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CTREATREG: STATA MODULE FOR ESTIMATING DOSE-RESPONSE MODELS UNDER EXOGENOUS AND ENDOGENOUS TREATMENT

Giovanni Cerulli

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CTREATREG: Stata module for estimating dose-response models under exogenous and endogenous treatment

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ABSTRACT: This paper presents `ctreatreg`, a Stata module for estimating a dose-response function when: (i) treatment is continuous, (ii) individuals may react heterogeneously to observable confounders, and (iii) selection-into-treatment may be endogenous. Two estimation procedures are implemented: OLS under Conditional Mean Independence, and Instrumental-Variables (IV) under selection endogeneity. A Monte Carlo experiment to test the reliability of the proposed command is finally set out.

Keywords: Stata commands; treatment effects, dose-response function, continuous treatment, Monte Carlo, R&D support

JEL Codes: C21, C87, D04

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1. INTRODUCTION

This paper presents a Stata routine, `ctreatreg`, for estimating a dose-response function through a regression approach when: (i) treatment is continuous, (ii) individuals may react heterogeneously to observable confounders, and (iii) selection-into-treatment may be potentially endogenous. In this context, the dose-response function is equal to the "Average Treatment Effect, given the level of treatment *t*" (i.e. ATE(*t*)). But also other causal parameters of interest, such as the unconditional Average Treatment Effect (ATE), the Average Treatment Effect on Treated (ATET), the Average Treatment Effect on Non-Treated (ATENT) are estimated by `ctreatreg`, along with those effects conditional on the vector (**x**; *t*), where **x** is a vector of pre-determined variables.

Such a routine seems of worth, as in many socio-economic and epidemiological contexts, interventions take the form of a continuous exposure to a certain type of treatment. Indeed, from a program evaluation perspective, what is relevant in many settings is not only the binary treatment status, but also the level of exposure (or "dose") provided by a public agency.

This is also in tune with the language of epidemiology, where dose-response functions are usually estimated in order to check patients' resilience to different levels of drug administration (Robertson et al., 1994; Royston and Sauerbrei, 2008).

To fix ideas, consider a policy program where the treatment is assigned not randomly (i.e., according to some "structural" rule), and where – after setting who is treated and who is not – the program provides a different "level" or "exposure" to treatment ranging from 0 (no treatment) to 100 (maximum treatment level). Two groups of units are thus formed: (i) *untreated*, whose level of treatment (or dose) is zero, and (ii) *treated*, whose level of treatment is greater than zero.

We are interested in estimating the causal effect of the treatment variable *t* on an outcome *y* within the observed sample, by assuming that treated and untreated units may respond differently both to specific observable confounders (that we collect in a vector **x**), and to the "intensity" of the treatment *t*. We wish to estimate a dose-response function of *y* on *t*, either when the treatment is assumed to be exogenous (i.e., selection-into-treatment depends only on observable-to-analyst factors) or endogenous (i.e., selection-into-treatment depends both on observable and unobservable-to-analyst factors).

Compared with similar models - and in particular the one proposed by Hirano and Imbens (2004) implemented in Stata by Bia and Mattei (2008)[1] - this model does not need a full normality assumption, and it is well-suited when many individuals have a zero-level of treatment ("spike" or no-nil probability mass at zero as in Royston et al. (2010)). Additionally, it may account for treatment "endogeneity" by exploiting an

[1] See also Bia, Flores and Mattei (2011) generalizing the Hirano-Imbens (2004) model by allowing for a nonparametric estimation of the Dose-Response Function. Furthermore, see Guardabascio and Ventura (2013) for an extension of the Hirano-Imbens model allowing for various non-normal distributions of the continuous-treatment variable.

Instrumental-Variables (IV) estimation in a continuous treatment context.

The reliability of the model and of its Stata implementation via `ctreatreg` is then checked by a Monte Carlo experiment, proving that the model and the routine lead to expected theoretical results.

The routine provides also an interesting graphical representation of results by optionally plotting both the conditional effects' distribution and the dose-response function along with its confidence intervals.

The paper is organized as follows: section 2 and 3 present the model, its assumptions and formulas, as well as the related estimation techniques (section 3); section 4 presents and explains the use of the Stata routine `ctreatreg`; then, the paper goes on by showing, in section 5, an application of `ctreatreg` on real data; section 6 sets out the results from a related Monte Carlo experiment to test the routine's reliability; section 7, finally, concludes the paper. At the end of the paper, Table1A reports `ctreatreg`'s help-file.

2. THE MODEL

We set out with some notation. Consider two different and exclusive outcomes: one referring to a unit *i* when she is treated, y_{1i}; and one referring to the same unit when she is untreated, y_{0i}.

Define w_i as the treatment indicator, taking value 1 for treated and 0 for untreated units, and $\mathbf{x}_i = (x_{1i}, x_{2i}, x_{3i}, \dots , x_{Mi})$ as a row vector of *M* exogenous and observable characteristics (confounders) for unit $i = 1, \dots , N$. Let *N* be the number of units involved in the experiment, N_1 be the number of treated units, and N_0 the number of untreated units with $N = N_1 + N_0$.

Define two distinct functions, $g_1(\mathbf{x}_i)$ and $g_0(\mathbf{x}_i)$, as the unit *i*'s responses to the vector of confounding variables $\mathbf{x}_i$ when the unit is treated and untreated respectively. Assume μ_1 and μ_0 to be two scalars, and e_1 and e_0 two random variables having zero unconditional mean and constant variance. Finally, define t_i – taking values within the continuous range [0;100] – as the continuous-treatment indicator, and $h(t_i)$ as a general derivable function of t_i.

In what follows, in order to simplify notation, we'll get rid of the subscript *i* when defining population quantities and relations.

Given previous notation, we assume a specific population generating process for the two exclusive potential outcomes[2]:

$$\begin{cases} w = 1: \;\; y_1 = \mu_1 + g_1(\mathbf{x}) + h(t) + e_1 \\ w = 0: \;\; y_0 = \mu_0 + g_0(\mathbf{x}) + e_0 \end{cases} \tag{1}$$

where the *h*(*t*) function is different from zero only in the treated status. Given this, we can also define the causal parameters of interests.

Indeed, by defining the treatment effect as the difference TE = ($y_1 - y_0$), We define the causal parameters of interests, as the population Average Treatment Effects (ATEs) conditional on **x** and *t*, that is:

[2] Such a model is the representation of a *treatment random coefficient regression* as showed by Wooldridge (1997; 2003). See also Wooldridge (2010, Ch. 18). For the sake of simplicity, as we refer to the population model, here we avoid to write the subscript *i* referring to each single unit *i*'s relationships.

