2019 Stata Conference
Gleacher Center
The University of Chicago Booth School of Business
Chicago, July 11-12

Extending the difference-in-differences (DID) to settings with many treated units and same intervention time:
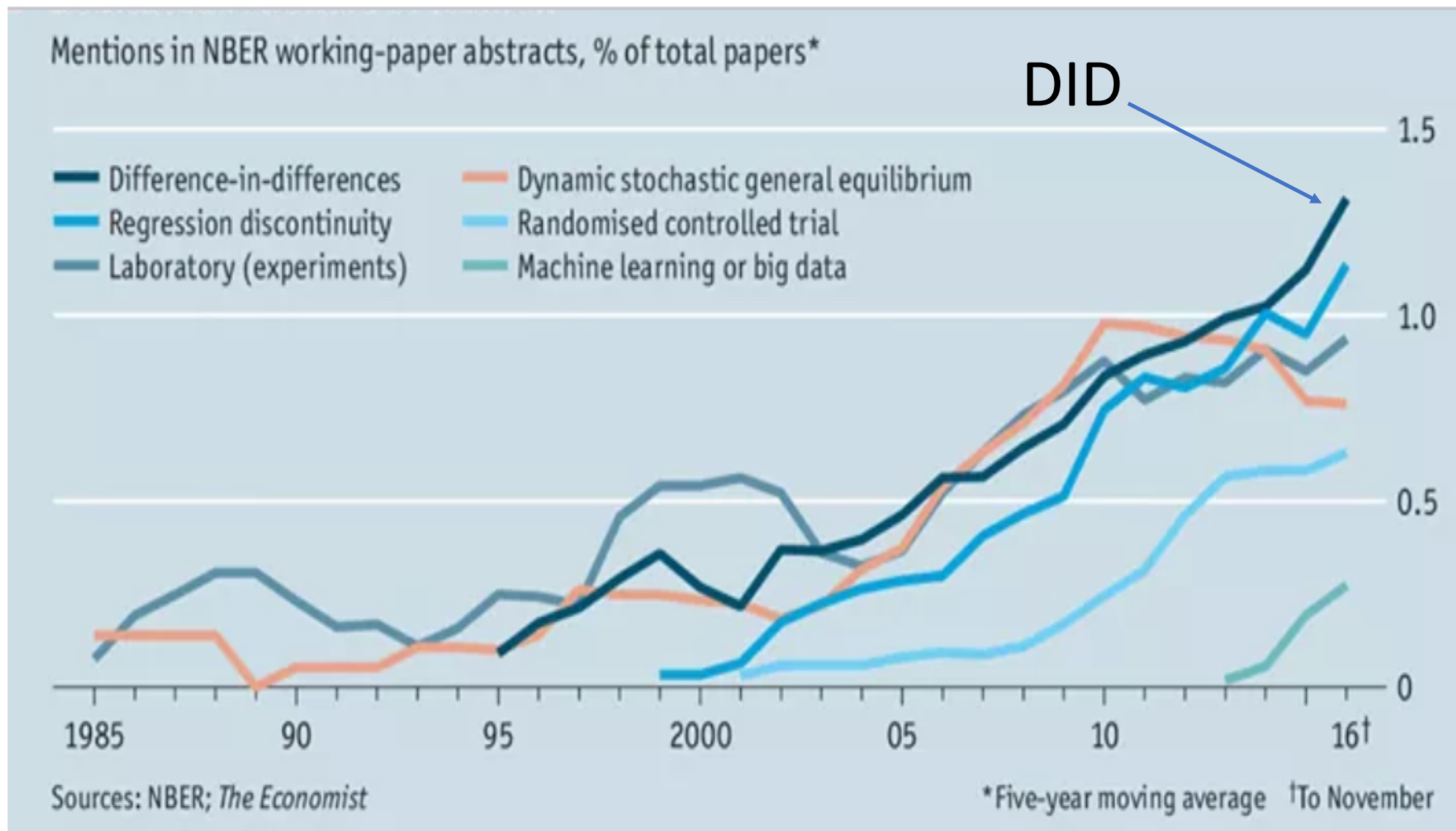## Model and Stata implementation

