Using simulation to inspect the performance of a test

in particular tests of the parallel regressions assumption in ordered logit models

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Outline

Parallel lines assumption in Ordered logit

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- So we can look at the effect of a variable X on the comparison 1 versus 2 and 3 and the comparison 2 versus 3.
- An ordered logit results in one effect of X by assuming that these effects are the same
- A generalized version of this model allows some or all of these effects to be different. This model is implemented by Richard Williams in `gologit2`.

5 Tests of the parallel lines assumption after ordered logit

Tests of the parallel lines assumption compare the ordered logit model with a full generalized ordered logit model. There are 5 tests implemented in Stata (soon) in `oparallel`

- likelihood ratio test
- Wald test
- score test
- Wolfe-Gould test (approximate likelihood ratio test)
- Brant test (approximate Wald test)

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5. It is the probability of drawing a sample that is at least as ‘weird’ as the observed data if the null hypothesis is true

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  1. I am going to change my data such that the null hypothesis is true
  2. I am going to draw many samples from this 'population' and perform the test in each of these samples
  3. I am going to compare the p-value returned by that test with the proportion of samples that are more extreme than that sample.

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- If we find a p-value of α, than the probability of drawing a dataset with a p-value $\leq \alpha$ if the null hypothesis is true should itself be α, and this should be true for all possible values of α.
- So the sampling distribution of the p-values if the null hypothesis is true should be a standard uniform distribution.

Outline

The basic simulation (preparation)

```
clear all
use "http://www.indiana.edu/~jslsoc/stata/spex_data/ordwarm2.d
ologit warm white ed prst male yr89 age

predict double pr1 pr2 pr3 pr4, pr
forvalues i = 2/3 {
    local j = `i' - 1
    replace pr`i' = pr`i' + pr`j'
}
replace pr4 = 1
gen pr0 = 0
keep if e(sample)

gen ysim = .
gen u = .
```

The basic simulation (actual simulation)

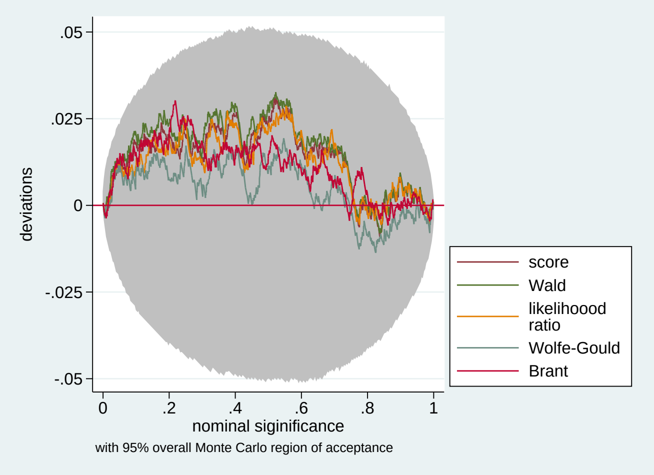
```
program define sim, rclass
    replace u = runiform()
    forvalues i = 1/4 {
        local j = `i' - 1
        replace ysim = `i' if u > pr`j' & u < pr`i'
    }
    ologit ysim white ed prst male yr89 age
    oparallel
return scalar s = r(p_s)
return scalar w = r(p_w)
return scalar lr = r(p_lr)
return scalar wg = r(p_wg)
return scalar b = r(p_b)
end

simulate s=r(s) w=r(w) lr=r(lr) wg=r(wg) b=r(b), reps(1000) :
```

The basic simulation (interpret the results)

```
simpplot s w lr wg b,             ///
 main1opt(ms(none) c(l) sort ) ///
 main2opt(ms(none) c(l) sort ) ///
 main3opt(ms(none) c(l) sort ) ///
 main4opt(ms(none) c(l) sort ) ///
 main5opt(ms(none) c(l) sort ) ///
 legend(order(2 "score"        ///
              3 "Wald"         ///
  4 "likelihoood"  ///
    "ratio"        ///
  5 "Wolfe-Gould"  ///
  6 "Brant" ))     ///
 overall reps(100000)          ///
 scheme(s2color)               ///
 ylab(-.05(.025).05,angle(horizontal))
```

The basic simulation (interpret the results)



