,s) rule regression models

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Contents

Introduction

- There are many economic variables such as product prices, or firm employment levels that exhibit infrequent adjustments.
- These outcomes can occur when there are costs associated with making changes (e.g. menu costs), which lead agents to adopt an (S,s) decision rule.
- Such rules are characterized by a band of inaction, where agents tolerate some deviation from an optimal frictionless outcome, provided the deviation is not too large.
- This presentation describes a new command `xtss` for estimating state dependent (S,s) models, for panel data applications. This is based on Fougere et al. (2010) and Dhyne et al. (2011).

Literature

- Fougere et al. (2010) estimate a price rigidity model to assess the impacts of the minimum wage on prices in French restaurants. This is based on a flexible (S,s) model where the thresholds vary over time and across restaurants.
- Dhyne et al. (2011) use price observations on various goods in Belgium and France to study the importance of real and nominal rigidities in price adjustments. They find that asymmetry in price adjustments are caused by trends in cost and not asymmetry in the (S,s) bounds.
- Gautier and Saout (2015) use a similar specification to Fougere et al. (2010) to examine the speed at which refined oil prices are passed-through to gasoline prices.

The model

- Consider the case where the dependent variable is the price of product $i = 1,..,N$ in period $t = 1,..,T$ denoted p_{it}.
- Letting p^*_{it} denote the latent desired or frictionless price, the price decision rule is given by:

$$p_{it} = \begin{cases} p^*_{it} & \text{if} \quad p^*_{it} - p_{it-1} > c^u_{it} \\ p^*_{it} & \text{if} \quad p^*_{it} - p_{it-1} < -c^d_{it} \\ p_{it-1} & \text{if} \quad -c^d_{it} \leq p^*_{it} - p_{it-1} \leq c^u_{it}. \end{cases} \tag{1}$$

  where c^u_{it} and $-c^d_{it}$ are stochastic (S,s) bounds which differ across i and t and allow for asymmetric menu costs.
- Price equals the frictionless value when the difference $p^*_{it} - p_{it-1}$ is above c^u_{it} (price rise) or below $-c^d_{it}$ (price fall).

The model

- In the analysis of prices, the thresholds measure the extent to which changes are costly and represent nominal rigidity.
- Dhyne et al. (2011) assumes these are normality distributed with time-invariant means; however this leads to a non-zero probability of a price rise and a price fall.
- For example if $p^*_{it} - p_{it-1} = 0.1$, $c^u_{it} = -0.2$ and $-c^d_{it} = 0.2$, the gap is above the upper bound but also below the negative of the lower bound.
- Such outcomes are inconsistent with the price decision rule where the thresholds must be non-negative.

The model

- To avoid this issue, I modify their model and allow the thresholds to have a normal distribution truncated at zero:

$$\begin{aligned} c_{it}^{u} &\sim N^{+}(\mu_{it}^{u}, \sigma_{c}^{2}) \\ c_{it}^{d} &\sim N^{+}(\mu_{it}^{d}, \sigma_{c}^{2}) \end{aligned} \tag{2}$$

- Following Gautier and Saout (2015) this also allows the means to depend on explanatory variables z_{it} that modify the timing of the price adjustment:

$$\begin{aligned} \mu_{it}^{u} &= z_{it}^{'}\lambda_{u} \\ \mu_{it}^{d} &= z_{it}^{'}\lambda_{d} \end{aligned} \tag{3}$$

The model

- The final equation is for the frictionless price, which is observed when prices are amended.
- Letting x_{it} denote a vector of exogenous explanatory variables:

$$p^*_{it} = x^{'}_{it}\beta + u_i + \epsilon_{it}, \quad \epsilon_{it} \sim iidN(0, \sigma^2_\epsilon) \tag{4}$$

  where $u_i \sim iidN(0, \sigma^2_u)$ is an individual specific random effect that contains unobserved product heterogeneity.