Giovanni Cerulli,
IRCrES-CNR
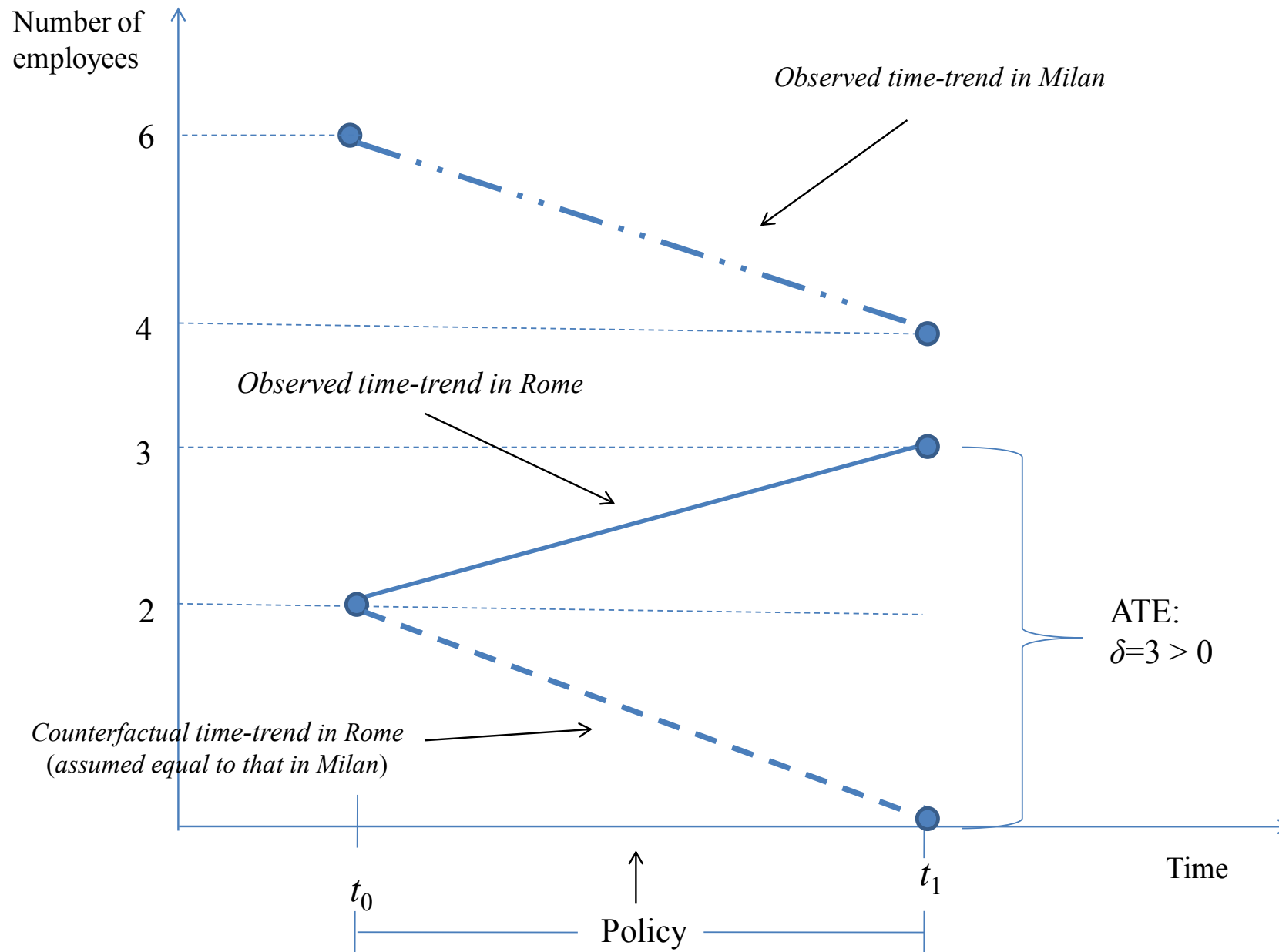National Research Council of Italy

Outline

- Providing an original model extending the **Difference-In-Differences (DID)** to the case of a **binary treatment** having a **time-fixed** nature

- Overcoming the **Synthetic Control Model limitations** on **inference**

- Providing a **test** the **common-trend** assumption

- Presenting **`tfdiff`**: **Stata routine** to implement this model

Diffusion of the DID in Economics

Mentions in NBER working-paper abstracts, % of total papers*

DID

- Difference-in-differences
- Regression discontinuity
- Laboratory (experiments)
- Dynamic stochastic general equilibrium
- Randomised controlled trial
- Machine learning or big data

1.5
1.0
0.5
0

1985   90   95   2000   05   10   16†

Sources: NBER; *The Economist*

*Five-year moving average   †To November

Number of employees

Observed time-trend in Milan

Observed time-trend in Rome

Counterfactual time-trend in Rome
(assumed equal to that in Milan)

ATE:
$\delta=3 > 0$

t_0

t_1

Time

Policy

DID modelling: a taxonomy

		TREATMENT TIMING	
		TIME-FIXED	**TIME-VARYING**
NUMBER OF TREATED UNITS	**ONE**	**Synthetic Control Method** (Abadie et al., 2010; Cerulli, 2019)	**?**
	MANY	**TFDIFF** (Cerulli, 2019)	**TVDIFF** (Autor, 2003; Cerulli and Ventura, 2019)

DID taxonomy tree

Many untreated → **DID**

One treated **Many treated**

Parametric **Nonparametric** **Time-varying** **Time-fixed**

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Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program

Alberto Abadie, Alexis Diamond & Jens Hainmueller

The Stata Journal (2019)
19, Number 3, pp. 1–15

Estimation of pre- and posttreatment average treatment effects with binary time-varying treatment using Stata

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ELSEVIER

A flexible Synthetic Control Method for modeling policy evaluation ☆

Giovanni Cerulli

Estimation of pre– and post–treatment Average Treatment Effects with binary time–fixed treatment*

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Modeling
TFDIFF

The TFDIFF model and its Stata implementation

o **Generalization** of the Difference–in–Differences estimator in a longitudinal data setting

o Treatment is **binary** and **fixed at a given time**

o Many **pre– and post–intervention periods** are assumed available

o **Stata routine** implementing this model in an **automatic** way:

- graphical representation of the estimated causal effects

- Testing *parallel-trend* assumption for the necessary condition of the identification of causal effects

Economics

In 2001, some European countries have adopted a common currency, the Euro. We would like to know whether this important economic reform has had an impact on adopters by comparing their economic performance over time with that of countries that did not adopt the Euro

Medicine

At a given point in time, some patients affected by too high blood pressure were exposed to a new drug developed to be more effective than previous ones in stabilizing blood pressure. We are interested in assessing the effect of this new drug by comparing follow-up blood pressure of treated people with that of a placebo group. We might be also interested in detecting effect duration over the follow-up time span

Environment

A group of regions decide to sign an agreement for reducing CO_2 emissions by promoting solar energy solutions. After some years, we are interested in assessing whether the level of CO_2 emissions in those regions is sensibly lower than the emissions in regions that did not sign the agreement

TFDIFF counterfactual setting

Longitudinal dataset with a **time-fixed** **treatment** at *t* = **01**.

1	2	3	4	5	6	7	8	9	10	11	12	13
id	t	w	D_0	D_1	D_2	y_{00}	y_{01}	y_{02}	y	y^1	y^0	$(y^1 - y^0)$
1	00	1	1	0	0	$y_{1,00}$	0	0	$y_{1,00}$	$y^1_{1,00} = y_{1,00}$	$y^0_{1,00} = \,.$	.
1	01	1	0	1	0	0	$y_{1,01}$	0	$y_{1,01}$	$y^1_{1,01} = y_{1,01}$	$y^0_{1,01} = \,.$	.
1	02	1	0	0	1	0	0	$y_{1,02}$	$y_{1,02}$	$y^1_{1,02} = y_{1,02}$	$y^0_{1,02} = \,.$	.
2	00	0	1	0	0	$y_{2,00}$	0	0	$y_{2,00}$	$y^1_{2,00} = \,.$	$y^0_{2,00} = y_{2,00}$	.
2	01	0	0	1	0	0	$y_{2,01}$	0	$y_{2,01}$	$y^1_{2,01} = \,.$	$y^0_{2,01} = y_{2,01}$	.
2	02	0	0	0	1	0	0	$y_{2,02}$	$y_{2,02}$	$y^1_{2,02} = \,.$	$y^0_{2,02} = y_{2,02}$	.

