xtoaxaca - Extending the Kitagawa-Oaxaca-Blinder Decomposition Approach to longitudinal data analyses

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Who would benefit from using xtoaxaca

- You have at least two time points
- You want a flexible way to decompose the level over time
- You want a (counterfactual) decomposition of change over time

Prior approaches

- Based on different research questions very many different decompositions can be chosen
- At the moment we cannot make a systematic comparison
- We present our preferred solution

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- $R = E + C + I$
- $R = [E(X_B) - E(X_A)]\beta_A + E(X_A)(\beta_B - \beta_A) + [E(X_B) - E(X_A)](\beta_B - \beta_A)$

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Research questions for the parts of the decomposition

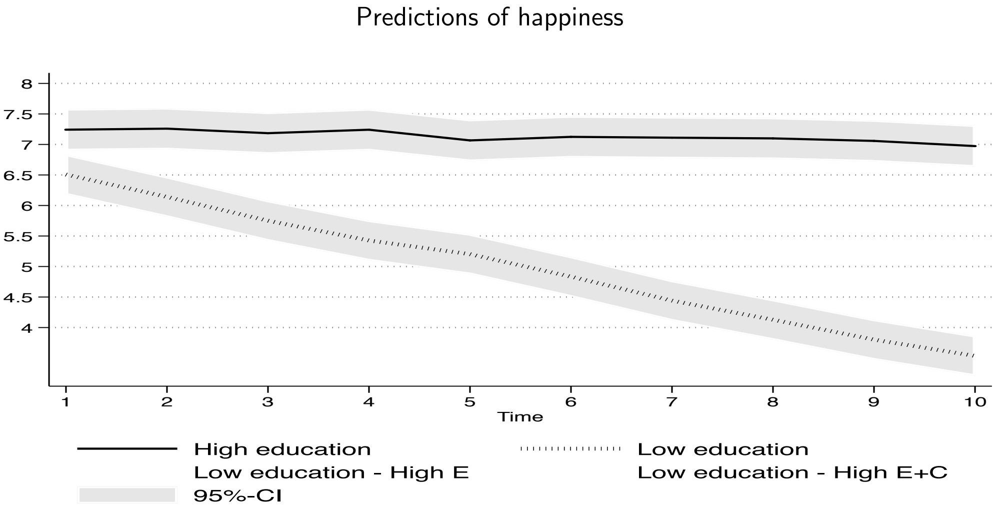
- **E**: How much smaller/bigger would the gap be, if the endowments of group A were the same as for group B?

Research questions for the parts of the decomposition

- **E**: How much smaller/bigger would the gap be, if the endowments of group A were the same as for group B?
- **C**: How much smaller/bigger would the gap be, if the effect of the explanatory variables of group A were the same as the effects for group B?

GC prediction approach

- $R(t) = E(Y_B(t) - Y_A(t))$
- Latent growth curve model for change in happiness
- parametric, semi-parametric, non-parametric


Predictions of happiness
8
7.5
7
6.5
6
5.5
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4.5
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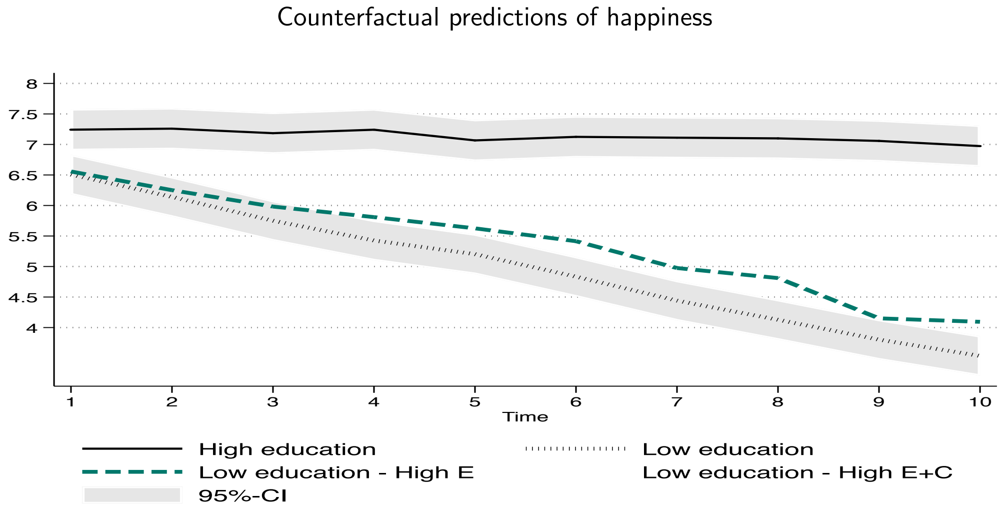
10
Time
High education
Low education
Low education - High E
Low education - High E+C
95%-CI


