Assessing the reasonableness of an imputation model

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Outline

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- Missing Data
- Multiple Imputation
- Weighting
  - theory
  - weightmis
- Application

Missing data

- two problems:
  1. Loss of information
  2. bias

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  - compare results with alternative method: weighting

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3. Not Missing At Random (NMAR)
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- The correction is based on the between dataset variance of the point estimates.

Multiple Imputation in Stata

- Within Stata the distribution of plausible values can be estimated with `ice` and `hotdeck`.
- Within Stata the estimates from the ‘complete’ datasets can be combined with `mim`.

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Application

Missing values for one x.

$$f(y|x, R_x) = \frac{f(y, x, R_x)}{f(x, R_x)}$$

Missing values for one x.

Bayes' Rule

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$$f(y|x, R_x) = \frac{f(y, x, R_x)}{f(x, R_x)} = \frac{\Pr(R_x|y, x)f(y|x)f(x)}{\Pr(R_x|x)f(x)}$$

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f(A,B,C) &= f(C|A,B)f(A|B)f(B)
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4. generate the weight by:
```
gen w = PrRx/PrRxGy
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Missing values for two *x*s and *y*.

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$$f(y|x_1, x_2, R_{x_1}, R_{x_2}, R_y) = \frac{f(y, x_1, x_2, R_{x_1}, R_{x_2}, R_y)}{f(x_1, x_2, R_{x1}, R_{x2}, R_y)}$$

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Missing values for two xs and y.

Observed

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Missing values for two xs and y.

Not observed if x_1 is missing

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$$f(y|x_1, x_2) = \frac{\Pr(R_{x_1}|x_2,R_{x_2},R_y)\Pr(R_{x_2}|x_1,R_y)}{\Pr(R_{x_1}|y,x_2,R_{x_2},R_y)\Pr(R_{x_2}|y,x_1,R_y)} f(y|x_1, x_2, R_{x_1}, R_{x_2}, R_y)$$

Estimating the weight $\frac{\Pr(R_{x_1}|x_2, R_{x_2}, R_y)\Pr(R_{x_2}|x_1, R_y)}{\Pr(R_{x_1}|y, x_2, R_{x_2}, R_y)\Pr(R_{x_2}|y, x_1, R_y)}$

1. The weight can be split up into two parts:

$$\frac{\Pr(R_{x_1}|x_2, R_{x_2}, R_y)}{\Pr(R_{x_1}|y, x_2, R_{x_2}, R_y)} \times \frac{\Pr(R_{x_2}|x_1, R_y)}{\Pr(R_{x_2}|y, x_1, R_y)}$$

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2. For both the first and the second part only use cases which are observed on y.
3. The first part can be estimated like before with `logit` and `predict`.
4. The second part can be estimated with `logit` and `predict`, but now with weights to correct for missing data in x_1.

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- In principle this method could be expanded for any number of *x*s with missing data,
- but the number of calls to `logit` rises very quickly with the number of variables.

number of variables	1	2	3	4	5	6
number of calls to `logit`	2	8	22	52	114	240

Number of variables

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- Often the same variable enters a regression equation multiple time, e.g.:
  - interaction terms
  - dummy variables
  - polynomials
  - splines
- These variables count as one variable, thus diminishing the computational load.

weightmis syntax

`weightmis` *varlist* [*if*] [*in*] [*pw*], command(*string*) [missing(*varlist*) observed(*varlist*) double#(*varlist*) generate(*string*) *]

example 1

Say, y, x_1, and x_2 contain missing values, and you want to estimate the following regression equation:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

```
weightmis y x1 x2, command(regress) /*
*/ missing(x1 x2)
```

example 2

Say, y, x_1, and x_2 contain missing values, and you want to estimate the following regression equation:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \varepsilon$$

```
weightmis y x1 x2 x2sq, command(regress) /*
/* missing(x1 x2) double2(x2sq)
```

example 3

Say, y, x_1, and x_2 contain missing values, and you want to estimate the following regression equation:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon$$

```
weightmis y x1 x2 x1x2, command(regress) /*
*/ missing(x1 x2) double1(x1x2) double2(x1x2)
```

Outline

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- The aim is to look at the strength of association between family background and child's highest achieved level of education

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- International Stratification and Mobility File (ISMF) on the Netherlands.
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  - Year in which the child is 12 (*byr*), and is added as a spline with three knots to allow for non-linearity,
  - an interaction between *fisei* and the splines of *byr*,
  - and interactions of all variables with *female*.

Summary of missing values using misschk

```
   #  Variable         # Missing   % Missing
--------------------------------------------
   1  educyr              1125         1.2
   2  fisei              10082        10.4
   3  female                 0         0.0
   4  byr                    0         0.0

