Structural Equation Modeling Using `gllamm`, `confa` and `gmm`

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Goals of the talk

1. Introduce structural equation models
2. Describe Stata packages to fit them:
   - `confa`: a 13mm hex wrench
   - `gllamm`: a Swiss-army tomahawk
   - `gmm`: do-it-yourself kit
   - `sem`: the promised land?
3. Example 1: daily functioning in NHANES
4. Example 2: experimental ecology data set

First, some theory

Structural equation modeling (SEM)

- Standard multivariate technique in social sciences
- Incorporates constructs that cannot be directly observed:
  - psychology: level of stress
  - sociology: quality of democratic institutions
  - biology: genotype and environment
  - health: difficulty in personal functioning
- Special cases:
  - linear regression
  - confirmatory factor analysis
  - simultaneous equations
  - errors-in-variables and instrumental variables regression

Origins of SEM

Path analysis of Sewall Wright (1918)

$\otimes$

Causal modeling of Hubert Blalock (1961)

$\otimes$

Factor analysis estimation of Karl Jöreskog (1969)

$\otimes$

Econometric simultaneous equations of Arthur Goldberger (1972)

Structural equations model

Latent variables:

$$\boldsymbol{\eta} = \boldsymbol{\alpha}_\eta + \boldsymbol{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta} \tag{1}$$

Measurement model for observed variables:

$$\boldsymbol{y} = \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y\boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{2}$$

$$\boldsymbol{x} = \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta} \tag{3}$$

$\boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{\varepsilon}, \boldsymbol{\delta}$ are uncorrelated with one another

Jöreskog (1973), Bollen (1989), Yuan & Bentler (2007)

Other re-expressions: Bentler & Weeks (1980), McArdle & McDonald (1984).

Implied moments

Denoting

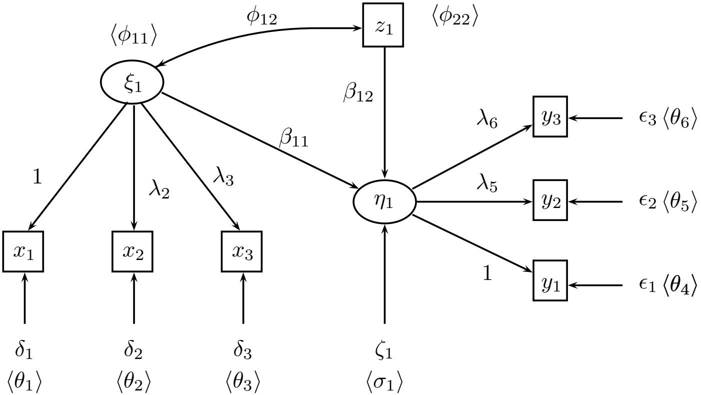
$$\mathbb{V}[\boldsymbol{\xi}] = \Phi, \quad \mathbb{V}[\boldsymbol{\zeta}] = \Psi, \quad \mathbb{V}[\boldsymbol{\varepsilon}] = \Theta_\varepsilon, \quad \mathbb{V}[\boldsymbol{\delta}] = \Theta_\delta,$$
$$R = \boldsymbol{\Lambda}_y (I - B)^{-1}, \quad z = \begin{pmatrix} x \\ y \end{pmatrix}$$

obtain

$$\mu(\boldsymbol{\theta}) \equiv \mathbb{E}\big[z\big] = \begin{pmatrix} \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y R \boldsymbol{\mu}_\xi \\ \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x \boldsymbol{\mu}_\xi \end{pmatrix} \tag{4}$$

$$\Sigma(\boldsymbol{\theta}) \equiv \mathbb{V}\big[z\big] = \begin{pmatrix} \boldsymbol{\Lambda}_x \Phi \boldsymbol{\Lambda}_x' + \Theta_\delta & \boldsymbol{\Lambda}_x \Phi \Gamma' R' \\ R \Gamma \Phi \boldsymbol{\Lambda}_x' & R(\Gamma \Phi \Gamma' + \Psi) R' + \Theta_\varepsilon \end{pmatrix} \tag{5}$$

Path diagrams



Identification

Before proceeding to estimation, the researcher needs to verify that the SEM is *identified*:

$$\mathbf{Pr}\{X : f(X,\theta) = f(X,\theta') \Rightarrow \theta = \theta'\} = 1$$

Different parameter values should give rise to different likelihoods/objective functions, either globally, or locally in a neighborhood of a point in a parameter space.