$$\begin{aligned}&\mathrm{ATE}(\mathbf{x};t)=\mathrm{E}(y_1-y_0\,|\,\mathbf{x},t)\\&\mathrm{ATET}(\mathbf{x};t>0)=\mathrm{E}(y_1-y_0\,|\,\mathbf{x},t>0)\\&\mathrm{ATENT}(\mathbf{x};t=0)=\mathrm{E}(y_1-y_0\,|\,\mathbf{x},t=0)\end{aligned} \tag{2}$$

where ATE indicates the overall average treatment effect, ATET the average treatment effect on treated, and ATENT the one on untreated units. By the law of iterated expectation (LIE), we know that the population unconditional ATEs are obtained as:

$$\begin{aligned}&\mathrm{ATE}=\mathrm{E}_{(\mathbf{x};t)}\{\mathrm{ATE}(\mathbf{x};t)\}\\&\mathrm{ATET}=\mathrm{E}_{(\mathbf{x};t>0)}\{\mathrm{ATE}(\mathbf{x};t>0)\}\\&\mathrm{ATENT}=\mathrm{E}_{(\mathbf{x};t=0)}\{\mathrm{ATE}(\mathbf{x};t=0)\}\end{aligned} \tag{3}$$

where $\mathrm{E}_{\mathbf{z}}(\cdot)$ identifies the mean operator taken over the support of a generic vector of variables **z**. By assuming a linear-in-parameters parametric form for $g_0(\mathbf{x})=\mathbf{x}\boldsymbol{\delta}_\mathbf{0}$ and $g_1(\mathbf{x})=\mathbf{x}\boldsymbol{\delta}_1$ the Average Treatment Effect (ATE) conditional on **x** and *t* becomes:

$$\begin{aligned}\mathrm{ATE}(\mathbf{x},t,w)=w\cdot[\mu+\mathbf{x}\boldsymbol{\delta}+h(t)]+\\(1-w)\cdot[\mu+\mathbf{x}\boldsymbol{\delta}]\end{aligned} \tag{4}$$

where μ=(μ_1-μ_0) and $\boldsymbol{\delta}$=($\boldsymbol{\delta}_1$-$\boldsymbol{\delta}_0$) and the unconditional Average Treatment Effect (ATE) related to model (1) is equal to:

$$\begin{aligned}\mathrm{ATE}=p(w=1)\cdot(\mu+\overline{\mathbf{x}}_{t>0}\boldsymbol{\delta}+\overline{h}_{t>0})+\\p(w=0)\cdot(\mu+\overline{\mathbf{x}}_{t=0}\boldsymbol{\delta})\end{aligned}$$

where $p(\cdot)$ is a probability, and $\overline{h}_{t>0}$ is the average of the response function taken over $t>0$. Since, by LIE, we have that ATE = $p(w=1)\cdot$ATET + $p(w=0)\cdot$ATENT, we obtain from the previous formula that:

$$\begin{cases}\mathrm{ATE}=p(w=1)(\mu+\overline{\mathbf{x}}_{t>0}\boldsymbol{\delta}+\overline{h}_{t>0})+p(w=0)(\mu+\overline{\mathbf{x}}_{t=0}\boldsymbol{\delta})\\\mathrm{ATET}=\mu+\overline{\mathbf{x}}_{t>0}\boldsymbol{\delta}+\overline{h}_{t>0}\\\mathrm{ATENT}=\mu+\overline{\mathbf{x}}_{t=0}\boldsymbol{\delta}\end{cases}$$

where the dose-*response function* is given by averaging ATE(**x**, *t*) over **x**:

$$\mathrm{ATE}(t)=\begin{cases}\mathrm{ATET}+(h(t)-\overline{h}_{t>0}) & \text{if} \quad t>0\\\mathrm{ATENT} & \text{if} \quad t=0\end{cases} \tag{6}$$

that is a function of the treatment intensity t. The estimation of equation (6) under different identification hypothesises is the main purpose of next sections.

3. THE REGRESSION APPROACH

In this section we consider the conditions for a consistent estimation of the causal parameters defined in (2) and (3) and thus of the dose-response function in (6).

What it is firstly needed, however, is a consistent estimation of the parameters of the potential outcomes in (1) – we call here "basic" parameters – as both ATEs and the dose-response function are functions of these parameters.

Under previous definitions and assumptions, and in particular the form of the potential outcomes in model (1), to be substituted into Rubin's potential outcome equation $y_i=y_{0i}+w(y_{1i}-y_{0i})$, the following *Baseline random-coefficient regression* can be obtained (Wooldridge, 1997; 2003):

$$y_i = \mu_0 + w_i \cdot \mathrm{ATE} + \mathbf{x}_i\boldsymbol{\delta}_\mathbf{0} + w_i \cdot (\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + w_i \cdot (h(t_i) - \bar{h}) + \eta_i \qquad (7)$$

where

$$\eta_i = e_{0i} + w_i \cdot (e_{1i} - e_{0i}) \, .$$

The equation sets out in (12), provides the baseline regression for estimating the basic parameters (μ_0, μ_1, $\boldsymbol{\delta}_0$, $\boldsymbol{\delta}_1$, ATE) and then all the remaining ATEs.

Both a semi-parametric or a parametric approach can be employed as soon as a parametric or a non-parametric form of the function $h(t)$ is assumed.

In both cases, however, in order to get a consistent estimation of basic parameters, we need some additional hypotheses. We start by assuming first *Unconfoundedness* or *Conditional Mean Independence* (CMI), showing that it is sufficient to provide parameters' consistent estimation.

Then we remove this hypothesis and introduce other identifying assumptions.

3.1 Estimation under Unconfoundedness

Unconfoundedness states that, conditional on the knowledge of the true exogenous confounders **x**, the condition for randomization are restored, and causal parameters become identifiable.

Given the set of random variables $\{y_{1i}, y_{1i}, w_i, \mathbf{x}_i\}$ as defined above, Unconfoundedness (or CMI) implies that:

$$\mathrm{E}(y_{ij} \mid w_i , \mathbf{x}_i) = \mathrm{E}(y_{ij} \mid \mathbf{x}_i) \quad \text{with } j = \{0,1\}$$

CMI is a sufficient condition for identifying ATEs and the dose-response function in this context.

Indeed, this assumption entails that, given the observable variables collected in **x**, both *w* and *t* are exogenous in equation (7), so that we can write the regression line of the response *y* simply as:

$$\mathrm{E}(y_i \mid w_i, t_i, \mathbf{x}_i) = \mu_0 + w_i \cdot \mathrm{ATE} + \mathbf{x}_i\boldsymbol{\delta}_\mathbf{0} + w_i \cdot (\mathbf{x}_i - \bar{\mathbf{x}})\boldsymbol{\delta} + w_i \cdot (h(t_i) - \bar{h}) \qquad (8)$$

and Ordinary Least Squares (OLS) can be used to retrieve consistent estimation of all parameters.

Once a consistent estimation of the parameters in (8) is obtained, we can estimate ATE directly from this regression, and ATET, ATENT and the dose-response function by plugging the estimated basic parameters into formula (5) and (6).

This is possible because these parameters are functions of consistent estimates, and thus consistent themselves.

Observe that standard errors for ATET and ATENT can be correctly obtained via bootstrapping (see Wooldridge, 2010, pp. 911-919).

To complete the identification of ATEs and the dose-response function, we finally assume a parametric form for $h(t)$:

$$h(t_i) = at_i + bt_i^2 + ct_i^3 \qquad (9)$$

where *a*, *b*, and *c* are parameters to be estimated in regression (8).

Under CMI, an OLS estimation of equation (8) produces consistent estimates of the parameters, we indicate as $\hat{\mu}_0, \hat{\boldsymbol{\delta}}_{\mathbf{0}}, \mathrm{A\hat{T}E}, \hat{\boldsymbol{\delta}}, \hat{a}, \hat{b}, \hat{c}$.

