How to assess the fit of multilevel logit models with Stata?

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"Models should not be true but it is important that they are applicable."

John W. Tukey

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1. What is the problem ?

Current situation in applied research:

- An increasing number of people uses multilevel logistic models for qualitative dependent variables with binary and ordinal outcome
- But users often complain that there are no fit measures for these models
- Neither Stata 14 / 15 nor SPSS 24 offer any fit measure for these models
- Let me demonstrate how to generalize the Pseudo R^2s for binary and ordinal logit model for the multilevel analysis

Which solutions does Stata provide?

- Indeed Stata estimates multilevel logit models for binary, ordinal and multinomial outcomes (melogit, meologit, gllamm) but it does not calculate any Pseudo R^2. It provides only the information criteria AIC and BIC (estat ic)
- Stata provides a Wald-test for the fixed-effects and a Likelihood-Ratio-χ^2 test for the random effects of the exogenous variables
- Even special purpose programs like HLM, MlwiN, MPLUS or SuperMix do not calculate any Pseudo R^2

What can we learn from multilevel literature?

- Raudenbush & Bryk (2002), Heck & Thomas (2009) and Rabe-Hesketh & Skrondal (2013) do not mention Peudo R^2s
- Snijder & Bosker(2012) propose a variation of McKelvey & Zavoina Pseudo R^2 for random-intercept- and intercept-as-outcome logit models. It is not implemented in any program
- Hox (2010) discusses the McFadden, Cox & Snell, Nagelkerke and McKelvey & Zavoina Pseudo R^2. He recommends the last one to assess the model fit

2. Summary of the econometric Monte-Carlo studies for testing Pseudo R^2s

- Econometricians made a lot of Monte-Carlo studies in the early 90s:
  - Hagle & Mitchell 1992
  - Veall & Zimmermann 1992, 1993, 1994
  - Windmeijer 1995
  - DeMaris 2002
- They tested systematically the most common Pseudo-R^2s for binary and ordinal probit / logit models

Which Pseudo R^2s were tested in these studies?

- Likelihood-based measures:
  - Maddala / Cox & Snell Pseudo R^2 (1983 / 1989)
  - Cragg & Uhler / Nagelkerke Pseudo R^2 (1970 / 1992)
- Log-Likelihood-based measures:
  - McFadden Pseudo-R^2 (1974)
  - Aldrich & Nelson Pseudo R^2 (1984)
  - Aldrich & Nelson Pseudo R^2 with the Veall & Zimmermann correction (1992)
- Basing on the estimated probabilities:
  - Efron / Lave Pseudo R^2 (1970 / 1978)
- Basing on the variance decomposition of the estimated Probits / Logits:
  - McKelvey & Zavoina Pseudo R^2 (1975)

Results of the Monte-Carlo-Studies for binary and ordinal logits or probits

- The McKelvey & Zavoina Pseudo R^2 is the best estimator for the "true R^2" of the OLS regression
- The Aldrich & Nelson Pseudo R^2 with the Veall & Zimmermann correction is the best approximation of the McKelvey & Zavoina Pseudo R^2
- Lave / Efron, Aldrich & Nelson, McFadden and Cragg & Uhler Pseudo R^2 severely underestimate the "true R^2" of the OLS regression
- My personal advice:
  - Use the McKelvey & Zavoina Pseudo R^2 to assess the fit of binary and ordinal logit models

3. The generalization of the McKelvey & Zavoina Pseudo R^2 for the binary and ordinal multilevel logit model

- The multilevel logit model is a systematic extension of the classical binary and ordinal logit model for clustered subsamples (contextual units j)

  - The variance of the estimated logits is decomposed into ▸ Fixed effects, ▸ Random effects and ▸ Level-1 Error variance $\sigma^2(r_{ij})$
  - Because of its own heteroscedasticity the variance of level 1 residua $\sigma^2(r_{ij})$ can not be estimated. It is replaced by the variance of the logistic density function $(\pi^2 / 3)$