Counterfactual predictions of happiness
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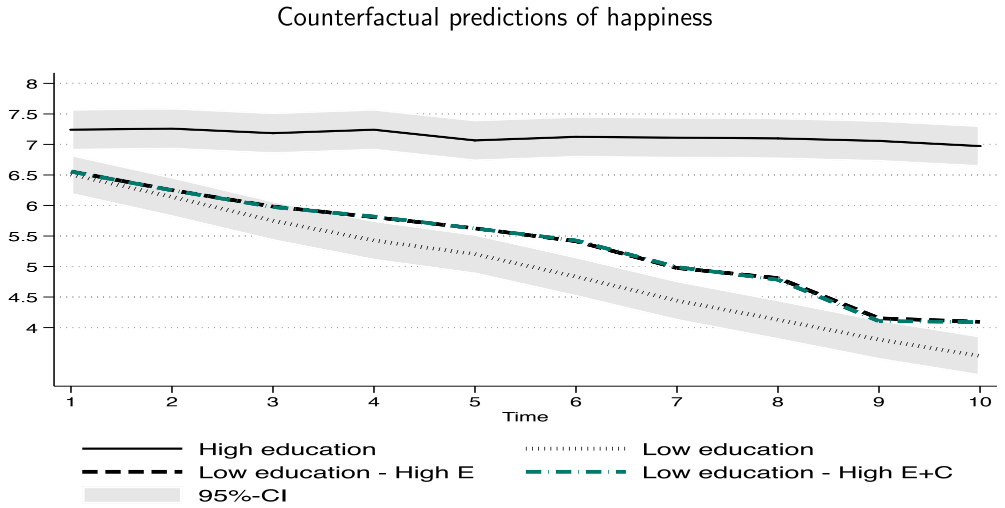