With these parameters at hand, we can finally estimate consistently the dose-response function as:

$$\mathrm{A\hat{T}E}(t_i) = w[\mathrm{A\hat{T}ET} + \hat{a}(t_i - \frac{1}{N}\sum_{i=1}^{N} t_i) + \hat{b}(t_i^2 - \frac{1}{N}\sum_{i=1}^{N} t_i^2) + \hat{c}(t_i^3 - \frac{1}{N}\sum_{i=1}^{N} t_i^3)] + (1-w)\,\mathrm{AT\hat{E}NT} \qquad (10)$$

where:

$$\mathrm{A\hat{T}E\,T}(t_i) = \mathrm{A\hat{T}E}(t_i)_{t_i>0}$$

A simple plot of the curve $\mathrm{A\hat{T}E}(t_i)_{t_i>0}$ over the support of t returns the pattern of the dose-response function.

Moreover, for each level of the dose t, it is also possible to calculate the α-confidence interval around the dose-response curve. Indeed, by defining T_1=t-E(t), T_2=t^2-E(t^2) and T_3= t^3-E(t^3), the standard error of the dose-response function is equal to[3]:

$$\hat{\sigma}_{\mathrm{A\hat{T}E}(t)} = \left\{T_1^2\hat{\sigma}_a^2 + T_2^2\hat{\sigma}_b^2 + T_3^2\hat{\sigma}_c^2 + 2T_1T_2\hat{\sigma}_{a,b} + 2T_1T_3\hat{\sigma}_{a,c} + 2T_2T_3\hat{\sigma}_{b,c}\right\}^{1/2} \qquad (11)$$

This means that the α-confidence interval of $\mathrm{A\hat{T}E}(t)$ for each t is then given by:

$$\left\{ \mathrm{A\hat{T}E}(t) \pm Z_{\alpha/2} \cdot \hat{\sigma}_{\mathrm{A\hat{T}E}(t)} \right\}$$

that can be usefully plotted along the dose-response curve for detecting visually the statistical significance of the treatment effect along the support of the dose t.

3.2 Estimation under treatment endogeneity

When w (and thus t) are endogenous, CMI hypothesis does not hold anymore, and the OLS estimate of regression (8) becomes biased.

This occurs because the orthogonality condition implied by Unconfoundedness fails, so that:

$$\mathrm{E}\left(\eta_i \middle| w_i, t_i, \mathbf{x}_i\right) = \mathrm{E}\left(e_{0i} + w_i \cdot (e_{1i} - e_{0i}) \middle| w_i, t_i, \mathbf{x}_i\right) \neq 0 \qquad (12)$$

where it is clear that inequality depends on the endogeneity of w_i (and t_i), being $\mathbf{x}_i$ assumed to be pre-determined.

In such a case, however, an Instrumental-Variables (IV) estimation may be implemented to restore consistency.

To this aim, it is sufficient to express previous model in a semi-structural form, that is:

[3] This comes from the variance/covariance properties where T_1 T_2 T_3 are taken as constant and a, b and c as random variables.

$$\begin{cases} y_i = \mu_0 + \mathbf{x}_i\boldsymbol{\delta}_\mathbf{0} + w_i\mathrm{ATE} + w_i[\mathbf{x}_i - \bar{\mathbf{x}}]\boldsymbol{\delta} + w_iT_{1i} + bw_iT_{2i} + cw_iT_{3i} + \eta_i & (13.1) \\ w_i^* = \mathbf{x}_{w,i}\boldsymbol{\beta}_w + \varepsilon_{wi} & (13.2) \\ t_i' = \mathbf{x}_{t,i}\boldsymbol{\beta}_t + \varepsilon_{ti} & (13.3) \end{cases}$$

where: T_{1i}=t_i-E(t_i), T_{2i}=t_i^2-E(t_i^2) and T_{3i} = t_i^3-E(t_i^3); w_i^* represent the latent unobservable counterpart of the binary variable w_i (for instance, w_i^* might be seen as the net benefit - cost minus return - of an agency choosing to finance specific subjects); t_i is fully observed only when w_i=1 (and t_i=t_i'), otherwise it is supposed to be unobserved (although put equal to zero); $\mathbf{x}_{wi}$ and $\mathbf{x}_{ti}$ are two sets of exogenous regressors, and ε_{iw}, ε_{it} and η_i are error terms supposed to be freely correlated one another with zero unconditional mean.

Equation (13.2) – the *selection* equation – defines the regression explaining the net benefit indicator w^*. The vector of covariates $\mathbf{x}_{1i}$ are the selection criteria used, for instance, by an agency to set the treated and untreated group. In turn, equation (13.3) – the *treatment-level* equation – defines how the level of unit treatment is decided, and it regards only units that were considered eligible for treatment.

The vector of covariates $\mathbf{x}_{2i}$ are those exogenous variables thought of as determining exactly the treatment level.

In equation (13.1), w_i and T_{1i}, T_{2i} and T_{3i} are endogenous, being these latter ones functions of the endogenous t. In general, with two endogenous variables, the identification of the linear system (13) would require the availability of at least two instrumental variables $z_{w,i}$ and $z_{t,i}$ supposed to be: (*i*) correlated with w_i^* and t_i', respectively; (*ii*) uncorrelated with $\varepsilon_{w,i}$, $\varepsilon_{t,i}$ and η_i. This leads naturally to the following specification of the exogenous confounders in system (13):

$$\mathbf{x}_{w,i} = [\mathbf{x}_i; z_{w,i}]$$

$$\mathbf{x}_{t,i} = [\mathbf{x}_i; z_{t,i}]$$

(14)

Practical estimation of system (13) starts from recognizing that the two last equations – i.e., (13.2) and (13.3) – represents a bivariate sample-selection model or type-2 tobit model (Heckman, 1979). Generally, such a model is estimated by invoking some distributive assumptions regarding the error terms. As usual, we assume that the error terms in (13.2) and (13.3) are jointly normally distributed and homoskedastic:

$$\begin{bmatrix} \varepsilon_{wi} \\ \varepsilon_{ti} \end{bmatrix} \sim N\left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} 1 & \sigma_{wt} \\ \sigma_{wt} & \sigma_t^2 \end{bmatrix}\right]$$

where the normalization σ_w=1 is used because only the sign of w_i^* is observed. Given this additional assumption, all the ingredients to provide a procedure for estimating system (13) consistently are available:

1. *First*: estimate equations (13.2)-(13.3) jointly by a type-2 tobit model.

Comment. As said, this can be achieved by a Heckman two-step procedure (Heckman, 1979). The Heckman two-step procedure performs a probit of w_i on $\mathbf{x}_{1i}$ in the first step using only the N_1 selected observations, and an OLS regression of t_i' on $\mathbf{x}_{2i}$, augmented by the Mills' ratio obtained from the probit in the second step, using all the N observations as predictions are made also for the censored data. However, because of the errors' joint normality, a maximum-likelihood (ML) estimation can be also employed; ML leads to more efficient estimates of β_w and β_t.