Missing for |
      which |
 variables? |      Freq.     Percent        Cum.
------------+-----------------------------------
       12__ |        330        0.34        0.34
       1___ |        795        0.82        1.16
       _2__ |      9,752       10.08       11.24
       ____ |     85,884       88.76      100.00
------------+-----------------------------------
      Total |     96,761      100.00
```

Imputation model

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Imputation model

- Regress *fisei* on *educyr*, *female*, *byr* (in dummies), dummies for survey, and all interactions.
- For each missing value of *fisei* draw a random value from a normal distribution whose mean is the predicted value of *fisei* and and whose standard deviation is the standard deviation of the errors.
- Predictions can be improved by adding other variables, like father's education (*feducyr*), mother's education(*meducyr*), child's occupational status (*isei*).

Imputation model

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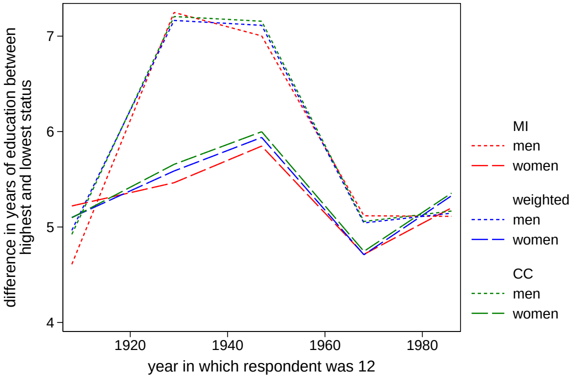
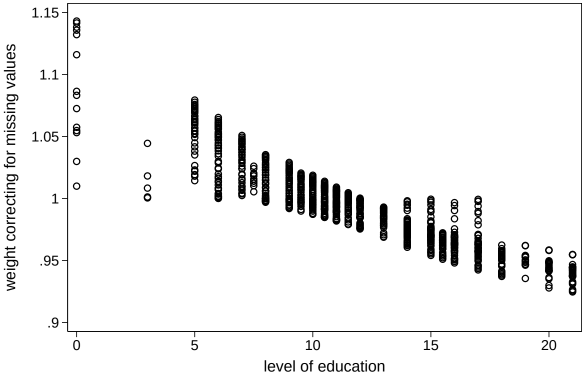
Imputation model

- In practice the interactions with survey number, *female*, and *byr* are modeled by estimating separate models for each combination of survey, gender, and three year birthcohort.
- *feducyr*, and *meducyr* are only used if they were asked in that survey.
- Imputations are only made if enough complete observations are available (number of variables + 2).
  - Of 10,082 missing cases for *fisei* 191 could not be imputed.
  - Of 1,145 missing cases for *educyr* 148 could not be imputed.

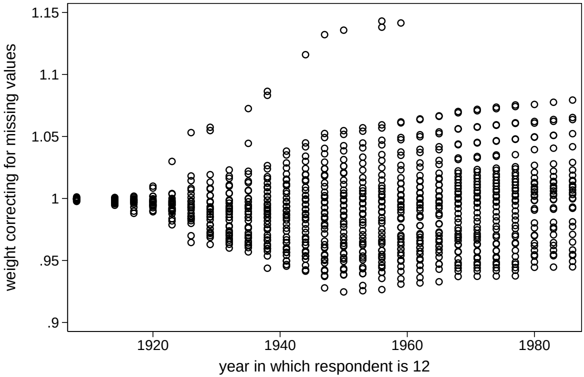
Trends in Inequality of educational opportunity



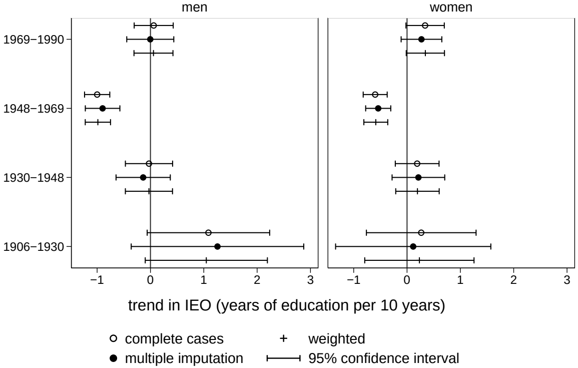
Weight versus level of education



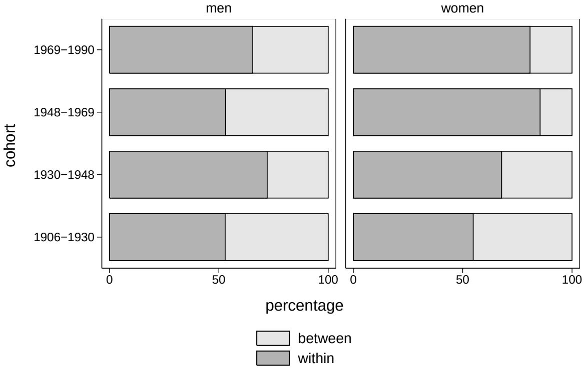
Weight versus cohort



Confidence intervals



Percentage of variance due to average variance across datasets and variance between datasets



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- The imputation model becomes part of the statistical model when using Multiple Imputation, and needs to be checked.
- One possible way of doing that is to compare the results with an alternative method that should also result in valid results.
- One such method is weighting, as (to be) implemented in `weightmis`

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