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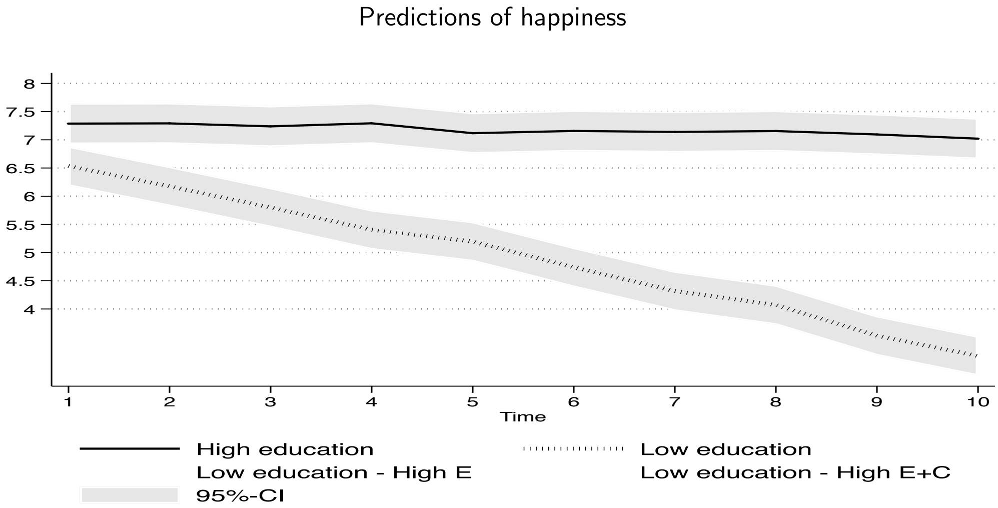
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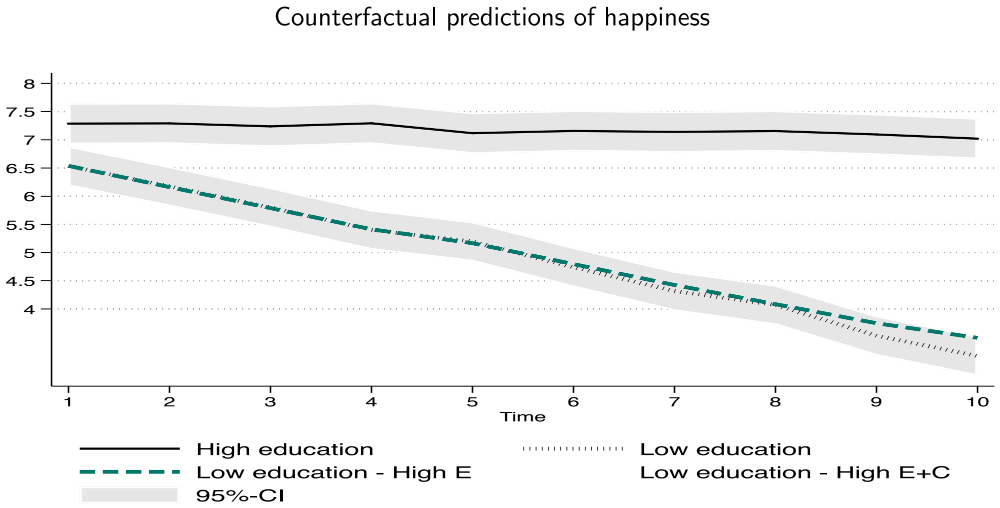

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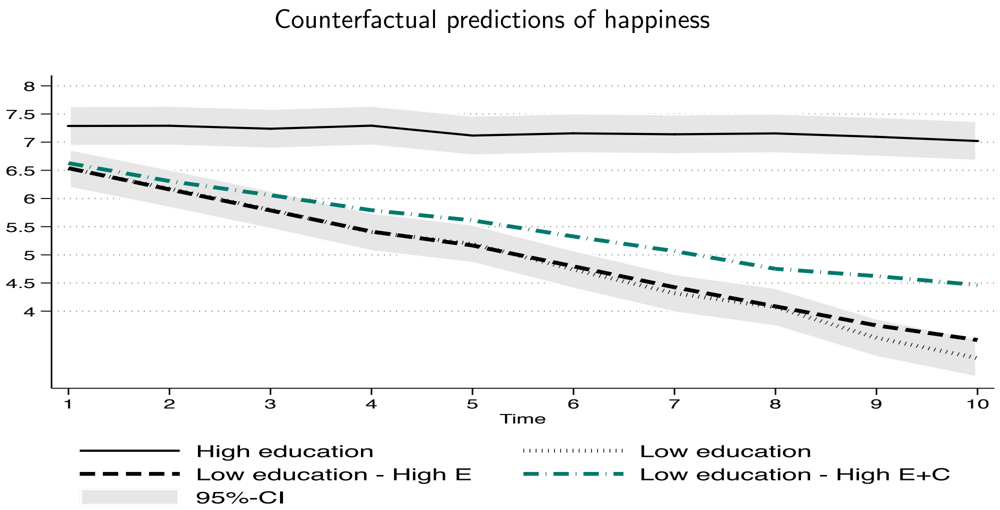

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Decomposition of change

- Decomposing the change in happiness
- it always needs two time points to compare

Decomposition of change

$$\begin{aligned}\Delta Y^I &= Y_t^I - Y_s^I \\ &= \bar{X}_t^I \beta_t^I - \bar{X}_s^I \beta_s^I\end{aligned}$$