2. *Second*: compute the predicted values of w_i (i.e. $\hat{p}_{wi}$) and t_i (i.e. $\hat{t}_i$) from the previous type-2 tobit estimation, and then perform a two-stage least squares (2SLS) for equation (13.1) using as instruments the following exogenous variables $(\mathbf{x}_i, \hat{p}_{wi}, \hat{p}_{wi}[\mathbf{x}_i - \bar{\mathbf{x}}], \hat{p}_{wi}\hat{T}_{1i}, \hat{p}_{wi}\hat{T}_{2i}, \hat{p}_{wi}\hat{T}_{3i})$

Comment. This 2SLS approach provides consistent estimation of the basic coefficients μ_0, $\boldsymbol{\delta}_0$, ATE, $\boldsymbol{\delta}$, a, b, c (Wooldridge, 2010, pp. 937-951)[4].

3. *Third*: once previous procedure estimates consistently the basic parameters in system (13), the causal parameters of interest - ATEs and the dose-response function - can be consistently estimated by the same plug-in approach used for the OLS case.

3.3 *Estimation of comparative dose-response functions*

Besides the dose-response function and the other causal parameters of interest as defined above, the previous model allows also for calculating the average comparative response at different level of treatment (as in Hirano and Imbens, 2004). This quantity takes this formula:

$$\mathrm{ATE}(t,\Delta) = \mathrm{E}[y(t+\Delta) - y(t)] \qquad (15)$$

Equation (15) identifies the average treatment effect between two states (or levels of treatment): t and $t+\Delta$. Given a level of $\Delta = \bar{\Delta}$, we can get a particular $\mathrm{ATE}(t,\bar{\Delta})$ that can be seen as the "treatment function at $\bar{\Delta}$".

4. THE STATA ROUTINE `CTREATREG`

The Stata routine `ctreatreg` estimates previous dose-response function both under CMI and under treatment endogeneity[5].

The complete Stata help-file of the routine showing the syntax along with the options as set out in Table A1, at the end of this paper.

Here, we just report the syntax and a comment on the main options.

[4] Observe that instruments used in the 2SLS are based on the orthogonal projection of w_i and t_i on the vector space generated by all the exogenous variables of system (13).

[5] For a Stata implementation when the treatment is binary see Cerulli (2012).

Syntax of `ctreatreg`

```
ctreatreg outcome treatment [varlist] [if] [in] [weight],
    model(modeltype) ct(treat_level) [hetero(varlist_h)
    estype(model) iv_t(instrument_t) iv_w(instrument_w)
    delta(number) ci(number) graphate graphdrf conf(number)
    vce(robust) const(noconstant) head(noheader) beta]
```

This routine appears rather straightforward to use and useful to provide suitable graphical representations of results. In particular, it provides a plot of the dose-response function (along with its confidence interval curves) and of the density of ATE(x,*t*), ATET(x,*t*) and ATENT(x,*t*). The main `ctreatreg`'s options with a comment of their function are reported below:

`model`(*`modeltype`*) specifies the treatment model to be estimated, where *modeltype* must be one of the following two models: "ct-ols", "ct-iv". It is always required to specify one model.

`ct`(*`treat_level`*) specifies the treatment level (or dose). This variable takes values in the [0;100] interval, where 0 is the treatment level of non-treated units. The maximum dose is thus 100.

`hetero`(*`varlist_h`*) specifies the variables over which to calculate the idiosyncratic Average Treatment Effect ATE(x), ATET(x) and ATENT(x), where x=*varlist_h*. It is optional for all models. When this option is not specified, the command estimates the specified model without heterogeneous average effect. Observe that *varlist_h* should be the same set or a subset of the variables specified in *varlist*. Observe however that only numerical variables may be considered.

`estype(`*`model`***`)`** specifies which type of estimation method has to be used for estimating the type-2 tobit model in the endogenous treatment case. Two choices are available: "twostep" implements a Heckman two-step procedure; "ml" implements a maximum-likelihood estimation. This option is required only for "ct-iv".

`iv_t(`*`instrument_t`***`)`** specifies the variable to be used as instrument for the continuous treatment variable t in the type-2 tobit model. This option is required only for "ct-iv".

`iv_w(`*`instrument_w`***`)`** specifies the variable to be used as instrument for the binary treatment variable w in the type-2 tobit model. This option is required only for "ct-iv".

`delta`(*`number`*) identifies the average treatment effect between two states: t and t+delta. For any reliable delta, we can obtain the response function ATE(t;delta)=E[y(t)-y(t+delta)].

`ci`(*`number`*) sets the significant level for the dose-response function, where number may be 1, 5 or 10. This option is mandatory when option `graphdrf` is called.

`graphate` allows for a graphical representation of the density distributions of ATE(x;t) ATET(x;t) and ATENT(x;t). It is optional for all models and gives an outcome only if variables into `hetero`() are specified.

`graphdrf` allows for a graphical representation of the Dose Response Function (DRF) and of its derivative. By default, it plots also the 95% confidence interval of the DRF over the dose levels.

Finally, `ctreatreg` generates some useful variables for post-estimation analysis and returns the estimated treatment effects into scalars so to get, for instance, bootstrapped standard errors for ATET and ATENT that do not have a standard analytical form (see Table A1).

5. AN INSTRUCTIONAL APPLICATION

To see how to use `ctreatreg` in practice, we consider the Stata 13 example-dataset "nlsw88.dta" collecting data from the National Longitudinal Survey of Young Women of 1988, containing information on women's labor conditions such as wages, educational level, race, marital status, etc.. As an example, we aim at studying the impact of the variable "tenure" (job tenure) on "wage" (wages in dollars per hour) conditional on a series of other covariates (i.e., observable confounders) referring to each single woman.

The variable tenure is a good candidate to be exploited as continuous-treatment (i.e., dose) for such a model, having a (small) spike at zero:

```
. sum tenure

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      tenure |      2231     5.97785    5.510331          0   25.91667

. count if tenure ==0
   51
```

The dataset description is set out below:

```
. sysuse nlsw88.dta

. describe


Contains data from C:\Program Files\Stata13\ado\base/n/nlsw88.dta
  obs:         2,246                          NLSW, 1988 extract
 vars:            51                          1 May 2011 22:52
 size:       345,884                          (_dta has notes)
----------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
------------------------------------------------------------------------------
idcode          int     %8.0g                 NLS id
age             byte    %8.0g                 age in current year
race            byte    %8.0g      racelbl    race
```

```
married          byte    %8.0g      marlbl     married
never_married    byte    %8.0g                 never married
grade            byte    %8.0g                 current grade completed
collgrad         byte    %16.0g     gradlbl    college graduate
south            byte    %8.0g                 lives in south
smsa             byte    %9.0g      smsalbl    lives in SMSA
c_city           byte    %8.0g                 lives in central city
industry         byte    %23.0g     indlbl     industry
occupation       byte    %22.0g     occlbl     occupation
union            byte    %8.0g      unionlbl   union worker
wage             float   %9.0g                 hourly wage
hours            byte    %8.0g                 usual hours worked
ttl_exp          float   %9.0g                 total work experience
tenure           float   %9.0g                 job tenure (years)
```

We consider a model where the outcome, the treatment and the controls are defined as follows:

- *outcome y*: "wage"
- *treatment w*: "tenure"
- *controls* **x**: "age", "race", "married", "collgrad", "south", "industry", "occupation", "union

Furthermore, we consider two (potential) instrumental variables to use in the IV estimation (when assuming endogenous treatment):

- *instrument* for *w*: "c_city"
- *instrument* for *t*: ttl_exp

Notice, however, that the goodness of these instruments is just assumed and neither discussed, nor tested, being this just an instructional example.

