Multilevel Selection Models using `gllamm`

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Gllamm can be downloaded from
http://www.iop.kcl.ac.uk/iop/departments/biocomp/programs/gllamm.html

Outline

Brief introduction to GLLAMM

The Heckman model and extensions

The Hausman-Wise-Diggle-Kenward dropout model and extensions

Application: Cluster randomized study of sex education in Norwegian schools

Overview of GLLAMM models

- Response model: Generalised linear model conditional on latent variables
  - Linear predictor: latent variables as factors or random coefficients
  - Links and distributions
- Structural model: Equations for the latent variables
  - Regressions of latent variables on observed variables
  - Regressions of latent variables on other latent variables
- Distribution of the latent variables (disturbances)
  - Multivariate normal
  - Discrete

Linear Predictor in GLLAMM

$$\eta = \boldsymbol{\beta}'\mathbf{x} + \sum_{l=2}^{L}\sum_{m=1}^{M_l} u_m^{(l)} \boldsymbol{\lambda}_m^{(l)\prime}\mathbf{z}_m^{(l)} \quad \text{for identification, } \lambda_{m1}^{(l)} = 1$$

- Fixed part: $\boldsymbol{\beta}'\mathbf{x}$ as usual
- Random part:
  - $u_m^{(l)}$ is mth latent variable at level l, $m = 1, \cdots, M_l$, $l = 2, \cdots, L$
  - $u_m^{(l)}$ can be a **factor** or a **random coefficient**
  - $\mathbf{z}_m^{(l)}$ are variables and $\boldsymbol{\lambda}_m^{(l)}$ are parameters
  - Unless regressions for the latent variables are specified, latent variables at different levels are independent whereas latent variables at the same level may be correlated.

Random coefficient models in GLLAMM

- One covariate multiplies each latent variable,

$$u_m^{(l)} z_m^{(l)} \quad (\lambda_m^{(l)} = 1)$$

- e.g. Latent growth curve model for individuals j (level 2) observed at times t_{ij}, $i = 1, \cdots, n_j$ (level 1)

$$\eta_{ij} = \beta_1 + \beta_2 t_{ij} + u_{1j}^{(2)} + u_{2j}^{(2)} t_{ij}$$

β_1, β_2 : mean intercept and slope

$u_{1j}^{(2)}$, $u_{2j}^{(2)}$: random deviations of the subject-specific intercepts and slopes from their means

- The model can also be defined as

$$\begin{aligned} \eta_{ij} &= b_{1j} + b_{2j} t_{ij} \\ b_{1j} &= \beta_1 + u_{1j} \\ b_{2j} &= \beta_2 + u_{2j} \end{aligned}$$

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Factor models in GLLAMM

- A linear combination of dummy variables for the items multiplies each latent variable,

$$u_m^{(l)} \boldsymbol{\lambda}_m^{(l)\prime} \mathbf{z}_m^{(l)}$$

- e.g. One-factor model for items i, $i = 1, \cdots, I$ (level 1) and subjects j (level 2)

$$\begin{aligned} \eta_{ij} &= \beta_1 \delta_{1i} + \cdots + \beta_I \delta_{Ii} + u_j^{(2)} (\delta_{1i} + \lambda_2^{(2)} \delta_{2i} \cdots + \lambda_I^{(2)} \delta_{Ii}) \\ &= \beta_i + u_j^{(2)} \lambda_i^{(2)} \end{aligned}, \quad \delta_{pi} = \begin{cases} 1 & \text{if } p = i \\ 0 & \text{otherwise} \end{cases}$$

β_i: intercept for item i

$u_j^{(2)}$: common factor

$\lambda_i^{(2)}$: factor loading for item i, $\lambda_1^{(2)} = 1$

unit j	item i	δ_{1i}	δ_{2i}	$\cdots$	δ_{Ii}	y_{ij}
1	1	1	0	$\cdots$	0	y_{11}
1	2	0	1	$\cdots$	0	y_{21}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
1	I	0	0	$\cdots$	1	y_{I1}