Decomposition of change

$$\Delta Y^l = Y^l_t - Y^l_s$$
$$= \bar{X}^l_t \beta^l_t - \bar{X}^l_s \beta^l_s$$

and the change of the group difference over time then is

$$\Delta Y = \Delta Y^A - \Delta Y^B$$
$$= (\bar{X}^A_t \beta^A_t - \bar{X}^A_s \beta^A_s) - (\bar{X}^B_t \beta^B_t - \bar{X}^B_s \beta^B_s)$$
$$= \bar{X}^A_t \beta^A_t - \bar{X}^A_s \beta^A_s - \bar{X}^B_t \beta^B_t + \bar{X}^B_s \beta^B_s$$

Change due to endowment

$$\Delta Y_E = (\bar{X}_t^A - \bar{X}_s^A)\beta_s^A - (\bar{X}_t^B - \bar{X}_s^B)\beta_s^B$$

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Change due to coefficients

$$\Delta Y_C = (\beta_t^A - \beta_s^A)\bar{X}_s^A - (\beta_t^B - \beta_s^B)\bar{X}_s^B$$

Change due to coefficients

$$\Delta Y_C = (\beta_t^A - \beta_s^A)\bar{X}_{\color{red}{s}}^A - (\beta_t^B - \beta_s^B)\bar{X}_{\color{red}{s}}^B$$

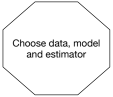
Research questions for the parts of the decomposition

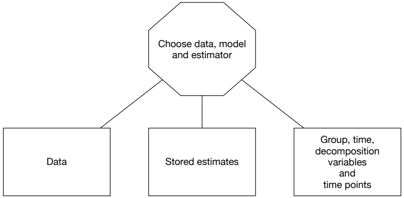
- **dE**: How much smaller/bigger would the **change** in the gap be, if the endowments of group A had **changed** in the same way as for group B (and the difference in coefficients had stayed the same)?

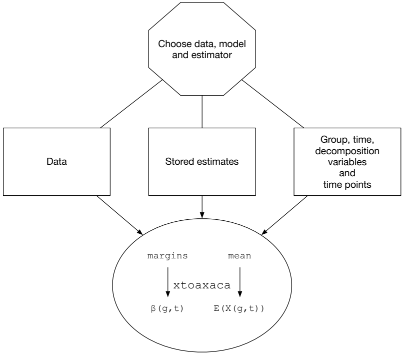
Research questions for the parts of the decomposition

- **dE**: How much smaller/bigger would the **change** in the gap be, if the endowments of group A had **changed** in the same way as for group B (and the difference in coefficients had stayed the same)?

- **dC**: How much smaller/bigger would the **change** in the gap be, if the coefficients of group A had **changed** in the same way as for group B (and the difference in endowments had stayed the same)?


Choose data, model and estimator


Choose data, model and estimator
Data
Stored estimates
Group, time, decomposition variables and time points



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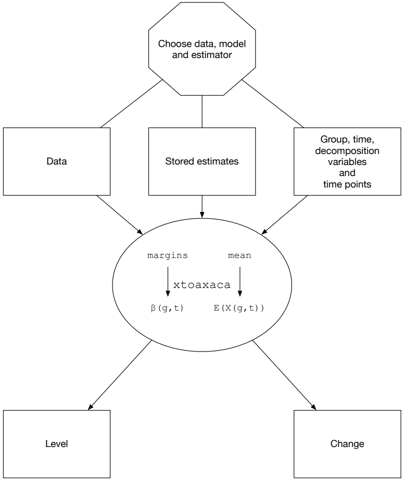
margins
mean
xtoaxaca
β(g,t)
E(X(g,t))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margins
mean
xtoaxaca
β(g,t)
E(X(g,t))
Level
Change

Quick note on the use of `margins`

- all decompositions are done at means or other specified values
- calculation of average over the population not necessary
- margins conducted only on one observation
- speed of the calculation of `margins` (not `mean`) independent of sample size

xtoaxaca in action

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xtoaxaca exp, groupvar(group2) groupcat(1 2) timevar(time)
times(1 3 5 ) /// model1(base) model2(control) ///
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Stata example
```

General problems with decomposition

- Reference group
- Reference group of decomposition variables

Extensions?

- Different forms of change decomposition (e.g. Kim, Makepiece)
- Different counterfactual scenarios
- Reduce to one model estimation
- Extension for SEM


DANKE!