# Estimating Markov-switching regression models in Stata

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# ARMA models

- Time series data are autocorrelated due to the dependence with past values.
- Autoregressive moving average (ARMA) class of models is a popular tool to model such autocorrelations.
- The AR part models the current value as a weighted average of past values with some error.

$$y_t = \phi y_{t-1} + \varepsilon_t$$

where

  - $y_t$ is the observed series
  - $\phi$ is the autoregressive parameter
  - $\varepsilon_t$ is an IID error with mean 0 and variance $\sigma^2$

# ARMA(1,1) model

- The MA part models the current value as a weighted average of past errors.

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

where $\theta$ is the moving average parameter.

- The AR and MA models generate completely different autocorrelations.
- Combining these lead to a flexible way to capture various correlation patterns observed in time series data.

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

# Linear ARMA models

- Current value of the series is linearly dependent on past values
- The parameters do not change throughout the sample
- This precludes many interesting features observed in the data

# Examples

- In economics, the average growth rate of gross domestic product (GDP) tend to be higher in expansions than in recessions. Furthermore, expansions tend to last longer than recessions
- In finance, stock returns display periods of high and low volatility over the course of years
- In public health, incidence of infectious disease tend be different under epidemic and non-epidemic states

# Nonlinear models

- In all these examples, the dynamics are state-dependent.
  - The states may be recession and expansion, high volatility and low volatility, or epidemic and non-epidemic states
  - Parameters may be changing according to the states
- Nonlinear models aim to characterize such features observed in the data

# Markov-switching model

- Hamilton (1989)
- Finite number of unobserved states
  - Suppose there are two states 1 and 2
  - Let $s_t$ denote a random variable such that $s_t = 1$ or $s_t = 2$ at any time
- $s_t$ follows a first-order Markov process
  - Current value of $s_t$ depends only on the immediate past value
  - We do not know which state the process is in but can only estimate the probabilities
- The process can switch between states repeatedly over the sample

# Features

- Estimate the state-dependent parameters
- Estimate transition probabilities
  - $P(s_t = j | s_{t-1} = i) = p_{ij}$
  - Probability of transitioning from state $i$ to state $j$
- Estimate the expected duration of a state
- Estimate state-specific predictions

# Background

- Consider the following state-dependent AR(1) model

$$y_t = \mu_{s_t} + \phi_{s_t} y_{t-1} + \varepsilon_t$$

  where $\varepsilon_t \sim N(0, \sigma^2_{s_t})$
- $s_t$ is discrete and denotes the state at time $t$
  - The parameters $\mu$, $\phi$, and $\sigma^2$ are state-dependent
  - The number of states are imposed apriori
- For example, a two-state model can be expressed as

$$y_t = \begin{cases} \mu_1 + \phi_1 y_{t-1} + \varepsilon_{t,1} & \text{if } s_t = 1 \\ \mu_2 + \phi_2 y_{t-1} + \varepsilon_{t,2} & \text{if } s_t = 2 \end{cases}$$

# Assumptions on the state variable

- Recall the two-state model

$$y_t = \begin{cases} \mu_1 + \phi_1 y_{t-1} + \varepsilon_{t,1} & \text{if } s_t = 1 \\ \mu_2 + \phi_2 y_{t-1} + \varepsilon_{t,2} & \text{if } s_t = 2 \end{cases}$$

- If the timing when the process switches states is known, we could
  - Create indicator variables to estimate the parameters in different states.
  - For example economic crisis may alter the dynamics of a macroeconomic variable.

# States are unobserved

- $s_t$ is drawn randomly every period from a discrete probability distribution
  - Switching regresssion model
  - The realization of $s_t$ at each period are independent from that of the previous period
- $s_t$ follows a first-order Markov process
  - The current realization of the state depends only on the immediate past
  - $s_t$ is autocorrelated

# mswitch regression command in Stata

- Markov-switching autoregression

  `mswitch ar` *depvar* [*nonswitch_varlist*] [*if*] [*in*] , `ar(`*numlist*`)` [*options*]

- Markov-switching dynamic regression

  `mswitch dr` *depvar* [*nonswitch_varlist*] [*if*] [*in*] [, *options*]