Before estimation, however, we first generate the binary treatment variable, we call "treatment":

```
* Generate the binary treatment
. cap drop treatment
. gen treatment=0 if tenure==0
. replace treatment=1 if tenure >0 &
tenure !=.
. tab treatment , mis
```

and then we generate the continuous-treatment (dose), we call "tenure2"

```
* Generate the continuous-treatment
(ranging between 0 and 100)
. cap drop tenure2
. qui sum tenure , detail
.  gen  tenure2=(tenure-0)/(r(max)-
0)*100
. sum tenure2
```

We have now all the ingredients to apply `ctreatreg` to this example. We start with estimating the "ct-ols" model (by assuming Unconfoundedness), and then the "ct-iv" model (by assuming treatment endogeneity).

Firstly, however, we put variables into proper global macros:

```
. global xvars age i.race i.married
i.collgrad    i.south    i.industry
i.occupation i.union
```

```
. global xvarh age married
```

1. Applying `ctreatreg` using "ct-ols" (Unconfoundedness):

```
. xi: ctreatreg wage treatment $xvars  ,  graphrf   ///
delta(10) hetero($xvarh) model(ct-ols) ct(tenure2)  ci(1)

      Source |       SS       df       MS              Number of obs =    1851
-------------+------------------------------           F( 36,  1814) =   31.98
       Model |  12500.2091    36  347.228032           Prob > F      =  0.0000
    Residual |  19693.1797  1814  10.8562181           R-squared     =  0.3883
-------------+------------------------------           Adj R-squared =  0.3761
       Total |  32193.3889  1850  17.4018318           Root MSE      =  3.2949

--------------------------------------------------------------------------------
          wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
     treatment |  -.9830216   .5226014    -1.88   0.060    -2.007985    .0419421
           age |   .2534269   .1523452     1.66   0.096    -.0453635    .5522173
      _Irace_2 |  -.2183432   .1938451    -1.13   0.260    -.5985263    .1618399
      _Irace_3 |   .4435454   .6846921     0.65   0.517    -.8993225    1.786413
   _Imarried_1 |   1.673357   1.137141     1.47   0.141    -.5568866    3.903602
  _Icollgrad_1 |   2.897919   .2261756    12.81   0.000     2.454327     3.34151
     _Isouth_1 |  -.9020501    .166936    -5.40   0.000    -1.229457   -.5746431
  _Iindustry_2 |   .8564371   2.577174     0.33   0.740    -4.198104    5.910978
  _Iindustry_3 |   2.053313   1.322305     1.55   0.121     -.540087    4.646714
  _Iindustry_4 |   .7290251   1.115645     0.65   0.514    -1.459059    2.917109
  _Iindustry_5 |   3.530271   1.152409     3.06   0.002     1.270084    5.790458
  _Iindustry_6 |  -1.227708   1.106073    -1.11   0.267    -3.397019    .9416029
  _Iindustry_7 |   1.205707   1.124255     1.07   0.284    -.9992633    3.410677
  _Iindustry_8 |   .0125544   1.173701     0.01   0.991    -2.289393    2.314502
  _Iindustry_9 |  -.5871449   1.212847    -0.48   0.628    -2.965868    1.791578
 _Iindustry_10 |   .6133445   1.407516     0.44   0.663    -2.147178    3.373867
 _Iindustry_11 |  -.6954708   1.102368    -0.63   0.528    -2.857514    1.466572
 _Iindustry_12 |   .8412632   1.125167     0.75   0.455    -1.365496    3.048023
 _Ioccupatio_2 |    .383661   .3211564     1.19   0.232    -.2462143    1.013536
 _Ioccupatio_3 |  -2.338188   .2647344    -8.83   0.000    -2.857404   -1.818971
 _Ioccupatio_4 |  -1.239092   .4720051    -2.63   0.009    -2.164823   -.3133615
 _Ioccupatio_5 |  -2.120975   .5446508    -3.89   0.000    -3.189184   -1.052767
 _Ioccupatio_6 |  -3.692821   .3946061    -9.36   0.000    -4.466752   -2.918891
 _Ioccupatio_7 |  -3.825877   .9565287    -4.00   0.000     -5.70189   -1.949863
 _Ioccupatio_8 |  -2.814455   .3311778    -8.50   0.000    -3.463985   -2.164925
 _Ioccupatio_9 |  -3.769752   3.483959    -1.08   0.279    -10.60275    3.063242
_Ioccupatio_10 |  -4.184864   1.639078    -2.55   0.011    -7.399542   -.9701856
_Ioccupatio_11 |  -3.131495   1.001367    -3.13   0.002    -5.095449   -1.167541
_Ioccupatio_12 |  -4.126322   3.321699    -1.24   0.214    -10.64108    2.388436
_Ioccupatio_13 |  -2.298906   .3491133    -6.58   0.000    -2.983613     -1.6142
     _Iunion_1 |   .9275427   .1960457     4.73   0.000     .5430437    1.312042
```

```
        _ws_age |  -.2517413    .154501    -1.63   0.103    -.5547598    .0512772
    _ws_married |  -1.875545   1.146583    -1.64   0.102    -4.124306    .3732158
             Tw |   .0592733   .0282068     2.10   0.036     .0039521    .1145946
            T2w |  -.0002733   .0008544    -0.32   0.749     -.001949    .0014024
            T3w |  -1.23e-06   7.13e-06    -0.17   0.863    -.0000152    .0000128
          _cons |   -1.19245   6.197508    -0.19   0.847    -13.34745    10.96255
------------------------------------------------------------------------------
```

Results show a good R-squared with a negative and significant ATE, equal to around –.98. It means that, on average on all values taken by job tenure, the effect of tenure on wage is negative. However, `ctreatreg` is able to plot the Dose Response Function (Fig. 1), showing that the relation is first weakly increasing and then decreasing with a maximum around a dose level of 70.

The relation is quite strongly significant (at 1%).

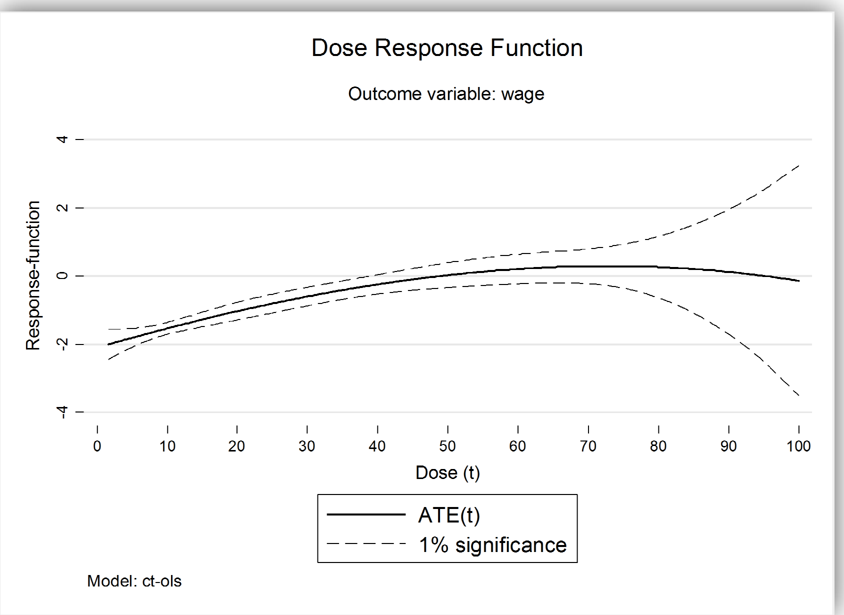


Figure 1. Dose response function of “job tenure” on “wage”. Exogenous treatment case.