Let’s have a short look at the lucky winner

- McKelvey & Zavoina Pseudo R^2 (M & Z Pseudo R^2)

$$M\ \&\ Z\ Pseudo\ R^2 = \frac{Var\left(\hat{y}^*\right)}{Var\left(\hat{y}^*\right) + Var\left(\varepsilon\right)} = \frac{\dfrac{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2}{n}}{\dfrac{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2}{n} + \dfrac{\pi^2}{3}}$$

Range: $0 \leq$ M & Z-Pseudo $R^2 \leq 1$

Legend:

$Var\left(\hat{y}^*\right)$: Variance of the estimated logits (latent variable Y^*)

$\hat{y}_i^*$: Estimated logit of case i

$\overline{\hat{y}^*}$: Expected value of the estimated logits

$\frac{\pi^2}{3}$: Variance of the logistic density function

Generalization to the 2-level logit model 2

- Prediction of the latent variable Y* (estimated binary or cumulative logit) in two ways
  - Population-Average Prediction with the fixed effects of the exogenous variables (all random effects hold at zero)
    - Stata-command: predict newvar1 if e(sample), xb
  - Unit-Specific Prediction of the fixed and random effects of the exogenous variable
    - Stata-command: predict newvar2 if e(sample), eta
- Therefore, the variance of the estimated logits (Y*) can be calculated in two different ways
  - Only for the fixedeffects of the exogenous variables
  - For the fixed and random effects of the exogenous variables

Generalization to the 2-level logit model 3

- Therefore we get two different McKelvey & Zavoina Pseudo R^2s
  - "Population-Average" M & Z Pseudo R^2 (fixed effects)
  - "Unit-Specific" M & Z PseudoR^2 (fixed- & random effects)

- For the "Unit-Specific" M & Z Pseudo R^2 uses estimated fixed and random effects for prediction, it assesses the fit more realistically as its "Population-Average" counterpart

Let's have a short look at the lucky loser

- McFadden-Pseudo R^2 (1974)

$$McFadden\ Pseudo\ R^2\left(\rho^2\right) = 1 - \left[\frac{\log L_A}{\log L_0}\right]$$

Range: $0 \leq$ McFadden Pseudo $R^2 < 1$

but ρ^2 does not reach the maximum of 1.0

Rule of thumbl: $0.20 \leq$ McFadden Pseudo $R^2 \leq 0.40$ marks an excellent fit (McFadden 1979: 307)

Legend: $\log L_A$: Log-Likelihood of the actual model
$\log L_0$: Log-Likelihood of the zero model

Generalization to the 2-level logit model 4

- Conditions of application
  - Maximum-Likelihood estimation of the fixed and random effects of the exogenous variables
  - Actual and zero model has to use the same sample
  - Choice of the "appropriate zero model" (M_0) depends on our knowlege to which context the respondent belongs
    - Membership known: Random-Intercept-Only Logit model estimates the proportion of Y* which can be maximally explained by the context (= ANOVA model)
    - Membership unkown: Fixed-Intercept-Only Logit model estimates only the marginal distributeion of Y* (= true zero model)