Likelihood

- Normal data $\Rightarrow$ likelihood is the function of sufficient statistic $(\bar{z}, S)$:

$$
\begin{aligned}
-2\log L(\theta, Y, X) \sim n \ln \det\bigl(\Sigma(\theta)\bigr) + n\,\mathrm{tr}[\Sigma^{-1}(\theta)S] \\
+n(\bar{z} - \mu(\theta))'\Sigma^{-1}(\theta)(\bar{z} - \mu(\theta)) \to \min_{\theta}
\end{aligned}
\tag{6}
$$

- Generalized latent variable approach for mixed response (normal, binomial, Poisson, ordinal, within the same model):

$$
-2\log L(\theta, Y, X) \sim \sum_{i=1}^{n} \ln \int f(y_i, x_i | \xi, \zeta; \theta) \mathrm{d}F(\xi, \zeta | \theta) \tag{7}
$$

Bartholomew & Knott (1999), Skrondal & Rabe-Hesketh (2004)

Estimation methods

- Normal theory MLE
- Weighted least squares:

$$s = \operatorname{vech} S, \quad \sigma(\theta) = \operatorname{vech} \Sigma(\theta)$$

$$F = (s - \sigma(\theta))' V_n (s - \sigma(\theta)) \to \min_\theta \tag{8}$$

  where V_n is weighting matrix:
  - Optimal $\hat{V}_n^{(1)} = \hat{\mathbb{V}}[s - \sigma(\theta)]$ (Browne 1984)
  - Simplistic: least squares $V_n^{(2)} = I$
  - Diagonally weighted least squares: $\hat{V}_n^{(3)} = \operatorname{diag} \hat{\mathbb{V}}[s - \sigma]$
- Model-implied instrumental variables limited information estimator (Bollen 1996)
- Bounded influence/outlier-robust methods (Yuan, Bentler & Chan 2004, Moustaki & Victoria-Feser 2006)
- Empirical likelihood

Goodness of fit

- The estimated model $\Sigma(\hat{\theta})$ is often related to the "saturated" model $\Sigma \equiv S$ and/or independence model $\Sigma_0 = \operatorname{diag} S$
- Likelihood formulation $\Rightarrow$ LRT test, asymptotically χ^2_k
- Non-normal data: LRT statistic $\sim \sum_j w_j \chi^2_1$, can be Satterthwaite-adjusted towards the mean and variance of the appropriate χ^2_k (Satorra & Bentler 1994, Yuan & Bentler 1997)
- Analogies with regression R^2 attempted, about three dozen fit indices available (Marsh, Balla & Hau 1996)
- Reliability of indicators: R^2 in regression of an indicator on its latent variable
- Signs and magnitudes of coefficient estimates

Now, some tools

sem?

As announced earlier this week, Stata 12 will be released on 25 July 2011 and will have a full-fledge `sem` estimation routine.

sem

Viewer - view sem2.smcl

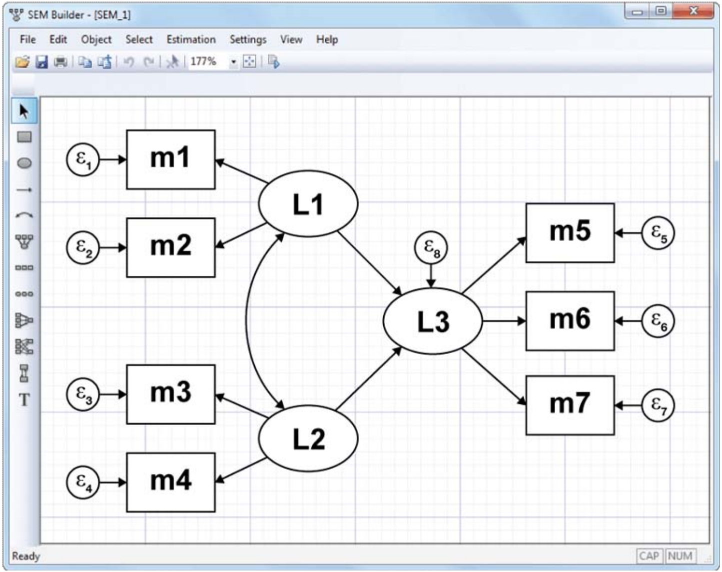
File Edit History Help

view sem2.smcl

Dialog | Also See | Jump To

```
Endogenous variables

Measurement:  anomia67 pwless67 anomia71 pwless71 educ66 occstat66
Latent:       Alien67 Alien71

Exogenous variables

Latent:       SES

Structural equation model                       Number of obs      =       932
Estimation method  = ml
Log likelihood     = -15246.469

 ( 1)  [anomia67]Alien67 = 1
 ( 2)  [anomia71]Alien71 = 1
 ( 3)  [educ66]SES = 1

------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  Alien71 <- |
     Alien67 |   .7046345   .0533512    13.21   0.000     .6000681    .8092008
         SES |  -.1744151   .0542489    -3.22   0.001     -.280741   -.0680891
  -----------+----------------------------------------------------------------
  Alien67 <- |
```