2. Applying `ctreatreg` using "CT-IV" (Treatment endogeneity):

```
xi: ctreatreg wage treatment $xvars  ,  graphrf   ///
delta(10) hetero($xvarh) model(ct-iv) ct(tenure2) ci(1)  ///
estype(twostep) iv_t(ttl_exp) iv_w(c_city)
```

```
*** First step ***

Heckman selection model -- two-step estimates   Number of obs      =      2231
(regression model with sample selection)        Censored obs       =        51
                                                Uncensored obs     =      2180

                                                Wald chi2(5)       =    325.86
                                                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
tenure2      |
         age |   .2443289   .5733199     0.43   0.670    -.8793574    1.368015
    _Irace_2 |   4.352386   5.732858     0.76   0.448    -6.883809    15.58858
    _Irace_3 |  -5.507249   8.302224    -0.66   0.507    -21.77931    10.76481
 _Imarried_1 |   1.249892    2.81451     0.44   0.657    -4.266446    6.766231
     ttl_exp |   2.711141    .151083    17.94   0.000     2.415024    3.007258
       _cons |  -20.02397    19.7768    -1.01   0.311    -58.78578    18.73784
-------------+----------------------------------------------------------------
treatment    |
         age |  -.0428734   .0193969    -2.21   0.027    -.0808907   -.0048561
    _Irace_2 |  -.4157974   .1334837    -3.11   0.002    -.6774207   -.1541741
    _Irace_3 |   4.201402          .        .       .            .           .
 _Imarried_1 |  -.1936928   .1339983    -1.45   0.148    -.4563246    .0689389
      c_city |   .0404156   .1371767     0.29   0.768    -.2284458     .309277
       _cons |   3.941515   .7921321     4.98   0.000     2.388965    5.494065
-------------+----------------------------------------------------------------
mills        |
      lambda |  -34.00867   110.9056    -0.31   0.759    -251.3797    183.3623
-------------+----------------------------------------------------------------
         rho |   -1.00000
       sigma |  34.008667
------------------------------------------------------------------------------


*** Second step ***

Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =    2231
-------------+------------------------------           F(  9,  2221) =   19.96
       Model | -7979.49202     9 -886.610224           Prob > F      =  0.0000
    Residual |  82081.3197  2221  36.9569201           R-squared     =       .
-------------+------------------------------           Adj R-squared =       .
       Total |  74101.8276  2230  33.2295191           Root MSE      =  6.0792
```

```
------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   treatment |   2.382129   28.81026     0.08   0.934    -54.11573    58.87999
     _ws_age |   2.483951   3.440528     0.72   0.470    -4.263037     9.23094
          Tw |   .4324907   .1644757     2.63   0.009     .1099485    .7550328
         T2w |   -.006349   .0051065    -1.24   0.214    -.0163629    .0036649
         T3w |   .0000279   .0000414     0.67   0.500    -.0000533    .0001091
         age |  -2.531013   3.396567    -0.75   0.456    -9.191792    4.129767
    _Irace_2 |   -1.90372    .732588    -2.60   0.009    -3.340349   -.4670908
    _Irace_3 |   .9560915   1.293234     0.74   0.460    -1.579983    3.492166
 _Imarried_1 |  -.7593091   .3741212    -2.03   0.043    -1.492973   -.0256451
       _cons |   105.4548   154.0347     0.68   0.494    -196.6122    407.5218
------------------------------------------------------------------------------
Instrumented:  treatment _ws_age Tw T2w T3w
Instruments:   age _Irace_2 _Irace_3 _Imarried_1 probw _ps_age T_hatp
               T_hat2p T_hat3p
```

We see that ATE becomes now positive (2.38), but no longer significant. However, the Dose Response Function (Fig. 2) sets out a pattern similar to the previous model, with still a slight parabolic form, getting the maximum at a dose level around 45.

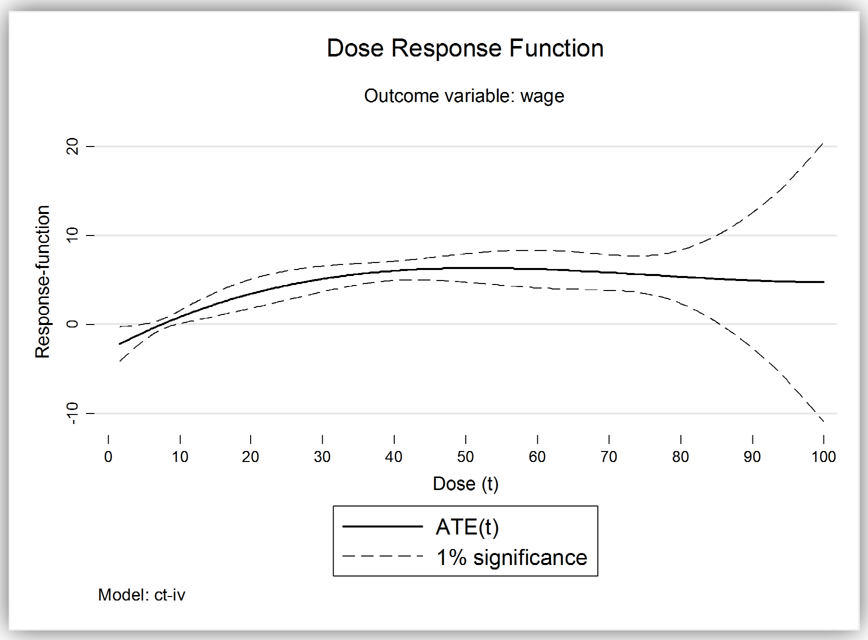


Figure 2. Dose response function of "job tenure" on "wage". Endogenous treatment case.

Of course, such results have to be taken just as instructional, as we have no idea about instruments' goodness.