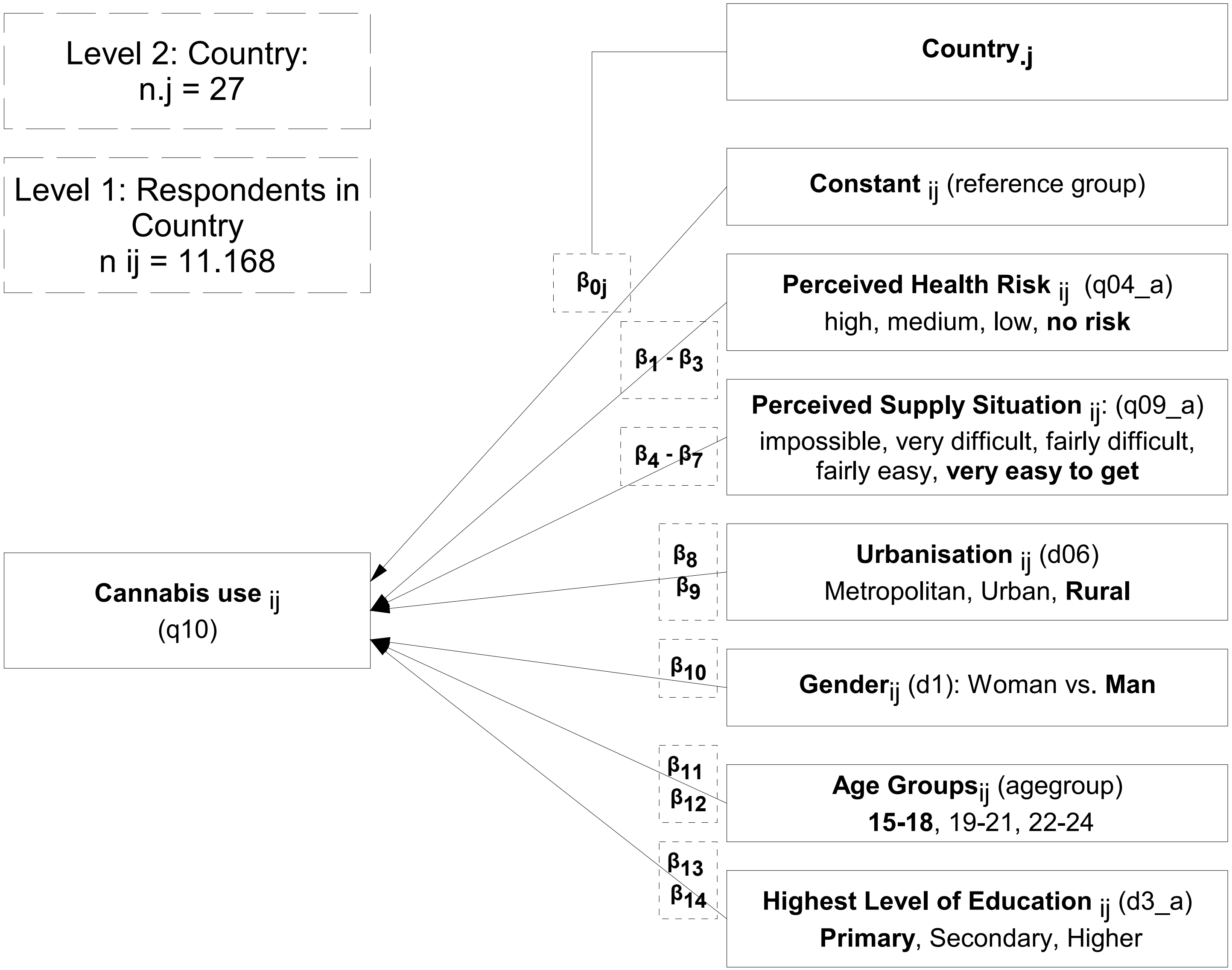
Generalization to the 2-level logit model 5

- Calculation of McFadden Pseudo R^2 is possible in two different ways using the following as a zero model
  - Random-Intercept-Only Logit-Model
    - It measures the proportional reduction of the log likelihood of the actual model caused by the fixed effects of the exogen-ous variables in comparison to the RIOM
    - Its Likelihood-Ratio χ^2 test refers to all fixed effects of the exogenous level 1 and level 2 variables
  - Fixed-Intercept-Only Logit-Model
    - It measures the proportional reduction of the log likelihood of the actual model caused by fixed and random effects of all exogenous variables in comparison to the FIOM
    - Its Likelihood-Ratio χ^2 test refers to all fixed and random effects of the exogenous level 1 and level 2 variables

4. Example of application

- Flash Eurobarometer No 330 about youth attitudes on drugs (2011)
  - WebCATI-Survey of n_{ij} = 12.313 respondents (aged 15 - 24) in $n_{.j}$ = 27 EU member states (contextual units j)
  - My focus:
    - prevalence of cannabis use by juveniles and young adults (q10): Have you used cannabis by yourself?
      - **1) never**
      - **2) more than 12 months ago**
      - **3) less than 12 months ago**
      - **4) in the last 30 days**
  - Let us have a look at the exogenous variables in the following diagram