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Heckman selection model

- Selection equation (probit regression):

$$\begin{aligned} y_{1j}^* &= \boldsymbol{\gamma}' \mathbf{z}_j + \epsilon_{1j}, \quad \epsilon_{1j} \sim N(0,1), \\ y_{1j} &= \mathrm{I}(y_{i1}^* > 0) \end{aligned}$$

- Substantive equation (linear regression):

$$y_{2j} = \begin{cases} \boldsymbol{\alpha}' \mathbf{w}_j + \epsilon_{2j}, \ \epsilon_{2j} \sim N(0, \sigma^2) & \text{if } y_{i1} = 1 \\ \text{missing} & \text{if } y_{i1} = 0 \end{cases},$$

- Correlation

$$\mathrm{cor}(\epsilon_{1j}, \epsilon_{2j}) = \rho$$

- Missingness or non-selection is
  - Completely at random if $\boldsymbol{\gamma} = 0$ and $\rho = 0$
  - At random if $\rho = 0$
  - Informative if $\rho \neq 0$

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Heckman selection model as a GLLAMM model

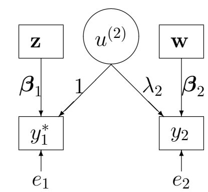
- Parameterize random part as

$$\epsilon_{ij} = u_j^{(2)} \lambda_i + e_{ij}, \quad \mathrm{var}(u_j^{(2)}) = 1, \ \lambda_1 = 1, \ \mathrm{var}(e_{ij}) = \nu^2, \ \mathrm{cov}(e_{1j}, e_{2j}) = 0$$

- Write as a GLLAMM model

$$\eta_{ij} = \boldsymbol{\beta}_1' \mathbf{z}_{ij} \delta_{1i} + \boldsymbol{\beta}_2' \mathbf{w}_{ij} \delta_{2i} + u_j^{(2)} (\delta_{1i} + \lambda_2 \delta_{2i}), \quad \mathrm{var}(u_j^{(2)}) = 1,$$

$$y_{ij} | \eta_{ij} \sim \begin{cases} \text{Bernouilli}(\Phi(\eta_{ij}/\nu)) \text{ if } i = 1 & \text{(Binomial with scaled probit link)} \\ N(\eta_{ij}, \nu^2) \text{ if } i = 2 & \text{(Gaussian with identity link)} \end{cases}$$



- Equivalences:

$$\sigma^2 = \lambda_2^2 + \nu^2, \ \rho = \frac{\lambda_2}{\sqrt{(\lambda_2^2 + \nu^2)(1 + \nu^2)}}, \quad \boldsymbol{\gamma} = \frac{\boldsymbol{\beta}_1}{\sqrt{1 + \nu^2}}, \quad \boldsymbol{\alpha} = \boldsymbol{\beta}_2$$

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Syntax for linear predictor in `gllamm`

gllamm [*varlist*] [if *exp*] [in *range*] , i(*varlist*) [nrf(*numlist*)
    eqs(*eqnames*) noconstant offset(*varname*) constraints(*numlist*)
    . . .

`i(`*varlist*`)` $L-1$ variables identifying the hierarchical, nested clusters, from level 2 to L, e.g., `i(pupil class school)`.

`nrf(`*numlist*`)` $L-1$ numbers specifying the numbers of latent variables M_l at each level.

`eqs(`*eqnames*`)` $M=\sum M_l$ equations for the $\boldsymbol{\lambda}_m^{(l)}\mathbf{z}_m^{(l)}$ multiplying each latent variable. No constant is assumed unless explicitly included in the equation definition.

`noconstant` no constant in the fixed part $\boldsymbol{\beta}'\mathbf{x}$.

`offset(`*varname*`)` variable in fixed part with regression coefficient set to 1.

`constraints(`*numlist*`)` list of linear parameter constraints defined using the `constraint define` command.