- In the context of price modeling, this process would arise as a log-linear expression under isoelastic demand and constant marginal costs where x_{it} contains factor prices.

Estimation

- Letting $I^u_{it} = 1$ indicate a price rise and $I^d_{it} = 1$ a price fall, the observed price in (1) is:

$$p_{it} = p_{it-1} + (I^u_{it} + I^d_{it})(p^*_{it} - p_{it-1})$$

- Letting $d_{it} = x'_{it}\beta + u_i - p_{it-1}$, from (4) the above becomes:

$$\Delta p_{it} = (I^u_{it} + I^d_{it})(d_{it} + \epsilon_{it}) \tag{5}$$

where the indicators

$$I^u_{it} = \begin{cases} 1 & \text{if} \quad d_{it} + \epsilon_{it} > c^u_{it} \\ 0 & \text{if} \quad d_{it} + \epsilon_{it} \leq c^u_{it}. \end{cases} \tag{6}$$

$$I^d_{it} = \begin{cases} 1 & \text{if} \quad d_{it} + \epsilon_{it} < -c^d_{it} \\ 0 & \text{if} \quad d_{it} + \epsilon_{it} \geq -c^d_{it}. \end{cases} \tag{7}$$

Estimation

- Analogous to a Tobit II model, OLS based on the amended prices will be inconsistent as $E[\epsilon_{it} \mid I^u_{it} = 1 \cup I^d_{it} = 1] \neq 0$. Instead the model is estimated by maximum likelihood.
- Given the first-order Markovian property of the model, the the contribution of product i to the likelihood given p_{i0} is:

$$L_i = \int_{-\infty}^{\infty} \prod_{t=2}^{T} f(\Delta p_{it} \mid p_{it-1}, u_i, x_{it}, z_{it}) f(u_i) du_i$$

- The integral in the above is approximated by Gauss-Hermite quadrature.
- To derive $f(\Delta p_{it} \mid p_{it-1}, u_i, x_{it}, z_{it})$ we distinguish between the cases where prices rise, prices fall and prices remain constant.

Price rise

- Suppressing the dependence on u_i, x_{it} and z_{it} to simplify the exposition, the contribution to the likelihood of a price rise is:

$$f(\Delta p_{it}, I_{it}^u = 1 \mid p_{it-1}) = Pr(c_{it}^u < \Delta p_{it} \mid \Delta p_{it}) f(\Delta p_{it} \mid p_{it-1})$$

- From (2) and (5), the components in the above are:

$$\begin{aligned} f(\Delta p_{it} \mid p_{it-1}) &= \frac{1}{\sigma_\epsilon} \phi\left(\frac{\Delta p_{it} - d_{it}}{\sigma_\epsilon}\right) \\ Pr(c_{it}^u < \Delta p_{it} \mid \Delta p_{it}) &= \frac{\Phi(\frac{\Delta p_{it} - \mu_{it}^u}{\sigma_c}) - \Phi(\frac{-\mu_{it}^u}{\sigma_c})}{\Phi(\frac{\mu_{it}^u}{\sigma_c})} \end{aligned}$$

Price fall

- The contribution to the likelihood of a price fall is:

$$f(\Delta p_{it}, I^d_{it} = 1 \mid p_{it-1}) = Pr(c^d_{it} < -\Delta p_{it} \mid \Delta p_{it}) f(\Delta p_{it} \mid p_{it-1})$$

- The first component in the above is:

$$Pr(c^d_{it} < -\Delta p_{it} \mid \Delta p_{it}) = \frac{\Phi(\frac{-\Delta p_{it} - \mu^d_{it}}{\sigma_c}) - \Phi(\frac{-\mu^d_{it}}{\sigma_c})}{\Phi(\frac{\mu^d_{it}}{\sigma_c})}$$