# MSAR with 4 lags

- Hamilton (1989) models the quarterly growth rate of real GNP as a two state model
- The dataset spans the period 1951q1 - 1984q4
- The states are expansion and recession

$$\texttt{rgnp}_t = \mu_{s_t} + \phi_1(\texttt{rgnp}_{t-1} - \mu_{s_{t-1}}) + \phi_2(\texttt{rgnp}_{t-2} - \mu_{s_{t-2}}) + \\ \phi_3(\texttt{rgnp}_{t-3} - \mu_{s_{t-3}}) + \phi_4(\texttt{rgnp}_{t-4} - \mu_{s_{t-4}}) + \varepsilon_t$$

# Quarterly growth rate of US RGNP

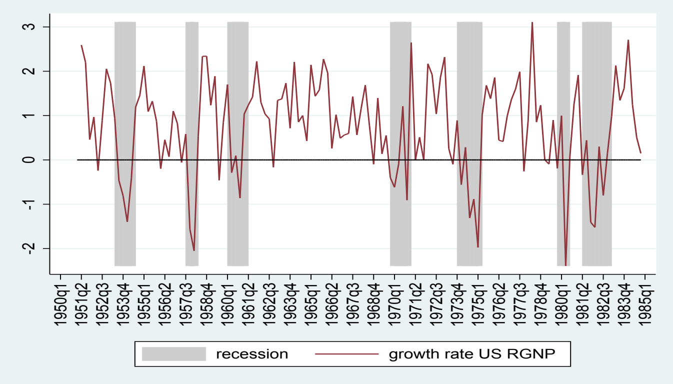


Figure : Quarterly growth rate of US RGNP

# Markov-switching autoregression

```
. mswitch ar rgnp, ar(1/4) nolog
Performing EM optimization:
Performing gradient-based optimization:
Markov-switching autoregression
Sample: 1952q2 - 1984q4                         No. of obs      =        131
Number of states =     2                        AIC             =     2.9048
Unconditional probabilities: transition         HQIC            =     2.9851
                                                SBIC            =     3.1023
Log likelihood = -181.26339
```

| rgnp | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| rgnp | | | | | | |
| ar | | | | | | |
| L1. | .0134871 | .1199941 | 0.11 | 0.911 | -.2216971 | .2486713 |
| L2. | -.0575212 | .137663 | -0.42 | 0.676 | -.3273357 | .2122933 |
| L3. | -.2469833 | .1069103 | -2.31 | 0.021 | -.4565235 | -.037443 |
| L4. | -.2129214 | .1105311 | -1.93 | 0.054 | -.4295583 | .0037155 |
| State1 | | | | | | |
| _cons | -.3588127 | .2645396 | -1.36 | 0.175 | -.8773007 | .1596753 |
| State2 | | | | | | |
| _cons | 1.163517 | .0745187 | 15.61 | 0.000 | 1.017463 | 1.309571 |
| sigma | .7690048 | .0667396 | | | .6487179 | .9115957 |
| p11 | .754671 | .0965189 | | | .5254555 | .8952432 |
| p21 | .0959153 | .0377362 | | | .0432569 | .1993221 |

# Transition probabilities

- State 1 is recession and State 2 is expansion.
- Let $P$ denote a transition probability matrix for 2 states. The elements of P are

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.75 & 0.25 \\ 0.1 & 0.9 \end{bmatrix}$$

  such that $\sum_j p_{ij} = 1$ for i,j = 1,2.
- $p_{11}$ denotes the probability of transitioning to recession in the next period given that the current state is in recession.

# Predicting the probability of recession

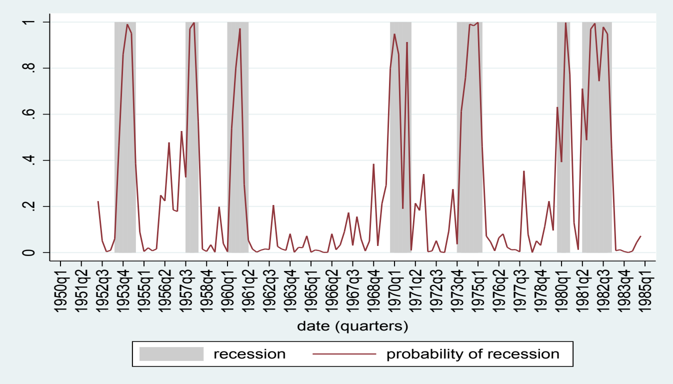


Figure : Probability of recession

# Expected duration

- Compute the expected duration the series spends in a state
- Let $D_i$ denote the duration of state $i$
  - $D_i$ follows a geometric distribution
  - The expected duration is