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Heckman selection model - linear predictor in `gllamm`

- `heckman` command in Stata

```
heckman y2 w, select(y1 = z w)
```

- Linear predictor in `gllamm`

$$\eta_{ij}=\beta_1 z_{ij}\delta_{1i}+\beta_2 w_{ij}\delta_{1i}+\beta_3\delta_{1i}+\beta_4 w_{ij}\delta_{2i}+\beta_5\delta_{2i}+u_j^{(2)}(\delta_{1i}+\lambda_2\delta_{2i})$$

- Data manipulation

```
gen id = _n
reshape long y, i(id) j(var)
tab var, gen(i)                  /* i1 = δ1i, i2 = δ2i */
gen z_i1 = z*i1
gen w_i1 = w*i1
gen w_i2 = w*i2
```

- `gllamm` command

```
eq load: i1 i2                   /* for u_j^(2)(δ1i + λ2δ2i) */
constraint define 1 [id1]i1 = 1  /* sets var(u_j^(2)) = 1 */
gllamm y z_i1 w_i1 i1 w_i2 i2, i(id) eqs(load) nocons constr(1) /*
    */  << more options >>
```

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Links and families in GLLAMM

- The conditional expectation of the response is 'linked' to the linear predictor

$$g(E[y|\mathbf{x},\mathbf{u},\mathbf{z}])=\eta$$

- The conditional distribution of the response is from the exponential family
- The response variables may be of mixed type - requiring mixed links and families:

Links
identity
reciprocal
logarithm
logit
probit
scaled probit
compl. log-log

Families
Gaussian
gamma
Poisson
binomial

Polytomous responses
ordinal logit
ordinal probit
ordinal compl. log-log
scaled ord. probit
multinomial logit

- Heteroscedasticity: The dispersion parameter for the Gauss and gamma families and the scale for the scaled probit link can depend on covariates: $\log\phi=\boldsymbol{\alpha}'\mathbf{z}^{(1)}$

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Options for links and families in `gllamm`

[· · · family(*families*) fv(*varname*) link(*links*) lv(*varname*) nats
    s(*eqname*) · · ·]

`family(`*families*`)` family or families to be used.

`fv(`*varname*`)` variable whose values indicate which family applies to which observation.

`link(`*links*`)` and `lv(`*varname*`)` analogous to `family(`*families*`)` and `fv(`*varname*`)`.

`nats` option to estimate the scale parameter directly instead of its logarithm.

`s(`*eqname*`)` equation for (log) scale parameter.

- Heckman selection model: links and families

```
reshape long y, i(id) j(var)   /* var = 1 for y=y1 and 2 for y=y2 */
tab var, gen(i)
     << more data manipulation >>
gllamm y z_i1 w_i1 i1 w_i2 i2, nocons i(id) eq(load) constr(1) /*
  */ family(binom gauss) fv(var) link(sprobit ident) lv(var)   /*
  */ nip(10) adapt
```

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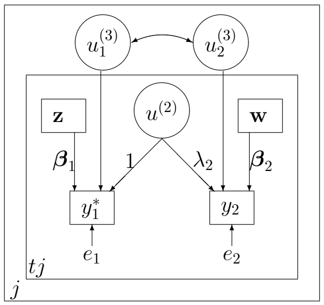
Multilevel Heckman selection model

- Example: longitudinal data with observations at times t on subjects j where data are missing intermittently
- Add correlated subject level random effects $u_{j1}^{(3)}$ for the selection model and $u_{j2}^{(3)}$ for the substantive model:

$$\eta_{itj} = \boldsymbol{\beta}_1{}'\mathbf{z}_{tj}\delta_{1i} + \boldsymbol{\beta}_2{}'\mathbf{w}_{tj}\delta_{2i} + u_{tj}^{(2)}(\delta_{1i} + \lambda\delta_{2i}) + u_{j1}^{(3)}\delta_{1i} + u_{j2}^{(3)}\delta_{2i}, \quad \text{var}(u_{tj}^{(2)}) = 1$$



- The variances of $u_1^{(3)}$ and $u_2^{(3)}$ are identified through the intraclass correlations in the selection and substantive models respectively.