6. A MONTE CARLO EXPERIMENT FOR TESTING CTREATREG'S RELIABILITY

In this section we provide a Monte Carlo experiment to check whether `ctreatreg` complies with predictions from the theory and to assess its correctness from a computational point of view. The first step is that of defining a data generating process (DGP) as follows:

$$\begin{cases} w = 1[50 + 60x_1 + 30x_2 + 60z + a > 0] \\ y_0 = 0.1 + 0.2x_1 + 0.3x_2 + e \\ y_1 = 0.3 + 0.6x_1 + 0.3x_2 + e \\ t = 0.4x_1 + 0.6x_2 + u \end{cases}$$

where we have assumed, for simplifying the model, that e_1=e_0=e and:

$$\begin{cases} x_1 \sim U(0;1)\cdot 100 \\ x_2 \sim U(0;1)\cdot 100 \\ z_w \sim N(15,1) \\ z_t \sim N(100,1) \end{cases}$$

with:

$$(a,u) \sim N(\mathbf{0};\mathbf{\Omega})$$

$$\mathbf{\Omega} = \begin{pmatrix} \sigma_a^2 & \sigma_{a,u} \\ & \sigma_u^2 \end{pmatrix} = \begin{pmatrix} \sigma_a^2 & \rho_{a,u}\sigma_a\sigma_u \\ & \sigma_u^2 \end{pmatrix}$$

$$\sigma_a^2 = 1, \quad \sigma_u^2 = 6.5, \quad \rho_{a,u} = 0.8$$

Finally, we suppose that the correlation between a and e_0 can be either equal or different from zero. In the latter case, w is endogenous. Therefore, we assume the following DGP[6]:

$$\begin{aligned} & e = \eta + \gamma a + v \\ & v \sim N(0;1) \\ & \gamma = \sqrt{\rho^2/(1-\rho^2)} \\ & \rho = \mathrm{corr}(e;a) \\ & \eta = 0.0001 \end{aligned}$$

When ρ=0 the model "ct-ols" would be the appropriate one; otherwise, the model "ct-iv" should be employed. By z_w and z_t, we indicate the instrumental variable for w and t, directly correlated with w and t respectively, but (directly) uncorrelated with y_1 and y_0.

Given these assumptions, the DGP is completed by the potential outcome equation $y_i = y_{0i} + w_i\,(y_{1i} - y_{0i})$, generating the observable outcome (or response) y.

The DGP is simulated 200 times using a sample size of 10,000. For each simulation we get a different data matrix (x_1, x_2, y, w, t, z_w, z_t) on which we apply the two models ("ct-ols" and "ct-iv") implemented by `ctreatreg`.

Case 1. Exogeneity

We start by assuming ρ=0, that is, zero correlation between the error term of the outcome equation (e) and the error term of the selection equation (a). Under this assumption, w is exogenous.

[6] The coefficient γ is equal to $(\rho^2/(1-\rho^2))^{-1/2}$, where ρ=corr(e_0;a). To get this result put x=e and y=a. We know that corr(x;y)=cov(x;y)/sd(x)sd(y). We can see that, while var(y)=1 by assumption, var(x)=γ^2+1. Moreover,cov(x;y)=cov(η+γa+v;a)=cov(η+γa;a)+cov (v;a)=cov(η+γa;a)=cov(γa;a)=γcov(a;a)=γvar(a)=γ. Thus, ρ=γ/$(\gamma^2+1)^{-1/2}$, that implies that γ=$(\rho^2/(1-\rho^2))^{-1/2}$.

Table 1. Mean test of ATE from Monte Carlo results using `ctreatreg`*.*
Exogenous selection is assumed.

	Mean	*Std. Err.*	*[95% Confidence Interval]*	
ATE (true value)	9.22	-	-	-
ATE - CT-OLS	9.21	0.01	9.19	9.22
ATE - CT-IV	9.20	0.01	9.19	9.22
% BIAS of OLS	0.81	0.04	0.73	0.90
% BIAS of IV	0.86	0.04	0.77	0.94

Note: ρ=0. Number of observations 10,000. Number of simulations 200.

Moreover, we assume a strong correlation between the *selection* and the *dose* equation, as implied by a correlation between a and u equal to 0.8. Results are set out in Table 1. It is immediate to see that the value of ATE obtained by the "ct-ols" estimator is really close to the true ATE (9.22) and that the confidence interval at 5% of significance for this estimator strictly contains that value. But also the percentage bias of "ct-iv" is very low (0.86%) and comparable with CT-OLS (0.81%) and sufficient to imply that the 5% of significance contains the true ATE even in this case.

These results confirm what was expected, thus showing that the option "ct-ols" of `ctreatreg` behaves correctly. As a conclusion, when the analyst assumes exogeneity, he/she may reliably use `ctreatreg` with the option "ct-ols".

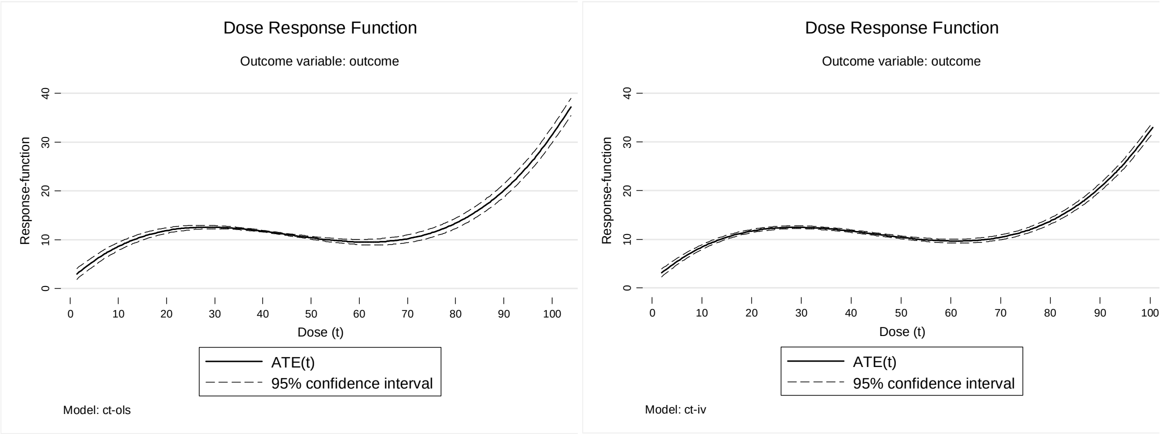
Case 2. Endogeneity

If we assume that ρ=0.7, that is, a high positive correlation between the error term of the outcome equation (e) and the error term of the selection equation (a), then w becomes endogenous. For the sake of comparison, we still assume the same strong correlation between the *selection* and the *dose* equation (0.8).Table 2 shows that results are - also in this case - coherent with the theoretical predictions. Indeed, the percentage bias of model "ct-ols" is rather high and equal to around 18%, whereas the bias of "ct-iv" is around 1%. Furthermore, and more importantly, the 95% mean test confidence interval for "ct-iv" contains the true ATE.

Table 2. Mean test of ATE from Monte Carlo results using `ctreatreg`*.*
Endogenous selection is assumed.

	Mean	*Std. Err.*	*[95% Confidence Interval]*	
ATE (true value)	9.22	-	-	-
ATE - CT-OLS	7.53	0.01	7.51	7.55
ATE - CT-IV	9.22	0.01	9.20	9.24
% BIAS of OLS	18.26	0.11	18.05	18.48
% BIAS of IV	1.28	0.07	1.15	1.41

Note: ρ=0.7. Number of observations 10,000. Number of simulations 200.


MODEL CT-OLS (under exogeneity)

MODEL CT-IV (under endogeneity)

Figure 3. Graphical representation of the dose-response function using the `ctreatreg` *option CT-OLS and CT-IV under exogeneity and endogeneity respectively.*

As expected, this implies that "ct-iv" is an unbiased estimator in presence of selection endogeneity, thus leading to a reliable estimation of the true value of ATE.

Overall, these results confirm the reliability of both the model and `ctreatreg` by allowing for a trustful use of this model and its related Stata implementation either under selection exogeneity or endogeneity. Finally, Fig. 3 plots the dose-response function along with the 95% interval confidence lines for both models. This is done by exploiting the "graphdrf" option of `ctreatreg`. Results clearly confirm our predictions.