Price constancy

- The contribution from no change in price occurs when both $p^*_{it} - p_{it-1} < c^u_{it}$ and $p^*_{it} - p_{it-1} > -c^d_{it}$; from (6)-(7) this is:

$$Pr(I^u_{it} = 0, I^d_{it} = 0 \mid p_{it-1}) = Pr(\underbrace{\epsilon_{it} - c^u_{it}}_{\epsilon_{1it}} < -d_{it}, \underbrace{\epsilon_{it} + c^d_{it}}_{\epsilon_{2it}} > -d_{it} \mid p_{it-1})$$

- As $\epsilon_{1it} \leq \epsilon_{2it}$, the above simplifies to:

$$= Pr(\epsilon_{1it} < -d_{it} \mid p_{it-1}) - Pr(\epsilon_{2it} < -d_{it} \mid p_{it-1}) \tag{8}$$

- Evaluating the above requires the CDF of the sum of a normal and truncated normal random variable.

Price constancy

- The pdf's of ϵ_1 and ϵ_2 can be derived using the convolution formula. Focusing on ϵ_1, after considerable algebra this yields:

$$f(\epsilon_1) = \frac{1}{s\Phi(\mu^u/\sigma_c)}\Phi\left(a + b\frac{\epsilon_1 + \mu^u}{s}\right)\phi\left(\frac{\epsilon_1 + \mu^u}{s}\right) \quad (9)$$

- The components in the above expression are:

$$\begin{aligned} s &= \sqrt{\sigma_\epsilon^2 + \sigma_c^2} \\ a &= \mu^u\frac{s}{\sigma_\epsilon\sigma_c} \\ b &= -\frac{\sigma_c}{\sigma_\epsilon} \end{aligned} \quad (10)$$

Price constancy

- Making the substitution $z = (\epsilon_1 + \mu^u)/s$ in (9) and integrating yields CDF:

$$Pr(\epsilon_1 \leq m) = \frac{1}{\Phi(\mu^u/\sigma_c)} \int_{-\infty}^{\frac{m+\mu^u}{s}} \Phi\left(a + bz\right) \phi\left(z\right) dz$$

- Based on in Owen (1980), the above is:

$$Pr(\epsilon_1 \leq m) = \frac{1}{\Phi(\mu^u/\sigma_c)} \Phi_2 \left[\frac{a}{\sqrt{1+b^2}}, \frac{m+\mu^u}{s}, \rho = \frac{-b}{\sqrt{1+b^2}} \right]$$

where $\Phi_2(x, y, \rho)$ is the bivariate normal CDF.

Price constancy

- For ϵ_2 the result is based on μ^d, the term $z = (\epsilon_2 - \mu_d)/s$ and correlation coefficient is negative.
- Substituting the expressions for s, a, b in (10), the probability of no change in price in (8) is given by:

$$\frac{\Phi_2\left[\frac{\mu^u_{it}}{\sigma_c}, \frac{\mu^u_{it}-d_{it}}{\sqrt{\sigma^2_\epsilon+\sigma^2_c}}, \frac{\sigma_c}{\sqrt{\sigma^2_\epsilon+\sigma^2_c}}\right]}{\Phi(\mu^u_{it}/\sigma_c)} - \frac{\Phi_2\left[\frac{\mu^d_{it}}{\sigma_c}, \frac{-\mu^d_{it}-d_{it}}{\sqrt{\sigma^2_\epsilon+\sigma^2_c}}, -\frac{\sigma_c}{\sqrt{\sigma^2_\epsilon+\sigma^2_c}}\right]}{\Phi(\mu^d_{it}/\sigma_c)}$$

- This completes the derivation of $f(\Delta p_{it} \mid p_{it-1}, u_i)$. The resulting MLE will be consistent if N or $T \to \infty$.

Stata Command: xtss

`xtss` *depvar* [*indepvars*] [*if*] [*in*] [, `thold(`*varlist*`)` `diff` `re` `intpoints(#)` `level(#)` `noconstant`]

`thold(`*varlist*`)` allows the upper and lower threshold mean parameters to depend on *varlist*. A constant is always included.