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Multilevel Heckman selection model in `gllamm`

- Linear predictor

$$\eta_{itj} = \beta_1 z_{tj}\delta_{1i} + \beta_2 w_{tj}\delta_{1i} + \beta_3\delta_{1i} + \beta_4 w_{tj}\delta_{2i} + \beta_5\delta_{2i} + u_{tj}^{(2)}(\delta_{1i} + \lambda\delta_{2i}) + u_{j1}^{(3)}\delta_{1i} + u_{j2}^{(3)}\delta_{2i}$$

- `gllamm` command

`eq load: i1 i2` `/* for` $u_{tj}^{(2)}(\delta_{1i} + \lambda_2\delta_{1i})$ `*/`

`eq i1: i1` `/* for` $u_{j1}^{(3)}$ `*/`

`eq i2: i2` `/* for` $u_{j2}^{(3)}$ `*/`

`constraint define 1 [t1]i1 = 1` `/* sets` $\text{var}(u_{tj}^{(2)}) = 1$ `*/`

```
gllamm y z_i1 w_i1 i1 w_i2 i2, nocons i(t id) nrf(1 2) eq(load i1 i2) /*
  */ constr(1) family(binom gauss) fv(var) link(sprobit ident) lv(var) /*
  */ nip(19 15) ip(m) adapt
```

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Hausman-Wise-Diggle-Kenward dropout model

- Longitudinal data at times $t = 1, 2, 3$ for subjects j. Subjects drop out at some time $t > 1$ and never return
- Substantive model (without autocorrelated errors)

$$y_{tj} = \boldsymbol{\beta}'\mathbf{x}_{tj} + u_j + \epsilon_{tj}, \quad \epsilon_{tj} \sim N(0, \sigma^2), \quad u_j \sim N(0, \tau^2)$$

- Dropout model ($d_{tj} = 1$ if subject j drops out at time $t > 1$)

$$\text{logit}(\Pr(d_{tj} = 1)) = \alpha_0 + \alpha_1 y_{tj}^* + \alpha_2 y_{t-1,j}, \quad y_{tj}^* = \begin{cases} \text{observed } y_{tj} & \text{if } d_{tj} = 0 \\ \text{unobserved } y_{tj} & \text{if } d_{tj} = 1 \end{cases}$$

- Dropout is
  - Completely at random if $\alpha_1 = \alpha_2 = 0$
  - At random if $\alpha_1 = 0$
  - Informative if $\alpha_1 \neq 0$

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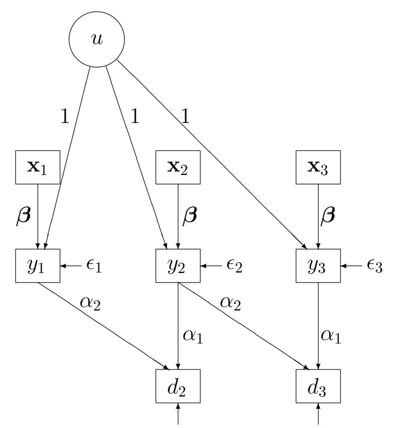
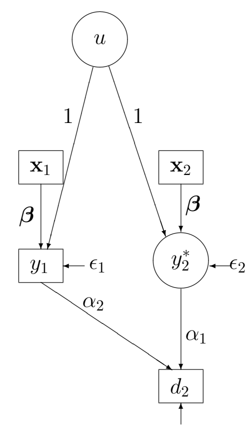
Diagram for dropout model

Complete data ($d_2 = d_3 = 0$)



Dropout at time 2 ($d_2 = 1$)



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Structural model in GLLAMM

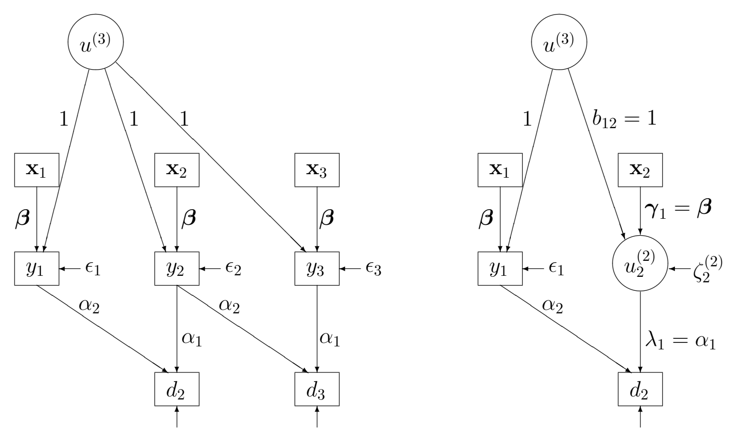
Regressions of latent variables on other latent and explanatory variables

$$\mathbf{u} = \mathbf{B}\mathbf{u} + \mathbf{\Gamma}\mathbf{w} + \boldsymbol{\zeta}$$