- Columns 11 and 12 set out the potential outcomes
- Column 13 shows the treatment effect.

Econometric set-up

According to Table 1, we can write the observed outcome $y_{i,t}$ using the following identity:

$$y_{i,t} = y_{i,00} + (y_{i,01} - y_{i,00}) \cdot D_{1,it} + (y_{i,02} - y_{i,00}) \cdot D_{2,it} \tag{1}$$

Given $y_{i,t}^w$, we can define the average treatment effect conditional on t as:

$$\text{ATE}(t) = \text{E}(y_{i,t}^1 - y_{i,t}^0 | t) \tag{2}$$

Average treatment effect at time t

We assume the **potential outcomes** to take on this form ($w = 0,1$):

$$y_{i,t}^{w} = \mu_t^{w} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \varepsilon_{i,t}^{w} \tag{3}$$

where μ_t^{w} is a time-varying component dependent on the treatment status, θ_i an idiosyncratic fixed–effect, λ_t a time fixed–effect, $g(\mathbf{x}_{it})$ a component depending on the characteristics of the unit i at time t, and $\varepsilon_{i,t}^{w}$ a random shock with finite variance and zero mean.

By taking the mean of Eq. (3), we have:

$$\mathrm{E}[y_{i,t}^{w}|w, t, \theta_i, \mathbf{x}_{it}] = \mu_t^{w} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) \tag{4}$$

which entails that:

$$\mathrm{ATE}(t) = \mu_t^{1} - \mu_t^{0}$$

$\longrightarrow$ **Average treatment effect at time t**

The potential outcomes have a similar expression as the one of the observed outcome in Eq. (1). Thus, for both the treated and untreated status, we have:

$$y_{i,t}^1 = \mu_{00}^1 + (\mu_{01}^1 - \mu_{00}^1) \cdot D_{1,it} + (\mu_{02}^1 - \mu_{00}^1) \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + e_{it}^1 \quad (6)$$

$$y_{i,t}^0 = \mu_{00}^0 + (\mu_{01}^0 - \mu_{00}^0) \cdot D_{1,it} + (\mu_{02}^0 - \mu_{00}^0) \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + e_{it}^0 \quad (7)$$

where $e_{it}^w = \varepsilon_{i,00}^w + (\varepsilon_{i,01}^w - \varepsilon_{i,00}^w) \cdot D_{1,it} + (\varepsilon_{i,02}^w - \varepsilon_{i,00}^w) \cdot D_{2,it}$, for $w = 0, 1$.

Using the Rubin's potential outcome identity, i.e.:

$$y_{i,t} = y_{i,t}^0 + w_{i,t} \cdot (y_{i,t}^1 - y_{i,t}^0) \quad (8)$$

Baseline regression

By substitution, we get:

$$y_{i,t} = \mu_{00}^0 + (\mu_{01}^0 - \mu_{00}^0) \cdot D_{1,it} + (\mu_{02}^0 - \mu_{00}^0) \cdot D_{2,it} + (\mu_{00}^1 - \mu_{00}^0) \cdot w_{i,t} +$$

$$w_{i,t} \cdot [(\mu_{01}^1 - \mu_{01}^0) - (\mu_{00}^1 - \mu_{00}^0)] \cdot D_{1,it} +$$

$$w_{i,t} \cdot [(\mu_{02}^1 - \mu_{02}^0) - (\mu_{00}^1 - \mu_{00}^0)] \cdot D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

$$y_{it} = \mu + \beta_1 D_{1,it} + \beta_2 D_{2,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \gamma_2 w_{it} D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

Baseline **fixed-effect** regression

Recovering ATE(*t*) from the baseline regression

$$y_{it} = \mu + \beta_1 D_{1,it} + \beta_2 D_{2,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \gamma_2 w_{it} D_{2,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it}$$

Causal effects

$$\text{ATE}(00) = \phi$$

$$\text{ATE}(01) = \phi + \gamma_1$$

$$\text{ATE}(02) = \phi + \gamma_2$$

A generalization of the previous model to $(T+1)$ times, i.e. $\mathcal{T} = \{0, 1, 2, \ldots, T-1, T\}$ with t^* the year of treatment is as follows:

$$y_{it} = \mu + \beta_1 D_{1,it} + \cdots + \beta_T D_{T,it} + \phi \cdot w_{it} + \gamma_1 w_{it} D_{1,it} + \cdots + \gamma_T w_{it} D_{T,it} + \theta_i + \lambda_t + g(\mathbf{x}_{it}) + \eta_{it} \quad (14)$$

with

$$\text{ATE}(0) = \phi$$

$$\text{ATE}(1) = \phi + \gamma_1$$

$$\ldots$$

$$\text{ATE}(t^*) = \phi + \gamma_{t^*}$$

$$\ldots$$

$$\text{ATE}(T) = \phi + \gamma_T$$

Causal effects over time

Testing the common–trend assumption

The previous model allows for a simple test for the common–trend assumption. This can be done by accepting this null:

$$H_0 : \phi = \gamma_1 = \cdots = \gamma_{(t*-1)} = 0$$

If accepted, this test leads to accept the presence of a common-trend between treated and untreated units, although the test is valid only under no–anticipatory behaviors.
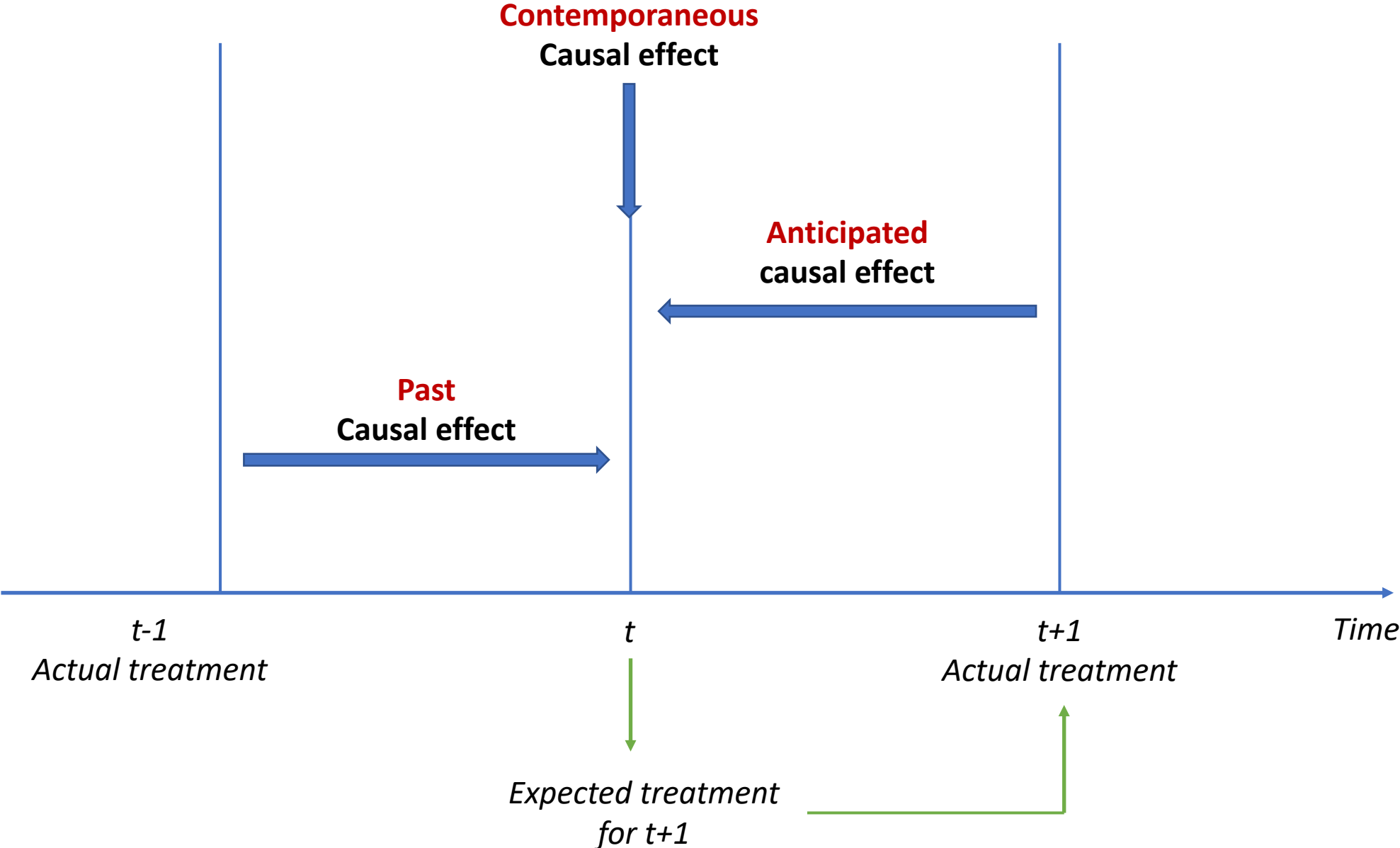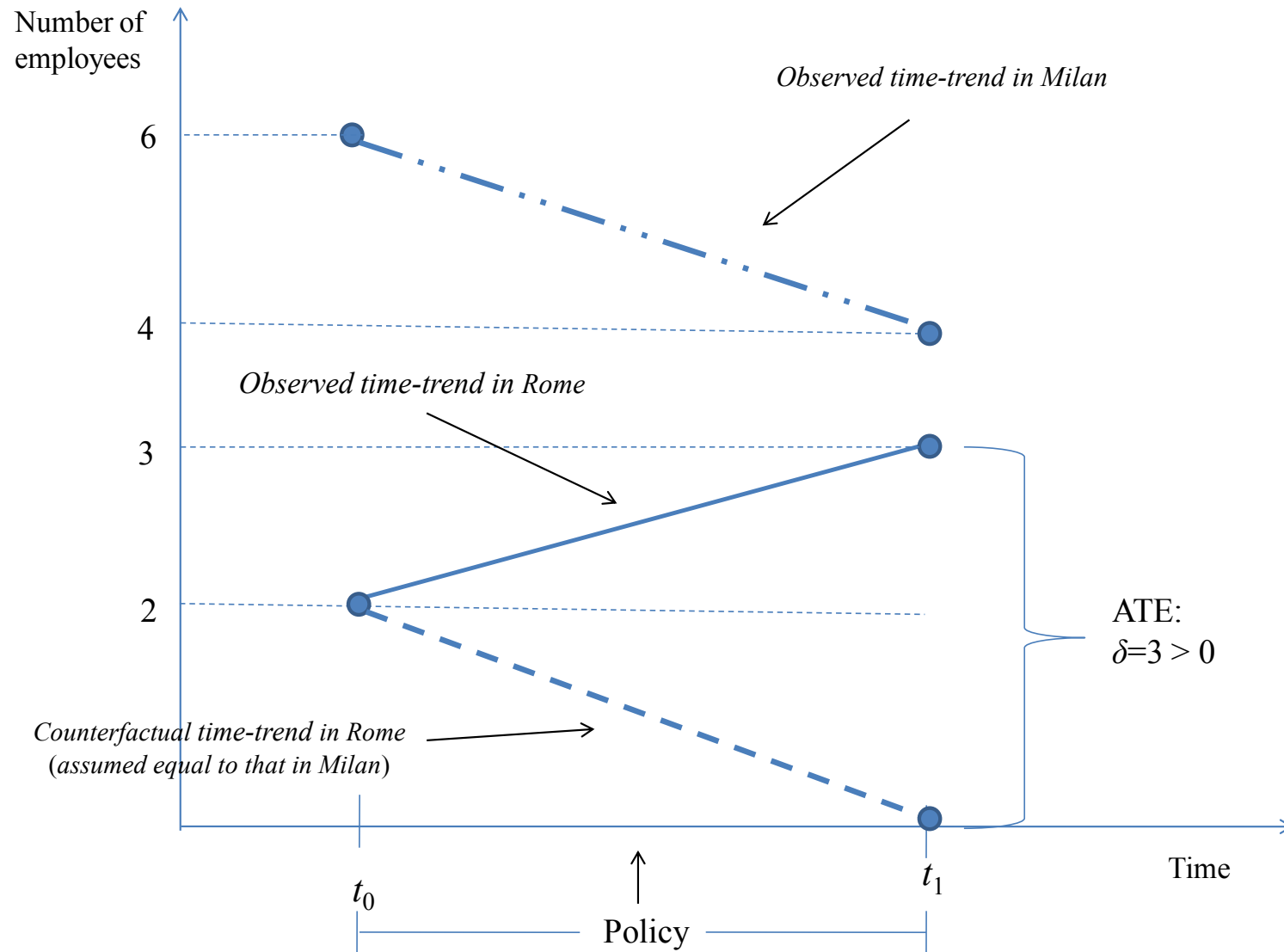
Testing the parallel-trend (or common-trend) assumption