$$E[D_i] = \frac{1}{1 - p_{ii}}$$

- The closer $p_{ii}$ is to 1, the higher is the expected duration of state $i$

# Estimating duration of a state

```
. estat duration
Number of obs = 131
```

| Expected Duration | Estimate | Std. Err. | [95% Conf. | Interval] |
|---|---|---|---|---|
| State1 | 4.076159 | 1.603668 | 2.107284 | 9.545916 |
| State2 | 10.42587 | 4.101873 | 5.017005 | 23.11772 |

# Equivalent AR specifications

- Consider the following equivalent AR(1) models:

$$y_t - \delta = \phi(y_{t-1} - \delta) + \varepsilon_t$$
$$y_t = \mu + \phi y_{t-1} + \varepsilon_t$$

- The unconditional means for the above models are related: $\delta = \frac{\mu}{1-\phi}$

# MSAR and MSDR specifications

- This equivalence is not possible if the mean is state-dependent

$$y_t = \delta_{s_t} + \phi(y_{t-1} - \delta_{s_{t-1}}) + \varepsilon_t \qquad \text{(AR)}$$
$$y_t = \mu_{s_t} + \phi y_{t-1} + \varepsilon_t \qquad \text{(DR)}$$

- A one time change in the state leads to an immediate shift in the mean level in the AR specification.
- A one time change in the state leads to the mean level changing smoothly over several time periods in the DR specification.

# State vector of MSAR

- The observed series depends on the value of states at time $t$ and $t-1$.
  - A two-state Markov process becomes a four-state Markov process.
  - In general, AR specification increases the state vector by the factor $K^{p+1}$, where $p$ is the number of lags.
- Used for modeling data with smaller frequency such as quarterly, annual, etc.

# Markov-switching model of interest rates

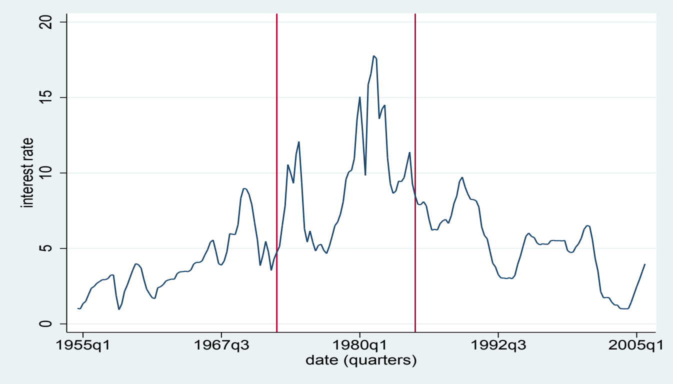


Figure : Short term interest rate

# Estimating interest rates

- Estimate using data for the period 1955q3–2005q4
- Assume the following specification for interest rates

$$\texttt{intrate}_t = \mu_{s_t} + e_{s_t}$$

where

- intrate is the interest rate
- $e_{s_t} \sim N(0, \sigma^2_{s_t})$
- $\mu$ and $\sigma^2$ is state-dependent

# Estimate the model using mswitch dr

```
. mswitch dr intrate, varswitch nolog
Performing EM optimization:
Performing gradient-based optimization:
Markov-switching dynamic regression
Sample: 1954q3 - 2005q4                     No. of obs        =        206
Number of states =    2                     AIC               =     4.4078
Unconditional probabilities: transition     HQIC              =     4.4470
                                            SBIC              =     4.5048
Log likelihood = -448.00658
```

| intrate | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| State1 | | | | | | |
| _cons | 2.650457 | .1260721 | 21.02 | 0.000 | 2.40336 | 2.897554 |
| State2 | | | | | | |
| _cons | 7.445134 | .2649754 | 28.10 | 0.000 | 6.925792 | 7.964477 |
| sigma1 | .9704124 | .0880692 | | | .8122805 | 1.159329 |
| sigma2 | 2.958272 | .1824307 | | | 2.621478 | 3.338336 |
| p11 | .9789357 | .0160089 | | | .9102967 | .9953235 |
| p21 | .0193584 | .0116402 | | | .0059 | .0616132 |