- $\mathbf{u} = (u_1^{(2)}, u_2^{(2)}, \cdots, u_{M_2}^{(2)}, \cdots, u_1^{(l)}, \cdots, u_{M_l}^{(l)}, \cdots, u_{M_L}^{(L)})'$ (M elements)
  - factors
  - random coefficients
- $\mathbf{B}$ is an upper diagonal $M \times M$ matrix of regression coefficients
- $\mathbf{\Gamma}$ is an $M \times p$ matrix of regression coefficients
- $\mathbf{w}$ is a p dimensional vector of explanatory variables
- $\boldsymbol{\zeta}$ is an M dimensional vector of errors/disturbances (same level as corresponding elements in $\mathbf{u}$).

Hausman-Wise-Diggle-Kenward dropout model in GLLAMM

Complete data ($d_2 = d_3 = 0$)   Dropout at time 2 ($d_2 = 1$)



Hausman-Wise-Diggle-Kenward dropout model in GLLAMM

- Linear predictor

$$\begin{aligned}\eta_{itj} &= \delta_{1i}\left(\boldsymbol{\beta}'\mathbf{x}_{tj} + u_j^{(3)}\right) + \delta_{2i}\left(\alpha_0 + \alpha_1 y_{tj}(1-d_{tj}) + \lambda_1 u_{tj}^{(2)} d_{tj} + \alpha_2 y_{i-1,j}\right)\\ &= \boldsymbol{\beta}'\mathbf{x}_{tj}\delta_{1i} + \alpha_0\delta_{2i} + \alpha_1 y_{tj}\delta_{2i}(1-d_{tj}) + \alpha_2 y_{i-1,j}\delta_{2i} + u_{tj}^{(2)}\lambda_1\delta_{2i}d_{tj} + u_j^{(3)}\delta_{1i}\end{aligned}$$

- Response process

$$y_{itj}|\eta_{itj} \sim \begin{cases} N(\eta_{itj}, \sigma^2) \text{ if } i = 1 & \text{(Gaussian with identity link)} \\ \text{Bernouilli}\left(\frac{\exp(\eta_{itj})}{1+\exp(\eta_{itj})}\right) \text{ if } i = 2 & \text{(Binomial with logit link)} \end{cases}$$

- Structural model

$$\begin{bmatrix} u_{tj}^{(2)} \\ u_j^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & b_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{tj}^{(2)} \\ u_j^{(3)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\gamma}_1{}' \\ \mathbf{0}' \end{bmatrix} \mathbf{x}_{tj} + \begin{bmatrix} \zeta_{tj}^{(2)} \\ \zeta_j^{(3)} \end{bmatrix}$$

- Constraints

$$\lambda_1 = \alpha_1, \quad \boldsymbol{\gamma}_1 = \boldsymbol{\beta}, \quad \text{var}(\zeta_{tj}) = \sigma^2$$

Options for the structural model

[··· `bmatrix`(*matname*) `geqs`(*eqnames*) `frload`(*numlist*) ···]

`bmatrix`(*matrix*) $M \times M$ matrix of 1s and 0s. Elements equal to 0 indicate that the corresponding element in $\mathbf{B}$ is 0; elements equal to 1 that the corresponding element in $\mathbf{B}$ should be estimated.

`geqs`(*eqnames*) equations for regressions of latent variables on explanatory variables. The second character of each equation name indicates which latent variable is regressed on the predictors.

`frload`(*numlist*) frees first factor loading for latent variables corresponding to *numlist*.