o The **common-trend** assumption is at the basis of DID identification

o In general it is **untestable**

o If a sufficiently long times-series is available, the common-trend can be "assessed", under **no-anticipatory effects**

Number of employees

6

4

3

2

Observed time-trend in Milan

Observed time-trend in Rome

Counterfactual time-trend in Rome
(assumed equal to that in Milan)

ATE:
$\delta=3 > 0$

t_0

t_1

Time

Policy

Stata

implementation via

`tfdiff`

$\texttt{tfdiff}$ *outcome treatment* [*varlist*] [*if*] [*in*] [*weight*], **model**(*modeltype*) **t**(*year*) [**test_tt graph save_graph**(*graphname*) **vce**(*vcetype*)]

*fweight*s, *iweight*s, and *pweight*s are allowed;

where:

- *outcome*: is the target variable measuring the impact of treatment.

- *treatment*: is the binary treatment variable taking 1 for treated, and 0 for untreated units.

- *varlist*: is the set of pre-treatment (or observable confounding) variables.

Options of `tfdiff`

Options

- **model**(*modeltype*) specifies the estimation model, where modeltype must be one out of these two alternatives: "fe" (fixed effects), or "ols" (ordinary least squares). It is always required to specify one model.

- **t**(*year*) specifies the year of treatment

- **test_tt** allows for performing the parallel–trend test using the time–trend approach. The default is to use the leads.

- **graph** allows for a graphical representation of results. It uses the coefplot command implemented by Jann (2014).

- **save_graph**(*graphname*) permits to save the graph as *graphname*.

- **vce**(*vcetype*) allows for robust and clustered regression standard errors in model's estimates.

Simulation of the TFDIFF model

Time span: 2000-2020
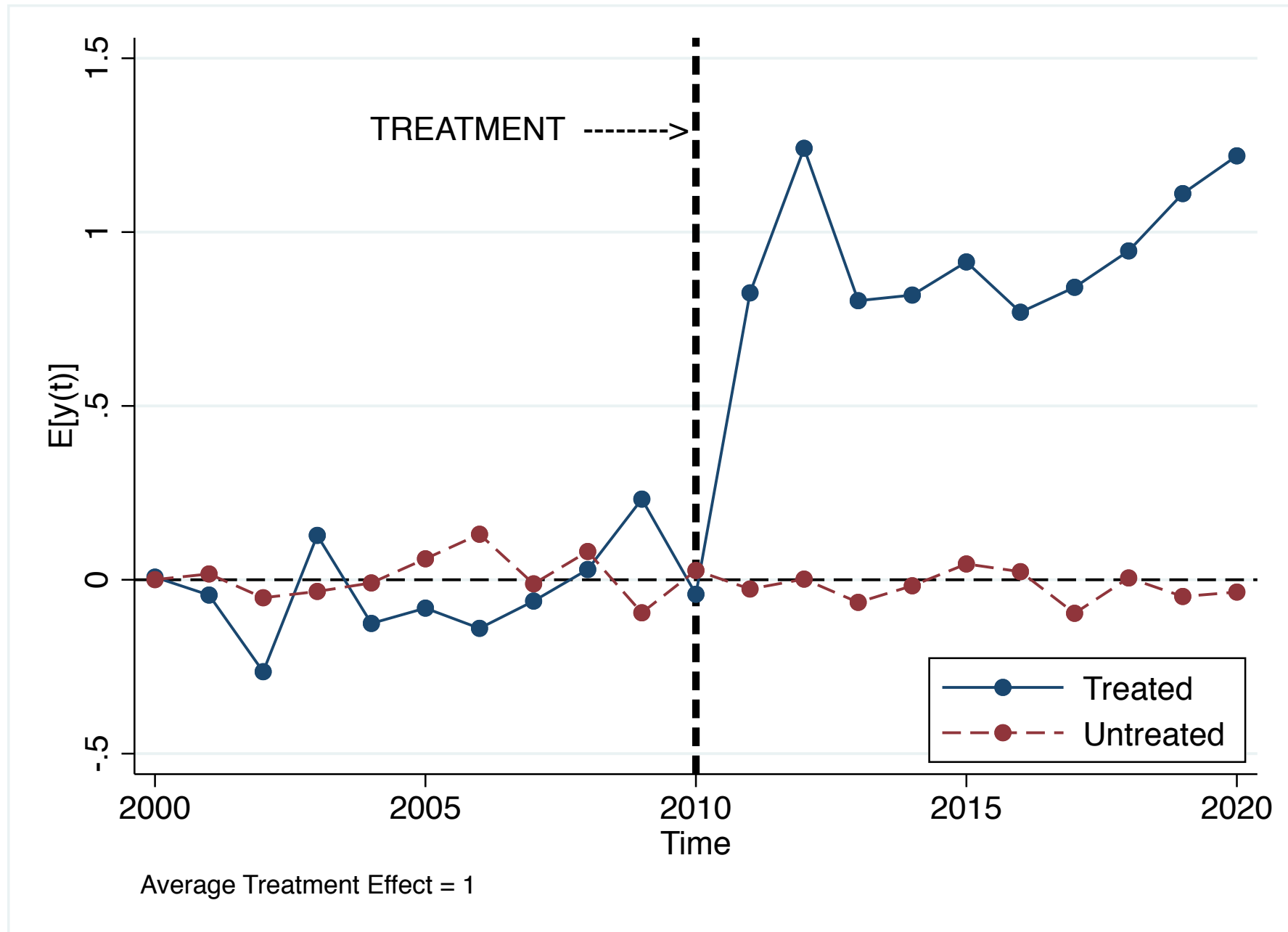
Number of years: 21