7. CONCLUSION

The paper has presented `ctreatreg`, a Stata module for estimating dose-response functions through a regression approach where: (i) treatment is *continuous*, (ii) individuals may react heterogeneously to observable confounders, and (iii) selection-into-treatment may be endogenous.

Two estimation procedures are contemplated by this routine: one based on OLS under Conditional Mean Independence (or CMI), and one based on Instrumental-variables (IV), when assuming selection endogeneity.

An application to real data, for testing in an instructional example the impact of job tenure on wages, has been set out. Finally, in order to test the reliability of the formulas and of their associated Stata implementation, a Monte Carlo experiment has been performed.

Monte Carlo results show that the model's formulas and the Stata routine accompanying it are both reliable as estimates consistently fit expected results.

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Table A1. Stata help-file for `ctreatreg`.

```
help ctreatreg
-------------------------------------------------------------------------------------------------------------------
-

Title

    ctreatreg -  Dose-Response model with "continuous" treatment, endogeneity and heterogeneous response to
                   observable confounders


Syntax

ctreatreg outcome treatment [varlist] [if] [in] [weight], model(modeltype) ct(treat_level) [hetero(varlist_h)
estype(model) iv_t(instrument_t) iv_w(instrument_w) delta(number) ci(number) graphate graphdrf conf(number)
vce(robust) const(noconstant) head(noheader) beta]


          fweights, iweights, and pweights are allowed; see weight.


Description

    ctreatreg estimates the dose-response function (DRF) of a given treatment on a specific target variable, within
    a model where units are treated with different levels. The DRF is defined as the “average treatment effect,
    given the level of the treatment t” (i.e. ATE(t)).  The routine also estimates other “causal” parameters of
    interest, such as the average treatment effect (ATE), the average treatment effect on treated (ATET), the
    average treatment effect on non-treated (ATENT), and the same effects conditional on t and on the vector of
    covariates x.The DRF is approximated by a third degree polynomial function.  Both OLS and IV estimation are
    available, according to the case in which the treatment is not or is endogenous. In particular, the implemented
    IV estimation is based on a Heckman bivariate selection model (i.e., type-2 tobit) for w (the yes/no decision to
treat     a given unit) and t (the level of the treatment provided) in the first step, and a 2SLS estimation for the
outcome (y)
    equation in the second step.  The routine allows also for a graphical representation of results.


Options

    model(modeltype) specifies the treatment model to be estimated, where modeltype must be one of the following two
        models: "ct-ols", "ct-iv".  it is always required to specify one model.

    ct(treat_level) specifies the treatment level (or dose).  This variable takes values in the [0;100] interval,
        where 0 is the treatment level of non-treated units. The maximun dose is thus 100.

    hetero(varlist_h) specifies the variables over which to calculate the idiosyncratic Average Treatment Effect
        ATE(x), ATET(x) and ATENT(x), where x=varlist_h. It is optional for all models. When this option is not
        specified, the command estimates the specified model without heterogeneous average effect. Observe that
        varlist_h should be the same set or a subset of the variables specified in varlist.  Observe however that
        only numerical variables may be considered.

    estype(model) specifies which type of estimation method has to be used for estimating the type-2 tobit model in
the
        endogenous treatment case. Two choices are available: "twostep" implements a Heckman two-step procedure; "ml"
        implements a maximum-likelihood estimation. This option is required only for "ct-iv".

    iv_t(instrument_t) specifies the variable to be used as instrument for the continuous treatment variable t in the
        type-2 tobit model. This option is required only for "ct-iv".

    iv_w(instrument_w) specifies the variable to be used as instrument for the binary treatment variable w in the
type-2
        tobit model. This option is required only for "ct-iv".

    delta(number) identifies the average treatment effect between two states: t and t+delta. For any reliable delta,
        we can obtain the response function ATE(t;delta)=E[y(t)-y(t+delta)].

    ci(number) sets the significant level for the dose-response function, where number may be 1, 5 or 10.

    graphate allows for a graphical representation of the density distributions of ATE(x;t) ATET(x;t) and
        ATENT(x;t). It is optional for all models and gives an outcome only if
        variables into hetero() are specified.

    graphdrf allows for a graphical representation of the Dose Response Function (DRF) and of
        its derivative. It plots also the 95% confidence interval of the DRF over the dose
        levels.

    vce(robust) allows for robust regression standard errors. It is optional for all models.

    beta reports standardized beta coefficients. It is optional for all models.

    const(noconstant) suppresses regression constant term. It is optional for all models.

    conf(number) sets the confidence level equal to the specified number.  The default is number=95.


  modeltype_options            description
  -----------------------------------------------------------------------------------------------------------------
-
  Model
  ct-ols                       Control-function regression estimated by ordinary least squares
  ct-iv                        IV regression estimated by Heckman bivariate selection model and 2SLS
  -----------------------------------------------------------------------------------------------------------------
-


    ctreatreg creates a number of variables:

        _ws_varname_h are the additional regressors used in model's regression when hetero(varlist_h) is specified.

        _ps_varname_h are the additional instruments used in model's regression when hetero(varlist_h) is specified
        in model "ct-iv".

        ATE(x;t) is an estimate of the idiosyncratic Average Treatment Effect.

        ATET(x;t) is an estimate of the idiosyncratic Average Treatment Effect on treated.

        ATENT(x;t) is an estimate of the idiosyncratic Average Treatment Effect on Non-Treated.
```

ATE(t) is an estimate of the dose-response function.

ATET(t) is the value of the dose-response function in t>0.

ATENT(t) it is the value of the dose-response function in t=0.

probw is the predicted probability from the Heckman selection model (estimated only for model "ct-iv").

mills is the predicted Mills' ratio from the Heckman selection model (estimated only for model "ct-iv").

t is a copy of the treatment level variable, but only in the sample considered.

t_hat is the prediction of the level of treatment from the Heckman bivariate selection model (estimated only for model "ct-iv").

der_ATE_t is the estimate of the derivative of the dose-response function.

std_ATE_t is the standardized value of the dose-response function.

std_der_ATE_t is the standardized value of the derivative of the dose-response function.

Tw, **T2w**, **T3w** are the three polynomial factors of the dose-response function.

T_hatp, **T2_hatp**, **T3_hatp** are the three instruments for the polynomial factors of the dose-response function when model "ct-iv" is used.

ctreatreg returns the following scalars:

r(N_tot) is the total number of (used) observations.

r(N_treated) is the number of (used) treated units.

r(N_untreated) is the number of (used) untreated units.

r(ate) is the value of the Average Treatment Effect.

r(atet) is the value of the Average Treatment Effect on Treated.

r(atent) is the value of the Average Treatment Effect on Non-treated.

Remarks

The variable specified in treatment has to be a 0/1 binary variable (1 = treated, 0 = untreated).

The standard errors for ATET and ATENT may be obtained via bootstrapping.

When using the option ct-iv in **modeltype()**, be sure that the number of variables included in **hetero()** is less than the number of variables included in *varlist*. This is because otherwise instruments are too much correlated and some emerging collinearity prevent to identify the estimates. For instance, when six covariates are specified in *varlist*, at most five are to be put into **hetero()**.

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