Estimating the Hausman-Wise-Diggle-Kenward model

- Data manipulation (data are in long form)

```
gen y0 = cond(y~=.,y,0)                       /* lag 0  y_tj  */
sort id t
qui by id: gen y1 = cond(_n>1,y[_n-1],0)  /* lag 1  y_t-1,j */
gen d = y == .                                /* d_tj */
sort id d t
qui by id d: drop if d==1&_n>1    /* drop records after first missing */
gen resp1 = y
gen resp2 = d
reshape long resp, i(id t) j(var) /* var = 1,2 if resp = y,d */
drop if var == 2 & t == 1         /* no dropout at time 1 */
tab var, gen(i)                   /* i1 = δ_1i, i2 = δ_1i */
gen x_i1 = x*i1
gen y0_i2d0 = y0*i2*(1-d)         /* y_tj δ_2i (1 - d_tj) */
gen y1_i2 = y1*i2
gen i2d1 = i2*d
```



```
. list id var t resp y0_i2d0 y1_i2 i2 in 1/12
```

	id	var	t	resp	y0_i2d0	y1_i2	i2d1
1.	1	1	1	2.657621	0	0	0
2.	1	2	2	1	0	2.657621	1
3.	2	1	1	1.423789	0	0	0
4.	2	2	2	1	0	1.423789	1
5.	3	1	1	-.7317839	0	0	0
6.	3	1	2	-.597519	0	0	0
7.	3	1	3	.8041697	0	0	0
8.	3	2	2	0	-.597519	-.7317839	0
9.	3	2	3	0	.8041697	-.597519	0
10.	4	1	1	.4663057	0	0	0
11.	4	1	2	1.797121	0	0	0
12.	4	2	2	0	1.797121	.4663057	0

- Linear predictor

$$\eta_{itj} = \boldsymbol{\beta}'\mathbf{x}_{tj}\delta_{1i} + \alpha_0\delta_{2i} + \alpha_1 y_{tj}\delta_{2i}(1-d_{tj}) + \alpha_2 y_{i-1,j}\delta_{2i} + u_{tj}^{(2)}\lambda_1\delta_{2i}d_{tj} + u_j^{(3)}\delta_{1i}$$

- Syntax for `gllamm`

```
eq u_2: i2d1            /* for eqs(): u_tj^(2) λ_1 δ_2i d_tj */
eq u_3: i1              /* for eqs(): u_j^(3) δ_1i */
matrix B=(0,1\0,0)      /* for bmatrix() */
gen one = 1
eq f1: x one             /* for geqs(): γ_2 x_tj  +  γ_1 */

constraint def 1 [b1_2]_cons = 1            /* set b_12 = 1 */
constraint def 2 [f1]one = [resp]i1         /* set γ_1 = β_1 */
constraint def 3 [f1]x = [resp]x            /* set γ_2 = β_2 */
constraint def 4 [t1]i2d1 = [s1]_cons       /* set var(ζ_jt^(2)) = var(ε_jt) */
constraint def 5 [t1l]i2d1 = [resp]y0_i2d0  /* set λ_1  =  α_1 */

gllamm resp x_i1 i1 y0_i2d0 y1_i2 i2, i(t id) eqs(u_2 u_3)        /*
  */ nocons family(gauss binom) fv(var) link(ident probit) lv(var) /*
  */ bmat(B) geqs(f1) frload(1) nats constr(1/5) nip(7) adapt
```


Extensions of the Hausman-Wise-Diggle-Kenward model

Application

- Cluster randomised study of sex education in Norway
- Schools were randomised to receive sex education or not
- Assessments pre randomisation, 6 months and 18 months post randomisation
- Three ordinal outcomes (5-point scale) measuring readiness to use contraception: "If my partner and I were about to have intercourse without either of us having mentioned contraception ...
  - I would have no problems saying that I have no contraception"
  - I would have no problems asking my partner whether he/she has contraception"
  - it would be easy for me to produce a condom (if I brought one)"
- 46 schools and 1183 pupils contributed to the analysis

Model

- Factor model with ordinal logit link for three outcomes i at time t for pupil j in school k

$$y^*_{itjk} = \beta_i + u^{(2)}_{tjk}\lambda_i + [u^{(3)}_{jk}0 + u^{(4)}_{k}0] + \epsilon_{itjk},\ \beta_1 = 0$$

$$y_{itjk} = s \text{ if } \kappa_{s-1} < y^*_{itjk} \le \kappa_s,\ s = 1, \cdots 5,\ \infty = \kappa_0 < \kappa_1 < \cdots < \kappa_5 = \infty$$

- Substantive model: structural model for latent outcome $u^{(2)}_{tjk}$

$$\begin{aligned} u^{(2)}_{tjk} &= \gamma_1 x_{Ttij} + \gamma_2 x_{Itij} + \gamma_3 x_{Ttij} x_{Itij} + u^{(3)}_{jk} + \zeta^{(2)}_{tjk} \\ u^{(3)}_{jk} &= u^{(4)}_{k} + \zeta^{(3)}_{jk} \end{aligned}$$

  where x_{Ttij} is time (0,1,3) and x_{Itij} is an indicator for the intervention group.