Theoretical 2-level-model: RIM



Stata-Output Version14

- Fixed effects
- Thresholds
- Random effect

```
Mixed-effects ologit regression                 Number of obs     =     11,168
Group variable:         country                 Number of groups  =         27

                                                Obs per group:
                                                              min =        211
                                                              avg =      413.6
                                                              max =        490

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(14)     =    2142.78
Log likelihood = -7410.7117                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------------------
                    q10ord |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------------+--------------------------------------------------------------
                     q4_a |
                high risk |  -2.670499   .1092326   -24.45   0.000    -2.884591   -2.456407
              medium risk |  -1.696693   .0730464   -23.23   0.000    -1.839861   -1.553525
                 low risk |  -.7425748   .0611709   -12.14   0.000    -.8624676    -.622682
                          |
                     q9_a |
               impossible |  -3.006983   .1899514   -15.83   0.000    -3.379281   -2.634685
           very difficult |  -2.191986   .1207629   -18.15   0.000    -2.428677   -1.955295
         fairly difficult |  -1.555672   .0870857   -17.86   0.000    -1.726357   -1.384987
              fairly easy |  -.6291072   .0553719   -11.36   0.000    -.7376341   -.5205803
                          |
                       d6 |
        metropolitan zone |   .3536598   .0713306     4.96   0.000     .2138545    .4934652
 other town/urban centre  |    .196061   .0606935     3.23   0.001     .0771039     .315018
                          |
                       d1 |
                   female |  -.4654088   .0504709    -9.22   0.000    -.5643298   -.3664877
                          |
                 agegroup |
                  19 - 21 |   .4924681    .073827     6.67   0.000     .3477699    .6371663
                  22 - 24 |   .6847313   .0797637     8.58   0.000     .5283974    .8410652
                          |
                     d3_a |
      secondary education |  -.0345302   .0753855    -0.46   0.647    -.1822832    .1132227
         higher education |  -.0415283    .099673    -0.42   0.677    -.2368837    .1538271
---------------------------+--------------------------------------------------------------
                    /cut1 |  -.4269461   .1329725    -3.21   0.001    -.6875674   -.1663248
                    /cut2 |   .6715688    .133064     5.05   0.000     .4107681    .9323695
                    /cut3 |   1.857033   .1357061    13.68   0.000     1.591053    2.123012
---------------------------+--------------------------------------------------------------
country                   |
                var(_cons)|   .2623196   .0849424                      .1390597     .494835
------------------------------------------------------------------------------------------
LR test vs. ologit model: chibar2(01) = 222.09        Prob >= chibar2 = 0.0000
```

What does Stata offer to assess the fit?

- Akaike (AIC) und Schwarz Bayesian Information Criterion (BIC)
  - Decision rule: Choose the model with the lowest AIC or BIC

```
. estimates stats fiom riom rim

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
        fiom |  11,168          .   -9326.802      3      18659.6    18681.57
        riom |  11,168          .   -9033.234      4     18074.47    18103.75
         rim |  11,168          .   -7410.712     18     14857.42     14989.2
-----------------------------------------------------------------------------
               Note: N=Obs used in calculating BIC; see [R] BIC note.
```

  - Looking at AIC and BIC, the rim fits best of all bad models
  - But we do not know how well the rim fits

Output of my fit_melogit_2lev.ado 1

- Assessing the fit by the McKelvey & Zavoina-Pseudo R^2s and the Intra-Class-Correlation

```
. fit_meologit_2lev
Fit-measures for the MELOGIT/MEOLOGIT-model:

McKelvey&Zavoina-Pseudo-R2 (fixed&random effects)=   0.5137

McKelvey&Zavoina-Pseudo-R2 (fixed effects only)=   0.4774


Just estimating the Random-/Fixed Intercept Only Logit-Model

Intra-Class-Correlation (Level 2) =    0.1507
```