Year of treatment: 2010
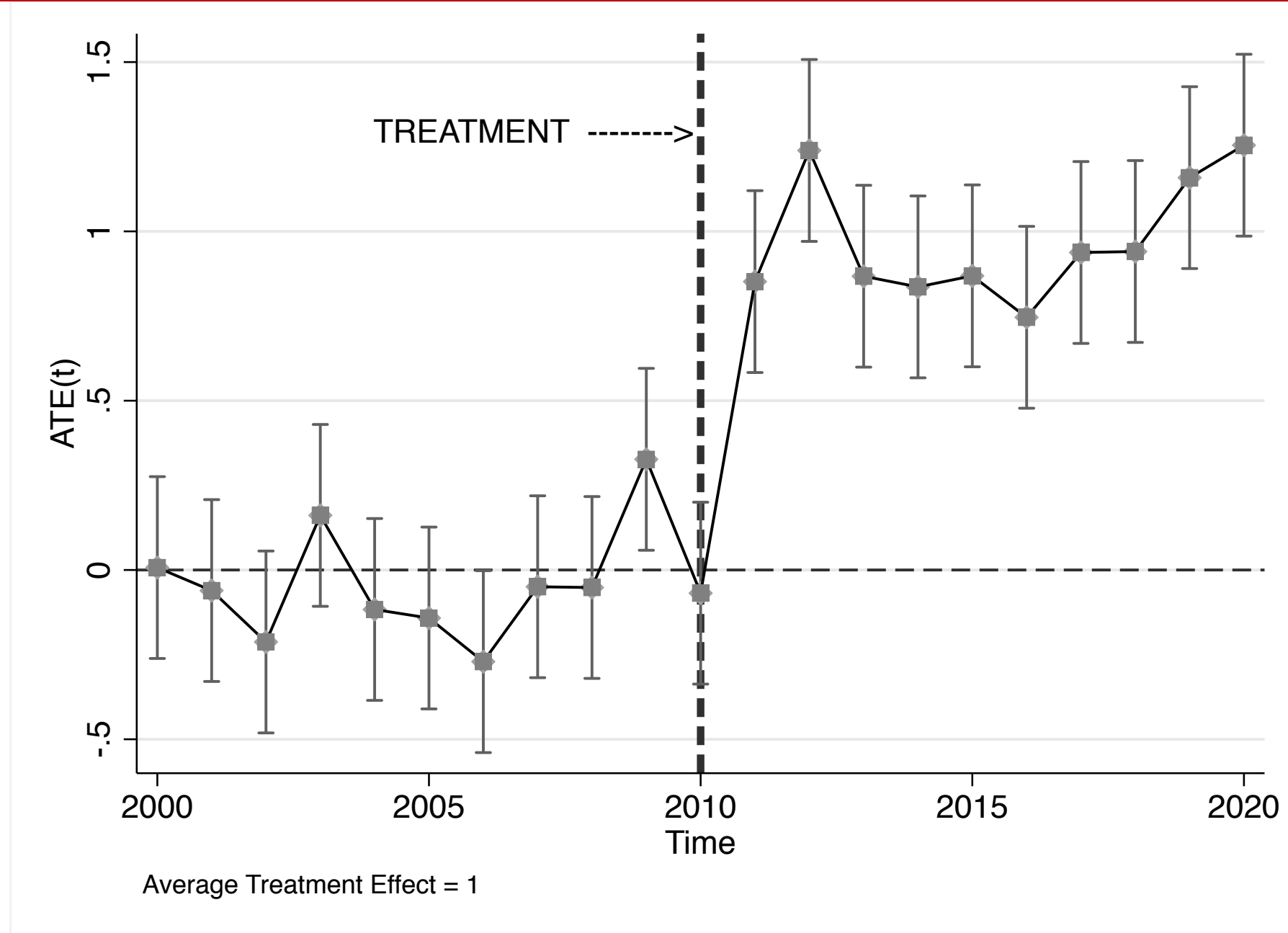
Treated units: 21 (441)

Untreated units: 79 (1,659)