sem


SEM Builder - [SEM_1]
File
Edit
Object
Select
Estimation
Settings
View
Help
177%
m1
m2
m3
m4
m5
m6
m7
L1
L2
L3
ε1
ε2
ε3
ε4
ε5
ε6
ε7
ε8
Ready
CAP
NUM

`gllamm`

Generalized Linear Latent And Mixed Models (Skrondal & Rabe-Hesketh 2004, Rabe-Hesketh, Skrondal & Pickles 2005, Rabe-Hesketh & Skrondal 2008)

- Exploits commonalities between latent and mixed models
- Adds GLM-like links and family functions to them
- Allows heterogeneous response (different exponential family members)
- Allows multiple levels
- Maximum likelihood via numeric integration of random effects and latent variables (Gauss-Newton quadrature, adaptive quadrature); hence one of the most computationally demanding packages ever

`gllamm`

- One line of data per dependent variable $\times$ unit
- Requires `reshape long` transformation of indicators for latent variable models
- Measurement model: `eq()` option
- Structural model: `geq()` `bmatrix()` options
- Families and links: `family()` `fv()` `link()` `lv()`
- Tricks that Stas commonly uses:
  - make sure the model is correctly specified: `trace noest` options
  - good starting values speed up convergence: `from()` option
  - number of integration points gives tradeoff between speed and accuracy: `nip()` option
  - get an idea about the speed: `dot` option

`confa` package

- CONfirmatory Factor Analysis models, a specific class of SEM
- Maximum likelihood estimation
- Arbitrary # of factors and indicators; correlated measurement errors
- Variety of standard errors (OIM, sandwich, distributionally robust)
- Variety of fit tests (LRT, various scaled tests)
- Post-estimation:
  - fit indices;
  - factor scores (predictions)
  - Bollen & Stine (1992) bootstrap

gmm

Estimation command `gmm` introduced in Stata 11:

- Estimation by minimization of

$$g(X,\theta)' V_n \, g(X,\theta) \to \min_{\theta}$$

- Evaluator vs. "regression+instruments"
- Variety of weight matrices V_n
- Asy efficient estimator: $V_n = \widehat{\mathbb{V}} g(X, \hat\theta)$
- Homoskedastic/`unadjusted`, heteroskedastic/`robust`, `cluster`'ed and HAC-consistent standard errors
- Overidentification (goodness of fit) J-test via `estat overid`

One possible set up for SEM

1. Write a program to compute the implied moment matrix $\Sigma(\theta)$
2. Form observation-by-observation contributions to the moment conditions
$g(X,\theta) = \mathrm{vech}\big[(x_i-\bar{x})(x_i-\bar{x})' - \Sigma(\theta)\big]$
3. Feed into `gmm` using moment evaluator function

Some of these steps were simplified by the author's `sem4gmm` which will be obsolete in Stata 12.

Another possible set up for SEM

- Rather than relying on covariance representation of SEM, one can use regression representation instead
- Latent variables are measured with error $\Rightarrow$ need to use the techniques to account for that
- Observed indicators of latent variables are endogenous variables in the model

Account for endogeneity by instrumental variables

- Econometric technique of instrumental variables adapted to SEM by Bollen (1996)
- An instrumental variable:
  - correlated with regressors
  - not correlated with the error term
- Single equation: `ivregress`
- Simultaneous equations: all earlier determined variables can serve as instruments
- Full structural equation model: tracing rules Bollen & Bauer (2004)
- Can be implemented using the "interactive" version of `gmm`
- Tests of model specification: by equation and for the system as a whole