`diff` lets the upper and lower stochastic thresholds have different mean parameters. The default is they are the same.

`re` specifies that the that the latent variable model includes a random effect

`intpoints(#)` specifies the number of integration points for the Gauss-Hermite quadrature method, which is used to integrate out the random effects. The default is `intpoints(12)`.

Numerical example

- In this example, data is simulated on product prices supplied by various firms from the following DGP:

$$
\begin{aligned}
\log p^*_{it} &= \alpha_j + \log material_{it} + 0.2 cartel_t + u_i + N(0, 0.1) \\
u_i &\sim N(0, 0.1)
\end{aligned}
$$

  where α_j is a firm fixed effect, $material_{it}$ are material costs and $cartel_t = 1$ indicates collusion between firms.

- Collusion also impacts the number of price changes, reducing the threshold for a rise and increasing the threshold for a fall:

$$
\begin{aligned}
c^u_{it} &\sim N^+(0.2 - 0.1 cartel_t, 0.1) \\
c^d_{it} &\sim N^+(0.2 + 0.1 cartel_t, 0.1)
\end{aligned}
$$

Description of variables

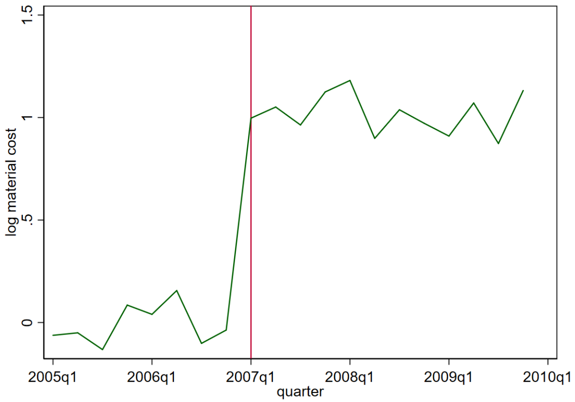
- Data is simulated for 150 products and 4 firms between 2005Q1 and 2009Q4 and the cartel operates from 2007Q1.

```
. use stickyprices.dta,clear
. describe

Contains data
  obs:         3,000
 vars:             9
 size:       108,000
--------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
--------------------------------------------------------------------------------
id              float   %9.0g                 product id
quarter         float   %tq                   quarter
firm            float   %9.0g      firm       firm
ln_price        float   %9.0g                 log price
ln_materials    float   %9.0g                 log material cost
cartel          float   %9.0g      cartel     cartel indicator
price_growth    float   %9.0g                 % change in price
direction       float   %9.0g      direction
                                              price direction
amend           float   %9.0g
--------------------------------------------------------------------------------
Sorted by: id  quarter
     Note: Dataset has changed since last saved.
```

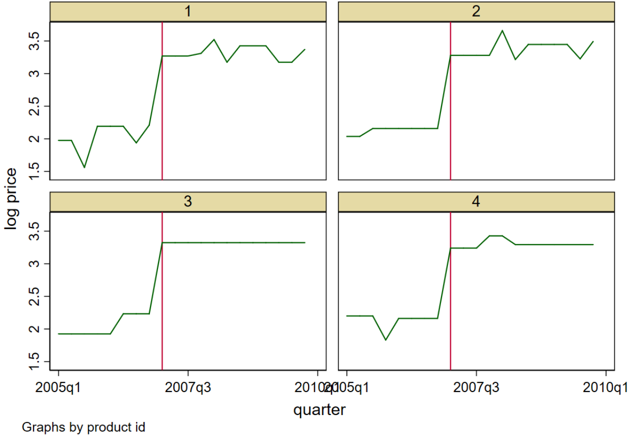
Path of material costs

- Collusion leads to a 20% rise in potential prices. The formation of the cartel is triggered by a large rise in material costs over the same period.



Path of prices

- The price paths are plotted for products 1 to 4. Prices are sticky with infrequent changes.