- Selection model

$$\text{logit}(\Pr(d_{tjk} = 1)) = \beta_6 + \alpha_0 u^{(2)}_{tjk} + \alpha_1 u^{(2)}_{t-1jk} + \alpha_2 u^{(2)}_{t-2jk}$$

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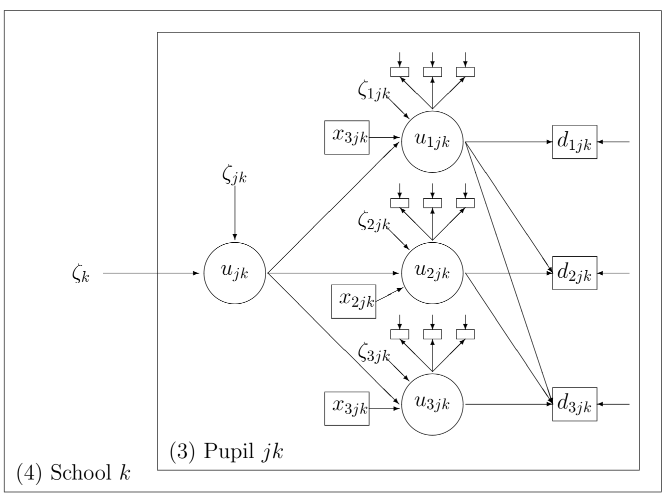

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	Model 1		Model 2		Model 3		Model 4	
	estimate	se	estimate	se	estimate	se	estimate	se
Selection model								
β_6	-1.99	0.26	-2.10	0.29	-0.85	0.04	-1.09	0.06
α_0	0.77	0.09	0.64	0.09	–	–	–	–
α_1	-0.10	0.04	–	–	–	–	-0.07	0.03
α_2	-0.20	0.05	–	–	–	–	-0.25	0.04
Substantive model								
γ_1 (time)	0.32	0.10	0.45	0.09	-0.06	0.09	-0.39	0.09
γ_2 (interv.)	-0.91	0.26	-0.25	0.23	-0.28	0.24	-1.46	0.21
γ_3 (time by interv.)	**0.48**	**0.11**	**0.34**	**0.10**	**0.20**	**0.11**	**0.56**	**0.11**
var($\zeta^{(1)}_{tjk}$)	6.74	0.60	7.29	0.68	4.57	0.41	5.04	0.45
var($\zeta^{(2)}_{jk}$)	5.30	0.57	4.29	0.98	3.72	0.43	3.51	0.39
Measurement model				Not shown				
log-likelihood	-8624.49		-8631.63		-8680.35		-8657.93	

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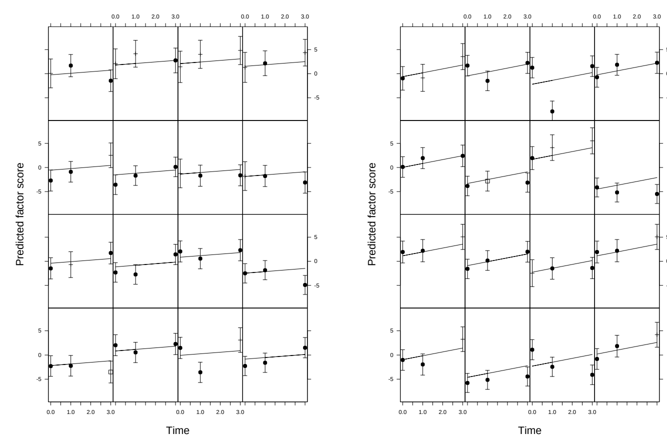
Empirical Bayes predictions for control (left) and intervention group (right) with standard errors



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