Comparison of functionality

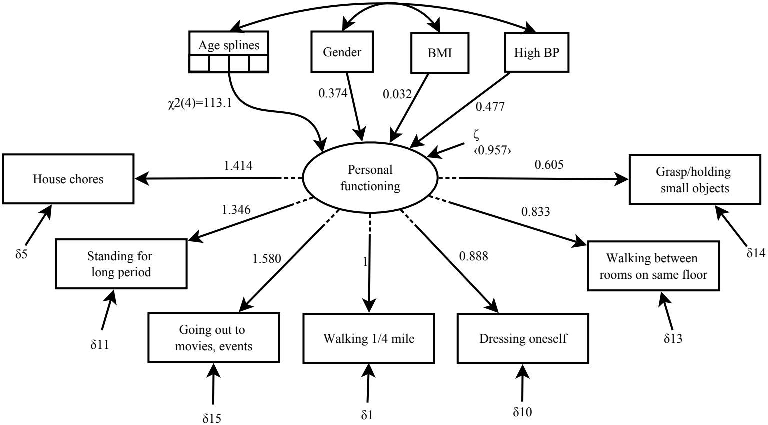
	gllamm	confa	gmm $+\Sigma(\theta)$	gmm + IV
General SEM	$\ldots$	–	$\surd$	$\ldots$
Estimation	$\surd$	$\surd$	$\surd$	$\ldots$
Overall test	–	$\surd$	$\surd$	$\surd$
Fit indices	–	$\ldots$	–	–
Prediction	$\surd$	$\ldots$	–	–
Ease of use	–	$\surd$	–	$\ldots$
Speed	–	$\ldots$	–	$\surd$

Finally, examples

NHANES data

- NHANES 2007–08 data
- Personal functioning section: *"difficulty you may have doing certain activities because of a health problem"*
- 17 questions: Walking for a quarter mile; Walking up ten steps; Stooping, crouching, kneeling; Lifting or carrying; House chore; Preparing meals; Walking between rooms on same floor; Standing up from armless chair; Getting in and out of bed; Dressing yourself; Standing for long periods; Sitting for long periods; Reaching up over head; Grasp/holding small objects; Going out to movies, events; Attending social event; Leisure activity at home
- Response categories: "No difficulty", "Some difficulty", "Much difficulty", "Unable to do"
- Research questions: How to summarize these items? What's the relation between individual demographics and health?

Path diagram



A multiple indicators and multiple causes (MIMIC) model

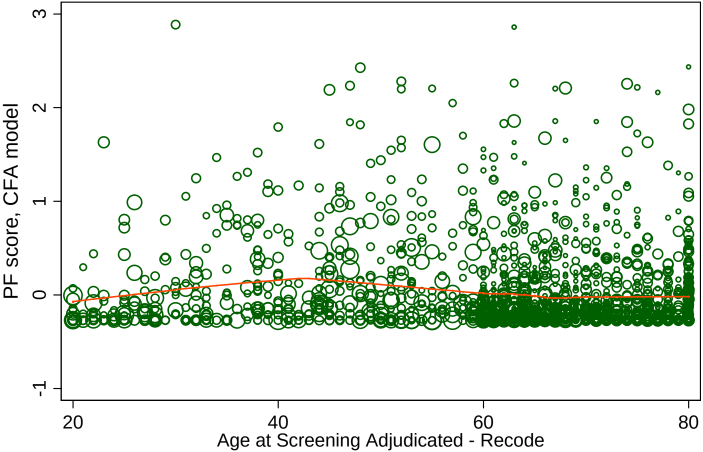
NHANES example using confa

Only the measurement model can be estimated with `confa`, as a preliminary step in gauging the performance of this part of the model.

```
. confa (difficulty: pfq*), from(iv)

. confa (difficulty: pfq*), from(iv)
> missing
```

Show results: `estimates use cfa; cfa_miss_fromcfa; cfa_miss_fromiv`

Factor scores



NHANES example via `gllamm`

Data management steps for `gllamm`:

1. Rename `pfq061b`↦`pfq1`, `pfq061c`↦`pfq2`, `...pfq061s`↦`pfq17`
2. `reshape long pfq, i(seqn) j(item)`
3. Generate binary indicators `q1-q17` of the items
4. Produce binary outcome measures: `bpfq`k'` = !(“No difficulty”) of `pfq`k'`

Model setup steps:

1. Define loading equations: `eq items: q1 q2 ...q17`
2. Come up with good starting values

NHANES example via `gllamm`

Syntax of `gllamm` command:

`gllamm ///`	
`bpfq ///`	single dependent variable
`q1 - q17, nocons ///`	item-specific intercepts
`i(seqn) ///`	"common factor"
`f(bin) l(probit) ///`	link and family
`eq(items) ///`	loadings equation
`from(...) copy`	starting values