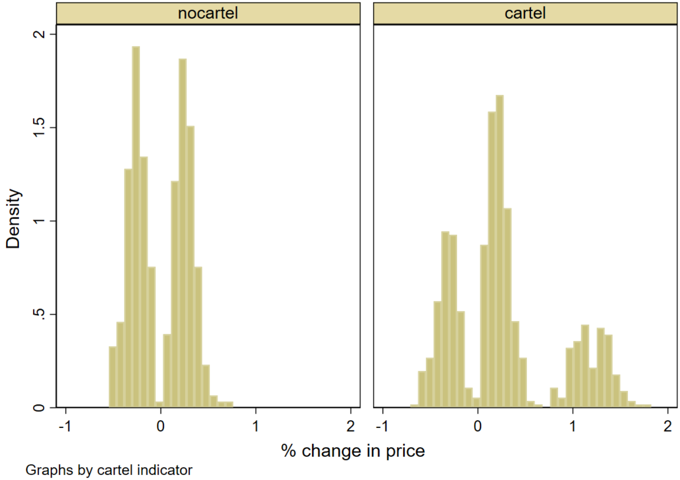
Frequency of price changes

- Prices remains constant for 65% of the sample, increases for 22% and falls for 13%.
- The frequency of price rises increases during the cartel as threshold for a rise is reduced.

```
. tab direction cartel, nofreq col
```

price direction	cartel indicator nocartel	cartel	Total
rise	15.50	27.11	22.47
fall	15.58	11.06	12.87
constant	68.92	61.83	64.67
Total	100.00	100.00	100.00

Histogram of % price changes



Maximum likelihood estimates

```
. xtss ln_price i.firm ln_materials cartel , thold(cartel) diff re

(output omitted)

ML random effects regression                    Number of obs     =      2,850
                                                Wald chi2(4)      =   59062.83
                                                Prob > chi2       =     0.0000
Log likelihood  = -446.50523
```

ln_price	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Model						
firm						
firm2	1.015957	.022717	44.72	0.000	.9714329	1.060482
firm3	1.029548	.020269	50.79	0.000	.9898214	1.069274
ln_materials	.986478	.0252836	39.02	0.000	.936923	1.036033
cartel	.2121362	.0261593	8.11	0.000	.160865	.2634074
_cons	.9832395	.0166212	59.16	0.000	.9506626	1.015816
Lower_threshold						
cartel	.0957907	.0106661	8.98	0.000	.0748855	.1166959
_cons	.1920669	.0082463	23.29	0.000	.1759045	.2082294
Upper_threshold						
cartel	-.0795848	.0122898	-6.48	0.000	-.1036724	-.0554972
_cons	.1918862	.0082121	23.37	0.000	.1757908	.2079816
/lnsigma_c	-2.436115	.0622327	-39.15	0.000	-2.558089	-2.314142
/lnsigma_e	-2.318582	.018449	-125.67	0.000	-2.354741	-2.282422
/lnsigma_u	-2.214386	.0571133	-38.77	0.000	-2.326326	-2.102446
sigma_c	.0875001	.0054454			.0774526	.098851
sigma_e	.0984131	.0018156			.0949181	.1020368
sigma_u	.1092206	.0062379			.0976539	.1221573

Comparison with other estimators

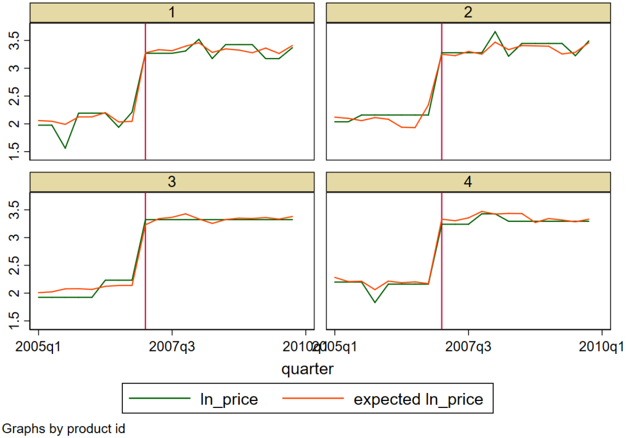
- The frictionless price model (4) is estimated by OLS/RE and FE using amended prices (`direction!="constant"`).
- The estimates are biased with no significant cartel overcharge.