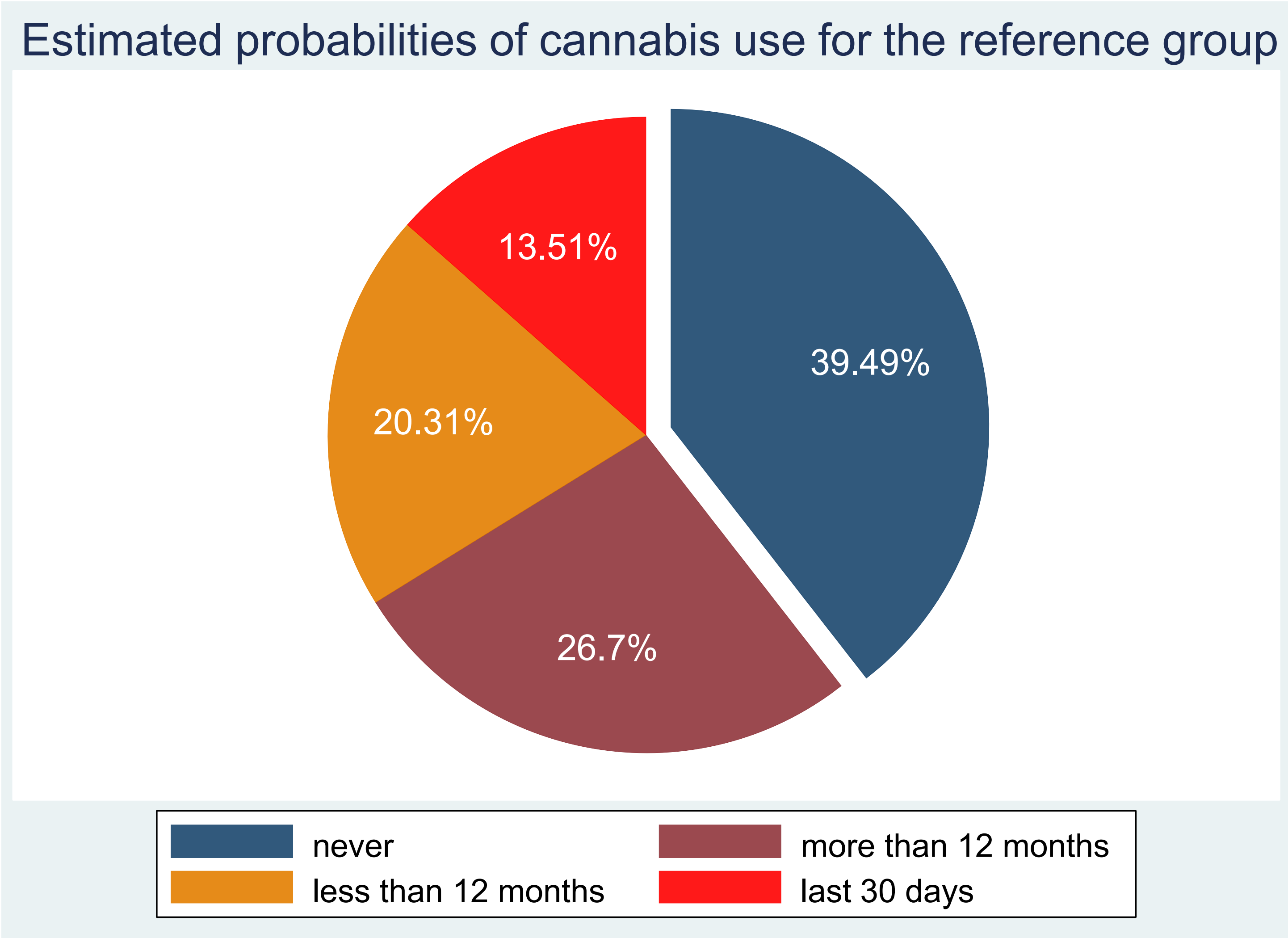
Output of my fit_melogit_2lev.ado 2

- McFadden Pseudo R^2s and corresponding Likelihood-Ratio-χ^2 tests

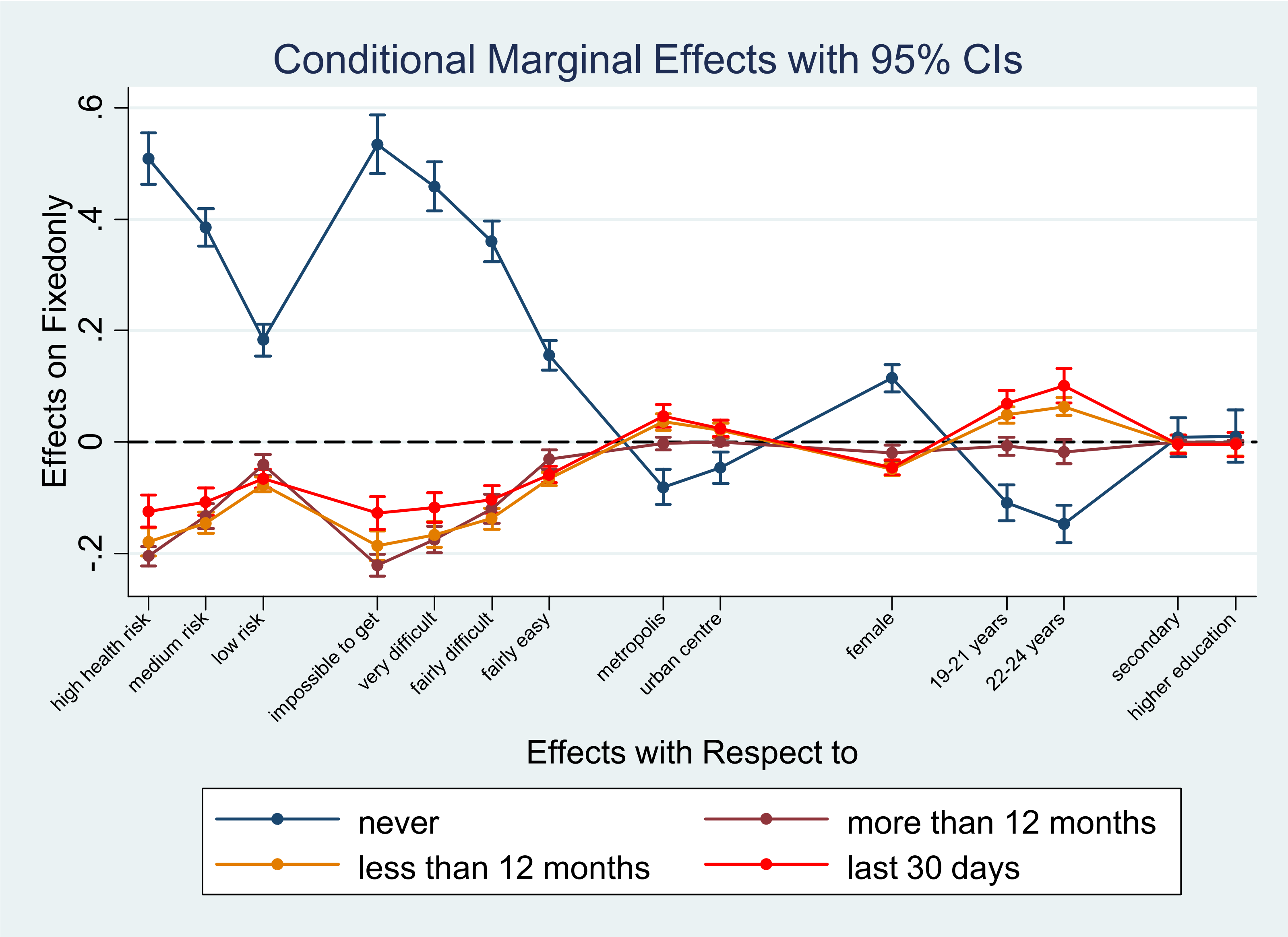
```
McFadden Pseudo-R2 (M_A vs. Random-Intercept-Only-Logit Model) = 0.1796

McFadden Pseudo-R2 (M_A vs. Fixed-Intercept-Only-Logit Model) = 0.2054


Likelihood-Ratio-chi2-Test (H0: All fixed effects = 0)

Likelihood-ratio test                                 LR chi2(14) =   3245.04
(Assumption: riom nested in ma)                       Prob > chi2 =    0.0000

Likelihood-Ratio-chi2-Test (H0: All fixed & random effects = 0)

Likelihood-ratio test                                 LR chi2(15) =   3832.18
(Assumption: fiom nested in ma)                       Prob > chi2 =    0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the
      parameter space.  If this is not true, then the reported test is conservative.
```

How does the effects look like?