The "common factor" is a latent variable that is constant across the `i()` panel, but can be modified with loadings

Show results in Stata: `est use cfa_via_gllamm; gllamm`

MIMIC model

Additional estimation steps:

1. Store the CFA results: `mat hs_cfa = e(b)`
2. Define the explanatory variables for functioning:
   `eq r1: female bmi highbp` *age splines*
3. Extend the earlier command:
   `gllamm ..., geq(r1) from( hs_cfa, skip )`

Parameter “complexity”:

1. fixed effects
2. loadings
3. latent regression slopes
4. latent (co)variances

Show results in Stata: `est use mimic_bmi; gllamm`;
show the diagram again.

NHANES example via gmm

Full model:

- 1 latent variable $\Rightarrow$ 1 variance
- 17 indicators $\Rightarrow$ 17 loadings, 17 variances
- 7 explanatory variables $\Rightarrow 7 \cdot 8/2$ covariances, 7 regression coefficients
- Total: 70 parameters, 300 moment conditions

Trimmed model:

- 1 latent variable $\Rightarrow$ 1 variance
- 5 indicators $\Rightarrow$ 5 loadings, 5 variances
- 4 explanatory variables $\Rightarrow 4 \cdot 5/2$ covariances, 4 regression coefficients
- Total: 25 parameters, 45 moment conditions

NHANES example: syntax and results

Show syntax: `nhanes-def-sem-reduced.do`, `nhanes-gmm-est-reduced.do`

Show results:

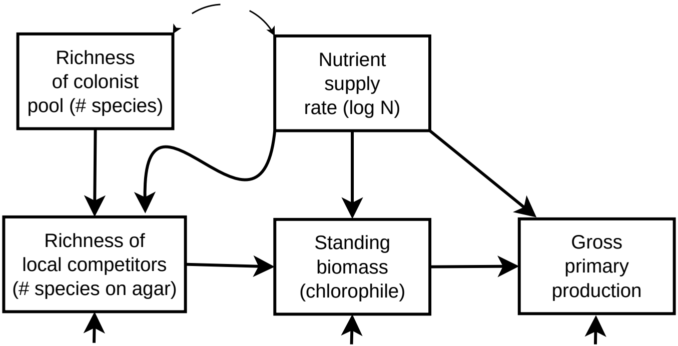
```
foreach eres in r_uls_homosked
r_uls_heterosked r_dwls_2step_heterosked
r_effls_2step_heterosked
r_effls_igmm_heterosked {
    est use `eres'
    gmm
    est store `eres'
}
estimates table, se stats(J)
```

Ecology example: observed variables

SEM in ecology

- Truly continuous variables, rather than Likert scales
- Observed and/or composite variables
- Small sample sizes (you're lucky if you have a few dozen)
- Methodology is at early stages of adoption
- Existing textbooks: Shipley (2000), Pugesek, Tomer & von Eye (2002)

Richness vs. productivity



Cardinale, Bennett, Nelson & Gross (2009)

First step: `regress`

```
regress ///
    dependent var ///
    its predictors from the path diagram
```

Account for endogeneity: `ivregress`

```
ivregress 2sls ///
   dependent var ///
   its exogenous predictors ///
      from the path diagram ///
   (its sl endogenous predictors = ///
      variables before them ///
      in the path model)
```

```
estat overid
```

Systemwide estimation: `reg3`

```
reg3 ///
    (depvar1 explvars1) ///
    (depvar2 explvars2) ///
```

Stata figures out the instrumental variables as all exogenous variables.
It will also implicitly correlate the errors to improve efficiency.

Systemwide estimation: gmm

gmm ///
    (explicit equation for first regression) ///
    (explicit equation for first regression) ///
    ... ///
    , winitial(id) wmatrix(robust) [igmm] ///
    instruments(1: instruments for first regression)
///
    ...

estat overid

Mediation, direct and indirect effects

- Is the effect of N on production mediated by biomass?
- Direct effect: regression coefficient
- Indirect effect: influence of N propagates through its effects on richness of local competition and biomass
- Algebraic expressions available, so this is the job for `nlcom`

What I covered was. . .

References I

Bartholomew, D. J. & Knott, M. (1999), *Latent Variable Models and Factor Analysis*, Vol. 7 of *Kendall's Library of Statistics*, 2nd edn, Arnold Publishers, London.