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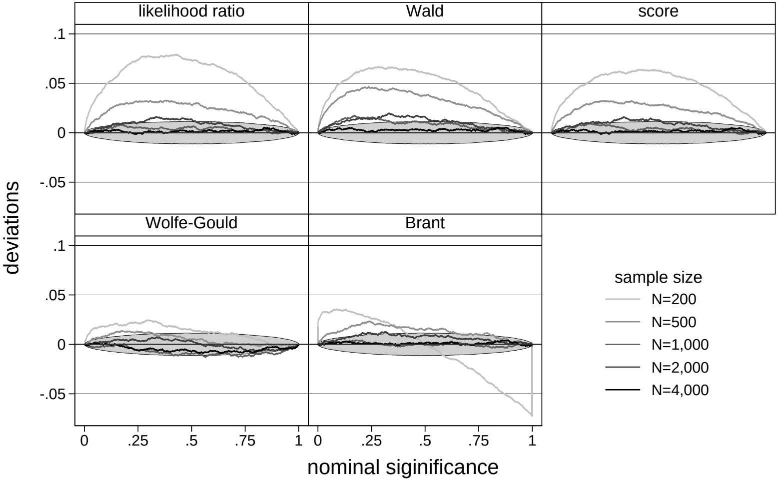
- What if I have a smaller dataset?
- Adapt the basic example by sampling say 200 observations, like so:

```
<prepare data>
save prepared_data
program define sim, rclass
    use prepared_data
    bsample 200
    ...
```

sample size



with 95% overall Monte Carlo region of acceptance

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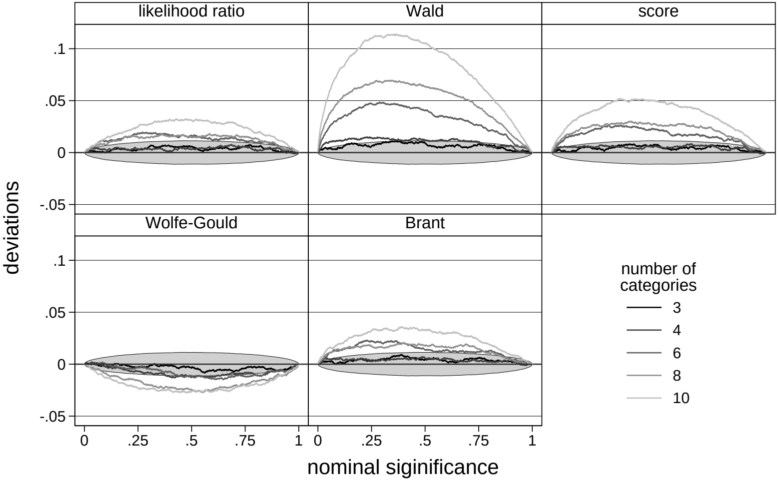
- We looked at 3, 4, 6, 8, and 10 categories, by changing the constants.
- These constants were chosen such that the proportion of observations in each of these categories are all the same

number of categories



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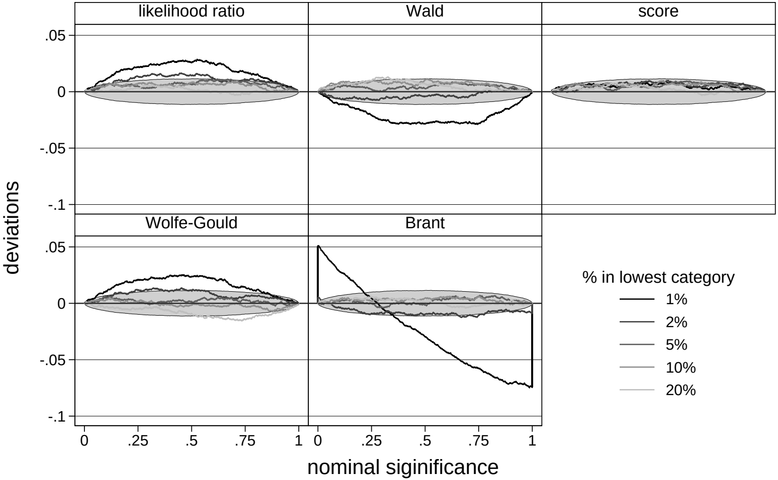
- Such sparse categories are also common in real data and often cause trouble.
- We fix the number of categories at 4 but change the first constant such that the proportion of observations in the first two categories change
- We do that in such a way that the first category contains 1%, 2%, 5%, 10%, or 20% of the observations

size of categories



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Introduction

simulations

Alternative

conclusion

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- This is a bootstrap test
- This is implemented in `oparallel` as the `asl` option

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- The probability of finding 1 or less samples that are more extreme than the observed data is $\frac{2}{B+1}$
- In general, the probability of finding k or less samples that are more extreme than the observed data is $\frac{k+1}{B+1}$

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- The mean of this distribution is $i/(B + 1) = (k + 1)/(B + 1)$.

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- The two are very similar

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- Surprisingly the Wolfe-Gould test seems to work best

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- The purpose is than to check whether the p-values follow a standard uniform distribution

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- That is the bootstrap test, and it is a general idea. It has been applied in: `asl_norm` and `propcnsreg`