CEM: Coarsened Exact Matching for Stata

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joint work with

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- (Medical experiments are the reverse: small-n with random treatment assignment; don't match unless something goes wrong)

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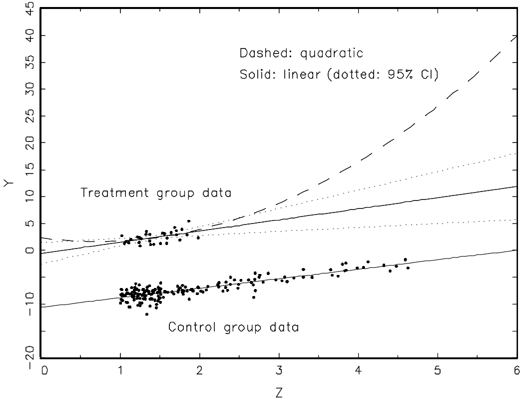

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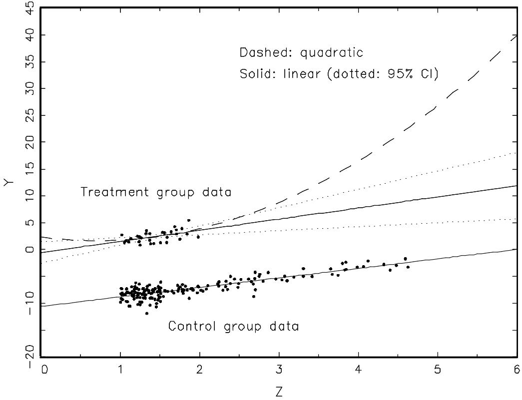

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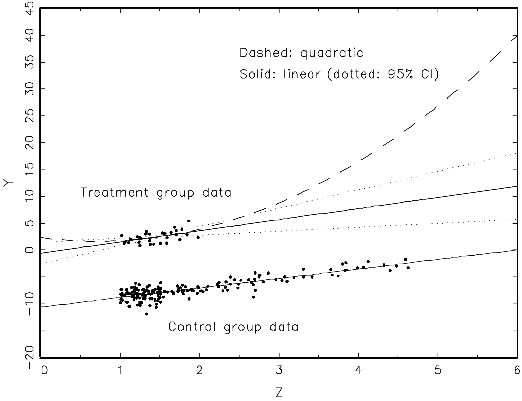

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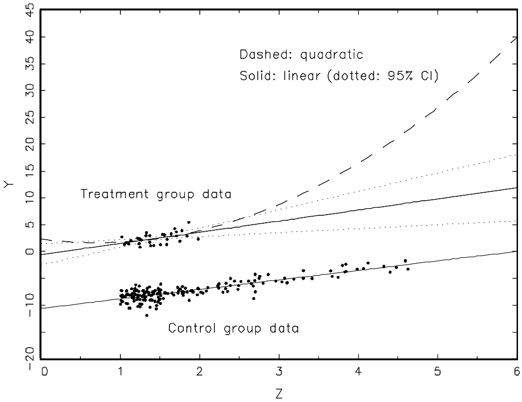

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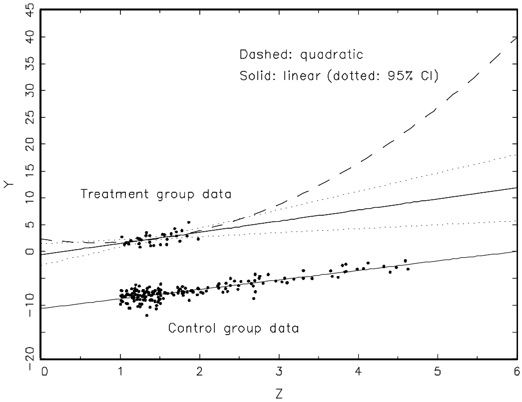

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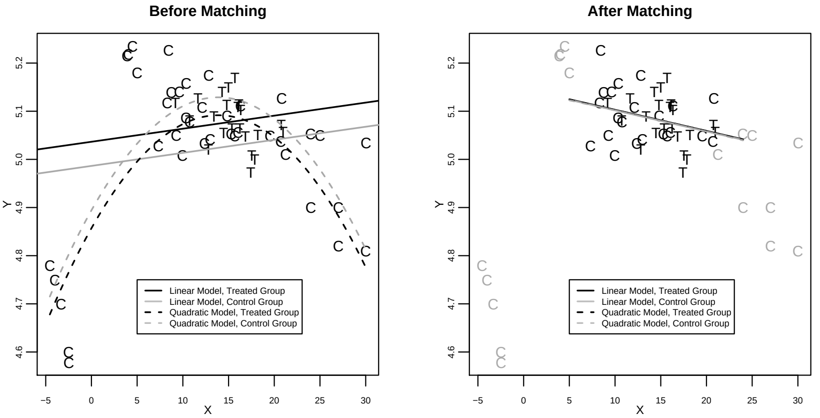