N = 100 (2,100)
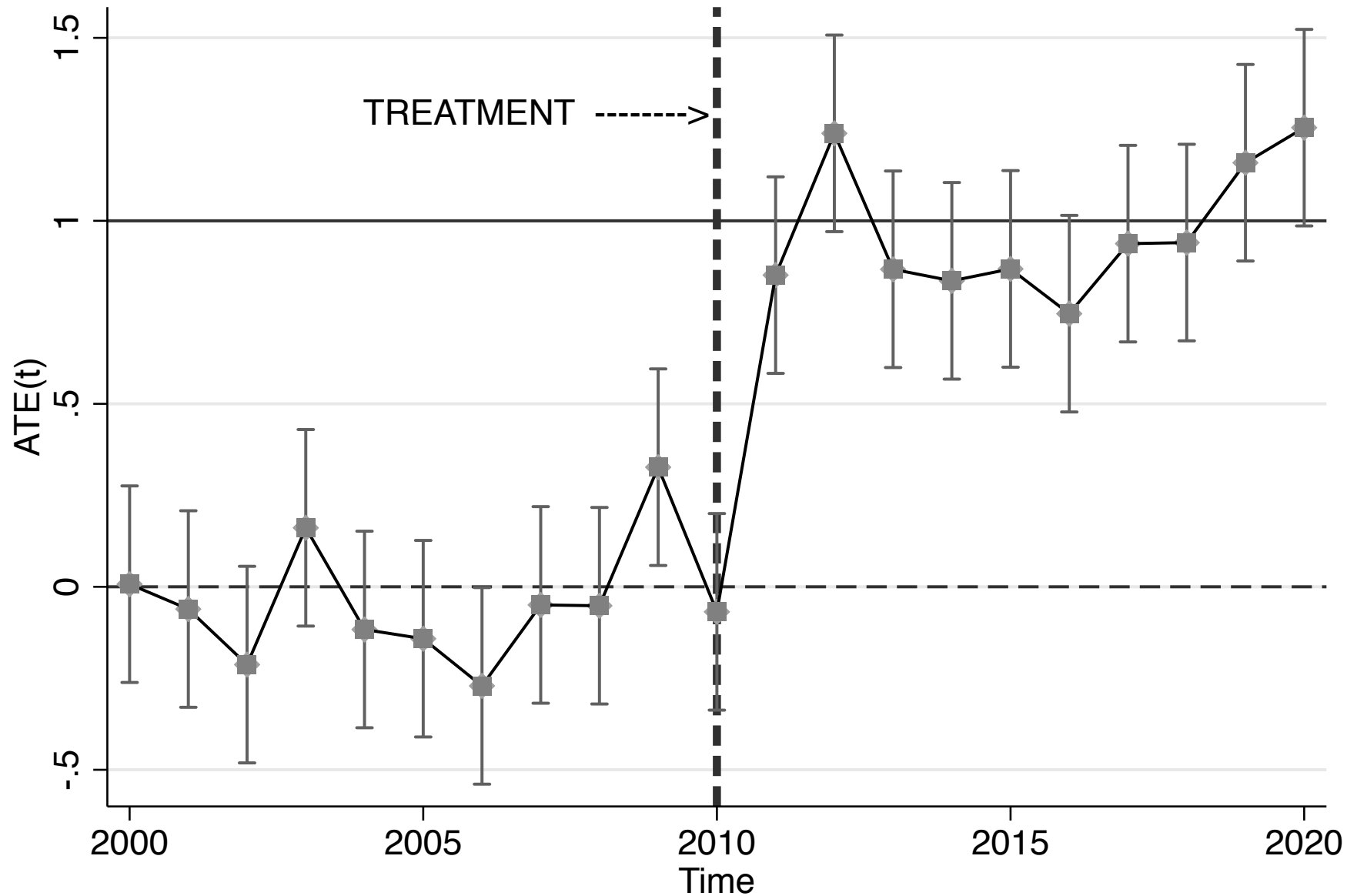
Outcome: Rate of GDP growth

Potential Outcomes Means: estimates for ATE = 1

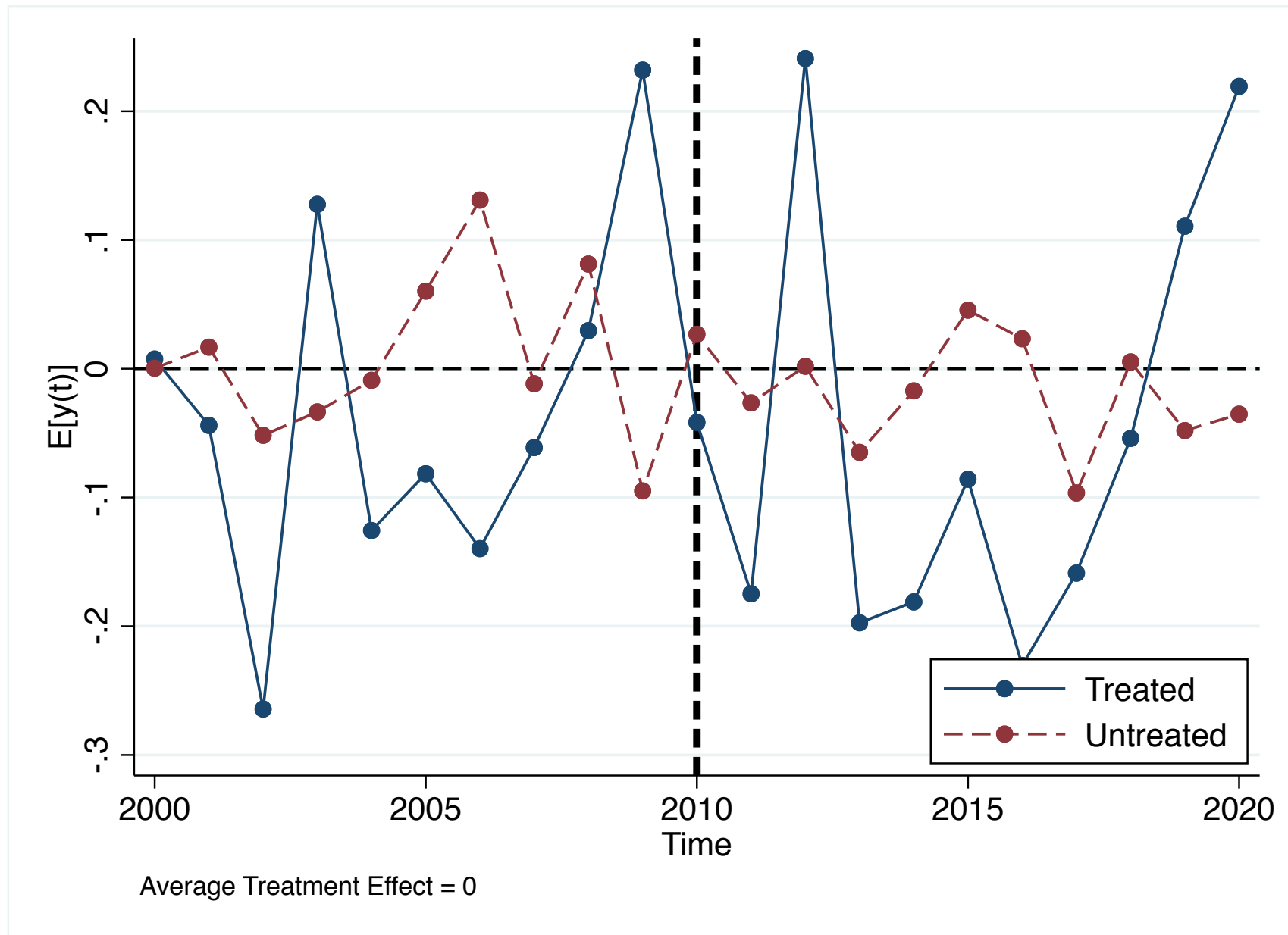# Average Treatment Effects at each *t*: estimates for ATE = 1