	(1) SS	(2) OLS	(3) RE	(4) FE
main				
1.firm	0	0	0	0
	(.)	(.)	(.)	(.)
2.firm	1.016***	1.002***	1.012***	
	(44.72)	(41.08)	(41.73)	
3.firm	1.030***	1.002***	1.006***	
	(50.79)	(45.76)	(46.41)	
ln_materials	0.986***	1.167***	1.211***	1.219***
	(39.02)	(34.78)	(46.80)	(47.13)
cartel	0.212***	0.0376	-0.00753	-0.0167
	(8.11)	(1.09)	(-0.28)	(-0.62)
_cons	0.983***	0.992***	0.988***	1.707***
	(59.16)	(54.72)	(56.40)	(401.46)
N	2850	1060	1060	1060

t statistics in parentheses
* p<0.1, ** p<0.05, *** p<0.01

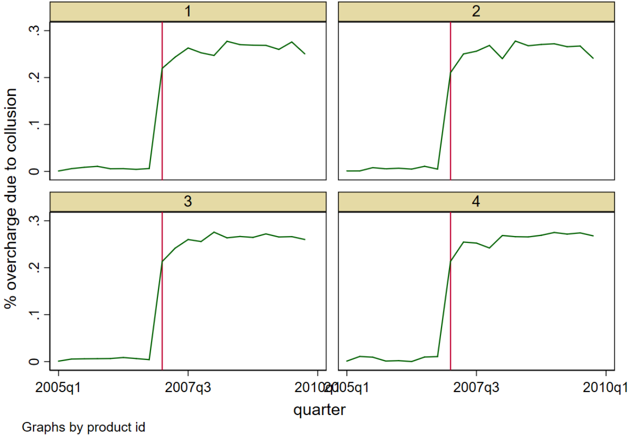
Model predictions

- Predicted prices are the average of 2000 simulated trajectories using draws of ϵ_{it}, u_i, c^u_{it} and c^d_{it} at the parameter estimates.



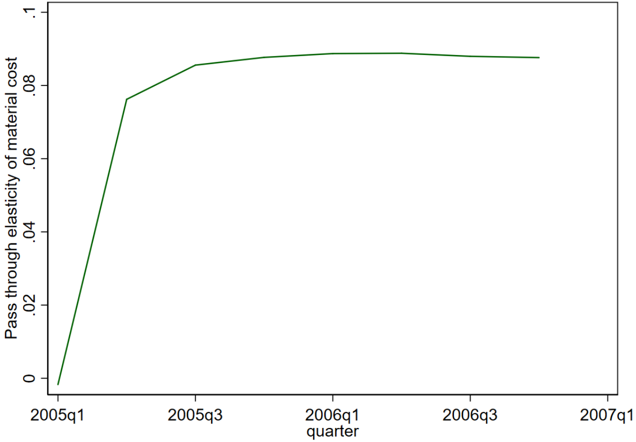
Price overcharge due to collusion

- The following plots the difference with and without the cartel. The maximum is above 20% as the frequency of rises increase.



Pass through of material costs

- The following shows the dynamic response across all products from a permanent 1% rise in material costs.



Conclusion

- This presentation has described a new Stata command `xtss` for estimating state dependent (S,s) models based on Fougere et al. (2010) and Dhyne et al. (2011).
- This estimator is appropriate when the decision to amend a variable occurs when the deviation from a frictionless outcome is outside the stochastic (S,s) thresholds
- As per sample selection models, usual estimators of the outcome equation (4) fitted to the amendments will be inconsistent as the amendment decision is endogenous.

References

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