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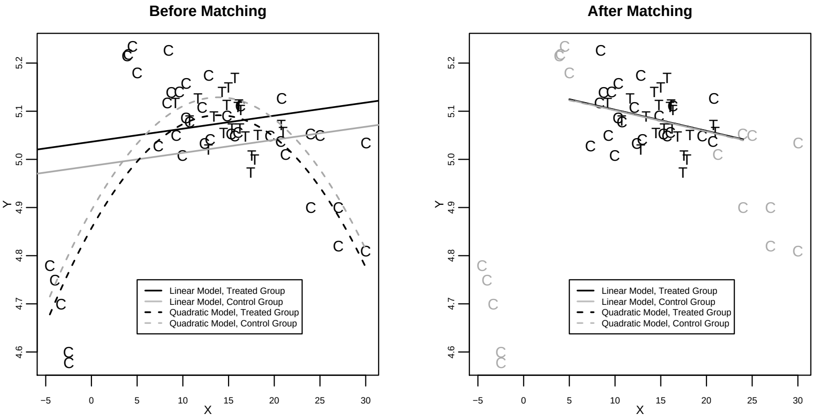

Matching reduces model dependence, bias, and variance

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- Sample Average Treatment effect on the Treated:

$$\text{SATT} = \frac{1}{n_T} \sum_{i \in \{T_i = 1\}} \text{TE}_i$$

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- Goal: changing balance on 1 variable should not harm others
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  (a) X drawn randomly from a specified population $\mathbf{X}$,
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- EPBR Definition: Matched sample size set ex ante, and

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  - (In practice, we're lucky if univariate mean imbalance is reduced)

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"Imbalance" given chosen distance metric

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One tuning parameter ϵ_j, one for each X_j

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If $\boldsymbol{\epsilon}$ is reduced, $\boldsymbol{\gamma}(\boldsymbol{\epsilon})$ decreases & $\gamma(\epsilon)$ is unchanged

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  - Age: infant, child, adolescent, young adult, middle age, elderly

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- Simple to teach: coarsen, then exact match

CEM in Stata – An example

```
. cem age education black nodegree re74, tr(treated)

Matching Summary:
-----------------
Number of strata: 205
Number of matched strata: 67

             0     1
      All  425   297
  Matched  324   228
Unmatched  101    69

Multivariate L1 distance: .46113967

Univariate imbalance:

               L1       mean   min   25%      50%      75%      max
      age  .13641   -.17634     0     0        0        0       -1
education  .00687    .00687     1     0        0        0        0
    black  3.2e-16  -2.2e-16    0     0        0        0        0
 nodegree  5.8e-16   4.4e-16    0     0        0        0        0
     re74  .06787    34.438     0     0   492.23   39.425   96.881
```

Imbalance Measures

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Variable-by-Variable Difference in Global Means

$$I_1^{(j)} = \left| \bar{X}_{m_T}^{(j)} - \bar{X}_{m_C}^{(j)} \right|, \quad j = 1, \ldots, k$$

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Local Imbalance by Variable (given strata fixed by matching method)

$$I_2^{(j)} = \frac{1}{S} \sum_{s=1}^{S} \left| \bar{X}_{m_T^s}^{(j)} - \bar{X}_{m_C^s}^{(j)} \right|, \quad j = 1, \ldots, k$$

Estimating the Causal Effect from cem output

```
. reg re78 treated [iweight=cem_weights]

      Source |       SS       df       MS              Number of obs =     552
-------------+------------------------------           F(  1,   550) =    3.15
       Model |   128314324     1   128314324           Prob > F      =  0.0766
    Residual |  2.2420e+10   550  40764521.6           R-squared     =  0.0057
-------------+------------------------------           Adj R-squared =  0.0039
       Total |  2.2549e+10   551  40923414.2           Root MSE      =  6384.7

------------------------------------------------------------------------------
        re78 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     treated |   979.1905   551.9132     1.77   0.077    -104.9252    2063.306
       _cons |    4919.49   354.7061    13.87   0.000     4222.745    5616.234
------------------------------------------------------------------------------
```

Choosing a custom coarsening

```
. table education

----------------------
education |      Freq.
----------+-----------
        3 |          1
        4 |          6
        5 |          5
        6 |          7
        7 |         15
        8 |         62
        9 |        110
       10 |        162
       11 |        195
       12 |        122
       13 |         23
       14 |         11
       15 |          2
       16 |          1
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High school	9–12
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- Improve Existing Matching Methods Applying other methods within CEM strata

For papers, software, tutorials, etc.

http://GKing.Harvard.edu/cem