Bentler, P. M. & Weeks, D. G. (1980), 'Linear structural equations with latent variables', *Psychometrika* **45**, 289–308.

Blalock, H. M. (1961), 'Correlation and causality: The multivariate case', *Social Forces* **39**(3), 246–251.

Bollen, K. A. (1989), *Structural Equations with Latent Variables*, Wiley, New York.

Bollen, K. A. (1996), 'An alternative two stage least squares (2SLS) estimator for latent variable models', *Psychometrika* **61**(1), 109–121.

Bollen, K. A. & Bauer, D. J. (2004), 'Automating the selection of model-implied instrumental variables', *Sociological Methods Research* **32**(4), 425–452.

References II

Bollen, K. A. & Stine, R. (1992), ‘Bootstrapping goodness of fit measures in structural equation models’, *Sociological Methods and Research* **21**, 205–229.

Browne, M. W. (1984), ‘Asymptotically distribution-free methods for the analysis of the covariance structures’, *British Journal of Mathematical and Statistical Psychology* **37**, 62–83.

Cardinale, B. J., Bennett, D. M., Nelson, C. E. & Gross, K. (2009), ‘Does productivity drive diversity or vice versa? a test of the multivariate productivity-diversity hypothesis in streams’, *Ecology* **90**(5), 1227–1241.

Goldberger, A. S. (1972), ‘Structural equation methods in the social sciences’, *Econometrica* **40**(6), 979–1001.

Jöreskog, K. G. (1969), ‘A general approach to confirmatory maximum likelihood factor analysis’, *Psychometrika* **34**(2), 183–202.

References III

Jöreskog, K. G. (1973), A general method for estimating a linear structural equation system, *in* A. S. Goldberger & O. D. Duncan, eds, 'Structural Equation Models in the Social Sciences', Academic Press, New York, pp. 85–112.

Marsh, H. W., Balla, J. R. & Hau, K.-T. (1996), An evaluation of incremental fit indices: A clarification of mathematical and empirical properties, *in* G. Marcoulides & R. Schumaker, eds, 'Advanced Structural Equation Modeling Techniques', Erlbaum, Mahwah, NJ, pp. 315–353.

McArdle, J. J. & McDonald, R. P. (1984), 'Some algebraic properties of the reticular action model for moment structures.', *The British Journal of Mathematical and Statistical Psychology* **37**, 234–251.

Moustaki, I. & Victoria-Feser, M.-P. (2006), 'Bounded-influence robust estimation in generalized linear latent variable models', *Journal of the American Statistical Association* **101**(474), 644–653.

References IV

Pugesek, B. H., Tomer, A. & von Eye, A., eds (2002), *Structural Equation Modeling: Applications in Ecological and Evolutionary Biology*, Cambridge University Press.

Rabe-Hesketh, S. & Skrondal, A. (2008), 'Classical latent variable models for medical research', *Statistical Methods in Medical Research* **17**(1), 5–32.

Rabe-Hesketh, S., Skrondal, A. & Pickles, A. (2005), 'Maximum likelihood estimation of limited and discrete dependent variable models with nested random effects', *Journal of Econometrics* **128**(2), 301–323.

Satorra, A. & Bentler, P. M. (1994), Corrections to test statistics and standard errors in covariance structure analysis, *in* A. von Eye & C. C. Clogg, eds, 'Latent Variable Analysis', Sage, Thousands Oaks, CA, chapter 16, pp. 399–419.

Shipley, B. (2000), *Cause and correlation in Biology: A user's guide to path analysis, structural equations and causal inference*, Cambridge Unversity Press, Cambridge, UK.

References V

Skrondal, A. & Rabe-Hesketh, S. (2004), *Generalized Latent Variable Modeling*, Chapman and Hall/CRC, Boca Raton, Florida.

Wright, S. (1918), 'On the nature of size factors', *Genetics* **3**, 367–374.

Yuan, K.-H., Bentler, P. & Chan, W. (2004), 'Structural equation modeling with heavy tailed distributions', *Psychometrika* **69**(3), 421–436.

Yuan, K.-H. & Bentler, P. M. (1997), 'Mean and covariance structure analysis: Theoretical and practical improvements', *Journal of the American Statistical Association* **92**(438), 767–774.

Yuan, K.-H. & Bentler, P. M. (2007), Structural equation modeling, *in* C. Rao & S. Sinharay, eds, 'Handbook of Statistics: Psychometrics', Vol. 26 of *Handbook of Statistics*, Elsevier, chapter 10.