TREATMENT ------->

Average Treatment Effect = 1

Average Treatment Effects at each *t*: estimates for ATE = 1
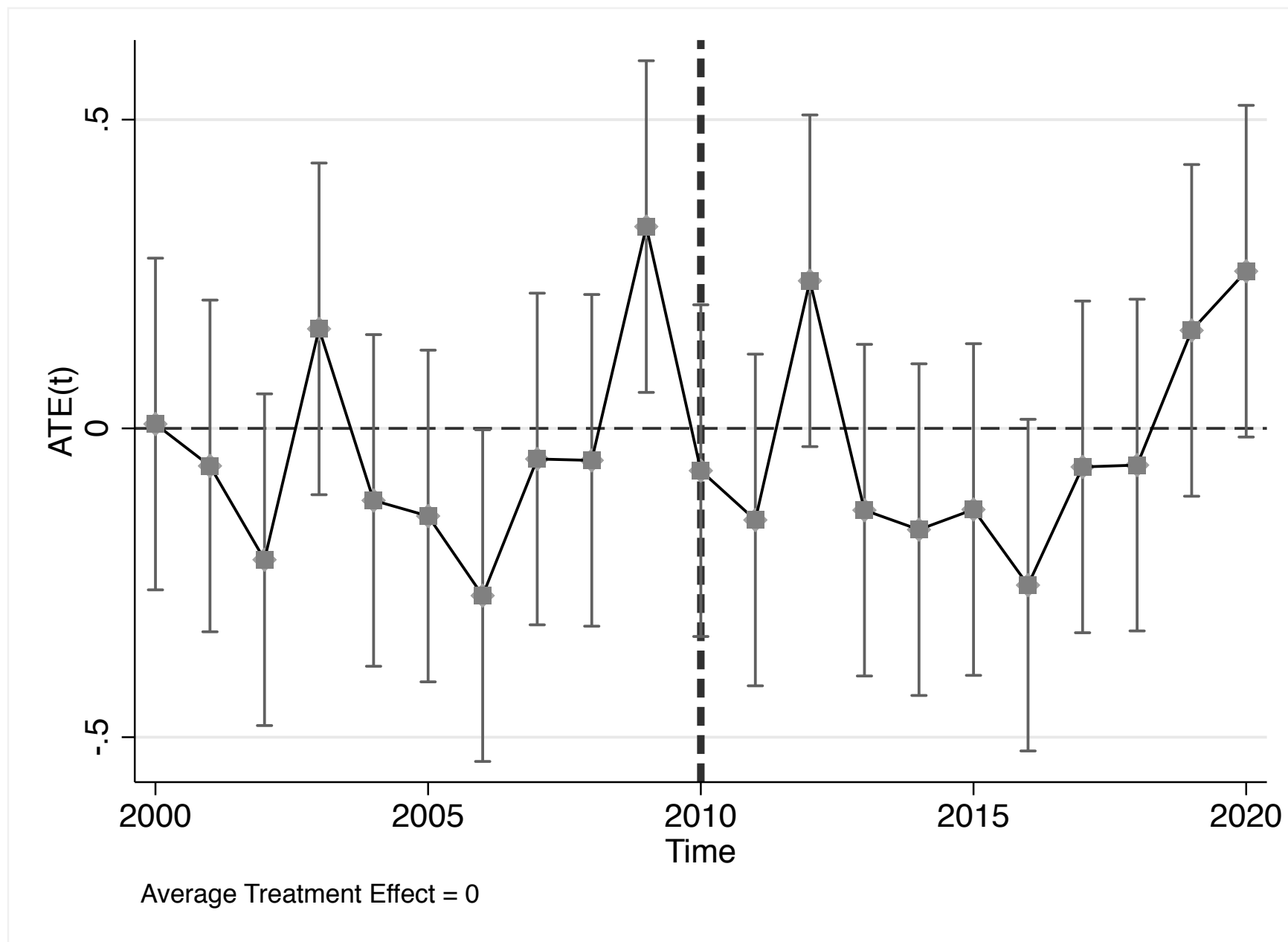
Potential Outcomes Means: estimates for ATE = 0 (*no-effect setting*)

Average Treatment Effects at each *t*: estimates for ATE = 0 (*no-effect setting*)

Average Treatment Effect = 0

Conclusions

o The model **TFDIFF** accommodates a large set of treatment/policy situations in several fields of application

o Compared to the SCM which considers only **one treated unit**, **TFDIFF** uses **many treated units** and provides a more **robust inference** on the causal effects over time – i.e. ATE(t) - than SCM

o Under *no-anticipation*, **TFDIFF** provides a straightforward way to test the common-trend

o The Stata command I developed – `tfdiff` – is simple to use and provides a nice graphical representation of the results

References

o Abadie, A., Diamond, A., and Hainmueller, J., 2010. Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program, **Journal of the American Statistical Association**, 105, 490, 493-505.

o Autor, D. 2003. Outsourcing at Will: The Contribution of Unjust Dismissal Doctrine to the Growth of Employment Outsourcing, **Journal of Labor Economics**, 21(1).

o Cerulli, G., and Ventura, M. 2019. TVDIFF: Stata module to compute pre- and post-treatment estimation of the Average Treatment Effect (ATE) with binary time-varying treatment, **Stata Journal**, forthcoming.

o Cerulli, G. 2019. A flexible Synthetic Control Method for modeling policy evaluation, **Economics Letters**, forthcoming.