- The baseline



The joint marginsplot for the 4 categories



5. Conclusions 1

- Known
  - The Monte-Carlo-simulation studies show that the McKelvey & Zavoina Pseudo R^2 is the best fit measure for binary and ordinal logit models
- New
  - Generalization of the M & Z-Pseudo R^2 to binary and ordinal multilevel logit models. The prediction of estimated logits bases upon the fixed effects only or upon fixed and random effects of exogenous variables
  - The McFadden-Pseudo R^2 bases upon the fixed effects only or upon fixed and random-effects of the exogenous variables using a context-independent zero model

5. Conclusions 2

- New
  - Simultaneous Likelihood-Ratio-χ^2 test for the estimated fixed effects using the random-intercept-only (RIOM) as the zero model
  - Simultaneous Likelihood-Ratio-χ^2 test for the estimated fixed and random effects using the fixed-intercept-only (FIOM) as the zero model
- That's why
  - I suggest to use my fit_meologit_2lev.ado and fit_meologit_3lev.ado to assess the fit of 2- and 3-level logit models with binary and ordinal outcome

Closing words

- Thank you for your attention
- Do you have some questions?

Contact

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Stata code for fit_meologit_2lev.ado 1

```
program fit_meologit_2lev, rclass
version 14

tempvar plgt1

quietly estimates store ma

quietly predict `plgt1' if e(sample), eta

quietly sum `plgt1'

display as text "Fit-measures for the MELOGIT/MEOLOGIT-model:"
display as text "                                        "
display as text "McKelvey&Zavoina-Pseudo-R2 (fixed&random effects)=   "  as result %6.4f  ///
 abs(r(Var)*r(N)-1) / ((r(N)*(_pi^2 / 3)+ (r(Var)*r(N)-1)))
display as text "                                        "
drop `plgt1'

tempvar plgt2
quietly predict `plgt2' if e(sample), xb

quietly sum `plgt2'

display as text "McKelvey&Zavoina-Pseudo-R2 (fixed effects only)=   "  as result %6.4f  ///
 abs(r(Var)*r(N)-1) / ((r(N)*(_pi^2 / 3)+ (r(Var)*r(N)-1)))

drop `plgt2'

dis "                                        "

capture drop llma

tempvar llma

gen llma=`e(ll)'

dis as text "                                        "
dis as text "Just estimating the Random-/Fixed Intercept Only Logit-Model"
dis as text "                                        "
```

Stata code for fit_meologit_2lev.ado 2

```
* Schaetzung des RIOM
quietly: `e(cmd2)' `e(depvar)' if e(sample), || `e(ivars)':

quietly: estimates store riom

* Berechnung der Intra-Class-Correlation (ICC)
display as text "Intra-Class-Correlation (Level 2) =    " as result %6.4f  ///
 (_b[var(_cons[`e(ivars)']):_cons]) / (_b[var(_cons[`e(ivars)']):_cons] + (_pi^2 / 3))

dis as text "                                        "
dis as text "McFadden Pseudo-R2 (M_A vs. Random-Intercept-Only-Logit Model) = " ///
    as result %6.4f abs(1- (llma /`e(ll)'))
dis as text "                                        "

* Schätzung des FIOM
quietly: `e(cmd2)' `e(depvar)' if e(sample)

quietly: estimates store fiom

dis as text "McFadden Pseudo-R2 (M_A vs. Fixed-Intercept-Only-Logit Model) = " ///
    as result %6.4f abs(1- (llma /`e(ll)'))
dis as text "                                        "

drop llma

dis as text "                                        "
dis as text "Likelihood-Ratio-chi2-Test (H0: All fixed effects = 0)  "
lrtest riom ma

dis as text "                                        "
dis as text "Likelihood-Ratio-chi2-Test (H0: All fixed & random effects = 0)  "
lrtest fiom ma

exit
```