# Predicted probability of State 2

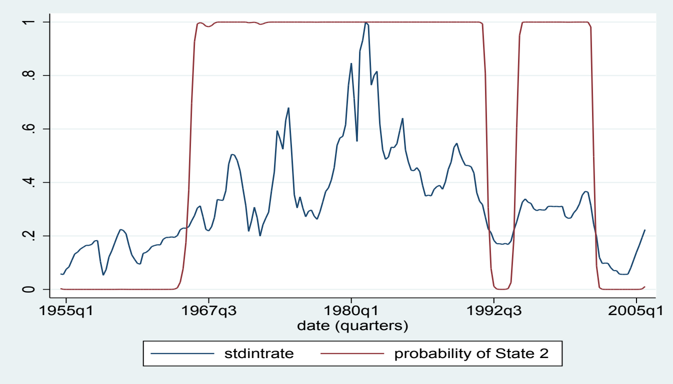


Figure : Predicted probabilities using MSDR model

# Dynamic forecasting with MSAR

- Estimate using data for the period 1955q3-1999q4
- Assume the following specification for interest rates

$$\texttt{intrate}_t = \mu_{s_t} + \rho\ \texttt{intrate}_{t-1} + \phi_{s_t}\texttt{inflation}_t + \gamma_{s_t}\texttt{ogap}_t + e_t$$

  where

  - `intrate` is the interest rate
  - `inflation` is the inflation rate
  - `ogap` is the output gap
  - $e_t \sim N(0, \sigma^2)$
  - $\rho$ is constant
  - $\mu$, $\phi$, and $\gamma$ are state-dependent
- Out-of-sample forecasting from period 2000q1 - 2007q1

# Estimate the model using mswitch dr

```
. mswitch dr intrate L.intrate if tin(,1999q4), switch(inflation ogap) nolog
Performing EM optimization:
Performing gradient-based optimization:
Markov-switching dynamic regression
Sample: 1955q3 - 1999q4                          No. of obs       =        178
Number of states =     2                         AIC              =     2.3301
Unconditional probabilities: transition          HQIC             =     2.4025
                                                 SBIC             =     2.5088
Log likelihood = -197.375
```

| intrate | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| **intrate** | | | | | | |
| intrate L1. | .8503947 | .0991269 | 8.58 | 0.000 | .6561096 | 1.04468 |
| **State1** | | | | | | |
| inflation | -.0392848 | .1298901 | -0.30 | 0.762 | -.2938646 | .215295 |
| ogap | .1473233 | .0528794 | 2.79 | 0.005 | .0436816 | .250965 |
| _cons | .7403998 | .2041607 | 3.63 | 0.000 | .3402522 | 1.140547 |
| **State2** | | | | | | |
| inflation | .2688704 | .0798215 | 3.37 | 0.001 | .1124232 | .4253177 |
| ogap | -.0075103 | .0856139 | -0.09 | 0.930 | -.1753105 | .1602899 |
| _cons | .2173127 | .4685576 | 0.46 | 0.643 | -.7010433 | 1.135669 |
| sigma | .6138084 | .0367645 | | | .54582 | .6902655 |
| p11 | .7459455 | .2512815 | | | .1792104 | .9752993 |
| p21 | .2061723 | .0956226 | | | .0763309 | .4494157 |

# Out-of-sample dynamic forecasts

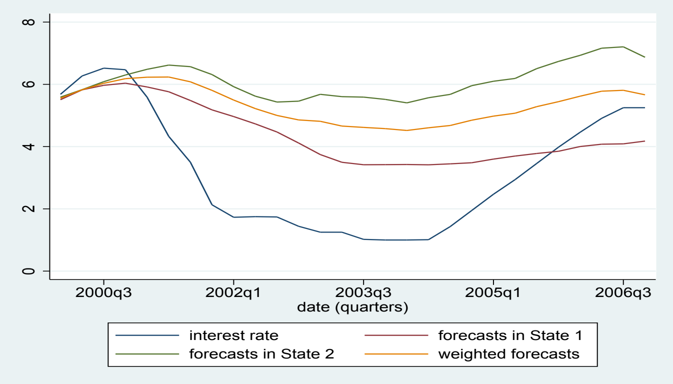


Figure : Forecasts using MSDR model

Thank you !

Hamilton, J. D. (1989), 'A new approach to the economic analysis of nonstationary time series and the business cycle', *Econometrica* **57**(2), 357–384.