Appendix

Multilevel ordered logit model 1

- Equations of the 2-level-ordered logit model

Level 2: Between-Context Regression

2a) Logistic Intercept-as-Outcome-Model:

$$\beta_{0j} = 0 + \gamma_{01} \times Z_{.j} + u_{0j}$$

2b) Logistic Slope-as-Outcome-Model:

$$\beta_{1j} = \gamma_{10} + \gamma_{11} \times Z_{.j} + u_{1j}$$

Level 1: Within-Context Regression

1) $$\ln\left[\frac{\mathrm{P}(\mathrm{Y} > k)}{\mathrm{P}(\mathrm{Y} \leq k)}\right] = \beta_{0j} + \beta_{1j} \times X_{ij} - \sum_{K=1}^{k-1} \delta_k \{+r_{ij}\}$$

Single equation notation: 2a) and 2b) in 1)

$$\ln\left[\frac{\mathrm{P}(\mathrm{Y} > k)}{\mathrm{P}(\mathrm{Y} \leq k)}\right] = \left(0 + \gamma_{01} \times Z_{.j} + u_{0j}\right) + \left(\gamma_{10} \times X_{ij} + \gamma_{11} \times X_{ij} \times Z_{.j} + u_{1j} \times X_{ij}\right) - \sum_{k=1}^{K-1} \delta_k \{+r_{ij}\}$$

Notation of Raudenbush&Bryk (2002):

γ: fixed-effect estimator
Z: exogenous level 2 variable
β: random-effect estimator
X: exogenous level 1 variable
u_{0j}: residuum random-intercept
u_{1j}: residuum random-slope
r_{ij}: residuum of within-context-logistic regression
δ_k: threshold for kategory k of Y

Multilevel ordered logit model 2

- Interpretation of the residua of the Between- Context-Regression

$$3a)\;\; u_{0j} = \beta_{0j} - \left[\gamma_{00} + \gamma_{01} \times Z_{.j}\right] = \beta_{0j} - \widehat{\beta_{0j}}$$

$$3b)\;\; u_{1j} = \beta_{1j} - \left[\gamma_{10} + \gamma_{11} \times Z_{.j}\right] = \beta_{1j} - \widehat{\beta_{1j}}$$

- Assumptions for the residua of the logistic 2-level logit model

Level 1:

1.1) r_{ij} is binomial distributed with an expected value of zero

and a variance $\sigma^2_{r_{ij}} = \widehat{P_{ij}}\left(Y=1\right) \times \left(1 - \widehat{P_{ij}}\left(Y=1\right)\right)$

1.2) Heteroscedasticity of r_{ij} in all contextual units j

Multilevel ordered logit model 3

- Implication for the level 1 residuum r_{ij}
  - Because of its own heteroscedasticity the variance $\sigma^2(r_{ij})$ can not be estimated. It is replaced by the variance of the logistic density function ($\pi^2 / 3$)

- Residua of level 2

2.1) u_{kj} is normal distributed with an expected value of zero and a covariance matrix T of the residua

$$E\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{bmatrix} \qquad \sigma^2_{u_{0j}} = \tau_{00} \qquad \sigma^2_{u_{1j}} = \tau_{11}$$

$$\sigma_{u_{0j},u_{1j}} = \tau_{10} = \tau_{01}$$

2.2) The residua of level 1 and level 2 are not correlated:

$$\sigma_{u_{0j},r_{ij}} = \sigma_{u_{1j},r_{ij}} = 0$$

Alternative in Stata: Information criteria

- Calculation of Akaike- (AIC) and Schwarz Bayesian-Information-Criteria (BIC)

$$AIC = -2 \times \log L_{M_A} + 2 \times k$$

$$BIC = \underbrace{-2 \times \log L_{M_A}}_{\text{deviance}} + \underbrace{\log N \times k}_{\text{complexity of the model}}$$

$$\text{Range:} \qquad 0 < AIC \leq +\infty$$

$$0 < BIC \leq +\infty$$

$Legend:$

$\log$: *Logarithmus naturalis*

k: *Number of estimated parameters*

N: *Sample size*

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